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JOINT ICTP-INFM SCHOOL/WORKSHOP ON "ENTANGLEMENT AT THE NANOSCALE"

(28 October – 8 November 2002)

"How to turn a SQUID into (Schrödinger) cat"

presented by:

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How to Turn a SQUID into (Schrödinger's) Cat

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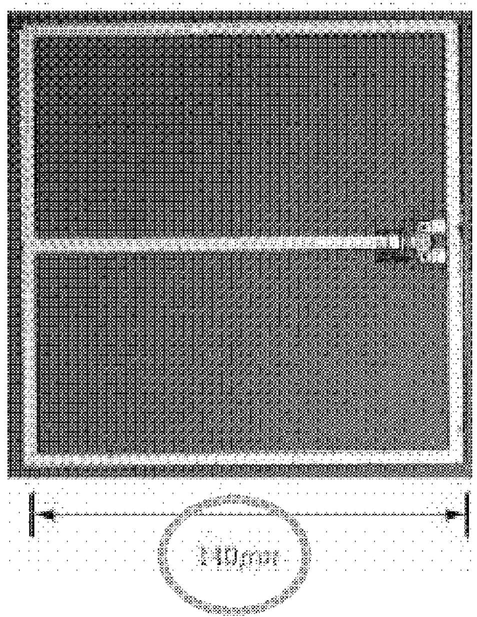
Sergey Tolpygo
Dmitri Averin

Wei Chen
James Lukens

"Entanglement" at the "Nanoscale"

$$\left|\circlearrowleft\right\rangle + \left|\circlearrowright\right\rangle$$

$$= \underbrace{\left|\uparrow\uparrow\cdots\uparrow\right\rangle}_{\substack{\sim 10^{9}\, e^{-}\text{'s} \\ \sim 10^{10}\,\mu_B}} + \left|\downarrow\downarrow\cdots\downarrow\right\rangle$$



The world's most entangled state!

Why Superconductors Make Good Cats

- Macroscopic Order Parameter (Wave function)

  $\psi = \sqrt{\rho} e^{i\varphi}$ $\rho =$ density of Cooper pairs

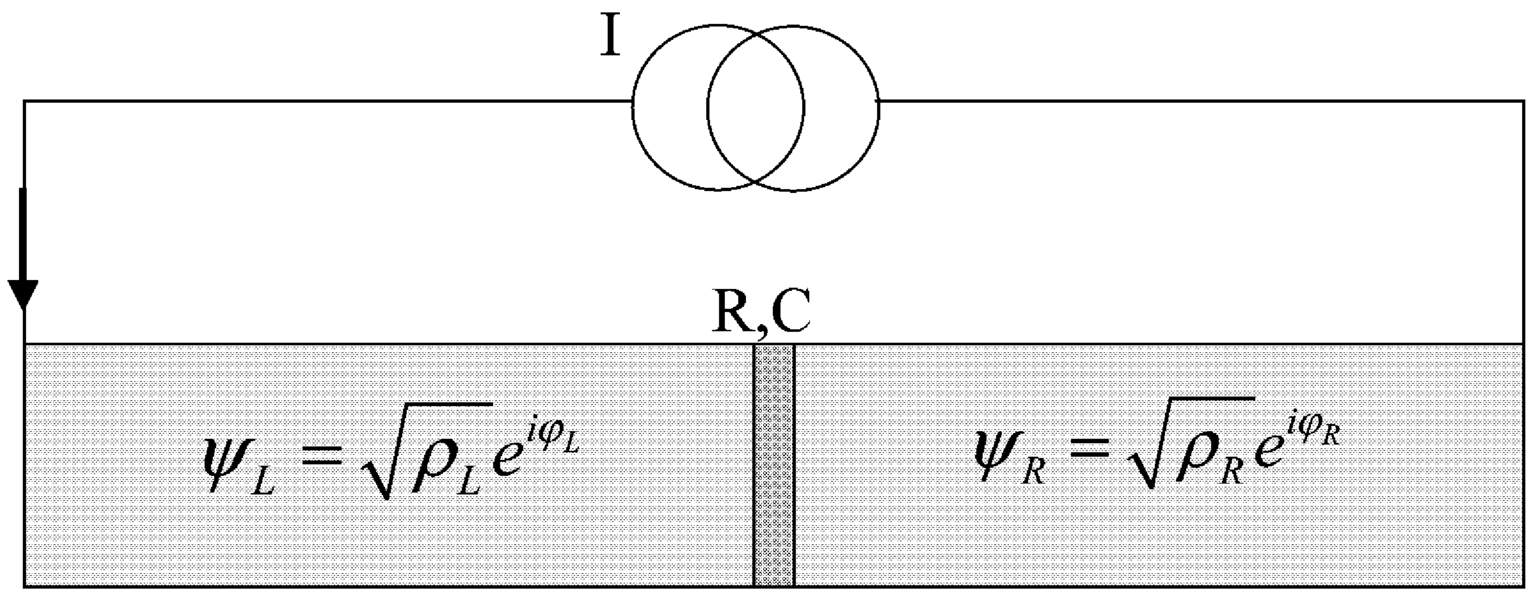
  All Cooper pairs have the same phase.

- Superconducting Energy Gap Δ

  When $T \ll \Delta$, excitations (broken pairs) are exponentially suppressed.

  For Niobium, $\Delta = 18$ K Experiments: <100 mK

What is a Josephson Junction?

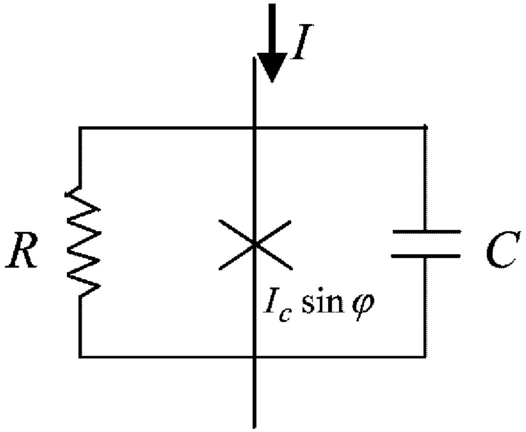


The Josephson relations:

$$I_{super} = I_c \sin(\varphi_L - \varphi_R) = I_c \sin \varphi$$

$$V = \frac{\hbar}{2e} \frac{d\varphi}{dt}$$

Resistively Shunted Junction (RSJ) Model



$$I = I_c \sin\varphi + V/R + C\frac{dV}{dt}$$
$$= I_c \sin\varphi + \frac{\hbar}{2eR}\frac{d\varphi}{dt} + \frac{\hbar C}{2e}\frac{d^2\varphi}{dt^2} \quad \left(\text{using } V = \frac{\hbar}{2e}\frac{d\varphi}{dt}\right)$$

Equation of motion for particle of "mass" $\left(\frac{\hbar}{2e}\right)^2$ C, "position" φ and friction coefficient $\left(\frac{\hbar}{2e}\right)^2 R^{-1}$.

potential energy:

$$U(\varphi) = \underbrace{-I_c\frac{\hbar}{2e}\cos\varphi}_{E_J\cos\varphi} - I\frac{\hbar}{2e}\varphi$$

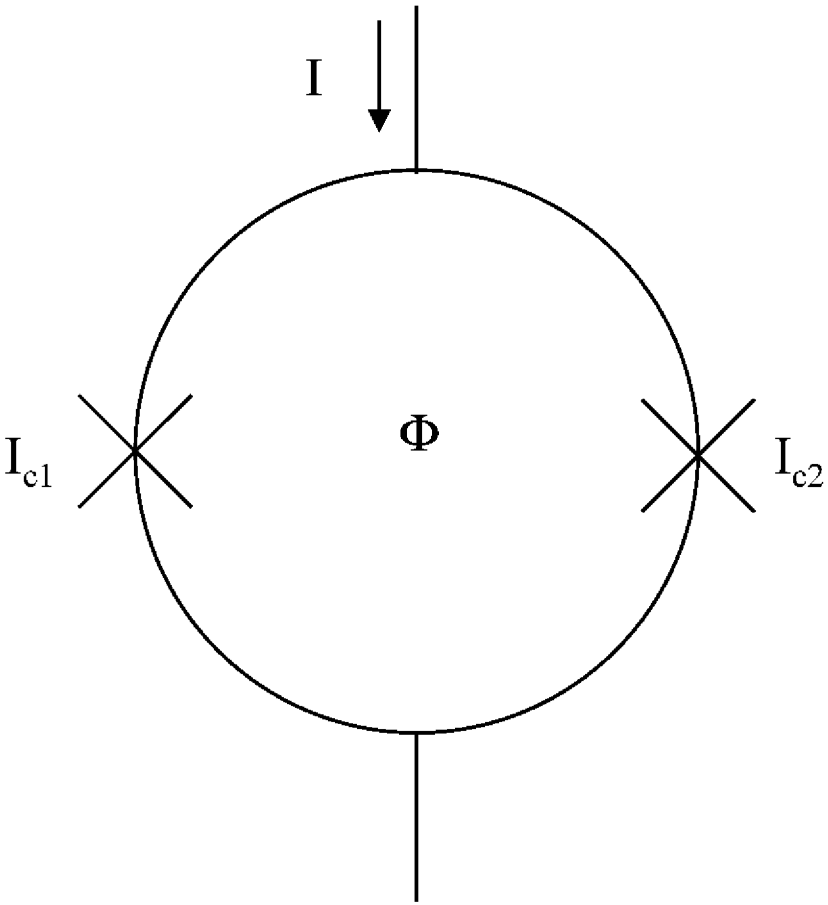
The "Josephson potential"

kinetic energy:

$$\tfrac{1}{2}\left(\frac{\hbar}{2e}\right)^2 \mathrm{C}\left(\frac{d\varphi}{dt}\right)^2 = \tfrac{1}{2}CV^2$$

SQUID Basics: The dc-SQUID

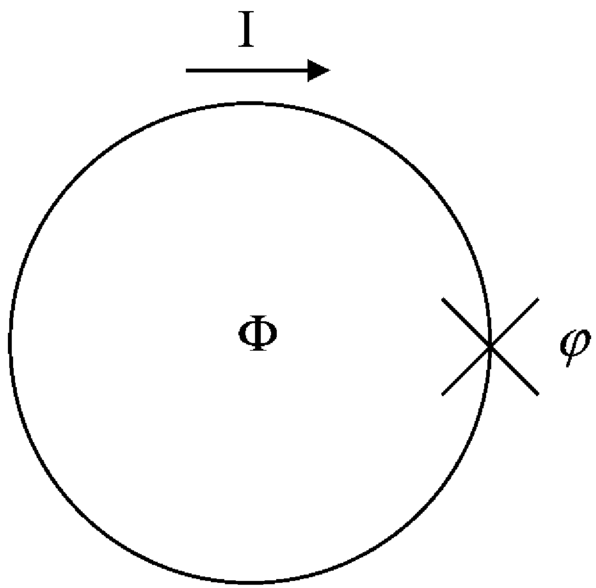
SQUID: Superconducting QUantum Interference Device



Take $I_{c1} = I_{c2}$

$$I_{c,total} = 2I_c \left|\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\right|$$

SQUID Basics: The rf-SQUID



Fluxoid quantization:

In a SQUID, flux is not strictly quantized.

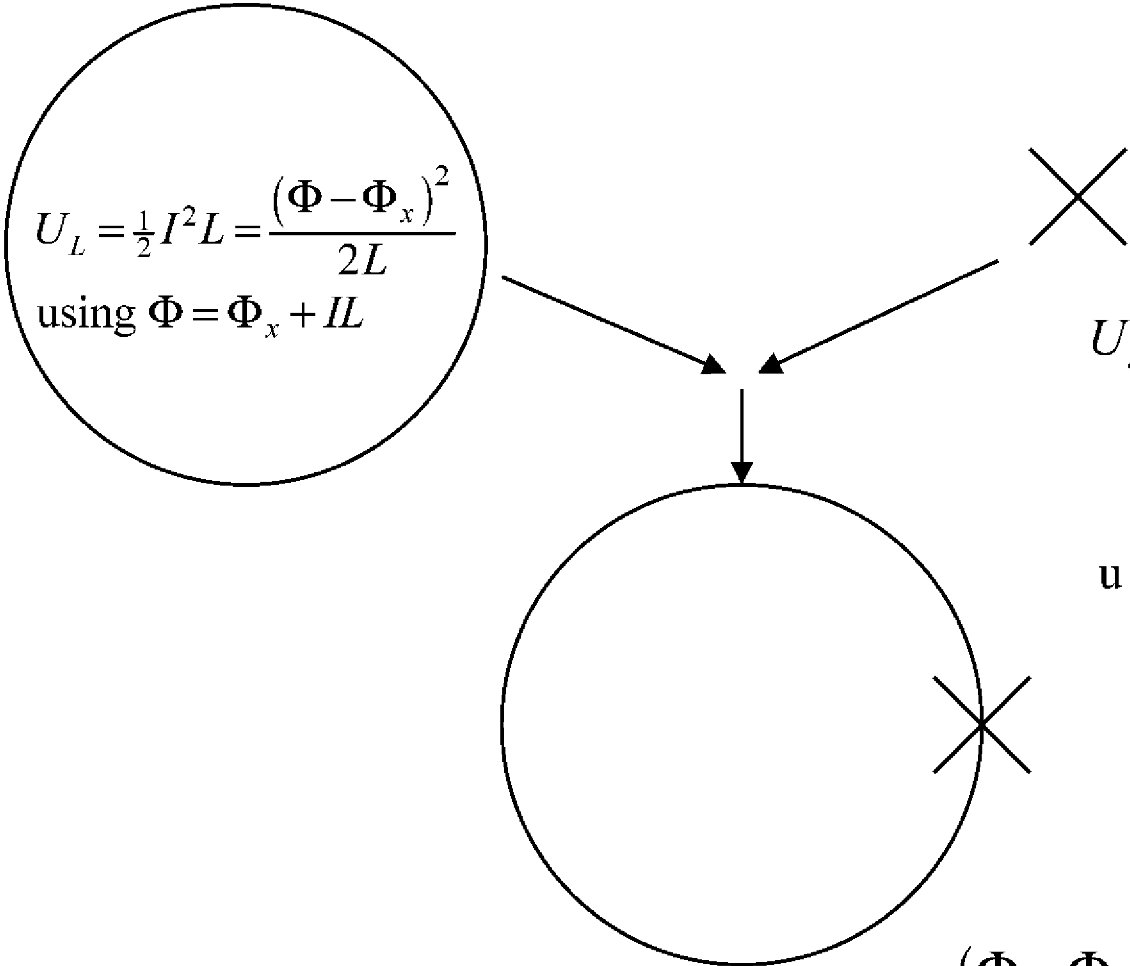
$$2\pi\Phi / \Phi_0 + \varphi = 2\pi n$$

$$\Phi = \Phi_x + LI$$

Potential Energy of an rf-SQUID

$$U_L = \tfrac{1}{2} I^2 L = \frac{(\Phi - \Phi_x)^2}{2L}$$

using $\Phi = \Phi_x + IL$



$$\begin{aligned} U_J &= -E_J \cos\varphi \\ &= -E_J \cos(2\pi(n - \Phi/\Phi_0)) \\ &= -E_J \cos(2\pi\Phi/\Phi_0) \end{aligned}$$

using $2\pi\Phi/\Phi_0 + \varphi = 2\pi n$

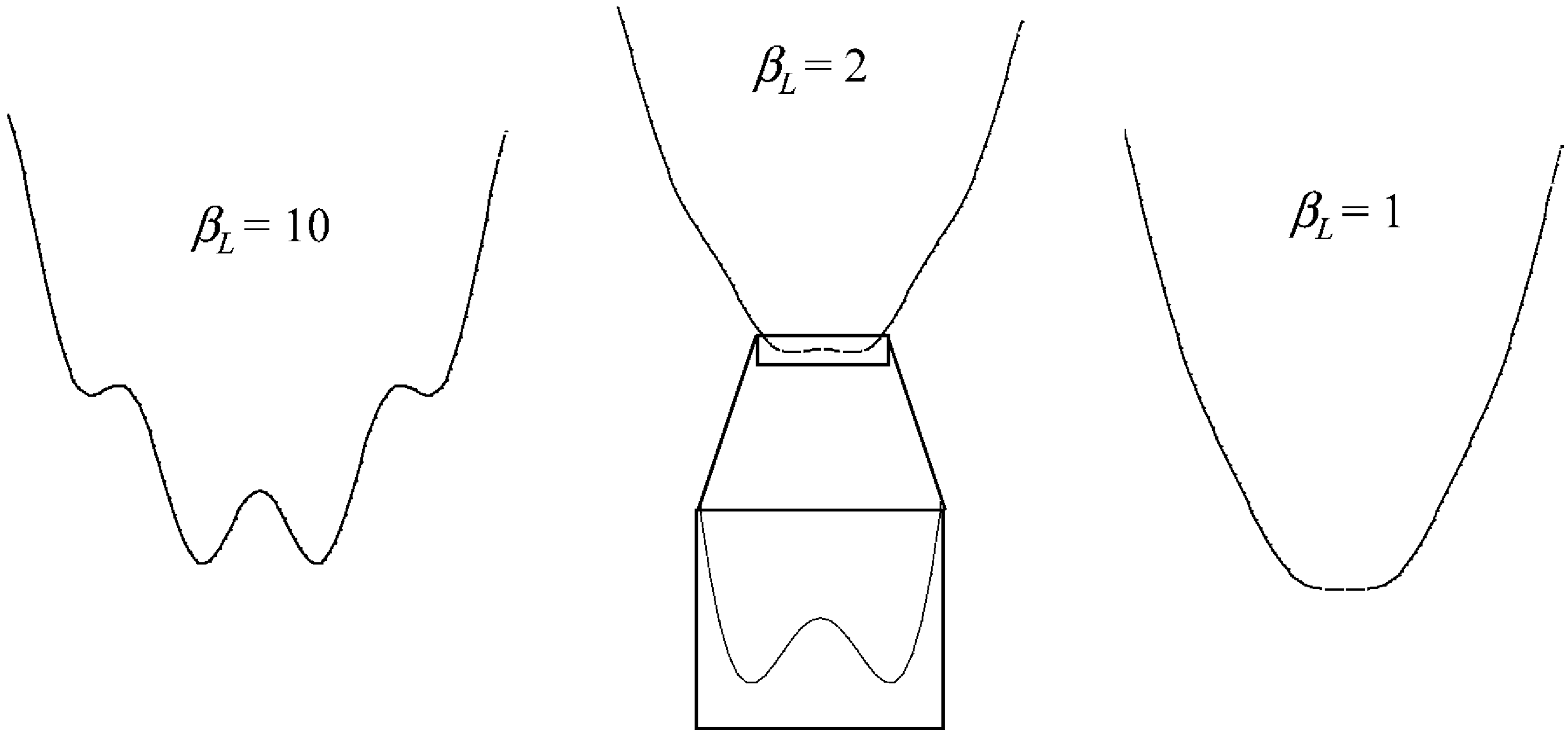
$$U = \frac{(\Phi - \Phi_x)^2}{2L} - \frac{I_c \Phi_0}{2\pi} \cos(2\pi\Phi/\Phi_0)$$

SQUID Potential

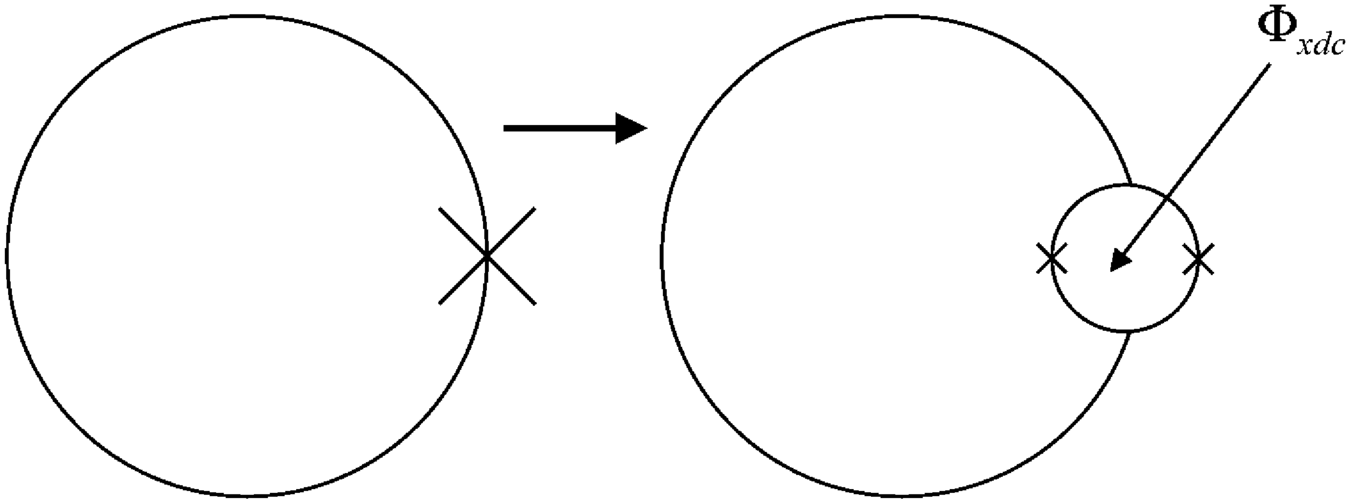
$$U = \frac{(\Phi - \Phi_x)^2}{2L} - \frac{I_c \Phi_0}{2\pi} \cos(2\pi\Phi/\Phi_0)$$

$$= \frac{\Phi_0^2}{4\pi^2 L}\left(\frac{(2\pi(\Phi - \Phi_x)/\Phi_0)^2}{2} - \beta_L \cos(2\pi\Phi/\Phi_0) \right)$$

$$\beta_L = \frac{2\pi L I_c}{\Phi_0}$$



Variable-Barrier SQUID



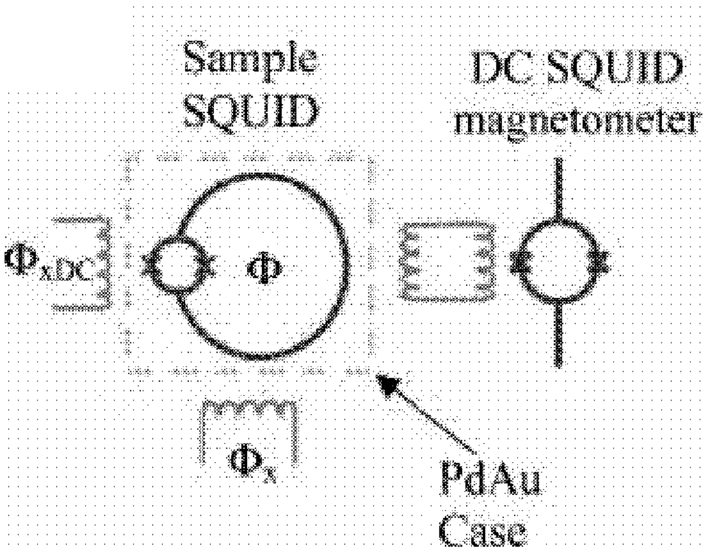
$$\beta_L = \beta_{L0} \cos\left(\frac{\pi \Phi_{xdc}}{\Phi_0}\right)$$

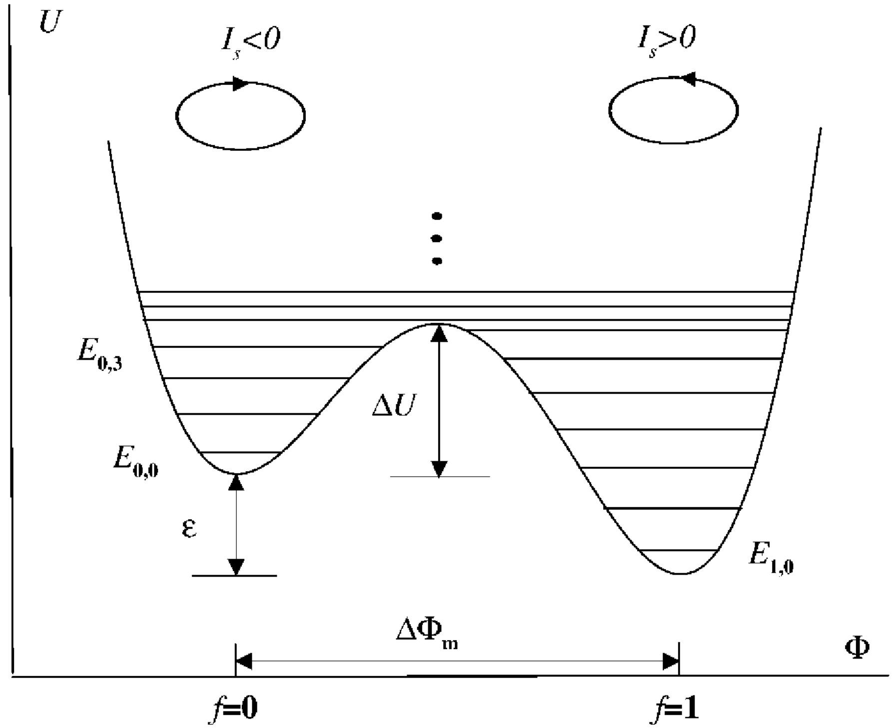
SQUID Potential with Energy Levels

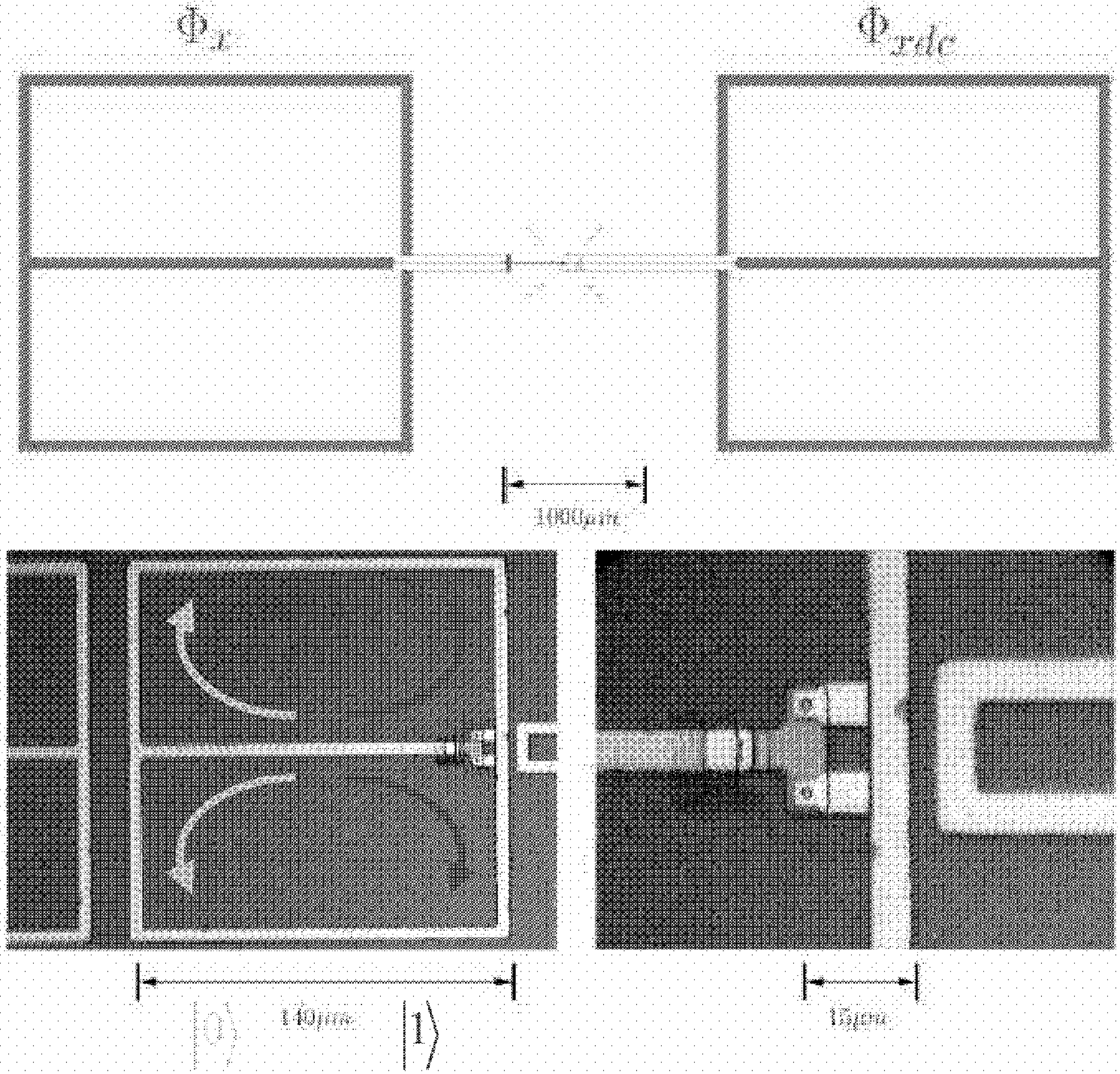
- Full Hamiltonian

$$H = \frac{Q^2}{2C} + \frac{\Phi_0^2}{4\pi^2 L}\left(\frac{\left(2\pi\left(\Phi - \Phi_x\right)/\Phi_0\right)^2}{2} - \beta_{L0}\cos\left(\frac{\pi\Phi_{xdc}}{\Phi_0}\right)\cos\left(\frac{2\pi\Phi}{\Phi_0}\right)\right)$$

- Two Control Fluxes
  - Φ_x tilts the potential
  - Φ_{xdc} adjusts the barrier height

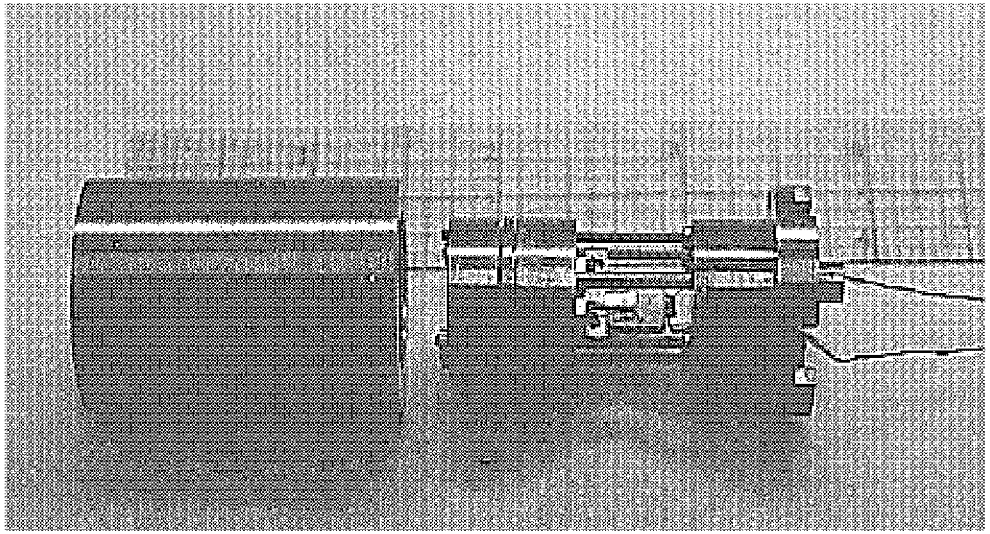




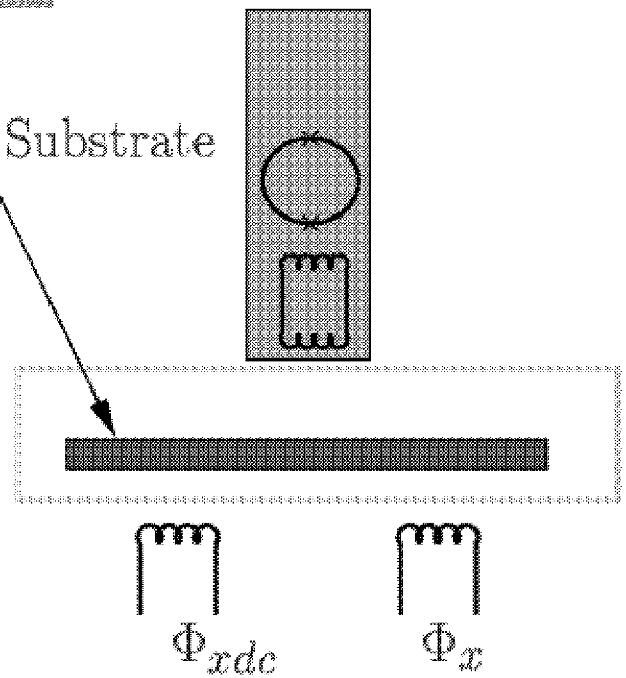


Φ_x
Φ_{xdc}
$1000\mu m$
$|0\rangle$
$140\mu m$
$|1\rangle$
$15\mu m$

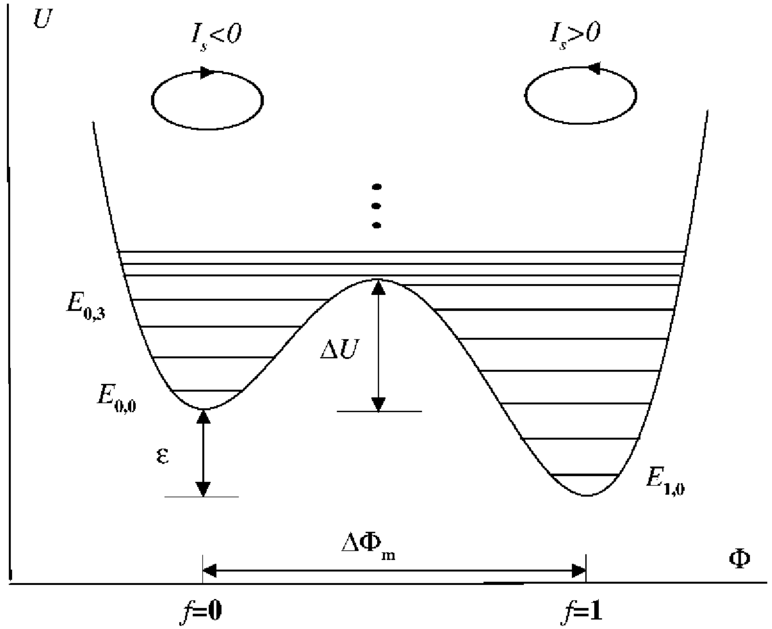
MQT Sample Cell

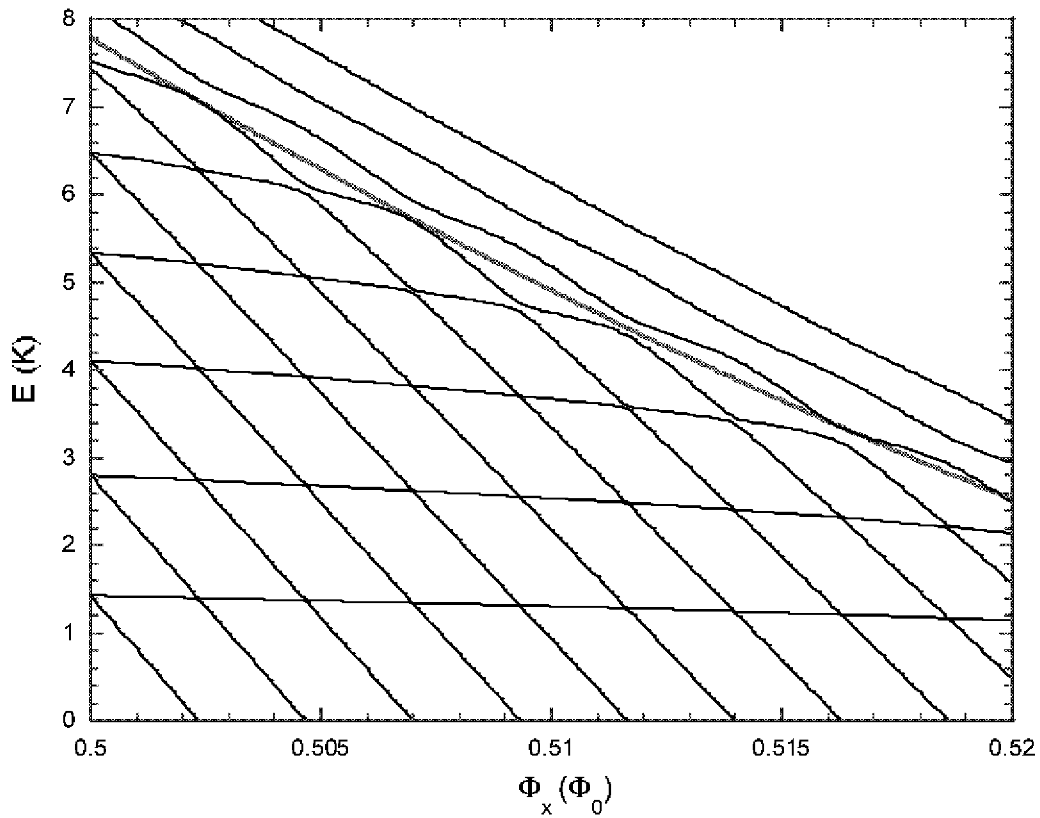


assembled

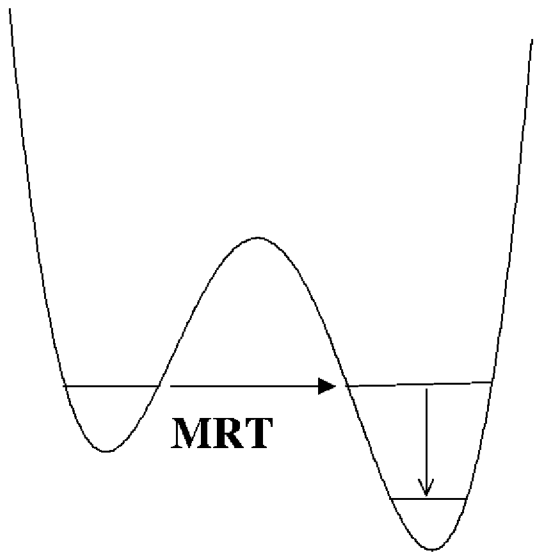


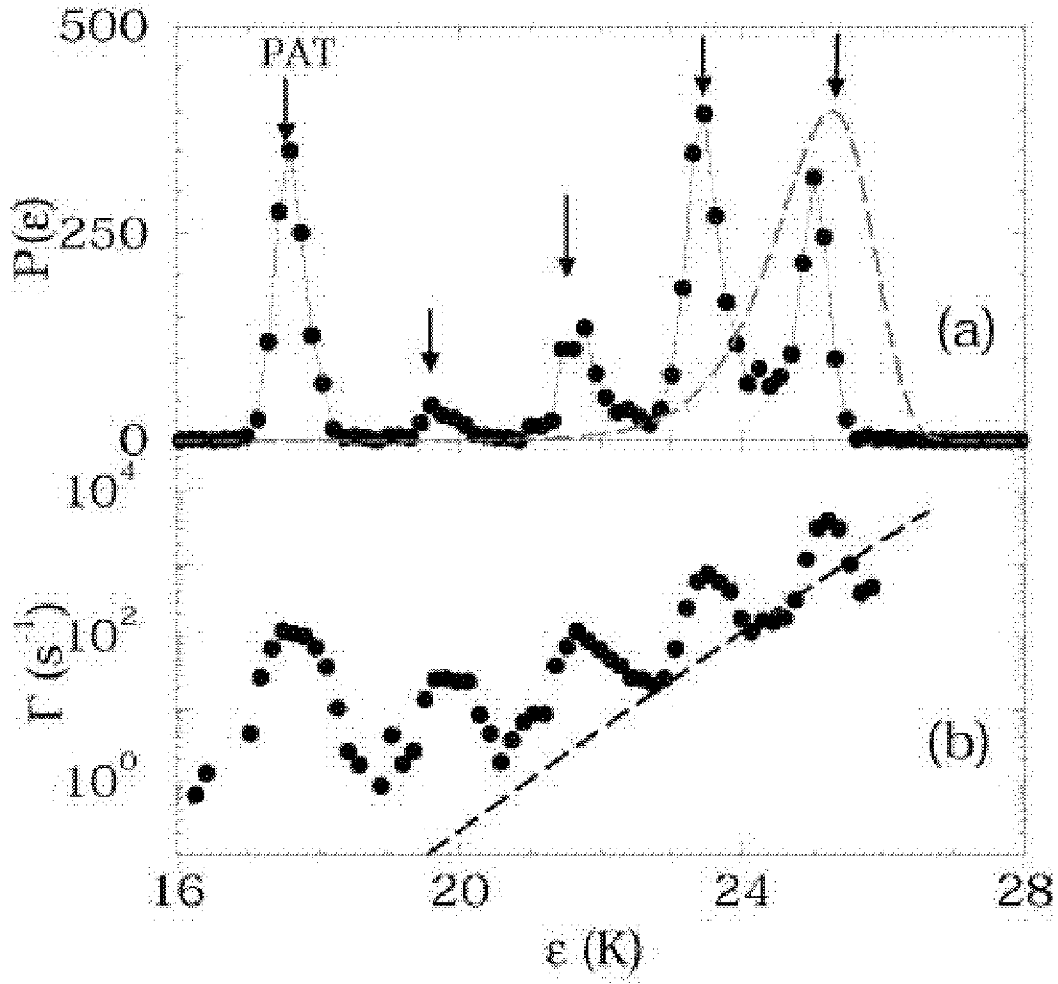
SQUID Energy Levels





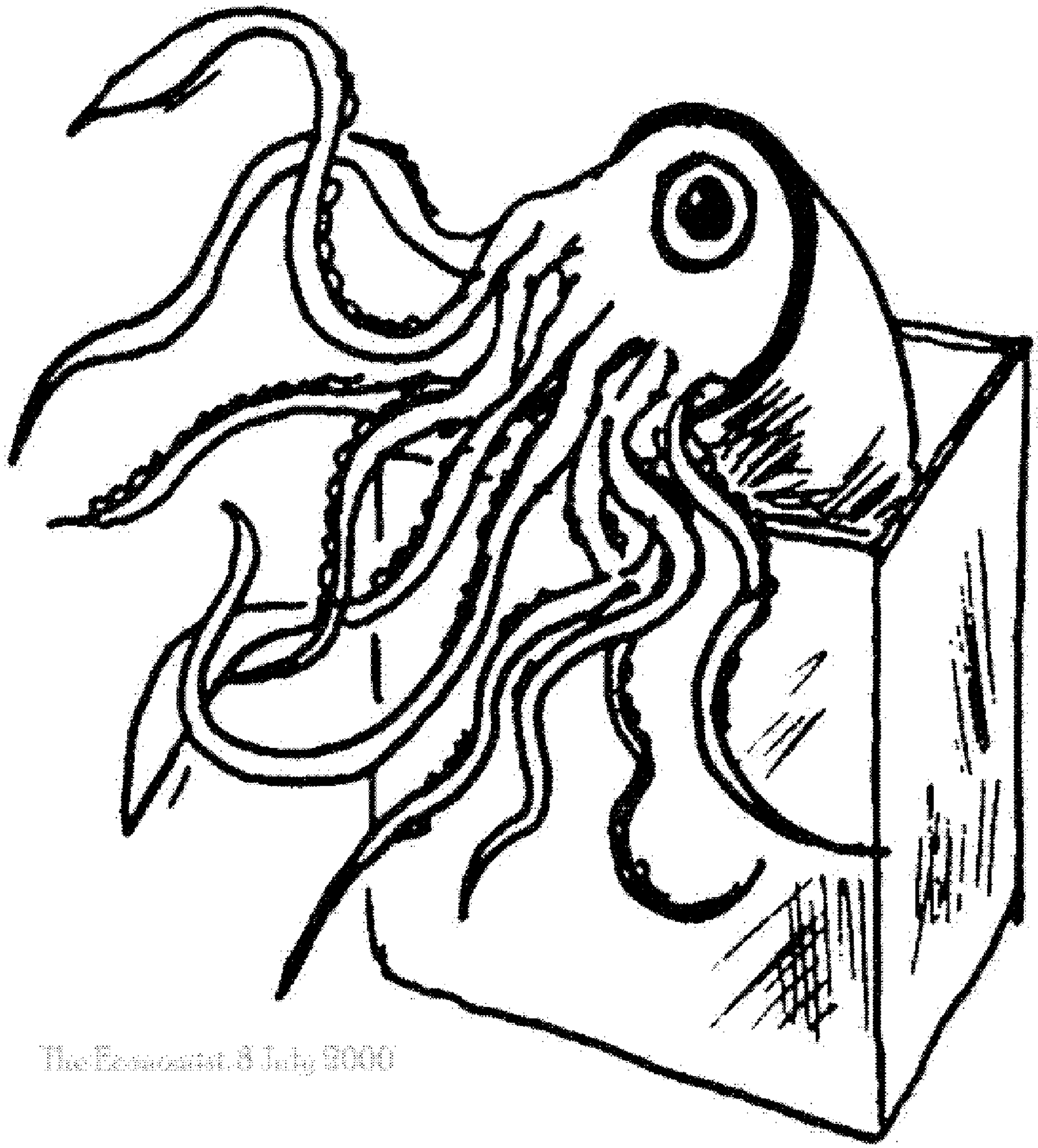
Macroscopic Resonant Tunneling





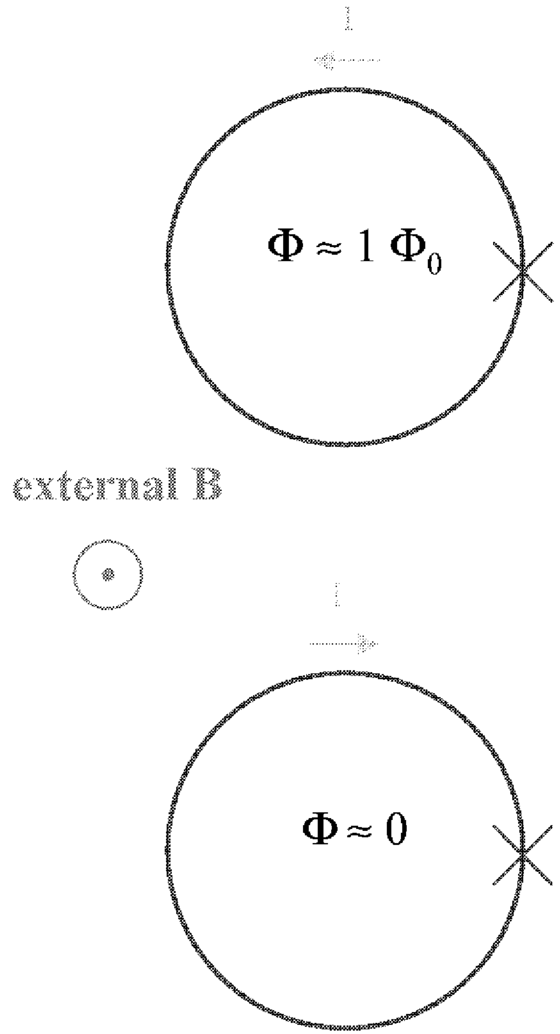
(Rouse, Han & Lukens, 1995)

Making a (Schrödinger's) Cat out of a SQUID

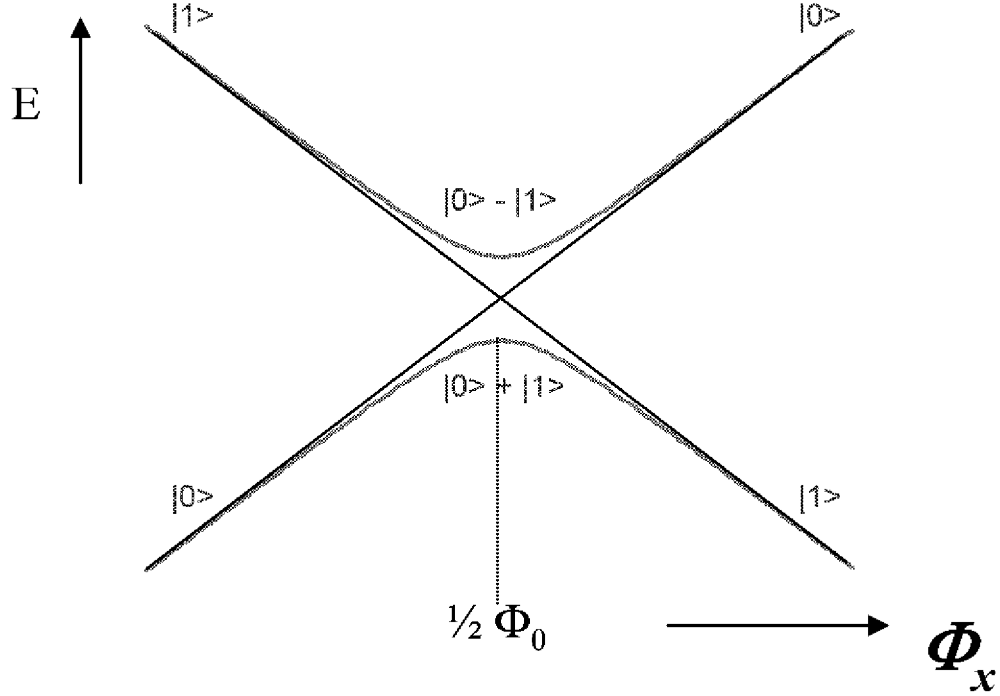


The Economist, 8 July 2000

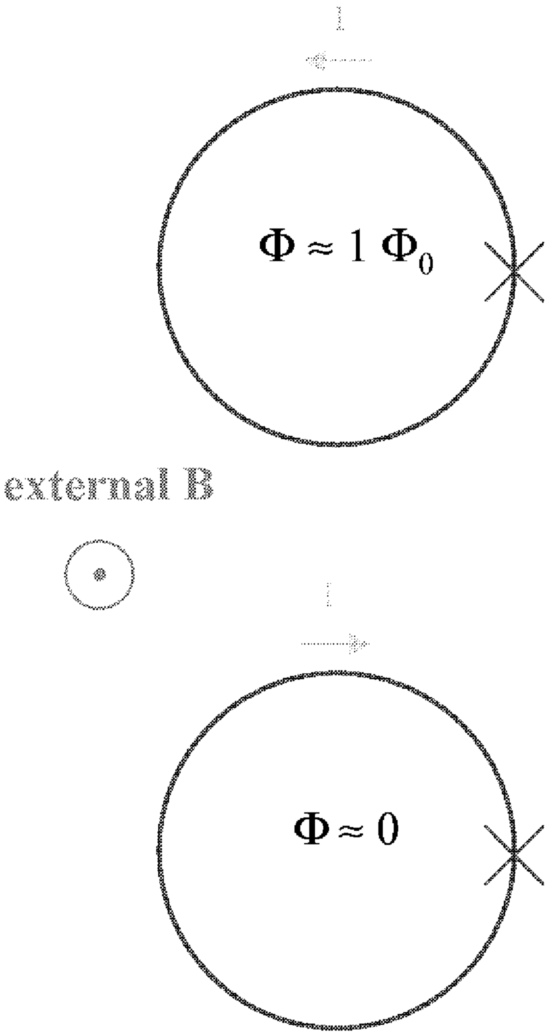
Making a (Schrödinger's) Cat out of a SQUID



What if $\Phi_x \simeq \frac{1}{2}\Phi_0$?



Making a (Schrödinger's) Cat out of a SQUID



What if $\Phi_x \simeq \frac{1}{2}\Phi_0$?

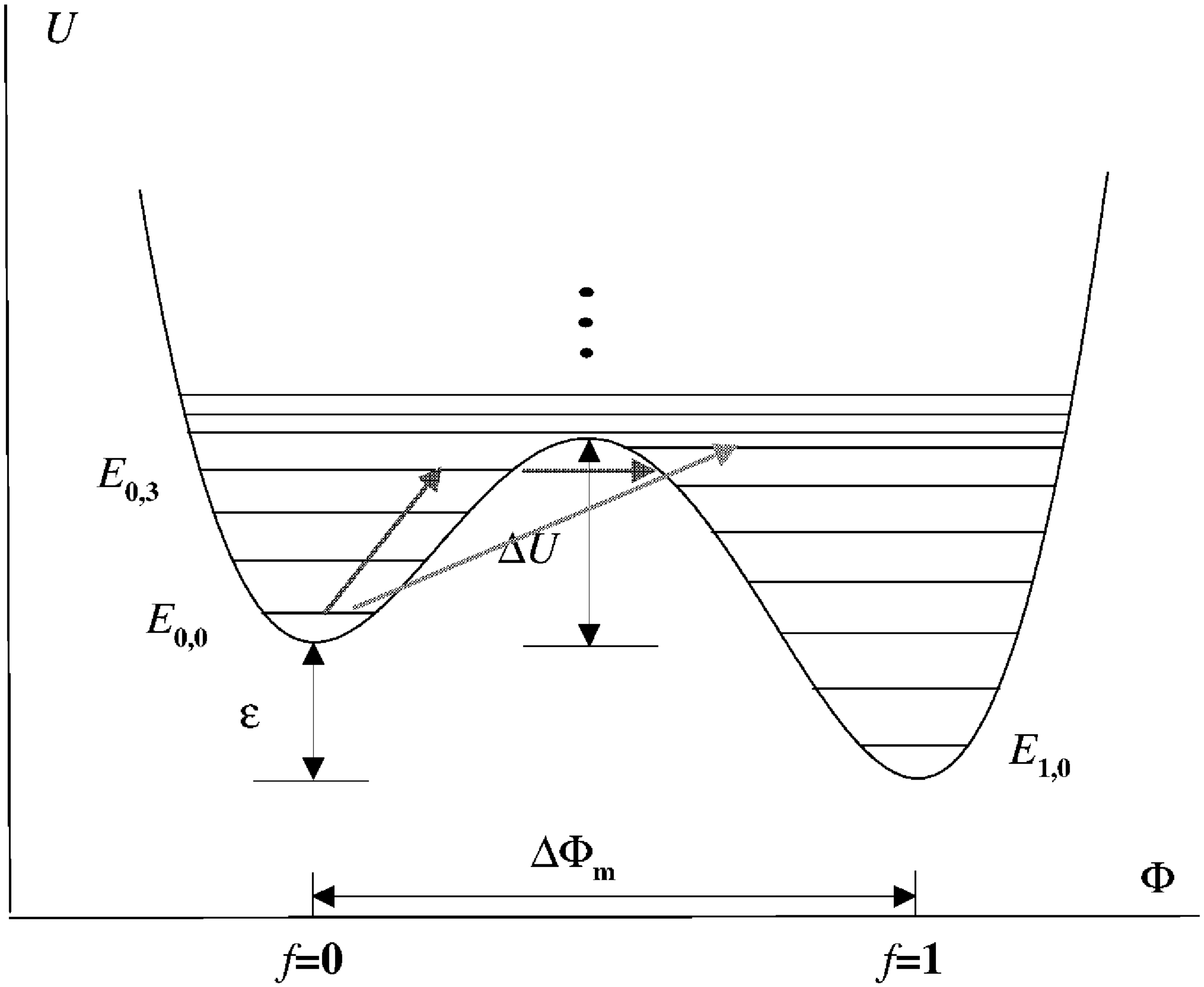
Macroscopic Cat

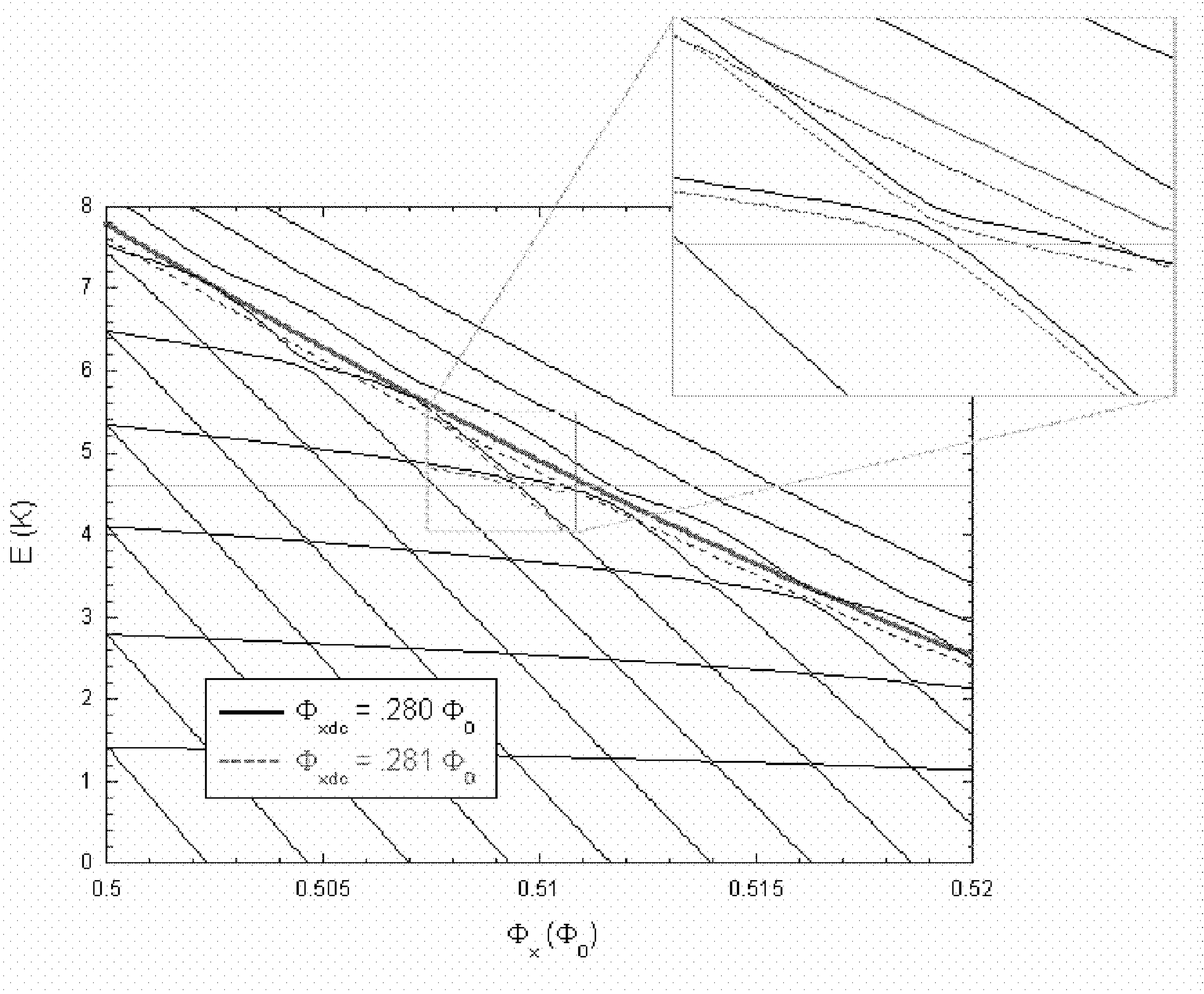
$I \approx 1\ \mu A$

10^9 circulating electrons

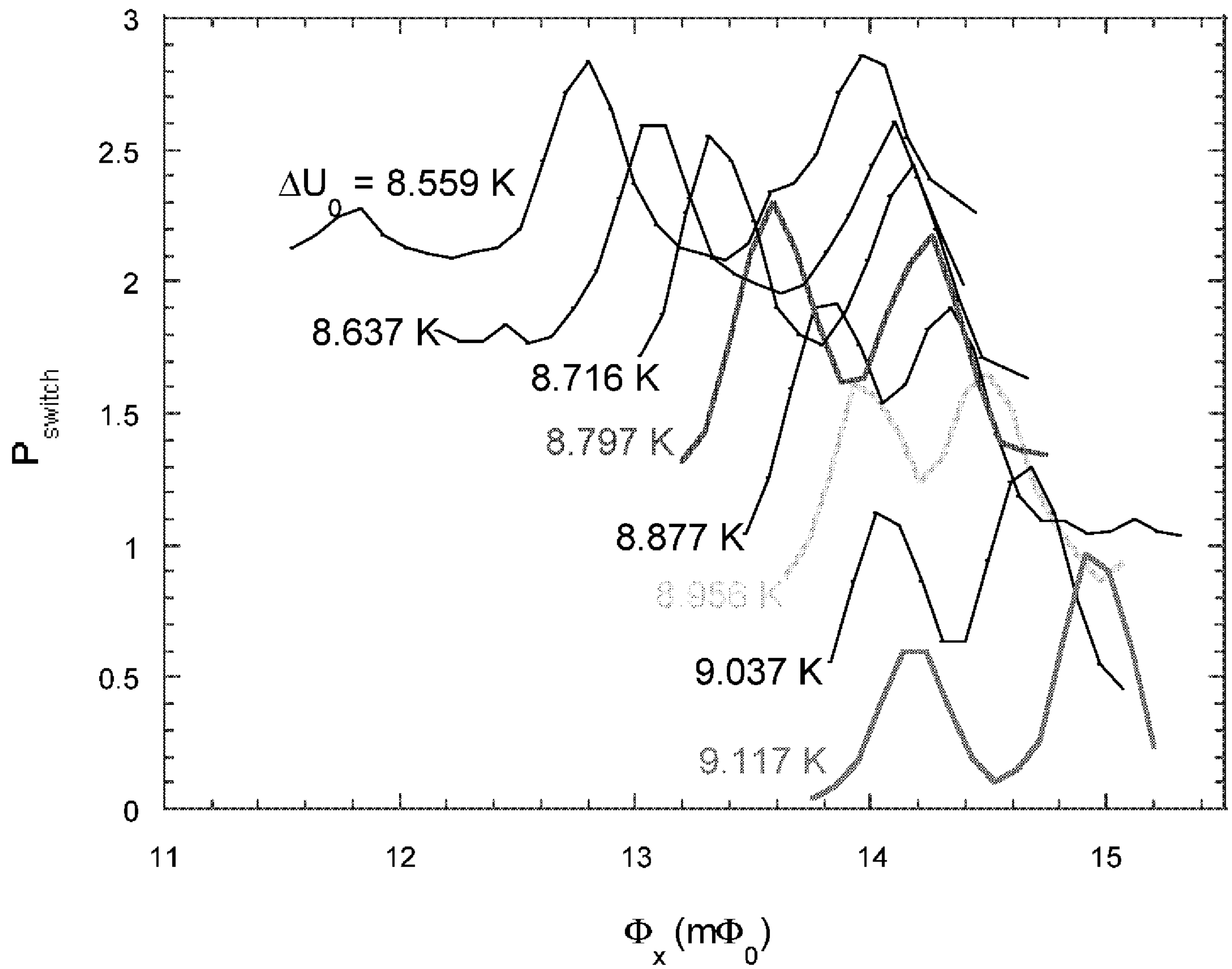
magnetic moment: $10^{10}\ \mu_B$

Photon-Assisted Tunneling



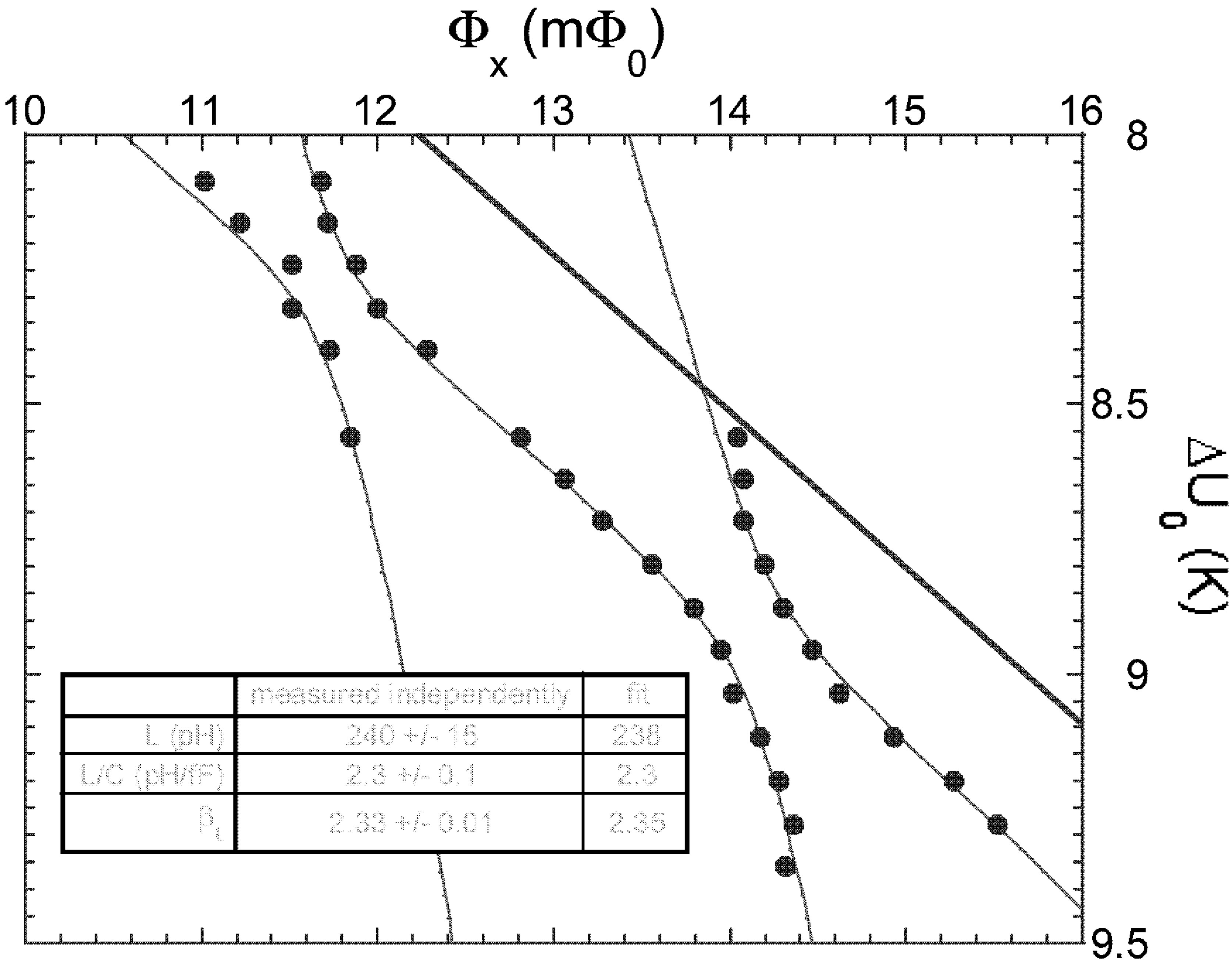


8
7
6
5
4
3
2
1
0
E (K)
0.5
0.505
0.51
0.515
0.52
$\Phi_x\,(\Phi_0)$
Φ_{xdc} = .280 Φ_0
Φ_{xdc} = .281 Φ_0

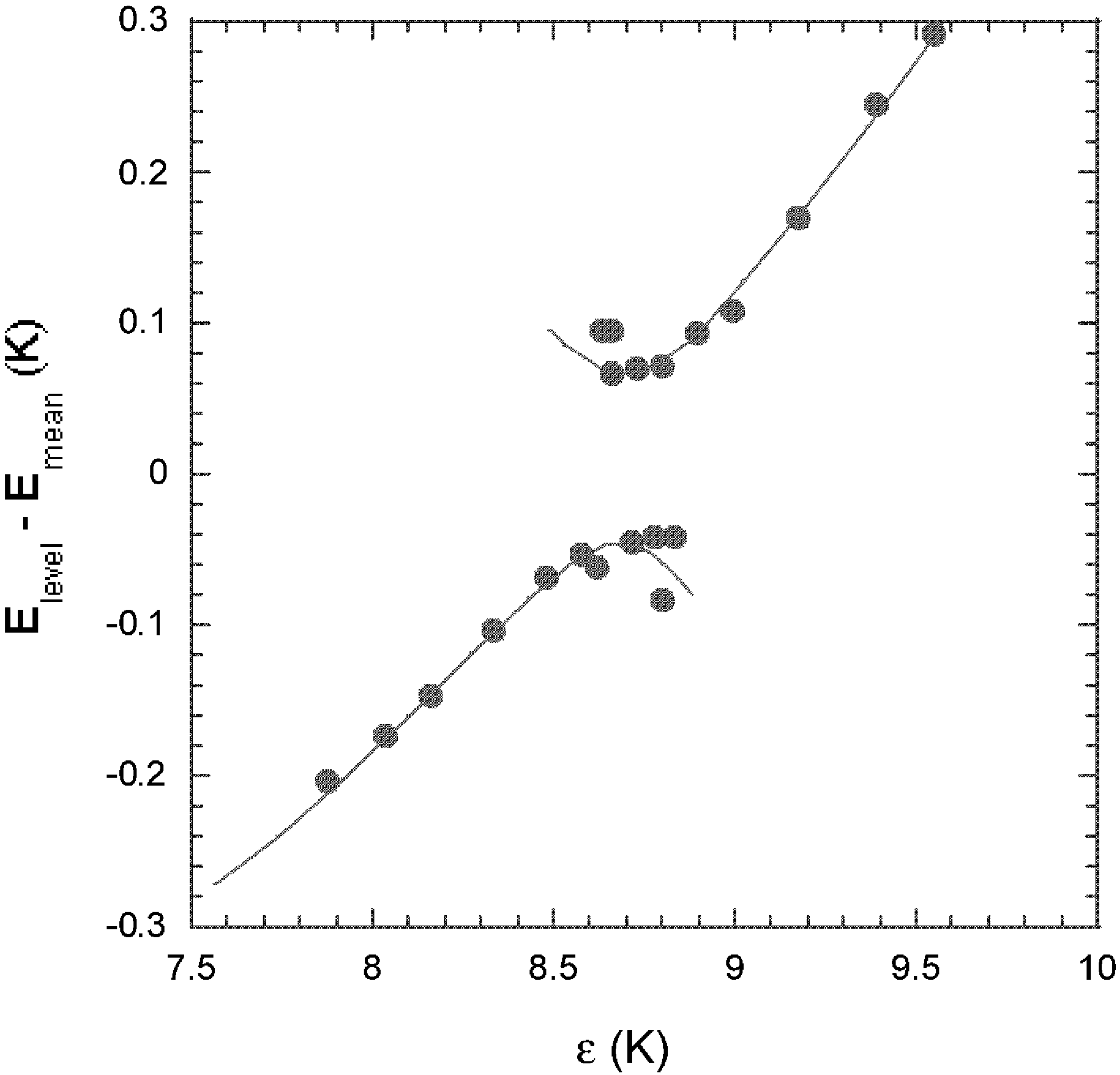


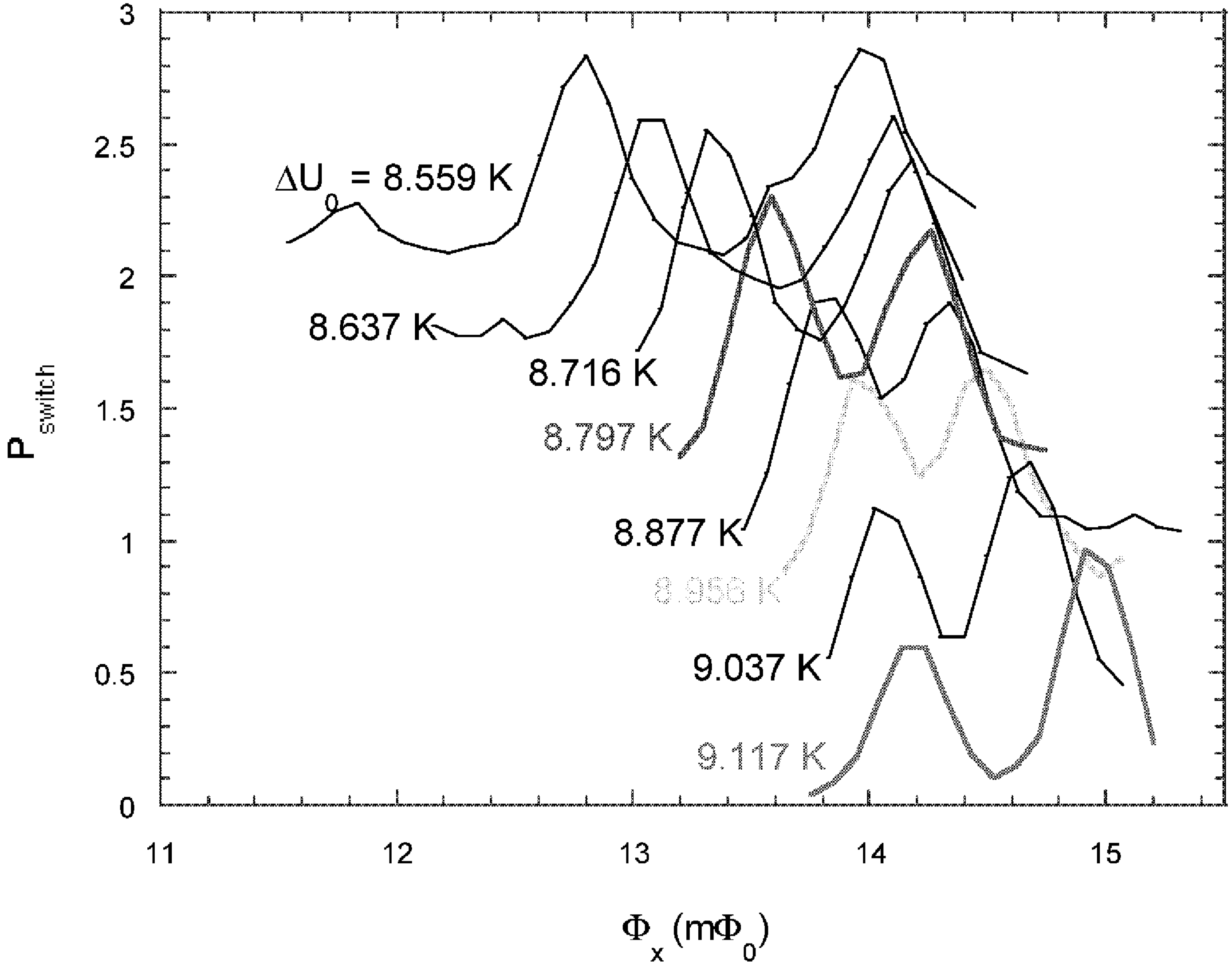
Friedman et al., Nature, 2000.

See also: van der Wal et al., Science, 2000.



	measured independently	fit
L (pH)	240 +/- 15	238
L/C (pH/fF)	2.3 +/- 0.1	2.3
β_L	2.33 +/- 0.01	2.35


0.3
0.2
0.1
0
-0.1
-0.2
-0.3
7.5
8
8.5
9
9.5
10
E_{level} - E_{mean} (K)
ε (K)



ΔU0 = 8.559 K
8.637 K
8.716 K
8.797 K
8.877 K
8.956 K
9.037 K
9.117 K
P switch
Φx (mΦ0)
0
0.5
1
1.5
2
2.5
3
11
12
13
14
15

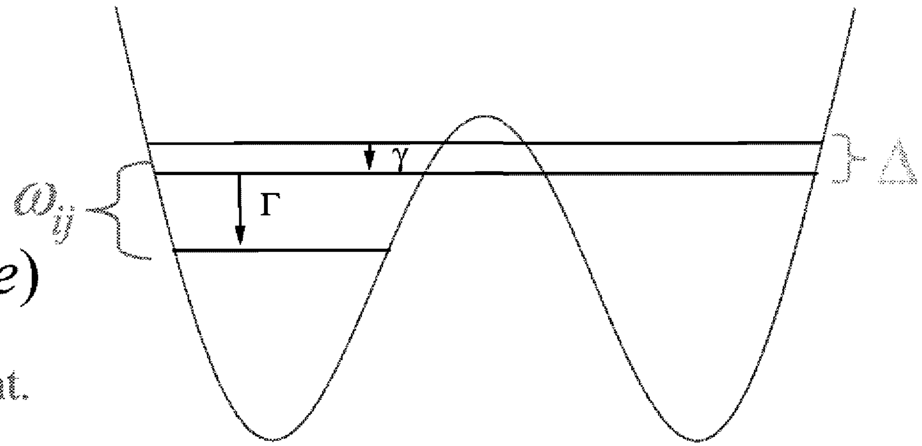
Damping

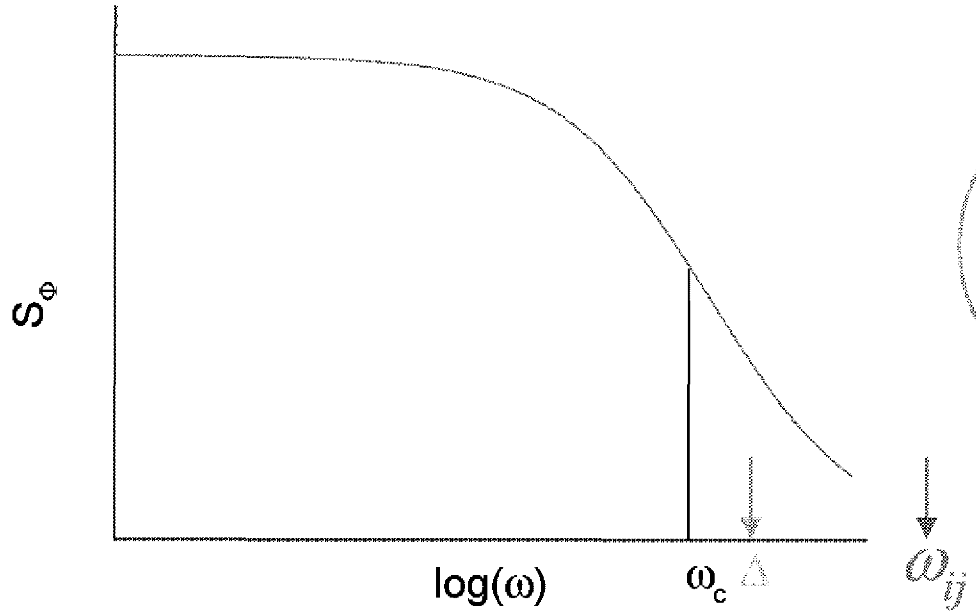
Intrinsic line width from interwell and intrawell relaxation (spontaneous decay) caused by resistive environment:

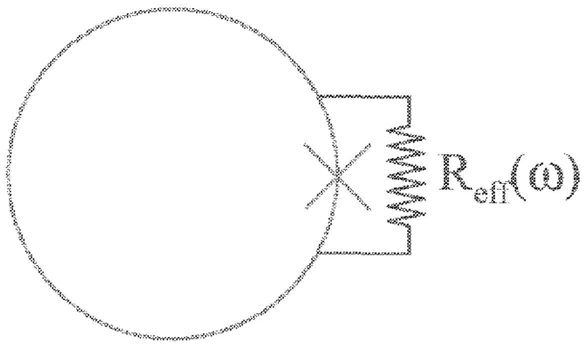
$$\Gamma = 2\pi\omega_{ij} \frac{R_Q}{R} \frac{\left|\Phi_{i,j}\right|^2}{\Phi_0^2}$$

$$\gamma = 4\pi\Delta \frac{R_Q}{R} \frac{\left|\delta\Phi\right|^2}{\Phi_0^2} \quad (on\ resonance)$$

The PdAu case is a non-ohmic environment.







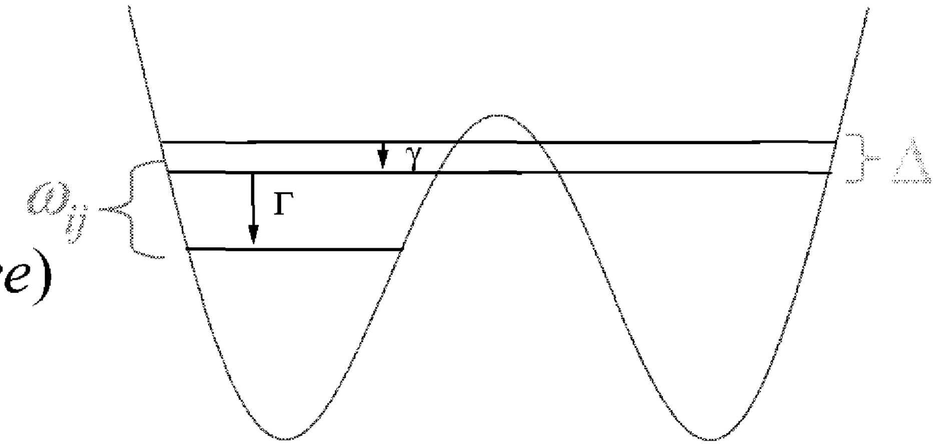
$$S_\Phi \propto 1/R_{eff}$$

Damping

Intrinsic line width from interwell and intrawell relaxation (spontaneous decay) caused by resistive environment:

$$\Gamma = 2\pi\omega_{ij}\frac{R_Q}{R}\frac{\left|\Phi_{i,j}\right|^2}{\Phi_0^2}$$

$$\gamma = 4\pi\Delta\frac{R_Q}{R}\frac{\left|\delta\Phi\right|^2}{\Phi_0^2} \quad (on\, resonance)$$



$$\text{for } \omega \gg \omega_c, \quad R_{eff}(\omega) \sim R_0\left(\frac{\omega}{\omega_c}\right)^2$$

In our case, $\omega_c \sim 0.6$ GHz and $R_0 = 3.4$ kOe.

$\omega = \omega_{ij} \approx 130$ GHz:
$R_{eff} \approx 210\ \mathrm{M\Omega} \Rightarrow \Gamma \approx 12$ kHz $\Rightarrow$ line width = 0.5 nΦ_0

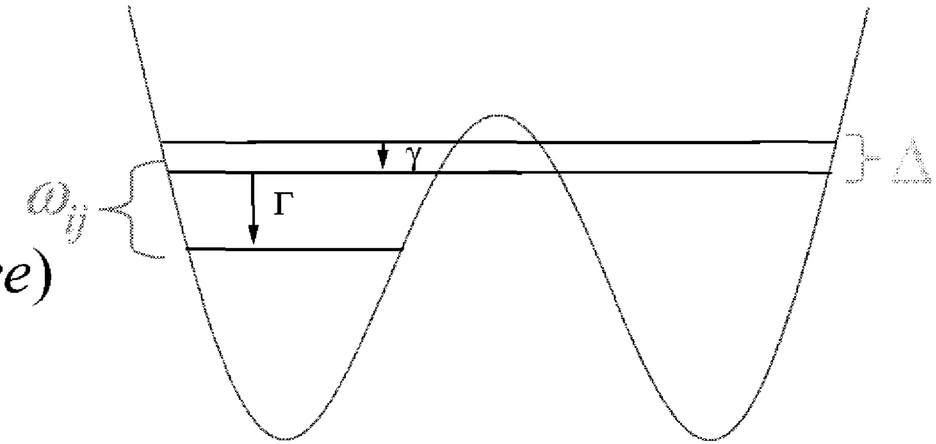
$\omega = \Delta \approx 7$ GHz:
$R_{eff} \approx 1.4\ \mathrm{M\Omega} \Rightarrow \gamma \approx 3.6$ MHz $\Rightarrow$ line width = 180 nΦ_0

Damping

Intrinsic line width from interwell and intrawell relaxation (spontaneous decay) caused by resistive environment:

$$\Gamma = 2\pi\omega_{ij} \frac{R_Q}{R} \frac{\left|\Phi_{i,j}\right|^2}{\Phi_0^2}$$

$$\gamma = 4\pi\Delta \frac{R_Q}{R} \frac{\left|\delta\Phi\right|^2}{\Phi_0^2} \quad (on\,resonance)$$

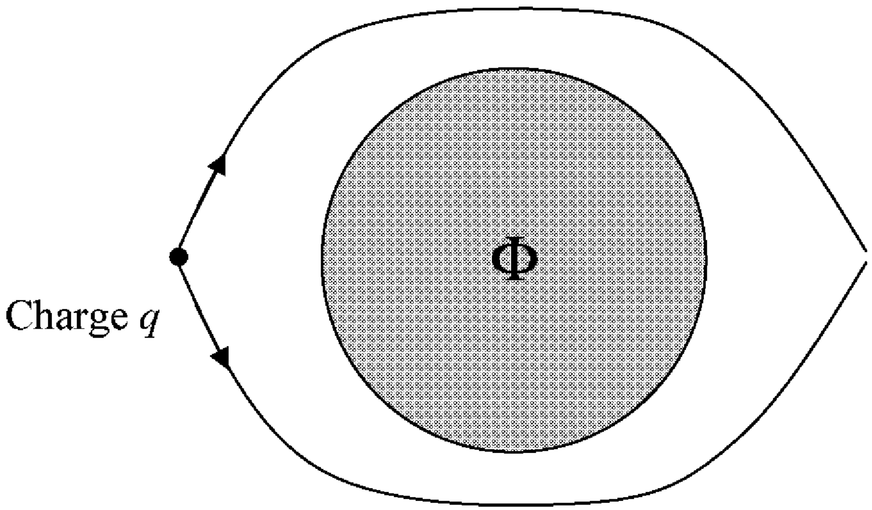


Inhomogeneous broadening due to low-frequency flux noise from PdAu case.

Calculated flux noise using SQUID geometry:

$0.1\ m\Phi_0 \approx$ measured width of peaks.

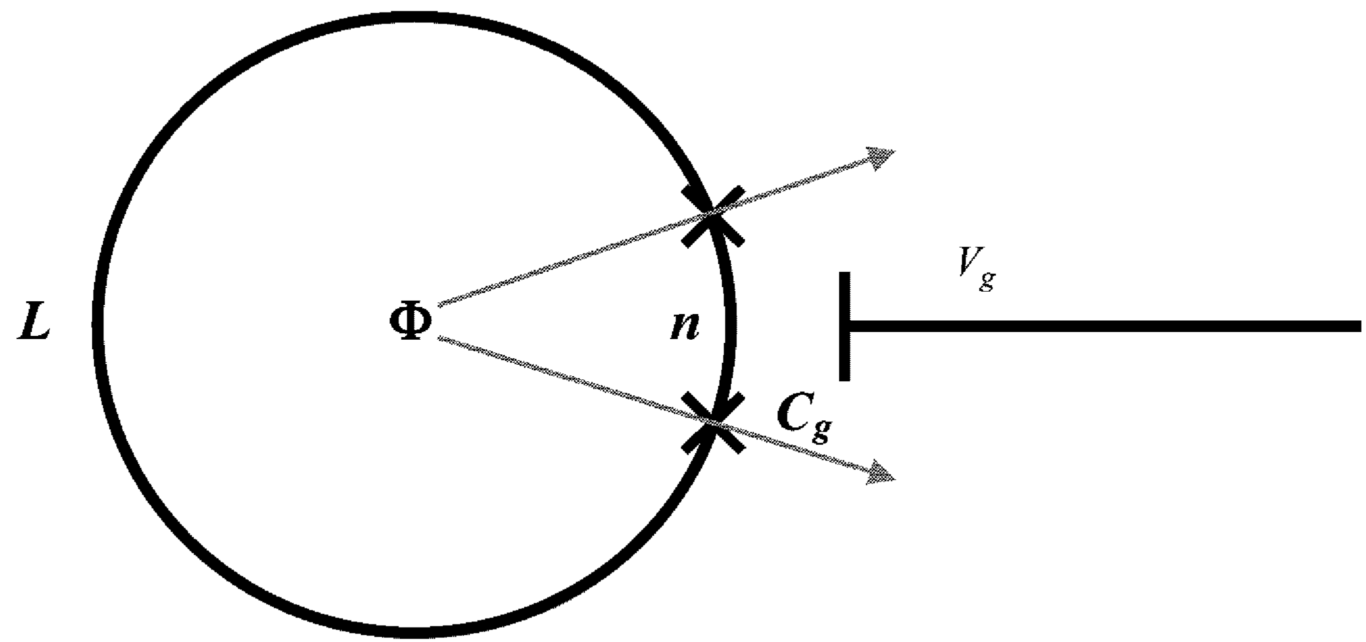
Aharonov-Bohm Effect



Interference pattern is modulated by flux Φ with period $\Phi_0 = hc/q$

$$H = \frac{(\mathbf{p} - q\mathbf{A}/c)^2}{2m} + \cdots$$

Aharonov-Casher Effect in a SQUID



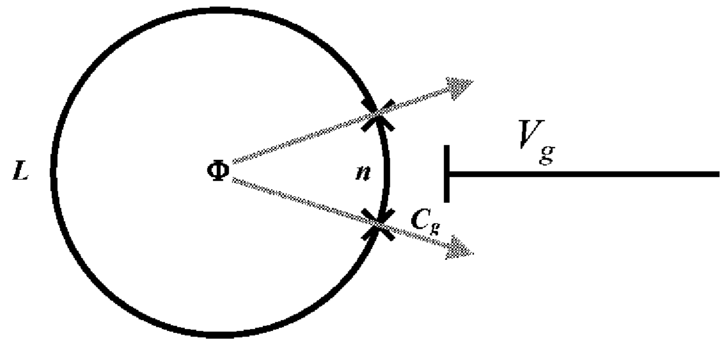
- Interference between tunneling paths leads to suppression of tunneling.
- Could be useful for qubits as a way of freezing the quantum state.

Hamiltonian for A-C-Effect SQUID

$$H = \frac{Q^2}{2C} + \frac{(2en-q)^2}{2C_\Sigma} + \frac{(\Phi-\Phi_x)^2}{2L} - E_{J1}\cos\phi_1 - E_{J2}\cos\phi_2 \qquad q \equiv C_g V_g$$

$$= \frac{Q^2}{2C} + \frac{(2en-q)^2}{2C_\Sigma} + \frac{(\Phi-\Phi_x)^2}{2L} - 2E_J\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\cos\theta, \qquad E_{J1} = E_{J2} = E_J$$

$$2\pi\Phi/\Phi_0 = \phi_1 + \phi_2 + 2\pi n \qquad \theta = (\phi_1 - \phi_2)/2$$



Hamiltonian for A-C-Effect SQUID

Induced charge.
Plays the role of a scalar gauge potential.

$$q \equiv C_g V_g$$

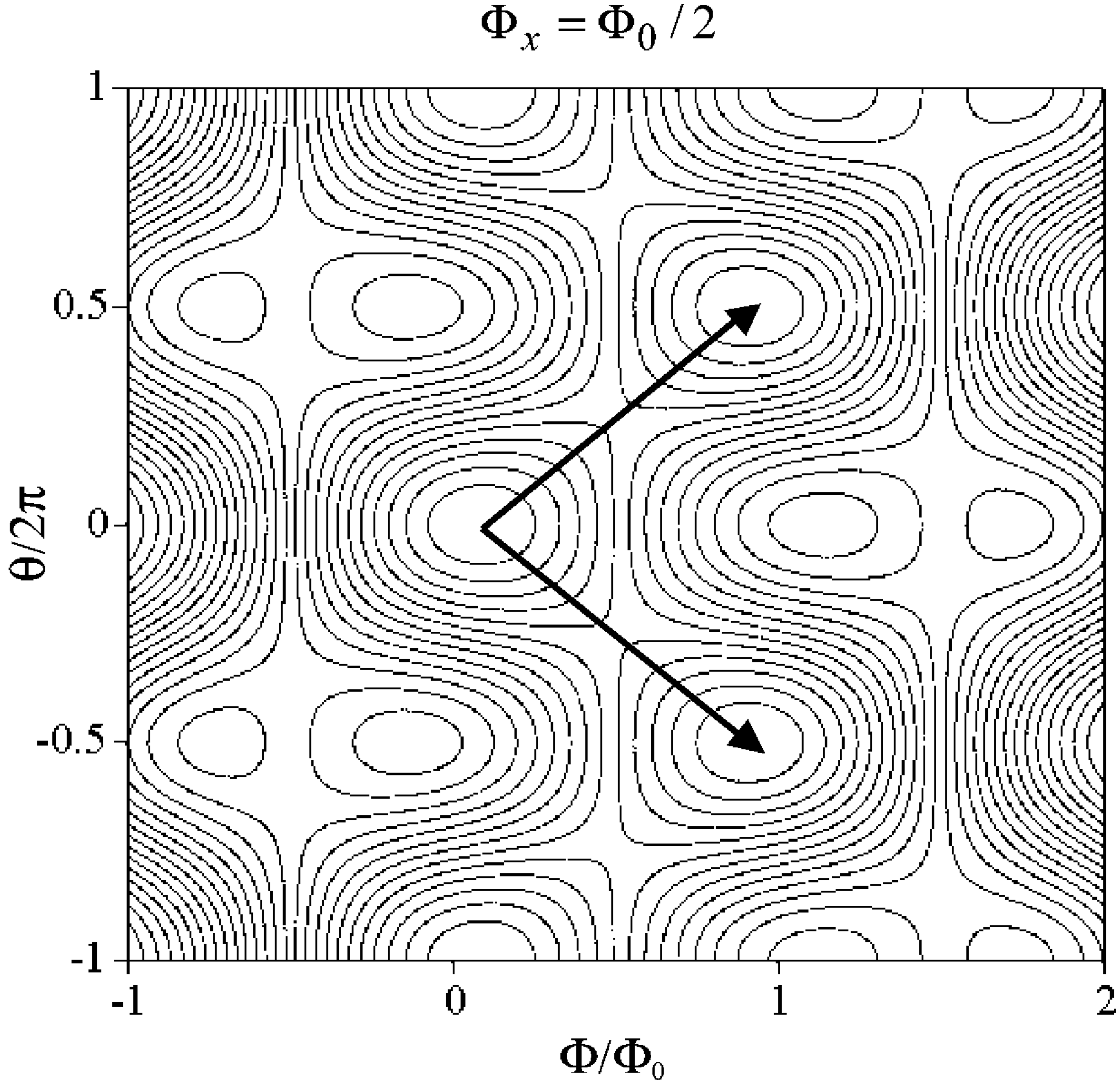
$$H = \frac{Q^2}{2C} + \frac{(2en-q)^2}{2C_\Sigma} + \frac{(\Phi-\Phi_x)^2}{2L} - 2E_J \cos\left(\frac{\pi\Phi}{\Phi_0}\right)\cos\theta$$

Two degrees of freedom:

$$[\Phi, Q] = i\hbar$$

$$[\theta, n] = i$$

2D Potential for A-C-Effect SQUID



$$U = \frac{(\Phi - \Phi_x)^2}{2L} - 2E_J \cos\left(\frac{\pi\Phi}{\Phi_0}\right)\cos\theta$$

Tight-binding model.
Valid for large E_J.

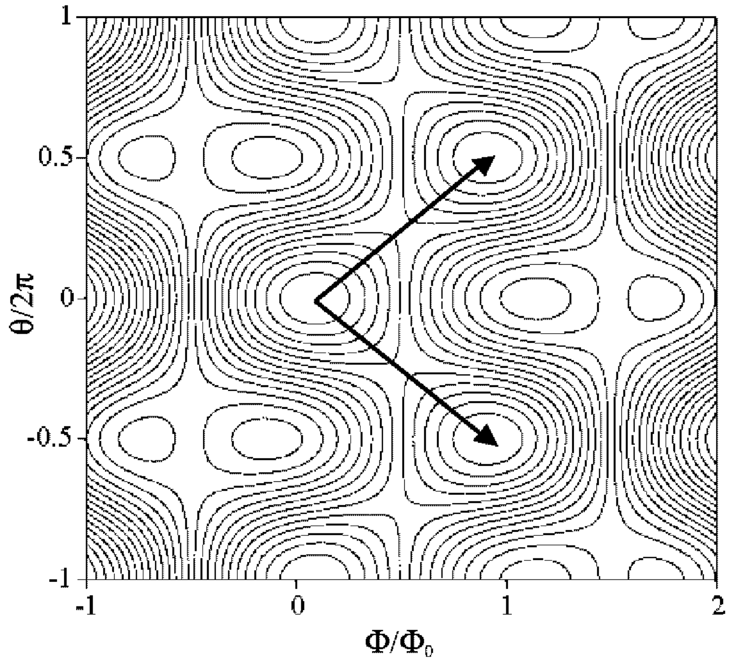
Instanton Calculation

Tunnel splitting:

$$\Delta = \sum_{paths\ j} \omega_j e^{-S_I^j/\hbar}$$

$$S_I^j = \int_{-\infty}^{\infty} \mathcal{L}_E(\tau)\, d\tau$$

$$\tau = it$$



$$\mathcal{L}_E(\tau) = \frac{C}{2}\dot{\Phi}^2 + \frac{C_\Sigma}{2}\left(\frac{\Phi_0\dot{\theta}}{2\pi}\right)^2 + \frac{(\Phi-\Phi_x)^2}{2L} - 2E_J\cos(\pi\Phi/\Phi_0)\cos\theta - iq\left(\frac{\Phi_0\dot{\theta}}{2\pi}\right)$$

Only this term is different for the two paths.

Instanton Calculation

$$S_{geo}^{1,2} = -iq\left(\frac{\Phi_0}{2\pi}\right)\int_{path\,1,2} \dot{\theta}\,d\tau$$

$$= \mp iq\left(\frac{\Phi_0}{2\pi}\right)\pi$$

$$= \mp i\pi\left(q/2e\right)\hbar$$

$$\Delta = \omega_0 e^{-\tilde{S}_I/\hbar} \sum_{paths\ j} e^{-S_{geo}^{j}/\hbar}$$

Suppression of Tunneling

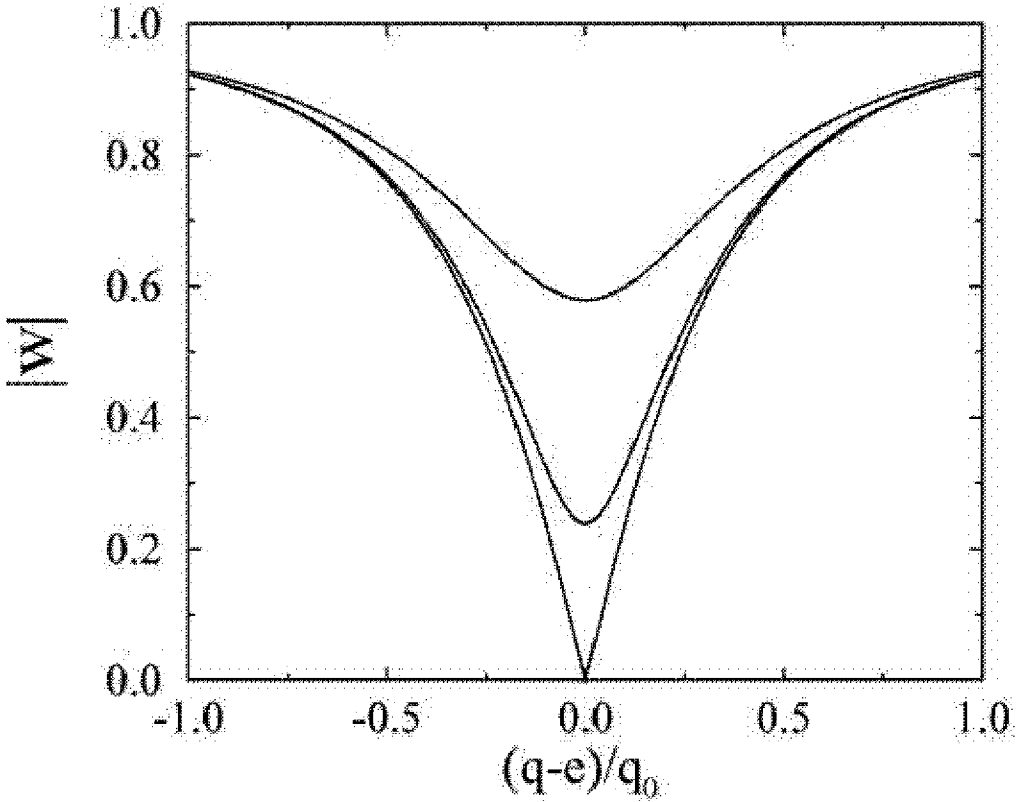
$$E_C = \frac{e^2}{2C_\Sigma} \ll E_J$$

$$\Delta = 2\Delta_0 \cos(q\pi / 2e)$$

Tunnel splitting changes sign at $q = e$. Ground and excited states interchange roles:

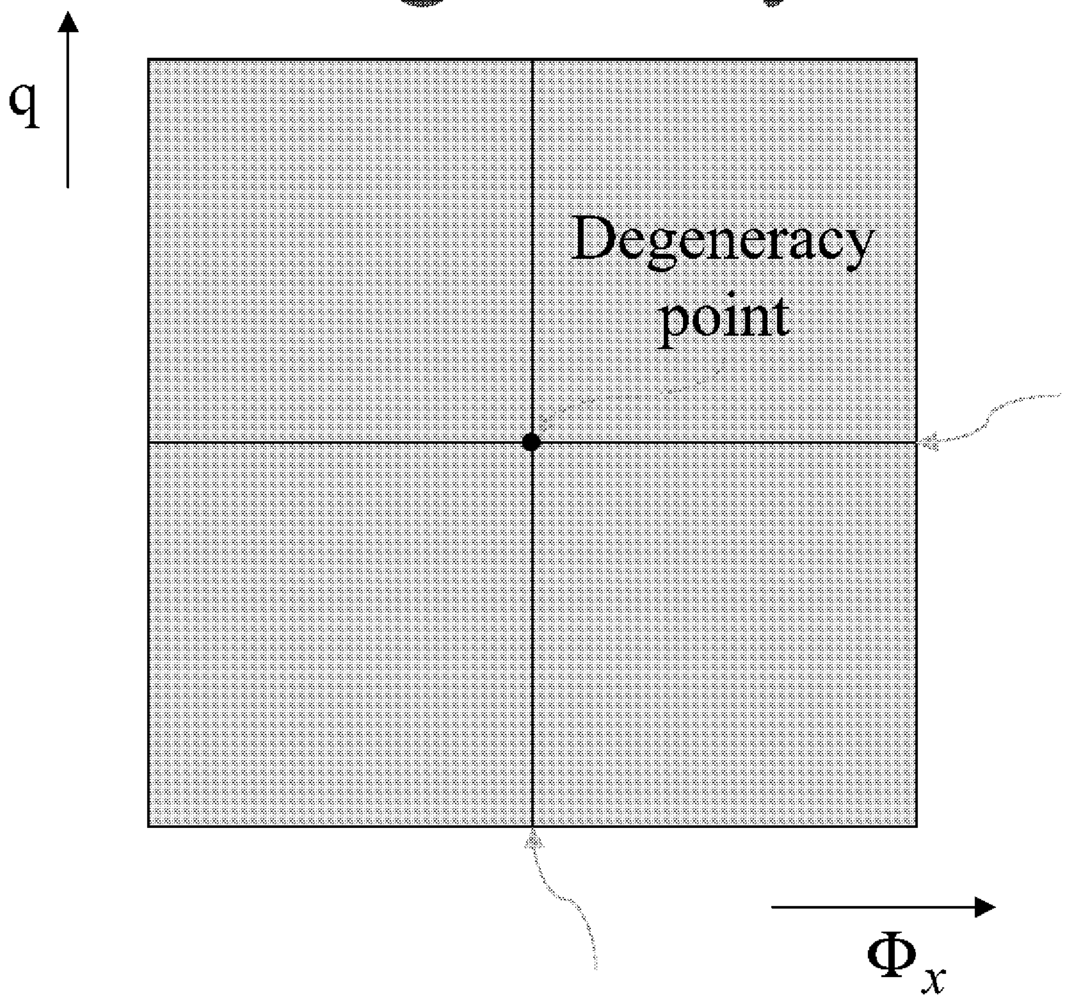
$$\left(|0\rangle + |1\rangle\right) \Leftrightarrow \left(|0\rangle - |1\rangle\right)$$

$$E_C \gg E_J$$



J. R. Friedman and D. V. Averin, PRL, 2002.

Degeneracy of Eigenstates



$q = e$

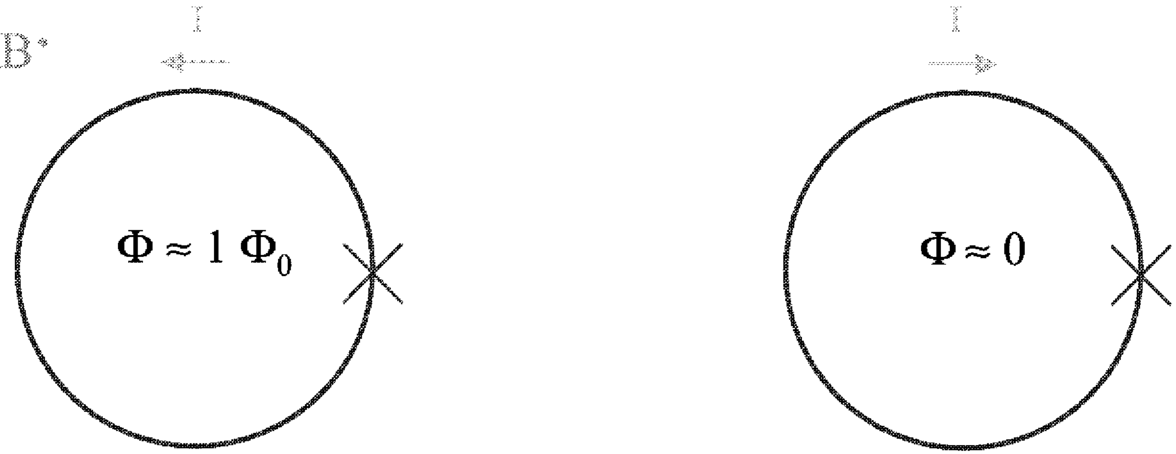
Eigenstates are symmetric and antisymmetric superpositions of charge states.

$\Phi_x = \Phi_0/2$

Eigenstates are symmetric and antisymmetric superpositions of flux states.

Conclusions

- Anticrossing of two excited states in an rf-SQUID.
- Schrödinger's Cat (Macroscopic Quantum Coherence) of flux states:
  - The flux states differ by ~ 1/4 Φ_0, ~1 μA and ~10^{10} μ_B.



- Future Experiment?: Suppression of Flux Tunneling by the Aharonov-Casher-Effect