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"Dissipation in solid state qubits"

presented by:

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Dissipation in Solid State Qubits

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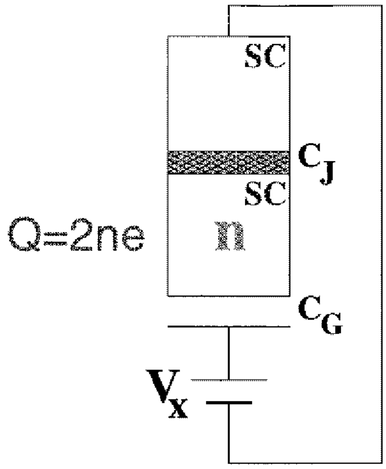
- **Possible sources of dissipation**
- **Models of environment**
- **Bloch-Redfield equations**
- **Ohmic environment**
- **Sub-Ohmic environment (1/f noise)**
- **Nonlinear coupling**
- **Dephasing vs. renormalizarion in nonadiabatic manipulations**

Josephson Charge Qubits

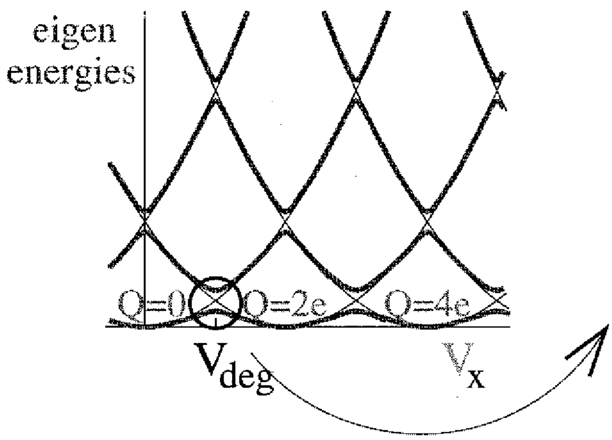


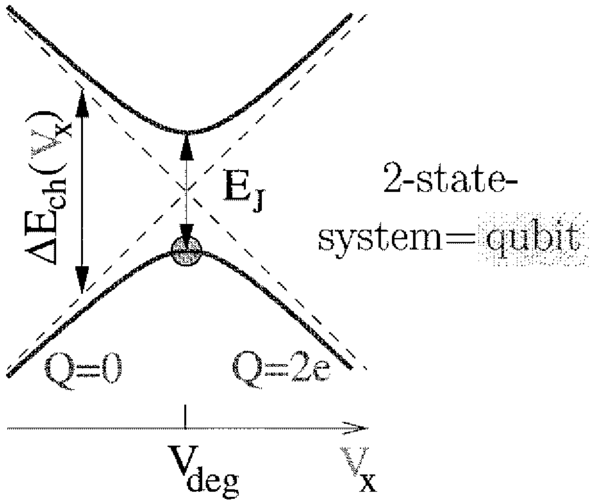
- charging energy (Cooper-pairs) $Q_g = C_g V_x$

$$E_{ch}(n, V_x) = \frac{(2ne - Q_g)^2}{2(C_g + C_J)}$$

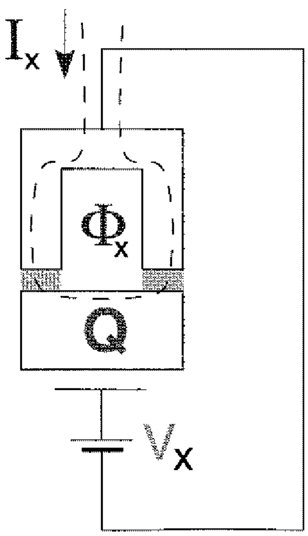
- Josephson coupling

$$E_J \cos\varphi \quad \rightarrow \quad \frac{E_J}{2} \, |n\rangle \, \langle n \pm 1|$$





$$\mathcal{H} = \quad \Delta E_{ch}(V_x) \, \hat{\sigma}_z + E_J \, \hat{\sigma}_x$$



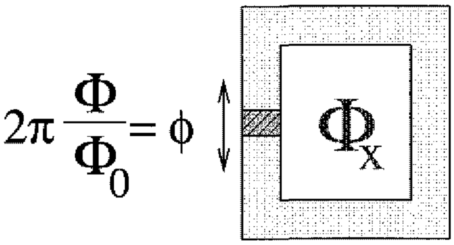
Flux-controlled qubit

controlled Josephson coupling

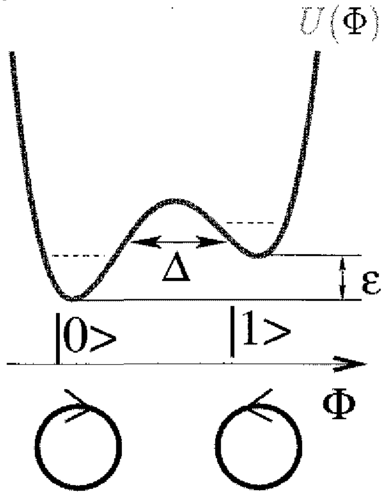
$$\mathcal{H} = \quad \Delta E_{ch}(V_x) \, \hat{\sigma}_z + E_J(\Phi_x) \, \hat{\sigma}_x$$

Josephson Flux Qubits

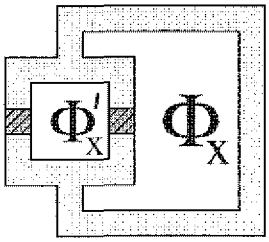
- SQUID type devices



$$\mathcal{H} = \frac{Q^2}{2C} - E_{\mathrm{J}} \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{(\Phi - \Phi_x)^2}{2L}$$

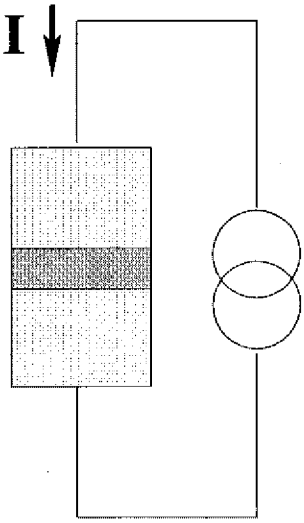


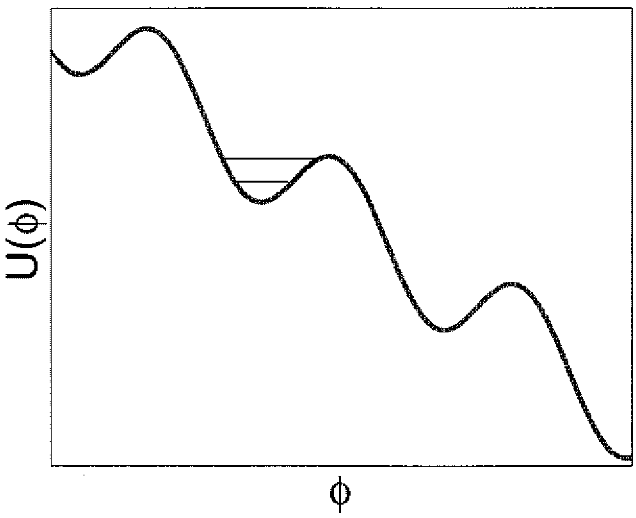
$$\mathcal{H} \approx \varepsilon(\Phi_{\mathrm{x}})\,\hat{\sigma}_z + \Delta\,\hat{\sigma}_x$$



$\Delta = \Delta(\Phi'_{\mathrm{x}})$ - controlled

- Current biased Josephson junctions

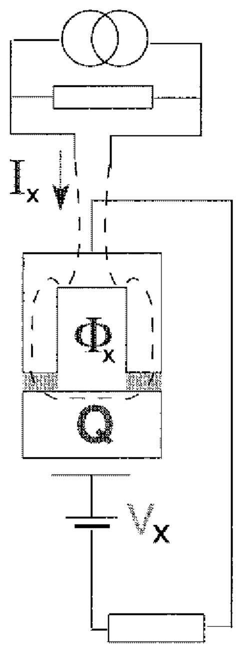




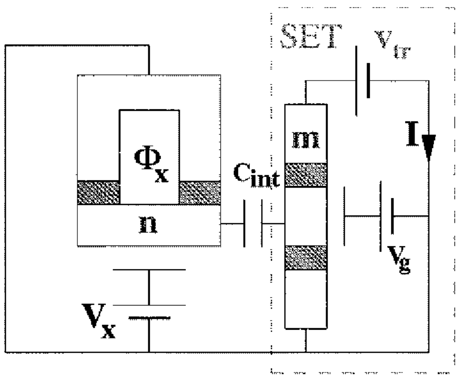
$$U(\Phi) = -E_{\mathrm{J}} \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) - I\Phi$$

Sources of Dissipation

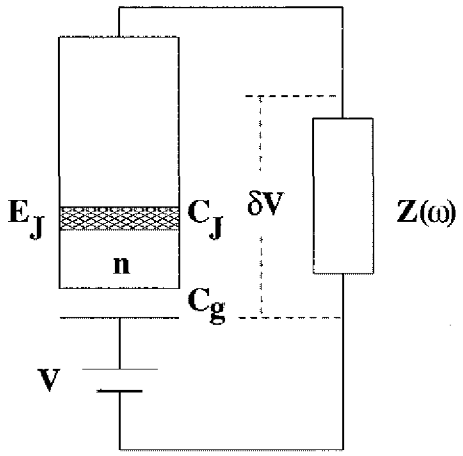
- Intrinsic sourses:
  - background charges & fluxes - very serious
  - quasiparticles - frozen
  - electromagnetic radiation - shielding
  - nuclear spins - almost static
  - etc.



- External (artificial) sources:
  - control (manipulations) circuitry
  - masuring devices - on-off



Charge qubit: spin-boson model



- Dissipationless Hamiltonian

$$H_0 = \frac{(2en - C_{\rm g}V)^2}{2(C_{\rm J}+C_{\rm g})} - E_{\rm J}\cos\Theta$$

- Include dissipation

$$V \to V + \delta V$$

- From FDT follows

$$\langle \delta V \delta V \rangle_\omega = {\rm Re}\{Z_{\rm t}(\omega)\}\hbar\omega \coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)$$

$$Z_{\rm t}(\omega) \equiv \left[i\omega C_{\rm t} + Z^{-1}(\omega)\right]^{-1}\ ,\quad C_{\rm t}^{-1} = C_{\rm J}^{-1}+C_{\rm g}^{-1}$$

- Model with dissipative bath (Caldeira & Leggett (1983))

$$H = H_0 - 2en\frac{C_{\rm t}}{C_{\rm J}}\sum_a \lambda_a x_a + \sum_a\left[\frac{p_a^2}{2m_a} + \frac{m_a\omega_a^2}{2}x_a^2\right] + \left(2en\frac{C_{\rm t}}{C_{\rm J}}\right)^2\sum_a\frac{\lambda_a^2}{2m_a\omega_a^2}$$

$$\delta V = \sum_a \lambda_a x_a\ ,\qquad J(\omega) \equiv \frac{\pi}{2}\sum_a\frac{\lambda_a^2}{m_a\omega_a}\delta(\omega-\omega_a) = \omega{\rm Re}\{Z_{\rm t}(\omega)\}$$

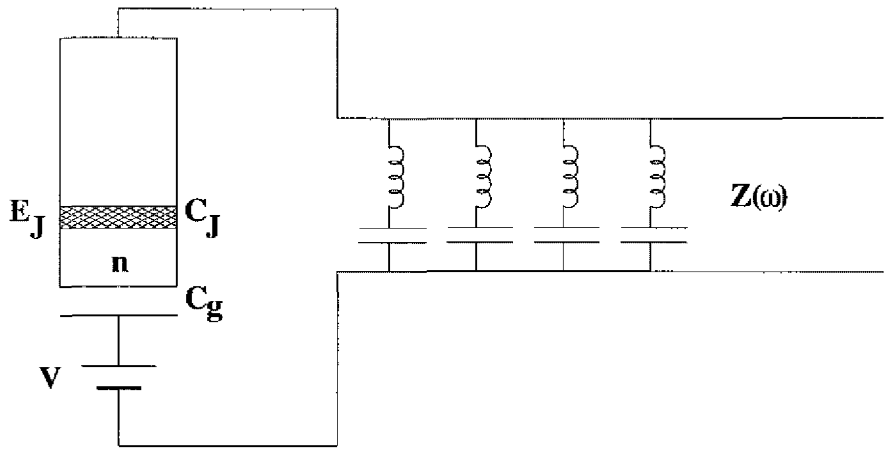
- Reduction to a two-level system, spin-boson model (Leggett et al. (1987), Weiss (1993))

$$n = 0,1\ ,\quad n = \frac{1}{2}(1+\sigma_z)\ ,\quad \cos\Theta = \frac{1}{2}\sigma_x$$

$$H = -\frac{1}{2}B_z(V)\,\sigma_z - \frac{1}{2}B_x\,\sigma_x + \sigma_z\sum_a\lambda_a x_a + H_{\rm bath}$$

$$J \leftarrow e^2\left(\frac{C_{\rm t}}{C_{\rm J}}\right)^2 J = \frac{\pi}{2}\alpha\hbar\omega\ ,\quad \alpha = \frac{4{\rm Re}\{Z_{\rm t}(\omega)\}}{R_{\rm K}}\left(\frac{C_{\rm t}}{C_{\rm J}}\right)^2 \approx 10^{-6}$$

Charge qubit: spin-boson model, alternative derivation



- Model any $Z(\omega)$

$$Z^{-1}(\omega) = \sum_a \left(i\omega L_a + \frac{1}{i\omega C_a} + \delta \right)^{-1} = \sum_\alpha \frac{i\omega}{L_a(\omega_\alpha^2 - \omega^2 + i\omega\tilde{\delta})}$$

- Derive Hamiltonian starting with Lagrangian of the whole system

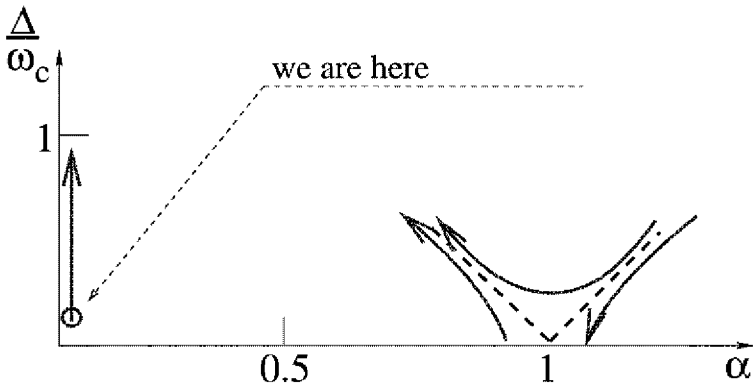
$$H = \frac{(2en - q)^2}{2C_\mathrm{J}} - E_\mathrm{J} \cos\Theta + \frac{q^2}{2C_\mathrm{g}} + H_Z(\phi + \textstyle\int dtV)$$

$$H_Z(\phi) \equiv \sum_a \left[\frac{q_a^2}{2C_a} + \frac{(\phi_a + \phi)^2}{2L_a} \right]$$

equivalent to the Caldeira-Leggett's Hamiltonain at $\omega \ll 1/RC_\mathrm{t}$

- Important points
  - weak dissipation $\alpha = \frac{4\mathrm{Re}\{Z_\mathrm{t}(\omega)\}}{R_\mathrm{K}} \left(\frac{C_\mathrm{t}}{C_\mathrm{J}}\right)^2 \approx 10^{-6}$
  - natural UV cut-off: $\mathrm{Re}Z(\omega) = \frac{R}{1+\omega^2 C_\mathrm{t}^2 R^2}$, $\quad \omega_c \approx 1/(RC_\mathrm{t})$

Bloch equations



F. Bloch (1946):

$$\frac{d}{dt}\vec{M} = \vec{B} \times \vec{M} - \frac{1}{T_1}(M_z\vec{z} - \vec{M}_0) - \frac{1}{T_2}(M_x\vec{x} + M_y\vec{y})$$

For 2-level system (spin 1/2)
$M_z = \langle \sigma_z \rangle = \rho_{11} - \rho_{22}$
$M_{x/y} = \langle \sigma_{x/y} \rangle = \mathrm{Re}/\mathrm{Im}\rho_{21}$

For $\vec{B} = B_z\vec{z}$ and $\vec{M}_0 || \vec{z}$

$$\dot{\rho}_{11} = -\Gamma_\uparrow \rho_{11} + \Gamma_\downarrow \rho_{22}$$

$$\dot{\rho}_{22} = \Gamma_\uparrow \rho_{11} - \Gamma_\downarrow \rho_{22}$$

$$\dot{\rho}_{12} = -iB_z\rho_{12} - \frac{1}{T_2}\rho_{12}$$

$\frac{1}{T_1} = \Gamma_\downarrow + \Gamma_\uparrow$ and $|M_0| = T_1(\Gamma_\downarrow - \Gamma_\uparrow)$

Microscopic derivation and generalizations:
F. Bloch (1957), A.G. Redfield (1957), ...

Relaxation and dephasing
$\Gamma_{\mathrm{rel}} \equiv T_1^{-1} \quad \Gamma_\varphi \equiv T_2^{-1}$

Golden Rule

$$H = -\tfrac{1}{2}B_z\,\sigma_z - \tfrac{1}{2}B_x\,\sigma_x + \sigma_z X + H_{\rm bath}\ , \qquad X \equiv \Sigma_a\,\lambda_a x_a$$

Rotation to eigenbasis

$$H = -\tfrac{1}{2}\Delta E\,\sigma_z + (\cos\eta\,\sigma_z - \sin\eta\,\sigma_x)\,X + H_{\rm bath}$$

$\uparrow$ $\uparrow$

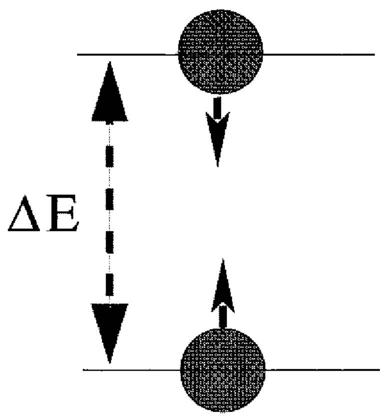
longitudinal and transverse coupling to bath

$$\Delta E \equiv \sqrt{B_z^2 + B_x^2} \qquad \tan\eta = B_x/B_z$$

$$\Gamma_\downarrow = \hbar^{-2}\ \sin^2\eta\ \langle X_\omega^2\rangle_{\omega=\Delta E/\hbar}$$

$$\Gamma_\uparrow = \hbar^{-2}\ \sin^2\eta\ \langle X_\omega^2\rangle_{\omega=-\Delta E/\hbar}$$

$$\langle X_\omega^2\rangle \equiv \int dt\, e^{i\omega t}\langle X(t)X(0)\rangle$$



$$\frac{1}{T_1} = \hbar^{-2}\ \sin^2\eta\ S_X(\omega = \Delta E/\hbar)$$

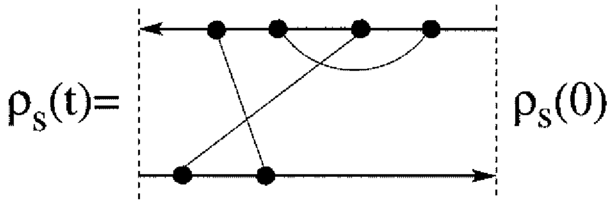
$$S_X(\omega) \equiv \langle X_\omega^2\rangle + \langle X_{-\omega}^2\rangle = 2\hbar J(\omega)\coth\frac{\hbar\omega}{2k_{\rm B}T}$$

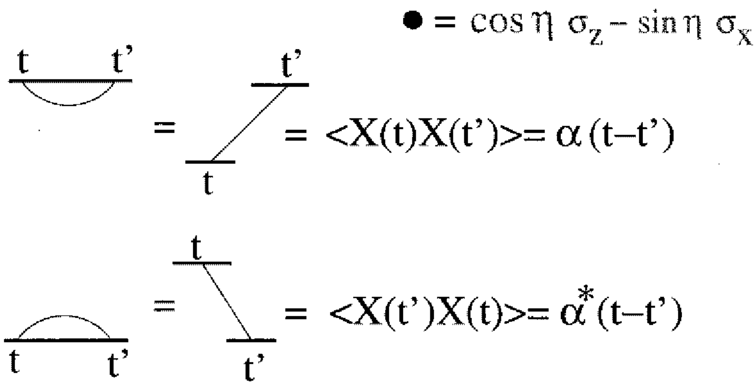
Diagrammatic technique

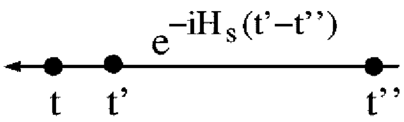
O.V. Konstantinov & V.I. Perel (1960), L.V. Keldysh (1965)
H. Schoeller & G. Schön (1994)

$$H = H_\mathrm{s} + H_\mathrm{bath} + H_\mathrm{int}\ ; \quad H_\mathrm{int} = (\cos\eta\ \sigma_z - \sin\eta\ \sigma_x)\, X = s\ X$$

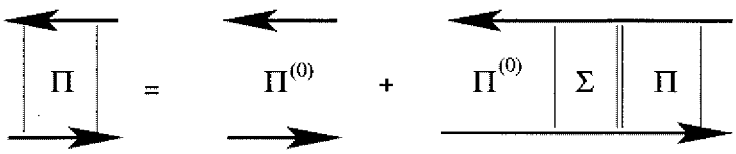
$$\rho(t) = Te^{-i\int_0^t H_\mathrm{int}(t)dt}\, \rho(0)\ \tilde{T}e^{i\int_0^t H_\mathrm{int}(t)dt}$$

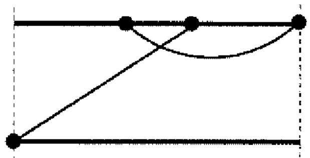






Propagator Π: $\rho_\mathrm{s}(t) = \Pi(t,0)\ \rho_\mathrm{s}(0)$.





Self−energy Σ

$$\frac{d}{dt}\rho_\mathrm{s}(t) = \frac{i}{\hbar}\left[\rho_\mathrm{s}(t), H_\mathrm{s}\right] + \int_0^t dt'\ \Sigma(t-t')\ \rho_\mathrm{s}(t')$$

Bloch-Redfield approximation

- Kinetic (master) equation

$$\frac{d}{dt}\rho_{\rm s}(t) = L_0 \rho_{\rm s}(t) + \int_0^t dt' \; \Sigma(t-t') \; \rho_{\rm s}(t') \; , \quad L_0 \equiv [1 \otimes iH_{\rm s} - iH_{\rm s} \otimes 1]$$

- Bloch-Redfield approximation - Markov approximation in rotating frame

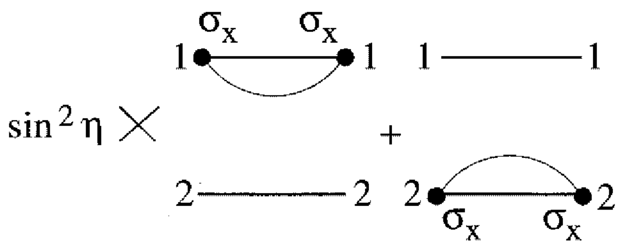
$$\rho_{\rm s}(t') \approx e^{-L_0(t-t')} \; \rho_{\rm s}(t)$$

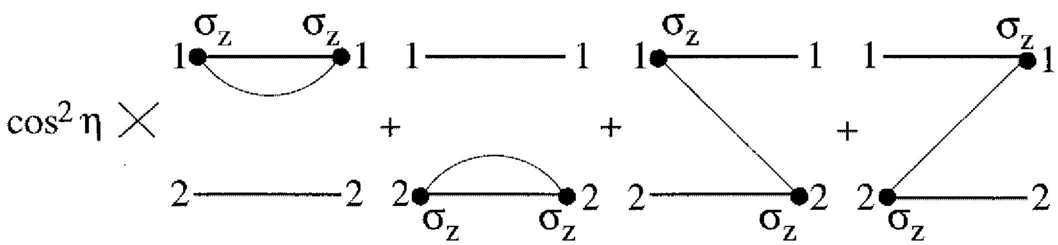
$$\frac{d}{dt}\rho_{\rm s}(t) = L_0 \rho_{\rm s}(t) + \hat{\Gamma} \; \rho_{\rm s}(t')$$

$$\hat{\Gamma} \equiv \int_0^\infty dt \; \Sigma(t) \; e^{-L_0 t} = \Sigma(s = L_0 + i\delta)$$

- RWA

consider only $\Gamma_{nn,mm}$ and $\Gamma_{nm,nm}$ (for example $\Gamma_{12,12}$)





$$\Gamma_{12,12} = -\frac{1}{2} \; \sin^2\eta \; S_X(\omega = \Delta E) - \cos^2\eta \; S_X(\omega = 0) + i\delta E(\omega_c)$$

$$\frac{1}{T_2} = \frac{1}{2}\,\frac{1}{T_1} + \cos^2\eta \; S_X(\omega = 0)$$

Decay of Rabi oscillations

Purely longitudinal coupling

$$H = -\frac{1}{2}B_z\,\sigma_z + \sigma_z X + H_{\rm bath}$$

$$\boxed{T_1^{-1} = 0} \qquad \boxed{T_2^{-1} = S_X(\omega \approx 0)}$$

Apply rotating field

$$H = -\frac{1}{2}B_z\,\sigma_z - \frac{1}{2}\Omega_{\rm R}\,(\cos\omega t\,\sigma_x + \sin\omega t\,\sigma_y) + \sigma_z X + H_{\rm bath}$$

In the rotating frame $\tilde{H} = \dot{U}U + UHU^\dagger$ $\quad U = \exp\left(-i\omega\frac{\sigma_z}{2}\,t\right)$

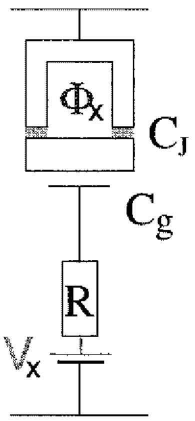
And at resonance $\omega = B_z$

$$H = -\frac{1}{2}\Omega_{\rm R}\,\sigma_x + \sigma_z\,X + H_{\rm bath}$$

$$\boxed{T_1^{-1} = S_X(\omega = \Omega_{\rm R})} \qquad \boxed{T_2^{-1} = (1/2)T_1^{-1}}$$

In general noise at $\omega = 0, B_z$, $\omega = B_z \pm \Omega_{\rm R}$, and $\omega = \Omega_{\rm R}$ may be involved

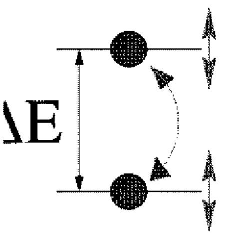
Dissipation in Ohmic Control Circuit



- gate voltage circuit is dissipative
  - characterized by $Z(\omega) = R$
  - induces voltage fluctuations

- model $$\mathcal{H}_{\text{diss}} = \frac{C_g}{C_J}\; e\,\delta V(t)\;\hat{\sigma}_z + \mathcal{H}_{\text{bath}}$$

$$\left\langle \delta V_\omega^2 \right\rangle = \hbar\omega R \coth\left(\frac{\hbar\omega}{2k_B T}\right)$$



incoherent transitions between eigenstates **and** fluctuation of eigenenergies

$$\tan\eta = \frac{E_J(\Phi_x)}{\Delta E_{ch}(V_x)}$$

$$T_1^{-1} = 4\pi \frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \sin^2\eta\;\frac{\Delta E}{\hbar}\;\coth\left(\frac{\Delta E}{2k_B T}\right)$$

$$T_2^{-1} = \frac{1}{2}T_1^{-1} + 4\pi\frac{R}{h/e^2}\left(\frac{C_g}{C_J}\right)^2 \cos^2\eta\;\frac{k_B T}{\hbar}$$

- choose: $R \ll h/e^2 \approx 26\text{k}\Omega$ e.g. $R = 100\Omega$
  $C_g \ll C_J$ (weak coupling to environment)

- $T_1, T_2 \approx 10^{-6} - 10^{-4}$s
- operation time $\tau_{op} \approx \hbar/E_J \approx 10^{-10}$s

Regime dominated by fluctuations, Zeno effect

Transverse coupling, $B_z \ll T$

$H = -\frac{1}{2}B_x\,\sigma_x + \sigma_z X + H_{\text{bath}}\ ,\qquad X \equiv \Sigma_a\,\lambda_a x_a\ ,\qquad S_X(\omega \ll T) \propto \alpha T$

In the eigenbasis of σ_x:

$$\frac{d}{dt}\begin{pmatrix}\rho_{11}\\ \rho_{22}\\ \rho_{12}\\ \rho_{21}\end{pmatrix} = \begin{pmatrix} -\alpha T & \alpha T & 0 & 0 \\ \alpha T & -\alpha T & 0 & 0 \\ 0 & 0 & -iB_x - \alpha T/2 & \alpha T/2 \\ 0 & 0 & \alpha T/2 & iB_x - \alpha T/2 \end{pmatrix}\begin{pmatrix}\rho_{11}\\ \rho_{22}\\ \rho_{12}\\ \rho_{21}\end{pmatrix}$$

If $\alpha T \gg B_x$ then Zeno effect (Harris & Stodolsky 1982)
"Motional narrowing" in NMR

$$\langle\sigma_z\rangle \propto \exp\left(-\frac{B_x^2}{\alpha T}\,t\right)$$

Compare longitudinal ("pure") dephasing rate $\Gamma_\varphi^* \equiv S_X(\omega = 0)$ and B_x

$B_z \gg \Gamma_\varphi^*$ - coherent (Hamiltonian dominated) regime
Natural description in eigenbasis (σ_x)

$B_z \ll \Gamma_\varphi^*$ - incoherent (dominated by fluctuations) regime
Natural description in the basis "observed" by the bath (σ_z)

Sub-Ohmic environment, $1/f$ noise

$H = -\frac{1}{2}\Delta E\, \sigma_z + (\cos\eta\, \sigma_z - \sin\eta\, \sigma_x)\, X + H_{\rm bath}$

• Sub-Ohmic spin-boson model

$$J(\omega) = \frac{\pi}{2}\alpha\hbar\omega_0^{1-s}\omega^s\ , \quad s < 1$$

$$S_X(\omega) = 2\hbar J(\omega)\coth\frac{\hbar\omega}{2k_{\rm B}T} \propto T\omega^{s-1} \rightarrow |_{\omega\to 0}\ \infty$$

• For $s = 0$ - 1/f noise at $\hbar\omega \ll k_{\rm B}T$

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}\ , \quad E_{1/f} \propto \sqrt{\omega_0 T}$$

T does not have to be the real temperature

• Longitudinal ($\eta = 0$) 1/f noise, classical treatment

$$\langle\sigma_+(t)\rangle \propto \langle e^{2i\int\limits_0^t dt'\, X(t')}\rangle = e^{-2\int\limits_0^t dt' \int\limits_0^t dt'' \langle X(t')X(t'')\rangle} = e^{-\int\frac{d\omega}{2\pi}S_X(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}}$$

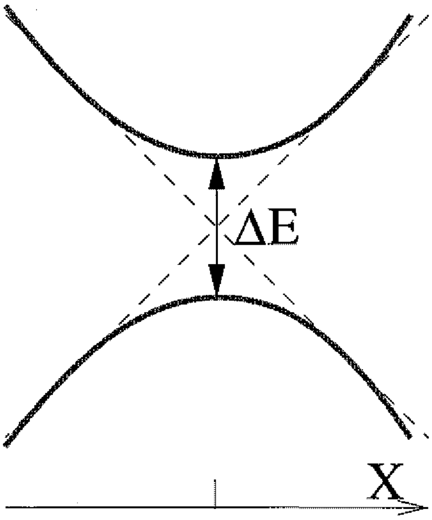
$$\langle\sigma_+(t)\rangle \propto \exp(-E_{1/f}^2 t^2 \ln t\omega_{\rm ir})\ , \quad T_2^{-1} \approx E_{1/f} \approx 10{\rm ns}$$

• Self-consistent method

$$\Gamma_\varphi = \Sigma(s = iL_0 + \Gamma_\varphi)\ \ \rightarrow\ \ \Gamma_\varphi = E_{1/f}^2/\Gamma_\varphi$$

• Non-Gaussian 1/f noise
See E. Paladino et al. (2002),
Gaussian approximation overestimates dephasing

$1/f$ noise, transverse coupling



To protect from $1/f$ noise - $\eta = \pi/2$

$$H = -\frac{1}{2}\Delta E\ \sigma_z - \sigma_x X + H_{\text{bath}}$$

Golden Rule (1-st order) for $E_{1/f} \ll \Delta E$

$$T_1^{-1} = S_X(\Delta E) = \frac{E_{1/f}^2}{\Delta E} \qquad T_2^{-1} = \frac{T_1^{-1}}{2}$$

Second order must be considered

At low frequencies (adiabatic approx.) $H = -\frac{1}{2}\Delta E(X)\ \sigma_z + H_{\text{bath}}$

$$\Delta E(X) = \sqrt{\Delta E^2 + 4X^2} \approx \Delta E + \frac{2X^2}{\Delta E}$$

$$S_{X^2}(\omega) = 2\int \frac{d\nu}{2\pi}\left\{\langle X^2_{\nu+\omega}\rangle\langle X^2_{-\omega}\rangle + \langle X^2_{\nu-\omega}\rangle\langle X^2_{-\omega}\rangle\right\} \sim \frac{E^4_{1/f}}{\omega}\ln\frac{\omega}{\omega_{\text{ir}}}$$

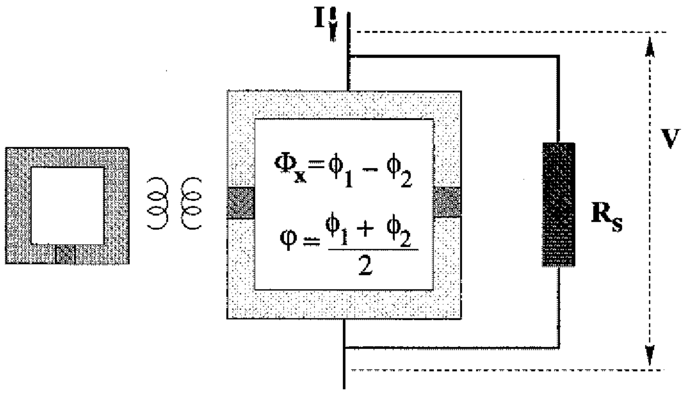
$$T_2^{-1} = a\frac{E^2_{1/f}}{\Delta E} \qquad a \sim \ln\frac{E^2_{1/f}}{\omega_{\text{ir}}\Delta E}$$

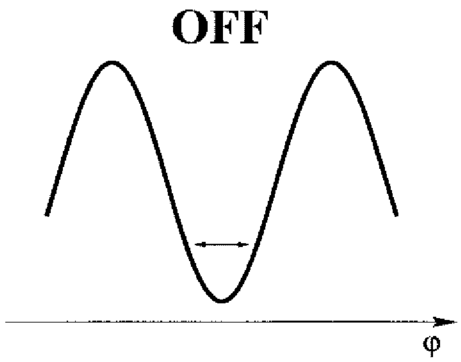
$$T_1/T_2 = 1/2 + a$$

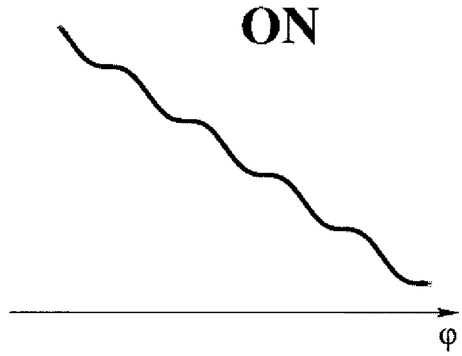
(≈ 3 in exp. of Saclay group)

Nonlinear coupling

Dephasing by symmetric dc-SQUID in the off state







$$H = \frac{\epsilon}{2}\hat{\sigma}_z + \frac{\Delta}{2}\hat{\sigma}_x - \frac{\Phi_0}{2\pi} I_c(\Phi_x)\cos\varphi - \frac{\Phi_0}{2\pi}\delta I_c \hat{\sigma}_z \cos\varphi$$

Overdamped oscillator: $\cos\varphi \approx 1 - \varphi^2/2$

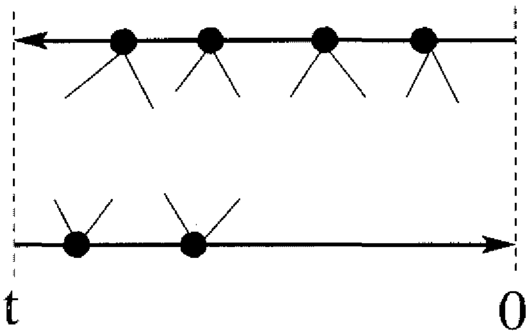
$$\langle \varphi_\omega^2 \rangle = \frac{1}{\omega}\frac{\mathrm{Re}Z(\omega)}{R_\mathrm{K}}\left(\coth\frac{\omega}{2T} + 1\right) \qquad Z(\omega) = \left(\frac{1}{R_s} + i\omega C + \frac{2\pi I_c}{i\omega\Phi_0}\right)^{-1}$$

$$\Gamma_\varphi^{\mathrm{off}} \propto \int d\omega \langle \varphi_\omega^2 \rangle \langle \varphi_{-\omega}^2 \rangle = \begin{cases} \left(\frac{\delta I_c}{I_c}\right)^2 \frac{T^3}{\alpha^2 E_\mathrm{J}^2} & T < \alpha E_\mathrm{J} \\ \left(\frac{\delta I_c}{I_c}\right)^2 \frac{T^2}{\alpha E_\mathrm{J}} & T > \alpha E_\mathrm{J} \end{cases} \qquad \alpha \equiv \frac{R_s}{R_\mathrm{K}}$$

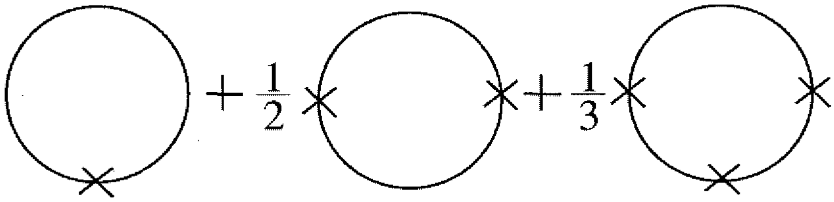
Nonlinear coupling

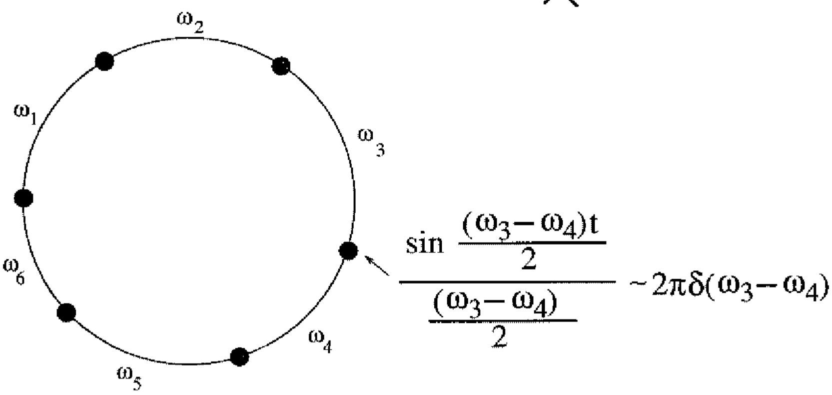
$$\mathcal{H} = -\tfrac{1}{2}\Delta E\ \hat{\sigma}_z + \hat{\sigma}_z \tfrac{X^2}{E_0} + H_{\rm bath}$$

$$\langle \sigma_+(t) \rangle = \langle \tilde{T} \exp\left(i \int_0^t \left(X^2/E_0 \right) dt \right)\ T \exp\left(i \int_0^t \left(X^2/E_0 \right) dt \right) \rangle$$



$$\ln\langle \sigma_+(t) \rangle = \sum_{n=1}^{\infty} \frac{1}{n} F_n =$$





$$\ln\langle \sigma_+(t) \rangle \approx t \int \frac{d\omega}{2\pi} \sum_n \frac{(-1)^n}{n} \frac{tr\left[\hat{\alpha}^n(\omega)\right]}{E_0^n} = -t \int \frac{d\omega}{2\pi}\, tr\, \ln\left(1 + \frac{\hat{\alpha}(\omega)}{E_0}\right)$$

$$\hat{\alpha} = \begin{pmatrix} \alpha^c & \alpha^< \\ \alpha^> & \alpha^{ac} \end{pmatrix} = \begin{pmatrix} -i\langle T X(t) X(0) \rangle & -i\langle X(0) X(t) \rangle \\ -i\langle X(t) X(0) \rangle & -i\langle \tilde{T} X(t) X(0) \rangle \end{pmatrix}$$

$$\ln\langle \sigma_+(t) \rangle \approx -t \int \frac{d\omega}{2\pi} \ln\left[(1 + \frac{\alpha^c}{E_0})(1 + \frac{\alpha^{ac}}{E_0}) - \frac{\alpha^> \alpha^<}{E_0^2} \right]$$

Nonlinear coupling

$$\ln\langle\sigma_+(t)\rangle \approx -t\int\frac{d\omega}{2\pi}\ln\left[(1+\frac{\alpha^c}{E_0})(1+\frac{\alpha^{ac}}{E_0})-\frac{\alpha^>\alpha^<}{E_0^2}\right]$$

The Green's functions

$$\begin{pmatrix}\alpha^c & \alpha^< \\ \alpha^> & \alpha^{ac}\end{pmatrix}=\begin{pmatrix}-i(S+iB) & -i(S-A) \\ -i(S+A) & -i(S-iB)\end{pmatrix}$$

$$A(\omega)=J(\omega)\qquad S(\omega)=A(\omega)\coth\frac{\omega}{2T}\qquad B(\omega)=\int\frac{d\nu}{\pi}\frac{A(\nu)}{\omega-\nu}$$

$$S_0(\omega)=S_{T=0}(\omega)=A(|\omega|)\qquad \delta S(\omega)=S-S_0=A(|\omega|)\left[\coth\frac{|\omega|}{2T}-1\right]$$

$$\ln\langle\sigma_+(t)\rangle \approx -t\int\frac{d\omega}{2\pi}\ln\left[(1+\frac{\alpha_0^c}{E_0})(1+\frac{\alpha_0^{ac}}{E_0})-\frac{2i\delta S}{E_0}\right]$$

At $T=0$

$$\Gamma_\varphi=\mathrm{Re}\int\frac{d\omega}{2\pi}\ln\left[(1+\frac{\alpha_0^c}{E_0})(1+\frac{\alpha_0^{ac}}{E_0})\right]=0$$

At $T>0$

$$\Gamma_\varphi=\mathrm{Re}\int\frac{d\omega}{2\pi}\ln\left[1-\frac{2i\delta S/E_0}{1-2iS_0/E_0-(S_0^2+B^2)/E_0^2}\right]$$

At $\omega\ll\omega_c$ $\quad B\sim\alpha\omega_c\quad B/E_0\sim(\alpha\omega_c/E_0)$ may be large

$$\Gamma_\varphi\approx\int\frac{d\omega}{2\pi}\frac{2(2S_0+\delta S)\delta S}{E_0^2(1-B^2/E_0^2)^2}=-2\int\frac{d\omega}{2\pi}\frac{\alpha^>\alpha^<}{E_0^2}\left(1-\frac{B^2}{E_0^2}\right)^{-2}$$

RG needed

Summary

- **Sources of dissipation**

  Electromagnetic environment

  Background charges and fluxes, 1/f noise

- **Models of dissipation**

  Derivation

  "Realistic models" useful

- **Master (Bloch) equations**

  Dephasing (T_2) and relaxation (T_1)

  Longitudinal vs. transverse coupling

  Dependence on noise spectrum and on manipulations

- **New questions**

  1/f noise

  Non-Gaussian effects

  Nonlinear coupling (quantum meter on-off)