Doppler effect and the thermal Hall conductivity of quasiparticles in d-wave superconductors

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At temperatures very close to absolute zero, all the electrons in a superconductor are paired. The collection of Cooper pairs constitutes the condensate or superfluid. At finite temperatures T, however, a few of the Cooper pairs break up to form a gas of 'singles' called Bogolyubov quasiparticles (the entropy of the quasiparticle gas lowers the sample's free energy). With increasing T, the quasiparticle population $n_{qp}(T)$ increases rapidly until all the pairs become singles at the critical transition temperature T_c.

The thermal conductivity κ has proved to be a fruitful way to study the transport properties of quasiparticles. In a temperature gradient $-\nabla T$, the flow of quasiparticles towards the cooler end generates a thermal current $\mathcal{J}_e$ (as the condensate itself has zero entropy, it does not contribute to the thermal conductivity). However, the gradient also produces a parallel phonon current $\mathcal{J}_{ph}$ which greatly complicates the interpretation of measurements of κ.

First, let us consider the normal state of a conventional metal such as Pb. κ is the sum of the electronic term κ_e and the phonon term κ_{ph}, viz. $\kappa = \kappa_e + \kappa_{ph}$ (κ_e is about 100 times larger than κ_{ph} in Pb, but κ_e is 5-8 times smaller than κ_{ph} in the cuprates). As T falls below T_c in Pb, the rapid (exponential) decrease of n_{qp} leads to a sharp decrease in κ_e. However, this is partially compensated by an increase in κ_{ph} because the decreasing n_{qp} results in a marked decrease in the scattering of phonons. Hence the total κ in a typical s-wave superconductor decreases rather gradually below T_c. If the sample is of exceptional purity, this gradual decrease is interrupted by a resurgent $\kappa \sim \kappa_{ph}$ which rises to a prominent peak below $\sim \frac{1}{2}T_c$. The phonons, now largely free of any scattering by quasiparticles, develop exceedingly long mean-free-paths (mfp) limited only by the size of the crystal. Eventually, at very low T, κ_{ph} vanishes as T^3, reflecting the specific heat of the phonon gas.

A major puzzle in the high-T_c cuprates became apparent shortly after the 1987 discovery of the '90-K' superconductor $YBa_2Cu_3O_7$. Instead of decreasing below T_c, κ is observed to rise sharply, reaching a peak value 2-3 times larger than the value just above $T_c = 93$ K (inset of Fig. 1). The origin of the giant anomaly in κ has been a source of debate. Recalling κ in Pb, many investigators initially identified the giant anomaly with a strong enhancement of the mfp of phonons. However, this identification was unconvincing because the quasiparticle population in the cuprates falls quite slowly below T_c (in contrast with Pb). How can we distinguish the phonon and quasiparticle currents in the cuprates?

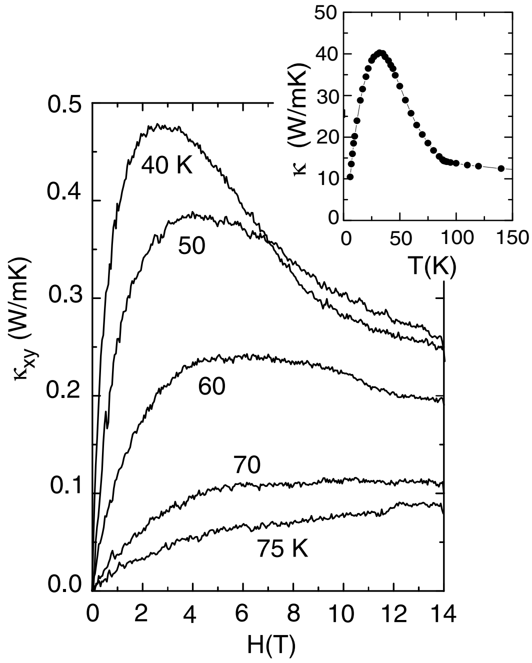

FIG. 1: (Main Panel) Curves of κ_{xy} versus the applied field H at selected temperatures in high-purity $YBa_2Cu_3O_7$. At high T, κ_{xy} is linear in H, but as T decreases, its field profile displays a prominent peak. The initial slope $\lim_{H\to 0} \kappa_{xy}/H$ increases by a factor of 1000 between 85 and 40 K. The inset shows the zero-field thermal conductivity κ vs. T. A prominent peak appears below $T_c = 93$ K [adapted from Zhang *et al.* [2]].

Let us consider applying a magnetic field **B** normal to the plane of the crystal. The field pierces the condensate as an array of vortices. Quasiparticles do not 'see' the applied magnetic field, but they scatter from the vortices (which act like impurities in the otherwise uniform condensate). A quasiparticle incident on a vortex line strongly scatters from the steep decrease in the pair potential at the core. More germane to our discussion, it interacts with the intense azimuthal supercurrent surrounding the core. Generally, the energy of a quasiparticle is shifted in the presence of a superflow. If the quasiparticle moves in a direction parallel to that of the local superfluid velocity (co-moving), its velocity relative to the lattice is increased (this so-called Doppler effect is analogous to viewing a pedestrian walking on a moving ramp). Conversely, a quasiparticle moving against the superfluid has its velocity lowered. The difference in kinetic energies implies that quasiparticles prefer to go around a vortex in the direction *against* the azimuthal superfluid velocity. If the superflow is clockwise (viewed from above), the incident quasiparticle is preferentially scattered to the left, whereas it is scattered to the right when the flow is counterclockwise. The resulting 'skew' scattering produces a net 'Hall entropy current' (if $-\nabla T$ is applied along $\hat{\mathbf{x}}$ with $\mathbf{H} \parallel \hat{\mathbf{z}}$, the Hall current is along $\hat{\mathbf{y}}$ and given by $\mathcal{J}_y = \kappa_{yx}(-\nabla T)$, where κ_{xy} is called the thermal Hall conductivity). Phonons, which are charge-neutral, do not experience this asymmetric scattering. Hence the thermal Hall effect acts very much like a selective filter that ignores the phonon current.

The thermal Hall effect was detected in high-purity crystals of $YBa_2Cu_3O_7$ and investigated in detail from $\sim$12 K to temperatures slightly above T_c [1, 2]. In weak fields, κ_{xy} increases linearly with H (main panel of Fig. 1). In high-purity crystals, this linear field dependence evolves into a profile with a prominent peak. The profile is consistent with a very long quasiparticle mean-free-path ℓ when the field is absent. If we plot the initial slope κ_{xy}/H ($H \rightarrow 0$) against T, we find that it displays a remarkable thousand-fold increase [2] between T_c and 30 K that dwarfs the corresponding increase in κ. This is consistent with an increase in ℓ of over 100 within this temperature interval. The steep increase is sufficient to produce a giant peak in κ_e that matches the observed giant anomaly. The thermal Hall effect presents the strongest evidence to date that the giant anomaly derives entirely from the quasiparticles.

[1] K. Krishana, J. M. Harris, and N. P. Ong, Phys. Rev. Lett. **75**, 3529 (1995).

[2] Y. Zhang *et al.*, Phys. Rev. Lett., **86**, 890 (2001).