united nations
educational, scientific
and cultural
organization

international atomic
energy agency

the
abdus salam
international centre for theoretical physics

SMR.1347 - 2

WORKSHOP ON STATISTICAL PHYSICS AND CAPACITY-APPROACHING CODES

ASPECTS OF SPIN GLASS THEORY

M. MEZARD
Universite' Paris Sud - L.P.T.M.S.
Laboratoire de Physique Theorique et Modeles Statistiques
Batiment 100 - Cedex 05
91405 Orsay, France

Please note: These are preliminary notes intended for internal distribution only.

Aspects of spin glass theory - Trieste, may 2001

M. Mézard, LPTMS Orsay

I - Spin glasses: general definition // Quenched disorder

II - Freezing transition: the Random Energy Model

III - Replicas in the REM

IV - Replicas in spin glasses

V - Cavity method: basic ideas

VI - Cavity method: the self consistent iteration with one state
TAP equations.

VII - Cavity with many states and free energy reshuffling

VIII - One word on dynamics

1- Spin glasses.

* N Ising spins $S_i = \pm 1$ (Boolean variables)

* Energy function $E_J(s_1 \cdots s_N)$, of order N, generally with some randomness.

eg: $E = \left(\sum_i h_i S_i\right)$ random field $P(h_i) = e^{-h_i^2/2}$

$\longrightarrow -\sum_{ij} J_{ij} S_i S_j$ $P(J_{ij}) = e^{-N J_{ij}^2/2}$

$-\sum_{i_1<i_p} J_{i_1 \cdots i_p} S_{i_1} \cdots S_{i_p}$ $P(J_{i_1 \cdots i_p}) = e^{-N^{p-1} \frac{J_{i_1 \cdots i_p}^2}{2}}$

s.g. discuss fully connected or diluted $\{J\}$ = geometry + structure of the graph.

scaling: $E = \sum_i S_i \underbrace{h_i(\{S_j\}, j \neq i)}_{\text{order } 1}$.

$h_i = \sum_j J_{ij} S_j$ should be $\sim 1 \rightarrow J_{ij} = \frac{1}{\sqrt{N}}$ if random (uniform $J_{ij}^2 = \frac{1}{N}$).

* Probability distribution

$$P_J(s_1 - s_N) = \frac{1}{Z} e^{-\beta E_J(s_1 - s_N)}$$

"J": sample $\beta = 1/T$ → Quenched vs annealed variables

$s = \{S_1 \cdots S_N\}$: configuration

$P_J(S_1 - S_N)$: Boltzmann weight.

* Pb: understand, at large N, ~~the one~~

- the structure of space of config

- How it evolves when β increases $\rightarrow$ optimization pbs.

Compute $\langle S_i \rangle$, $\langle S_i S_j \rangle$, $\langle E_J(S_1 \ldots S_N) \rangle = U_J$

$\sum_S -P_J(S) \operatorname{Log} P_J(S) = S_J$ counts the number of relevant configurations

- Is it the same for almost all J ? (self averageness)

define $\sum \left(\frac{U_J}{N} - u_0\right)^2 \lesssim \frac{1}{N^\alpha}$

2- Freezing phase transition : the REM

* Simplest case: the REM

$$E_J(S_1 \ldots S_N) = \text{iid random variables,}$$

$$P(E) = A e^{-E^2/N}$$

$$\rightarrow \{E_e\},\ e = 1, \ldots, 2^N$$

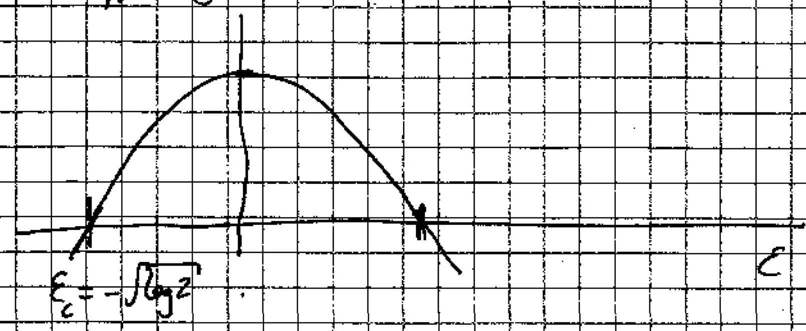
$$P(e) = \frac{e^{-\beta E_e}}{Z} \qquad Z = \sum_{e'} e^{-\beta E_{e'}}$$

* "Microcanonical" solution

$$\overline{\mathcal{N}(E)} = \underset{\substack{\uparrow \\ \text{normalisation of } P(E)}}{A}\, 2^N e^{-E^2/N} \simeq A\ e^{N[\operatorname{Log} 2 - \varepsilon^2]}$$

$\varepsilon = \frac{E}{N}$: "energy density"

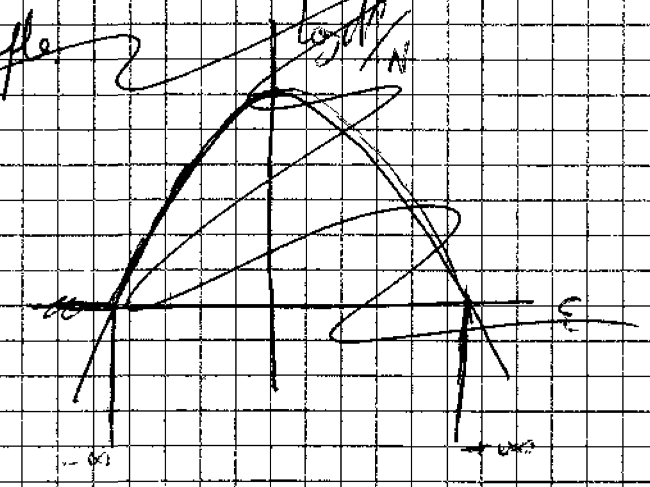
We are interested in $\frac{1}{N} \log \overline{\mathcal{N}(E)}$



If $\varepsilon^2 < \log 2 \Rightarrow \overline{\mathcal{N}} \sim e^{N} \simeq \mathcal{N}_{typique}$

If $\varepsilon^2 > \log 2 \Rightarrow \overline{\mathcal{N}} \sim e^{-N} \Rightarrow \mathcal{N}_{typ} = 0$; Occasionally (with proba $\sim e^{-N}$), $\exists$ sample where $\mathcal{N} \neq 0$.

~~Typical sample~~



Total weight of all configurations at energy E, when weighted with the Boltzmann weight

$$\to \mathcal{N}(E)\, e^{-\beta E} = e^{N[\log 2 - \varepsilon^2 - \beta \varepsilon]}$$

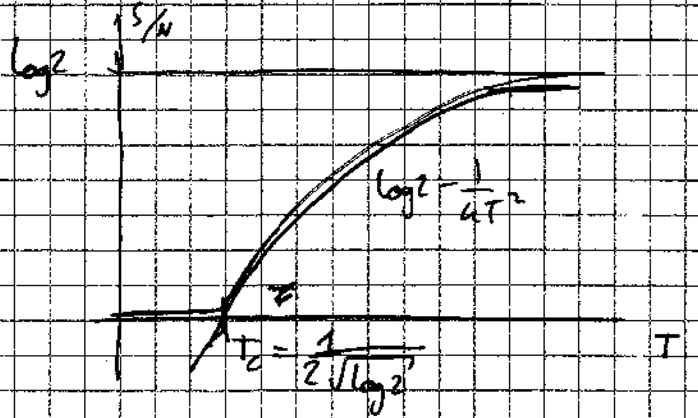
Saddle point $\Rightarrow$ measure concentrates on the ε such that $\log 2 - \varepsilon^2 - \beta\varepsilon$ is maximal $\Rightarrow \varepsilon = -\beta/2 = -\frac{1}{2T}$

If $\frac{\beta}{2} < \sqrt{\log 2} \rightarrow$ OK, exponentially large number of relevant configurations.

If $\frac{\beta}{2} > \sqrt{\log 2} \rightarrow$ impossible. The ~~leading~~ dominant configurations are those at $\varepsilon = \varepsilon_c$ ("sticking of the saddle point")

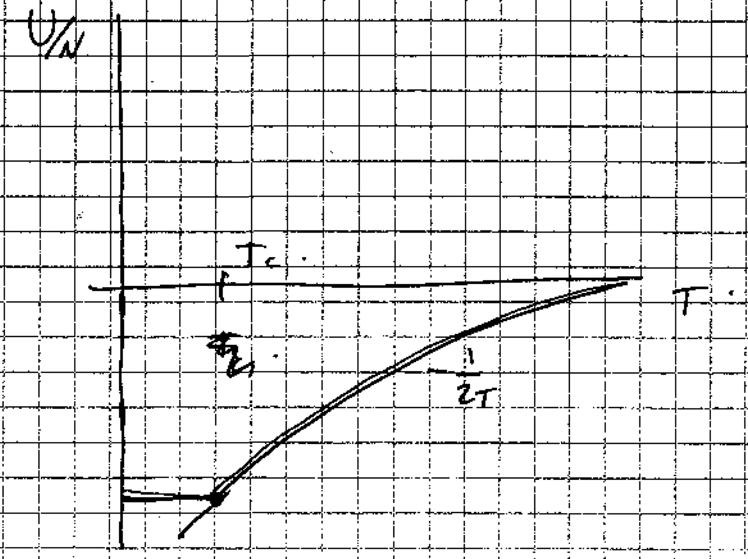
$\boxed{\beta_c = 2\sqrt{\log 2}}$

Entropy density: $S = \log 2 - \varepsilon^2 \Big|_{\varepsilon = -\frac{1}{2T}} = \log 2 - \frac{1}{4T^2}$.



~~NB: The~~

$\frac{U}{N} = -\frac{1}{2T}$



NB: behavior of almost all samples.

2) Phase transition, freezing.

3) Partial freezing can occur also

CT 5

Structure of the space of configurations at $\beta > \beta_c$

Weights concentrate on configurations with $E_{\mathcal{C}} = -N\sqrt{\log 2} + U_{\mathcal{C}}$.

non extensive correction

$$P(U_{\mathcal{C}}) \sim cte\, e^{\frac{U_{\mathcal{C}}}{T_c}} \quad \text{(Gumbel law for extremes)}.$$

Take many variables $U_{\mathcal{C}}$, ~~with~~ i.i.d with density $e^{\frac{U_{\mathcal{C}}}{T_c}}$

and compute $W_{\mathcal{C}} = \dfrac{e^{-U_{\mathcal{C}}\beta}}{\sum_{\mathcal{C}} e^{-U_{\mathcal{C}}\beta}}$ with $\beta > \beta_c = \dfrac{1}{T_c}$

$\Rightarrow$ the low energy config dominate.

One gets after an easy computation:

$$P(W) = \frac{W^{-1-\mu}(1-W)^{-1+\mu}}{\Gamma(\mu)\,\Gamma(1-\mu)} \quad \text{with } \mu = \frac{T}{T_c}.$$

In particular $\displaystyle\sum_{\mathcal{C}} W_{\mathcal{C}}^2 = 1 - \frac{T}{T_c}$.

condensation of the measure on a "finite" number of states, more pronounced when T goes to zero.

All these features are present in more complicated spin glass problems, but one needs to develop other techniques because the $E_{\mathcal{C}}$ are no longer independant $\rightarrow$ replicas, cavity.

3 - Replicas in the REM

$$Z_J = \sum_{\mathcal{C}} e^{-\beta E_{\mathcal{C}}} \quad \text{"partition function"}$$

$$= \int d\varepsilon \; \mathcal{N}_J(\varepsilon)\, e^{-\beta N \varepsilon}$$

$$\simeq \int_{-|\varepsilon_c|}^{|\varepsilon_c|} d\varepsilon \; e^{N\left[\log 2 - \varepsilon^2 - \beta\varepsilon\right]} \quad \text{for almost all } J.$$

* The knowledge of $\frac{1}{N}\log Z$ gives all interesting previous results, eg.

$$\frac{1}{N}\log Z = S - \beta U \quad (= -\beta F)$$

$$-\frac{\partial}{\partial\beta}\log Z = U.$$

* One expects $\frac{1}{N}\log Z_J$ to be the same for almost all samples.

* Annealed average:

$$\overline{Z_J} = 2^N \int e^{-\beta E}\, e^{-E^2/N} = 2^N e^{\beta^2/4N} \rightarrow \text{finds} \quad U = -\frac{1}{2T}$$

$$S = \log 2 - \frac{1}{4T^2}$$

without seeing the ε_c, ie the T_c.

for $T < T_c$: wrong because dominated by rare samples (remember $Z_J \sim e^{N\,\text{something}}$).

* Quenched average: RS solution

$$\overline{\log Z_J}$$

One expects: $\overline{\log Z_J} = \log(\overline{Z_J})$ if $T > T_c$

$\neq$ if $T < T_c$

$$\overline{\log Z_J} = \lim_{n \to 0} \frac{\overline{Z_J^n} - 1}{n}$$ replicas → very general method

$$Z_J^n = \sum_{\mathcal{C}_1 \dots \mathcal{C}_n} e^{-\beta (E_{\mathcal{C}_1} + \dots + E_{\mathcal{C}_n})}$$ n non interacting REM'S.

$$\overline{Z_J^n} = \sum_{\mathcal{C}_1 \dots \mathcal{C}_n} e^{-\beta \sum_{\mathcal{C}} E_{\mathcal{C}} \left(\sum_{a=1}^{n} \delta_{\mathcal{C}, \mathcal{C}_a} \right)}$$ $P(E_{\mathcal{C}}) = e^{-E_{\mathcal{C}}^2/N}$

$$= \sum_{\mathcal{C}_1 \dots \mathcal{C}_n} e^{\frac{\beta^2 N}{4} \sum_{a,b} \delta_{\mathcal{C}_a, \mathcal{C}_b}}.$$ → replica attraction. very general.

Call $Q_{ab} = \delta_{\mathcal{C}_a, \mathcal{C}_b}$ the "overlap matrix"

here $Q_{ab} \in \{0, 1\}$.

Then $$\overline{Z_J^n} = \sum_{\{Q_{ab}\}} e^{N \left[\frac{\beta^2}{4} \sum_{ab} Q_{ab} + S(Q) \right]}.$$

where $$e^{N S(Q)} = \sum_{\{\mathcal{C}_1 \dots \mathcal{C}_n\}} \prod_{a<b} \delta(Q_{ab}, \delta_{\mathcal{C}_a, \mathcal{C}_b}).$$

For n fixed, and N large: estimate $\overline{Z_J^n}$ by saddle point.

~~[illegible]~~

→ find a matrix Q which extremizes $\frac{\beta^2}{4}\sum_{ab} Q_{ab} + S(Q) = H(Q)$

symmetric under S_n

a priori: $H(Q) = H(Q^{\pi})$ → look for RS saddle pt.

$$Q_{ab} = \begin{pmatrix} 1 & & q \\ & \ddots & \\ q & & 1 \end{pmatrix}$$

two possibilities: $q=1$ or $q=0$

$\underline{q=1}$: all identical copy $e^{NS(Q)} = 2^N$.

$\overline{Z^n} \simeq e^{N\left[\frac{\beta^2}{4}n^2 + \log 2\right]}$ does not go to 1 when $n\to 0$

$\underline{q=0}$: $e^{NS(Q)} \simeq 2^N(2^N-1)\cdots(2^N-n+1) \sim 2^{Nn}$

$\overline{Z^n} = e^{N\left[\frac{\beta^2}{4}n + n\log 2\right]}$ ~~gives back~~

$= \overline{Z}^n$ gives back annealed result. Correct at ~~large~~ $\beta<\beta_c$, incorrect at $\beta>\beta_c$.

→ Need to break replica symmetry.

NB: For n integer one finds that the rs saddle pt are ok.

with $\begin{cases} q=0 & \left(\text{ie } \overline{Z^n} = \overline{Z}^n\right) \text{ if } T > \sqrt{n}\,T_c \\ q=1 & \left(\text{ie } \overline{Z^n} \sim e^{N\left[\frac{\beta^2}{4}n^2+\log 2\right]}\right) \text{ if } T<\sqrt{n}\,T_c \end{cases}$

$\gg \overline{Z}^n$

growth of moments too fast to reconstruct the $n\to 0$ analytic continuation from the ~~moments~~

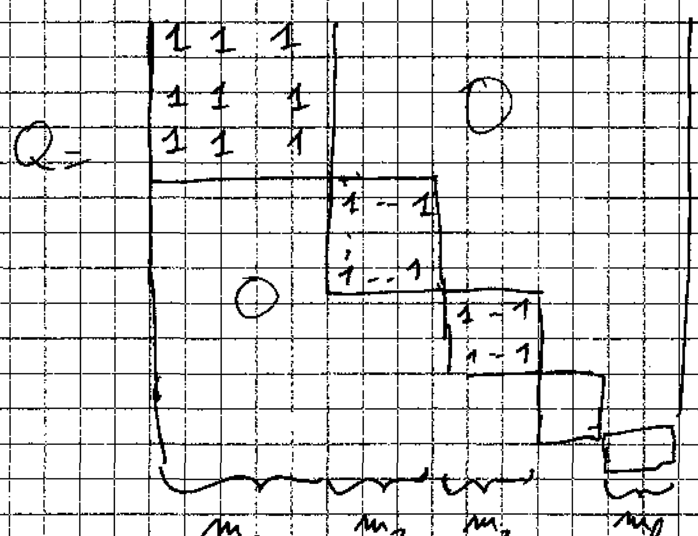
* Quenched average: rsb solution

Here it is simple:

$Q_{ab} \in \{0,1\}$.

If $Q_{ab}=1$ then $\forall c$: $Q_{ac}=Q_{bc}$ $\rightarrow$ the two lines (or columns) of the matrix are equal.

Renumber the replica indices such that



nb: $m_1+\dots+m_\ell = n$

Entropy: call ~~...~~

$$e^{NS(Q)} = (2^N)(2^N-1)\cdots(2^N-\ell+1) \simeq 2^{N\ell}$$

$\Rightarrow$ saddle point gives $\frac{\beta^2}{4}\sum_{ab} Q_{ab} + S(Q) = \frac{\beta^2}{4}(m_1^2+\dots+m_\ell^2) + \ell \log 2$

only acceptable saddle points for $n \to 0$: those with an action $\propto n$.

right solution: $m_1 = \dots = m_\ell = m$; $\ell = \frac{n}{m}$

action: $\frac{\beta^2}{4} n m + \frac{n}{m}\log 2$. ~~Maximal~~ Minimal for $m = \frac{T}{T_c}$ NB: $m<1$

→ gives back all the thermodynamics.

NB: ~~when~~ How concentrated is the measure at small temperature (in the glass phase)?

$$\overline{\sum_{\mathcal{C}} W_{\mathcal{C}}^2} = \overline{\sum_{\mathcal{C}} \frac{e^{-2\beta E_{\mathcal{C}}}}{\left(\sum_{\mathcal{C}'} e^{-\beta E_{\mathcal{C}'}}\right)^2}}$$

$$= \lim_{n\to 0} \overline{\sum_{\mathcal{C}_1 \dots \mathcal{C}_n} \delta_{\mathcal{C}_1 \mathcal{C}_2} \, e^{-\beta(E_{\mathcal{C}_1} + \dots + E_{\mathcal{C}_n})}}$$

$$= \lim_{n\to 0} \langle Q_{12} \rangle$$

The measure on Q_{ab} $\quad e^{N\left[\frac{\beta^2}{4}\sum_{ab} Q_{ab} + S(Q)\right]}$

$\Rightarrow \langle Q_{12} \rangle \simeq Q_{12}^{SP}$

But, when we have rsb, spontaneous breakdown of S_n symmetry $\Rightarrow$ $Q^{\text{Parisi}}_{\pi(a)\pi(b)}$ is also a saddle point.

$$\langle Q_{12} \rangle = \sum_{S.P} Q_{12}^{SP} = \frac{1}{n(n-1)} \sum_{a\neq b} Q_{ab}^{\text{Parisi}}$$

$$= \frac{1}{n(n-1)}\, n(m-1) = 1 - m$$

$$= 1 - \frac{T}{T_c}$$

agrees with direct computation.

Means that low-lying states have $e^{\beta m E}$ distribution $\quad$ %

NB: rsb is associated with the fact that, for large β, a few configurations start to dominate the ~~the~~ Boltzmann probability

IV Replicas in spin glasses

SK model: $E = -\sum_{i<j} J_{ij} S_i S_j$ $\qquad S_i \in \{\pm 1\}$

$P(J) \propto e^{-N J_{ij}^2/2}$

2^N configurations

$Z_J = \sum_{\{S\}} e^{\beta \sum_{i<j} J_{ij} S_i S_j}$, $\quad Z_J^n = \sum_{\{S^a\}} e^{\beta \sum_{i<j} J_{ij} \sum_a S_i^a S_j^a}$

$\downarrow$ short algebra

$$\overline{Z_J^n} = \int \prod_{a<b} dQ_{ab}\, e^{N\left[-\frac{\beta^2}{2}\sum_{a<b} Q_{ab}^2 + S(Q)\right]}$$

$$e^{S(Q)} = \int \sum_{\{S^a\}} e^{\beta \sum_{a<b} Q_{ab} S_a S_b}$$

NB1: Compare to the REM expression. Very similar

NB2: p-spin model: $E = -\sum J_{i_1 \ldots i_p} S_{i_1} \ldots S_{i_p}$ → very close to that with a Q_{ab}^p (in fact: $+\frac{\beta^2}{2}\sum_{a<b} Q_{ab}^p + \sum_{a<b} \lambda_{ab} Q_{ab} + S(\lambda)$)

→ back to the problem of finding the Q matrix.

→ hierarchical construction, Parisi solution,

again: $n \geq m_1 \geq m_2 \ldots \geq m_k \geq 1$ $\qquad$ stability AT, Almeida Thouless

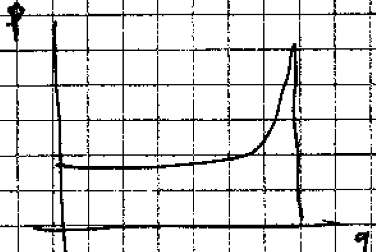
$\downarrow$

$n \to 0$: $\quad 0 \leq m_1 \leq m_2 \leq \ldots \leq m_k \leq 1$.

Order parameter

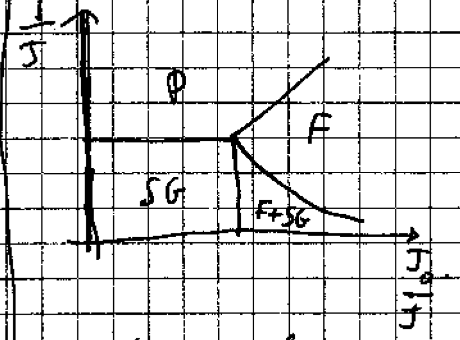
$$P(q) = \overline{\sum_{\theta\theta'} W_\theta W_{\theta'} \, \delta(q_{\theta\theta'} - q)}$$

$$= \lim_{n\to 0} \frac{1}{n(n-1)} \sum_{a\neq b} \delta(Q^{ab}_{\text{Parisi}} - q) \qquad \left(\text{after } \sum_{\text{perm}}\right)$$

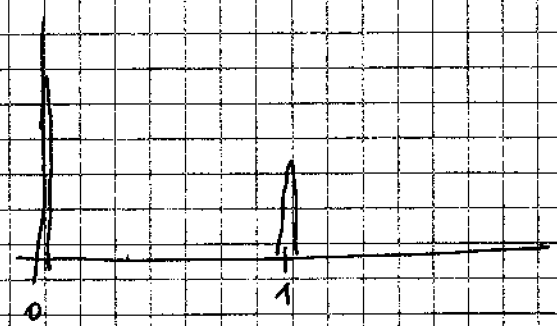


"full"

Phase diagram in SK model, with J_0:



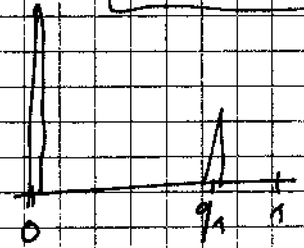
while in the REM we had $q \in \{0, 1\}$.



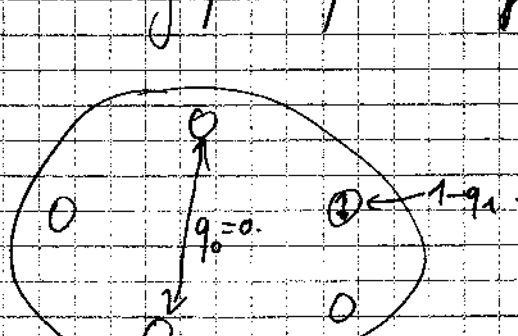
"one step"

explain all order parameters and phases

in general in the p-spin



Structure of phase space in the one step,



In the full → ultrametricity

5- Cavity method: basic ideas

Applies to various spin-glass or similar problems, where the graph of interactions is locally tree like (see also matching, TSP)

$$E = -\sum_{\langle ij\rangle} J_{ij}\,\sigma_i\,\sigma_j$$

$\langle ij\rangle$: links on a locally tree-like structure.

ex1: $n_{ij} = 1$ proba $\frac{c}{N}$; $n_{ij} = 0$ " $1-\frac{c}{N}$ $\Big) \rightarrow$ random graph with average connectivity c

ex2: $n_{ij} \in \{0,1\}$, $\sum_j n_{ij} = k+1$ random graph with fixed connectivity $k+1$

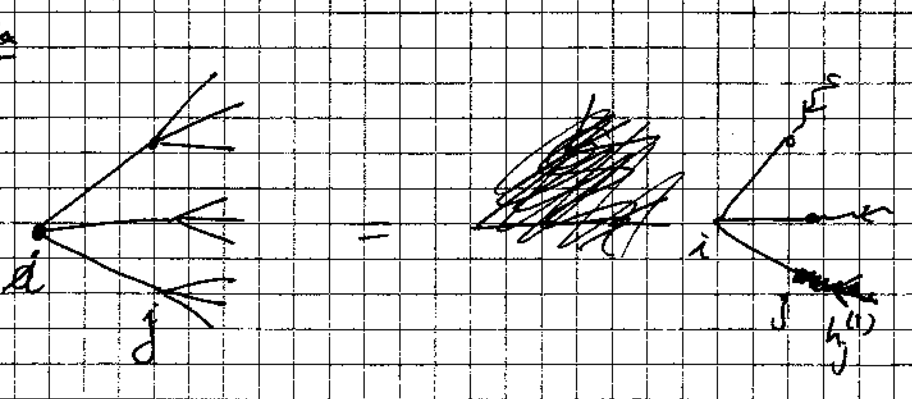
loops are present but rare, typical size $\sim \log N$

→ correct generalization of "Bethe lattice" concept for frustrated systems.
Cayley tree ⟺ Bethe lattice

Take for instance the fixed connectivity case.

* basic cavity mechanism:

one branch:

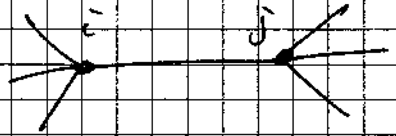


→ measure $e^{\beta \sigma_i \sum_j J_{ij}\sigma_j + \beta \sum_j h_j^{(i)}\sigma_j}$

$h_j^{(i)}$: cavity fields, i.e. field on j when i is absent.

• Thermodynamics:

Measure on a bond ij:



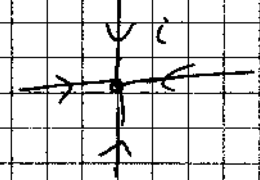
$$Z^{(2)}_{ij} = \sum_{\sigma_i \sigma_j} e^{-\beta\left(\sigma_i h_i^{(j)} + \sigma_j h_j^{(i)} + J_{ij}\sigma_i\sigma_j\right)}$$

allows to compute the energy of this link by ~~$U_{ij} = -\frac{\partial \log Z^{(2)}_{ij}}{\partial \beta}$~~.

$$U_{ij} = -\langle J_{ij}\,\sigma_i\sigma_j \rangle \text{ with the above measure.}$$

Measure on a site:

$$Z^{(1)}_i = \sum_{\sigma_i} e^{-\beta H_i \sigma_i}$$

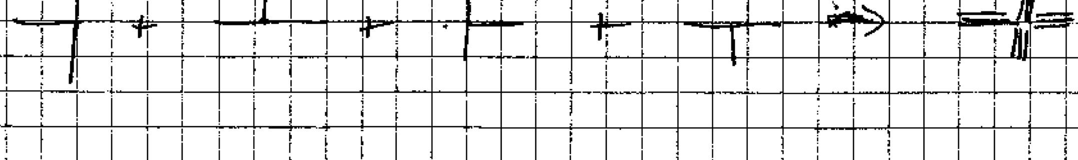


$$H_i = \frac{1}{k_i}\sum_j h_i^{(j)}$$

where k_i+1 is the connectivity of site i in the tree.

Becau.

=



CT 15

$$F = \sum_{\langle ij\rangle} F_{ij}^{(2)} - \sum_i k_i F_i^{(1)}$$ TAP free energy, exact on tree

where $-\beta F_i^{(1)} = \log Z_i^{(1)}$, $-\beta F_{ij}^{(2)} = \log Z_{ij}^{(2)}$

$\boxed{\text{Proof:}}$ * $\underline{T \to \infty} \Rightarrow Z_{ij}^{(2)} = 4$, $Z_{ij}^{(1)} = 2$.

one gets $F \to -TN\log 2$ ok.

* $$\underline{\frac{\partial \beta F}{\partial \beta}} = -\sum_{\langle ij\rangle} \langle \sigma_i h_i^{(j)} + \sigma_j h_j^{(i)} + J_{ij}\sigma_i\sigma_j \rangle$$

$$+ \sum_i k_i \langle H_i \sigma_i \rangle = -\sum_{\langle ij\rangle} J_{ij} \langle \sigma_i \sigma_j \rangle \quad \text{ok.}$$

NB: limit where ~~the~~ $k \to \infty$ and $J \sim \frac{1}{\sqrt{k}}$

then: $h_i^{(j)} \simeq \sum_{\ell=1}^{k} J_{i\ell} \, \mathrm{th}\beta h_i^{(\ell)}$

$$H_i \simeq \sum_{\ell=1}^{k+1} J_{i\ell} \, \mathrm{th}\beta h_i^{(\ell)} \simeq h_i^{(j)} - O\left(\frac{1}{\sqrt{k}}\right).$$

expanding in $\frac{1}{k}$, one gets:

$$-\beta F = \beta \sum_{\langle ij\rangle} J_{ij} m_i m_j + \frac{\beta^2}{2} \sum_{\langle ij\rangle} J_{ij}^2 (1-m_i^2)(1-m_j^2)$$

$$- \sum_i \left(\frac{1+m_i}{2} \log\frac{1+m_i}{2} + \frac{1-m_i}{2} \log\frac{1-m_i}{2} \right) + O\left(\frac{1}{\sqrt{k}}\right)$$

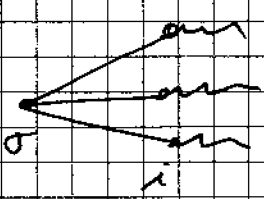
where $m_i = \mathrm{th}\beta H_i$ are local magnetizations.

→ "TAP equations". NB: the general case gives TAP equations in terms of the $h_i^{(j)}$ (= beliefs...).

→ $\boxed{\text{Discuss TAP states}}$

6. Cavity: the self consistent iteration with one state

Iterate the merging:



$$m = \langle \sigma \rangle$$

$$P(\sigma, \sigma_i) = e^{\beta \sigma \sum_i J_i \sigma_i + \beta \sum_i h_i \sigma_i}$$

$$\Rightarrow \langle \sigma \rangle = \mathrm{th}\,\beta \left[\sum_i u(J_i, h_i) \right]$$

$$u(J, h) = \frac{1}{\beta} \,\mathrm{Atanh}\left(\mathrm{th}\,\beta J \;\mathrm{th}\,\beta h \right)$$

$$h = \sum_i u(J_i, h_i) \rightarrow \text{self consistent equation for } P(h)$$

$$P(h) = E_J \int \prod_i dh_i \, P(h_i) \, \delta\left(h - \sum_i u(J_i, h_i) \right)$$

* e.g. $k \to \infty$ (SK limit) expect $P(h) = \dfrac{e^{-h^2/2q}}{\sqrt{2\pi q}}$

then: $u(J_i, h_i) \simeq J_i \,\mathrm{th}\,\beta h_i$ because $J_i \sim \frac{1}{\sqrt{k}}$

and: $\langle h^2 \rangle = \sum_i J_i^2 \,\mathrm{th}^2 \beta h_i$ with $\overline{J^2} = \frac{1}{k}$

$$\Rightarrow q = \int dh \, \frac{e^{-h^2/2q}}{\sqrt{2\pi q}} \,\mathrm{th}^2 \beta h \quad \text{usual SK result in RS case}$$

* for finite k: easy to solve by population dynamics $\rightarrow P(h) = \delta(h)$ for $T > T_c$, $P(h) \neq \delta(h)$ for $T < T_c$, where $E_J \,\mathrm{th}^2 \beta_c J = \frac{1}{k}$.

7- Self consistent ~~equation~~ iteration with many states: more complicated

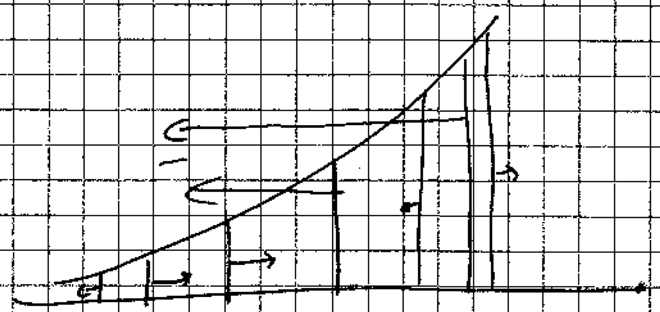
$$(1)\quad h_0^\alpha = \sum_{i=1}^{k} u(J_i, h_i^\alpha) \quad \text{in each state } \alpha$$

free energies: $F^\alpha \to F^\alpha + \Delta F^\alpha$.

Assume the F^α are iid with $\rho(F) \simeq e^{\beta m F}$ (remember REM!)

and the h_i^α are iid with $Q_i(h)$.

From (1) one gets a distribution $P(h_0^\alpha, \Delta F^\alpha)$ independent from α to α', but obviously h_0^α correlated to ΔF^α.



$$G^\alpha = F^\alpha + \Delta F^\alpha.$$

$$R(h_0, G) = \int dF\; e^{\beta m F} \int d(\Delta F)\, P(h_0, \Delta F)\, \delta(G - (F + \Delta F))$$

$$\propto e^{\beta m G} \underbrace{\int d(\Delta F)\, P(h_0, \Delta F)\, e^{-\beta m \Delta F}}_{\text{distribution } Q_0(h_0) \text{ after the reweighting and the selection of low lying states.}}$$

$$\{Q_1(h), \dots, Q_k(h)\} \to P(h_0, \Delta F) \to Q_0(h_0).$$

Order parameter:

$$\mathcal{P}\left[Q(h)\right] = \frac{1}{N}\sum_i \prod_h \delta\left(Q(h) - Q_i(h)\right).$$

functional - complicated.

but the process $Q_1(h) \ldots Q_k(h) \longrightarrow P(h_0, \Delta F) \rightarrow Q_0(h_0)$

can be followed by population dynamics again.

<u>ex</u> $k=5$, $J=\pm 1$, $T=.8$ $(T_c \simeq 2\ ?)$

$$U_{rs} = -1.8160 \pm .001$$

$$U_{1rsb} = -1.799 \pm .001$$

Simulation $\rightarrow$ $U = -1.799 \pm .001$

~~in the s~~

* <u>large k limit, simpler</u> (SK):

$$\rightarrow \cancel{Q_0(h_0) = \int \frac{dH}{\sqrt{2\pi q_0}} e^{-\frac{H^2}{2q_0}} \int dh}$$

$$Q_0(h_0) \propto e^{-\frac{(h_0-H)^2}{2(q_1-q_0)}} \left(2\,ch\,\beta h_0\right)^m.$$

fluctuates from site to site $\rightarrow$ H has distrib $e^{-\frac{H^2}{2q_0}}$

$\rightarrow$ 3 numbers q_0, q_1, m, cf Parisi Ansatz.

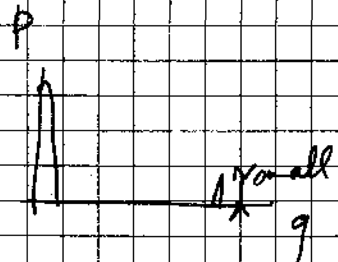
Possibility of higher order rsb in the cavity

also → gives back all results from replicas.

Allows to get rigorous results (e.g. Talagrand - paper).

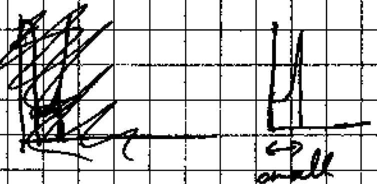
<u>Two main classes</u>

1 step rsb:



near T_c

full rsb:



near T_c.

* Basically in 1 step rsb systems ⇒ states preexist, at some T they start to contribute to thermodynamics

(Possible to see that by giving the TAP states $\{m_i(T)\}$ a weight $e^{-\frac{x}{T}F_{TAP}}$ where $x \neq 1$)

* in full rsb → states start to form exactly at T_c.

<u>Implications for Monte Carlo dynamics</u> → trapping of one step rsb systems in high energy config.