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SMR 1232 - 28

XII WORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

17 - 28 July 2000

ELECTRON EDGE STATES
IN QUASI-1D and QUASI-2D

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Electrons on Edge

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Outline

I. Localized electron state at the end of a one-dimensional system with a periodic potential

II. Chiral edge states in the quantum Hall regime of the quasi-one-dimensional organic conductors $(TMTSF)_2X$

III. Midgap Andreev bound states in the triplet superconducting state of $(TMTSF)_2X$. Relation to

- Solitons in polyacetylene
- Midgap Andreev bound states in d-wave cuprate superconductors

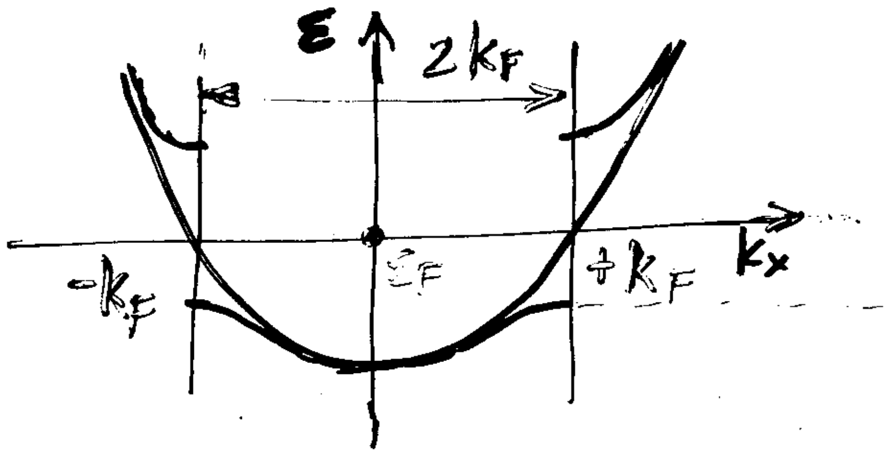
IV. Chiral edge states and spin quantum Hall effect in the triplet superconductor Sr_2RuO_4

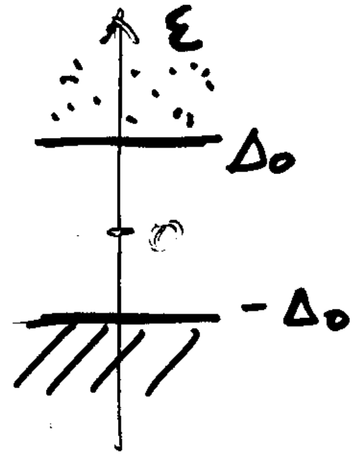
V. Work in progress

- Edge states in graphite and carbon nanotubes
- Entropy of black holes

Semi-infinite one-dimensional system with a periodic potential (CDW, SDW)

$$\hat{\mathcal{H}} = -\frac{1}{2m}\frac{\partial^2}{\partial x^2} - 2\Delta_0 \cos(2k_F x - \Theta)$$





$$\psi(x) = \psi_+(x)\, e^{ik_F x} + \psi_-(x)\, e^{-ik_F x}$$

$$\begin{pmatrix} -iv_F\partial_x & -\Delta_0 e^{-i\Theta} \\ -\Delta_0 e^{i\Theta} & iv_F\partial_x \end{pmatrix} \begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix} = \varepsilon \begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix}$$

Delocalized bulk states

$$\begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix} \propto e^{ikx} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}, \qquad \varepsilon = \pm\sqrt{(v_F k)^2 + \Delta_0^2}$$

Localized edge state

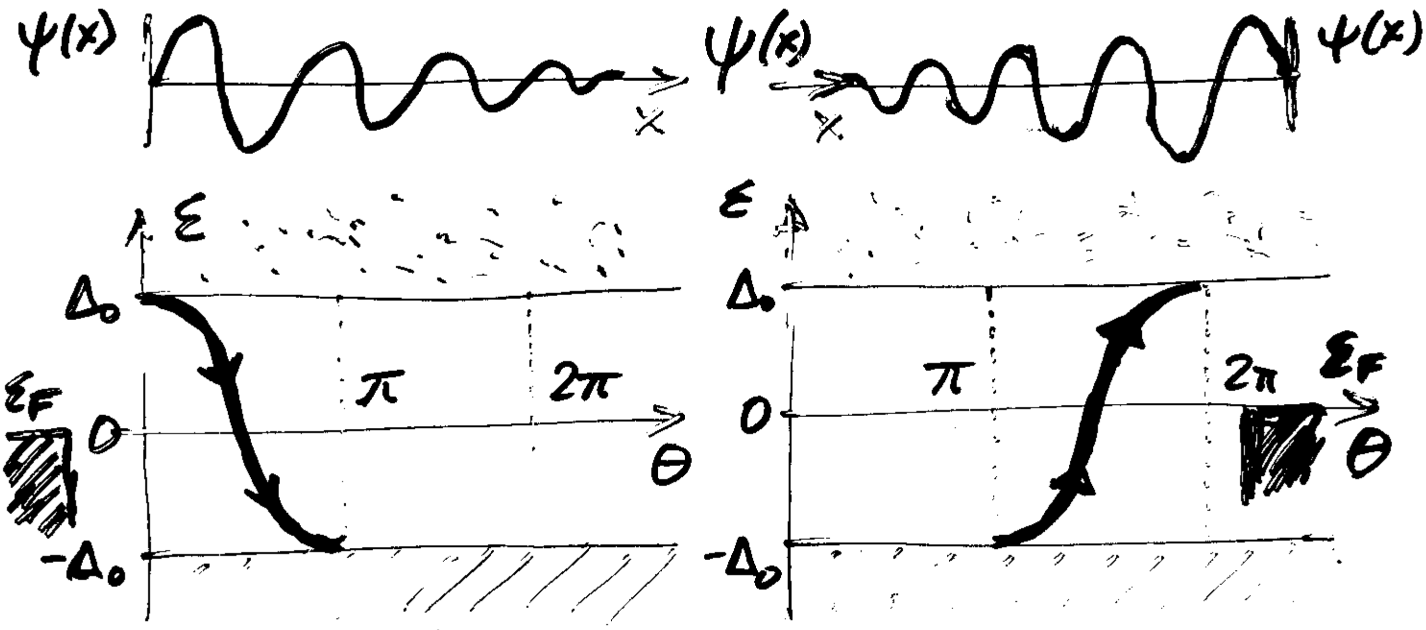
Boundary condition: $\psi(x=0) = 0 \;\Rightarrow\; \psi_+ = -\psi_-$

$$\begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix} \propto e^{-\kappa x} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \varepsilon = \Delta_0 \cos\Theta, \quad \kappa = \frac{\Delta_0}{v_F}\sin\Theta$$

Spectral flow of localized edge states

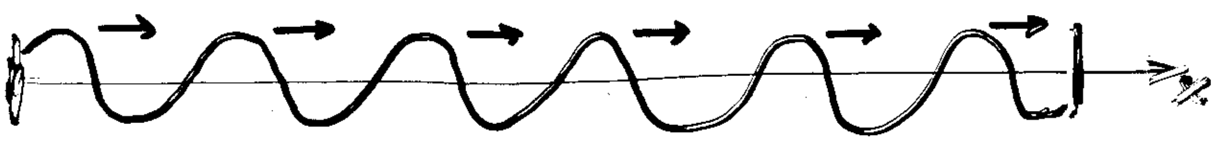
$$\varepsilon = \Delta_0 \cos\Theta, \quad \kappa = \frac{\Delta_0}{v_F}\sin\Theta, \quad \psi(x) = e^{-\kappa x}\sin(k_F x)$$

Left edge: $\kappa > 0$, $0 < \Theta < \pi$. Right edge: $\kappa < 0$, $\pi < \Theta < 2\pi$.



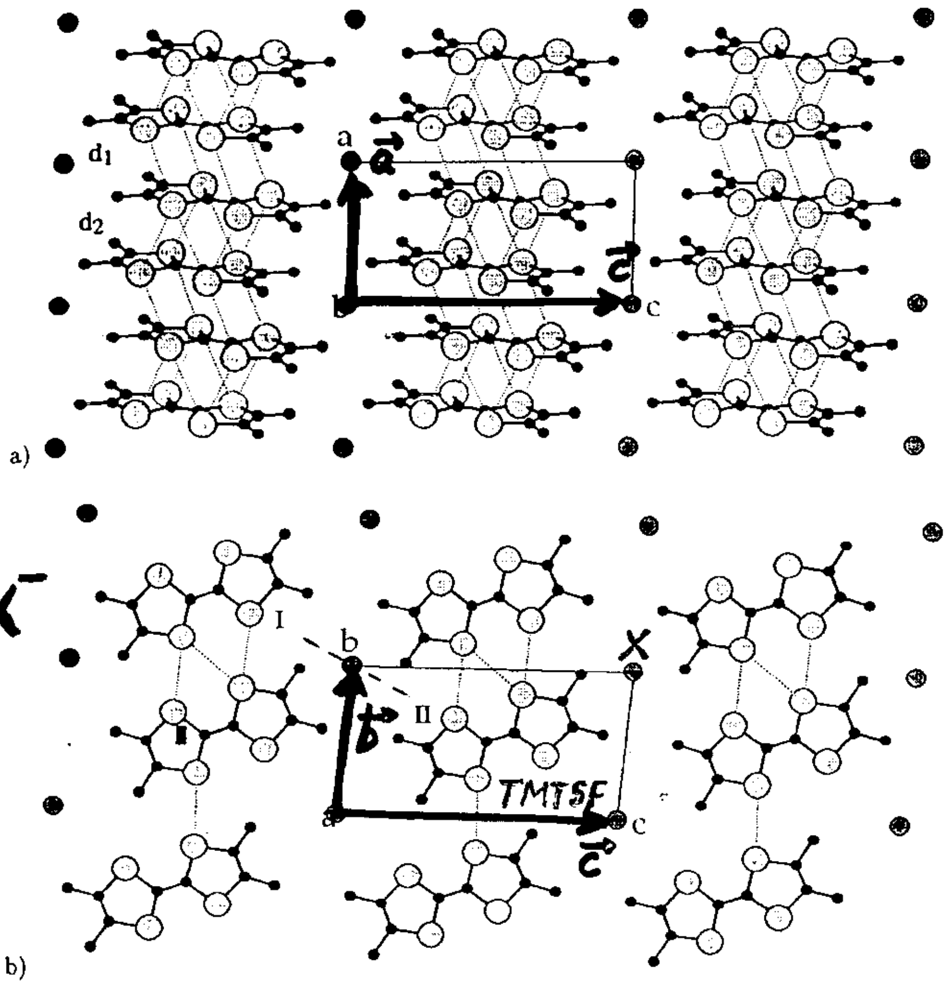
When Θ increases by 2π

- The periodic potential moves to the right by one period
- It moves one electron to the right (two for spins ↑ & ↓)
- **Spectral flow:** An electron state flows
  - from the upper to the lower band on the left edge
  - from the lower to the upper band on the right edge



Conversion of electric current from metallic leads into the collective Fröhlich current of a sliding charge-density wave (CDW) via spectral flow

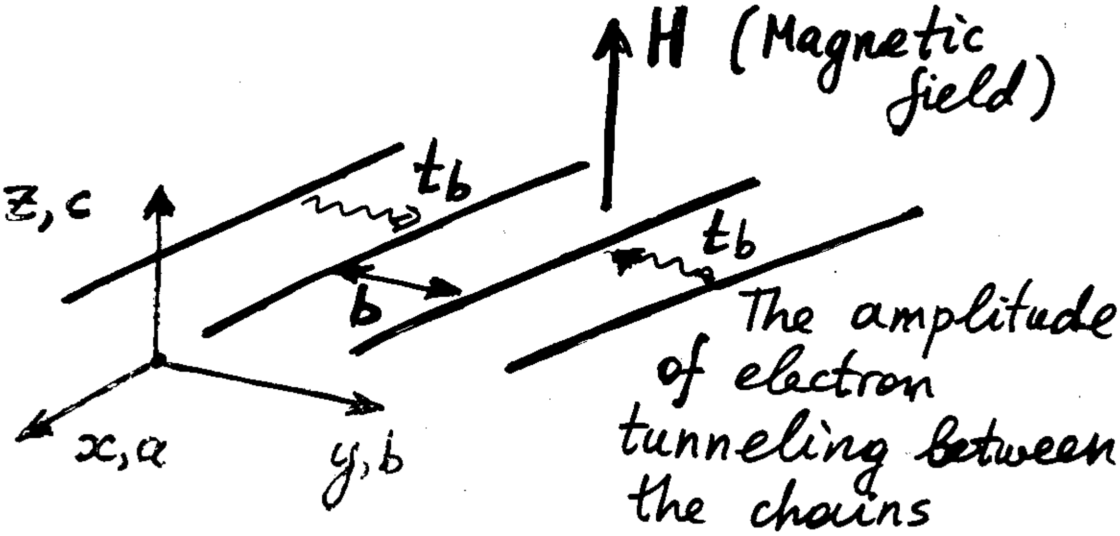
J.P. Pouget and S. Ravy. J. Phys. I France 6, 1501 (1996)



$(TMTSF)_2^+ X^-$

$X = PF_6$, ClO_4

Fig. 1. — Projections of the crystallographic structure of $(TMTSF)_2X$ in the (a, c) plane (a) and the (b, c) plane (b). Only the Se (large grey dots) and C (small black dots) atoms of the TMTSF molecule are represented. The grey dots of medium size between the molecules symbolizes the location of the anions X.



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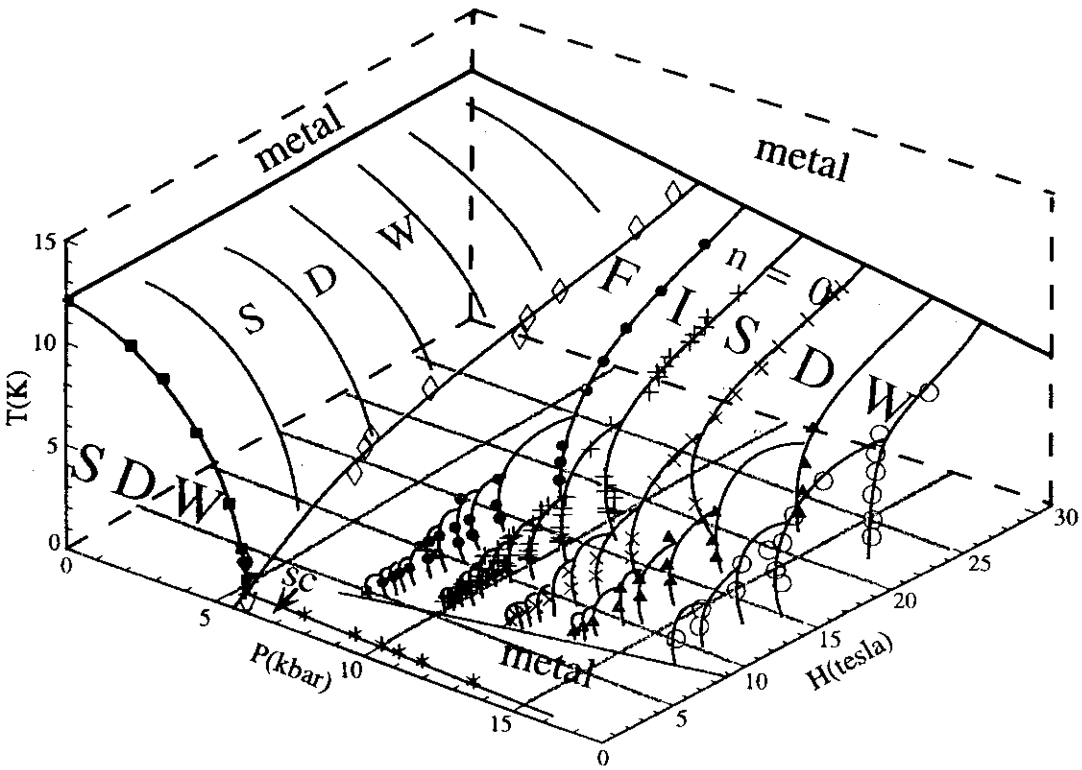
Phase Diagram of $(TMTSF)_2PF_6$



W. Kang et al

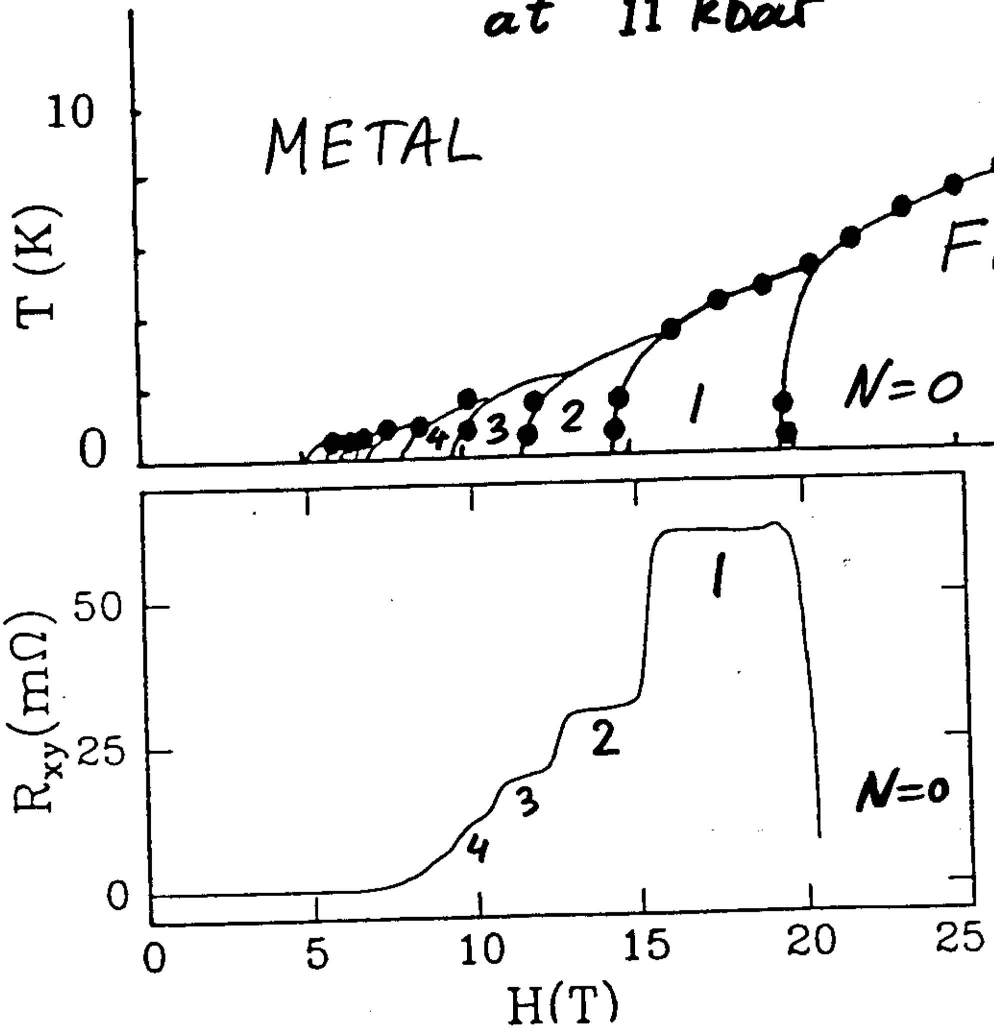
SC = Superconductivity

SDW = Spin-Density Wave

FISDW = Magnetic-Field-Induced Spin-Density Wave

P.M. Chaikin, W. Kang, S. Hannahs, and R.C. Yu
Physica B 177, 353 (1992)

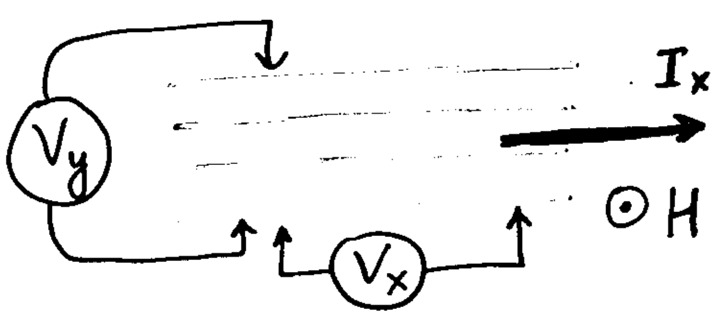
Phase diagram of $(TMTSF)_2PF_6$ at 11 kbar



$$\sigma_{xy} = \frac{2Ne^2}{h}$$

$$\hookrightarrow \propto \frac{1}{R_{xy}}$$

Hall resistance at T = 0.5K



$$R_{xx} = \frac{V_x}{I_x}$$ Longitudinal resistance

$$R_{xy} = \frac{V_y}{I_x}$$ Hall resistance

Quantum Hall effect in Q1D conductors

$$\hat{\mathcal{H}} = -\frac{1}{2m}\frac{\partial^2}{\partial x^2} - 2\Delta_0 \cos[(2k_F - NG)x] + 2t_b \cos(k_y b - Gx)$$

- Δ_0 - the FISDW order parameter
- N - an integer number characterizing FISDW
- $G = ebH/\hbar c$ - the magnetic wave vector
- H - the external magnetic field, use gauge $A_y = Hx$
- t_b - the interchain tunneling amplitude
- k_y - the electron momentum transverse to the chains
- b - the interchain distance

The effective Hamiltonian (after some transformations):

$$\begin{pmatrix} -iv_F\partial_x & -\Delta_N e^{-iNk_y b} \\ -\Delta_N e^{iNk_y b} & iv_F\partial_x \end{pmatrix} \begin{pmatrix} \psi_+(x,k_y) \\ \psi_-(x,k_y) \end{pmatrix} = \varepsilon(k_y) \begin{pmatrix} \psi_+(x,k_y) \\ \psi_-(x,k_y) \end{pmatrix}$$

The same as the Hamiltonian in part I with $\Theta \to Nk_y b$.

Topological winding number: The phase of the gap changes by $2\pi N$ when k_y traverses the Brillouin zone from 0 to $2\pi/b$.

The quantum Hall effect:

- Apply electric field $\mathcal{E}_y$ transverse to the chains
- Use gauge $A_y = -\mathcal{E}_y ct$ and substitute $k_y \to k_y - eA_y/c$
- The phase $\Theta \to Nk_y b + Ne\mathcal{E}_y bt$ becomes time-dependent
- That results in the Fröhlich current along the chains: $j_x = e\dot{\Theta}/\pi b$
- After substitution, we find the (integer) quantum Hall effect: $j_x = (2Ne^2/h)\,\mathcal{E}_y$

cond-mat/0006050

Chiral edge states in the quantum Hall regime of Q1D conductors

Use the results of part I with $\Theta \to N k_y b$.

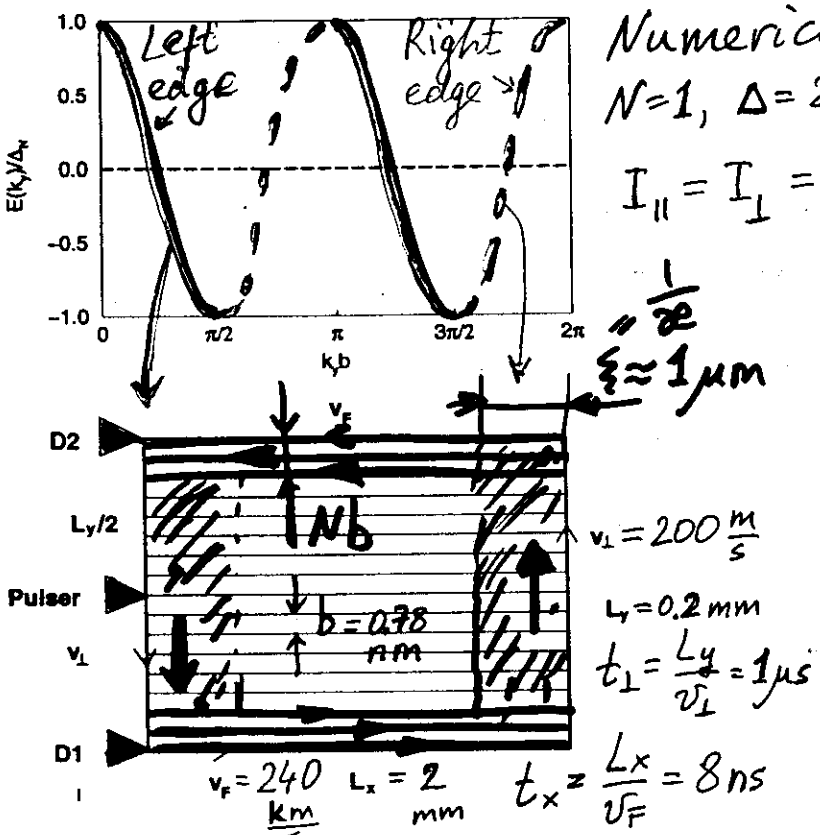
N chiral branches of edges states at the ends of the chains

$$\begin{pmatrix} \psi_+(x) \\ \psi_-(x) \end{pmatrix} \propto e^{i k_y y - \kappa x} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \kappa = \frac{\Delta_N}{v_F} \sin(N k_y b) = \frac{1}{\xi}$$

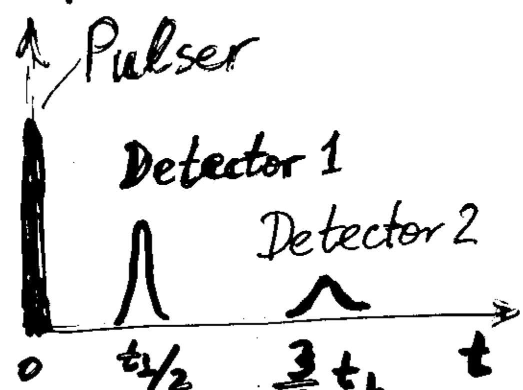
$$\varepsilon(k_y) = \Delta_N \cos(N k_y b), \quad v_\perp = \left. \frac{\partial \varepsilon(k_y)}{\partial k_y} \right|_{\varepsilon=0} = -N \Delta_N b$$

Numerical estimates for $N=1$, $\Delta = 2K$ ($2\Delta = 6K$ @ $25T$)

$$I_\parallel = I_\perp = \frac{e v_\perp}{\pi b} = 13 \text{ nA}$$



Proposed time-of-flight experiment



The edge contribution to the specific heat:

$$\frac{C_{\text{edge}}^{\text{FISDW}}}{T} = \frac{2N\pi k_B^2}{3\hbar}\left(\frac{L_y}{v_\perp} + \frac{L_x}{v_F}\right) \approx \frac{2\pi k_B^2 L_y}{3b\Delta_N}, \quad \frac{C_{\text{edge}}^{\text{FISDW}}}{C_{\text{bulk}}^{\text{normal}}} = \frac{2\xi_N}{L_x} \approx 10^{-3}$$

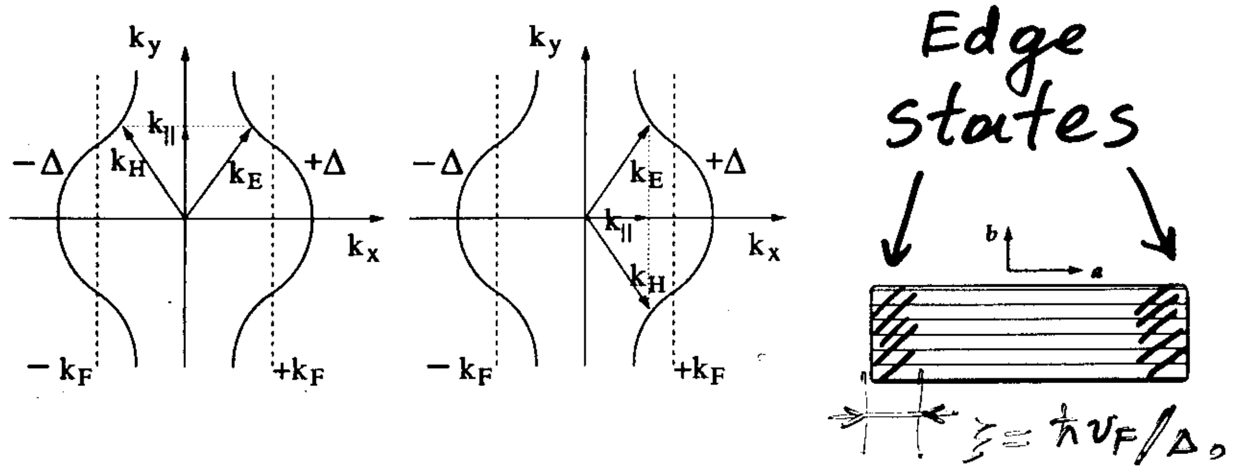
Edge states in the triplet superconducting state of Q1D conductor $(TMTSF)_2X$

Singlet pairing: s-wave: common; d-wave: high-T_c cuprates

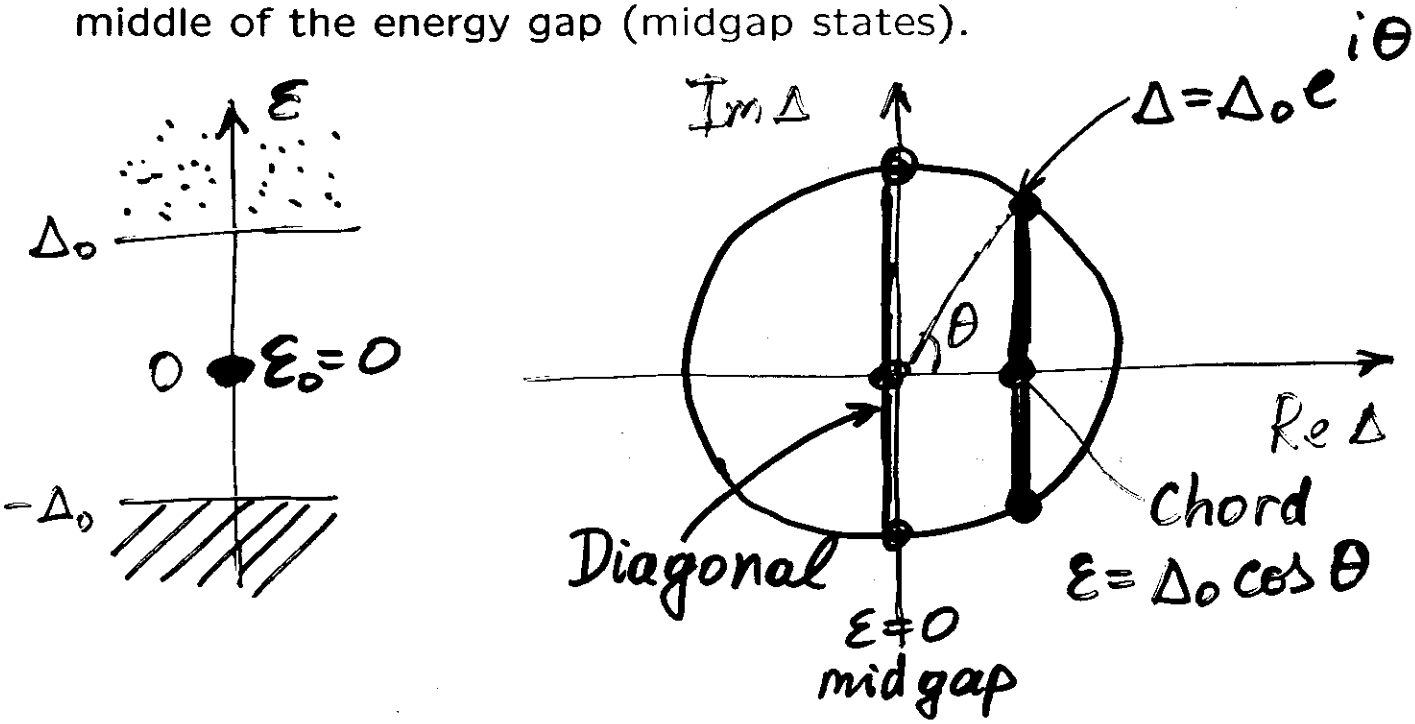
$$\langle\hat{\psi}_\alpha(k_F)\hat{\psi}_\beta(-k_F)\rangle=\epsilon_{\alpha\beta}\Delta(k_F)=i\sigma_y\Delta(k_F),\quad \Delta(+k_F)=\Delta(-k_F)$$

Triplet pairing: $(TMTSF)_2X$, p-wave: ^3He, Sr_2RuO_4

$$\langle\hat{\psi}_\alpha(k_F)\hat{\psi}_\beta(-k_F)\rangle=i\sigma_y(\mathbf{d}\cdot\boldsymbol{\sigma})\Delta(k_F),\quad \Delta(+k_F)=-\Delta(-k_F)$$



Because $\Delta(\pm k_F)$ changes sign, Andreev bound states form at the ends of the chains with the energy exactly in the middle of the energy gap (midgap states).



Midgap Andreev edge states

The wave function of a Bogolyubov quasiparticle in a Q1D superconductor:

$$\Psi(x,y) = e^{ik_y y}\left[Ae^{ik_F x}\begin{pmatrix}u_+(x)\\ v_+(x)\end{pmatrix} + Be^{-ik_F x}\begin{pmatrix}u_-(x)\\ v_-(x)\end{pmatrix}\right]$$

$[u_\pm(x), v_\pm(x)]$ obey the Bogolyubov-de Gennes equations:

$$\begin{pmatrix}\mp iv_F\partial_x & \pm\Delta \\ \pm\Delta & \pm iv_F\partial_x\end{pmatrix}\begin{pmatrix}u_\pm(x)\\ v_\pm(x)\end{pmatrix} = \varepsilon\begin{pmatrix}u_\pm(x)\\ v_\pm(x)\end{pmatrix}$$

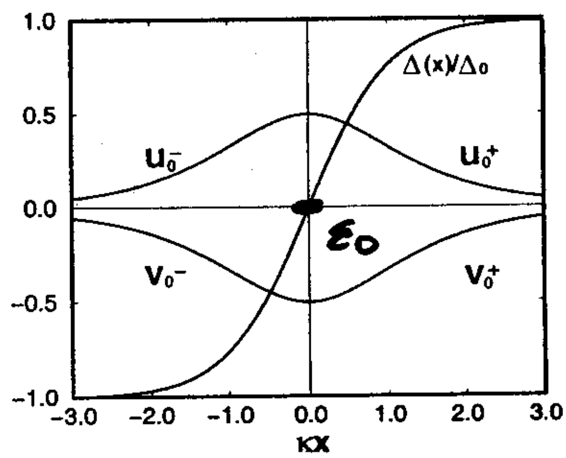
To satisfy the boundary condition $\Psi(0) = 0$, we choose $A = -B$ and $[u_+(0), v_+(0)] = [u_-(0), v_-(0)]$. Then we extend the problem to $-\infty < x < +\infty$:

$$\Psi(x) = \begin{cases}[u_+(x), v_+(x)], & x > 0\\ [u_-(-x), v_-(-x)], & x < 0\end{cases}$$

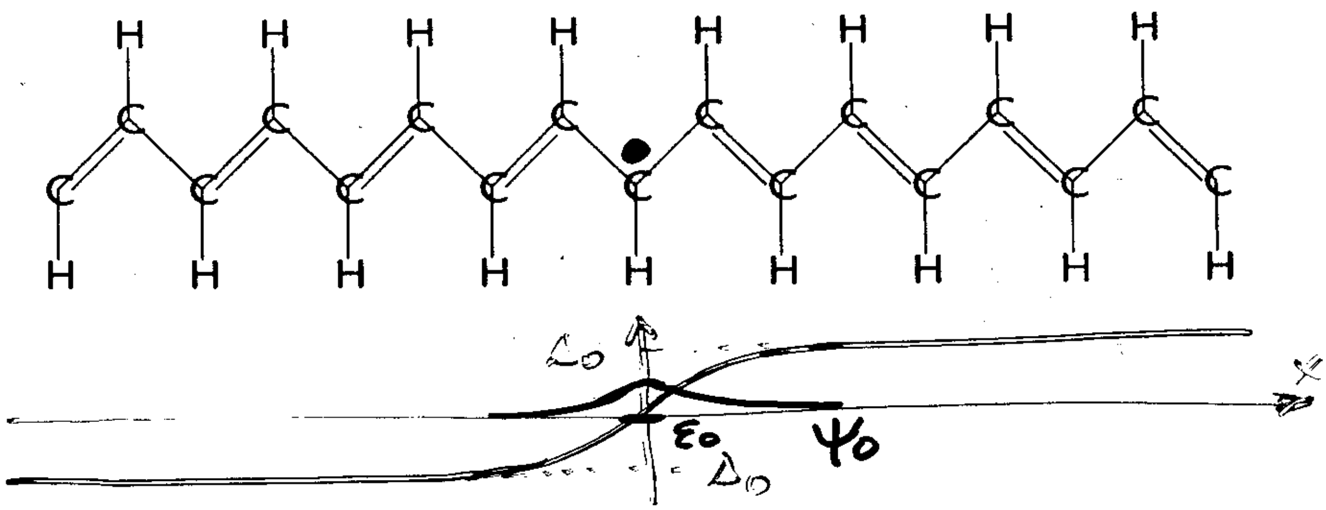
Self-consistent pair potential:
$\Delta(x) = \Delta_0 \tanh(\kappa x)$, $\kappa = \Delta_0/v_F$

Localized zero-energy state:

$$\varepsilon_0 = 0, \quad \begin{pmatrix}u_0(x)\\ v_0(x)\end{pmatrix} = \frac{\sqrt{\kappa}}{2\cosh(\kappa x)}\begin{pmatrix}1\\ -1\end{pmatrix}$$



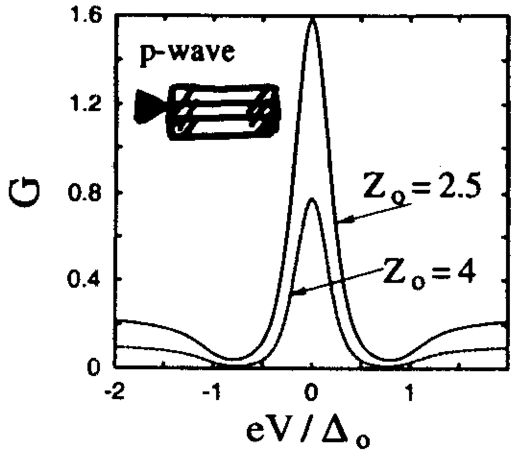
Exact mapping to the solitons in polyacetylene $(CH)_x$

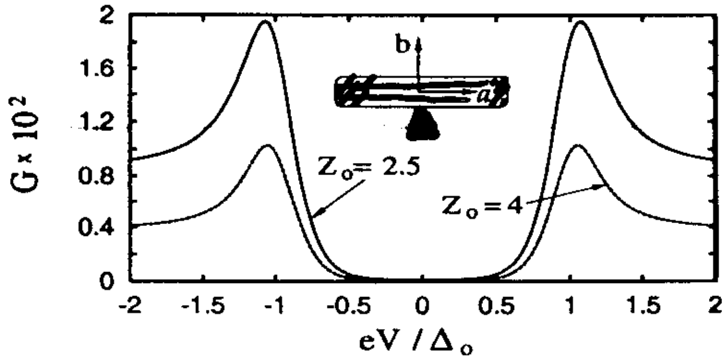


Tunneling into midgap Andreev edge states

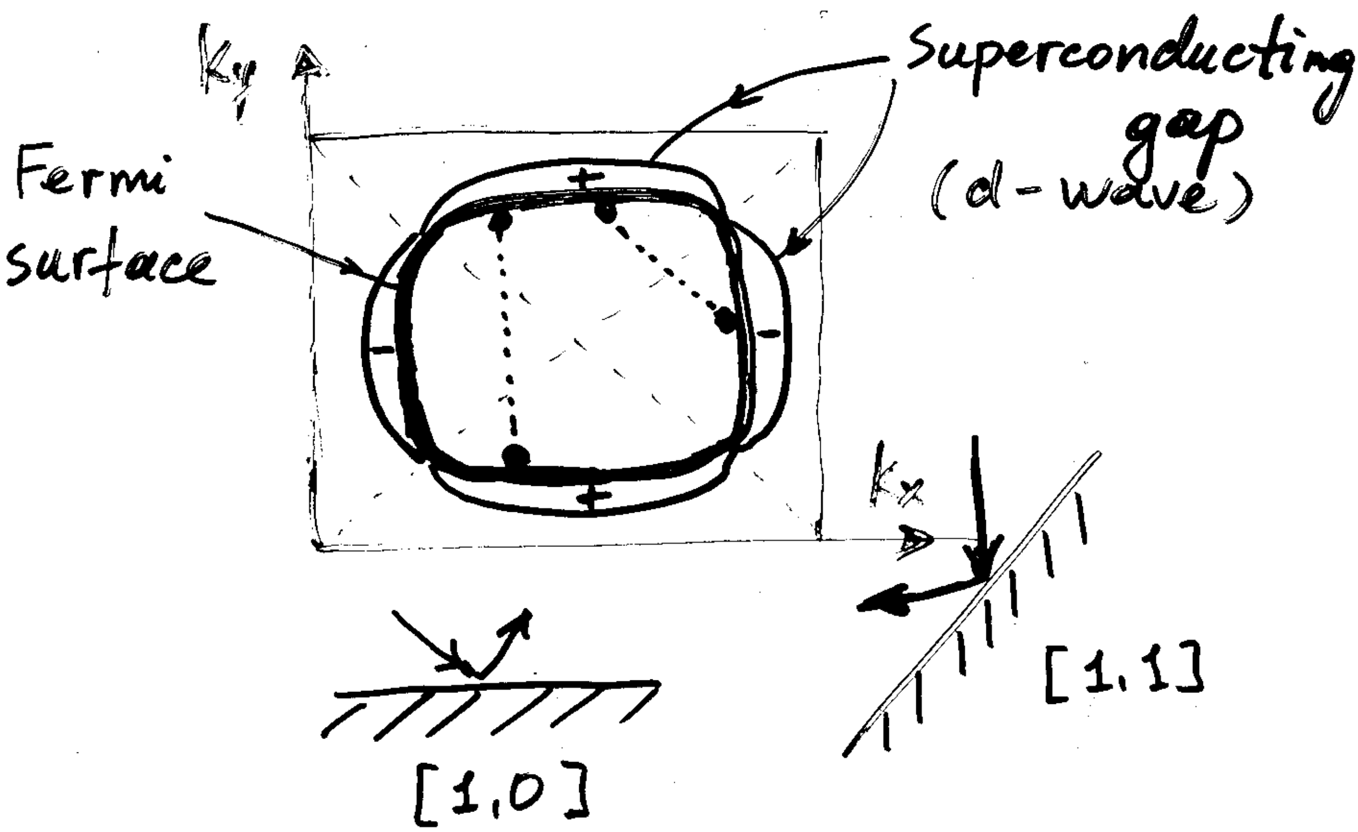
Tunneling along the chains:
Zero-bias conductance peak

Across the chains:
No zero-bias peak

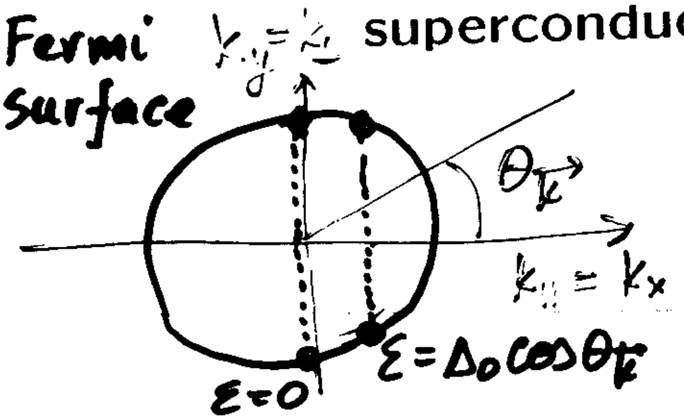


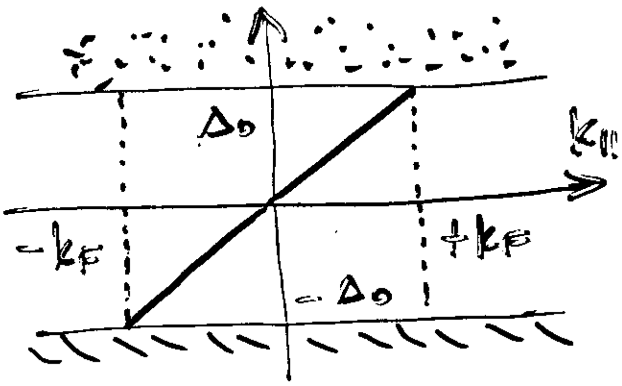


Similar to tunneling into the [1,1] and [1,0] edges for the d-wave superconductivity in high-T_c cuprates:



Chiral edge states in the triplet superconductor Sr_2RuO_4

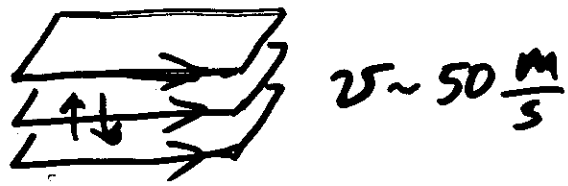




Sr_2RuO_4 is a layered Q2D superconductor similar to the high-T_c cuprates. $T_c = 1.5$ K

The superconducting state is believed to be triplet, of the "$p_x + ip_y$" type: $\Delta = \Delta_0\, e^{i\Theta_k}$.

The edge states are chiral, like in the quantum Hall systems: $\varepsilon = \Delta_0 \cos\Theta_{\mathbf{k}} = \Delta_0(k_\parallel/k_F)$, $v = \Delta_0/k_F \approx 50$ m/s



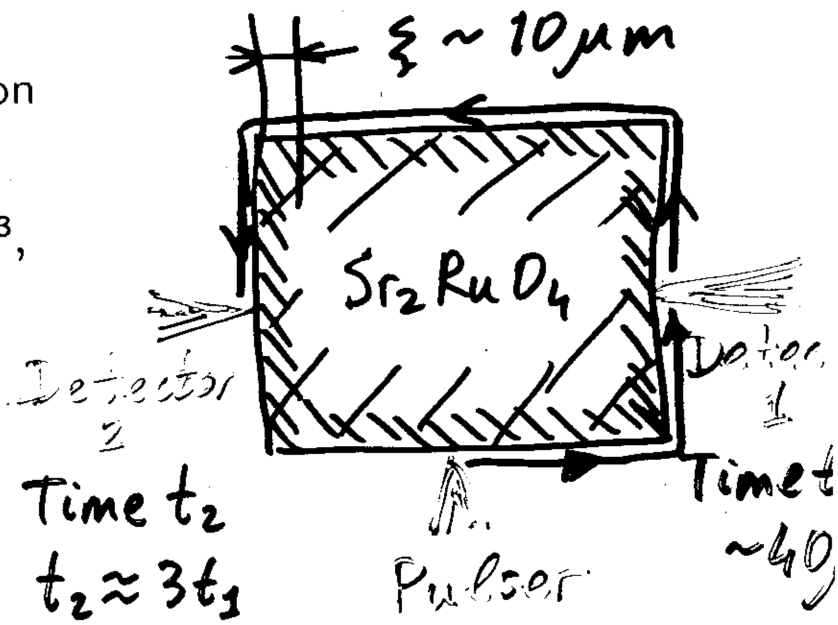
Edge states have no electric charge ($\rho = |u|^2 - |v|^2 = 0$), but they do carry spin 1/2.

Time-of-flight experiment with spin or temperature pulses: $t = L/v \approx 40\ \mu$s.

The edge contribution to the specific heat:

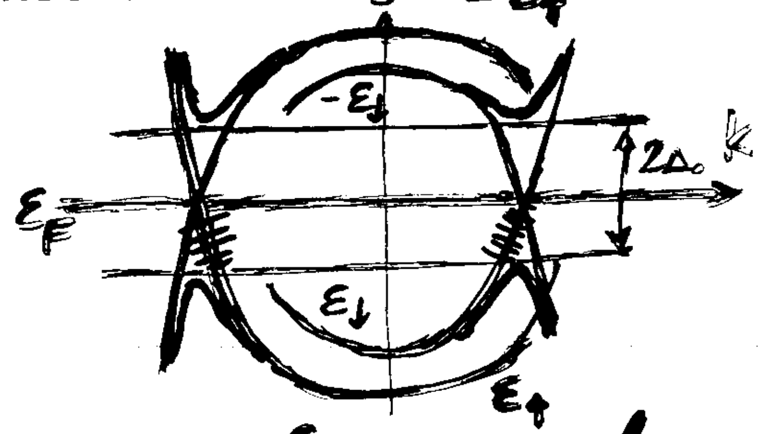
$$\frac{C_{\text{edge}}^{\text{super}}}{C_{\text{bulk}}^{\text{normal}}} \sim \frac{\xi}{L_x} \approx 5 \times 10^{-3},$$

for $\Delta_0 = 2.64$ K, $L = 2$ mm.

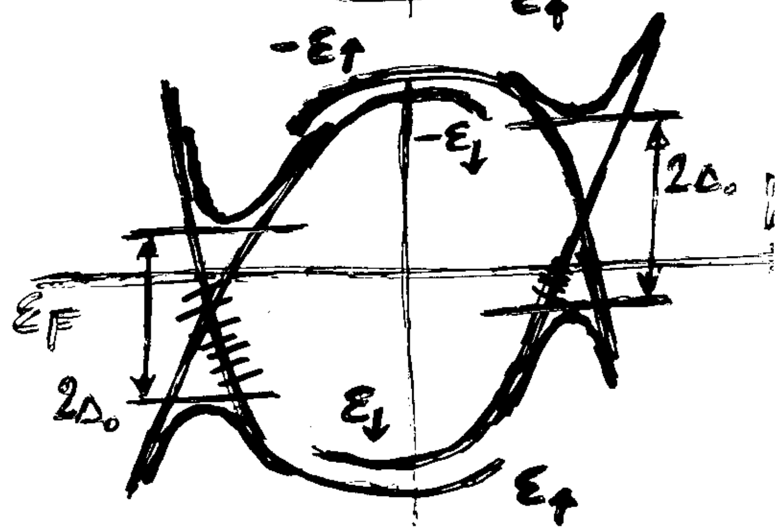


Spin response of the edge

For $\mathbf{d} \perp \mathbf{H} \parallel \hat{z}$:
$\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = \Delta_0$.
Zeeman magnetic field **induces** no net spin current **on the edge.**



For $\mathbf{d} \parallel \mathbf{H} \parallel \hat{z}$:
$\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta_0$.
Zeeman magnetic field splits the energies of the ↑ and ↓ edge states. That produces a net spin current on the edge.



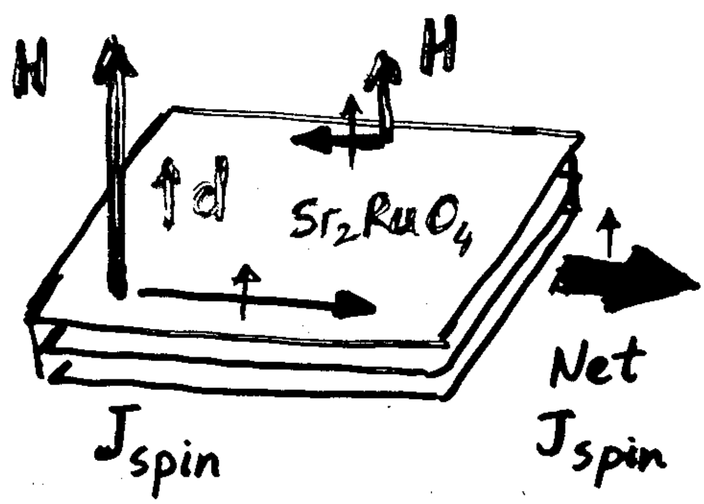
Spin quantum Hall effect

Spin current perpendicular to the gradient of a Zeeman magnetic field:

$$J_{a,\alpha}^{\text{spin}} = \frac{\mu_B}{2\pi} \epsilon_{ab} \frac{\partial H_\beta}{\partial x_b} d_\beta d_\alpha$$

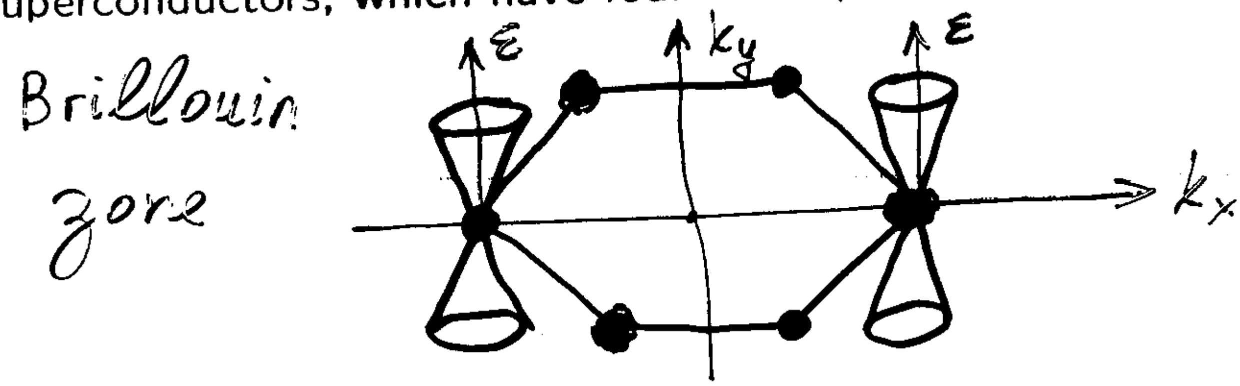
Only the component of **H** parallel to **d** produces edge spin current.

Problem: cancellation of the edge and bulk spin Hall currents.



Edge states in graphite

The band structure of graphite has two Dirac points (where the energy gap vanishes). That is similar to the d-wave superconductors, which have four Dirac points.



What are the energies of the edge states as a function of the angle Θ between the edge and a crystal axis?
Boundary conditions are tricky . . .

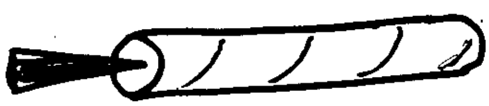
Edge states in carbon nanotubes

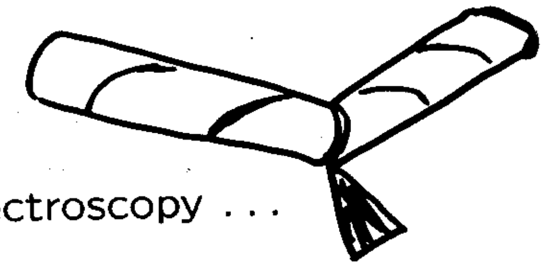
The effective Dirac equation for electrons in an insulating carbon nanotube:

$$\begin{pmatrix} \mp i v_F \partial_x & \Delta_0\, e^{\pm i\Theta} \\ \Delta_0\, e^{\mp i\Theta} & \pm i v_F \partial_x \end{pmatrix} \begin{pmatrix} u_\pm(x) \\ v_\pm(x) \end{pmatrix} = \varepsilon \begin{pmatrix} u_\pm(x) \\ v_\pm(x) \end{pmatrix},$$

where Θ is the angle between the nanotube axis and a crystal axis of graphite (the chiral angle).

Localized states
- at the ends of a nanotube?
- at a junction between nanotubes with different chiral angles Θ_1 and Θ_2?





Experiment: electron tunneling spectroscopy . . .

Entropy of black holes

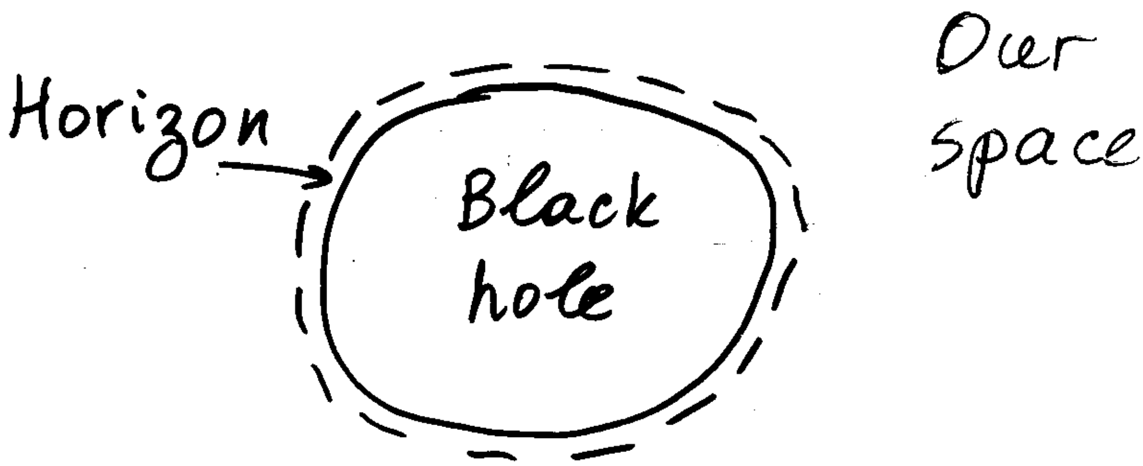
From $\delta E = T\,\delta S$, $\delta E = (\kappa/8\pi)\,\delta A$, and the Hawking radiation temperature $T = \kappa/2\pi$, it follows that

$$\text{Black hole entropy} = \frac{\text{Horizon area}}{4 \times (\text{Planck length})^2}$$

On the other hand,

$$\text{Entropy} \propto \log(\text{Number of states})$$

Thus, some states may be associated with the horizon area. What are those states?



- The volume inside the black hole horizon is effectively excluded from our space. In other words, the horizon serves as a boundary of our space. Any associated edge states?

- When horizon expands, our space looses some volume to the black hole. Thus, the number of available states must decrease: spectral flow?

- Anything changing sign at the horizon: index theorem?

Conclusions

1. • In the FISDW state of $(TMTSF)_2X$, there exist N chiral electron states bound near the edges of a sample with energies inside the gap.
   • The energies of the edge states have dispersion (velocity) as a function of the momentum tangential to the surface. The energies interpolate between the top and bottom of the gap.
   • Similar to the superconducting, p-wave Sr_2RuO_4

2. • In the superconducting state of $(TMTSF)_2X$, there exist bound midgap states with $\varepsilon=0$ at the ends of the chains.
   • Similar to d-wave superconductors (YBCO).

3. Tunneling or Josephson experiments on $(TMTSF)_2X$ are desirable.