

SMR/1438 - 10

JOINT ICTP-INFN SCHOOL/WORKSHOP ON  
"ENTANGLEMENT AT THE NANOSCALE"

(28 October – 8 November 2002)

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*"Adiabatic charge transport in arrays of Josephson junctions  
and the Cooper pair pump "*

presented by:

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Finland



# **Adiabatic charge transport in arrays of Josephson junctions and the Cooper pair pump**

Jukka Pekola, Helsinki University of Technology,  
Finland

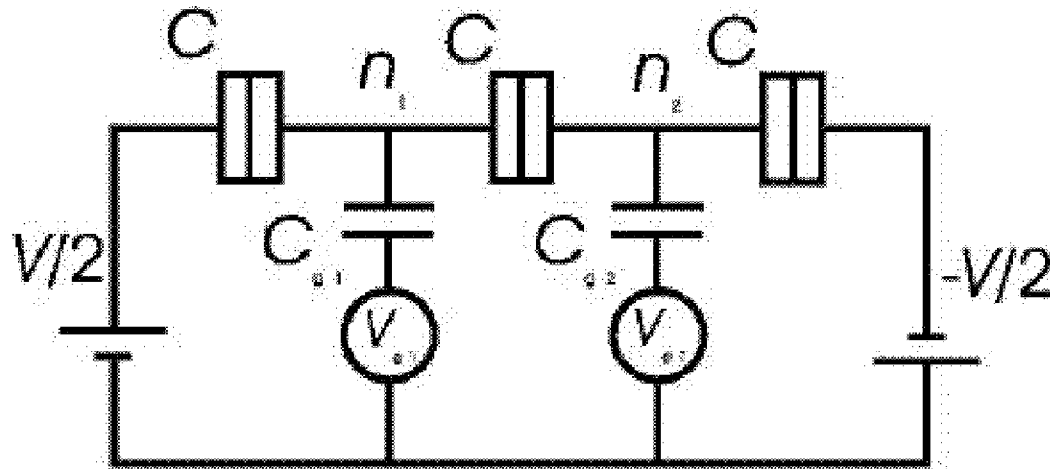
with Jussi Toppari and Matias Aunola, University  
of Jyväskylä, Finland

Dmitri Averin (SUNY), Rosario Fazio (Pisa),  
Frank Hekking (UJF Grenoble)

# Contents

1. Controlled charge transport in various systems
2. Focus on Josephson arrays and pumps
3. Adiabatic transport
4. Measurement of current
5. Experimental issues

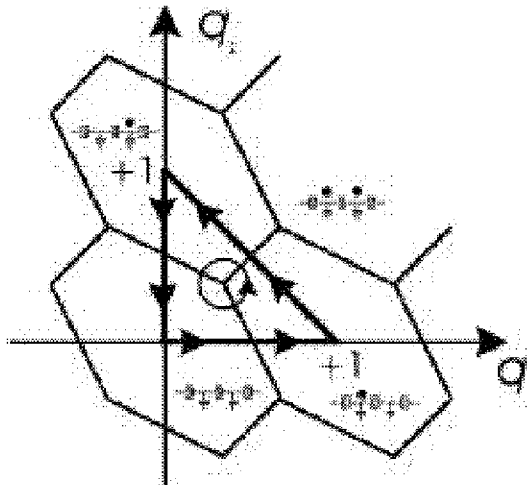
# Pumping of single electrons in a 3-junction pump



$$I \sim 1\text{pA}$$

$$(f \sim 10\text{MHz})$$

$$\delta I/I \sim 0.01$$



H. Pothier, P. Lafarge, C. Urbina, D. Esteve, M. Devoret, EPL 17, 249 (1992).

# Metrological charge pump

## Accuracy of electron counting using a 7-junction electron pump

Mark W. Keller<sup>a)</sup> and John M. Martinis

*National Institute of Standards and Technology, Boulder, Colorado 80303*

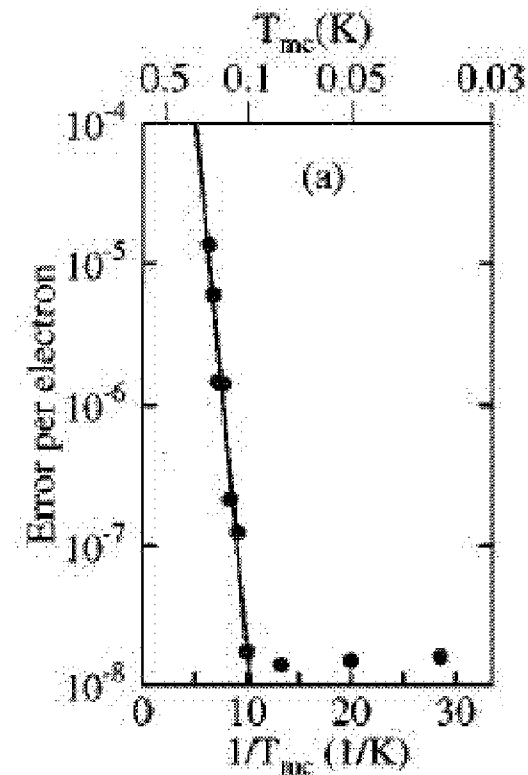
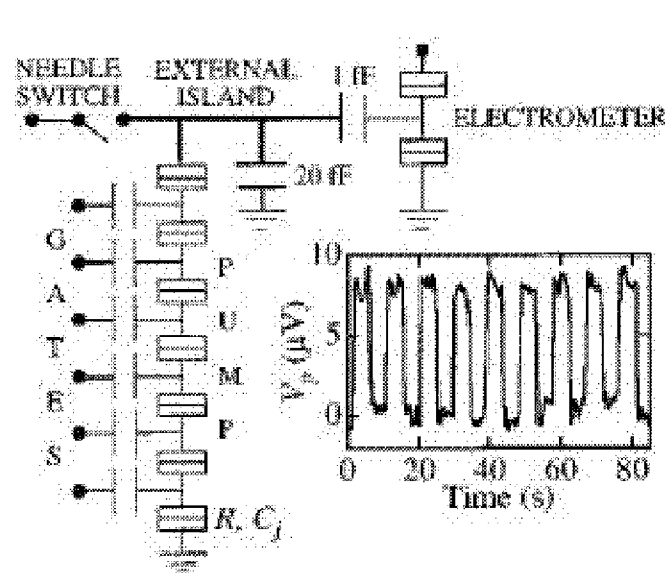
Neil M. Zimmerman

*National Institute of Standards and Technology, Gaithersburg, Maryland 20899*

Andrew H. Steinbach<sup>b)</sup>

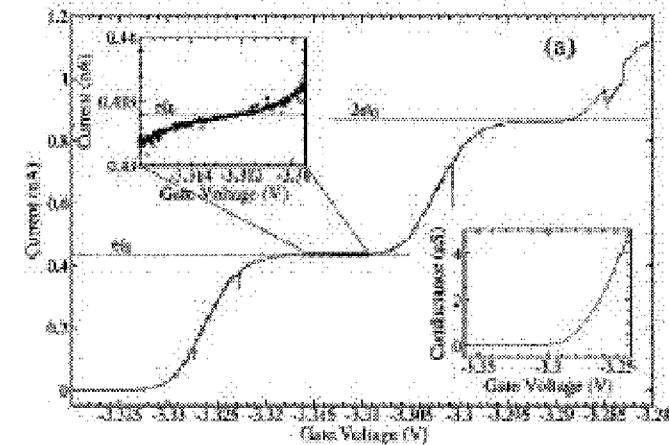
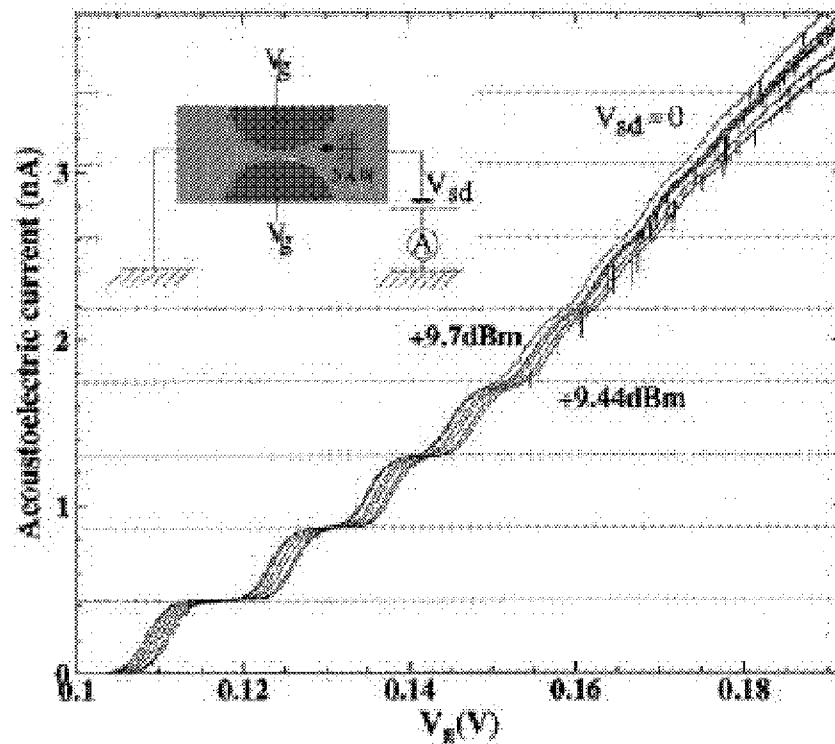
*National Institute of Standards and Technology, Boulder, Colorado 80303*

(Received 29 April 1996; accepted for publication 22 July 1996)



Appl. Phys. Lett. 69, 1804 (1996).

# Surface acoustic wave (SAW) driven $I = ef$



**J. Shilton et al., J. Phys. C 8, L531 (1996).**

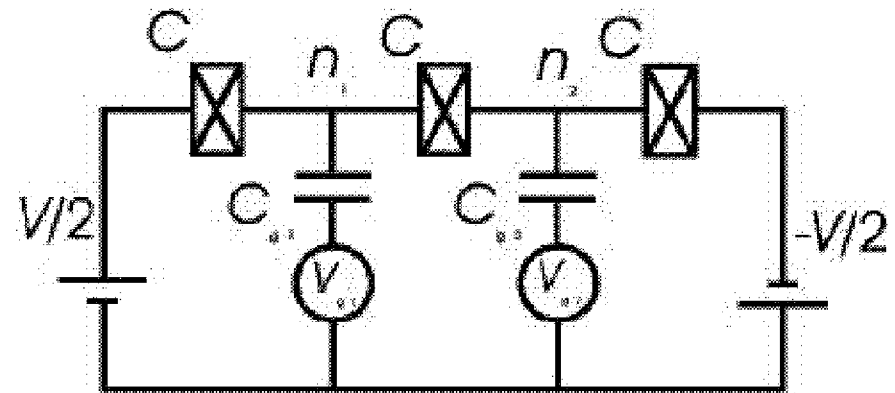
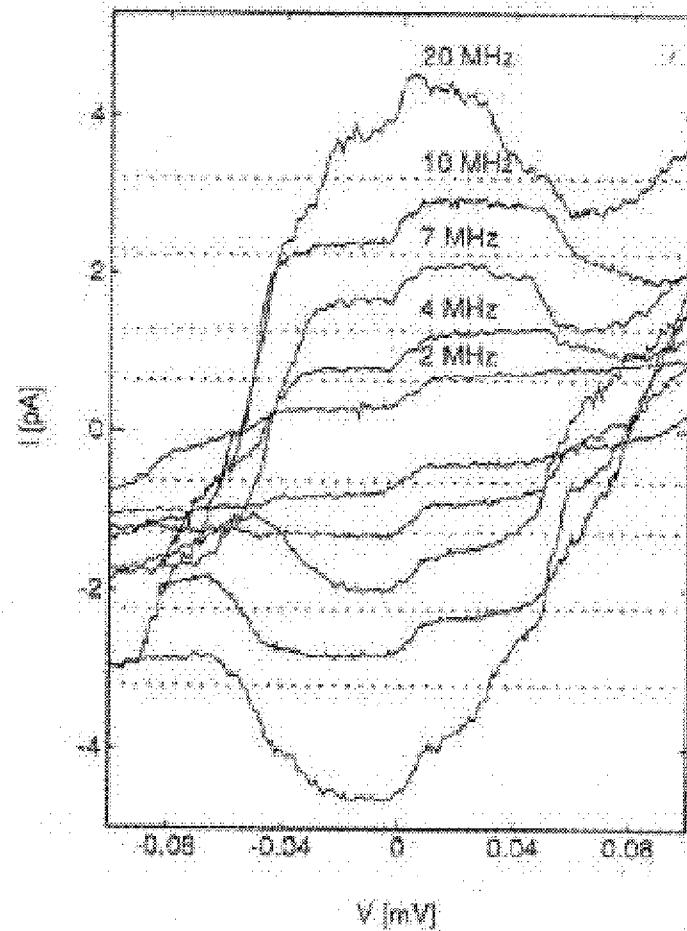
**V. Talyanskii et al., PRB 56, 15180 (1997).**

**J. Cunningham et al., PRB 62, 1654 (2000).**

**A. Robinson et al., PRB 65, 045313 (2002).**

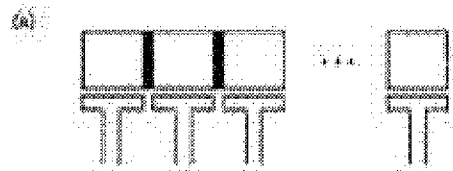
**J. Pekola et al., PRB 50, 11255 (1994).**

# 3 junction CPP

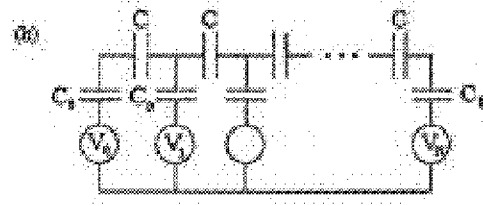


**L. Geerligs et al., Z. Phys. B: Condensed Matter 85, 349 (1991).**

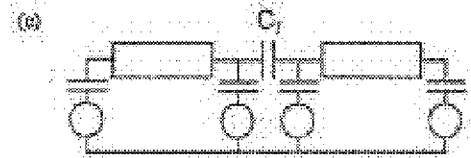
# Adiabatic manipulation of Cooper pairs in arrays of JJs



**Adiabatic quantum computing, D. Averin (1998).**



**Topologically stable Josephson qubits, Ioffe, Feigelman (2002).**

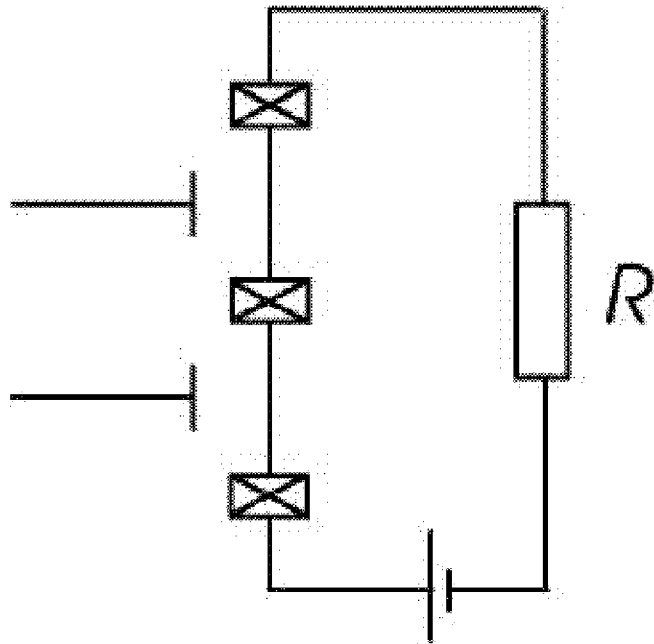


**Cooper pair manipulations in geometric qubits, Catania group (2000-).**

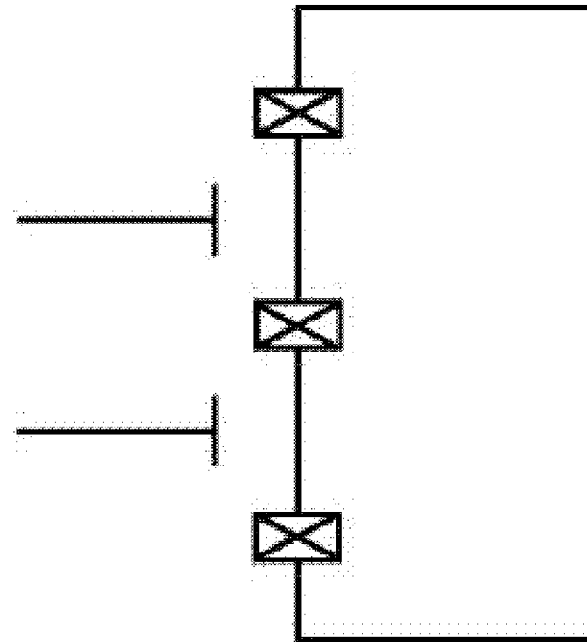
# **Coherent and incoherent Cooper pair pumping**

- 1. Ideal coherent and incoherent  
(adiabatic) pumping**
- 2. Phase fluctuations in dissipative  
environment**
- 3. Inductance limited phase  
fluctuations**

# Coherent vs incoherent Cooper pair pumping



R-pump,  $R \gg R_Q$



Coherent pump

S. Lotkhov, S. Bogoslovsky, A. Zorin, J. Niemeyer, APL 78, 946 (2001).

# **Why Cooper pair pump?**

- 1. New experimental method to measure dissipation and decoherence**
- 2. Multi-gate control of multi-junction qubits**
- 3. Metrological applications**

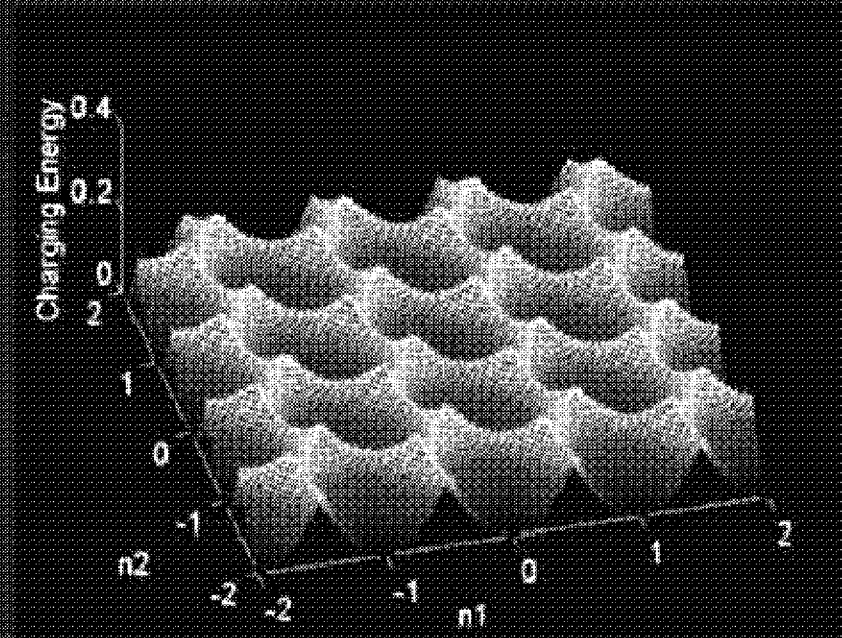
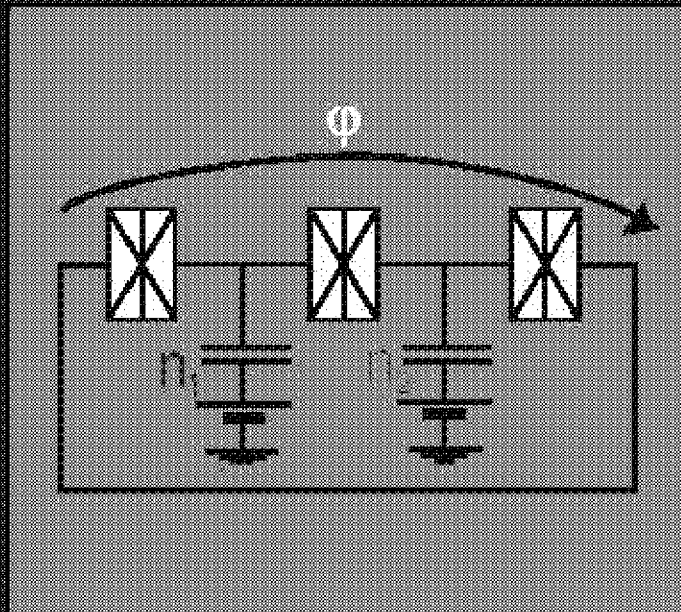
# Charging energy

## Cooper pair pump

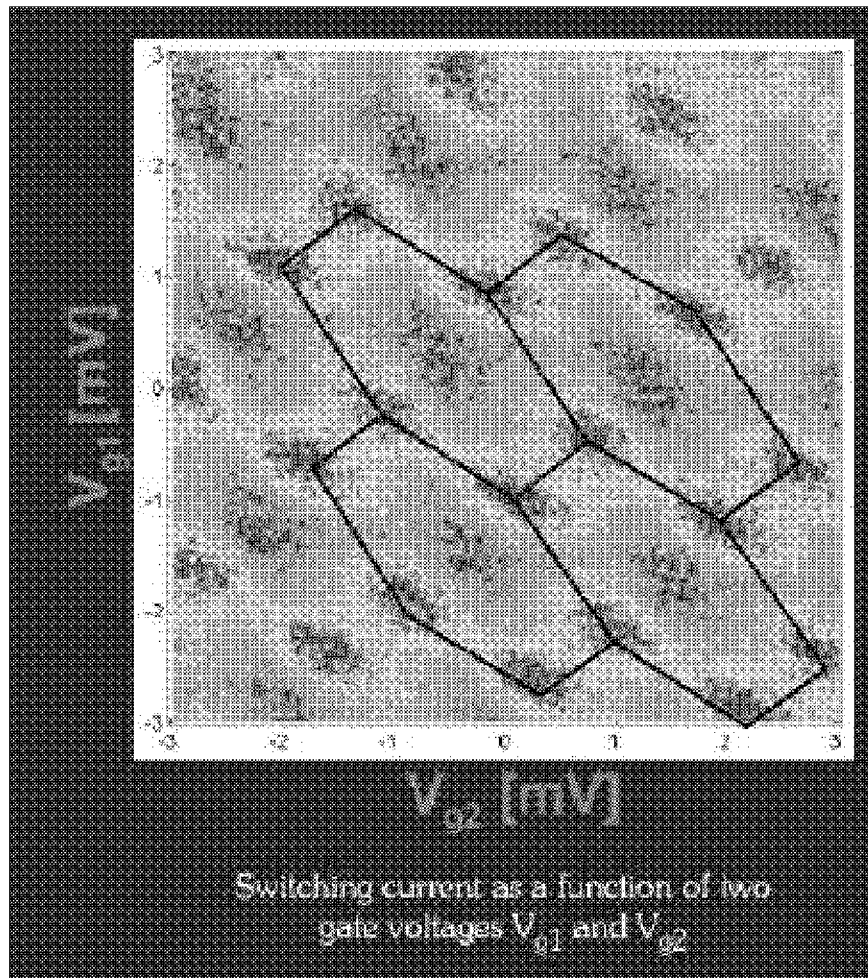
$$H_C = (2Ec/3)[(N_1 - n_1)^2 + (N_2 - n_2)^2 + (N_1 - n_1)(N_2 - n_2)]$$

Ground state energy in  $n_1, n_2$ -plane

*(Lecture 10 slide 11)*



# Experimental determination of the gate dependences



**J. Toppari et al., unpublished**

# Adiabatic transport of Cooper pairs

If the Hamiltonian of the system varies slowly enough, we may study its behaviour using the adiabatic approximation of quantum mechanics.

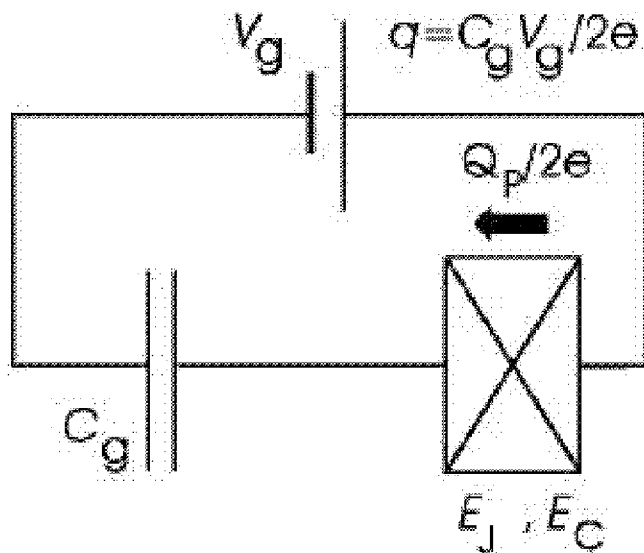
$\hat{H} = \hat{H}_C + \hat{H}_J$  varies due to gate operations

$$Q_P = 2\hbar \Im \left[ \sum_{n \neq m} \oint \frac{(\hat{I}_l)_{mn}}{E_m - E_n} \langle n | \partial_{\vec{q}} m \rangle \cdot d\vec{q} \right]$$

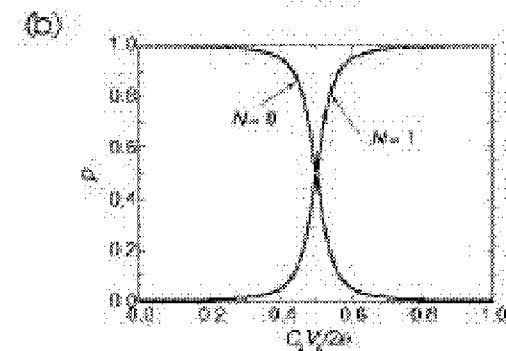
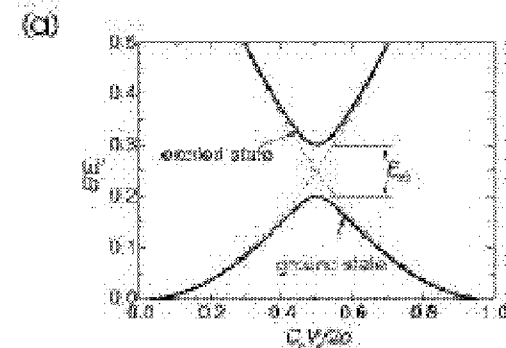
( $m = 0$  for transport in the ground state)

**J. Pekola, J. Toppari, M. Aunola, M. Savolainen, D. Averin, PRB 60, 9931 (1999).**

# Adiabatic transport in a Cooper pair box



$$Q_P = 2\hbar\Delta m \left[ \sum_{n \neq m} \oint \frac{(J_t)_{mn}}{E_m - E_n} \langle n | \partial_{\vec{q}} m \rangle \cdot d\vec{q} \right]$$

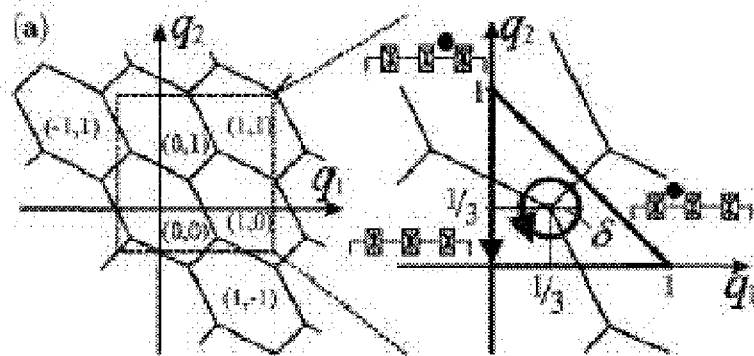


$$Q_P / (2e) = [\eta_f / \sqrt{1 + \eta_f^2} - \eta_i / \sqrt{1 + \eta_i^2}] / 2$$

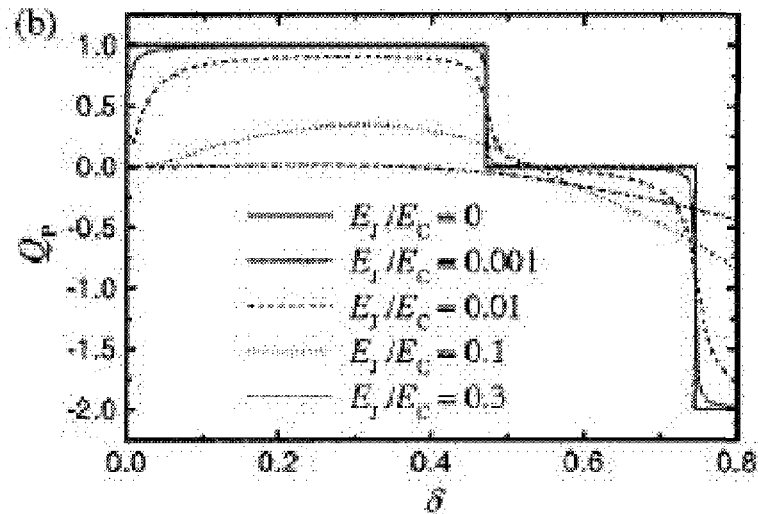
$$\eta_{i,f} = (E_1 - E_2)_{i,f} / E_J$$

(The same result can be seen directly from the composition of the ground state!)

# Coherent adiabatic CPP, basic results



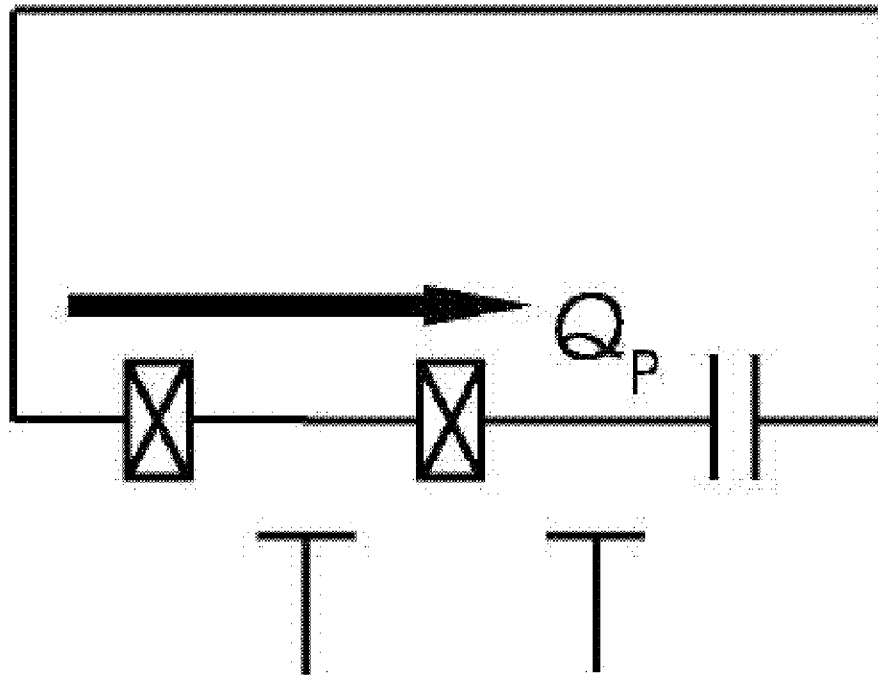
$$Q_P = 2\hbar\Im m \left[ \sum_{n \neq m} \oint \frac{(I_t)_{mn}}{E_m - E_n} \langle n | \partial_{\vec{q}} m \rangle \cdot d\vec{q} \right]$$



$$I_p/(2ef) = 1 - 9E_J/E_C \cos \varphi$$

(triangular path)

# Cooper pair trap



As a CPP, but instead of the third JJ there is a (classical) capacitor

$$\frac{Q_P}{2e} = 1$$

# Adiabatic transport vs. Landau-Zener band-crossing

Adiabatic approximation is  
valid, if

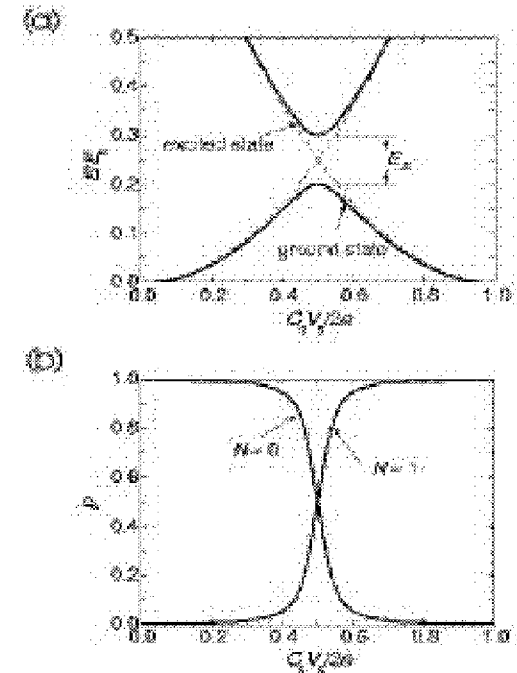
$$|\langle 1 | \dot{0} \rangle| \ll (E_1 - E_0) / \hbar$$

Band-crossing (LZ) becomes  
important, if this condition is  
violated.

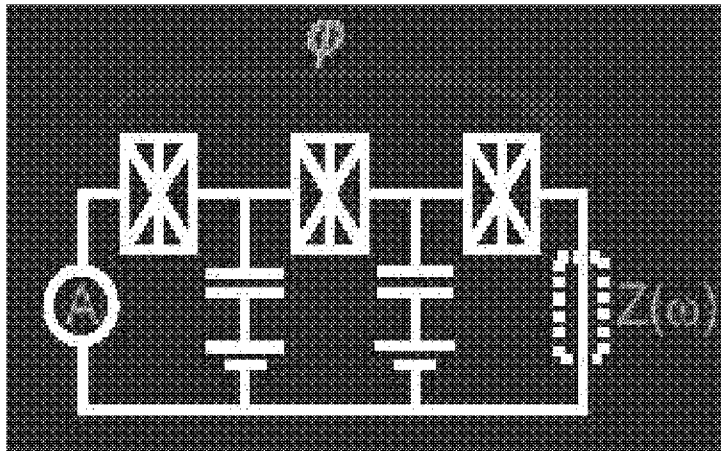
For periodic gate operation this  
happens at

$$f_{\text{LZ}} \sim \frac{E_J^2}{\hbar E_C}$$

$$H = H_J + H_C$$



# Phase fluctuations due to environment



J. Pekola and J. Toppari, PRB 64, 172509 (2001).

**Simple idea:**

**Determine the fluctuations of  $\varphi$  (in real time).**

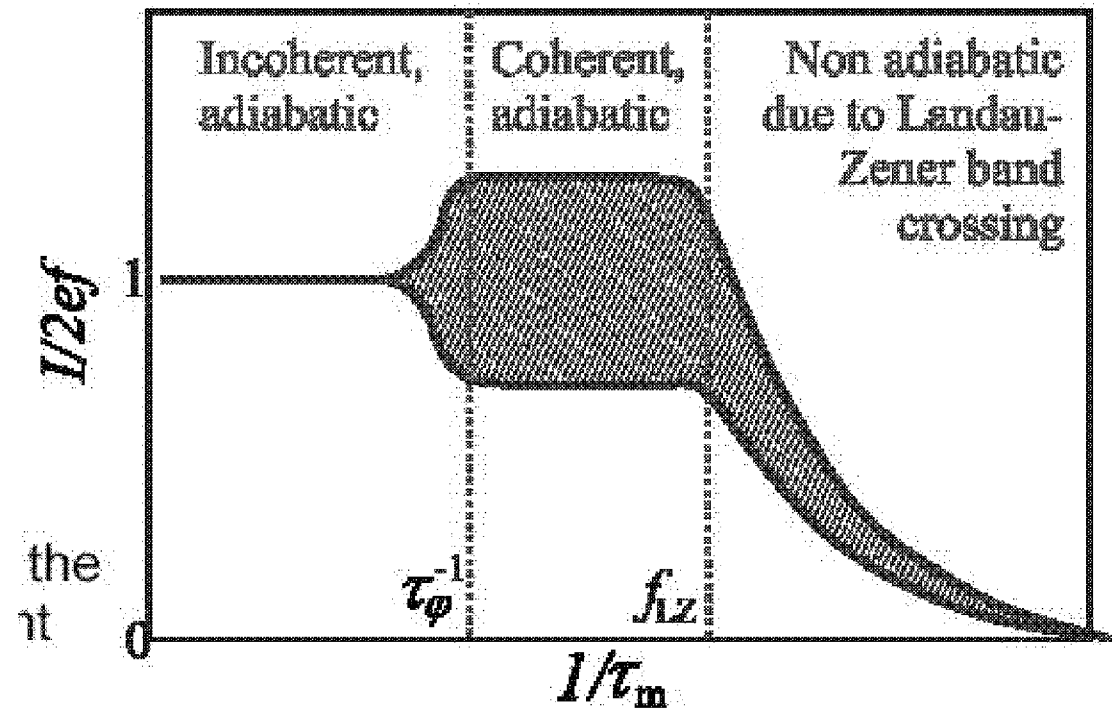
**If  $\langle (\Delta\varphi)^2 \rangle \ll (\pi/2)^2$ , pumping is coherent**

**If  $\langle (\Delta\varphi)^2 \rangle \gg (\pi/2)^2$ , pumping is incoherent**

**In the latter case:**

$$Q_P/2e = 1 - 9E_J/E_C \langle \cos \varphi \rangle \rightarrow 1$$

# Pumping regimes determined by Landau-Zener band-crossing and decoherence



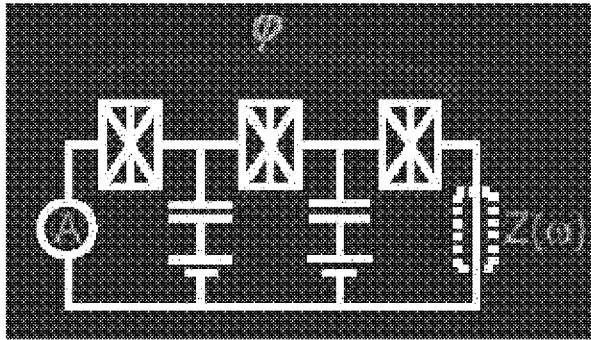
# Phase fluctuations due to environment

FDT: 
$$J(t) = 2 \int_0^\infty \frac{d\omega}{\omega} \frac{\text{Re} Z_L(\omega)}{R_K} \times \left\{ \coth\left(\frac{\hbar\omega}{2k_B T}\right) [\cos(\omega t) - 1] - i \sin(\omega t) \right\}$$

$$\Delta\varphi \equiv \varphi(t) - \varphi(0)$$

$$\langle (\Delta\varphi)^2 \rangle = -2 \text{Re} J(t)$$

# Phase fluctuations: results in resistive environment



$$T = 0$$

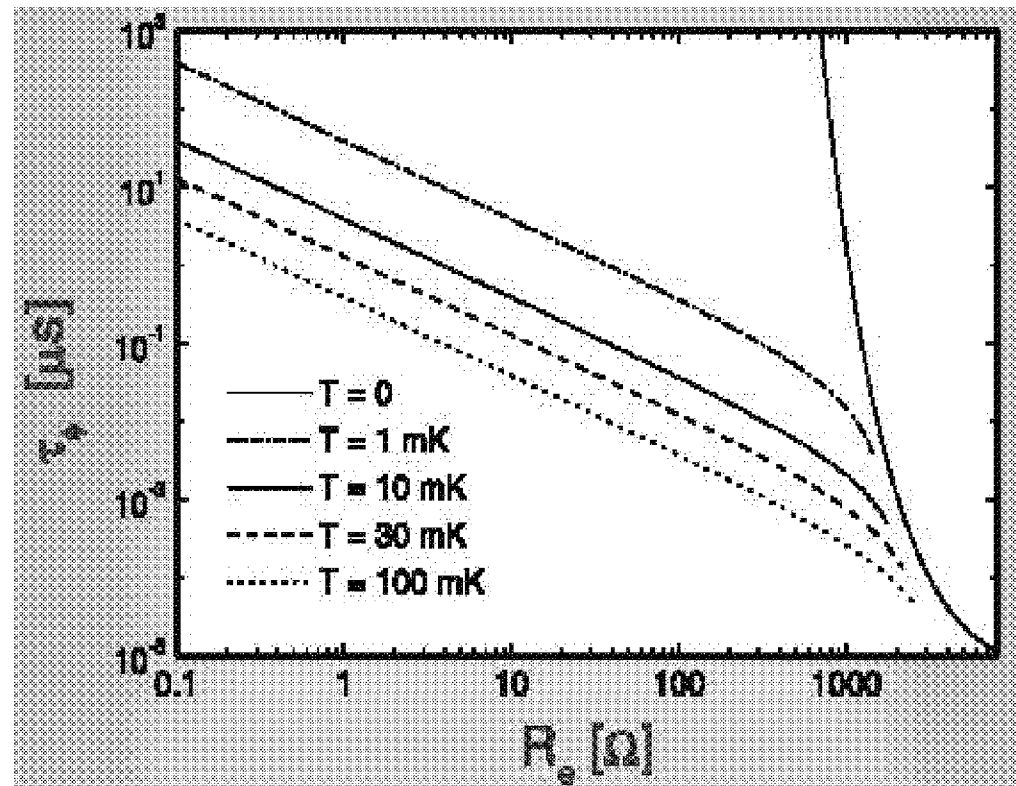
$$\langle (\Delta\varphi)^2 \rangle = 4 \frac{R_e}{R_K} [\ln(t/\tau) + \gamma]$$

$$T > 0$$

$$\langle (\Delta\varphi)^2 \rangle = 4 \frac{R_e}{R_K} \left[ \frac{\pi k_B T}{\hbar} t - \ln \left( \frac{2 \pi k_B T \tau}{\hbar} \right) + \gamma \right]$$

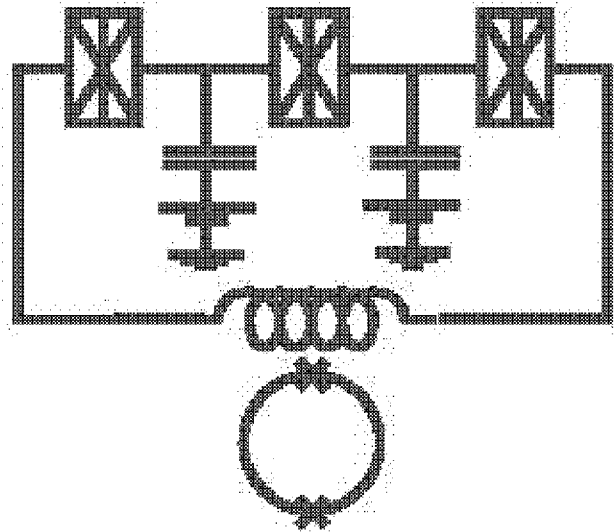
$$\tau_\varphi = \tau \exp \left( \frac{\pi^2}{16} \frac{R_K}{R_e} - \gamma \right) \quad (T=0)$$

$$\tau_\varphi \simeq \frac{\pi}{16} \frac{\hbar}{k_B T} \frac{R_K}{R_e} \quad (T>0)$$



$$\langle (\Delta\varphi)^2 \rangle = (\pi/2)^2 \text{ at } t = \tau_\varphi$$

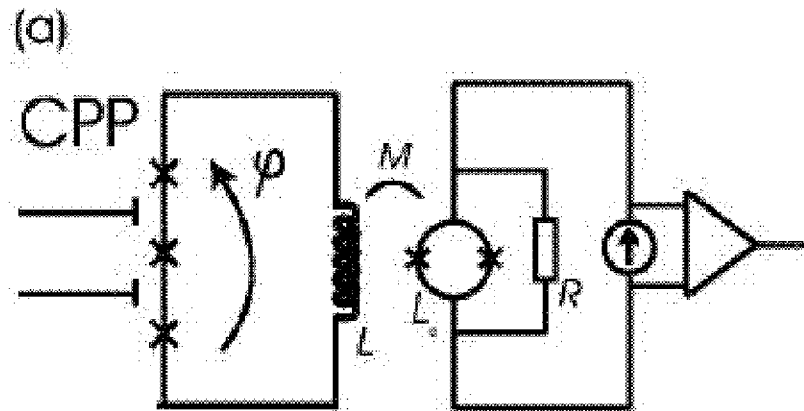
# Measurement schemes to avoid phase fluctuations?



Measuring current with  
SQUID ammeter ?

$$Q_P/(2e) = 1 - [9E_J/E_C - (2\pi^2/3)LE_J/\Phi_0^2] \cos \varphi$$

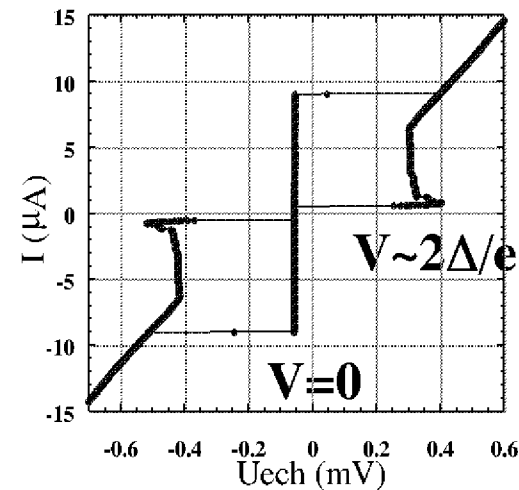
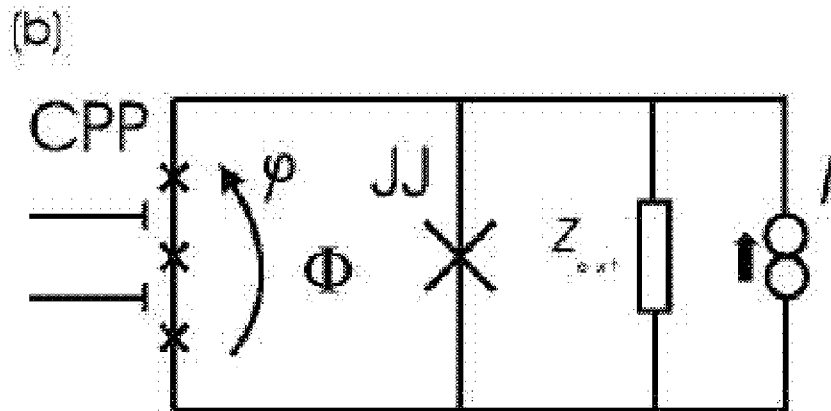
# SQUID amplifier or escape junction



Current measurement by:

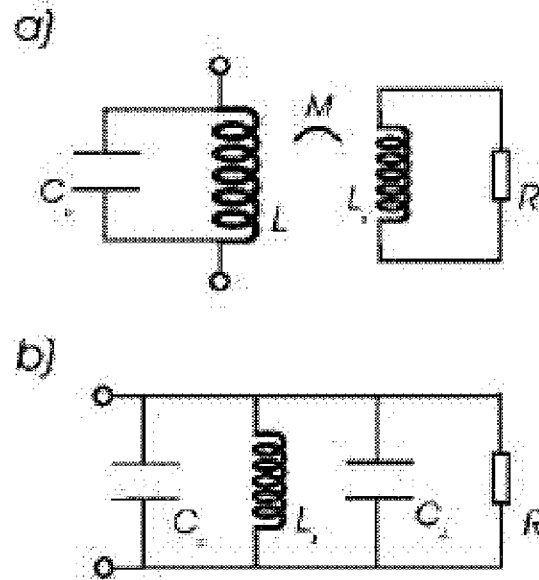
(a) SQUID amplifier

(b) Biased Josephson junction



F. Balestro, O. Buisson et al., Grenoble

# Circuit models of SQUID amplifier and escape measurement

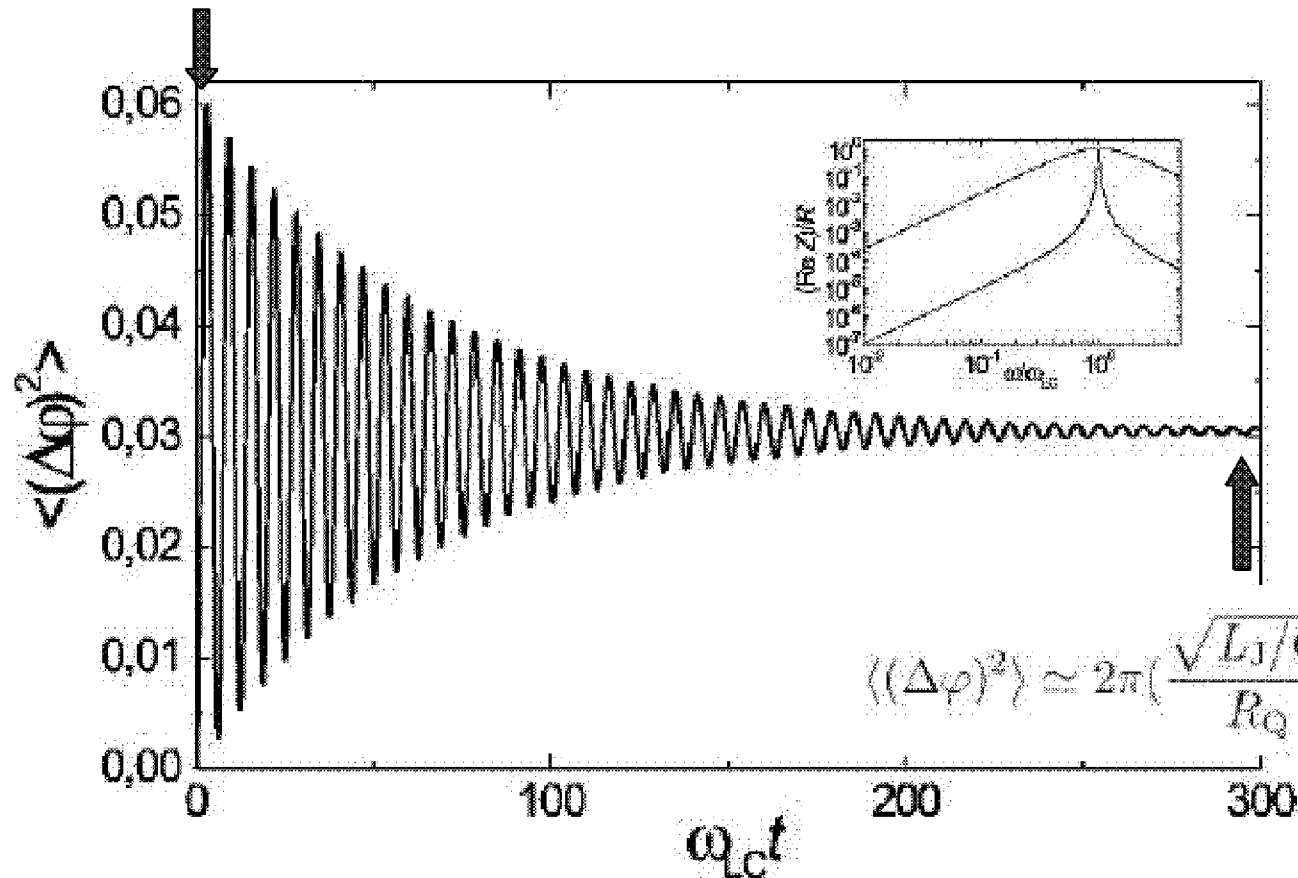


# Inductance limited dephasing (underdamped case)

$$\sqrt{L_J/C_J^*}/R \ll 1$$

$$\langle (\Delta\varphi)^2 \rangle \simeq 2\pi \left( \frac{\sqrt{L_J/C_J^*}}{R_Q} \right) \coth\left(\frac{\hbar\omega_{LC}}{2k_B T}\right) [1 - \cos(\omega_{LC}t)]$$

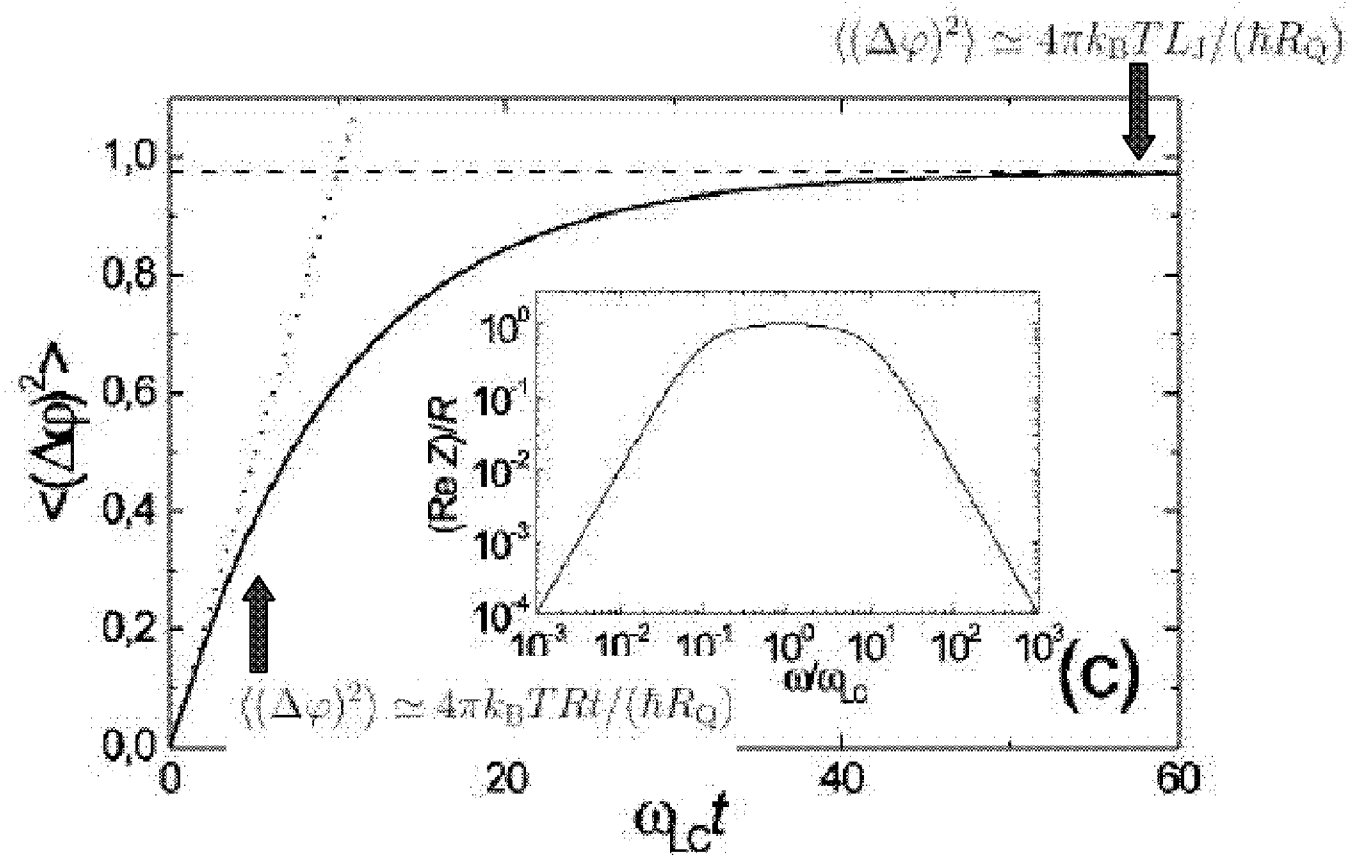
$$t_{\text{dec}} = 2RC_J^*$$



$$\langle (\Delta\varphi)^2 \rangle \simeq 2\pi \left( \frac{\sqrt{L_J/C_J^*}}{R_Q} \right) \coth\left(\frac{\hbar\omega_{LC}}{2k_B T}\right)$$

Fazio, Hekking, Pekola, unpublished (2002)

# Inductance limited dephasing (overdamped case)

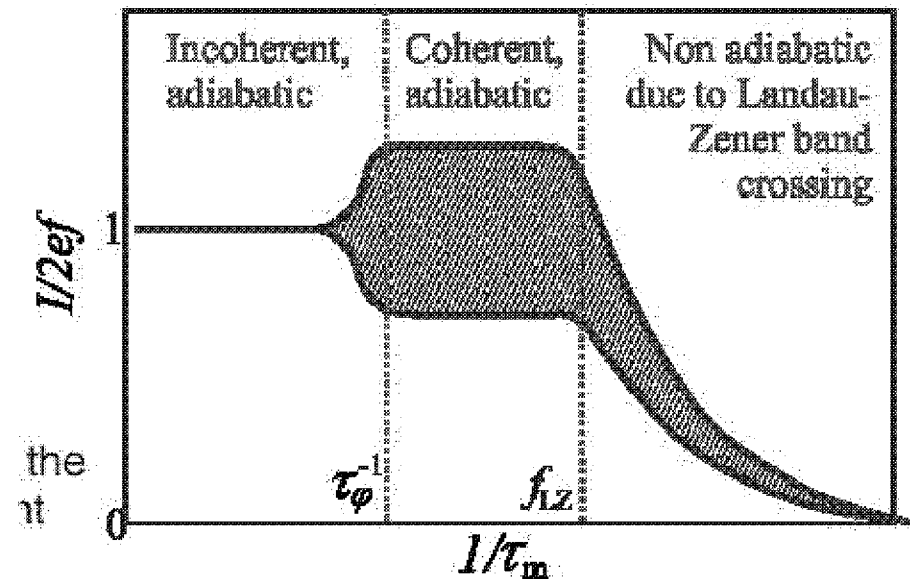


# Parameters of the planned measurement

Pumping frequency  $< 100$  MHz, current  $< 30$  pA (aluminium)

Niobium would allow  $> 1$  GHz and  $> 0.3$  nA (?)

Measurement bandwidth  $> \tau_\phi^{-1} \sim 1$  MHz



# Experiments on Nb structures

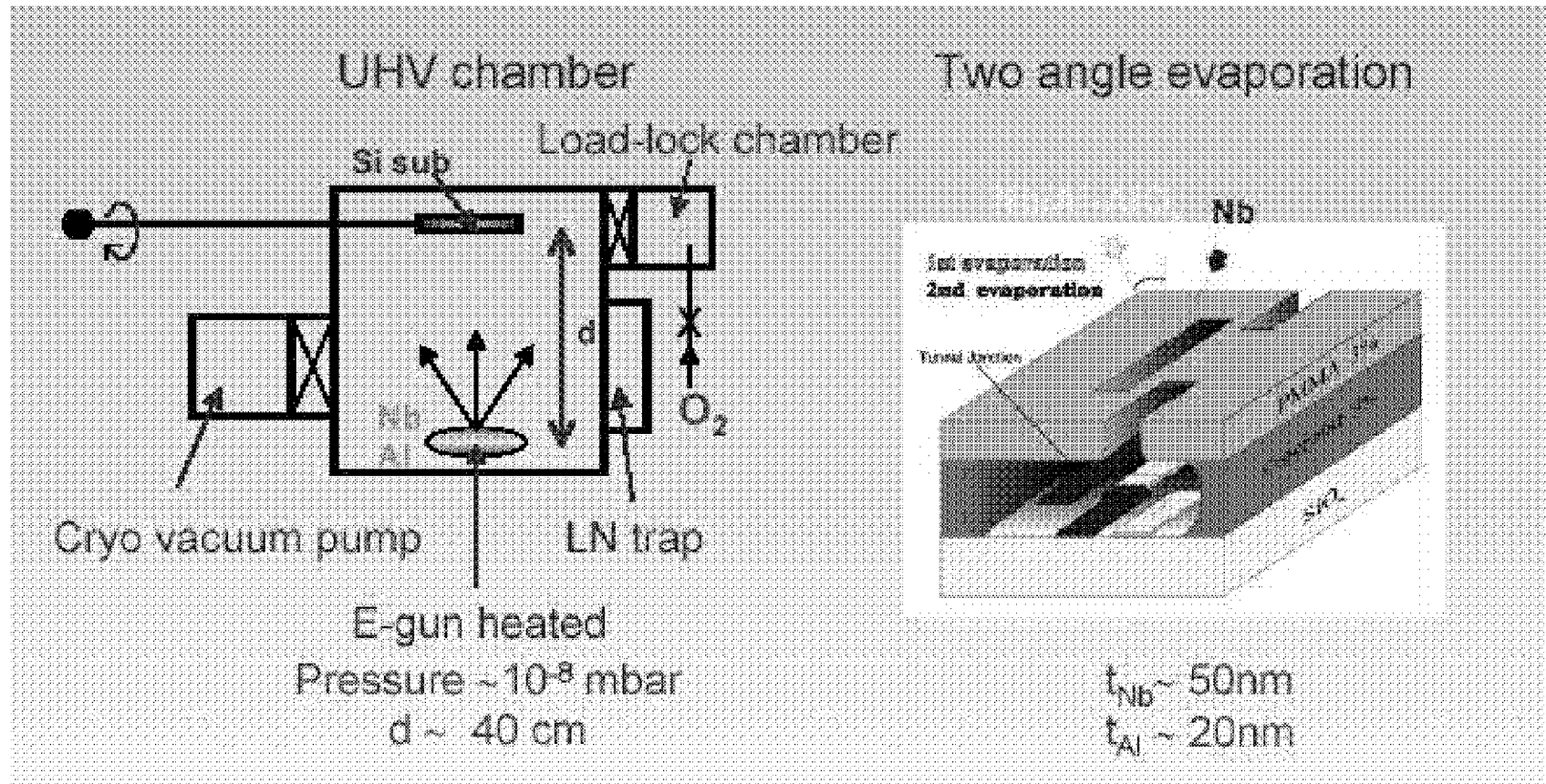
N. Kim, J. Toppari, L. Taskinen, S. Paraoanu, K. Hansen, J. P.

$$E_J \sim (R_Q/R) \cdot (\Delta/2)$$

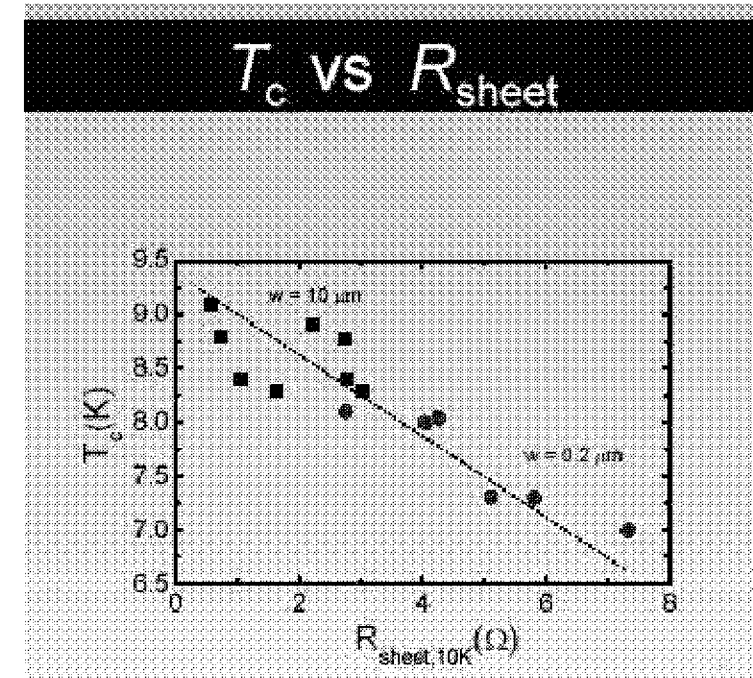
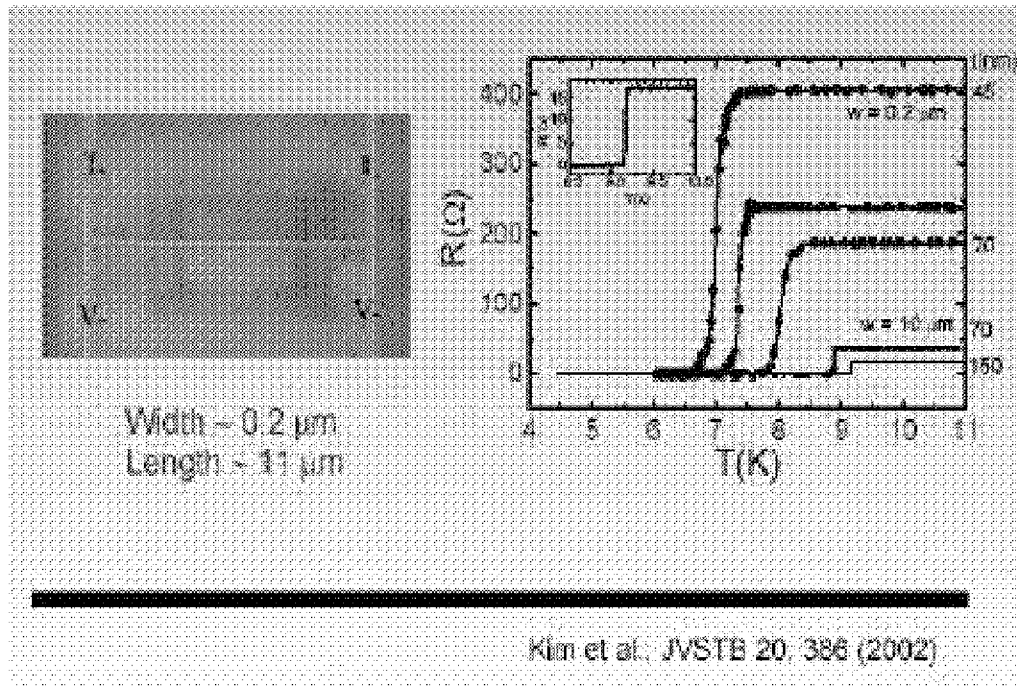
$$\Delta(\text{Nb}) = 1.5 \text{ meV} \approx 7.5 \Delta(\text{Al})$$

- multilayer technique  
Nb/(Al-)AlO<sub>x</sub>/Nb tri-layers.  
Pavolotsky *et al.* JVSTB(1994)
- two angle evaporation technique  
Nb/AlO<sub>x</sub>/Al, Harada *et al.* APL(1994)  
Nb/AlO<sub>x</sub>/Al, Dolata *et al.* APL(2002)

# Metallisation set-up



# Nb-wires



# Nb-SSETs

