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H4. SMR/1247
Lecture Note: 17

WORKSHOP ON PHYSICS OF MESOSPHERE-STRATOSPHERE-TROPOSPHERE INTERACTIONS WITH SPECIAL EMPHASIS ON MST RADAR TECHNIQUES

(13 - 24 November 2000)

LABORATORY WORK:
1. Beam pattern of a linear array antenna (antenna)
2. Expected detectability profile of atmospheric radars (detectability)
3. Basic characteristics of atmospheric radar signal (signal)

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LABORATORY WORK for S. Fukao's lecture

Prepared for the Workshop on Physics of Mesosphere-Stratosphere-Troposphere Interactions with Special Emphasis on MST Radar Techniques

At Abdus Salam International Center for Theoretical Physics
Trieste, Italy
13-24 November 2000

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For laboratory work, we have written three small programs to demonstrate some features of the atmospheric radar technique. They show

1. Beam pattern of a linear array antenna (antenna)
2. Expected detectability profile of atmospheric radars (detectability)
3. Basic characteristics of atmospheric radar signal (signal)

We have written simple documentation for each program. Please run them with different parameters to help you understand the atmospheric radars. It may be interesting to read the source program to find how they work. Each program is written by the combination of a C program and a GNUPLOT script. It is easy to modify or improve them.

Beam Pattern of Array Antenna

Function

This program demonstrates radiation pattern of a linear array antenna (Fig. 1). We assume that *n*-Dipoles or omni directional antennas are linearly located on a ground with a separation of d/λ.

Usage

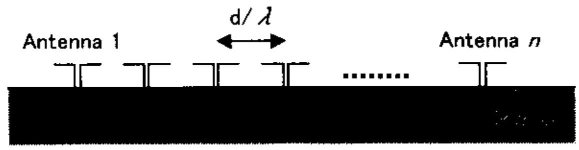


Fig. 1 Linear array antenna

1. Program is installed in the directory “antenna”. Go to the directory and run “a.out”.
2. Input parameters are follows
   A) Number of antennas: n
   B) Separation of antennas in the unit of wavelength: d/λ
   C) Phase difference between antennas in degrees: ψ
   D) Selection of each antenna: 1 = Dipole, and 0 = omni directional
   E) Selection of graphic output: 1 = polar plot, 0 = x-y plot
3. A new window appears, and radiation pattern shows up. Direction and width of the main beam are calculated and shown in the text window. When selecting a polar plot, a red line indicates radiation pattern in the visible range. A green line is for the invisible range (underground). An x-y plot only indicates radiation in the visible range. The angle θ =0 and 180 degrees correspond to the horizontal direction. θ =90 degrees is the zenith.
4. Hit carriage return to stop the program.

Calculation

Fig. 2 is a schematic of an antenna array. Number of antennas is *n*, and distance between each antenna is *d*. Radio wavenumber is $k = 2\pi/\lambda$ (λ: wavelength). Antennas are driven at the same intensity *M*, but phase is delayed from antenna 1 to *n* at ψ for each.

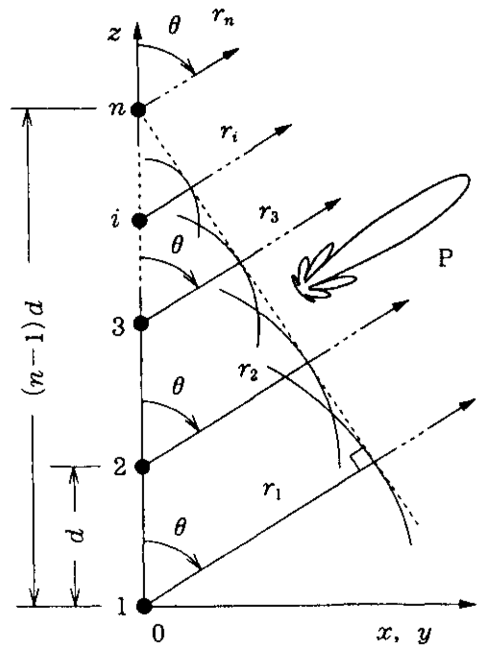


Fig. 2 Schematic of antenna array

Radiation from the array is calculated by summing electric field radiated from each array element at a very far point P which is in the direction of θ from the z-axis. The result is written as

$$E_t(r,\theta) = \frac{nM}{r} e^{-jkr} D(\theta) U(\theta),$$

$$U(\theta) = \frac{\sin(n\Phi/2)}{n\sin(\Phi/2)},$$

where r is distance from antenna to the point P, and $\Phi = kd\cos\theta - \psi$. Term $U(\theta)$ is called array factor. Another term $D(\theta)$ is the radiation pattern of each antenna. In our case, omni directional antenna has $D(\theta) = 1$, and Dipole antenna has $D(\theta) = \frac{\cos(\pi/2\cos\theta)}{\sin(\theta)}$. Since we display power radiation pattern in dB, the program calculates $|D(\theta)U(\theta)|^2$.

Notes

1. Beam pattern of Dipole antenna can be shown by setting n=1.
2. When antenna distance is larger than $\lambda/2$ ($d/\lambda > 0.5$), grating lobe appears. Setting $d/\lambda = 1$, grating lobes appear horizontally even when the main beam is in the zenith (ψ=0).
3. Check the relationship between direction of the main beam (θ), and the antenna phase difference (ψ).

Radar equation

$$P_{\mathrm{r}} = \frac{\pi P_{\mathrm{t}} A_{\mathrm{e}} \Delta r a^2 L}{64 r^2} \sigma$$

$$\overline{P_{\mathrm{r}}} = P_{\mathrm{r}} \frac{\tau}{T_{\mathrm{IPP}}} = \frac{\pi P_{\mathrm{t}} A_{\mathrm{e}} \Delta r^2 a^2 L}{32 c T_{\mathrm{IPP}} r^2} \sigma$$

$$\begin{aligned} \sigma &= 0.38 {C_{\mathrm{n}}}^2 \lambda^{-1/3} && \text{(for atmospheric turbulence)} \\ &= 284 Z \lambda^{-4} && \text{(for precipitation)} \end{aligned}$$

$\overline{P_{\mathrm{r}}}$:	Received echo power
σ:	Radar volume reflectivity (η)
C_{n}^2:	Refractivity turbulence structure constant
Z:	Radar refractivity index
c:	Light speed
r:	Range
P_{t}:	Peak transmitter power
A_{e}:	Antenna aperture
Δr:	Range resolution
a:	Efficiency of the radar antenna and transmission line
L:	Loss factor
T_{IPP}:	Inter-pulse period (IPP)
τ:	Pulse length
λ:	Wave length

Model profiles of C_n^2 and Z:

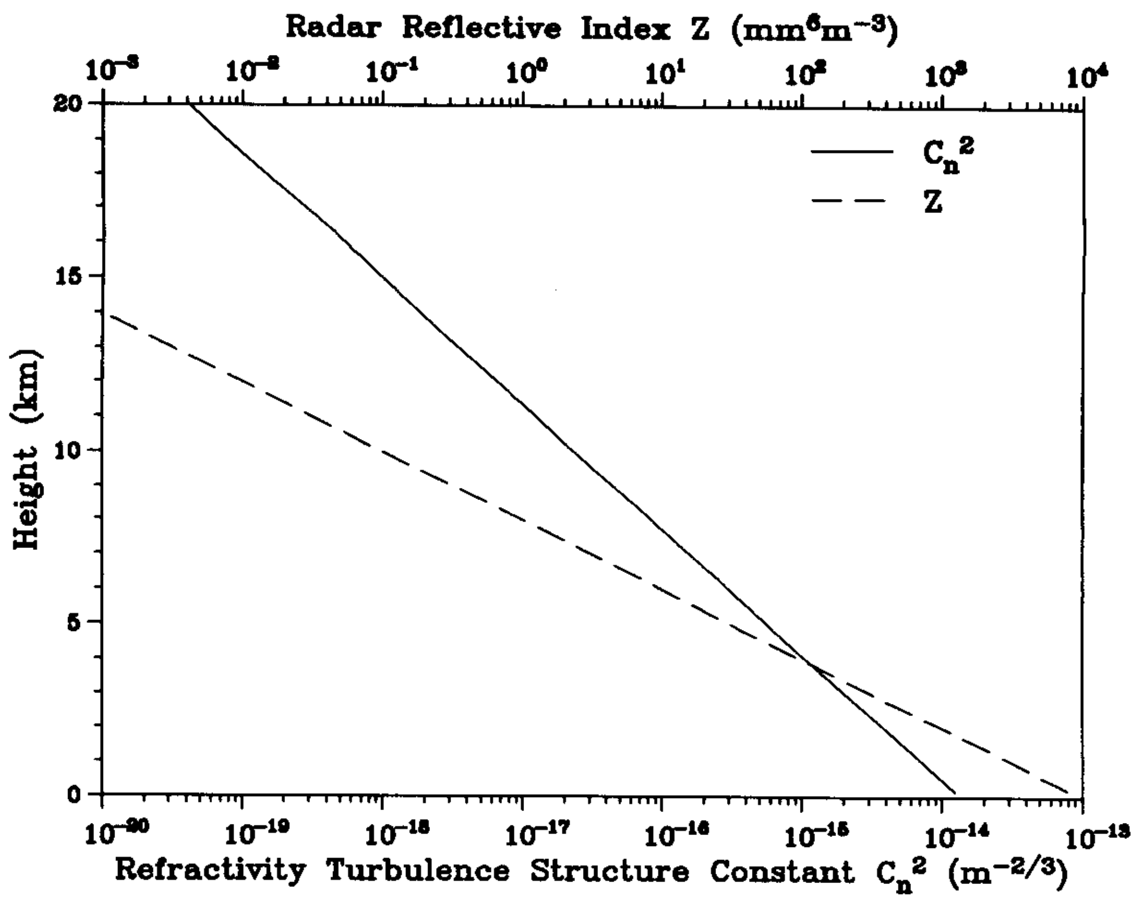
Refractivity turbulence structure constant [COST74]:

$$C_n^2(h) = 10^{-2.76\times 10^{-4}\times h - 13.862}\ [\mathrm{m}^{-2/3}]$$

Radar refractivity index:

$$Z(h) = 10^{-5\times 10^{-4}\times h + 4}\ [\mathrm{mm}^6\mathrm{m}^{-3}]$$

(which corresponds to 11.5 mm/hour rain intensity at the ground)

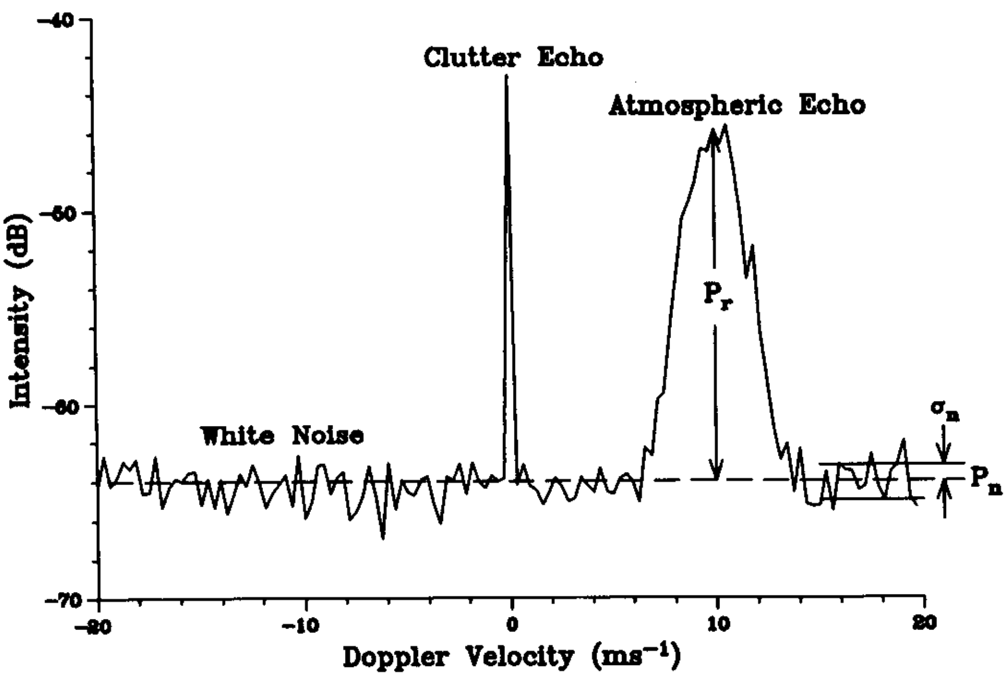


Detectability

$$D = \frac{P_r}{\sigma_n} = \frac{P_r}{P_n/\sqrt{L_i}} = \frac{\overline{P_r}}{\overline{P_n}} \frac{\sqrt{L_i}}{\delta f T_{IPP} N_c}$$

$$\overline{P_n} = kTB$$

P_r:	Maximum echo power
σ_n:	Standard deviation of the noise fluctuation
P_n:	Noise power density
$\overline{P_n}$:	Noise power
L_i:	Number of incoherent integrations
N_c:	Number of coherent integrations
δf:	Width of the atmospheric echo
k:	Boltzmann's constant
T:	Receiver noise temperature
B:	Band width of the integration filter



An example of the Doppler spectrum.

Calculation of Detectability

Parameters

Frequency ($f = c/\lambda$):	Variable
Antenna size (square shape) (A_e):	Variable
Peak transmitter power (P_t):	Variable
Integration time (T_{int}):	Variable
IPP (T_{IPP}):	200 μs
Range resolution (Δr):	150 m
Duty ratio (assuming 16-bit pulse compression):	8%
Efficiency (a):	1
Loss factor (L):	−2 dB
Number of FFT points (N_{FFT}):	128
Number of beams (N_{beam}):	5
Window size of Doppler spectra (V_{max}):	20 m/s

N_c and L_i are determined from the following relations.
Window size of Doppler spectra:

$$V_{max} = \frac{\lambda}{2T_{IPP}N_c}$$

Integration time:

$$T_{int} = T_{IPP}N_cN_{FFT}N_{beam}L_i$$

Characteristics of Atmospheric Radar Signal

Function

This is a simple simulation program to generate model signal of an atmospheric radar, and to demonstrate its statistical and spectral features. We assume NO NOISE case.

Usage

1. Program is installed in the directory "signal". Go to the directory and run "a.out".
2. Input parameters are follows
   A) Correlation time of the signal in the unit of data-point: τ
   B) Doppler frequency in the unit of cycles/timeseries: f_d
   C) Number of incoherent integration: L_i
3. A new window appears, and power spectrum of the generated model radar signal shows up in the right panel. One sample of real (red line) and imaginary (green line) components of timeseries, and distribution of whole real and imaginary components are shown in the left panels. Blue line in the distribution panel represents (0,1) Gaussian distribution.
4. Hit carriage return to stop the program.

Calculation

Model radar signal in this program is 256 points of complex timeseries. Power spectrum is calculated for each timeseries, and incoherently integrated for L_i times. Each model timeseries of data are generated by the following scheme.

1. Generate two sets of 512 random numbers with the Gaussian distribution of (0, 1). Two sets are used for real and imaginary components, respectively. (Fig. 1)
2. Calculate weighting function $w(t)$ of Gaussian function with standard deviation of τ, where

$$w(t) = \exp\left(-\frac{t^2}{2\tau^2}\right). \qquad \text{(See Fig. 2)}$$

In this program $w(t)$ is calculated as 256-point series. Real and imaginary parts of random numbers are convolved with $w(t)$

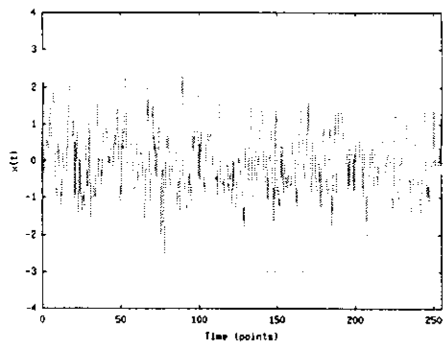


Fig. 1 Random numbers with Gaussian distribution (256 points)

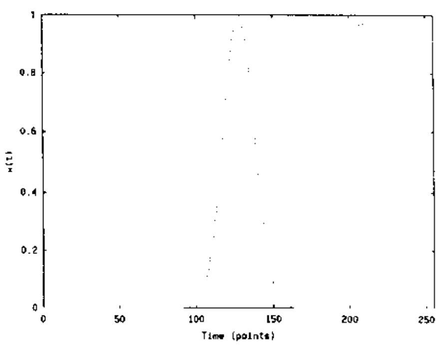


Fig. 2 Weighting function (τ =10 points)

separately. The result is 256 complex timeseries with correlation time of τ. (Fig. 3)

3. Add Doppler phase rotation to the complex timeseries by using the following transform.

$$\begin{pmatrix} a'(t) \\ b'(t) \end{pmatrix} = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},$$

where $x(t) = a(t) + jb(t)$ is the timeseries before adding mean Doppler shift at angular frequency ω.

4. Calculate power spectra of the complex timeseries, and process it through coherent integration.

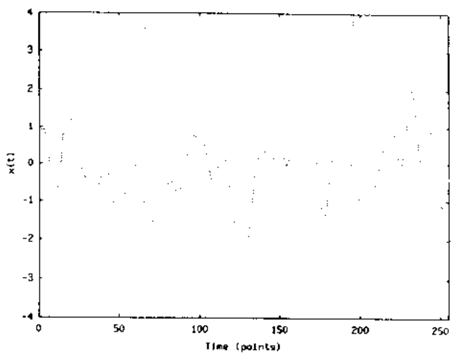


Fig. 3 Example of model signal (τ =3 points, no Doppler shift)

Notes

1. Random noise can be generated by setting τ=0.
2. Check the relationship between correlation time τ of the timeseries and spectral width σ of the power spectrum.
3. Model signal in this program is noise free. Fluctuation of power spectrum is owing to the stochastic characteristics of the signal. Check if power spectrum becomes smoother by increasing the number of incoherent integration.
4. Modify the program to consider noise.

Suggested Parameters

1. Antenna

A set of parameters to generate 13° width beam at the zenith angle of 30° .

Number of antennas n = 10
Separation of antennas $d/\lambda = 0.5$
Phase difference between antennas: $\psi = 90$
Diple or omni directional antenna: 1 or 0
Polar or x-y plot: 1 or 0

2. Detectability

A typical Boundary Layer Radar case.

Frequency (MHz): 900
Antenna size (m): 2
Peak power (kW): 2
Integration time (s): 60

3. Signal

A sample set of parameters to generate spectral peak at the frequency of 50.

Correlation time: $\tau = 3$
Doppler shift: fd = 50
Number of incoherent integration: Li = 10

Note: Li ＞100 may take long calculation time depending on a PC.