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#### SPRING COLLEGE ON NUMERICAL METHODS IN ELECTRONIC STRUCTURE THEORY

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#### "Electronic excitations and response functions in solids and reduced dimensional systems"

presented by:

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These are preliminary lecture notes, intended only for distribution to participants.

#### Electronic Excitations and Response Functions in Solids and Reduced Dimensional Systems

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# Outline

- Introduction
- Excited States and Spectroscopies
  - Quasiparticle excitations and the GW approximation
  - Excitonic effects in optical response and the Bethe-Salpeter equation
- Examples of Applications
  - Semiconductors and insulators
  - Conjugated polymers
  - Surfaces, clusters, and atoms
- Summary

## **Properties of Solids**

Ground-state Properties:

Cohesive Structural Vibrational Magnetic structure Phase transformations

- Etot [{Ri}]
- Density Functional Theory (LDA, GGA)

Spectroscopic Properties:

Photoemission Tunneling

. . .

- N+1 Particle Problem
- Quasiparticle Approach (GW)

Optical

- N+2 Particle Problem
- Electron-hole interaction



#### Quasiparticle band structure:

describes individually excited holes and electrons  $(\leftrightarrow \text{ One-particle Green function } G_1)$ 

Quasi-electron:  $N \rightarrow N+1$ (inverse photoemission; tunneling)

Quasi-hole:  $N \rightarrow N - 1$ (photoemission; tunneling)





#### Optical experiment (e.g., absorption):

- Creation of electron-hole pair:  $N \rightarrow N^*$
- Electron-hole interaction
- Requires a two-particle approach on top of the QP band structure



Quasiparticle (QP) band structure calculations

• Density-functional theory:

$$\left\{-\nabla^2 + V_{\text{ext}} + V_{\text{Coul}} + V_{\text{xc}}\right\}\psi_{nk}^{\text{DFT}} = \varepsilon_{nk}^{\text{DFT}}\psi_{nk}^{\text{DFT}}$$

Hohenberg, Kohn; and Sham 1965

• Green-function approach + QP approximation:

$$\left\{-\nabla^2 + V_{\text{ext}} + V_{\text{Coul}} + \Sigma(\varepsilon_{nk}^{\text{QP}})\right\}\psi_{nk}^{\text{QP}} = \varepsilon_{nk}^{\text{QP}}\psi_{nk}^{\text{QP}}$$

 $\Sigma = iG_1 \mathcal{W}$  GW approximation for the self energy

 $G_1$  one-particle Green function  $W = \epsilon^{-1}v$  screened Coulomb interaction Hedin 1965, Hybertsen and Louie 1985

$$W(\underline{\mathbf{x}},\underline{\mathbf{r}}',\omega) = \int \overline{\epsilon'(\underline{\mathbf{x}},\underline{\mathbf{r}}'',\omega)} \frac{e^{z}}{|\underline{\mathbf{x}}''-\underline{\mathbf{r}}'|} d\underline{\mathbf{r}}''$$

#### **Quasiparticle Excitations**

Kohn-Sham Eigenvalues ≠ QP Energies

### **Two simple examples:**

<u>Homogeneous</u> Interacting Electron System

Standard K-S equation:

$$\begin{bmatrix} -\frac{1}{2} \nabla^2 + V_{\rm H} + \frac{\delta E_{\rm xc}}{\delta \rho(r)} \end{bmatrix} \psi(r) = \varepsilon_{\rm KS} \psi(r)$$

 $V_{xc}(r) = \frac{\delta E_{xc}}{\delta \rho(r)} = \text{constant} \implies \text{Free electron dispersion}$  $(m^* = m_e, \text{etc.}) \approx \text{Lifetime}$ Incorrect!

Generalized K-S eigenvalues <u>not</u> unique
 Different K-S equations → new set of eigenvalues

GW Approximation 
$$\Sigma = \frac{1}{6}$$

$$\Sigma(\vec{r},\vec{r}';E) = \frac{1}{2\pi} \int W(\vec{r},\vec{r}',\omega)G(\vec{r},\vec{r}',E+\omega)e^{i\delta\omega}d\omega$$

with

$$W(\vec{r},\vec{r}',\omega) = \int v(\vec{r},\vec{r}'')\epsilon^{-1}(\vec{r}'',\vec{r}',\omega)d^{3}r''$$

$$G(\vec{r}, \vec{r}', \omega) = \sum_{n\vec{k}} \frac{\Psi_{n\vec{k}} (\vec{r}) \Psi_{n\vec{k}}^{*} (\vec{r}')}{\omega - E_{n\vec{k}} - i\delta_{n\vec{k}}}$$

### Require:

(1) Full dielectric matrix (local fields)

$$\varepsilon^{-1}(\vec{r}',\vec{r},\omega)$$
 or  $\varepsilon^{-1}_{\vec{G}\vec{G}'}(\vec{q},\omega)$ 

(2) Good starting  $\psi_{n\vec{k}}$  and  $E_{n\vec{k}}$  to construct the Green's function G.

1		None None None None None None None None	
	LDA	QP	Expt
diamond	3.9	5.6	5.48
Si	0.52	1.16	1.17
Ge	< 0	0.73	0.744
LICI	6.0	9.1	9.4
AIAs	1.18	2.01	2.24
GaAs	0.56	1.42	1.52







#### **Compare with Experiment**



#### **Density of States**

computed by GW method gaussian broadening 0.6 eV (the experimental resolution)

computed 
$$E_{d-d}^{gap} = 4.2 \text{ eV}$$
  
 $\Delta E_{peak} \approx 5.3 \text{ eV}$ 



Si(111) Surface.



# Surface State Band Gap Si(111) 2×1

#### DFT-LDA

#### 0.27 eV

#### Experiment

(a)  $0.75 \pm .1 \text{ eV}$ (b) 0.47 eV(c) 0.45 eV(d) ~0.60 eV

- (a) Photoemission + Inverse photoemission (Uhrberg etal.; Perfetti, et al.)
- (b) Differential reflectivity (Ciccaci et al.)
- (c) Photo-thermal deflection (Olmstead and Amer)
- (d) Scanning tunneling spectroscopy (Feenstra) reanalyze

#### DFT-LDA E

#### Experiment

### Bulk Si 0.5 eV 1.17 eV

Si(111) 2×1 Surface



Photoemission data
 Inverse photoemission data
 Theory
 Northrup, Hybertsent
 Louie

Lifetime au for a quasiparticle:

$$\tau_{n\mathbf{k}}^{-1} = 2\langle n\mathbf{k} | Im \mathbf{\Sigma}(E_{n\mathbf{k}}) | n\mathbf{k} \rangle$$



$$G_{n\mathbf{k}}(\omega) = \frac{1}{\omega - E_{n\mathbf{k}} + i\delta} \qquad \qquad W = \epsilon^{-1}v$$

• full frequency-dependent dielectric matrix  $\epsilon_{G,G'}(q,\omega)$  calculated within RPA

References:

M.S. Hybertsen and S.G. Louie, PRB **34** (1986) 5390 P.M. Echenique et al., Chem Phys. **251** (2000) 1

# QP lifetimes in graphite along several directions



Graphite AB stacking



#### Calculated average lifetime

•  $\tau^{-1}(E)$  calculated by averaging  $\tau_{n\mathbf{k}}^{-1}$  over all states having energy E.

• QP energies updated from DFT-LDA to GW.



• Energy dependence cannot be fit with simple power law  $(E - E_F)^n$ .

 Measured inverse lifetime can depend appreciably on the experimental setup.

\* S. Xu et al., PRL **76** (1996) 483 \*\* T. Hertel et al., in preparation



Time Dependent Density Functional Theory

TDLDA



#### **Optical Absorption Cross Section**

Solve the Bethe-Salpeter equation for the two-particle Green's function  $G_2 \implies$  electron-hole excitations:

• excitation energy  $\Omega_S$  of  $|N, S\rangle$ 

$$\Omega_{S} = E_{N,S} - E_{N,O}$$
$$|N,S\rangle = \sum_{m,n} A_{S}(m,n) a_{m}^{+} b_{n}^{+} |N,O\rangle$$

• Bethe-Salpeter equation for electron-hole amplitude

$$\chi_{S}(\underline{\mathbf{r}},\underline{\mathbf{r}'}) = \sum_{nm} A_{S}(m,n) \phi_{m}(\underline{\mathbf{r}}) \phi_{n}^{*}(\underline{\mathbf{r}'})$$

• dipole transition matrix element

$$\begin{split} \left< \vec{p}_{op} \right>_{S} &= \left< N, S \left| \vec{p}_{op} \right| N, 0 \right> \\ &= \sum_{nm} A_{S}(m, n) \left< \phi_{n} \left| \vec{p} \right| \phi_{m} \right> \end{split}$$

absorption cross section

$$f(E) \sim \sum_{S} \left| \left\langle \vec{p}_{op} \right\rangle_{S} \right|^{2} \delta(E - \Omega_{S})$$

#### **References:**

- G. Strinati, Phys. Rev. B 29, 5718 (1984)
- M. Rohlfing and S. G. Louie, Phys. Rev. Lett. 80, 3320 (1998)
- M. Rohlfing and S. G. Louie, Phys. Rev. Lett. 81, 2312 (1998)
- S. Albrecht, L. Reining, R. Del Sole, and G. Onida, Phys. Rev. Lett. 80, 4510 (1998)
- L. X. Benedict, E. L. Shirley, and R. B. Bohn, Phys. Rev. Lett. **80**, 4514 (1998)

#### **Theoretical Framework**

• Coupled electron-hole excitations:

$$\begin{split} |S\rangle &= \sum_{v}^{\text{elec hole}} A_{vc}^{S} \hat{a}_{v}^{\dagger} \hat{b}_{c}^{\dagger} |0\rangle \\ |0\rangle \text{ ground state of many-electron system} \\ \hat{a}_{v}^{\dagger}, \hat{b}_{c}^{\dagger} \text{ creates quasi-hole, -electron} \\ A_{vc}^{S} \text{ coupling coefficients} \end{split}$$

• The Bethe-Salpeter Equation for the two-particle Green's function G<sub>2</sub> yields:

$$(\varepsilon_{c}^{\mathsf{QP}} - \varepsilon_{v}^{\mathsf{QP}})A_{vc}^{S} + \sum_{v'c'} \langle vc|K^{eh}|v'c'\rangle A_{v'c'}^{S} = \Omega^{S}A_{vc}^{S}$$

G. Strinati, Phys. Rev. B 29, 5718 (1984).

 $\begin{pmatrix} c & c \\ c & c \end{pmatrix}$ 

 $\varepsilon_c^{\rm QP}$ ,  $\varepsilon_v^{\rm QP}$  single-quasiparticle energies  $K^{eh}$  electron-hole interaction

 $\implies \Omega^S$  excitation energies

•  $\Longrightarrow$  Optical absorption spectrum:  $\epsilon_2(\omega)$ 

#### **Computational Details**

- GW quasiparticle calculation ( $\Sigma = iG_1W$ ;  $W \stackrel{(\mathbf{r}, \mathbf{r}', \omega)}{=} \int e^{-i} \langle \mathbf{r}, \mathbf{r}', \omega \rangle \mathcal{T} \langle \mathbf{r}', \mathbf{r}'$ 
  - Electron-hole interaction kernel:

$$\begin{split} \langle vc|K^{eh}|v'c'\rangle &= \int d^3r d^3r' \ \psi_c^*(\mathbf{r})\psi_v(\mathbf{r}) \ v(\mathbf{r},\mathbf{r}') \ \psi_{c'}(\mathbf{r}')\psi_{v'}^*(\mathbf{r}') \\ & \text{exchange term} \end{split}$$

$$-\int d^3r d^3r' \,\psi_c^*(\mathbf{r})\psi_{c'}(\mathbf{r}) \,W^{[\Omega_S]}(\mathbf{r},\mathbf{r}') \,\psi_v(\mathbf{r}')\psi_{v'}^*(\mathbf{r}')$$

screened direct term

**Optical spectrum:** 

• Free transitions  $v\mathbf{k} \longrightarrow c\mathbf{k}$ :

$$\begin{split} \epsilon_2(\omega) &= \frac{4\pi e^2}{\omega^2} \sum_{vck} |M_{vck}|^2 \,\delta(\omega - (\varepsilon_{ck}^{\text{QP}} - \varepsilon_{vk}^{\text{QP}})) \\ M_{vck} &= \vec{\lambda} \cdot \langle v\mathbf{k} | \vec{V} | c\mathbf{k} \rangle \\ \vec{\lambda} \text{ polarization vector of the light} \\ \vec{V} \text{ velocity operator} \end{split}$$

• Coupled transitions  $|S\rangle$ :

$$\epsilon_2(\omega) \ = \ rac{4\pi e^2}{\omega^2} \ \sum\limits_S \ |M_S|^2 \ \delta(\omega-\Omega_S)$$

$$M_S = \vec{\lambda} \cdot \langle 0 | \vec{V} | S \rangle = \sum_{vck} A^S_{vck} M_{vck}$$

•  $\epsilon_2 \longrightarrow \epsilon_1, n, k, R, T, A, \dots$ 

#### Nonlocal Potentials, Electromagnetic Fields and Gauge Invariance

Nonlocal pseudopotential formalism:

$$H^{\circ} = \frac{\vec{p}^2}{2m} + V(\vec{r}) + V_{NL}(\vec{r}, \vec{r'})$$

With E-M perturbations: 
$$\vec{p} \rightarrow \vec{p} - \frac{q}{c}\vec{A}$$
  
 $V_{NL} \rightarrow \left[e^{\frac{iq}{c}\int_{\vec{r}'}^{\vec{r}}\vec{A} \cdot d\vec{\ell}}\right]V_{NL}(\vec{r},\vec{r}')$   
 $straight-line$   
 $path integral$   
Choose  $\vec{\nabla} \cdot \vec{A} = 0$ :  $H = H^{\circ} + H_{int}$ 

$$H_{int} = -\frac{q}{mc}\vec{A}\cdot\vec{p} + \frac{q^2}{2mc^2}\vec{A}^2 + \left[e^{\frac{iq}{c}\int_{\vec{r}'}^{\vec{r}}\vec{A}\cdot d\vec{\ell}} - 1\right]V_{NL}(\vec{r},\vec{r}')$$

For a slowly varying, small  $\vec{A}$  (to 2<sup>nd</sup> order in  $\vec{A}$ ):

$$H_{int} \approx -\frac{q}{c}\vec{A}(\vec{r})\cdot\vec{V} + \frac{q^2}{2c^2}\left[\frac{A^2(\vec{r})}{m} - [(\vec{r} - \vec{r}')\cdot\vec{A}]^2 V_{NL}(\vec{r},\vec{r}')\right]$$
  
where  $\vec{V} = [\vec{r}, H^\circ]/i\hbar = \frac{\vec{p}}{m} + [\vec{r}, V_{NL}]/i\hbar$ 



Atomic Magnetic Susceptibility  

$$\chi = -\frac{\partial^2 E}{\partial B^2} \Big|_{B=0} \vec{B} = \vec{\nabla} \times \vec{A}$$



Gauge: 
$$\vec{A} = -(0, 0, x + x_0) B$$
  $\vec{\nabla} \times \vec{A} = B\hat{g}$ 

Xo (a.u.)	H,	$H_1 + H_2$	H,+H2+H3	All Electron
	7.84	7.74	7.75	770
2	-9.71	7.34	7.75	7 1.15
4	-62.37	6.15	7.7.5	

# **Optical Absorption Spectrum of GaAs**



Calculation: 3 occupied, 6 empty bands Exp.: Aspnes and Sturge, Phys. Rev. B 27, 985 (1983).

GaAs: the joined density-of-states (JDOS)



Effects of the electron-hole interaction:

- Only marginal changes in the JDOS
- Coherent (constructive, destructive) coupling of the dipole moments

#### Optical Absorption Spectrum of GaAs at $E_q$



Includes LS interaction; 1000 out of 100M k-points. Exp.: M. D. Sturge, Phys. Rev. 127, 768 (1962)

# Optical Spectrum of SiO<sub>2</sub>







# Poly-Phenylene-Vinylene (PPV)

- Most important conducting polymer:
  - LED's, Optoelectronics
  - Photovoltaics







calculated at the X point; 0.6 Å above atomic plane



Exp.: D. A. Halladay et al., Synthetic Metals 55-57, 954 (1993).

#### Electron-hole Wavefunctions in PPV



At 2.4 eV (below E<sub>g</sub>): Exciton
At 4.0 eV and higher: Resonant states

# **Singlet and Triplet Excitons**

- Spin-singlet excitons (excitable by light): observe repulsive exchange interaction
- Spin-triplet excitons: No exchange interaction

Polyacetylene: Singlet Dipole

Triplet  $E_i^S \qquad E_g - E_i^S \qquad E_i^T \qquad E_g - E_i^T$  $\Delta_{S \Leftrightarrow T}$ 1.7 0.4 **0.9 1.2** 0.8 1.8 0.3 1.7 0.4 0.1

#### PPV:

	Singlet		Triplet		
Dipole	$E_i^S$	$E_g - E_i^S$	$E_i^T$	$E_g - E_i^T$	$\Delta_{S \nleftrightarrow T}$
۲	2.4	0.9	1.5	1.8	0.9
<b>460</b> 00000000	2.8	a may b d and a second a second a second a second a second a second a second a second a second a secon	2.7	0.6	0.1

 Even (
 <u>´</u> dipole-allowed) states: Electron and hole are close to each other ⇒ Strong exchange interaction





Österbacka, Wohlgenannt, Chimin + Vardeny (1999)

### Structure of the Si(111) 2x1 Surface

Side view





**Quasiparticle Surface-State Bands of Si(111)2x1** 



 Exp.: R. Uhrberg et al (1982); P. Perfetti et al (1987) Eg = 0.65 eV (< Eg(bulk) = 1.17 eV)
 *J* Surface state QP gap
 Rohlfing & Louie (1999)

#### Si(111) 2×1 Surface

Experimental differential reflectivity spectrum (Ciccacci et al., 1986)



• 
$$\Delta R/R = (R_{clean} - R_{ox})/R_{ox}$$

- Reflectivity associated with surface states
- QP results  $\rightarrow$  onset at 0.65 eV  $\pm 2 - peak structure$





Artificial broadening : 0.05 eV Exp.: P. Chiaradia et al., PRL **52**, 1145 (1984)

- Discrete exciton spectrum
- Optical spectrum dominated by lowest exciton
- Surface exciton binding energy: 0.25 eV
- Bulk exciton binding energy: 15 meV

Rohlfing & Louie (1999)

#### Surface Exciton Two-particle Amplitude - Side View (Distribution of electron relative to the hole for state at 0.43 eV)



#### Surface Exciton Two-particle Amplitude - Top View (Distribution of electron relative to the hole for state at 0.43 eV)



Across the chains - 8 A

Rohlfing & Louie (1999)

### Absorption Spectrum of Si Clusters



- melacing electron-hole interaction
- Huge excitonic binding energy:  $\sim 5 \text{ eV}$

# Optical Excitation Energies of SiH\_4 and Si\_2H\_6

	This work	Experiment
	[eV]	[eV]
SiH <sub>4</sub> :	9.0	8.8
	10.2	9.7
	11.2	10.7
$Si_2H_6$ :	anard y	7.6
	<b>9</b> .0	8.4
	9.6-9.8	9.5, 9.9

Exp.: U. Itoh, Y. Toyoshima, and H. Onuki, J. Chem. Phys. 85, 4867 (1986).

## Noble-gas Atom Ionization Energies and First Neutral Excitation Energies (in eV)

		This Work	Expt
He:	I	. 24.7	24.6
	E <sub>s</sub> E <sub>T</sub>	20.8 19.8	20.6 19.8
Ne:	I	21.5	21.6
	E <sub>s</sub> E <sub>T</sub>	16.9 16.7	16.9 16.7
Ar:	I	15.9	15.8
	E <sub>s</sub> E <sub>T</sub>	12.0 11.8	11.8 11.6

I = Ionization energy

 $E_s = Singlet excitation$ 

 $E_{T}$  = Triplet excitation

#### **Summary**

- Density functional theory provides valuable input to many-body perturbation theory calculation of excited-state properties.
- GW approximation yields highly accurate first-principles quasiparticle energies for many materials systems, to a level of ~0.1 eV.
- Evaluation of the Bethe-Salpeter equation provides *ab initio* and quantitative results on exciton states and optical response of crystals, surfaces, polymers, and clusters.
- Combination of DFT and MBPT → excited state properties.