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XII WORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

17 - 28 July 2000

# *RVB REDUX*

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# NAMES

RVB: The Originators

Z Zou, G Baskaran, T Hsu, PWA; I Affleck, G Kotliar, X-G Wen
R B Laughlin et al
KRS (Kivelson et al)

Inhabitants of the Big Tent--(spin gap = RVB)

PWA, Baskaran, Muthukumar
P A Lee, Wen, Nagaosa et al (Gauge Th.)
Fisher, Nayak, Senthil et al (Nodons)
Ting, Weng, Sheng et al (Schwinger Bosons)
Fukuyama et al
John, Berciu (instantons)

Fellow-travelers: (NFL, but......)

Varma, Abrahams (MFL)
Rice, Laughlin, Lederer, etc (Flux phases)
Chakravarty, Sudbo, et al (ILT)

Many Experimentalists --too many to mention but especially:

N-P Ong, T Timusk, H Drew and many other persistent, careful transport measurers
J-C Campuzano, Mike Norman and the Argonne PES group
P Johnson, Z-X Shen and many other PES workers

# RVB Redux
## The Theory that got lost

Outline of Talk

I Historical (1987–88) RVB

II Successes, realized and unrealized

III Reasons for Abandonment and (III a) why they were wrong

IV New RVB

V What about SC?

# I. Historical RVB of '87-
(Baskaran, Zou, Hsu.., 

Cuprates based on Mott insu-
lator with Hubbard gap: t-J model
from Hubbard Model

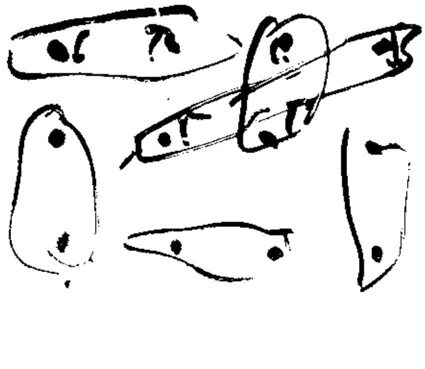
$$\underbrace{\sum_{\substack{\langle ij\rangle \\ \sigma}} t\, c^\dagger_{i\sigma} c_{j\sigma}}_{T} + \underbrace{U \sum n_{i\uparrow} n_{i\downarrow}}_{U}$$

$$\Longrightarrow \; PTP + \underbrace{J \sum_{\langle ij\rangle} S_i \cdot S_j}_{J}$$

P: ✗ ~~double occupancy~~ ✗

perturbation in $1/U$.

RVB: a hypothetical spin liquid of singlet pairs



+ etc.

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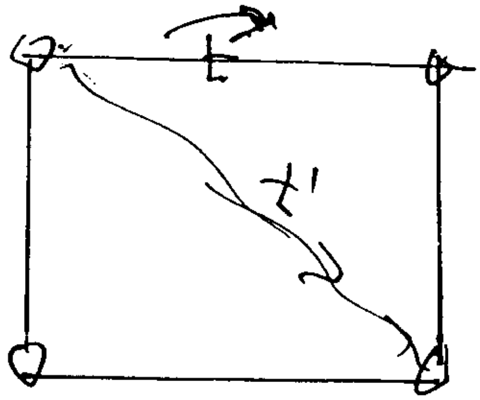
# THE HUBBARD MODEL--ONE BAND!




(T2) : U

$$H = t \sum_{\langle ij \rangle \sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + t' \sum_{\substack{\langle nnn\, ij \rangle \\ \sigma}} c^{\dagger}_{i\sigma} c_{j\sigma}$$

$$+ U n_{i\uparrow} n_{i\downarrow} \qquad U \gg 0, \quad > t$$

+ Hopping between planes, sometimes.

Justifiable from chemical reasoning--all but the essentials abstracted out. But we have NO reason to think more is needed!

## THE T-J TRANSFORM:

Antiferromagnetic exchange from a PROJECTIVE transformation. Take U as Ho, t as a perturbation, and eliminate double occupancy:

$$n_{i\uparrow} n_{i\downarrow} \longrightarrow 0$$

you get the "t-J" Hamiltonian

$$\mathcal{H} = P \sum_{ij\sigma} t_{ij} c^{+}_{i\sigma} c_{j\sigma} P + J S_i \cdot S_j$$

for large U, small x, J≈t2/U; it is a DERIVED quantity, always <t, U, somewhat x-dependent.

$$P = \underline{\text{projector}} \quad ; \quad \vec{S}_i = \sum_{\sigma\sigma'} s^{+}_{i\sigma} (\vec{\sigma})_{\sigma\sigma'} s_{i\sigma'}$$

You can't have AF exchange without the projection: FLT and AF J are logically incompatible. In FLT exchange is ferromagnetic.( SDW's are caused by nesting, not by.
s superexchange)

A possible ground state of Heisenberg Model

To construct: Bose Condensate of singlet pairs ≡ BCS wave function, <u>project</u> on

$\langle \tau_3 \rangle_i \equiv 0$ so $\tau_3$ $(= 1 - n_{i\uparrow} - n_{\downarrow})$ drops out.

All self-energy anomalous

$$\begin{matrix} \psi \\ \psi^{+} \end{matrix} \begin{pmatrix} 0 & \frac{1}{2}\Delta_1 + i\Delta_2 \\ \Delta_1 - i\Delta_2 & 0 \end{pmatrix}$$

Eigenfns $\psi \pm \psi^{+}$

<u>Real</u> Fermions

"spinons" = eigenexcitations = Majorana Fs

(projection means only created in pairs!)

ENORMOUS SYMMETRY: Local U(1)

3 (KOHN; BASKARAN)

(or $Z_2$ : ~ GB, Yulu, G Rosetti!)

(doesnt care about sign of w.f.)

Jalili

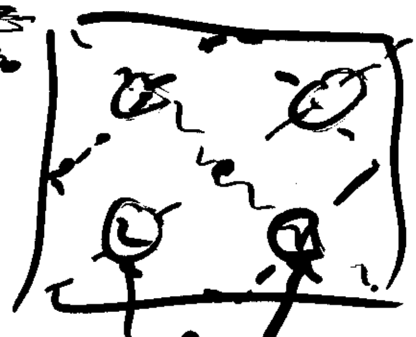
$\Delta_1$ and $\Delta_2$

eg $\cos k_x + \cos k_y \equiv \cos k_x - \cos k_y$

can use both : "s + id" = $\Delta_1 + i\Delta_2$

is best solution

variationally.



nodes

BAskeran: define an

$F(\Delta)$ with $U(1)$ symmetry

$$F = F_1(|\Delta|^2)$$

$+ F_2 \sum$ plaquet terms

$F_1 \rightarrow$ "s"

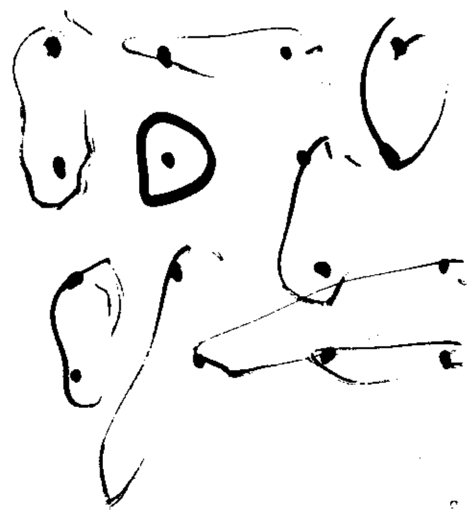
$\Delta_{12} \Delta^*_{23} \Delta_{34} \Delta^*_{41}$

plaquet term → "d" = resonan

4 (Diff of energy of symmetric vs anti || ≠ = )

spinon

holon



sc

For ½ full, symmetry says $F_1 \cong F_2 = J$

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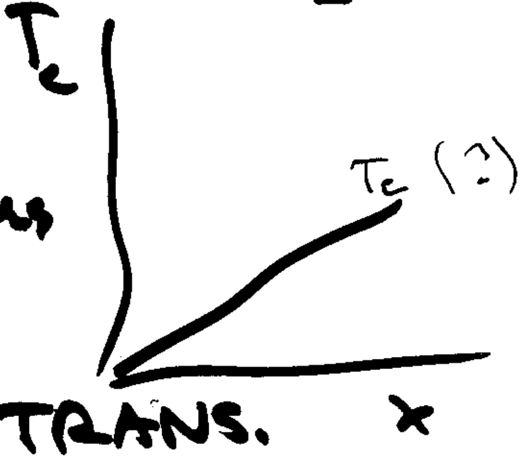
Now DOPE   $F_2$ No $T_c$ at x=0
phase stiffness <u>increases</u> with x

but $\Delta$ forms at $T \sim J$




NO PHASE TRANS.

II QUALITATIVE SUCCESSES OF RVB.

(1) <u>ABOVE ALL</u>: SOLUTION OF THE DEEP, QUALITATIVE PROBLEM: PAIRING WITHOUT PHONONS, PURELY FROM REPULSIVE ELECTRON-ELECTRON INTERACTION

(2) PREDICTED : PSEUDOGAP !

(3) " : NO PHASE TRANSITION TO PSEUDOGAP

(4) " : d-wave gap ! (Kotliar, Liu)

(5) "protectorate": insensitive to phonons, impurities (magnetic)

(6) Dep of $T_c$ on $x$

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III So why abandoned ?

(1) Heisenberg Model not RVB (though close ∑ - Hsu)

(2) Insensitivity of $T_c$ suggested s-wave (?!)

(3) Optimally doped Has Fermi surface : NFL theory

(4) $T_c$ = f (layer # ): ILT

(5) $T^*$, $E_g^*$ = f (x)

(6) Main : no formal th of $T_c$.

Redux: solve these problems

(1) Irrelevant: AF → RVB with doping (critical pt.). Soft mode = neutron resonance!

(2) d-wave! insensitivity → quantum protectorate

(3) combine RVB, NFL! see below

(4) Experimental red Herring? partly true

(5) see (3)

(6) Still a problem: but energy scales are clear.

NEW RVB

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(1) Charge - spin separation is generic in 2D.

Expt - ARPES EDC's
Transport theory; impurities

Theory - chiral anomaly, infrared catastrophe

Symmetry : $U(2) \to U(1) \times U(1) \times Z_2$ ← hole-particle

(2) Therefore : spin sector ~~Still~~ Lagrangian retains $U(1)$ invariance : Elitzur's theorem allows no phase transition at $T^*$, $E_g^*$

(3) Effective energy in spin sector $= F(\Delta_{ij}^2) + K \square$
d-wave term

F <u>increases</u> with doping

$F \propto J + xt$

K <u>decreases</u> with doping

+ sign. Weakening of Umklapp

$v_s$ increases because of ~~...~~

repeating paths - Brinkman-Rice effect.

(at ~~low~~ high doping $v_s \to v_F$)

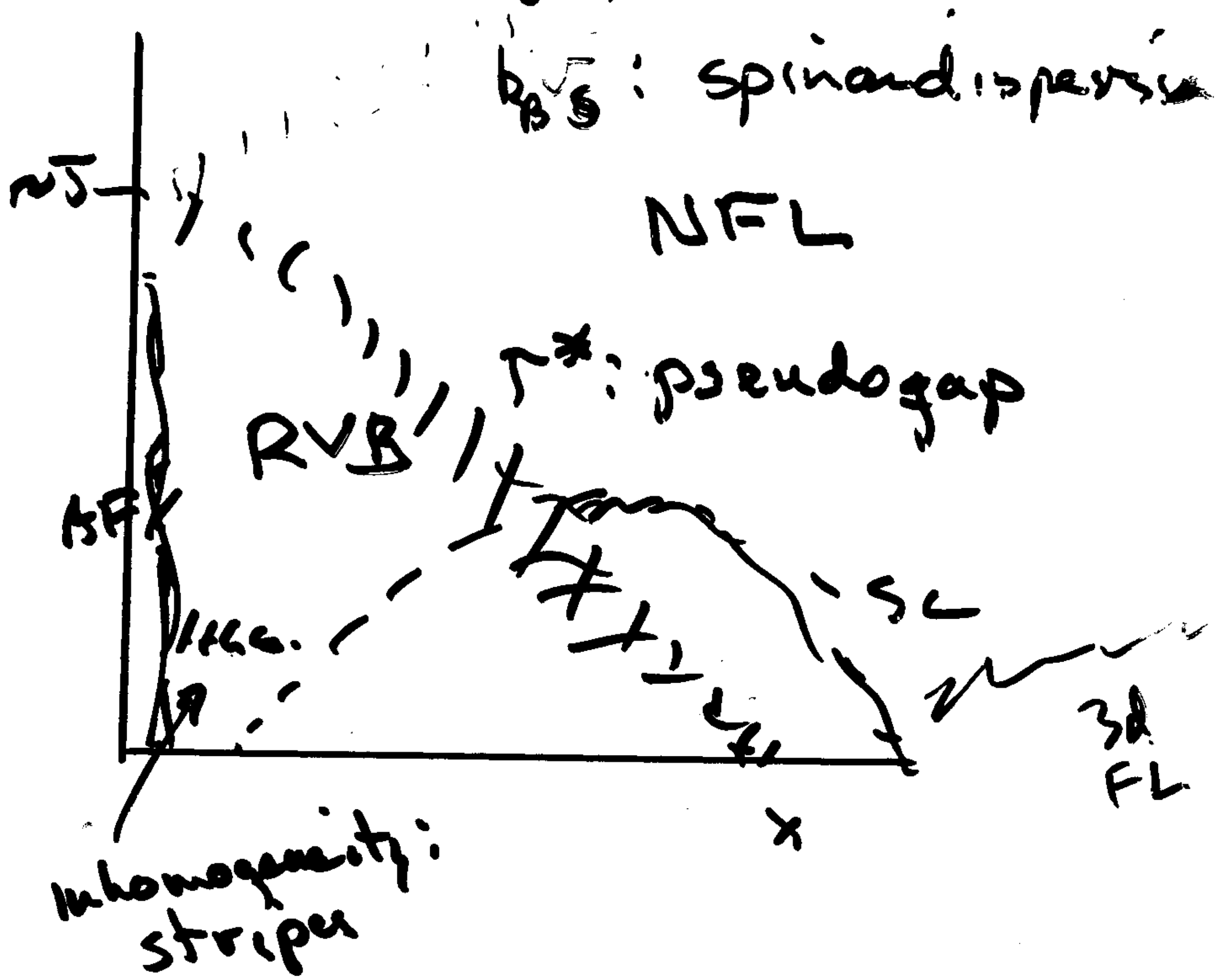
( How do we define $v_s$?
make an electron excitation
with charge in ground state,
this has defined momentum,
satisfies Luttinger thm.)

So we have

(1) pseudo gap, no transition

(2) neutron res in RVB state only

Phase Diagram




Why does $J_{eff}$ decrease?

Resonance energy decreases because loop paths of holes have ferromagnetic sign. Hole makes ⬭ → ▯ with

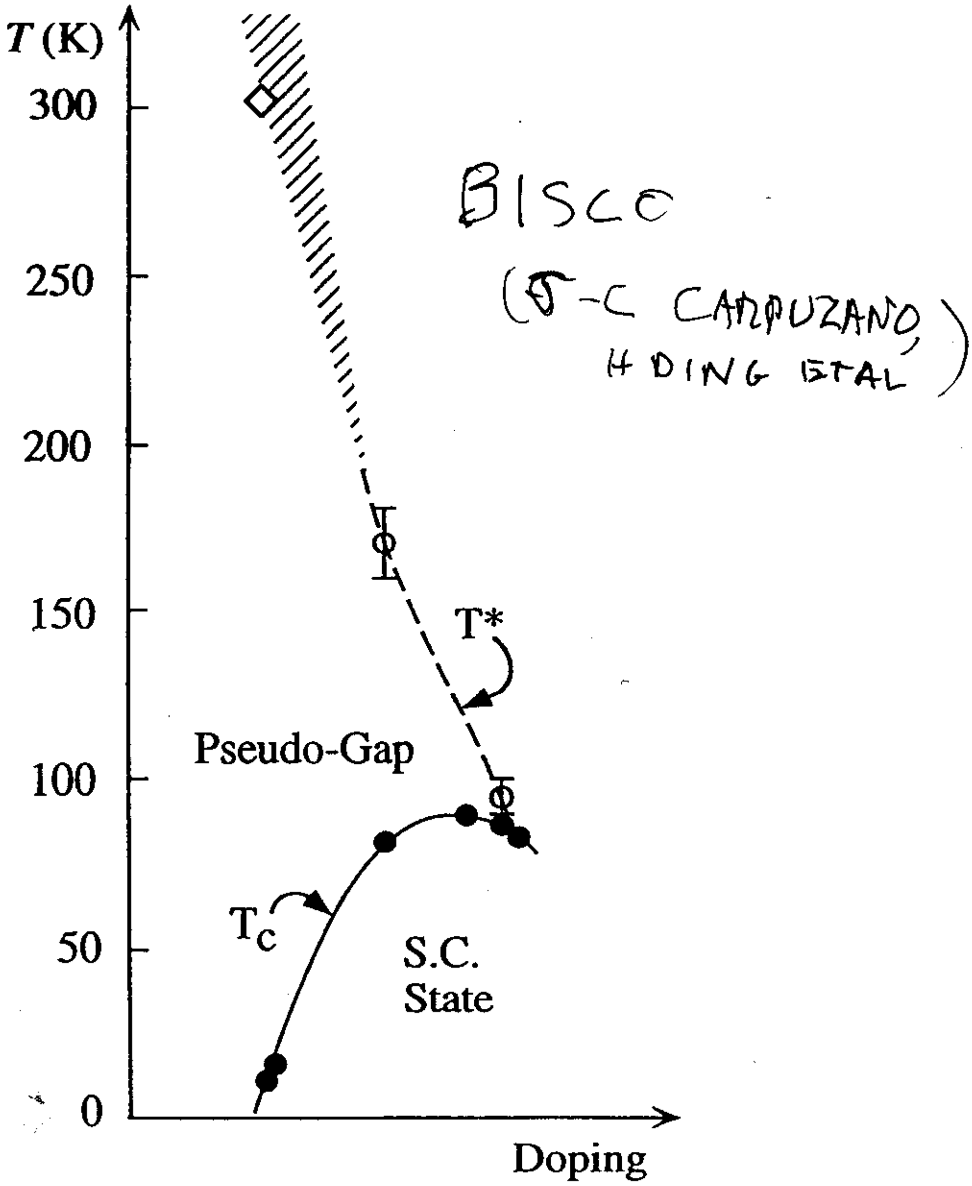

T (K)
300
250
200
150
100
50
0
Doping
Pseudo-Gap
S.C.
State
T*
Tc
BISCO
(J-C CAMPUZANO,
H DING ETAL)

Fig. 4

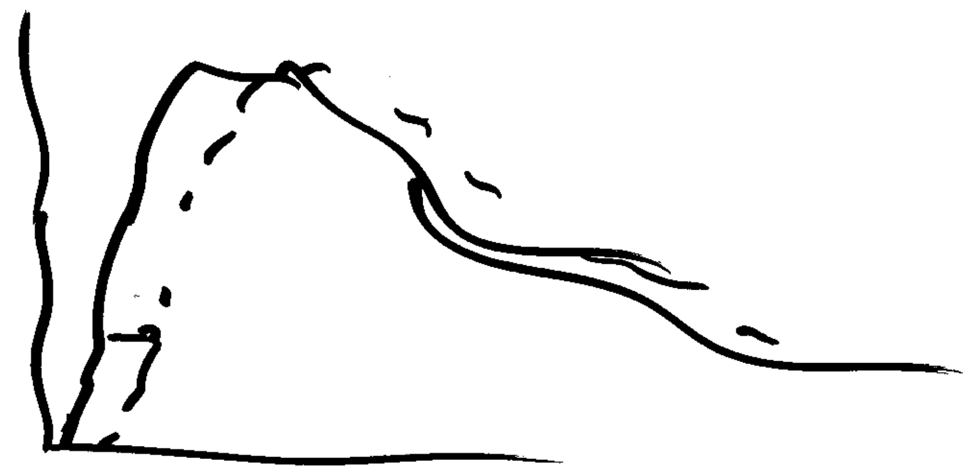
2. $T^*(x)$

V Why + How supercond-uctivity?

Why is easy! [redacted]

Spin gap causes big increase in kinetic energy.

ARPES



IR: BIG INCR IN $m^*$ (see Fig) W

TIMUSKI SECRET: KOHN SUM RULE SAYS K.E.

KE REGAINED BY COHERENT

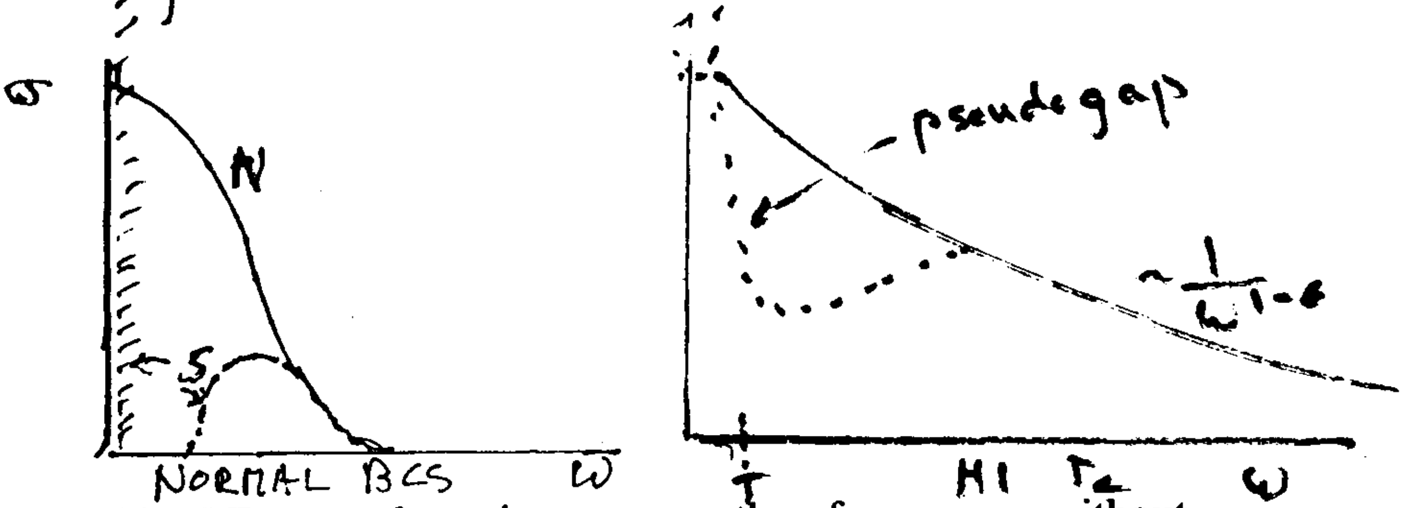
CHARGE FLUCTUATIONS AS IN

12 FERRELL - TINKHAM IDENTITY

# SUPERCONDUCTIVITY AT LAST

The role of the (Kohn)-Ferrell-Tinkham sum rule

$$\int \sigma \, d\omega = e^2 \times \text{Total K.E.}$$




In BCS, K.E.>>condensation energy; therefore no gap without SC--compensation of coherent pair hopping vs coherent single-particle motion is nearly exact.

But in the cuprate the conductivity is oc x, and much more widely distributed in frequency; so the gap in the conductivity is less severe and roughly oc x. This is why the spin gap can open for small x, ie why phase II exists.

But eventually, the kinetic energy wins and charge must condense into hole pairs: hence Tc. The K E can be both interlayer and intralayer in multilayers.

On the overdoped side, T* and Tc coincide--Tc is TWO transitions. Hole condensation is s-wave, hence not much affected by scattering? But may have isotope shift.

$$\sigma = \frac{ne^2}{i m^*(\omega)\omega + 1/\tau(\omega)}$$

$m^*$ Big in spin gap

Timusk's little secret.



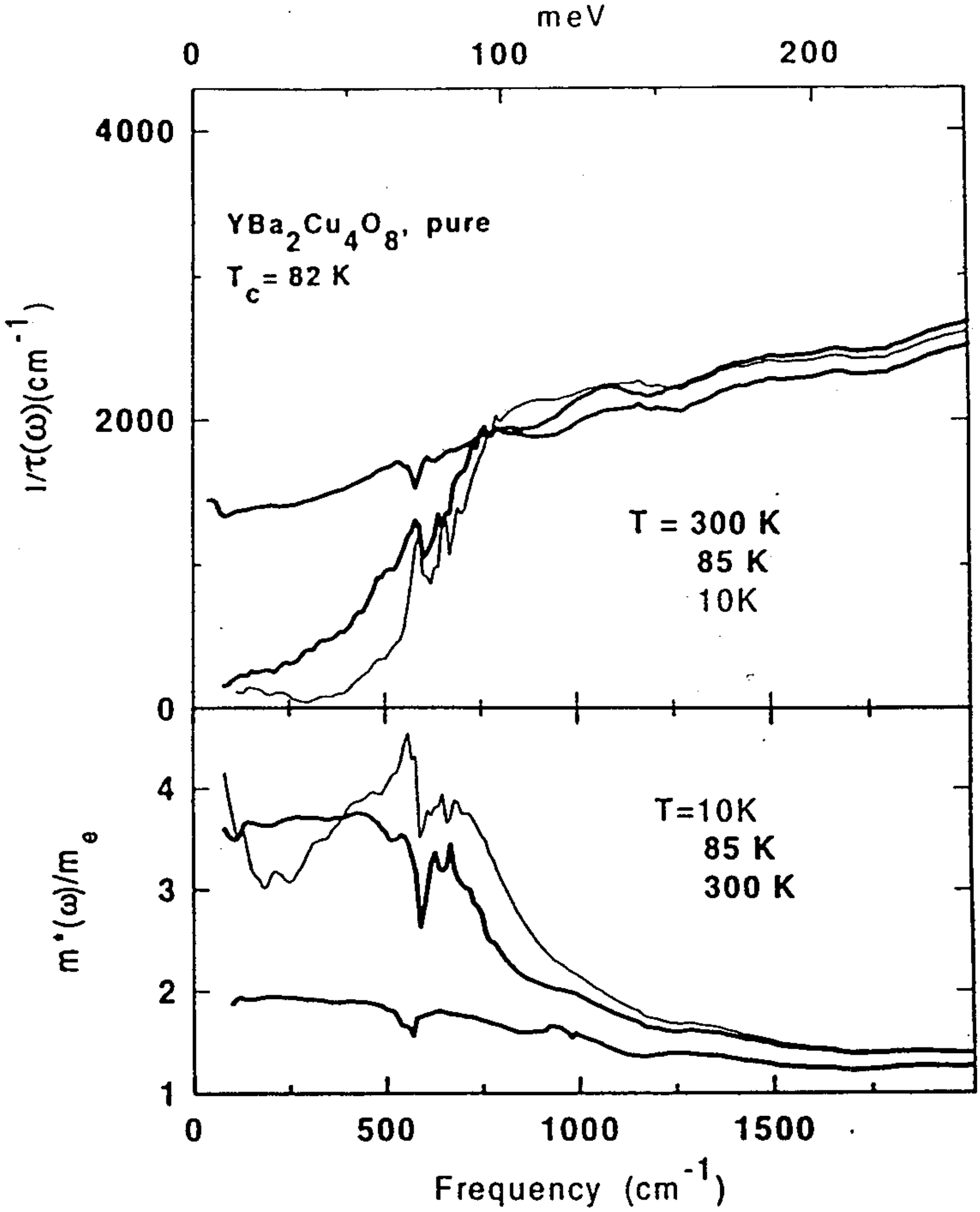
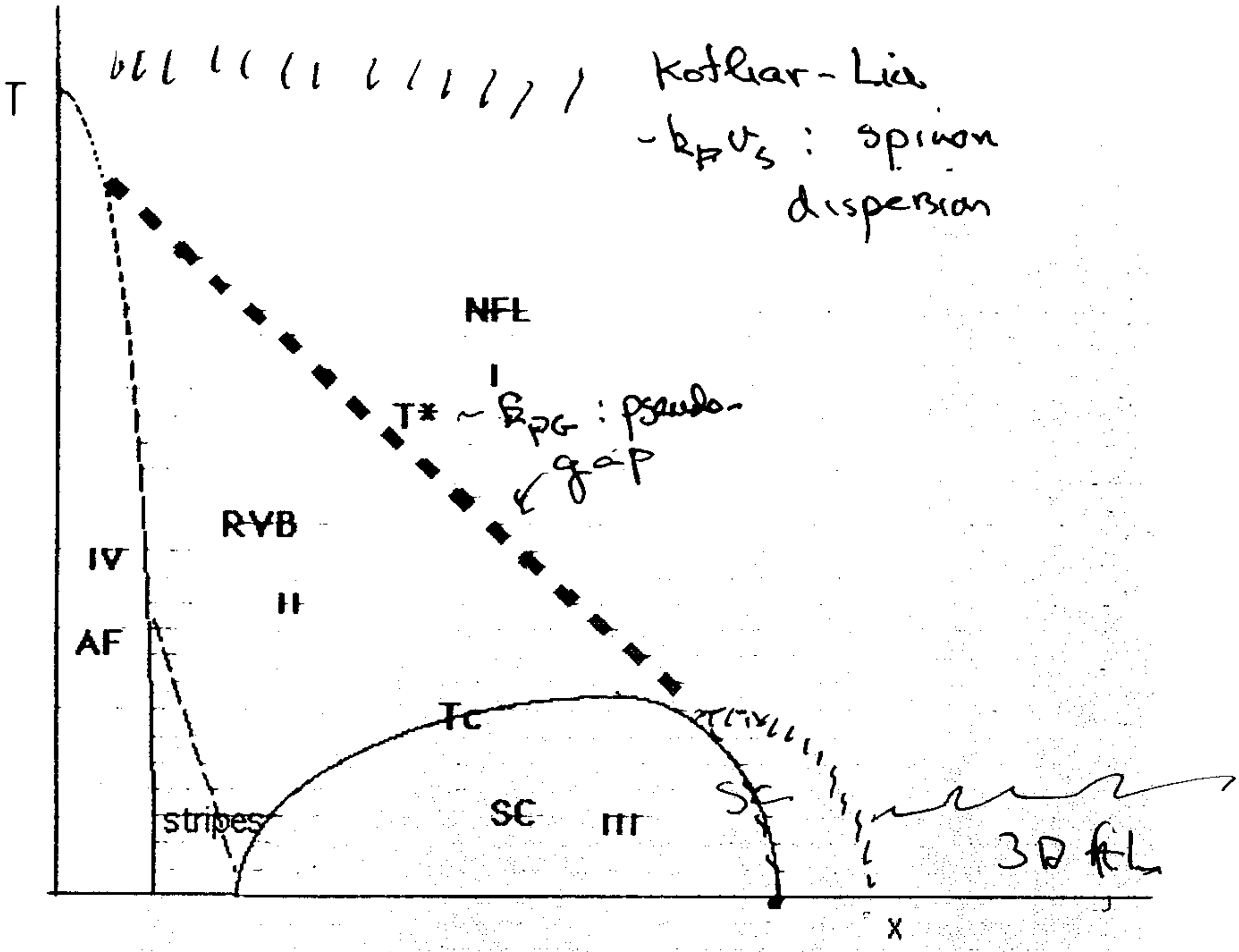



**Figure 28.** The frequency-dependent scattering rate and the effective mass of $YBa_2Cu_4O_8$. The scattering rate varies linearly at room temperature but develops a gap like depression in the normal state. At the same time the effective mass of the carriers develops a resonance peak at $\approx 600\ cm^{-1}$ (75 meV).




GENERALISED PHASE DIAGRAM

spin prefers to bind to pairs of holes

## CONCLUSIONS

Cuprates have unique properties:

single, narrow band, without self-trapping

proximity of an S=1/2 Mott Antiferromagnet

2D= non-Fermi liquid

allows AF->RVB with doping

THE CRUCIAL TRANSITION IS THE SPINON PAIRING AT T*

Superconductivity is then inevitable.

We were so close in 1987!!.