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Workshop on
EMERGENT MATERIALS AND HIGHLY CORRELATED ELECTRONS

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GAUGE FIELD THEORY AND METAL-INSULATOR CROSSOVER IN CUPRATE SUPERCONDUCTORS

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Gauge Field Theory and Metal Insulator Crossover in Cuprate Superconductors

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Outline

I. Motivation

II. Summary of basic ideas

III. Sketch of derivation

IV. M-I crossover in the absence of magnetic field

V. M-I crossover in magnetic field

VI. Calculations of other physical quantities

VII. Concluding remarks

Why Gauge Field Theory ?

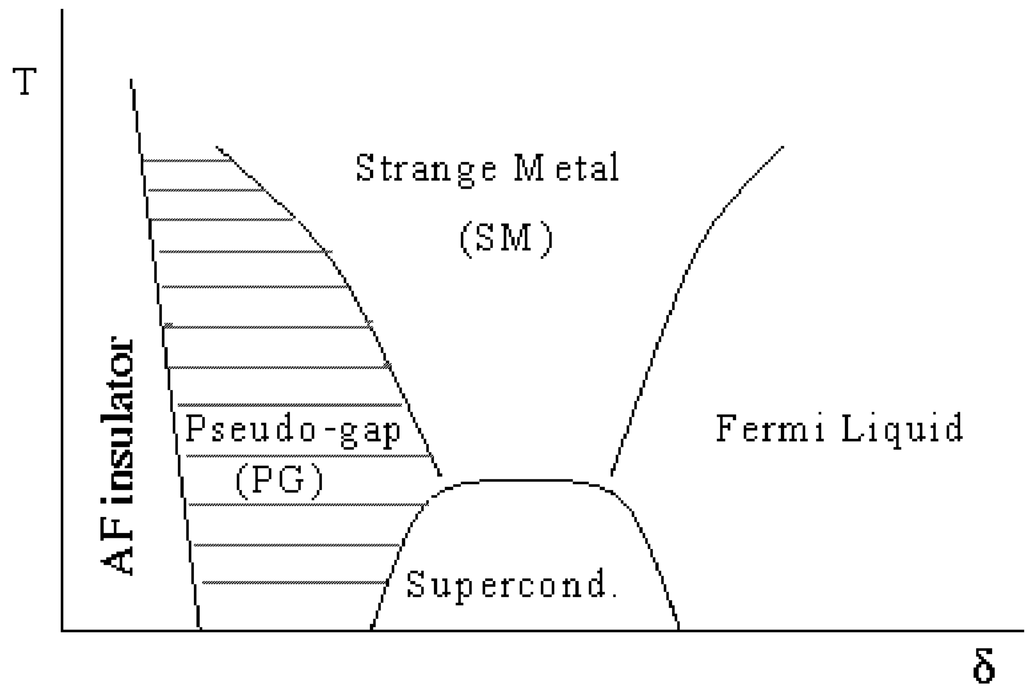
(some skepticism)

Perturbation not valid: Strong coupling

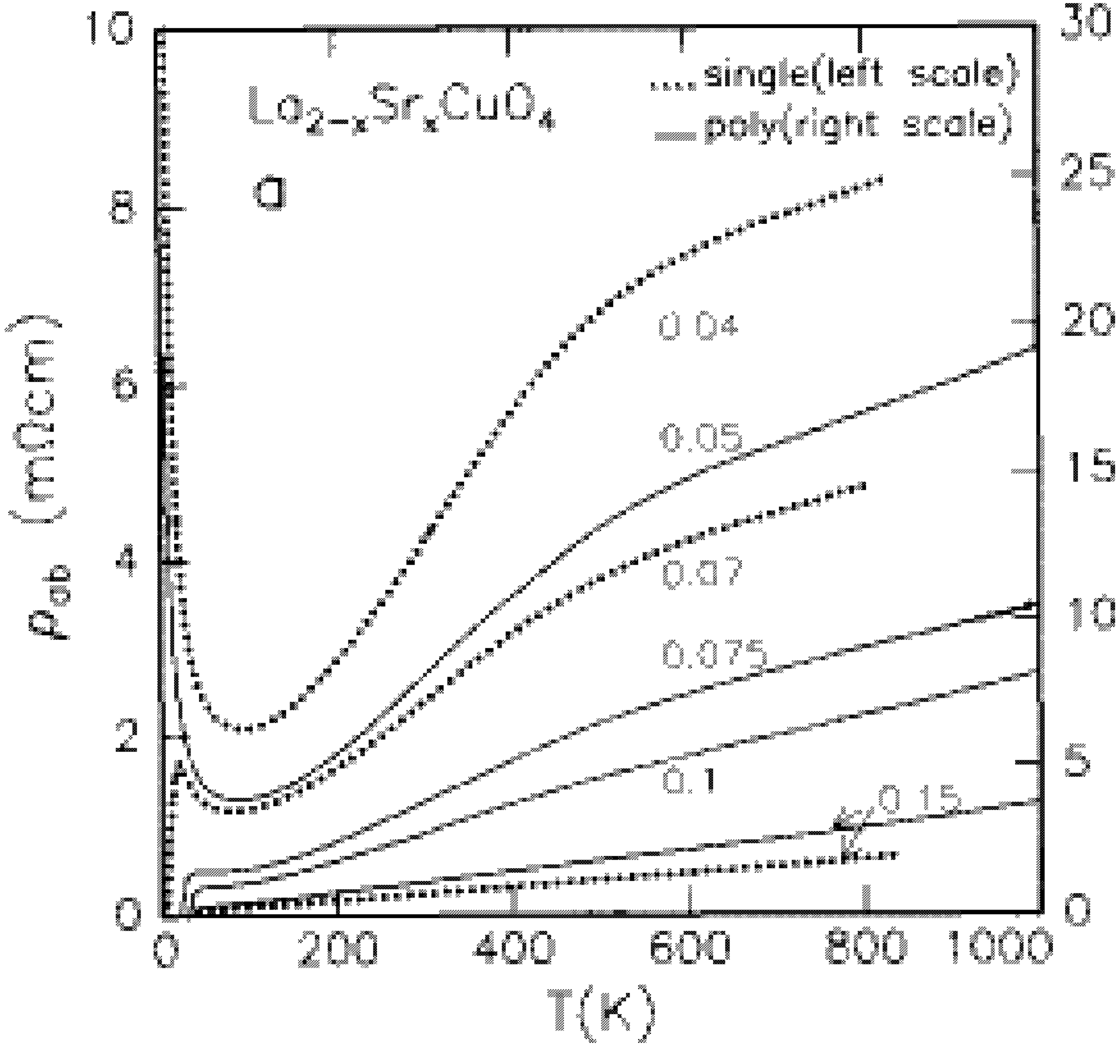
Extra degrees of freedom:
excess specific heat

Why another interpretation?

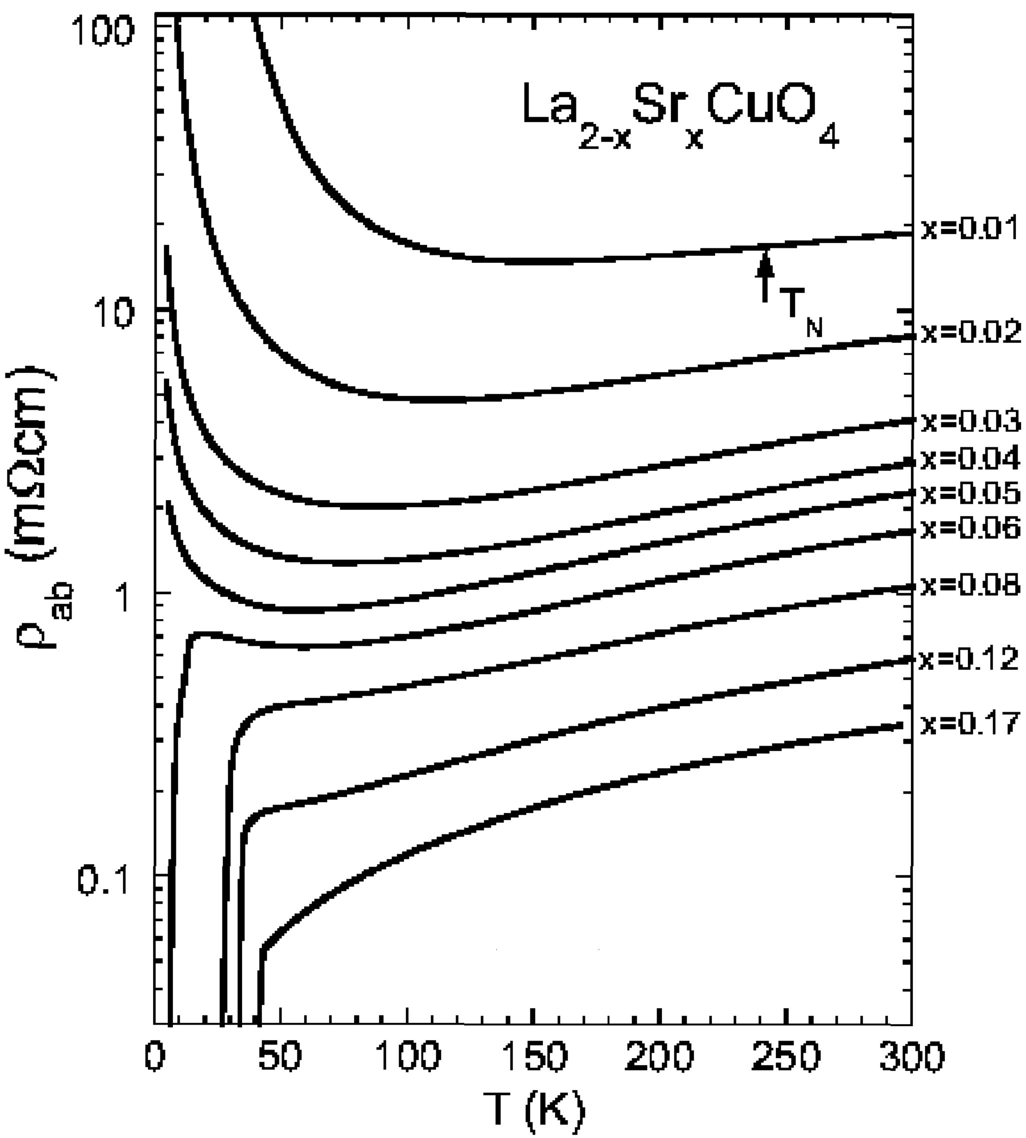
Open Issue: Metal-Insulator Crossover It is a generic phenomenon



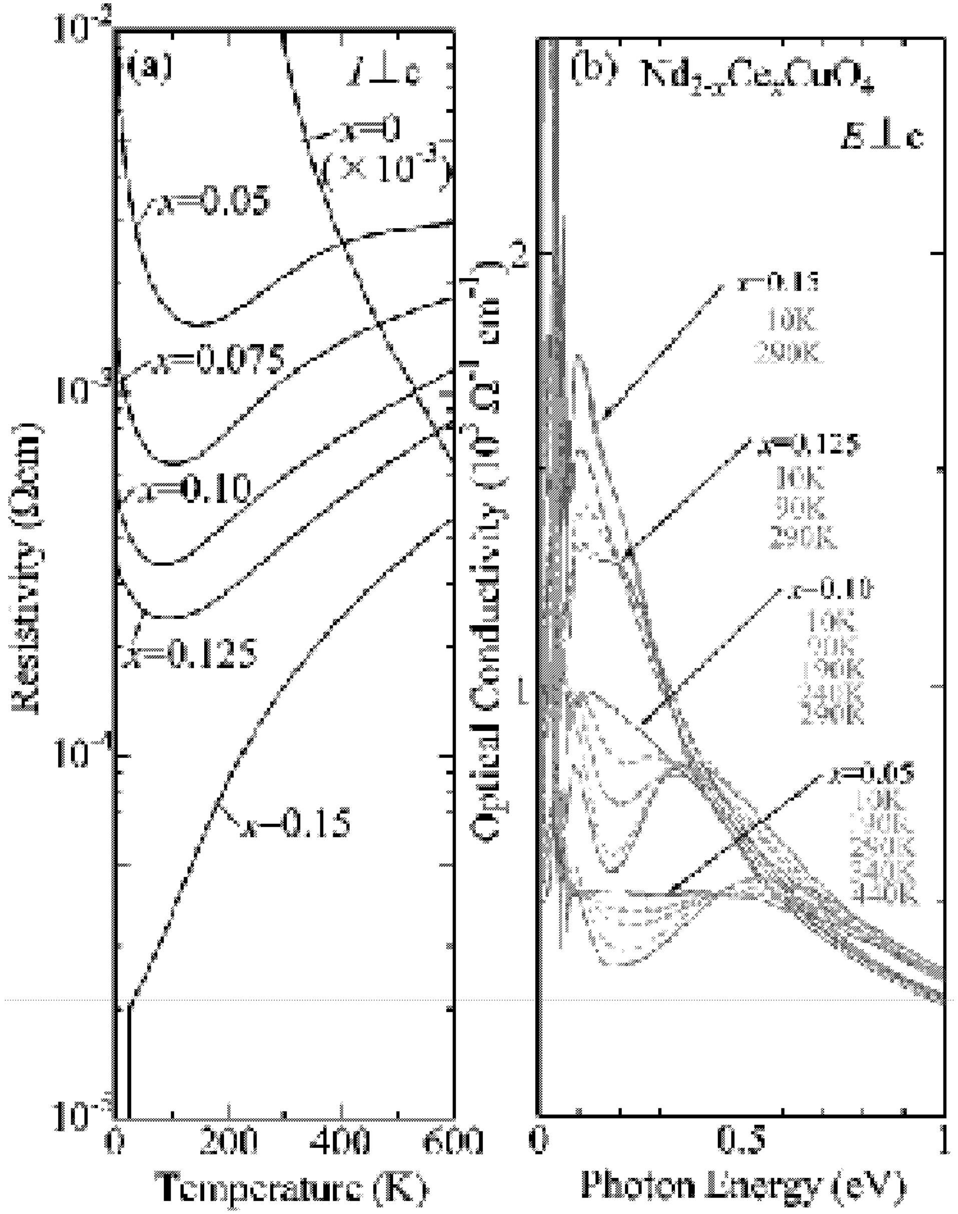
Schematic Phase Diagram



H. Takagi et al. Phys. Rev. Lett. **69**, 2975 (1992)

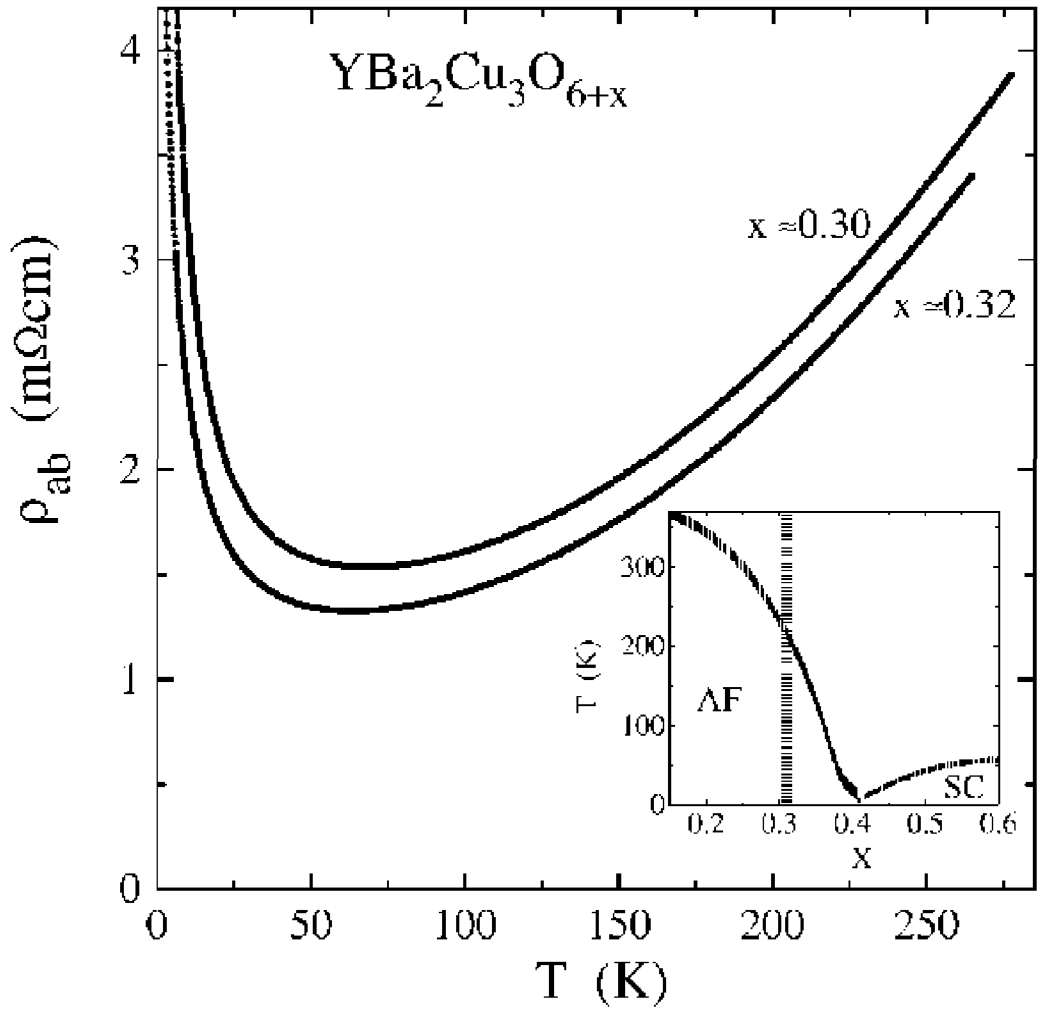


Y. Ando et al. Phys. Rev. Lett. **87**, 017001 (2001)

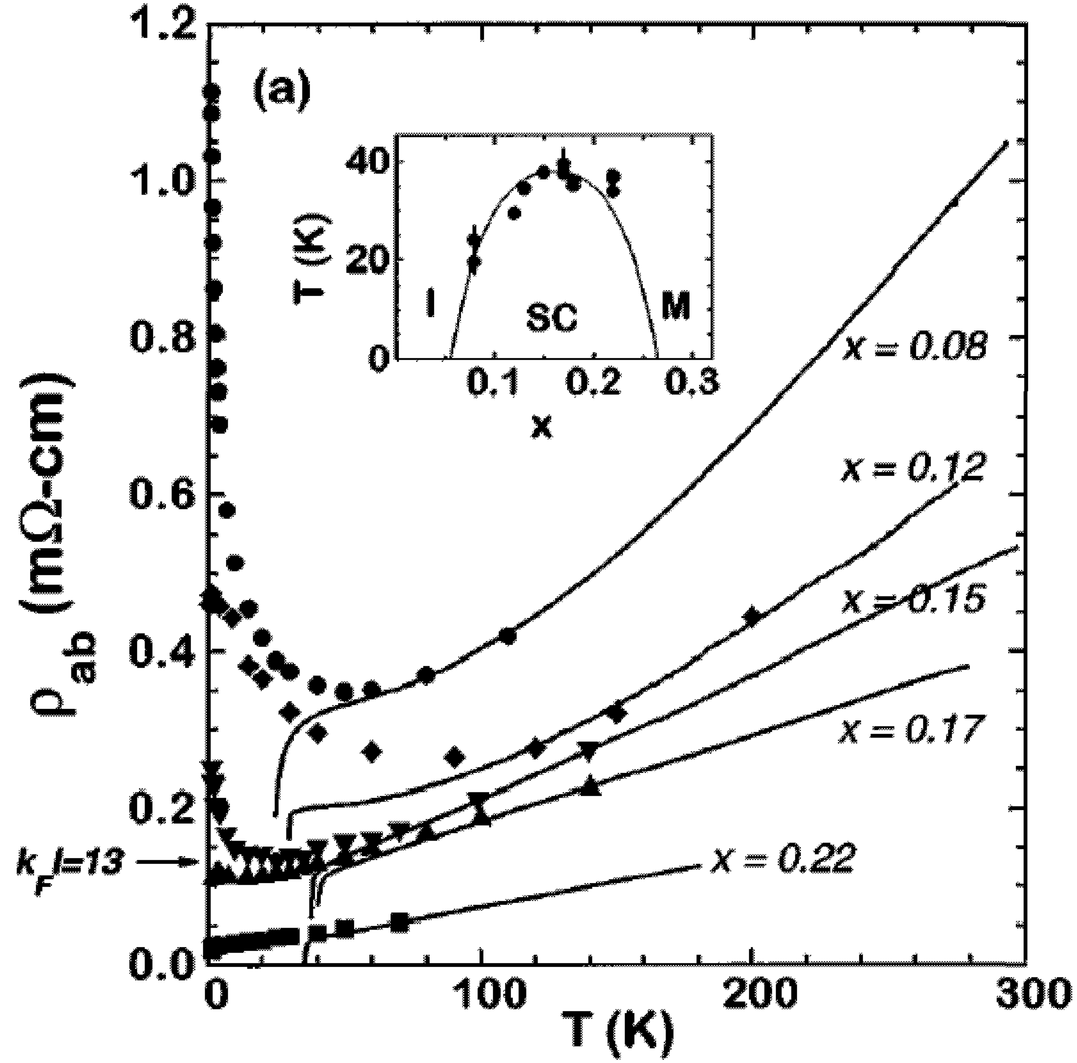


Electron-Doped NCCO

Y. Onose et al. Phys. Rev.Lett. **87**, 217001 (2001)

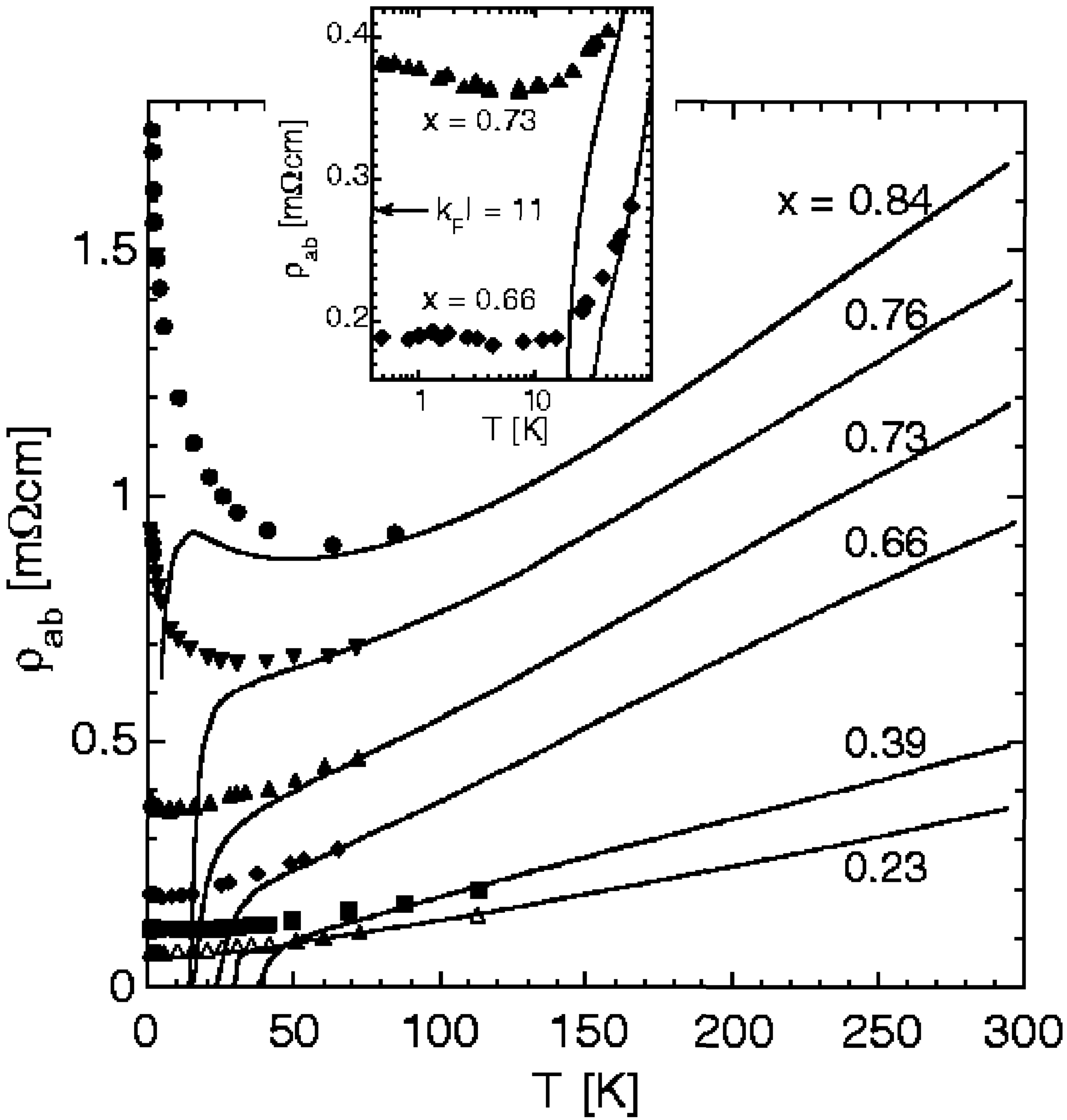


Y. Ando et al. Phys. Rev. Lett. **83**, 2813 (1999)



LSCO in 0T /60T magnetic field

G.S. Boebinger et al. Phys. Rev. Lett. **77**, 5417(1996)



$Bi_2Sr_{2-x}La_xCuO_{6+y}$ La-doped Bi - 2201 in 0T/60T magnetic field

S. Ono et al. Phys. Rev. Lett. **85**, 638 (2000)

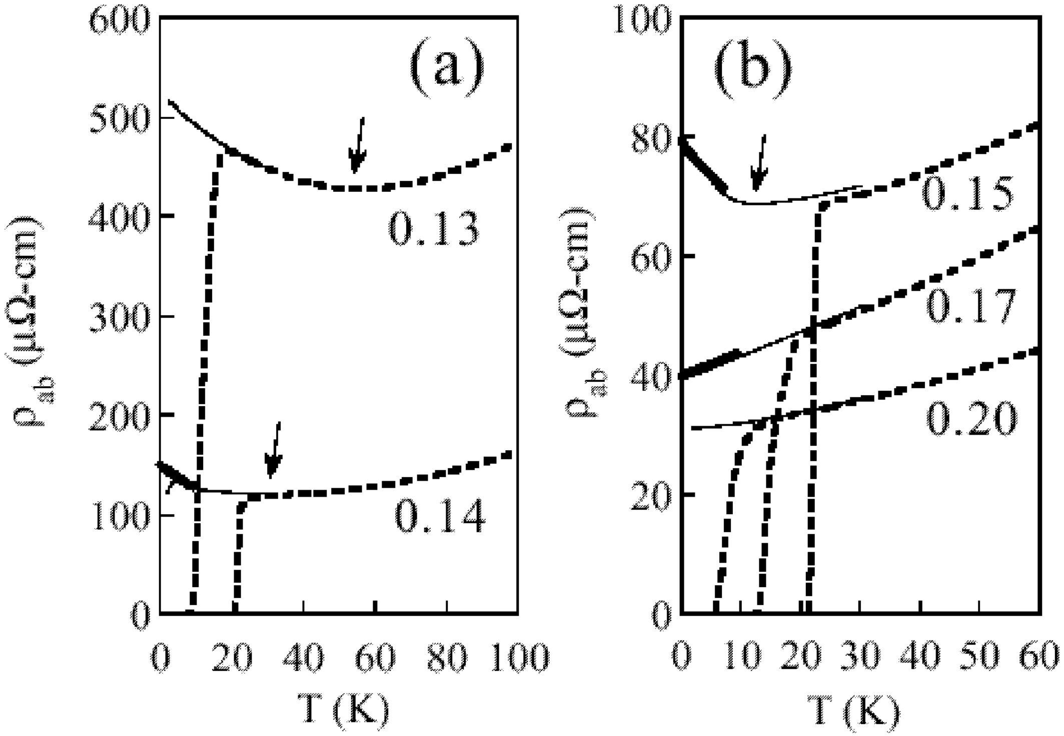


FIG. 1. Resistivity ρ_{ab} as a function of temperature for the c-axis oriented $Pr_{2-x}Ce_xCuO_4$ thin films in magnetic fields of 0 T (dashed lines), 8.7 T (thin lines), and 12 T (thick lines). (a) $x = 0.13$ and 0.14; (b) $x \geq 0.15$. The field is applied along the c axis.

Electron doped material, $k_F l \sim 25$

P. Fournier et al. Phys. Rev. Lett. **81**, 4720 (1998)

-- "Obvious" explanation as 2D localization
DOES NOT WORK!

Estimated $k_F l \sim 0.1$ for x= 0.01 LSCO w/o field
$\sim 12 - 25$ for M-I in SC samples

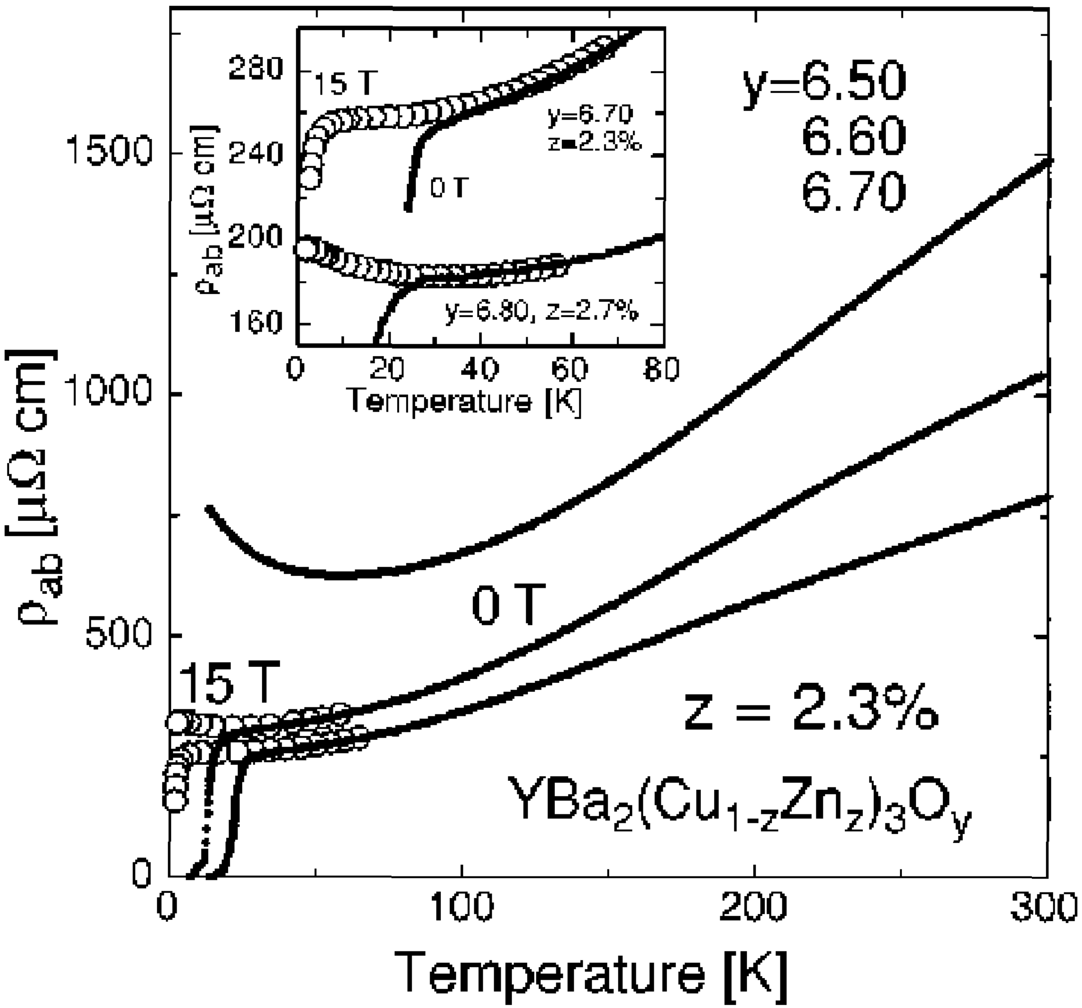
Not due to Cooper pair localization, either.

-- Proximity to quantum critical point?

In La-doped Bi-2001 only up to 1/8 doping, no signatures of stripe formation

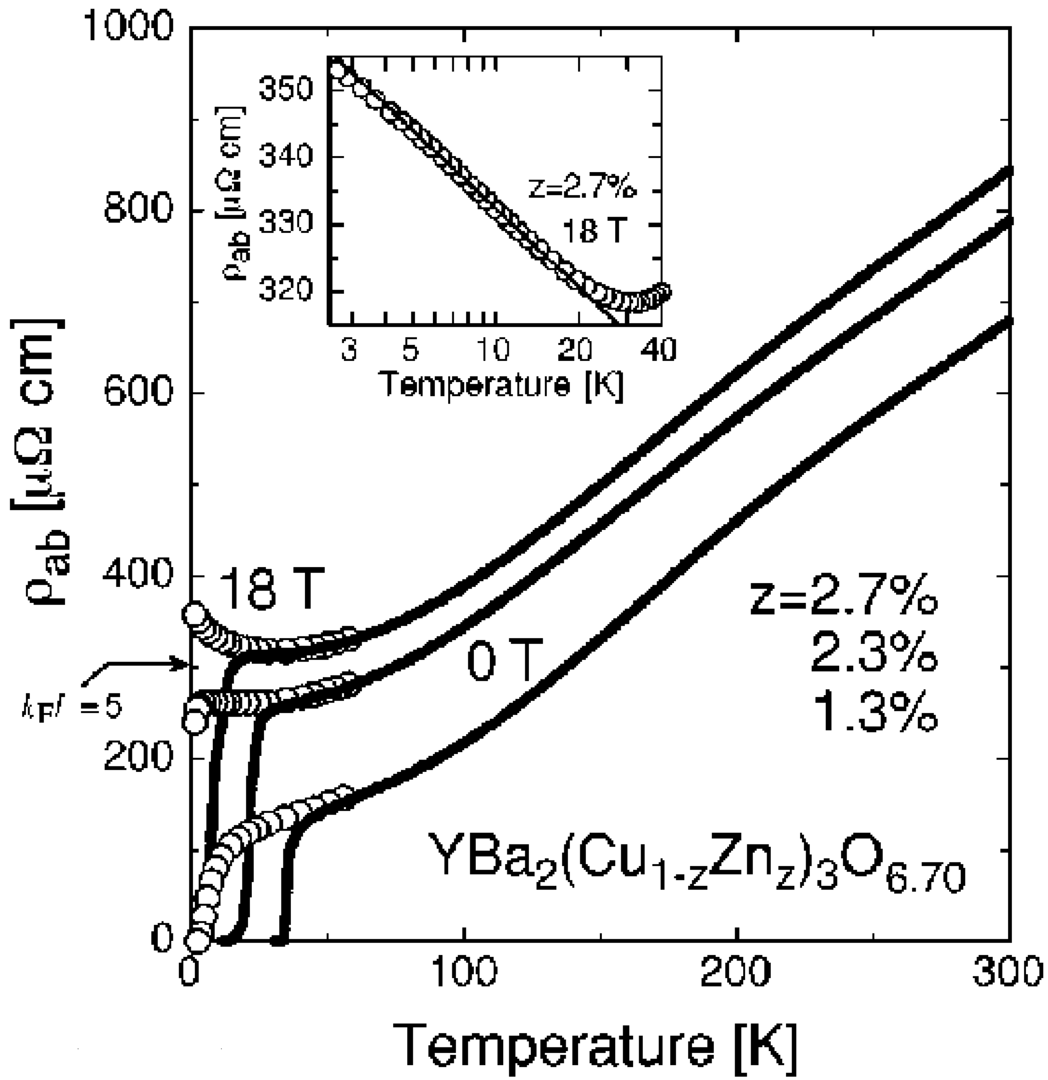
-- Two phenomena : M-I crossover with and w/o magnetic field are of the same origin?
YES or NO?

Continuous change in Zn-doped samples.



Continuous change from M-I w/o magnetic field to absence of M-I crossover

K. Segawa et al. Phys. Rev. B **59**, 3948 (1999)



K. Segawa et al. Phys. Rev. B **59**, 3948 (1999)

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Basic Considerations

- Insulating behavior (localization) is mainly due to interaction rather than disorder
- We start from AF long range order; doping converts LRO to SRO

holes disturb spin background

spin excitations acquire a gap (mass)

$$m_s \propto J(-\delta \ln\delta)^{1/2} \quad \text{(DERIVED!)}$$
$$\text{LRO} \Rightarrow \text{SRO} \quad \xi \propto m_s^{-1}$$

The gauge field has dissipation $\propto \delta T$:

Renormalized holes holes can diffuse

Competition of gap effect with dissipation
Localization versus diffusive motion

gives rise to metal-insulator crossover

$$T_{M-I} \propto t(-\ln\delta) \text{ for low } \delta$$

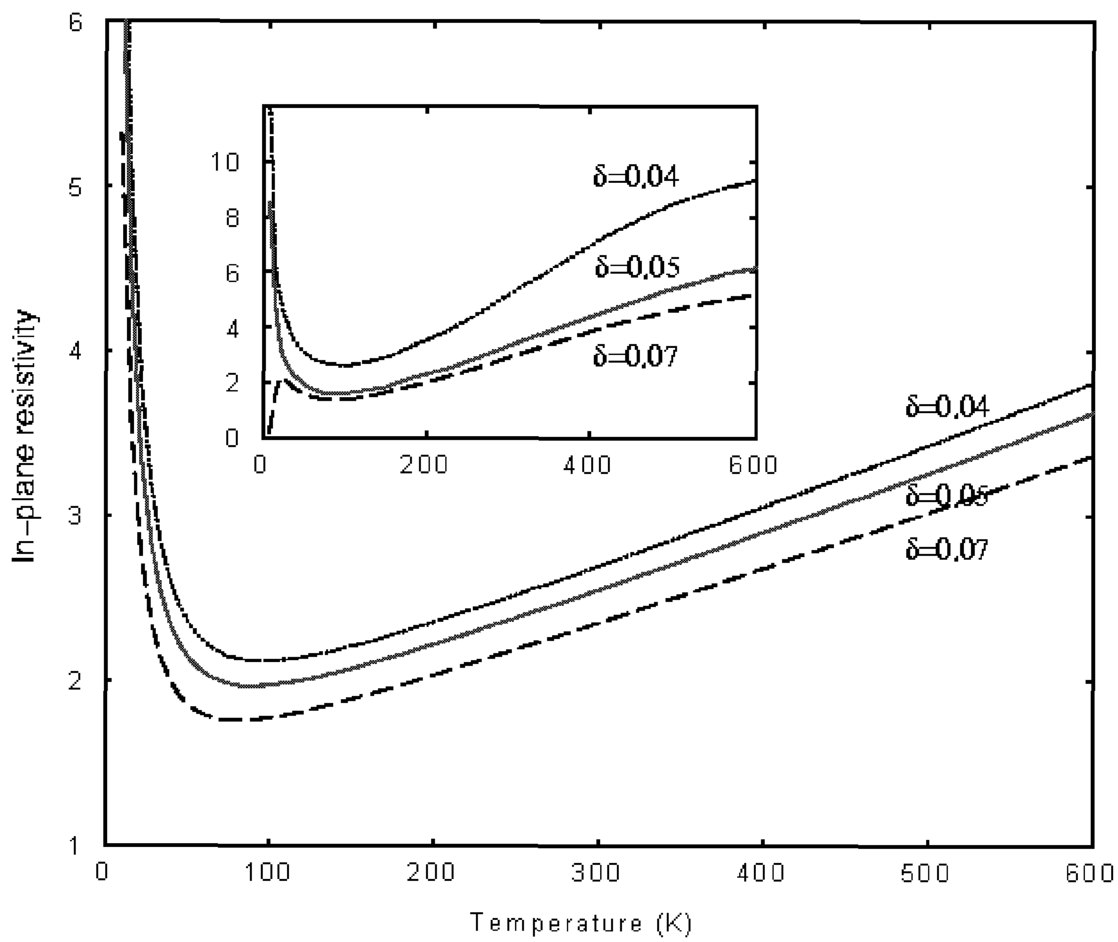
"peculiar" localization due to SR AF order. Magnetic correlation length $\xi^2 \propto m_s^{-2} \propto 1/(-\delta\ln\delta)$

Thermal de Broglie wave length: $\lambda^2 \propto 1/Tm_h \propto \chi/T$

χ diamagnetic susceptibility

If $T/\chi >> m_s^2$ or $\lambda << \xi$, the "magnetic localization" is not felt, system is metallic (only smallest scale matters)

Otherwise, $T/\chi << m_s^2$ or $\lambda >> \xi$, system is insulating



Data from H. Takagi et al. Phys. Rev. Lett. **69**, 2975 (1992)

Calculated resistivity compared with experiments, no adjustable parameters except for resistivity scale

Strong coupling approach

•Single occupancy constraint

"standard" form: $\psi^{+}_{i\sigma}\psi_{i\sigma} \leq 1$

•Spin-charge separation- Slave particles

$$\psi^{+}_{i\sigma} = h^{+}_{i} z_{i\sigma}$$

h_i - fermion - holon, carrying charge

$z_{i\sigma}$ - boson -spinon, carrying spin

Constraint:

$$h^{+}_{i} h_{i} + \sum_{\sigma} z^{+}_{i\sigma} z_{i\sigma} = 1$$

Gauge approach to treat correlations

- mismatch of degrees of freedom 4→ 4+2-1
- underlying gauge symmstry

$$z_{i\sigma} \Rightarrow z_{i\sigma} e^{i\phi_i}$$
$$h_i \Rightarrow h_i e^{i\phi_i}$$

Physical operator $\psi_{i\sigma}$ gauge invariant

•Introduce a spinon-holon gauge field
Physical observable-gauge invariant

•First enlarge Hilbert space, then eliminate extra degrees of freedom by "gauge fixing"

Baskaran & Anderson; Ioffe & Larkin
U(1): Lee & Nagaosa, many others
SU(2): Wen, Lee, Nagaosa & Ng

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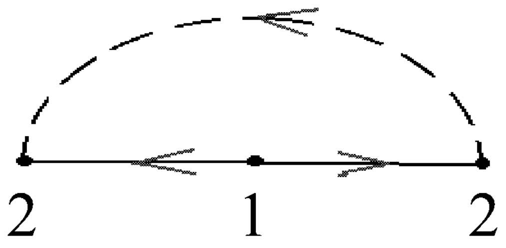
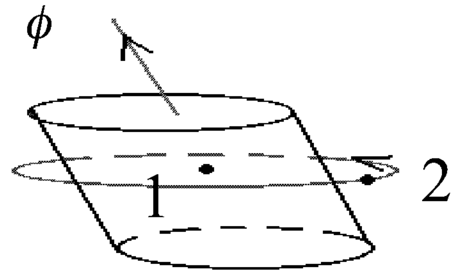
Sketch of derivation

- Chern-Simons bosonization
- Gauge fixing
- Optimization of holon partition on spinon background
- Effective (long wave length limit) spinon action: nonlinear σ model
- Effective holon action: Dirac structure
- Gauge field propagator: Reizer singularity

Basic Ideas of Chern-Simons Bosonization

Anyons in 2D





$$\psi(\phi+\pi) = e^{-i\theta}\psi(\phi), \text{ where } -\infty < \phi < \infty$$

$\theta = 0$ - boson, $\theta = \pi$ -fermion, arbitrary - anyons

Aharonov-Bohm effect:modification of wave function

$$\psi \to e^{iq\oint \vec{A}\cdot d\vec{l}}\psi = e^{iq\phi}\psi$$

Attaching flux changes statistics by $\delta\theta = \dfrac{q\phi}{2}$

2π flux converts boson into fermion and v.v.

π ($-\pi$) flux makes boson (fermion) “semion” $\theta = \pi/2$

•Jordan-Wigner transformation

$$c_j^+ = a_j^+ e^{-i\pi \sum_{i<j} a_i^+ a_i}$$

Express fermions in terms of hard-core bosons

•1D abelian bosonization: (Luther & Peschel, Mattis)

$$\psi_\alpha(x) \propto e^{\frac{i}{\sqrt{\pi}}\int_{-\infty}^{x} dy \dot{\phi}(y) - (-1)^\alpha i\sqrt{\pi}\phi(x)}$$

•Abelian bosonization in (2+1)D: (Mele, Semënoff, Fradkin, Baskaran,…)

•Analogue of J-W formula

$$\psi_\alpha(x) \propto \phi(x) e^{i\int_{\gamma_x} A_\mu(y) dy^\mu}$$

•Additional factor exp(-$kS_{c.s.}$) in path integral Chern-Simons action:

$$S_{c.s.} = \frac{1}{4\pi i}\int d^3x \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

With C.S. coefficient (level) $k = 1/(2l+1)$, $l = 0,1,2….$

For non-relativistic fermions

$$\psi(x) \propto \phi(x) e^{i(2l+1)\int d^2y \Theta(x-y)\phi^+(y)\phi(y)},$$

$$\Theta(x-y) = \arctan\frac{\mathrm{x}^2 - y^2}{x^1 - y^1}$$

Fractional statistics: Aharonov-Bohm phase
$2\pi\theta$, θ =0, 1/4, 1/2 $\Rightarrow$boson, semion, fermion

Non-abelian bosonization in (2+1)D:
Rewrite fermion partition function and correlation in terms of boson theory (Fröhlich, Kerler, Marchetti)

What is a good Mean Field Theory?

Gauge field approach can be derived as a MF approximation using different C.S. representations in 2D. For a suitable choice of group G and coefficient k_G we can replace fermionic action with single occupancy constraint

$$S(\psi) \rightarrow S(\chi, W) + k_G S_{c.s.}(W)$$

where $S(\chi, W)$ is obtained from $S(\psi)$ by substituting ψ with a new field χ, bosonic or fermionic, depending on G and k_G, minimally coupled to gauge field W with group G, and the C.S. action

$$S_{c.s.} = (1/4\pi i)\int d^3x\ \varepsilon^{\mu\nu\rho}\,\mathrm{Tr}(W_\mu \partial_\nu W_\rho + 2/3\, W_\mu W_\nu W_\rho)$$

The new action is exactly equivalent to the original fermionic action.

Each of these representations can be taken as a starting point of MFA.

For example,

$S(\psi)$ – action of the t-J model

$G = U(1)$, $k_{U(1)} = 1 \rightarrow$ slave boson theory

IDEA: Introduction of C.S. action of W
attaches a G vortex to each fermion (boson) described by $\chi \rightarrow$ Aharonov-Bohm effect guaranteeing the anticommutation relation of the original fermions.

The choice we made:
χ as fermion of spin 1/2; G as $U(1)\times SU(2)$

Introducing:

- a $U(1)$ gauge field B_μ
- an $SU(2)$ gauge field $V_\mu \equiv V_\mu^{(a)}\sigma_a/2$
- (acting on the spin space)

$\mu = 0,1,2$; σ_a -- Pauli matrices, $a = 1,2,3$

$$\Psi_{j\alpha} = \exp(-i\int_{\gamma j} B) H_j^+ (P \exp(i\int_{\gamma j} V))_{\alpha\beta} \Sigma_{j\beta},$$

H_j^+ is a spinless fermion
$\Sigma_{j\beta}$ are spin 1/2 hard-core boson with constraint

$$\Sigma_\alpha \Sigma_\alpha^+ \Sigma_\alpha = 1$$

Why $U(1)\times SU(2)$??

In 1+1D:
Electron is decomposed into holon and spinon plus attached "strings"

$$\psi_x = H_x e^{i\frac{\pi}{2}\sum_{l>x} H_l{}^* H_l} [e^{i\frac{\pi}{2}\sum_{l<x} b_l{}^* b_l} b_x]$$

$\uparrow$ $\uparrow$

Charged semion spin 1/2 semion

The "semionic" nature turns out to be essential:
spin 1/2 spinons are "deconfined"
spin and charge are separated

Exact critical exponents (Bethe Ansatz and conformal field theory) are reproduced at the "mean field" level.

t-J Model

$$H = P_G \sum_{<i,j>} [-t\psi_i^+\psi_j + h.c. + J(\psi_i^+ \frac{\vec{\sigma}}{2}\psi_i)(\psi_j^+ \frac{\vec{\sigma}}{2}\psi_j)]P_G$$

1D *t-J* model: → Exact spin-charge separation

2D *t-J* model:
What can we say about spinons and holons?

Free | Bound State | Confined

<u>OPEN QUESTION</u> !!!

Gauge fixing conditions

$U(1)$ - charge sector

Coulomb gauge $\partial_\mu B_\mu = 0$

Integration over B_0 $\longrightarrow$ $B = \overline{B} + \delta B(H)$

$\overline{B}$ $e^{i\int_{\partial p}\overline{B}} = -1$ π flux phase

$SU(2)$ - spin sector $\Sigma_j = \sigma_x^{|j|}\begin{pmatrix}1\\0\end{pmatrix}$

Néel gauge

Split V into "Coulomb" part and g

$$\begin{cases} V \to g^\dagger V^{(c)} g + g^\dagger \partial g \\ \mathcal{D}V \to \mathcal{D}V^{(c)}\mathcal{D}g \end{cases}$$

Integration over V_0 yields $V_\mu{}^c(g,H)$

Up to now no approximations

Counting degrees of freedom:

H (2) + g (3) =5, but the $U(1)$ spinon/holon gauge field to be fixed.

Maximization of partition function for a given holon configuration (mathematical physics proof)

Hint from 1D:

"happy" configuration: ↑↓↑↓↑↓○↑↓↑↓↑↓

"unhappy" configuration:

↑↓↑↓↑↓○↓↑↓↑↓↑

moving spins to form "squeezed" A-F chains

○↑↓↑○↓○○↑↓○○○↑↓○○

↑↓↑↓↑↓↑↓

each swap brings a "phase" ~π/2 for large *U*

"memory" is kept by "strings"

$$\psi_x = H_x e^{i\frac{\pi}{2}\sum_{l>x} H_l{}^* H_l} [e^{i\frac{\pi}{2}\sum_{l<x} b_l{}^* b_l} b_x]$$

For 2D the counterpart of "kink" is vortex of V field

$$V^c(g, H) = \bar{V}^c(H) + \delta V(g, H)$$

$\bar{V}^c(H)$ optimal configuration

$\delta V(g, H)$ fluctuations around MF

Mean field approximation:
neglect δV and δB responsible for semion statistics
keep feedback of holons on V

Justifiable for 2D:

No real spin-charge separation: bound state

Low energy action for spinons S_s

Taking continuum limit, integrate out ferromagnetic component, in CP^1 representation: $z_1^* z_1 + z_2^* z_2 = 1$

$$S_s = \int d^3x \frac{1}{g}\left[v_s^{-2} \left| (\partial_0 - A_0) z_\alpha \right|^2 + \left| (\partial_\mu - A_\mu) z_\alpha \right|^2 + m_s^2 z_\alpha^* z_\alpha \right]$$

g=8/J, v_s is J dependent spin velocity, A is $U(1)$ spinon/ holon gauge field. This $O(3)$ nonlinear σ model describes spin wave propagation in vortex fields V, and the "mean field" treatment gives

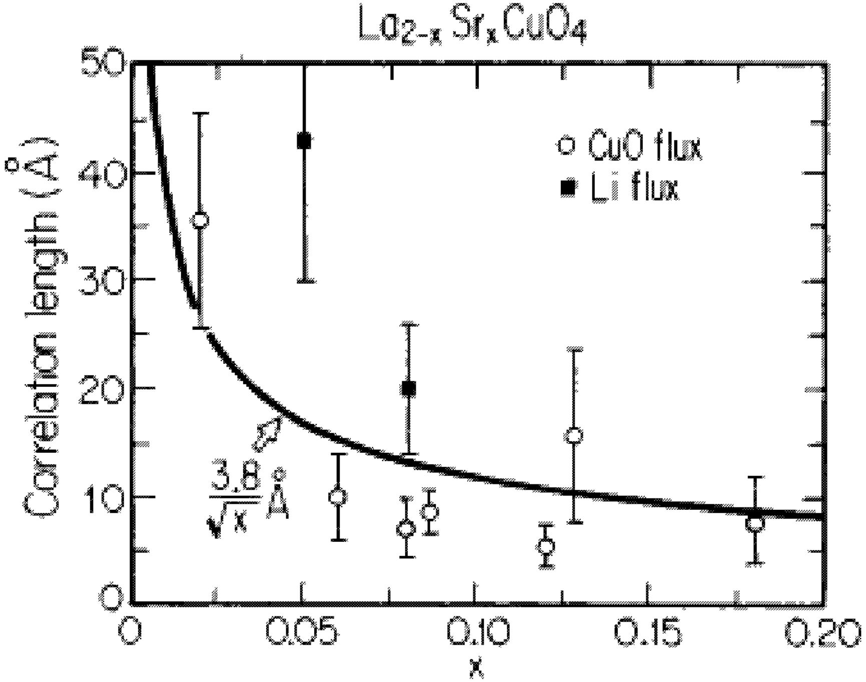
$$m_s^2 = \left\langle \overline{V}^2 \right\rangle \approx -\delta \, ln \, \delta$$

This can be justified for $J << t$

It is consistent with neutron scattering data

ln dependence gives rise to important consequences

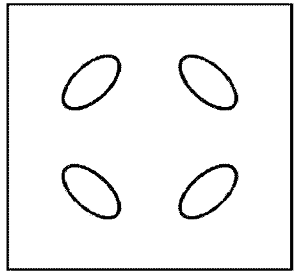
Early neutron data- interpretation now derived from theory:
$\xi \sim m_s^{-1} \sim (\delta)^{-1/2}$



R.T. Birgeneau et al. Phys.Rev. B **38,** 6614 (1988)

Low energy action for holons S_h

The π-flux per plaquette, mean field of B shifts energy minima to (±π/2,±π/2) two species of Dirac-like two component spinons ψ^r, r =1,2; e_r =± 1

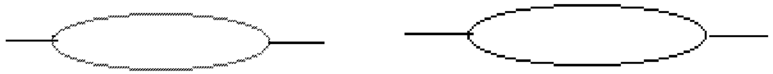


$$\int dx_0 d^2x \sum_r \bar{\psi}^{(r)}[\gamma^0(\partial_0 - e_r A_0 - \delta) + t(\not{\partial} - e_r \not{A})]\psi^{(r)},$$

$$\not{A} = \gamma_\mu A_\mu, \not{\partial} = \gamma_\mu \partial_\mu, \gamma_0 = \sigma_z, \gamma_\mu = (\sigma_y, \sigma_x),$$

The "Dirac" structure, especially presence of $\gamma^0\delta$ term is crucial
one species is gapless, FL-like $\varepsilon_F \sim t\delta$
another species gapful, mixing affects spectral weight

Gauge field propagator: Reizer singularity



Spinon loop Holon loop

If only spinons, massive particle -Maxwell-like action:
$(\omega^2 - v^2 q^2)$ - in real space ~ ln R logarithmic confinement
Presence of Fermi surface ⇒ dissipation ⇒ singular propagator

Analogy with skin effect:
Normal: dissipation q- independent, only length scale, skin depth

$$\delta \propto \frac{c}{\sqrt{2\pi\omega\sigma}} >> \ell$$

Anomalous skin effect: $\delta << \ell$ dissipation $\sim q^{-1}$

Here also anomalous dissipation and a new scale

$$\tilde{q} = (\frac{\kappa\omega}{\chi})^{1/3}$$

$$< A_\mu{}^{\perp} A_\nu{}^{\perp} > (q,\omega) \propto (i\omega\lambda_h(\vec{q}) - \chi|\vec{q}|^2)^{-1}$$

$$< A_0 A_0 > (q,\omega) \propto (\nu_h + \omega_p)^{-1}$$

$\lambda_h = \kappa / q, \sim 0(\delta)$ Landau damping

$\chi = \chi_h + \chi_s, \chi_h \sim m_h^{-1} \sim o(\delta^{-1}),$

$\chi_s = v_s m_s^{-1} \sim o((-\delta \ln \delta)^{-1/2})$

ν_h holon density, ω_p plasmon frequency

Gauge field not confining: bound state

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Calculation of conductivity

Coupling to external electromagnetic field: holons or spinons?

$$S_{eff}(A, A_{e.m.}) = S_s(z, A - \varepsilon A_{e.m.}) + S_h(\psi, A + (1-\varepsilon)A_{e.m.})$$

If quadratic in A, after integration over A

$$\Pi_{e.m.} = [\Pi_s^{-1} + \Pi_h^{-1}]^{-1}$$

Using Kubo formula $\sigma(\omega) \propto \mathrm{Im}\Pi(\omega)/\omega$

One can derive Ioffe-Larkin formula: $R = R_h + R_s$

Is it valid only in perturbation?
Effective action in scaling limit (long distance, low energy) is quadratic to all orders in coupling (Fröhlich, Götschmann, Marchetti, 1995)

Holon contribution

Lee & Nagaosa, Ioffe & Wiegmann

$$R_h \propto \delta[\frac{1}{\varepsilon_F \tau_{imp}} + (\frac{T}{\varepsilon_F})^{4/3}]$$

τ_{imp} impurity scattering time

It turns out to give minor contributions except for very high temperatures

Spinon contribution

$$\Pi_{\mu\nu} \propto \partial_\mu \partial_\nu < z^*(x) z(y) z^*(y) z(x) >_A$$

How to calculate correlation functions gauge-invariantly?
Feynman-Schwinger-Fradkin path integral

$$< T(z^*(x)z(y)) > = ((\partial_\mu - A_\mu)^2 + m^2)^{-1})(x,y) = i\int_0^\infty ds e^{-im^2 s} e^{is(\partial_\mu - A_\mu)^2 (x,y)}$$

Like a particle in field A_μ

$$= i\int_0^\infty ds e^{-im^2 s} \int_{q(0)=x, q(s)=y} Dq(t) e^{i\int_0^s dt [\dot{q}_\mu^2(t)/4 + \dot{q}_\mu(t) A^\mu(q(t))]}$$

Change variable $\qquad q \rightarrow \dot{q} = \phi$

$$\sim i\int_0^\infty ds e^{-im^2 s} \int D\phi^\mu(t) e^{\frac{i}{4}\int_0^s \phi_\mu^2(t) dt} e^{i\int_0^s \widetilde{A}_\mu(t)\phi^\mu(t) dt} \int d^3 p e^{ip_\mu (x^\mu - y^\mu - \int_0^s \phi^\mu(t) dt)}$$

where $\qquad \widetilde{A}_\mu = A_\mu(x + \int_0^t \phi(t') dt')$

Introducing the identity:

$$\int_0^s A_\mu\left(x+\int_0^t \phi_\mu(t')dt'\right)\phi^\mu(t)dt = \int_0^1 d\lambda (y^\mu - x^\mu) A_\mu[(1-\lambda)x+\lambda y]$$
$$-\int_0^1 d\lambda\lambda \int_0^s dt \int_0^t dt' \phi^\mu(t)\phi^\nu(t') F_{\mu\iota}[x+\lambda\int_0^t \phi(t')dt']$$

The field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

under the constraint $y - x = \int_0^s \phi(t)dt$

Denoting the first term on right as $\int_x^y \widetilde{A}$

$$G(x,y\,|\,A) = \exp\{i\int_x^y \widetilde{A}\}G(x,y\,|\,F)$$

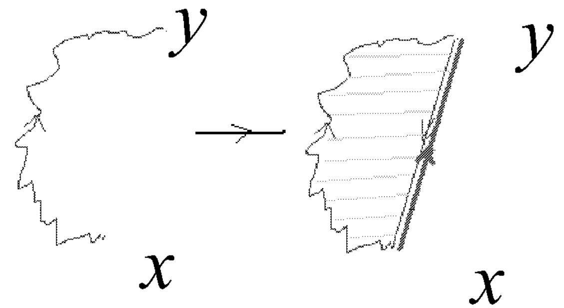
$\exp\{i\int_x^y \widetilde{A}\}$ along straight line, not gauge independent

$G(x,y\,|\,F)$ depends only on field strength *F*, is gauge invariant

$G(x,y\,|\,F) \to G(x,y\,|\,0)$ "Gor'kov" approximation

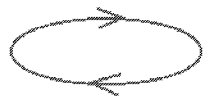
Pictorially:

$$< z^*(x) z(y) > = \sum_{\{x \to y\}} \int \gamma_{xy}$$

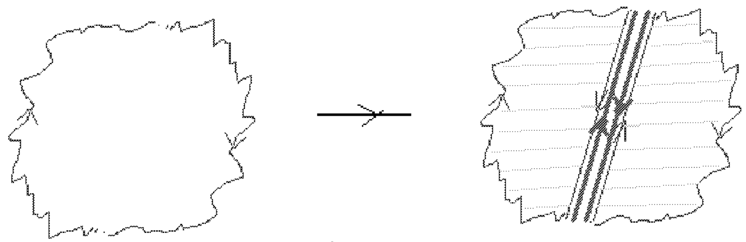


path {x→y} carries "gauge phase factor" $e^{i\int_{\gamma_{xy}} \tilde{A}}$

polarization "bubble"



$$\sim \partial_\mu \partial_\nu \int DA e^{A \Pi A} \sum_{\{x \to y\}} \int \sum_{\{y \to x\}} \int$$



gauge dependent prefactors cancel. Integration

$$\exp(i\int_0^1 d\lambda \lambda \int_0^s dt \int_0^t dt' \phi^\mu(t) \phi^\nu(t') F_{\mu\nu}[x + \lambda \int_0^t dt' \phi(t')])$$

can be preformed if action quadratic

Eikonal approximation: $$< e^{i\hat{O}(\phi)} >_{\phi} \Rightarrow e^{i<\hat{O}(\phi)>_{\phi}}$$

In calculating the gauge-invariant part we limit ourselves to Gaussian approximation

Multiple integrals are calculated using saddle point approximation

In the temperature range

$$m_s^2 \geq T/\chi \geq q_0 m_s, q_0 = (\frac{\kappa T}{\chi})^{1/3}$$

between tens and hundreds degrees q_0--typical inverse length scale as in anomalous skin effect

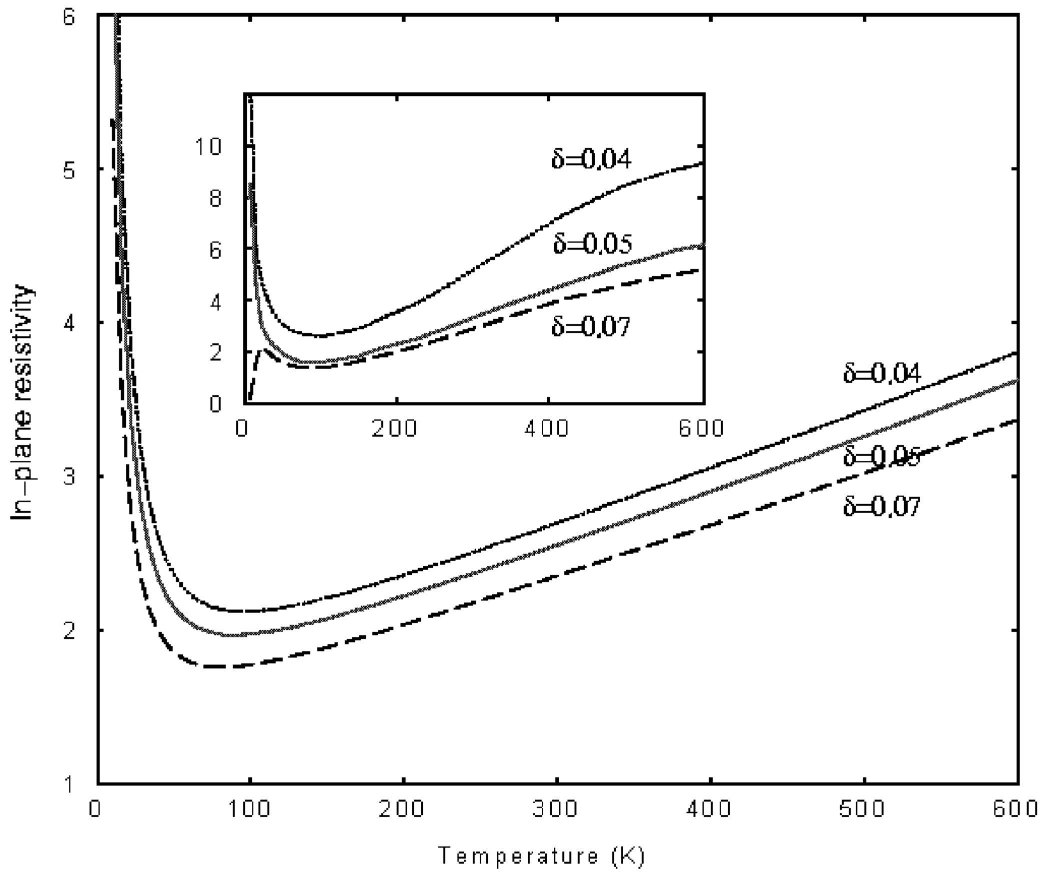
Spinon contribution

$$R_s \propto \frac{1}{\sqrt{\delta}} \frac{(m_s^4 + (\frac{cT}{\chi})^2)^{1/8}}{\sin[\frac{1}{4}\arctan(\frac{cT}{\chi m_s^2})]}$$

Apart from *R* scale, no adjustable parameters.

•At low *T*, the spin gap effect dominates, $R \propto 1/T$ insulating behavior

• At higher *T* the gap effect is less important and *R* grows due to gauge fluctuations



$R = R_s + R_h$

Data from H. Takagi et al. Phys. Rev. Lett. **69**, 2975 (1992)

$$\xi^2 \propto 1/\delta\,|\ln\delta\,|;\ \lambda^2 \propto \chi/T \propto 1/\delta T$$

As δ increases, both ξ and λ decrease, but $|\ln\delta|$ decreases, so ξ decreases less, and the M-I crossover temperaure goes down. The presence of ln is crucial.

Spin-relaxation rate

$$\frac{1}{T_1 T} \propto \lim_{\omega \to 0} \int d^2 q \, |f(q)|^2 \frac{\mathrm{Im}\chi_s(\vec{q},\omega)}{\omega}$$

$f(q)$ form factor peaked near $\vec{q} = (\pi,\pi)$

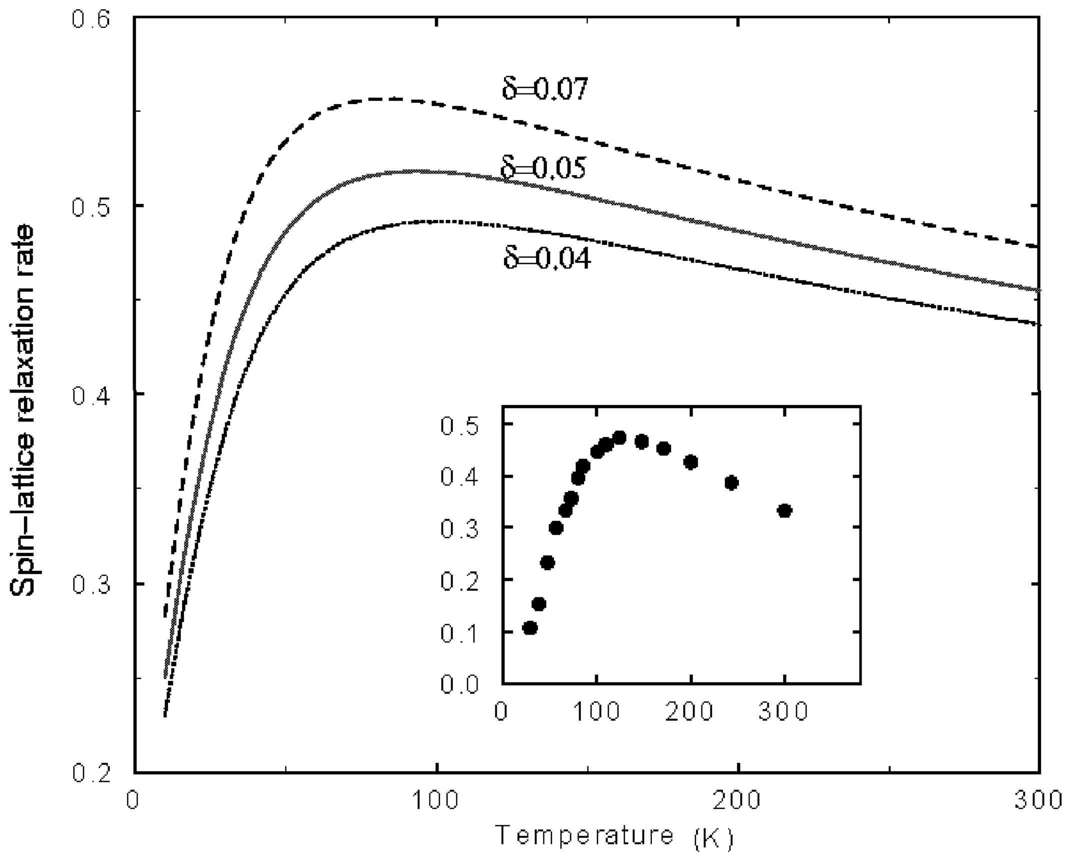
$$\langle \vec{S}(x)\vec{S}(0) \rangle \propto e^{i\pi|x|}(1-\delta)^2 \langle z^* \vec{\sigma} z(x) z^* \vec{\sigma} z(0) \rangle$$

$$\frac{1}{T_1 T} \propto (1-\delta)^2 (m_s^4 + (\frac{cT}{\chi})^2)^{-1/8}$$

$$\times \{ a\cos[\frac{1}{4}\arctan(\frac{cT}{\chi m_s^2})] + b\sin[\frac{1}{4}\arctan(\frac{cT}{\chi m_s^2})] \}$$

$$a/b \sim 10^{-1}$$

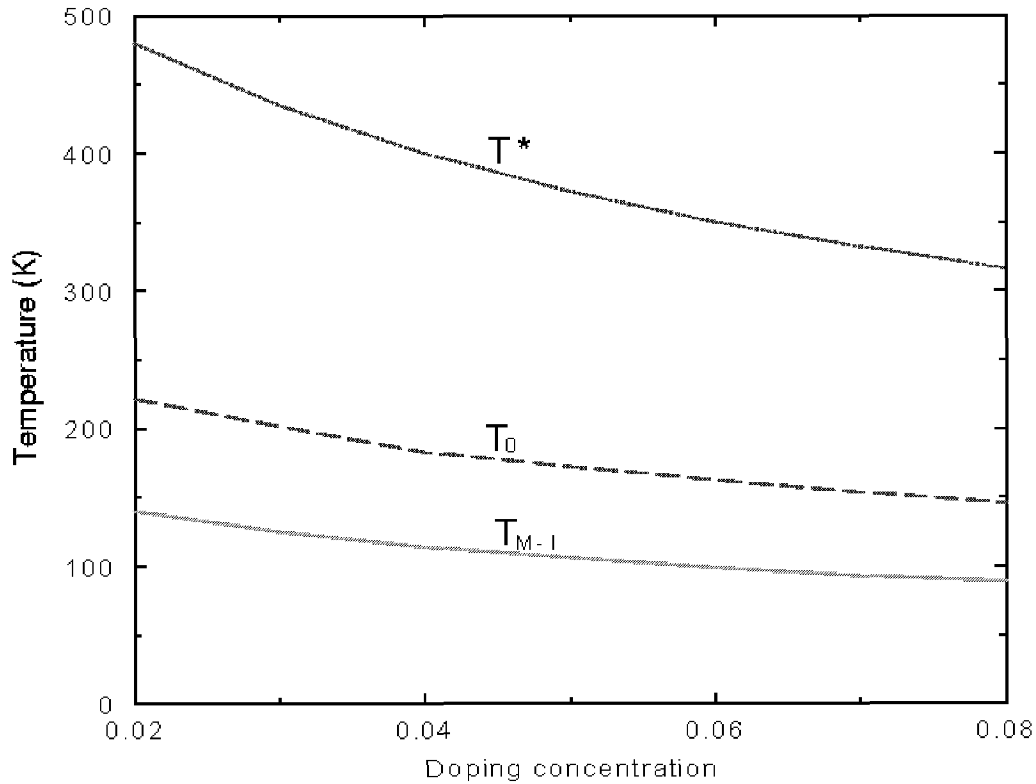
Maximum near the M-I crossover temperature



Maximum near the M-I crossover temperature

Data from C. Berthier et al. Physica C **235-240**,67 (1994)

Characteristic temperatures



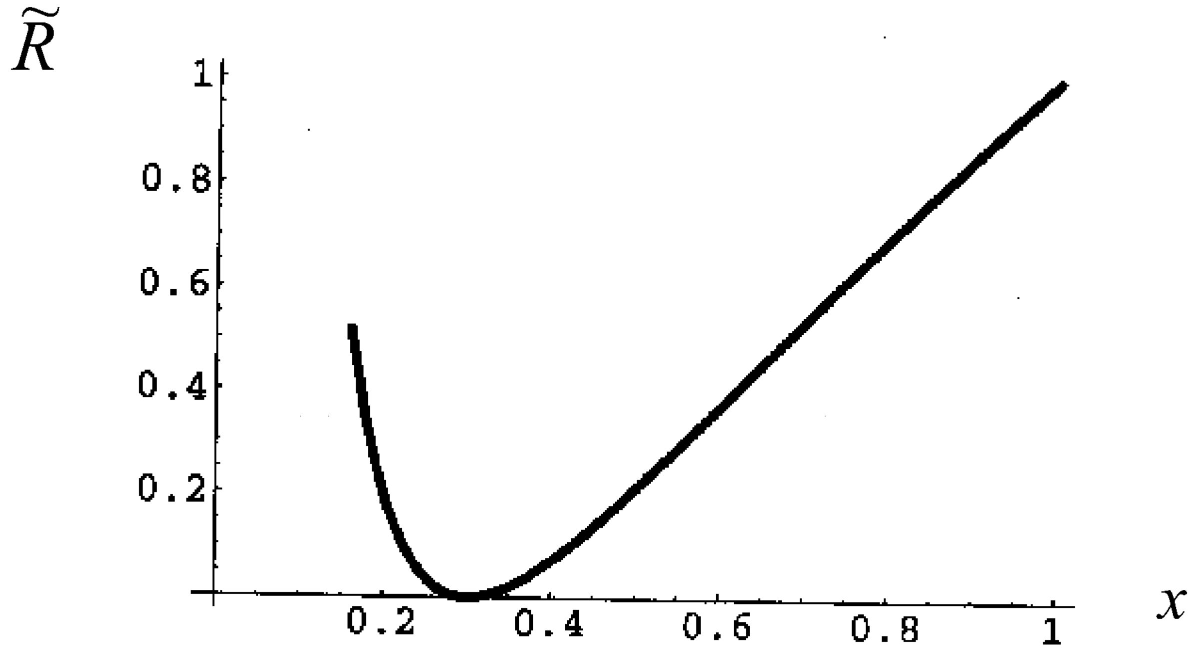
T^* defined as $d^2R/dT^2=0$, inflection point

T_0 defined as maximum of the NMR relaxation

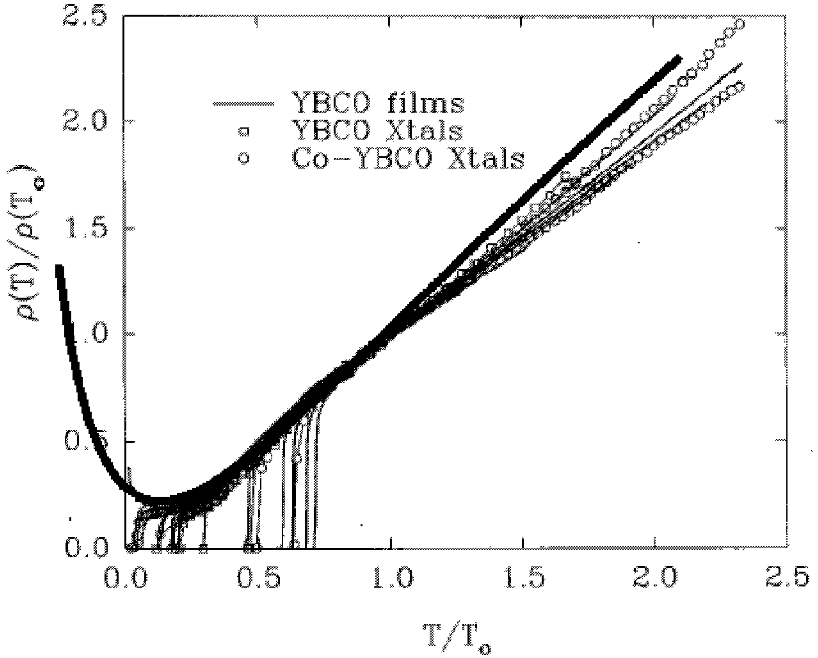
Universality of resistivity curve

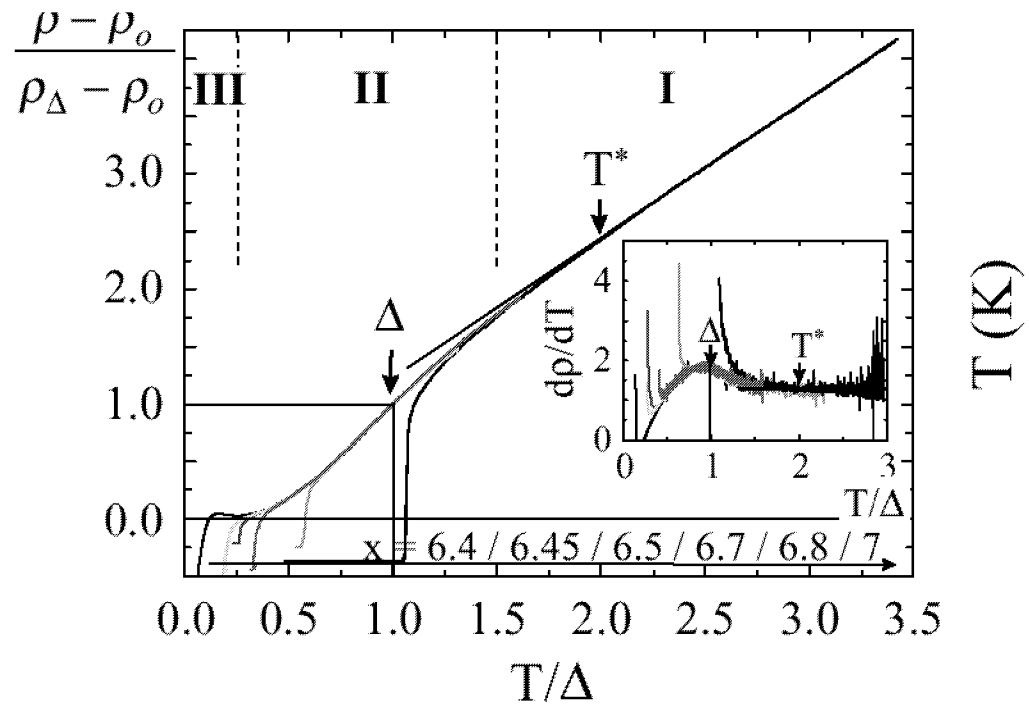
If we identify $T^*(\delta)$ with experimental T^*, both MI crossover temperature $T_{MI}(\delta)$ and $T^*(\delta)$ are in agreement with experiments in range $0.02 \leq \delta \leq 0.08$. R_s is function of $x = cT/\chi m_s^2 \cong T/T^*$ apart from factor $(|\ln\delta|)^{1/2}$. Universal function of x.

$$\tilde{R} \equiv [R - R(T_{MI})]/[R(T^*) - R(T_{MI})]$$



Experimental curves





B. Wuyts et al PRB **53**, 9818 (1996)

L. Trappeniers et al. Cond-mat/9910033

I. Motivation

II. Summary of basic ideas

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<u>V. M-I crossover in magnetic field</u>

VI. Calculations of other physical quantities

VII. Concluding remarks

Magnetic Field Effects

Ioffe-Larkin formula still holds: $R = R_h + R_s$

Two effects: classical (cyclotron) and quantum: $A_{e.m.}$ couples with $-\varepsilon$ to spinons, $(1-\varepsilon)$ to holons, $0 \leq \varepsilon \leq 1$

To be consistent with requirement:
$\chi^{-1} = (\chi_s^*)^{-1} + (\chi_h^*)^{-1}$, χ_s^*, χ_h^* renormalized susceptibilities

$\varepsilon = \chi_h^* / (\chi_h^* + \chi_s^*)$

Replacing them by unrenormalized values:

$$\varepsilon \approx \chi_s / (\chi_h + \chi_s) \approx \frac{J}{t} \sqrt{\delta / |\ln\delta|} << 1$$

Effective Action in Coulomb Gauge

$$S_{eff}(A) = \int dx^0 d^2x [\frac{i}{2}[A^0(\Pi_h^0 + \Pi_s^0)A^0 + (A^T - \varepsilon A_{e.m.})\Pi_s^{\perp}(A^T - \varepsilon A_{e.m.})$$
$$+ (A^T + (1-\varepsilon)A_{e.m.})\Pi_h^{\perp}(A^T + (1-\varepsilon)A_{e.m.})] + \frac{i\sigma_h(H)}{2\pi} A^0 \varepsilon_{ij}\partial^i A^j]$$

Minimal couplings to holons and spinons. C.S. breaks time reversal symmetry

Quantum effects: Renormalization of diamagnetic susceptibility in A^T effective action

$$\chi \to \chi(H) = \chi + \frac{\sigma_h^2}{4\pi^2\gamma}$$

Holon contribution R_h

(Ioffe, Kotliar) and (Ioffe, Wiegmann)

$$R_h = R_h^0[1 + (\frac{(1-\varepsilon)H\tau}{m_h})^2],$$

$$R_h^0 \propto \frac{m_h}{\tau} \propto \delta[\frac{1}{\varepsilon_F \tau_{imp}} + (\frac{T}{\varepsilon_F})^{4/3}]$$

τ transport relaxation time, τ impurity scattering time

Spinon contribution R_s

Renormalization of the spinon mass

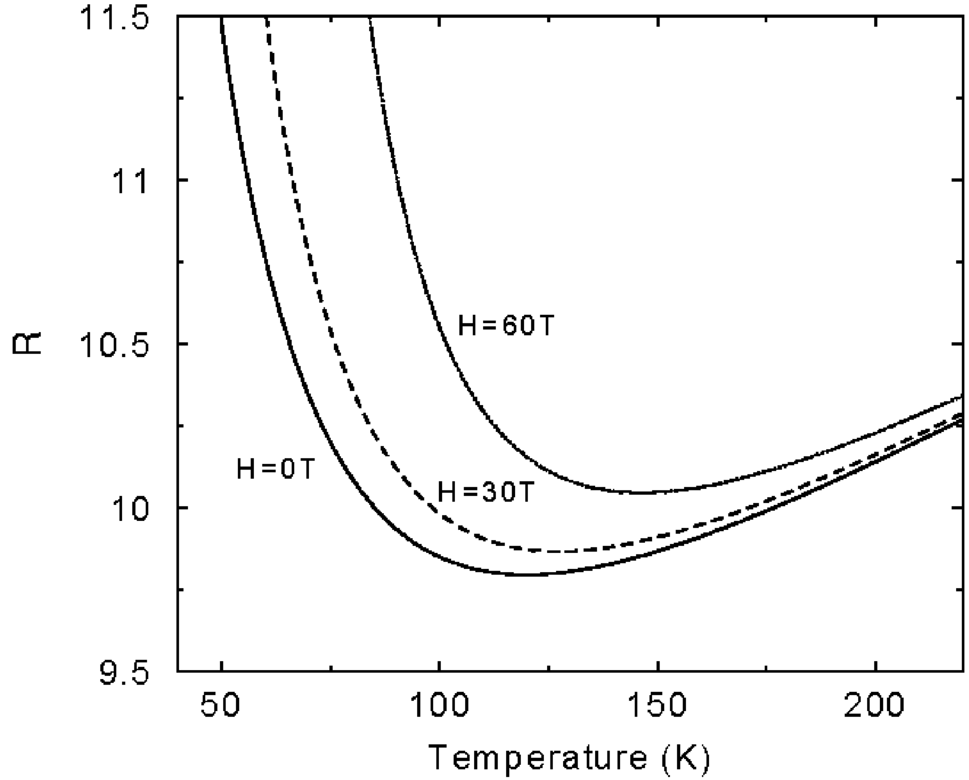
$$m_s^2(T,H) = m_s^2 - i\left[\frac{cT}{\chi(H)} - \frac{\varepsilon^2 H^2}{3q_0^2}\right], \qquad q_0 \propto \left(\frac{\delta^2 T}{t}\right)^{1/3}$$

$$R_s \propto Z(T,H)\frac{|m_s(T,H)|^{1/4}}{\sin(\theta(T,H)/4)},$$

$$m_s(T,H) = |m_s(T,H)|\, e^{i\theta(T,H)}, \quad Z(T,H) = \frac{c'T}{\chi(H) q_0^3} - \frac{2\varepsilon^2 H^2}{3q_0^5}$$

M-I crossover from M to I survives, $T_{M\text{-}I}(\delta, H)$ is decreasing with δ, increasing with H.

Magnetic Field Dependence of M-I Crossover



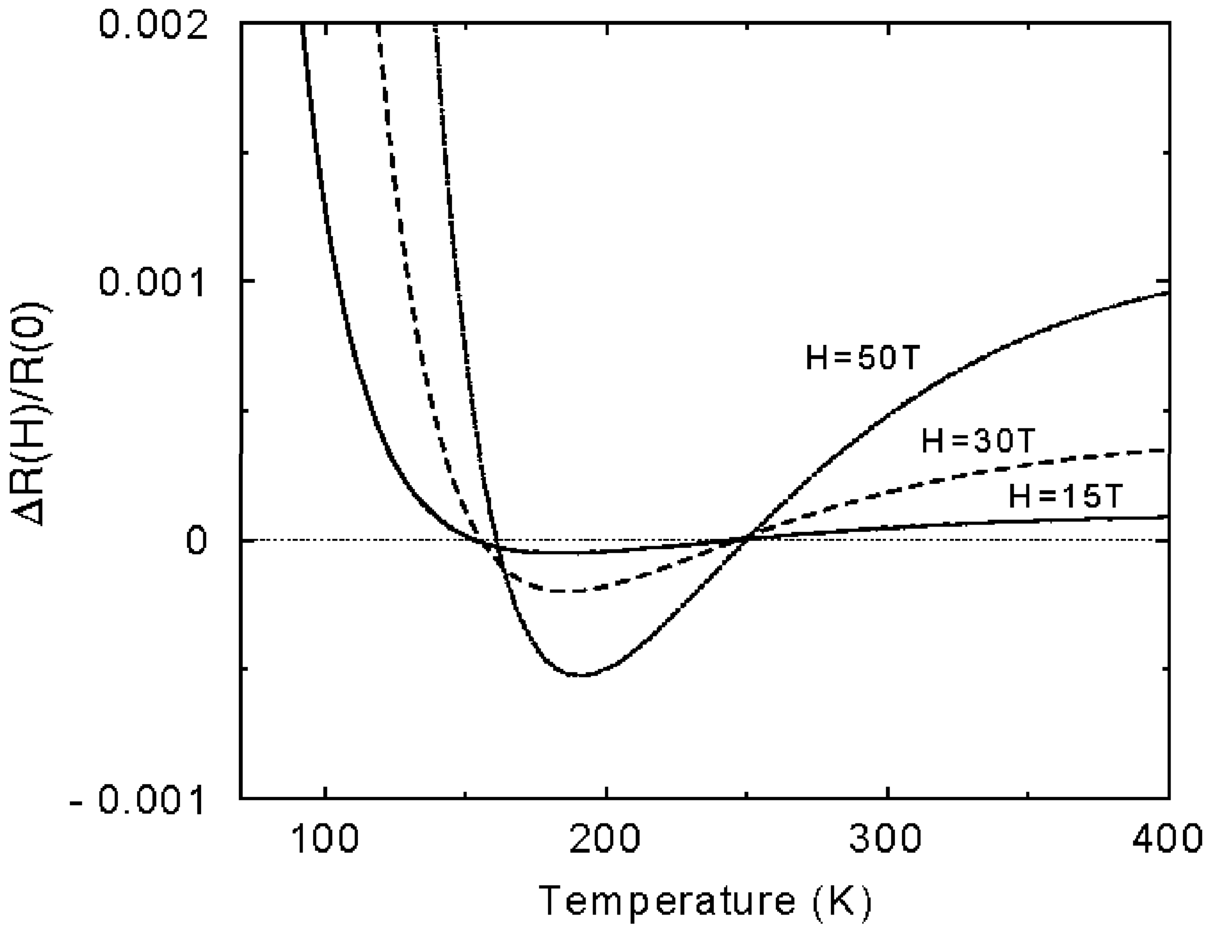
Diamagnetic susceptibility χ increases with field H, de Broglie wave length $\lambda^2 \propto \chi/T$ increases, while ξ remaining unchanged. M-I crossovertemperature goes up with field H.

Magnetoresistance $\Delta R = R(H) - R(0)$

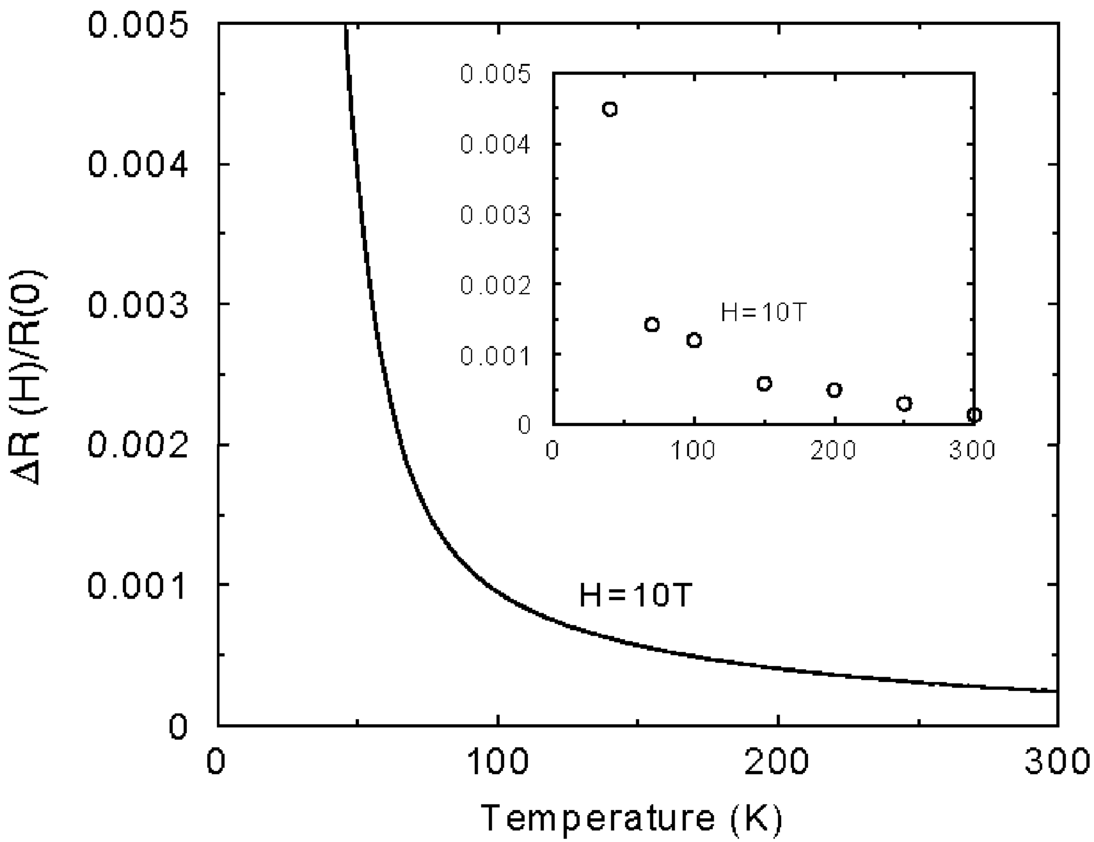
- Shift of $T_{M\text{-}I}$ leads to a large MR at low temperature, in agreement with experiments.

- Shift of χ induced by C.S. term and H^2 term due to minimal coupling reduce dissipation. In region when dissipation dominates, it leads to negative MR.

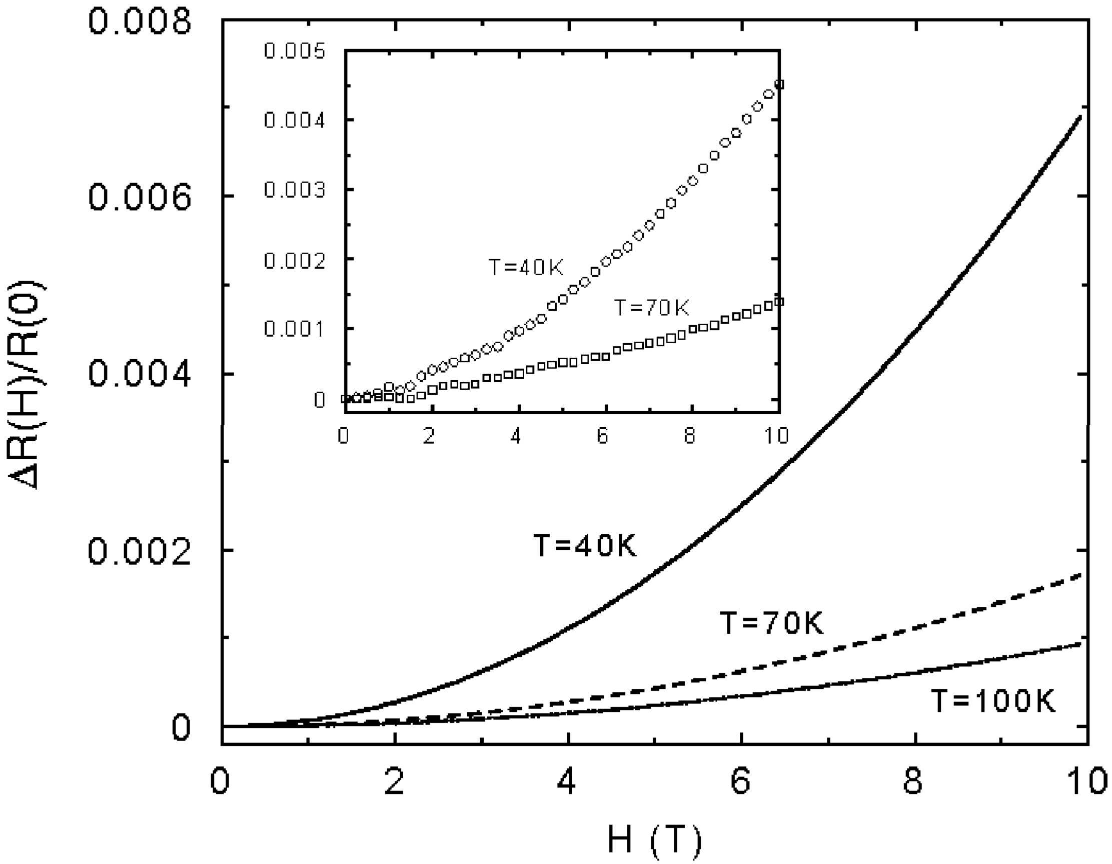
- Two possible MR curves: all positive, with a knee near $T_{M\text{-}I}$, or a negative region around $T_{M\text{-}I}$.



Calculated MR curves when quantum effects are strong. $\delta = 0.05$.



Calculated temperature dependence of MR for δ =0.075, compared with data on $La_{1.925}Sr_{0.075}CuO_{4+\varepsilon}$ from A. Lacerda et al. Phys. Rev. B **49**, 9097 (1994)



Calculated field dependence of MR for δ =0.075, compared with data on $La_{1.925}Sr_{0.075}CuO_{4+\varepsilon}$ from A. Lacerda et al. Phys. Rev. B **49**, 9097 (1994)

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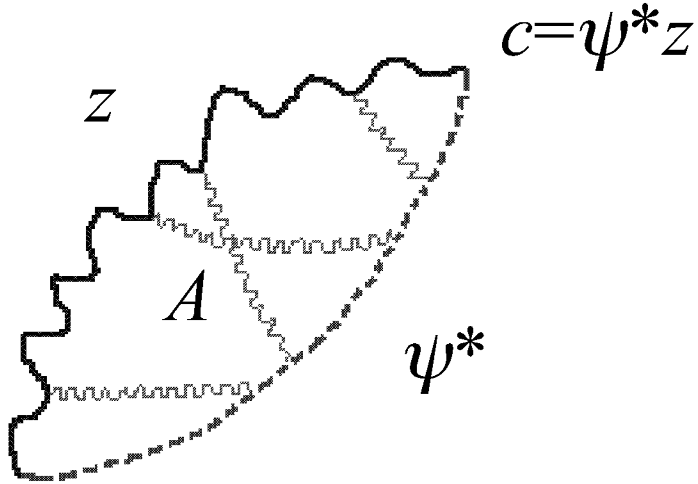
Calculation of electron Green's function and observables: ARPES and *c*-axis resistivity

$$G(x,y) = \langle \mathrm{T}(c(x), c^{+}(y)) \rangle$$

Electron field: $c(x)=\psi^{*}(x)z(x)$



$c=\psi^{*}z$

$$G(x,y) \propto \langle\, \langle \mathrm{T}(\psi^{*}(x)\psi(y)) \,\rangle \langle \mathrm{T}(z^{*}(y)z(x)) \,\rangle\, \rangle_{A}$$

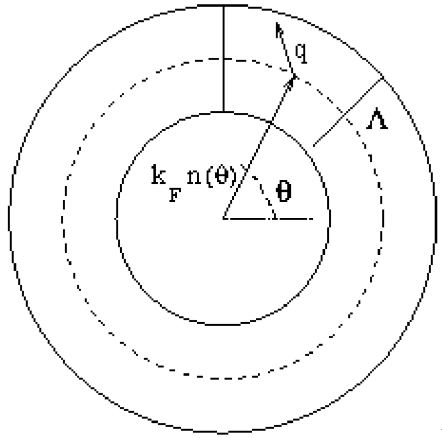
$$G^{e}_{\alpha}(x,y) = \frac{\int DA_{\mu} G^{h}(x,y|-A) G^{s}(x,y|A) e^{iS_{eff}(A)}}{\int DA_{\mu} e^{iS_{eff}(A)}}$$

Spinon propagator

$$G^s(x_0, |\vec{x}|) \sim \frac{1}{\sqrt{x_0^2 - |\vec{x}|^2}} e^{-i\sqrt{m_s^2 - \frac{T}{\chi} f(\alpha)}\sqrt{x_0^2 - |\vec{x}|^2}} e^{-\frac{T}{4\chi} q_0^2 g(\alpha) \frac{x_0^2 - |\vec{x}|^2}{m_s^2}}$$

Holon propagator: Finite Fermi Surface

Choose in K space a shell of thickness $\Lambda \ll k_F$, and decompose into quasi-1D systems with zero density



$$\boldsymbol{k} = k_F \boldsymbol{n}(\vartheta) + \boldsymbol{q} \qquad |\boldsymbol{q}| < \Lambda << k_F$$

$$\int d\boldsymbol{k} \quad \rightarrow \quad \int d\vartheta \int_\Lambda d\boldsymbol{q}$$

Electron Green's function

$$G(\omega, k_F\mathbf{n}(\vartheta) + \mathbf{q}) = B(\vartheta)\frac{Z}{\omega - \Gamma - \Delta\mathbf{q}\cdot\mathbf{n}(\vartheta)}$$

Z $\longrightarrow$ wave function renormalization constant

$\Re\Gamma$ $\longrightarrow$ renormalization of the chemical potential

$\Im\Gamma$ $\longrightarrow$ inverse lifetime for the "electron" on the FS

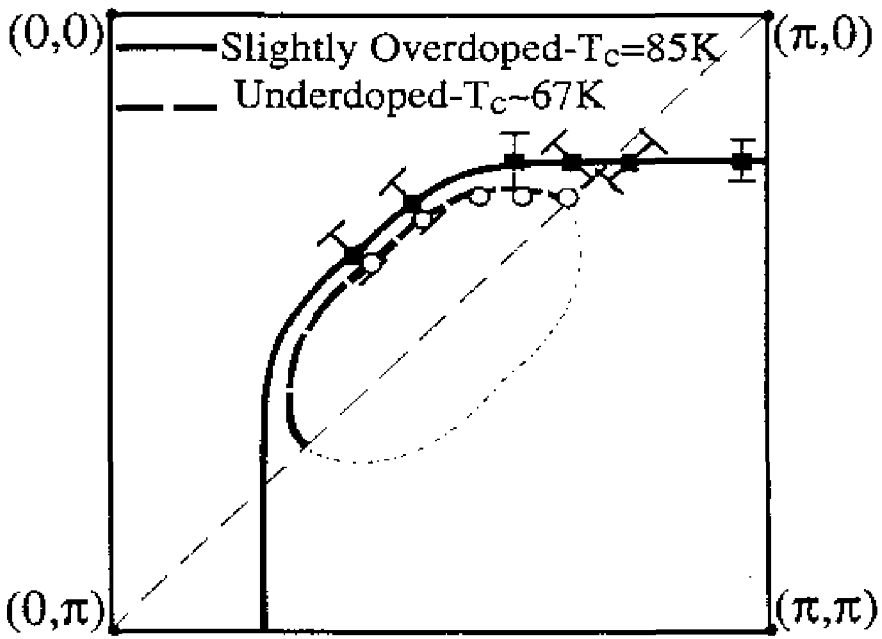
$B(\theta)$ $\longrightarrow$ "angular" modulation of the Green's function

Dependence on T:

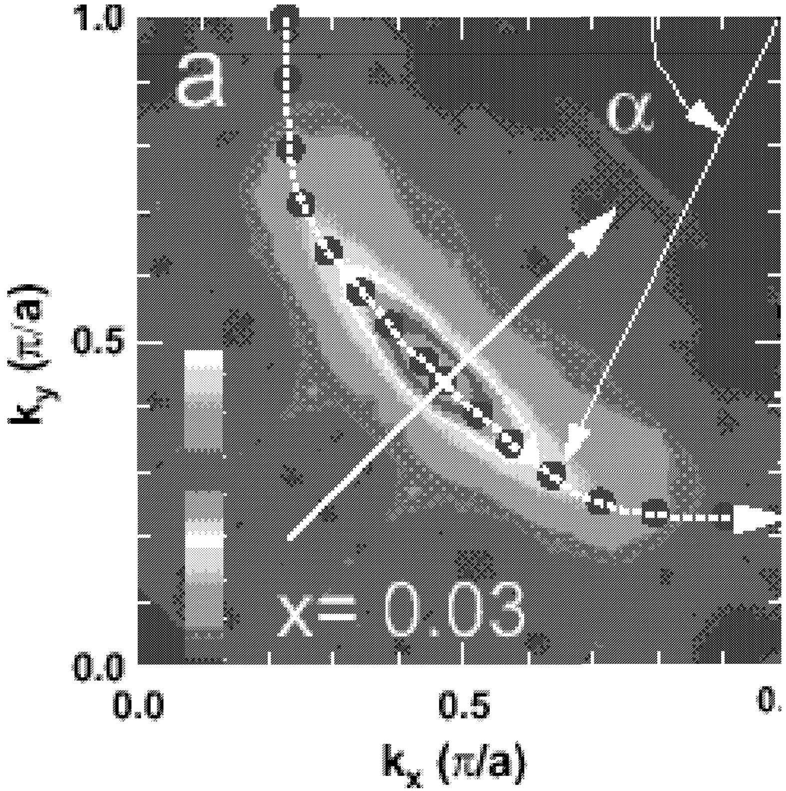
- $\Im\Gamma \sim T$, insulating phase
- $\Im\Gamma \sim T^{1/2}$, metallic phase

Temperature dependence of WF renormalization $\boldsymbol{Z} \sim T^{1/6}$

Fermi Surface

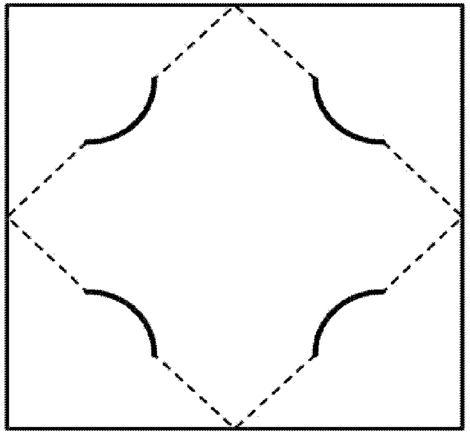


Early data on BSCO. D.S. Marshall et al. Phys. Rev. Lett. **76**, 4841 (1996)



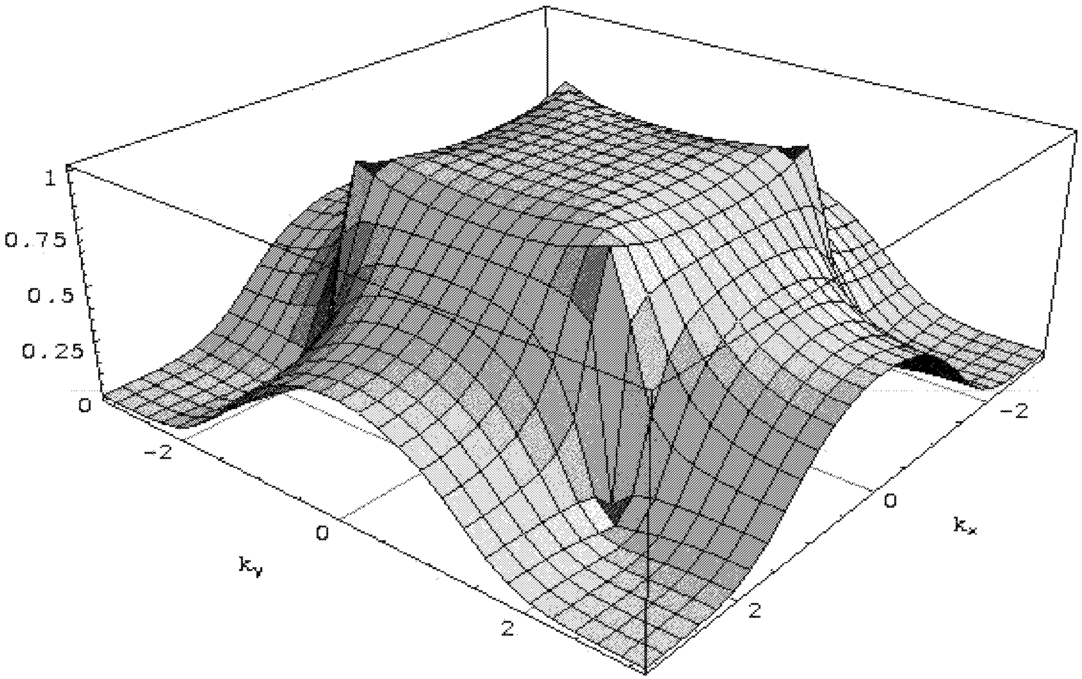
Fermi “arc” in 3% doped LSCO
T. Yoshida et al. Cond-mat/ 0206469

Calculated Fermi Surface



ARPES intensity $\sim \mathrm{Im}\, G(\boldsymbol{k},\omega) f(\omega)$

Fermi arc



Renormalization due to Dirac structure $B(\vartheta)$

$$B(\boldsymbol{k}) = \frac{1}{2}\left(1 + \frac{\cos k_x \cos}{\sqrt{2}\,\sqrt{\cos^2 \cos}}\right)$$

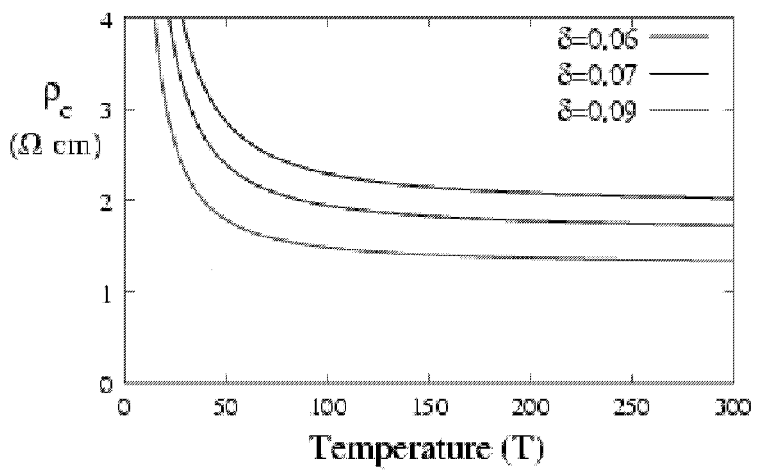
C-Axis Resisticity

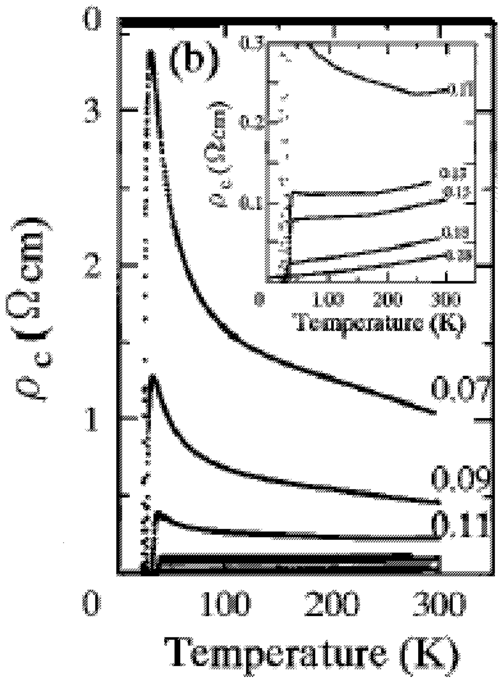
Kumar et al. Interlayer tunneling blocked by in-plane scattering

$$G^R(\omega,k) = \frac{Z}{\omega - \xi_k + i\Gamma}$$

$$\rho_c = \frac{const}{\nu(\varepsilon_F)}\left(\frac{1}{\Gamma} + \frac{\Gamma}{t_c^2 Z^2}\right)$$

Theory anticipates a kink from T^{-1} at low T to $T^{-1/2}$ at high T

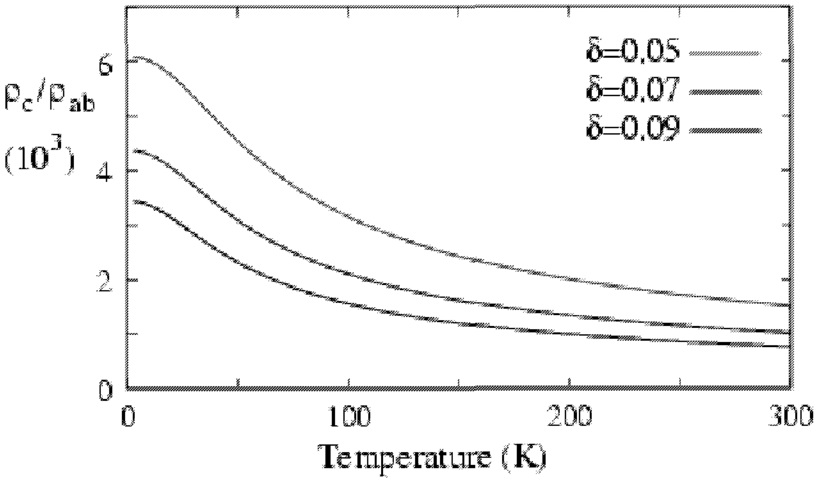


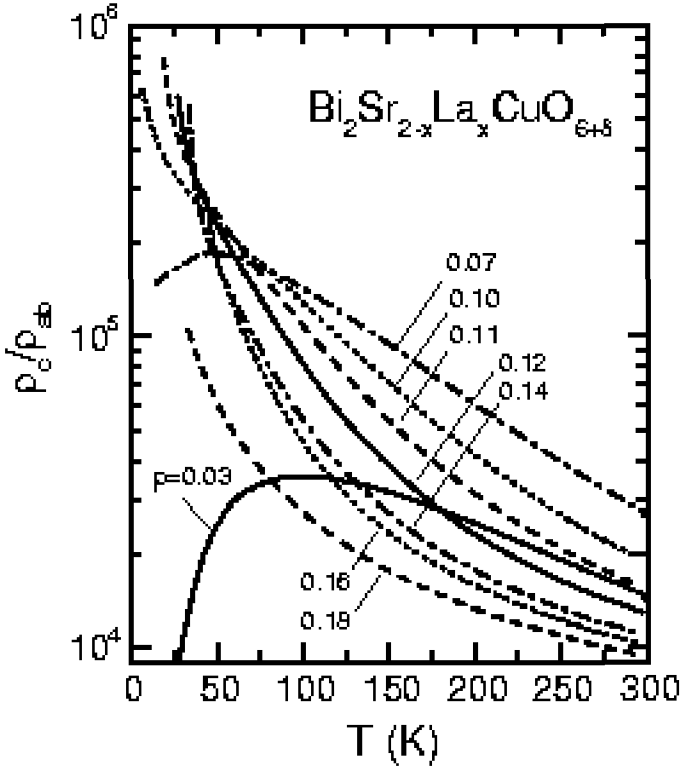


Kink in experiments. Kimura et al.'96

Anisotropy Ratio ρ_c / ρ_{ab}

Theoretical curve





Ono & Ando, cond-mat/ 0205305

Concluding remarks

- Gauge field approach can provide a consistent explanation for M-I crossover in and w/o magnetic field
- It can also explain a number of other experiments: NMR relaxation, ARPES, anisotropy ratio, etc.
- To be generalized to higher doping and SC state

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