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SMR 1232 - 33

XII WORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

17 - 28 July 2000

CROSSOVER FROM COHERENT TO INCOHERENT PAIRING IN THE SPIN FERMION MODEL FOR CUPRATES

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Crossover from coherent to incoherent pairing in the spin fermion model for cuprates

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collaborators:

Artem Abanov + Andrey Chubukov *(UW Madison)*
(spin fermion model)

SUMMARY OF THIS TALK

AF - QuantumCritical Point

↓

specific NON FERMI LIQUID

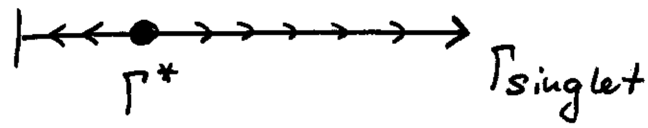
$Z_{q.p.} \sim \omega^{\eta}$; $\eta \simeq \frac{1}{2}$

$G^{-1}(\omega) \sim \omega^{1-\eta}$

↓

pairing problem Δ

Non F.L. unstable



$T_{inst.} \sim$ upper cut off

$\Rightarrow$ singlet gap Δ

↓

mean field theory
(1/N expansion)
at the QCP

- $\Delta \sim$ cut off (fermions are gapped)
- gapless spin mode
- excitations above Δ:
  eff. mass $\mu^* \to \infty$
  (no coherency peak, no S.C.)

mean field theory
away from QCP

- ground state : superconducting
- gapless mode at the QCP
  $\longrightarrow$ resonance mode, Ω_{res}
- excitations above the gap:

$$\mu^* \sim f\left(\frac{\Omega_{res}}{T}\right)$$

$$\sim \left(\frac{\Omega_{res}}{T}\right)^{-1}$$

$\left(\rho_s \sim \text{coherency peak} \sim \frac{\Omega_{res}}{T} \right)$

$\Rightarrow \boxed{T_c \sim \Omega_{res}}$

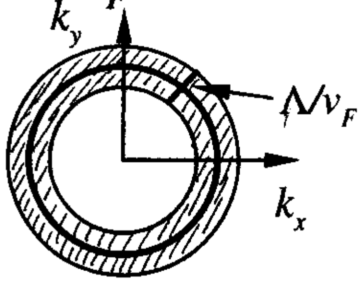
- spin excit.
  are insensitive to "T_c effects"

gap becomes visible
for $T \approx T^* > T_c$

spin fermion model

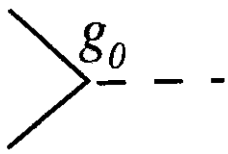
fermions

v_F FS



Fermi liquid for

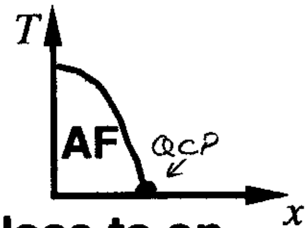
$g_0=0$



$g=g_0^2/\Lambda$

spin modes

(generated by the fermions) ξ

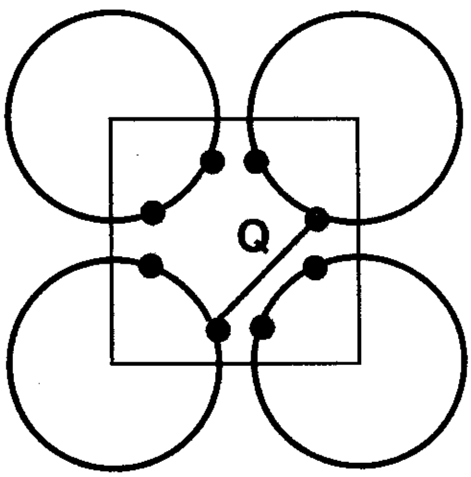


close to an antiferromagnetic transition, i.e.

$\xi \gg v_F \Lambda^{-1}$

"hot spots" of the Fermi surface

(Hlubina+Rice)

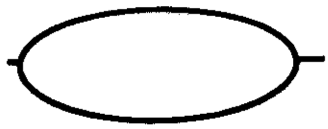


two parameters: g, v_F, ξ

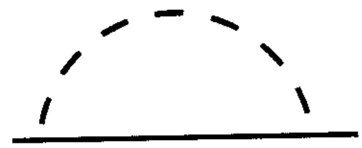
$\rightarrow$ g, v_F/ξ

alternatively: $\lambda = \frac{3g}{4\pi v_F \xi^{-1}}$, g

perturbation theory $\left(1/N - \text{expansion}\right)$

spins acquire dynamics: $\Pi(i\omega) = -$  $= |\omega|/\omega_{sf}$

$$\chi^{-1} = \xi^{-2} + (q-Q)^2 - \Pi(i\omega) \qquad \omega_{sf} \propto v_F^2/g\,\xi^{-2}$$

fermions become "quantum critical": $\Sigma_k(i\omega) =$ 

$$\Sigma_k(i\omega) = \frac{\lambda\, i\omega\, \phi_k(\omega)}{1+\sqrt{1+|\omega|/\omega_{sf}}} + \frac{6}{\pi N}\, \log(\lambda)\ \varepsilon(\mathbf{k}+\mathbf{Q})$$

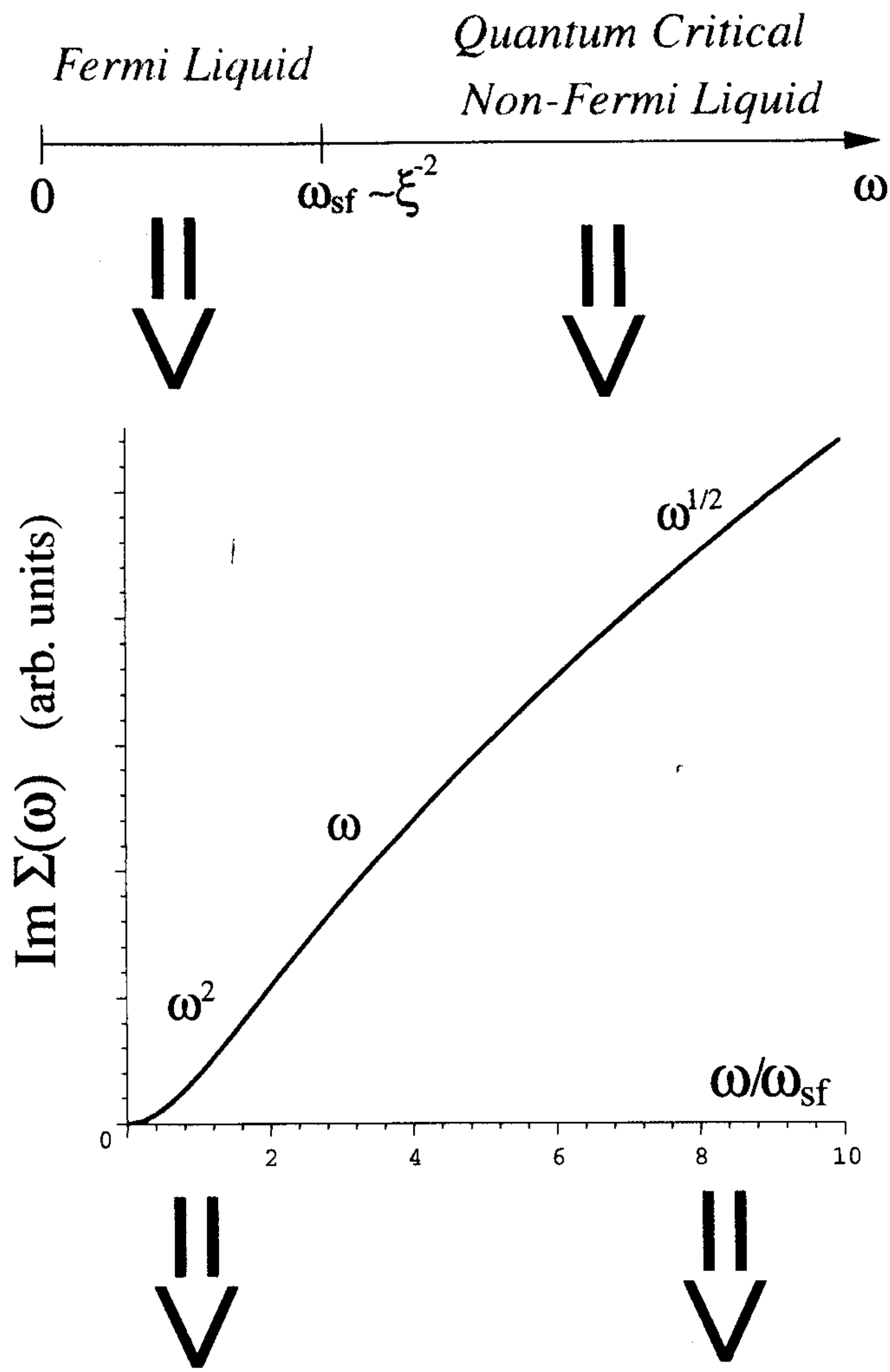
number of hot spots

at the hot spot $\phi_k(\omega) = 1$

dimensionless coupling constant $\lambda \propto g/v_F\,\xi$

$\omega < \omega_{\mathrm{sf}}$ **: Fermi liquid**

$\omega > \omega_{\mathrm{sf}}$ **: quantum critical** $\Sigma_k(i\omega) \propto \sqrt{g\omega}$



Fermions: ***FL***
Spin excitations: ***static***

$\chi^{-1}(q,\omega) \sim q^2 + \xi^2$

Fermions: ***QC NFL***
Spin excitations: ***relaxational***

$\chi^{-1}(q,\omega) \sim q^2 + i\,\omega/\omega$

$G^{-1}(\omega) \sim \sqrt{\omega}$

quantum critical behavior: arbitrary dimensions

$d > 3$: $\Sigma(i\omega) = -Z_0\, \lambda_0\, i\omega$; $\lambda_0 \cong g/(v_F k_F)$

$d \leq 3$: $\Sigma(i\omega) = -Z_0\, \lambda_0\, i\omega\, [\, \log(\xi) + \varepsilon \log^2(\xi)/2 + \ldots\,]$

$\Rightarrow$ $\varepsilon = 3-d$ **expansion, if** λ_0 **small**

$\Rightarrow$ **renormalized quasiparticle weight** $Z = \dfrac{Z_0}{1 - \partial\Sigma/\partial i\omega}$

$$\frac{dZ}{d\log\xi} = -\varepsilon Z - \lambda_0 Z^2 \quad \Rightarrow Z \propto e^{-\varepsilon \log \xi} \propto \omega^{\varepsilon/z}$$

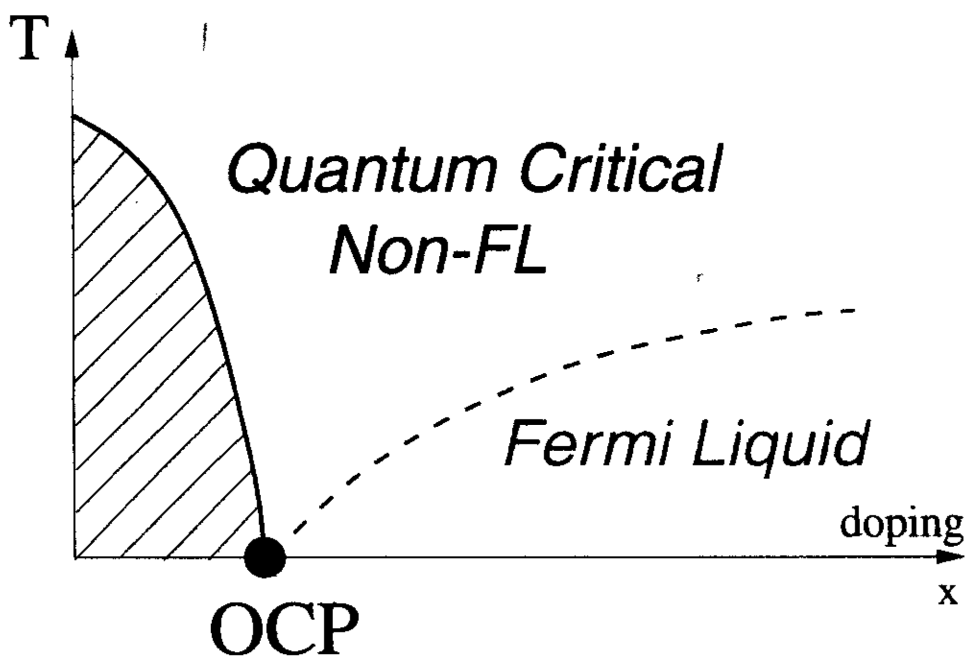
$$\Sigma(i\omega) \propto \omega^{1-\varepsilon/z}$$

$$d=2,\ z=2 \Rightarrow \Sigma \propto \omega^{1/2}$$

d=2 : $\delta\Gamma \propto \partial\Sigma/\partial\mathbf{k} \propto \log(\xi)/N$

$\Rightarrow$ expansion in the inverse # of hot spots, N
(Abanov+Chubukov, PRL vol 84 (2000).)

first (incorrect) attempt to draw a phase diagram.

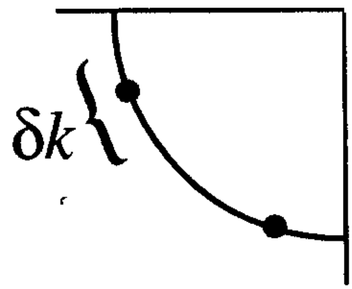


What is the size of a hot spot?

at the hot spot

$$\Sigma_k(i\omega) = \frac{-\lambda\, i\omega}{1+\sqrt{1+|\omega|/\omega_{sf}}}\,, \qquad \lambda \propto \xi$$

away from the hot spot



$$\Sigma_k(i\omega) = -\lambda_0\, i\omega\,, \quad \lambda_0 << \lambda$$

if $\delta k >> \xi^{-1}\,(1+\omega/\omega_{sf})^{1/2}$

$\Rightarrow$ $\omega=0$: $\delta k \propto \xi^{-1} \to 0$

hot spots are isolated, singular points

$\Rightarrow$ dc - transport cannot be due to h.s. anomalies

$\Rightarrow$ $\omega_{typ} \neq 0$: $\delta k \approx \sqrt{\omega_{typ}\, g}\,/v_F$

a large part of the BZ behaves like h.s.

$\Rightarrow$ ARPES, optical cond. $\omega>\Delta$

pairing instability ...

pairing problem

$d_{x^2-y^2}$ **pairing due to A.F. spin fluct.**

(in high T_c context: Moriya et al., Scalapino et al. Pines et al.)

Fermi liquid regime: $T_c < \omega_{sf} \quad (\lambda = g\xi/v_F < 1)$

$$\Phi(\omega') = \Phi_0 + \lambda^* \int_{\pi T_c} d\omega\, \Phi(\omega)\, \frac{1}{\omega} \frac{1}{\sqrt{1+|\omega-\omega'|/\omega_{sf}}}$$

F.L. quasi-particles

$$T_c \approx \omega_{sf}\, exp(-\lambda^{*\,-1})\,; \quad \lambda^* = \lambda\,(1+\lambda)^{-1}$$ **(BCS + McMillian)**

Q.C. regime: $T > \omega_{sf} \quad (\lambda = g\xi/v_F > 1)$

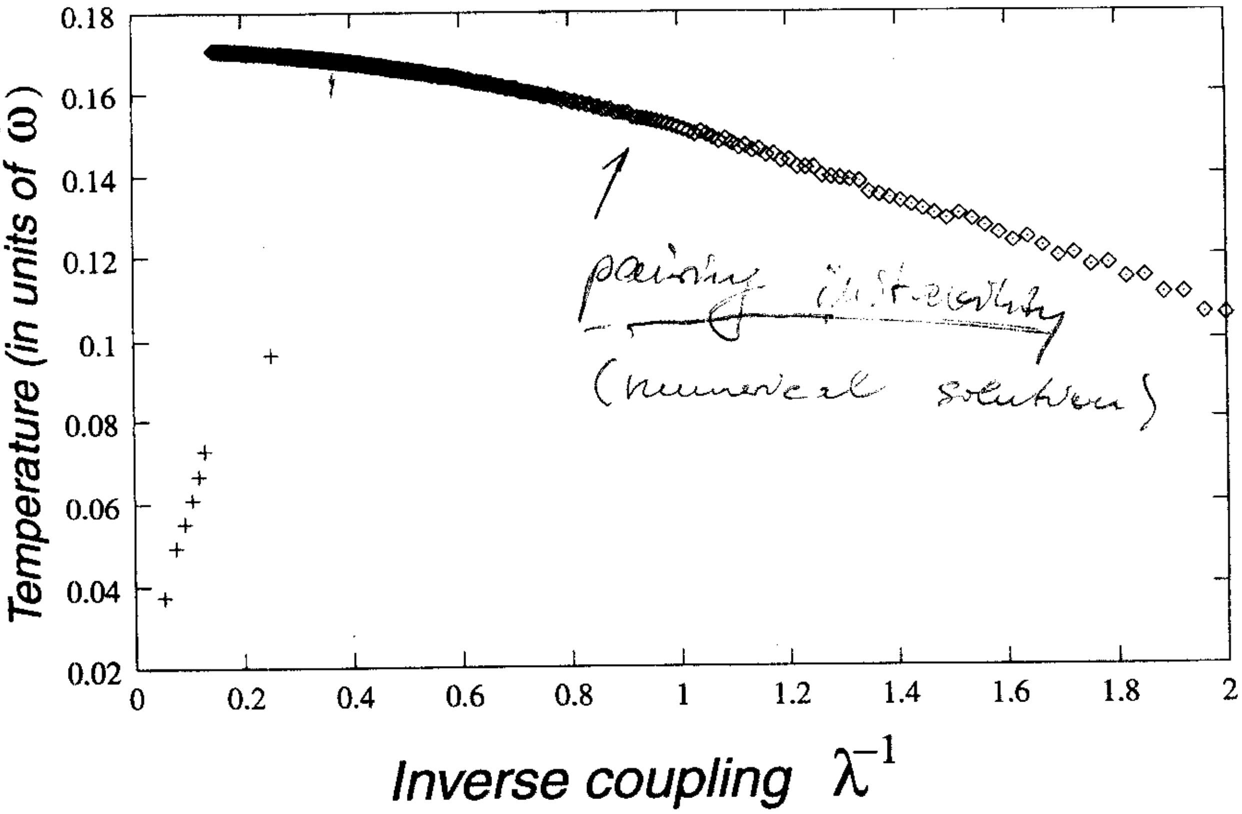
$$\frac{1}{\omega} \rightarrow \frac{1}{\sqrt{\omega}}$$ **Q.C. fermions**

$\Rightarrow$ **no perturbative solution of the gap equation**

$\Rightarrow$ **nonpert. solution exists with** $T_{ins} \propto g$

(Abanov, Chubukov +Finkel'stein, 2000)

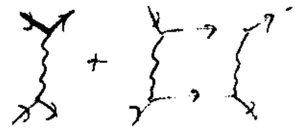
T_{ins} $\cancel{\text{and } T_c}$



ε – expansion for Q.C. pairing ($\varepsilon = 3\text{-}d$)

$$\Phi(\omega') = \Phi_0 + a_d \int_{\pi T_c} d\omega\, \Phi(\omega) \frac{1}{\omega^{1-\varepsilon/2}} \frac{1}{|\omega-\omega'|^{\varepsilon/2}}$$

flow of the pairing vertex $l = log(g/T_c)$

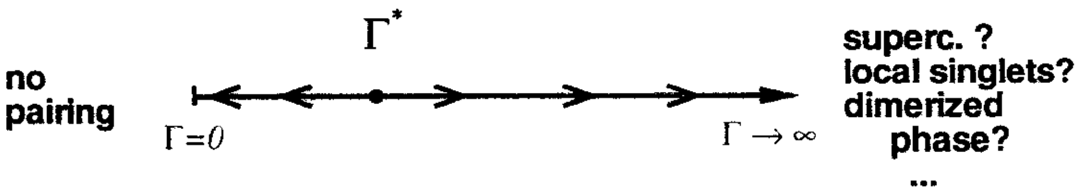


$$\frac{d\Gamma}{dl} = -\frac{\varepsilon}{4}\Gamma + a_d \Gamma^2$$

suppresses pairing for small Γ

favors pairing

$\Rightarrow$ **new fixed point:** $\Gamma^* = \varepsilon\, (4a_d)^{-1}$



instability temperature: $T_{ins} \propto g$

$d=2$: $\Gamma(l=0) > \Gamma^*$ **always flows to strong coupling**

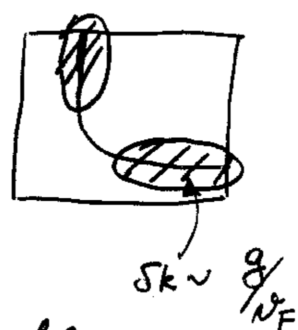
$\Rightarrow$ QC - Non-FL is unstable.

AT THE QCP: Summary

(1.) $T_{ins} \sim g \quad (0.17 g)$

- singlets melt at the upper cut off, not at the characteristic boson frequency

(2.) Singlet gap, Δ



(3.) gapless collective mode

$$\chi \sim \frac{1}{\omega^2 - c^2 q^2}$$

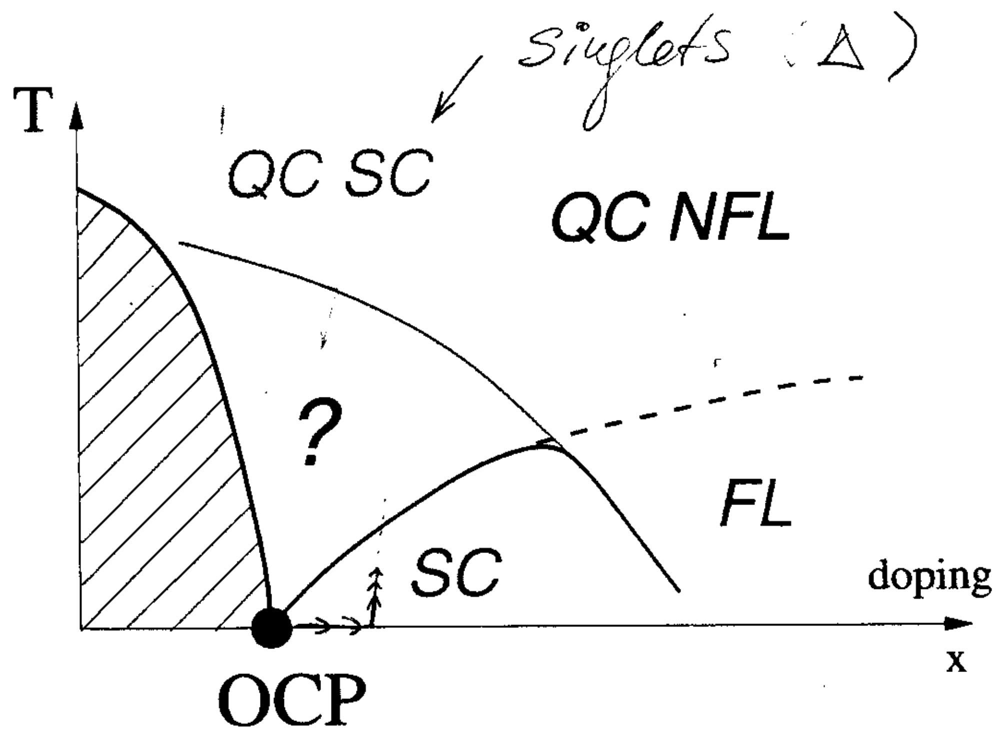
$\longrightarrow$ Strong scatt. with the excitations above the gap (mass μ^*)

(4.) $\mu^* \to \infty$

$\rho_s \to 0$; no coherency peak

Away from the QCP

- Ground state : convent. Superconductor



gapless mode

$\longrightarrow$ resonance mode, Ω_{res}

$$\chi_Q \sim \frac{1}{\xi^{-2}(1-\Pi(\omega))}$$

dynamics due to fermions (but now with s.c. gap)

$1/N$ expansion:

$$\Pi(\omega) = \frac{\pi}{8}\frac{\omega^2}{\Delta\omega_{sf}} + \mathcal{O}(i\omega^4)$$

$$\Longrightarrow \quad \Omega_{res} \sim \sqrt{\Delta\omega_{sf}} \sim \xi^{-1}$$

$$\operatorname{Im}\Pi(\omega) = 0 \qquad \omega < 2\Delta$$

$\Longrightarrow$ sharp mode for $\Omega_{res} < 2\Delta$

- resonance is inevitable
- single layer systems ??

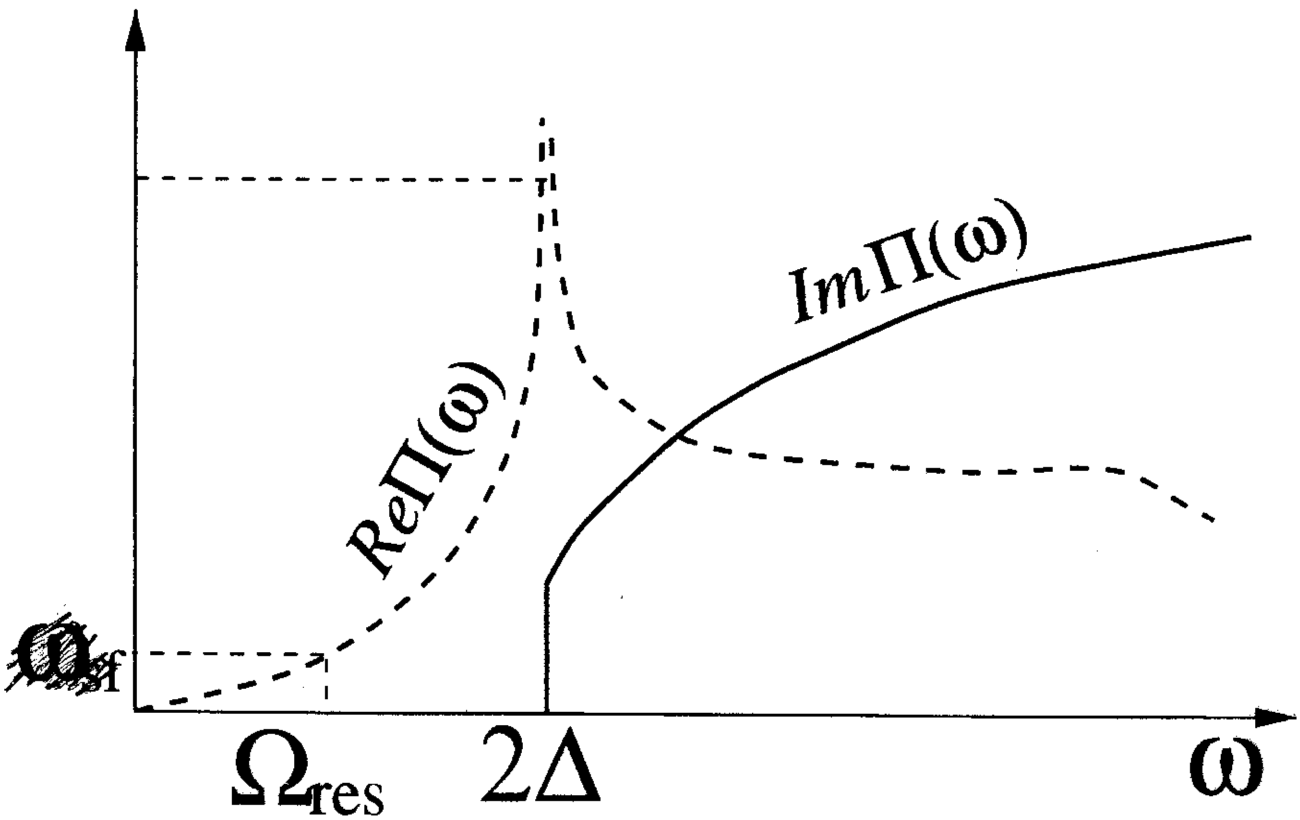
1. Opening of the pairing gap

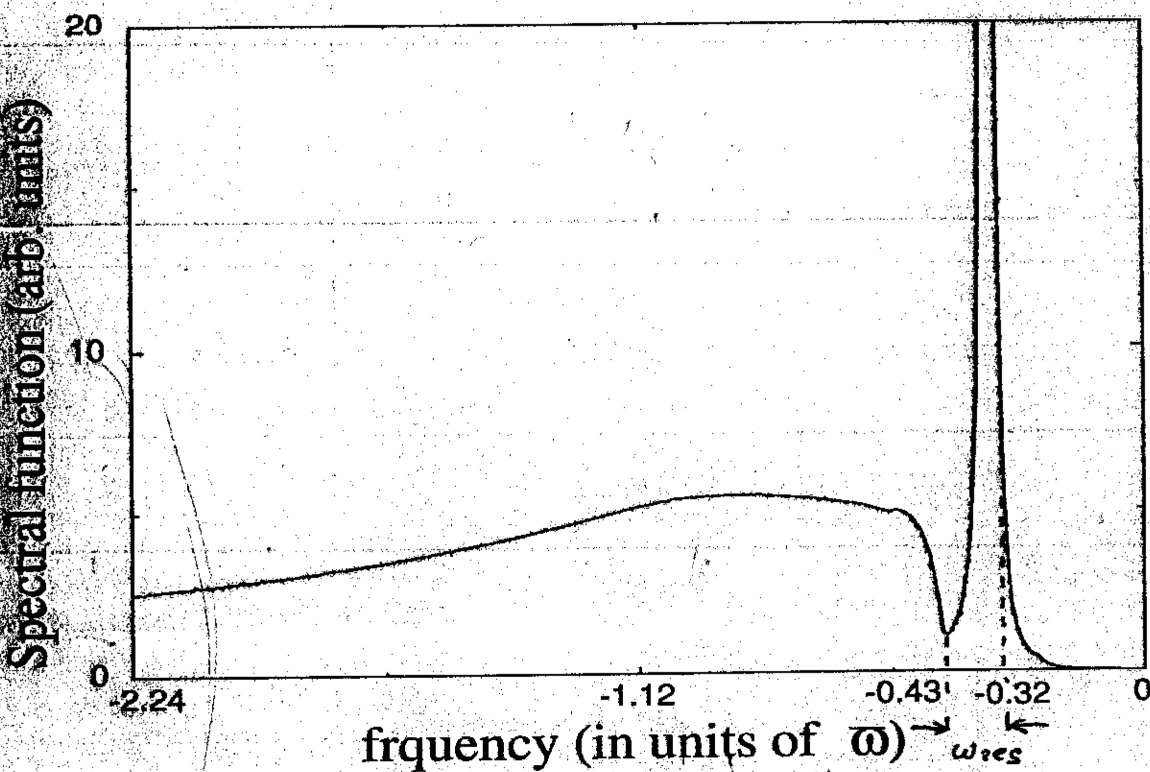
2. Proximity to an A.F. State

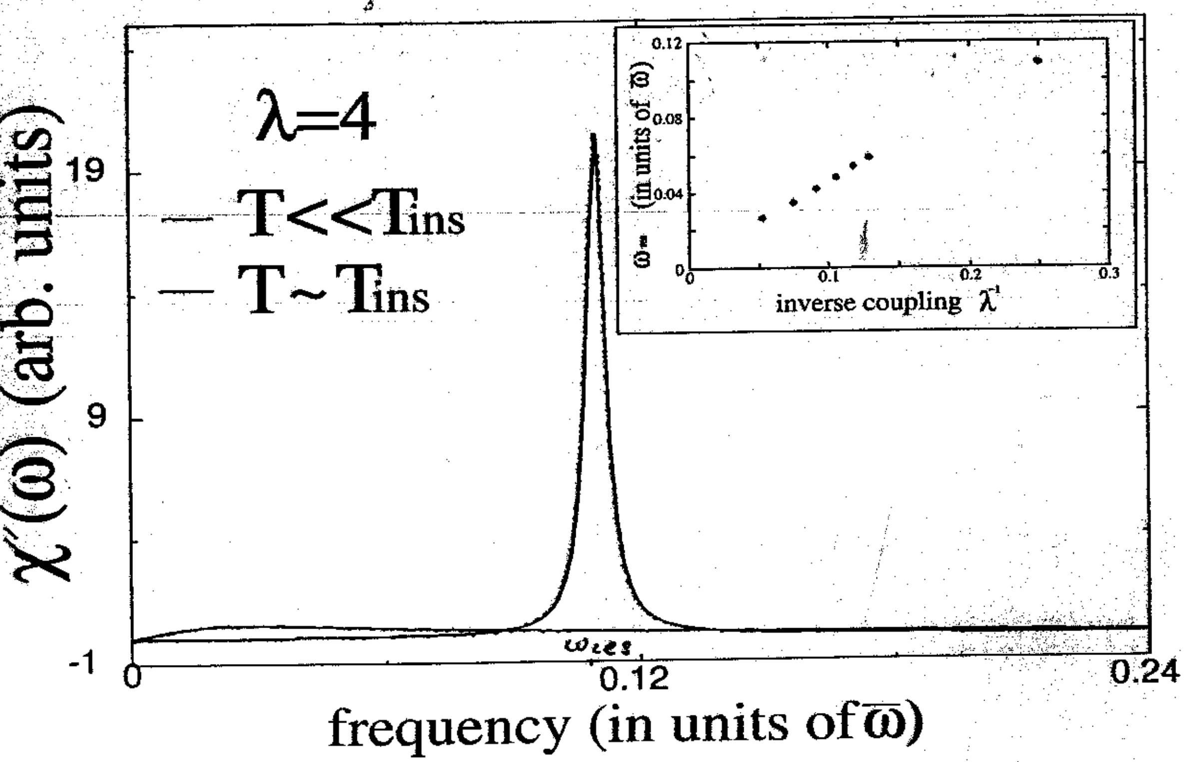
Resonance in the spin response at:

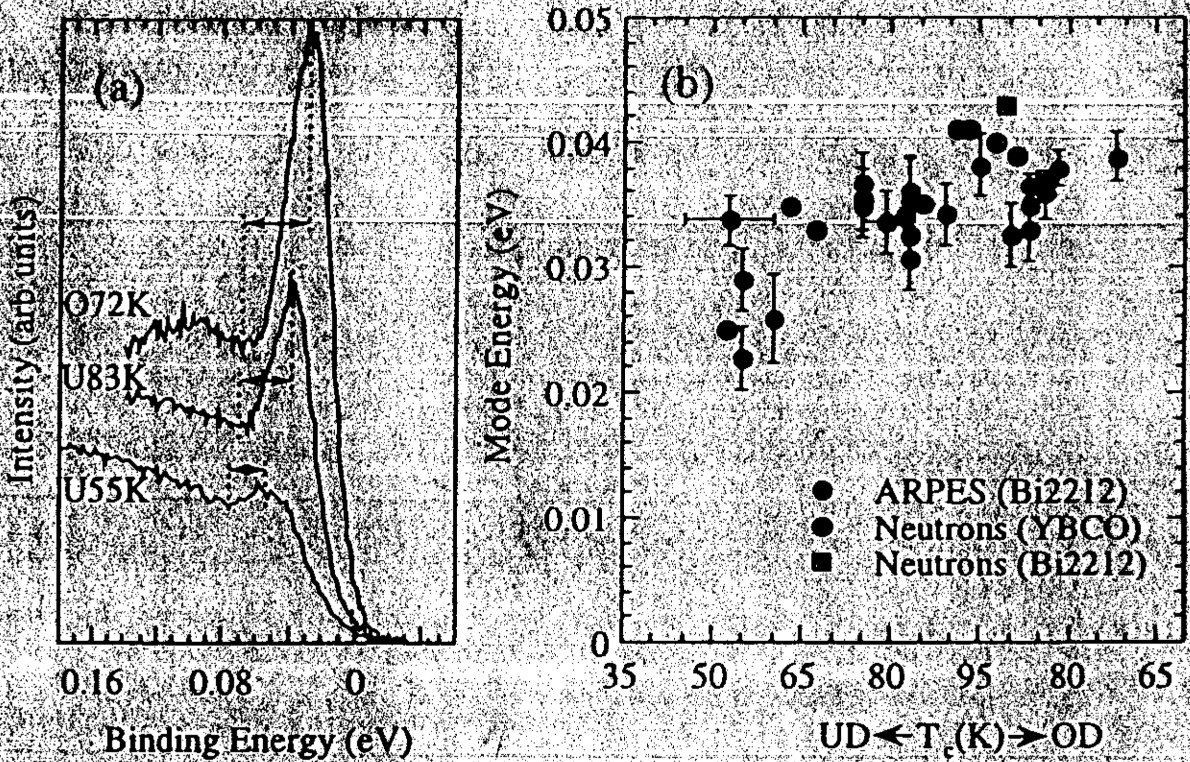
$$\Omega_{res} = (\omega_{sf}\Delta)^{1/2} \sim \xi^{-1}$$

(Abanov + Chubukov, PRL 1999)



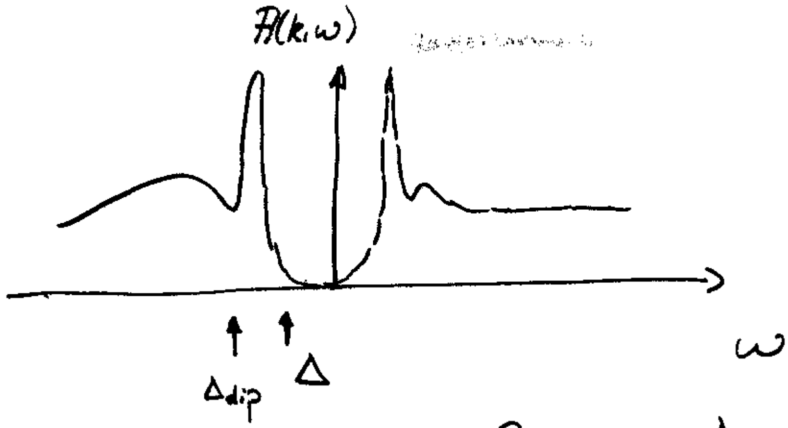

Spectral function (arb. units)
20
10
0
-2.24
-1.12
-0.43
-0.32
0
frquency (in units of $\overline{\omega}$)
ω_{res}


$\chi''(\omega)$ (arb. units)
λ=4
T<<Tins
T~Tins
19
9
-1
0
0.12
0.24
ω_{res}
frequency (in units of $\overline{\omega}$)
ω_{res} (in units of $\overline{\omega}$)
0.12
0.08
0.04
0
0.1
0.2
0.3
inverse coupling λ^{-1}

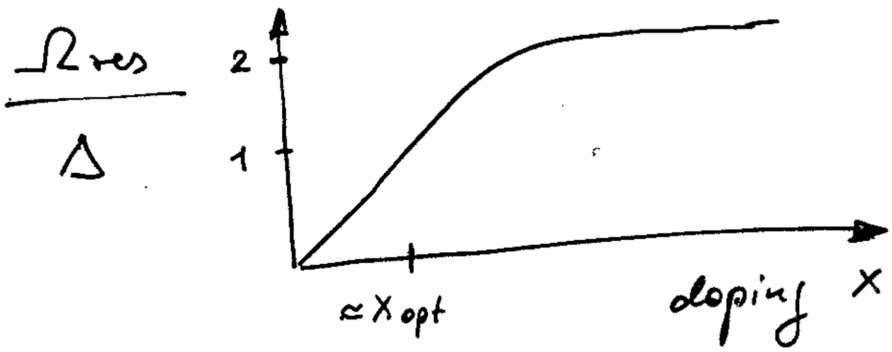


Campuzano et al

resonance as seen in ARPES / tunneling



$$\Omega_{res} = \Delta_{dip} - \Delta$$



coupling between <u>electrons</u>
and <u>resonance mode</u>

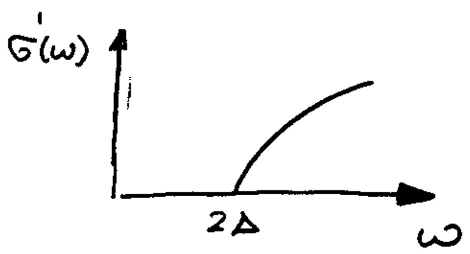
optical conductivity ($T \ll T_c$)

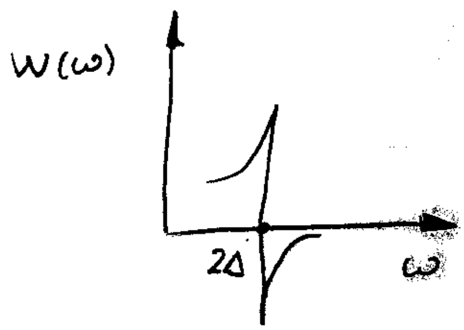
relation between $\sigma(\omega)$ and Ω_{res}

(D. Basov et al., J. Carbotte et al.)

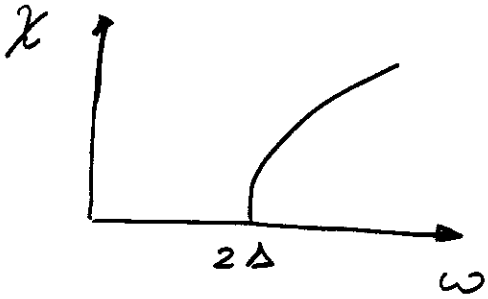
$$W(\omega) = \frac{d^2}{d\omega^2}\left(\omega\, \sigma^{-1}(\omega)\right)$$

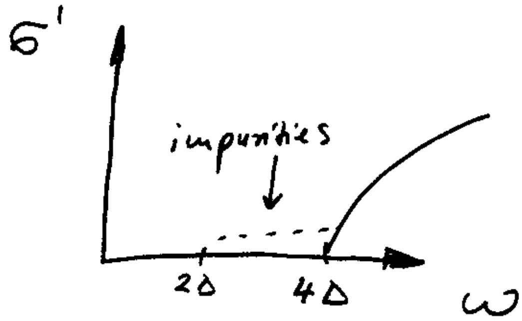
conventional
s-wave S.C. + <u>impurities</u>

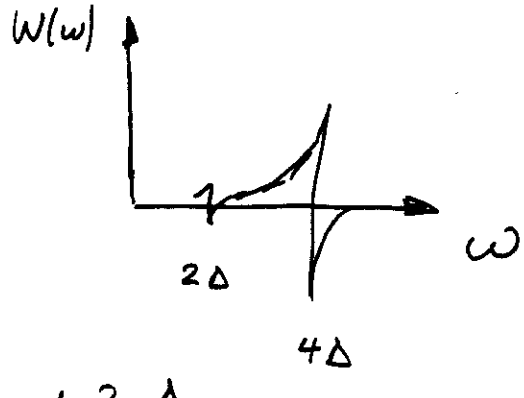




scatt. due to gapped p.-h. excitations
(Varma, Littlewood, MFL)

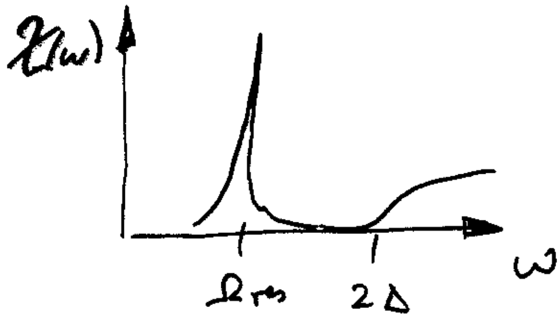


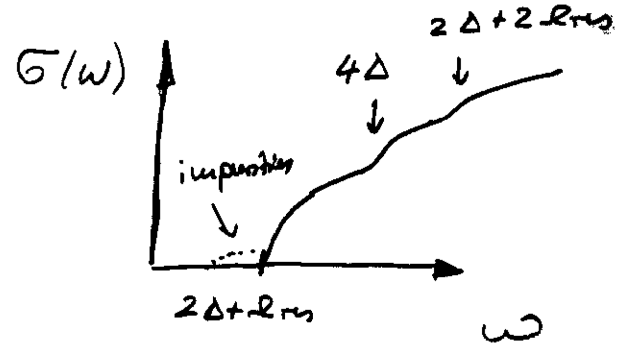


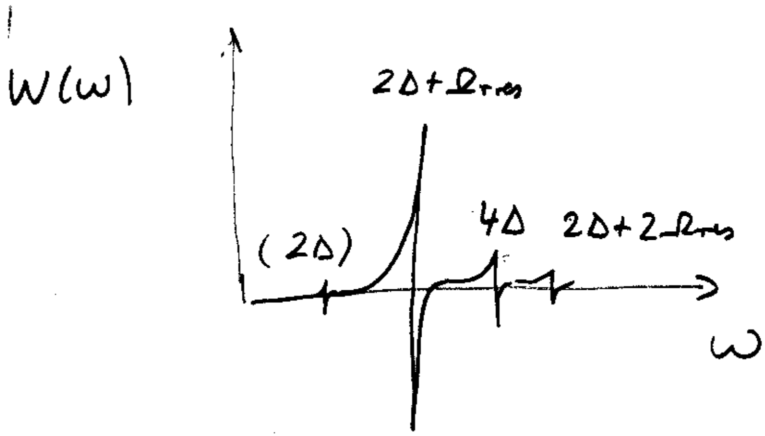


onset of scattering + 2Δ
$\longrightarrow$ peak in $W(\omega)$

charge carriers coupled to the resonance mode







if Δ is known $\Longrightarrow$ Ω_{res}

\+

scatt. is not impurity dominated

(we are at low T !!)

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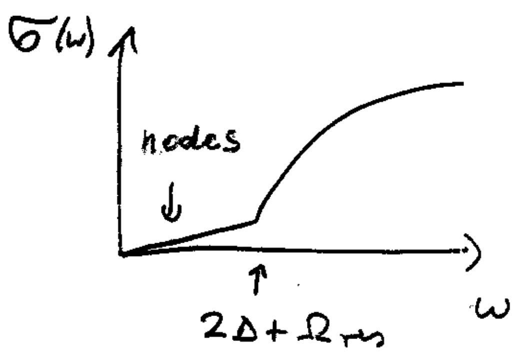
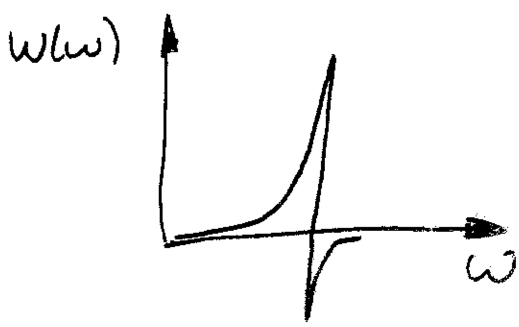
s wave $\Leftrightarrow$ d wave





(basically unchanged)

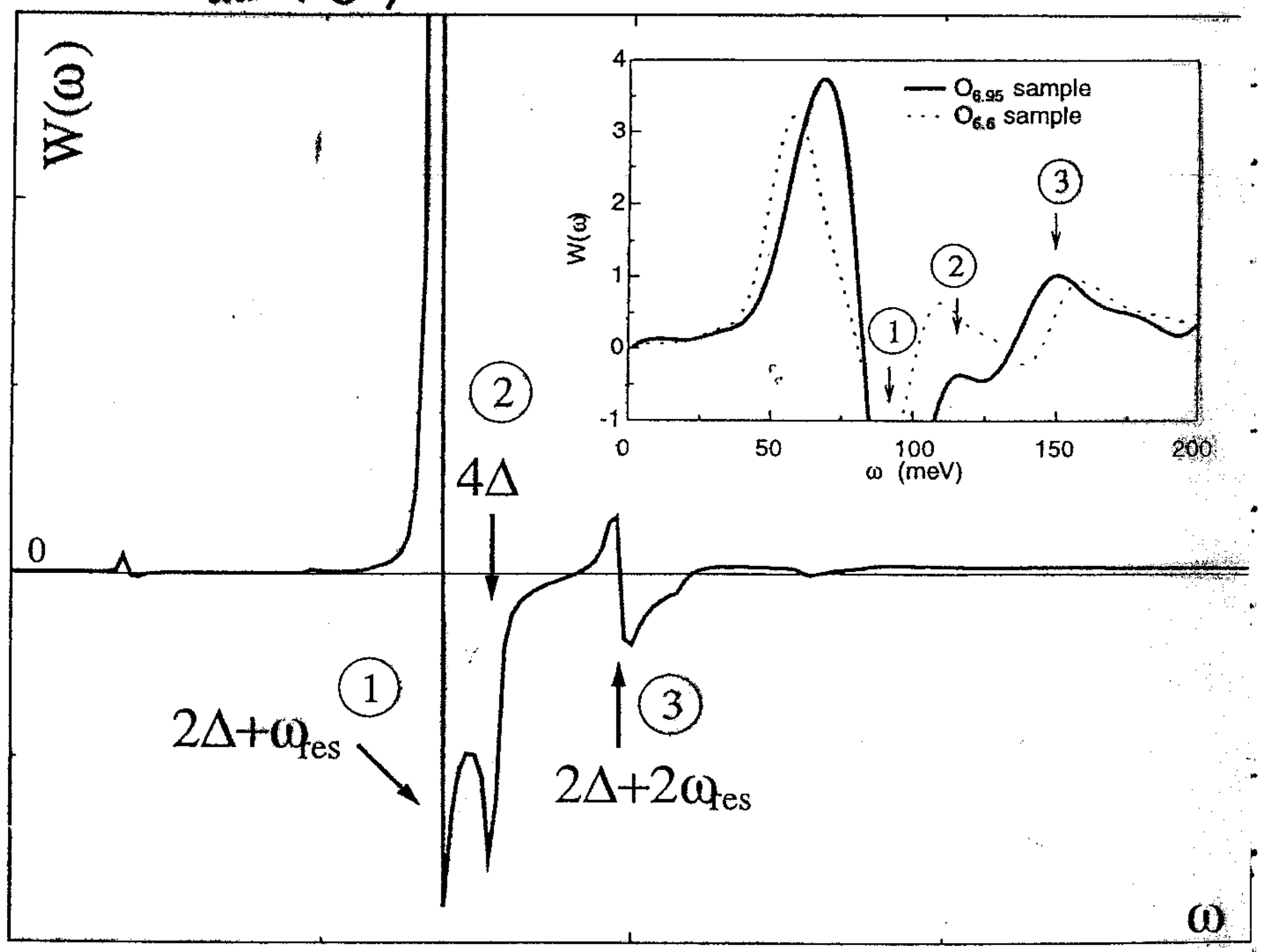
σ :  } from $(1/N)^0$ - exp.

$\lambda = 1 \quad ; \quad g = 2\pi \cdot 150\,meV$

$\Longrightarrow \quad \Delta = 30\,meV$

$\Omega_{res} = 37\,meV$

$W = \frac{d^2}{d\omega^2}\left(\frac{\omega}{\sigma'}\right)$



Abanov, Chubukov, Schmalian, preprint

finite TEMPERATURES

resonance mode, Ω_{res} is weakly T-depend.

$\Rightarrow$ if $\boxed{\pi T \sim \Omega_{res}}$

mode is thermally excited (elastic scattering)

conventional s.c. with elastic scattering
(P. W. Anderson '59 / A.A. Abrikosov + L.P. Gor'kov '59)

1. T_c is unchanged
2. ρ_s is strongly suppressed

$$\frac{\text{q.p. damping}}{\Delta} \rightarrow \frac{\pi T}{\Omega_{res}}$$

following P.W.A / A.A.A + L.P.G
classical contributions can be singled out

$$\phi = \tilde{\phi}\,\mu^* \quad ; \quad \Sigma = \tilde{\Sigma}\,\mu^*$$

$\tilde{\phi}, \tilde{\Sigma}$ behave like at $T=0$

$$\boxed{\mu^* = 1 + \frac{T}{T_{coh}}} \qquad \pi T_{coh} \simeq \Omega_{res}$$

⇒ SUPERFLUID DENSITY

$$\rho_s \sim \frac{1}{\mu^*} \sim \frac{\Omega_{res}}{\pi T}$$

vanishes on the scale Ω_{res}

$$\Rightarrow \boxed{T_C \simeq \frac{1}{\pi}\Omega_{res}}$$

s.c. transition is driven by $\rho_s \to 0$ for $T \sim \frac{1}{\pi}\Omega_{res}$

(gap, Δ, stays intact !!)

⇒ SPECTRAL FUNCTION

trivially : $G_K(\omega) = f(\Sigma, \phi) = \tilde{f}(\eta, \tilde{\Sigma}, \tilde{\phi})$

scaling of the spectral function

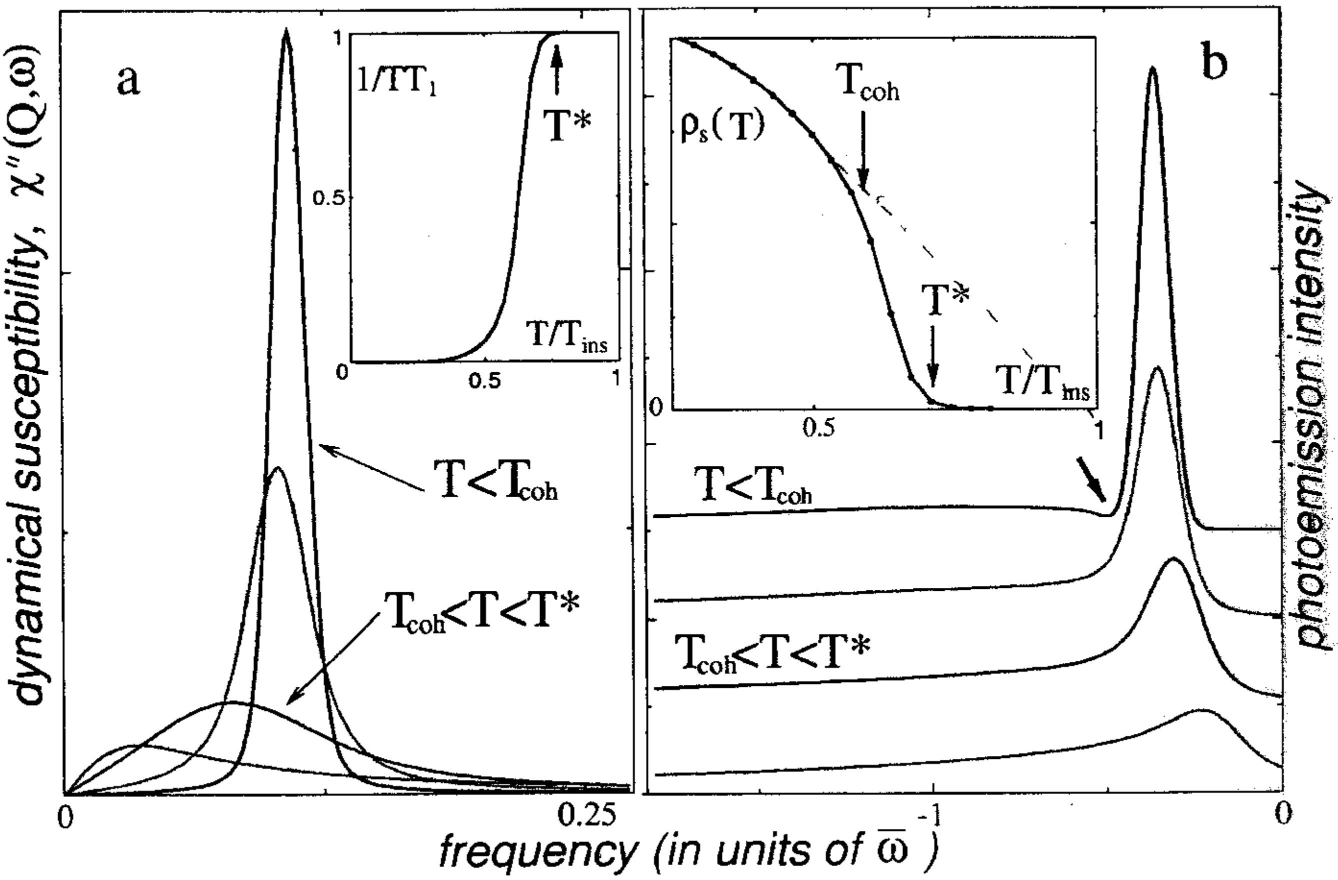
$$\boxed{G_K(\omega, T, \varepsilon_K) = \frac{1}{\mu^*} G_K(\omega, T=0, \varepsilon_K/\mu^*)}$$

•) coherency peak $\sim \frac{1}{\mu^*}$ vanishes at $T_c \sim \Omega_{res}$

:) " " scales with Ω_{res} for fixed T

⋮) " " scales with ρ_s $\forall_{dop., T}$

Resonance peak in neutron scattering



Spin dynamics

$\tilde{\Pi} = \Pi$ (unaffected by classical spin excitations)

low freq. spin response unchanged at T_c, eventhough spin dynamics is entirely due to fermion dynamics

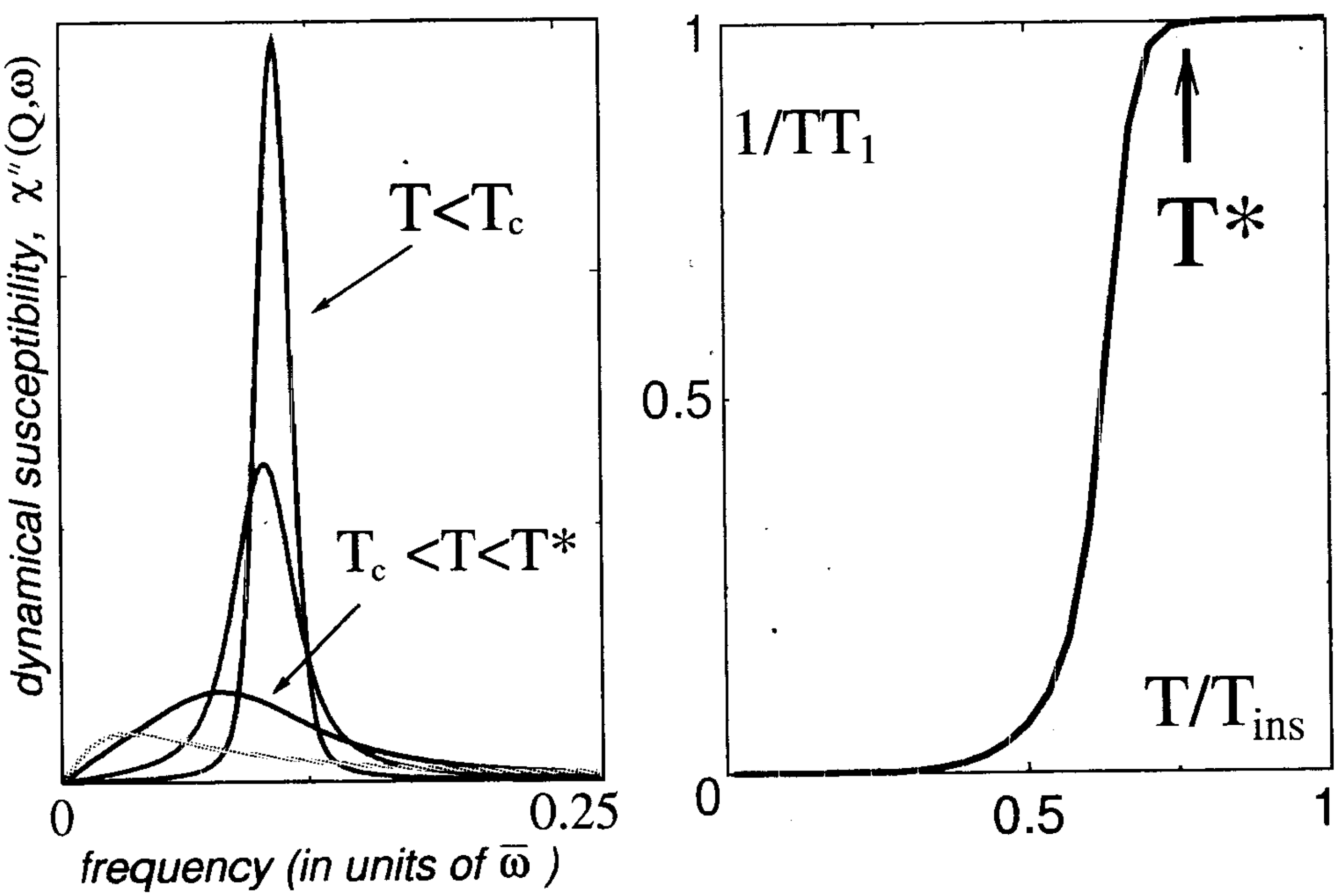
but: (we neglected phase fluctuations !!)

$$\Pi = G\,G + F\,F \underbrace{\left\langle e^{i(\phi-\phi')} \right\rangle}_{\text{sensitive to changes at } T_c}$$

•) if $\Sigma \sim \phi$ at high T

$\Rightarrow$ gap becomes invisible (singlets are still there)

$\Rightarrow T \sim T^*$ (non universal #



dynamical susceptibility, χ''(Q,ω)
T<Tc
Tc <T<T*
0
0.25
frequency (in units of ω̄)
1
0.5
0
1/TT1
T*
T/Tins
0.5
1

Crossover temperatures below the pairing instability: