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SMR 1232 - 36

XII WORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

17 - 28 July 2000

GLASSES, OPTIMIZATION AND COMPUTATION

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GLASSES, OPTIMIZATION

AND

COMPUTATION

L.B. IOFFE

PC

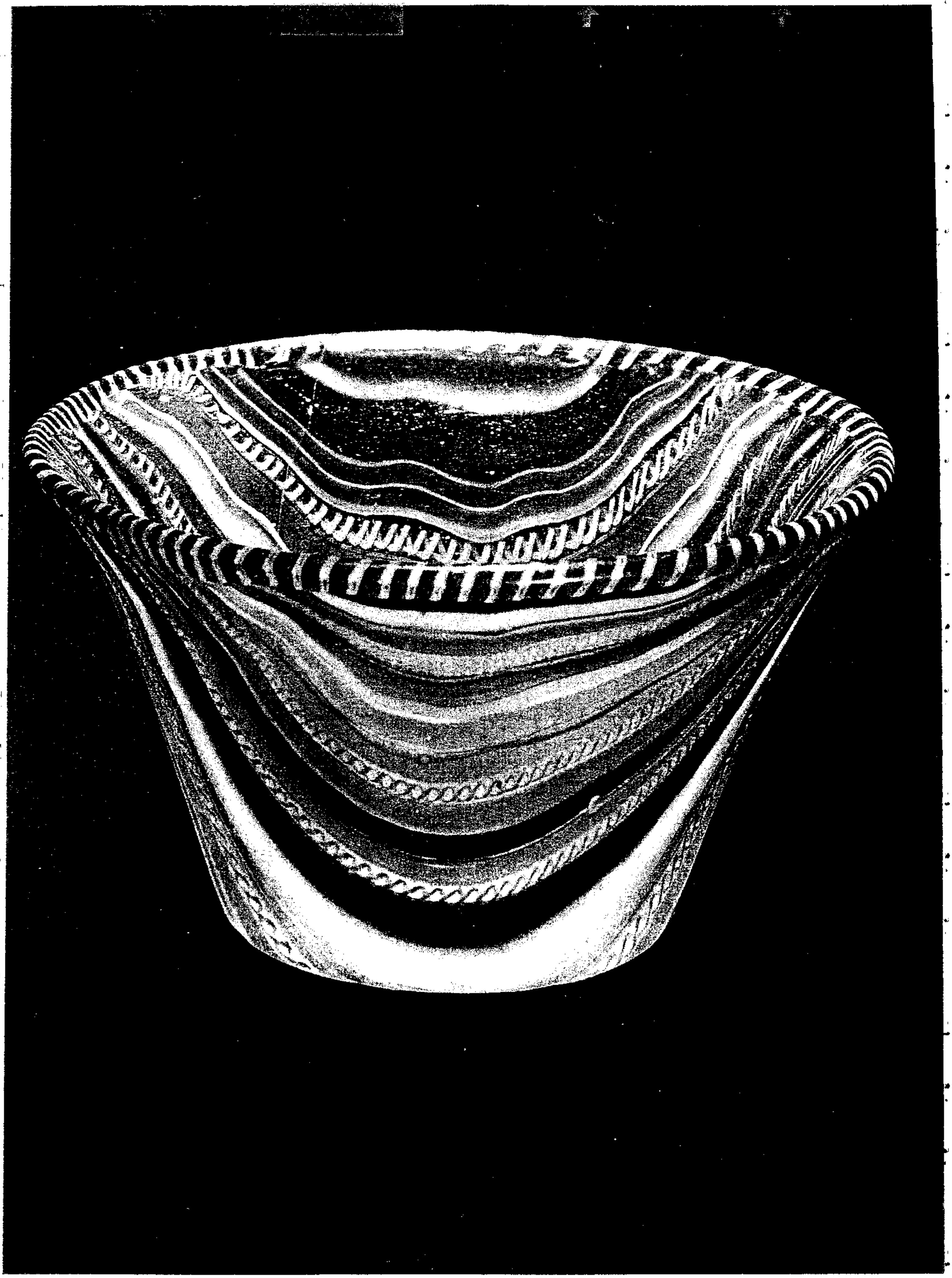
I. MOTIVATION

II. A MODEL

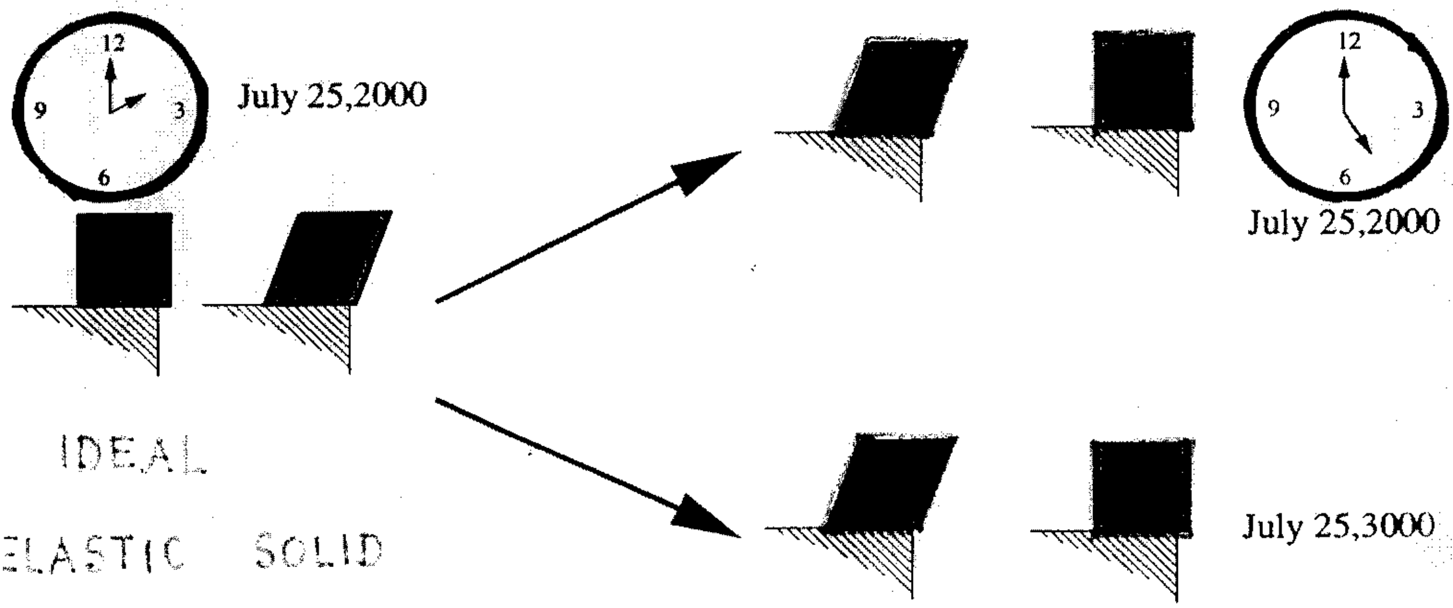
III. NEW APPROACHES TO OPTIMIZATIO[N]

IV. COMPUTATION?

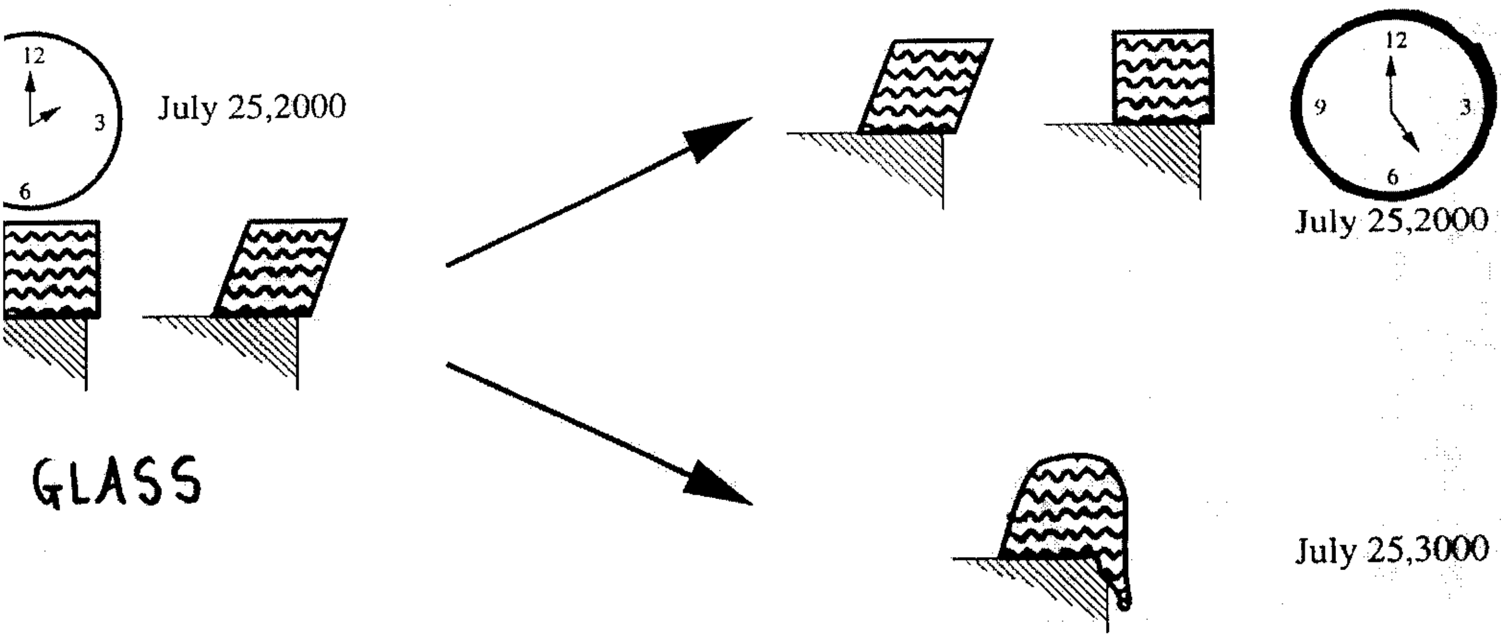
V. SUMMARY / ??s



RESPONSE TO SHEAR



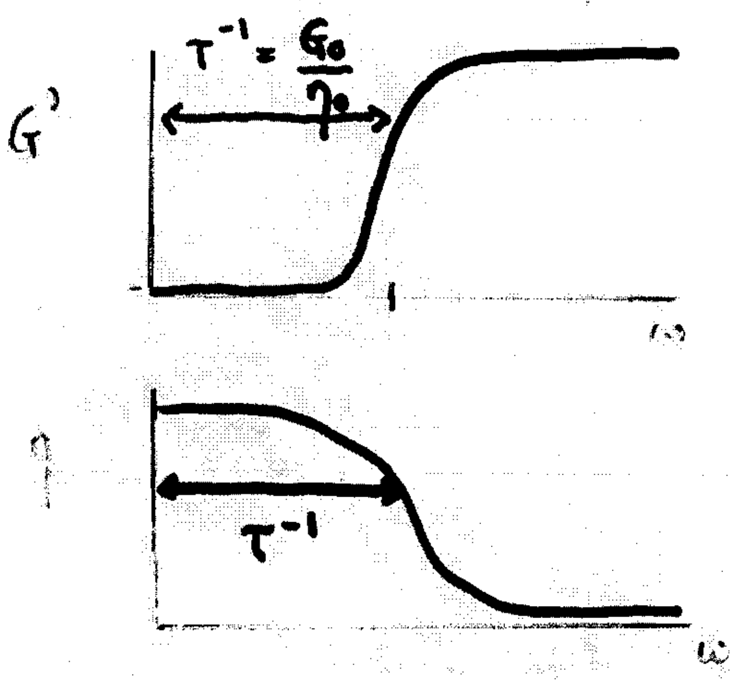
IDEAL ELASTIC SOLID

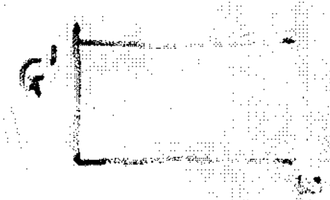


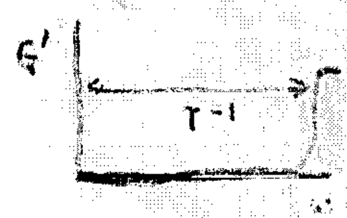
GLASS

	LIQUID → SOLID	METAL → SUPERCONDUCTOR
CONDUCTIVITY EQUATION	$\Pi = \eta \dot{u}$	$J = \sigma E = -\sigma \dot{A}$
"DRUDE"	$\eta = \eta_0 \frac{1}{1 - i\omega\tau}$	$\sigma = \frac{ne^2\tau}{m} \frac{1}{1-i\omega\tau}$
LONDON KERNEL	$\Pi = G u$	$J = -Q A$
KUBO RELATION	$G = -i\omega\eta$	$Q = -i\omega\sigma$

$$G = \frac{G_0}{\eta_0} \frac{-i\omega\tau}{1 - i\omega\tau}$$







WANTED : A REDUCTIONIST MODEL

(e.g. ISING MODEL)

FOR GLASSES

- HOW IS A SYSTEM SIMULTANEOUSLY OUT-OF-EQUILIBRIUM AND STABLE ON VERY LONG (e.g. MILLENIUM) TIME-SCALES ?

- HOW DOES ONE KEEP AN INTRINSICALLY NON-RANDOM SYSTEM AWAY FROM ITS THERMODYNAMIC GROUND-STATE ?

WEISS MODELS OF DISORDER-FREE GLASSINESS

MAIN DIFFICULTY : INEQUIVALENCE OF SITE

(IN A GIVEN STATE M_i VARIES FROM SITE TO SI

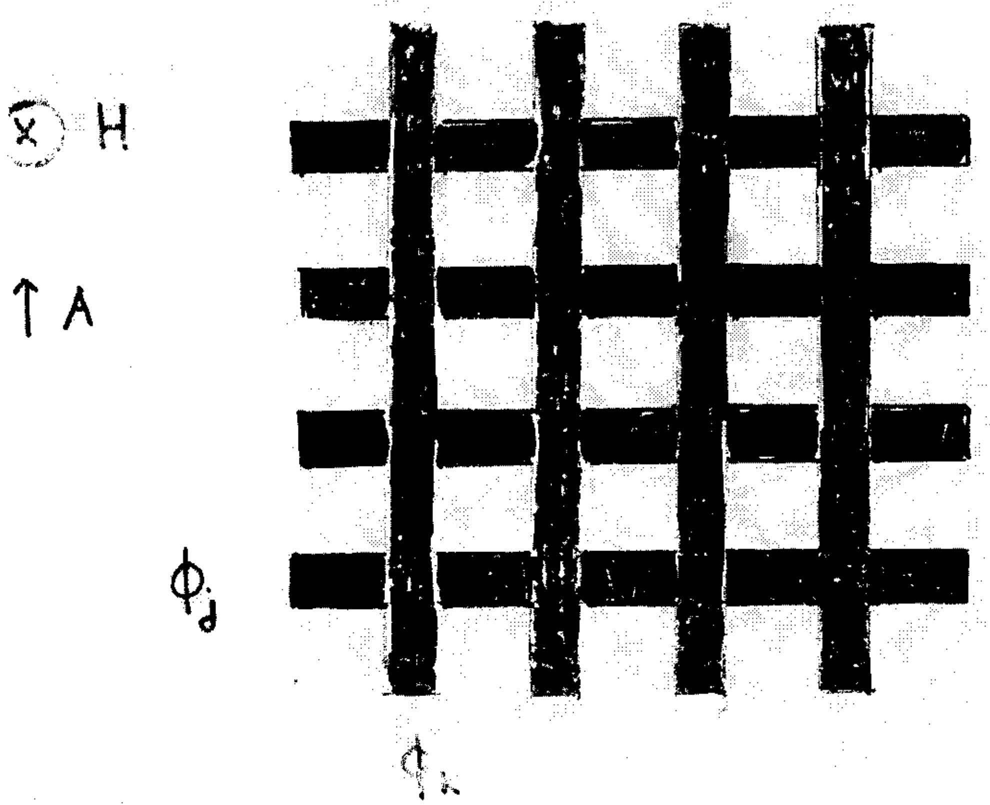
N.B. GLASSES WITH QUENCHED DISORDER $\Rightarrow$ EQUIVALENCE OF SITE AFTER DISORDER AVERAGING

CURRENT APPROACHES

- FIDUCIARY HAMILTONIAN
- REPLICA / CLONING
- DYNAMICS

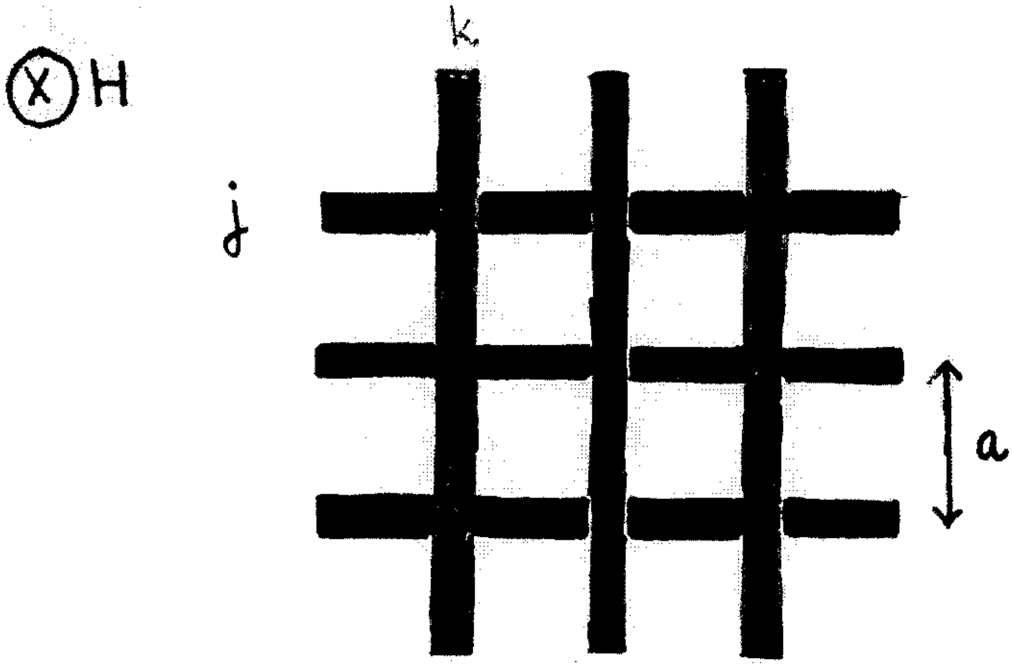
REVIEW: BOUCHAUD, CUGLIANDOLO, KURCHAN + MEZARD, IN YOUNG: <u>SPIN GLASSES AND RANDOM FIELDS</u> (WORLD SCIENTIFIC, SINGAPORE, 1998), pp. 161 - 225.

A MODEL



$$\boxed{Z = N}$$

$$\boxed{\begin{gathered} E = \rho_s \int (j_s)^2 \, dV \; - \; J_0 \sum_{jk} \cos \Delta \phi_{jk} \\[2em] j_s \cong \nabla \phi - \frac{2\pi}{\phi_0} \vec{A} \end{gathered}}$$



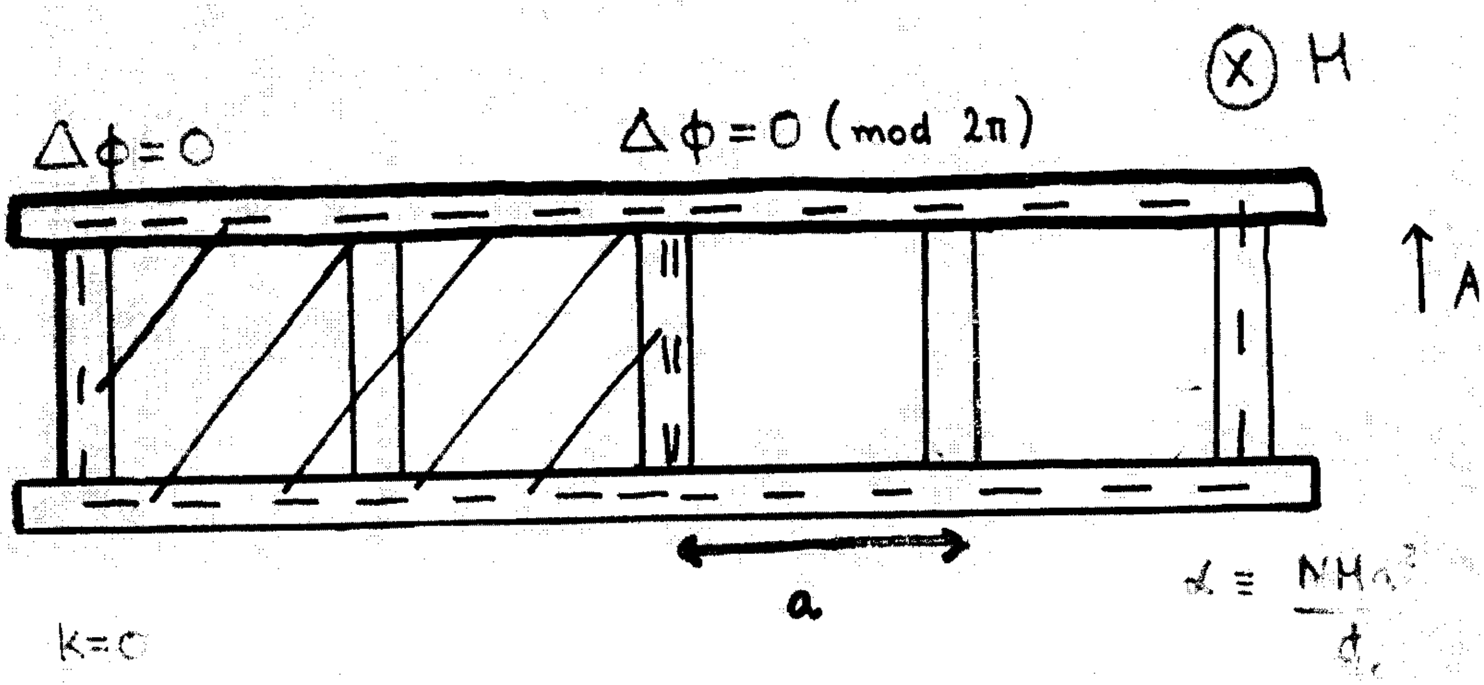
$$\alpha = \frac{NHa^2}{\Phi_0}$$

$$\mathcal{H} = -J \sum_{jk} \cos\left(\phi_j - \phi_k - \frac{2\pi \Phi_{jk}}{\Phi_0}\right)$$

$$= -\mathrm{Re} \sum_{jk} s_j^* \, J_{jk} \, s_k$$

$$s_j = e^{i\phi_j}$$

$$J_{jk} = J \exp \frac{2\pi i \Phi_{jk}}{\Phi_0} = J \exp \frac{2\pi i \alpha j k}{N}$$



$\alpha \equiv \frac{NHa^2}{\phi_0}$

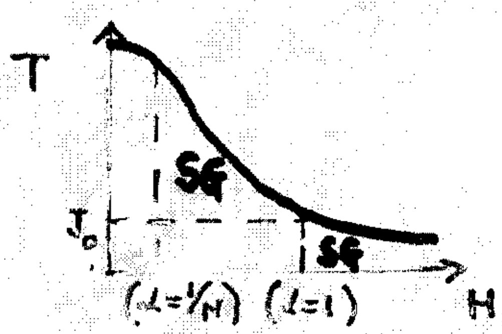
$$\sum_{k}^{N} J_{jk} J^{+}_{k\ell} = \sum_{k} \exp \boxed{\frac{2\pi i \alpha k (j-\ell)}{N}}$$

COMMENSURATE IFF

INTEGER

DISORDERED CASE

(VINOKUR, IOFFE, LARKIN, FEIGELMAN 1987)

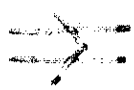


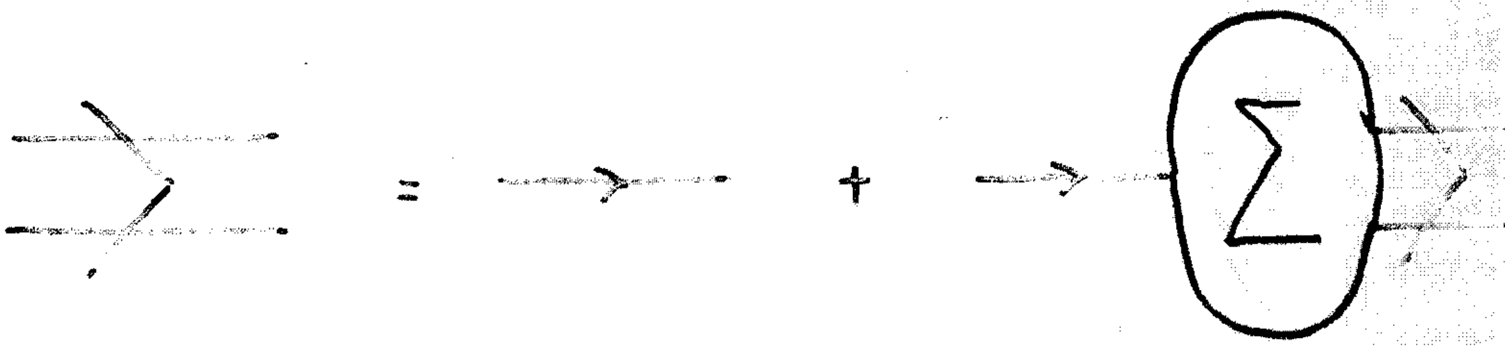
PERIODIC CASE $\quad 1/N < \alpha < 1$

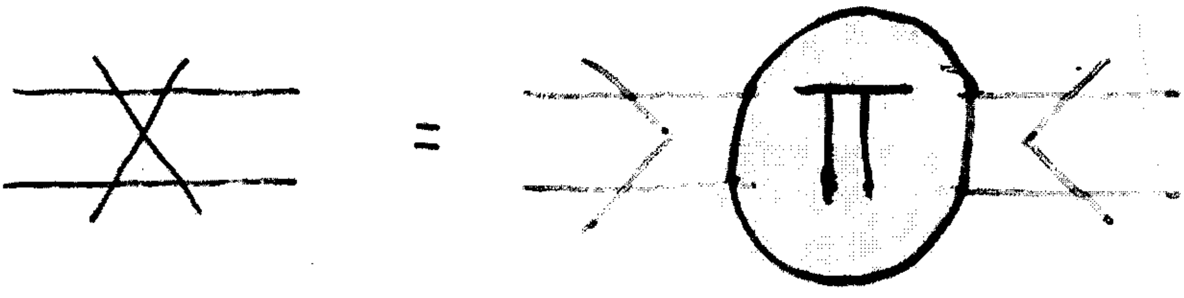
ABSENCE OF COMMENSURABILITY !!

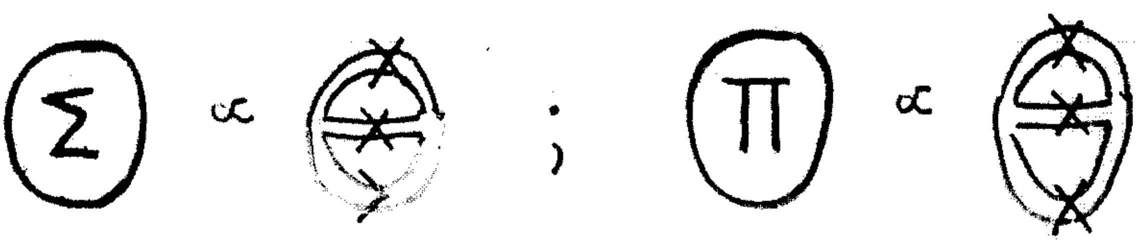
DYNAMICS (SOFT SPINS W/ LANGEVIN)

$$D(t,t') = $$  $$= \frac{1}{2N} \sum_{j=1}^{2N} \left\langle s_j(t)\, s_j(t') \right\rangle$$

$$G(t,t') = $$  $$= \frac{1}{2N} \sum_{j=1}^{2N} \left. \frac{\partial \langle s_j(t) \rangle}{\partial h_j(t')} \right|_{\substack{h_j = 0 \\ t > t'}}$$



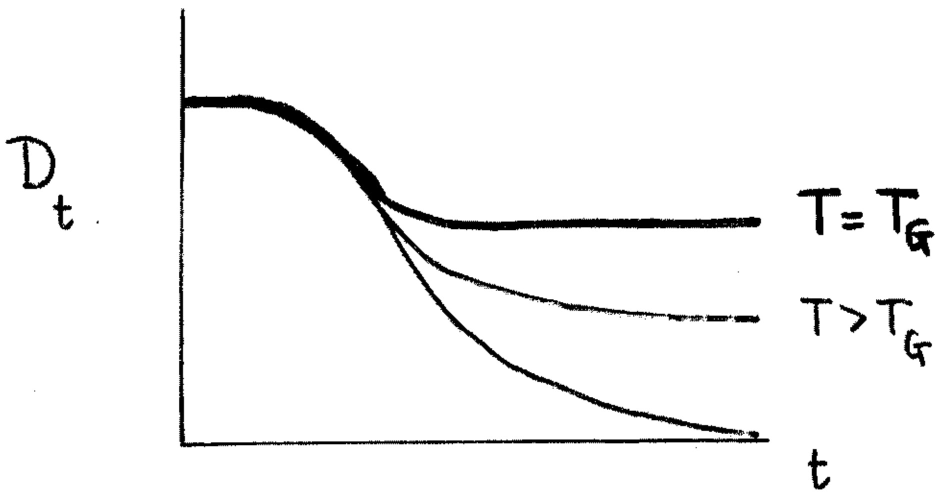




$T > T_G$ $\qquad G(t-t') = -\frac{\partial D}{\partial t}(t-t')\,\theta(t-t')$

$$\Downarrow$$

$$\ldots + \gamma \dot{D}_t + \Omega^2 D_t + \int_0^t \Pi(t-t')\,\dot{D}_{t'}\,dt' = 0 \qquad (*)$$



(*) SIMILAR TO MODE-COUPLING EQ. (DENSITY-DENSITY CORRELATIONS)

(*) IDENTICAL TO DYNAMICAL EQUATION FOR THE $P=4$ (DISORDERED) SPHERICAL MODEL ("SIMPLEST SPIN GLASS")

Transition Temperature of Josephson Junction Arrays with Long-Range Interaction

H. R. Shea and M. Tinkham

Department of Physics and Division of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02

(Received 17 June 1997)

We report measurements of the dependence on magnetic field and array size of the resistive transition of Josephson junction arrays with long-range interaction. Because every wire in these arrays has a large number of nearest neighbors (9 or 18 in our case), a mean-field theory should provide an excellent description of this system. Our data agree well with this mean-field calculation, which predicts that T_c (the temperature below which the array exhibits macroscopic phase coherence) shows very strong commensurability effects and scales with array size. [S0031-9007(97)04071-4]

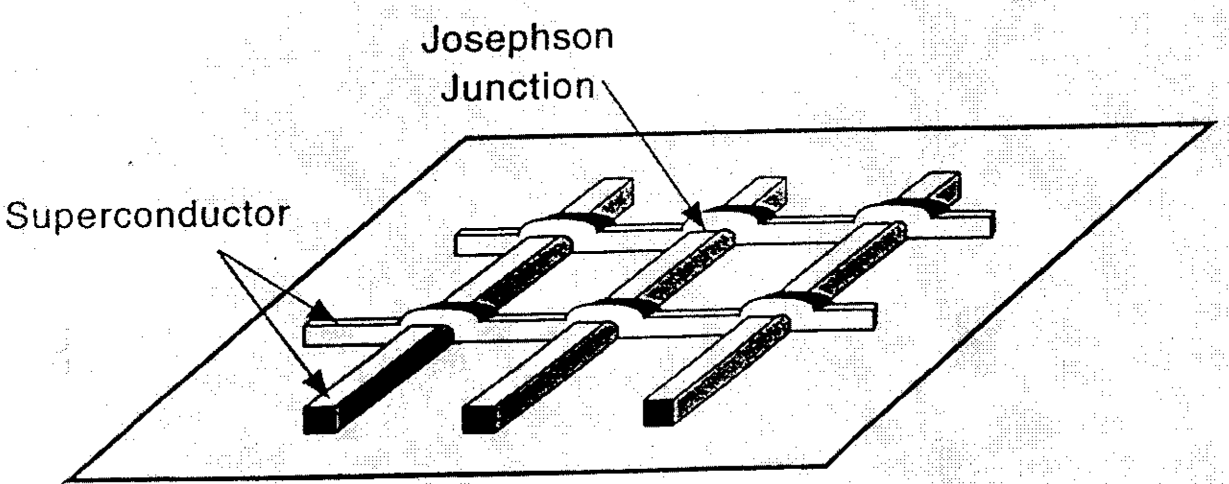
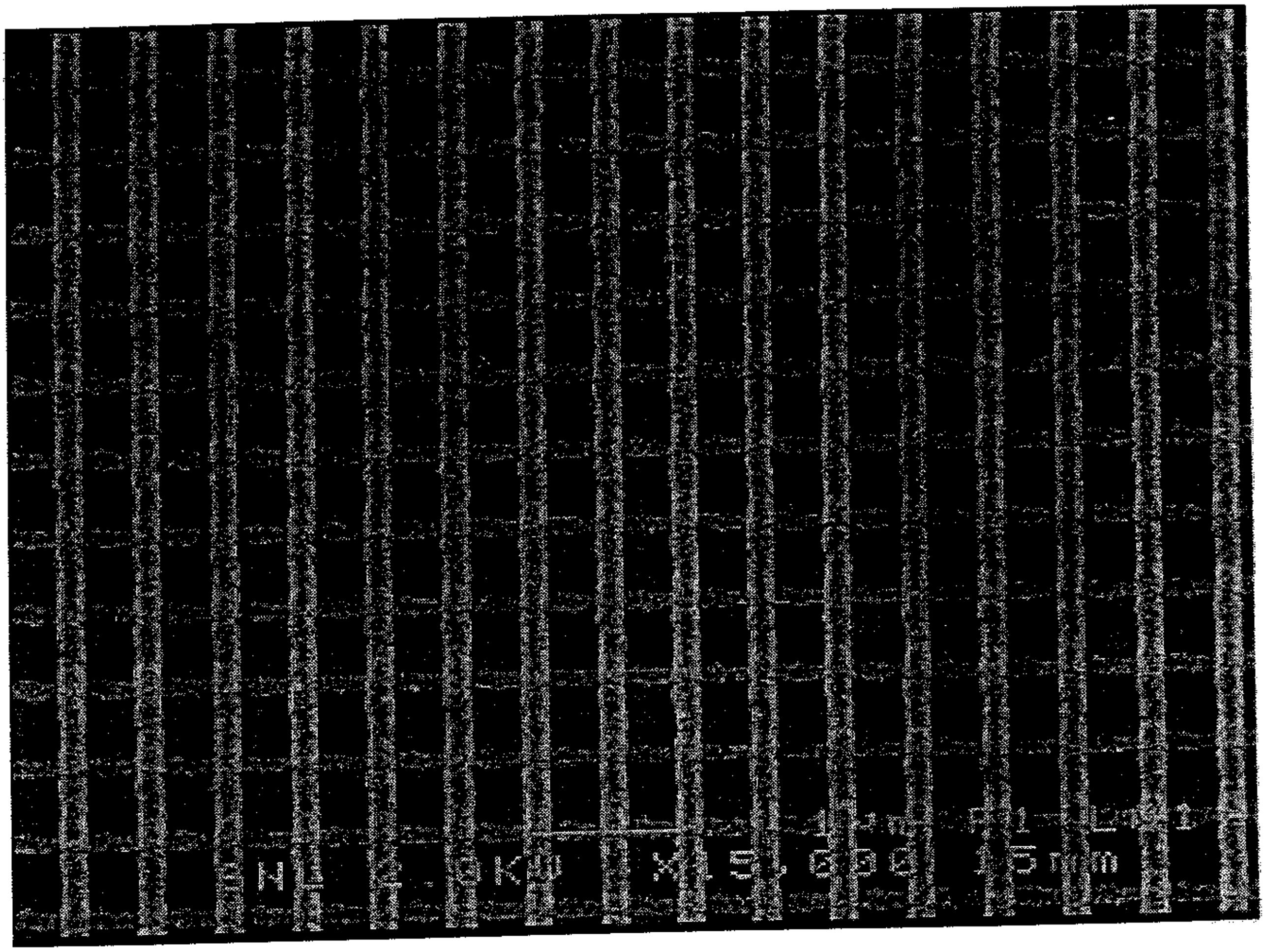


FIGURE 1

H.R. Shea and M. Tinkham
"Transition Temperature of Josephson Junction Arrays with Long-Range Interaction"



(M. GERSHENSON ET AL.)

cond-mat/9706179 17 Jun 1997

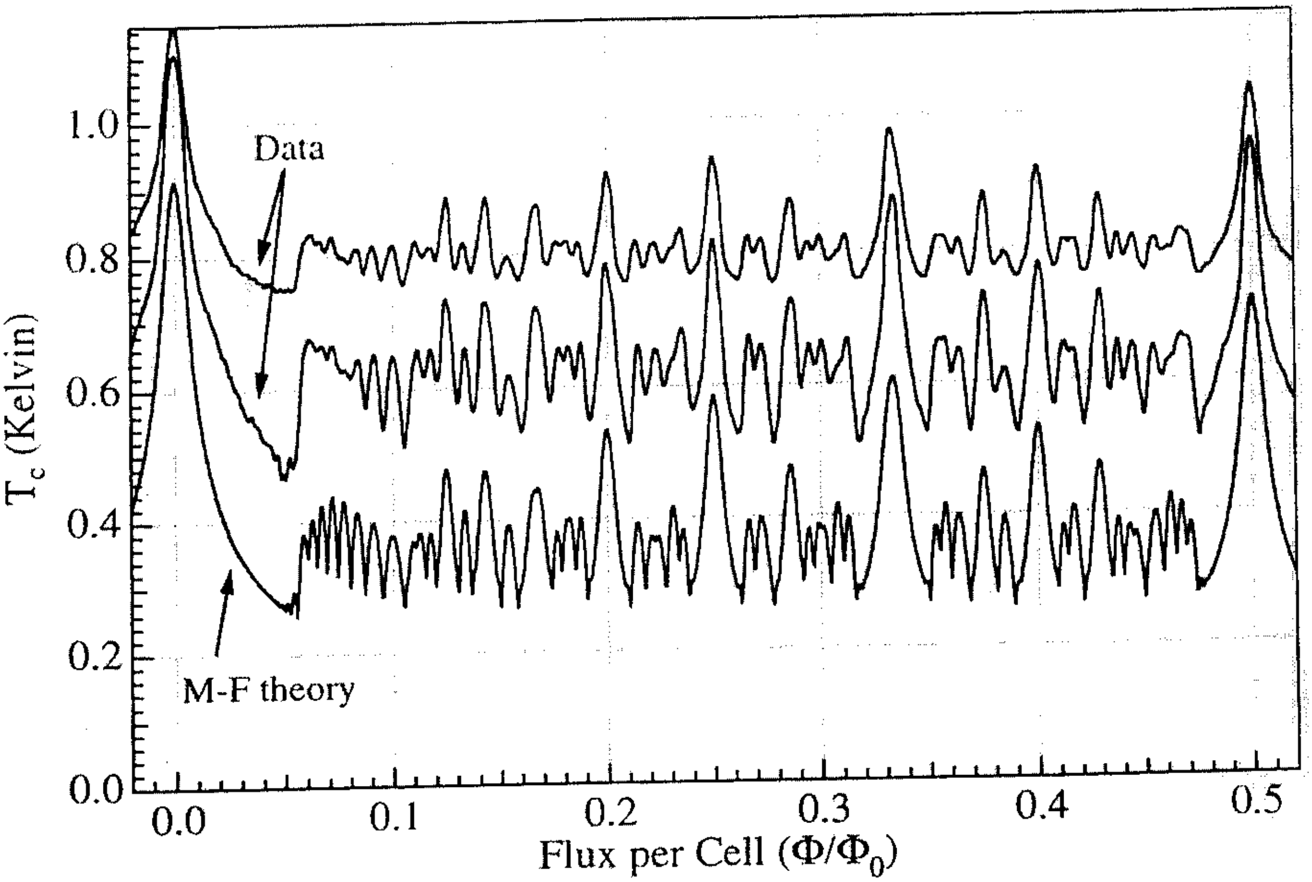


Figure 4

H.R. Shea and M. Tinkham
"Transition Temperature of Josephson Junction Arrays with Long-Range Interaction"

UNITARY CASE $(\alpha = 1)$

GROUND STATE

$$E_0 = -\sum_j |h_j|$$

$$\sum_k h_k^2 = N \implies \text{MINIMAL } E_0 \iff |h_k| = 1$$

GROUND STATES $\iff$ UNITARY SEQUENCE WITH FLAT FOURIER TRANSFORM

$$B_r \equiv \sum_{\ell} S^*_{\ell+r}\, S_r = 0$$

ZERO AUTOCORRELATIO

$$\text{SINCE} \quad \sum_k e^{2\pi i \frac{kr}{N}} \left| h_k^2 \right| = B_r = \delta_{0r}$$

FAMOUS PSEUDORANDOM SEQUENCES

GAUSS $\qquad s_k = e^{2\pi i \frac{k^2}{N}}$

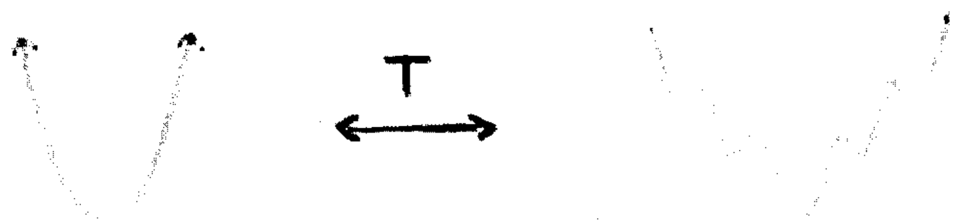
GALOIS $\qquad s_k = (-1)^{a_k} \qquad k = 1 \dots p$

⋮

OPTIMIZATION

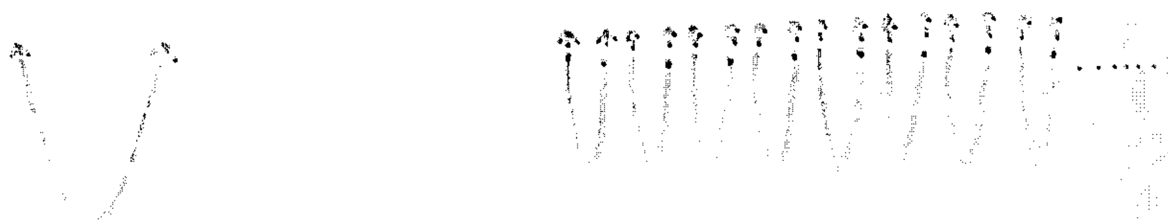
OPTIMAL STATE $\longleftrightarrow$ STATE W/ LARGE BASIN OF ATTRACTIO

E.G. SIMULATED ANNEALING



$T > T_c$ $\qquad\qquad$ $T < T_c$

HOWEVER....



$T > T^*$ $\qquad\qquad$ $T = T^*$

TUNING PARAMETER ?

FAMILY OF MODELS

$$\sqrt{x} \sum_{i_1 i_2} J_{i_1 i_2} s_{i_1} s_{i_2} + \sqrt{1-x} \sum_{i_1 i_2 i_3} J_{i_1 i_2 i_3} s_{i_1} s_{i_2} \ldots$$

$$\left(\sum_{i=1}^{N} s_i^2 = 1 \right)$$

$p > 2$

- ∞ RANGE
- ALL METASTABLE STATES APPEAR AT T_c AND ARE ORTHOGONAL (CRISANTI + SOMME KURCHAN, PARISI + VIRASORO)
- HISTORY-DEPENDANCE AND AGING (CUGLIANDOLO + KURCHAN)

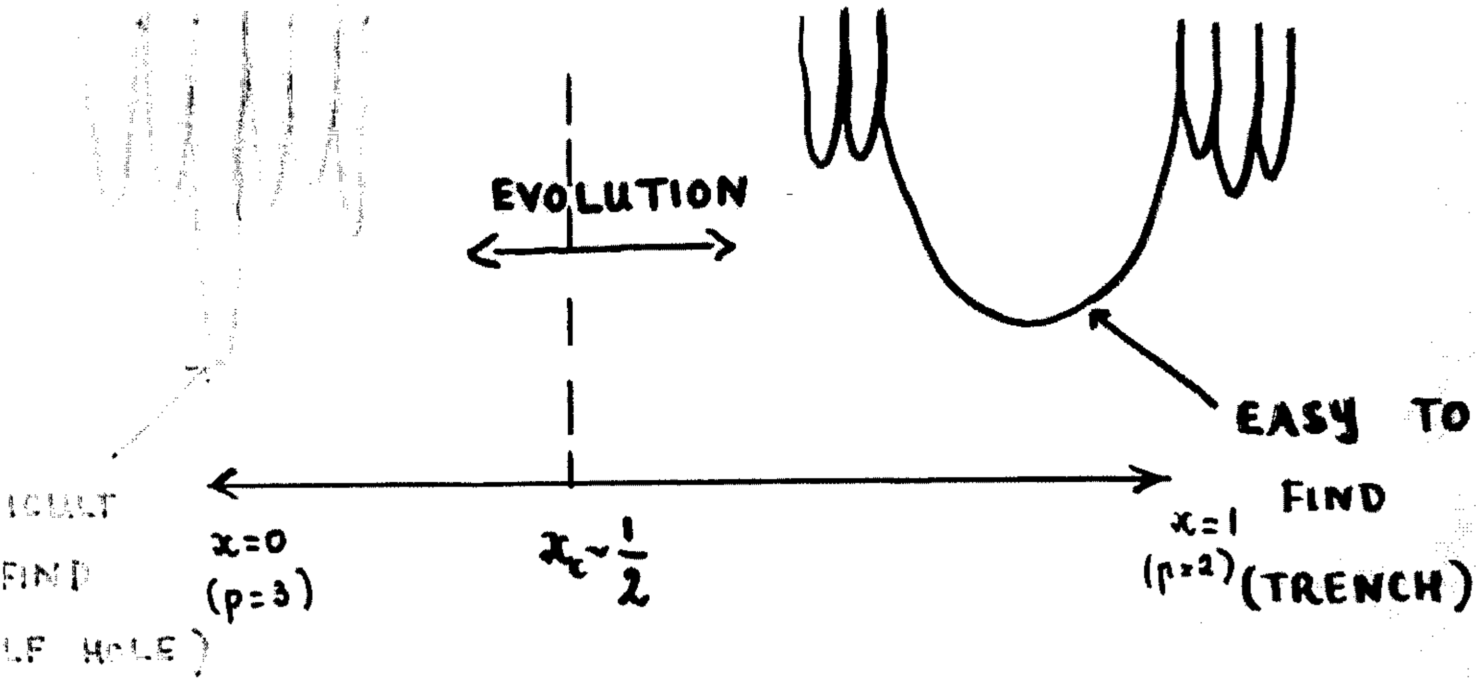
$x = 1$ $(p = 2)$ ONE SOLUTION (KOSTERLITZ)

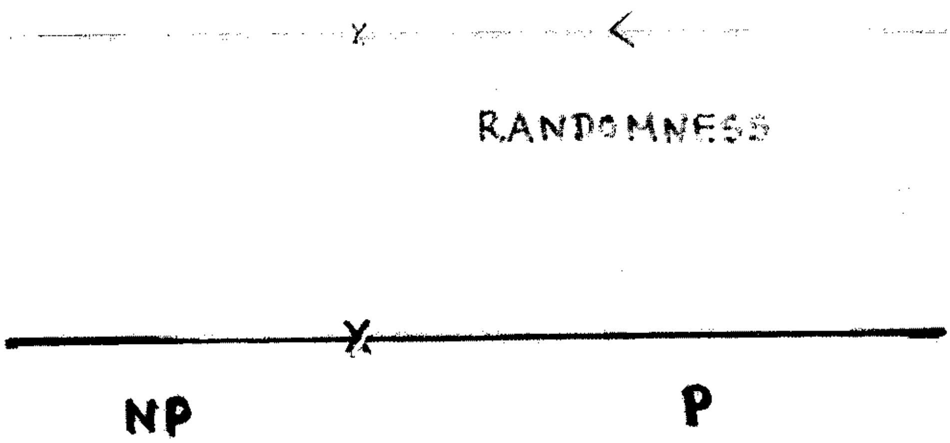
$x = 0$ $(p = 3)$ ATTRACTION BASINS EXPONENTIALLY SMALL (BARRAT + FRANZ)

$0 \leq x < 1$

- 1 STEP RSB
- $S_{\text{CONF}} \equiv \ln n_s \propto N$

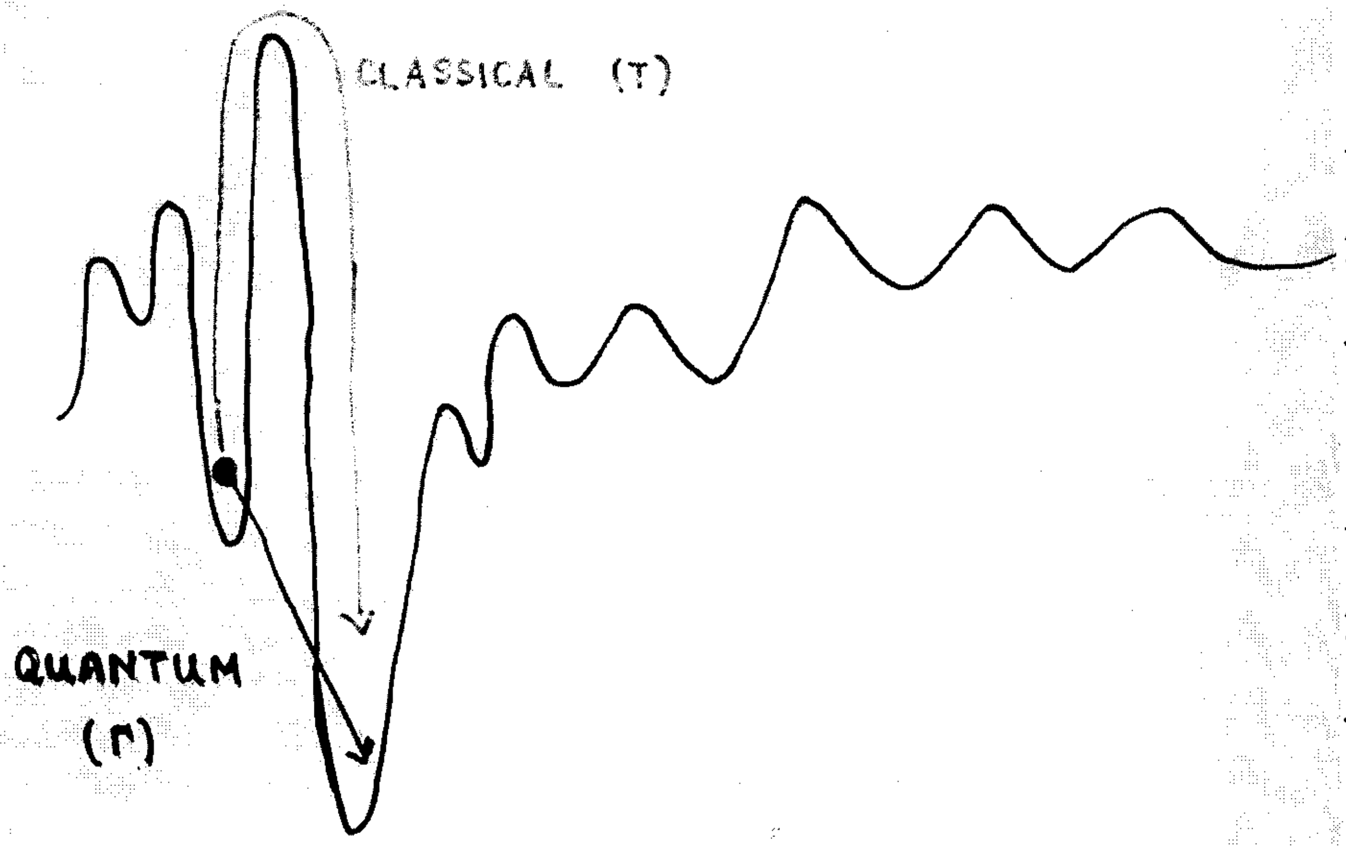
QUALITATIVE RESULTS





K-SAT PROBLEMS
MONASSON, ZECCINA, KIRKPATRICK
ET AL, NATURE 400, 133 (1999)

QUANTUM ANNEALING



$$H = \sum_{ij}^{N} J_{ij}\, \sigma_i^z\, \sigma_j^z \;-\; \Gamma \sum_i \sigma_i^x$$

- EXPERIMENT

  BROOKE ET AL. SCIENCE 284, 779 (99)

- THEORY

  KADOWAKI + NISHIMORI COND-MAT/9804280

  CUGLIANDOLO ET AL COND-MAT/003268

COMPUTATION ?

MANIPULATION OF STATES $\Longleftrightarrow$ ROBUST TO ENVIRONMENT

WANTED :

QUANTUM GLASS W/

"HALL LIQUID"

FEATURES

- SPIN LIQUID G. S.

- GAP

SUMMARY

- DISORDER-FREE GLASS MODELS

$\Updownarrow$

CODING SEQUENCES

- ALTERNATIVE OPTIMIZATION METHODS

QUESTIONS

- UNIFYING PRINCIPLES UNDERLYING SYSTEMS FAR FROM EQUILIBRIUM ?

  - DRIVEN VS. SPONTANEOUS

Q. DOTS	GLASSES
TURBULENCE	↑
	IMPORTANCE OF DISORDER ?

- COMBINED COMPLEXITIES OF CLASSICAL + QUANTUM MECHANICS ?

  (NONLINEARITY + SUPERPOSITION) ?