Abstracts

Belief Optimization: A stable Alternative to Loopy Belief Propagation
Max Welling in collaboration with Yee Whye Teh

Abstract

Recently, it has been shown that the fixed points of Belief Propagation correspond to the stationary points of the Bethe Free Energy. Although exact on a tree, Belief Propagation is not guaranteed to converge on loopy graphical models. In this talk we will explore the possibility to directly minimize the Bethe Free Energy using an algorithm we call Belief Optimization, which is guaranteed to converge to a local minimum. For binary undirected networks we derive an analytic expression to solve for the pairwise marginals in terms of the node marginals, while we perform gradient in the latter. We also show that the Bethe Free Energy is equal to the TAP Free Energy up to second order in the weights. For general discrete networks we present an algorithm which cycles through tree−structured subgraphs on which it performs "Iterative Proportional Fitting". We comment on how this procedure can be generalized to improve the approximation.

On polynomial invariants of linear codes
Alexander Barg

Abstract

We give linear−algebraic proofs of a few results relating polynomial invariants of linear codes, linear matroids and the partition function of the Potts model.

For a set of slides plus some other results and context:
http://cm.bell−labs.com/cm/ms/who/abarg/postscript/potts−talk.ps.gz

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Cluster expansions in dilute systems: applications to satisfiability problems and spin glasses
Guilhem Semerjian, Leticia F. Cugliandolo

Geometric programming, Bethe free energy minimization and the sum product

algorithm
and
Statistical Physics, convex optimization and the sum product algorithm
Mung Chiang and G. David Forney, Jr. Presented by Mung Chiang.

Abstract

Geometic programming is a particular type of convex optimization problem involving posynomial objectives and constraints. We show the relationships between the dual problem of geometric programming, message passing algorithms, and the minimization of the Bethe free energy, which is a non−convex objective function. The relationships build on fundamental results on conjugacy, duality and duality gap from convex optimization theory.

Monte Carlo Decoding of LDPC Codes
Radford M. Neal, University of Toronto

Abstract

I investigate the decoding of low density parity check codes using Markov chain Monte Carlo techniques. Two Markov chain update mechanisms are considered: the simple Gibbs sampling (heatbath) method, and an auxiliary variable method related to the Swendsen−Wang algorithm, based on sampling uniformly from a sub−code defined by a set of free bits and active checks. These methods are used in combination with annealing or tempering schemes in which the attention paid to the received data and/or the parity−check constraints varies. These Monte Carlo methods can achieve essentially the same decoding success as probability propagation, albeit at considerably greater computational cost. I discuss the possible implications of this for attempts to find decoding methods that go beyond the limits of probability propagation.

Slides for are available in compressed Postscript or PDF at one of the URLs below:

ftp://ftp.cs.utoronto.ca/pub/radford/mcdecode−talk.ps.Z
ftp://ftp.cs.utoronto.ca/pub/radford/mcdecode−talk.pdf

Zero Error Noise Thresholds for Gallager Code Ensembles
Yoshiyuki Kabashima

Abstract

We compare four different methods to estimate zero error noise thresholds for ensembles of

Gallager's linear code. Three of the methods are based on Gallager's methodology (1965); while Jensen's inequality is employed to upper bound the average over a given code ensemble, using somewhat similar inequalities with two and one optimization parameters (A and B respectively), we use the replica method (C) to carry out the average directly. The results obtained from using (A), (B) and (C) are compared to the 'typical pair analysis' (D) which was reintroduced recently by Aji et al (1999).

We show that the replica based approach (C) provides the tightest estimate to date and that (A) and (D) are equivalent. This work has been done in collaboration with Kazutaka Nakamura (TITECH), David Saad (Aston) and Jort van Mourik (Aston).

Another related paper can be downloaded from cond−mat/0010173

Cascading low density parity−check−codes with low complexity
Ido Kanter

Abstract

The efficacy of a specially constructed MN−type error−correcting code to communication in BSC channel and Gaussian channel is being examined. The construction is based on the introduction of complex matrices, used in both encoding and decoding, which comprise sub−matrices of cascading connection values. The following issues will be discussed: (a) cyclic matrices (b) degree of the code versus connectivity of the graph (c) breaking inversion symmetry (d) efficient simulations to estimate finite size scaling.

Statistical Physics of LDPC Codes
David Saad in collaboration with Yoshiyuki Kabashima (TITECH), Tatsuto Murayama (TITECH) and Renato Vicente (Aston)

Abstract

We study parity−check error−correcting codes using methods adopted from statistical physics, to discover their capabilities and limitations. We focus on Gallager's error−correcting code and a variation of it termed the MN code. In both frameworks, the codeword comprises products of the original message bits, selected using two randomly constructed sparse matrices. We investigate the typical performance of these codes for various matrix constructions, by deriving their free energy using the replica method as well as the iterative method of Husimi trees, to obtain typical case solutions.

The analysis exposes the differences between the two code types and provides insight to the mechanisms controlling their practical and theoretical performance. It shows that the number of non zero elements per row in Gallager's matrix construction should become infinite for saturating the theoretical limit, while finite connectivity is sufficient in the case

of MN codes. The practical performance is bounded by the noise rate for which sub−optimal solutions, other than the ferromagnetic solution, emerge. This also provides a handle for optimising the code's performance using specific construction of the relevant matrices.

Tree−based reparameterization for approximate estimation of stochastic processes on graphs with cycles

Martin Wainwright, Lab for Information & Decision Systems, MIT, Joint work with Tommi Jaakkola and Alan Willsky, MIT

Abstract

We develop a tree−based reparameterization framework that provides a new conceptual view of a large class of iterative algorithms for computing approximate marginals in graphs with cycles. Among them is belief propagation, otherwise known as the sum−product algorithm, which can be reformulated as a very local form of reparameterization. More generally, this class includes algorithms in which updates are more global, and involve performing exact computations over spanning trees of the full graph. On the practical side, we find that such tree reparameterization (TRP) algorithms typically converge more quickly than belief propagation (BP) with equivalent or lower cost per iteration; moreover, TRP often converges on harder problems for which BP fails.

Our perspective also provides new theoretical insights into approximate estimation. In particular, it leads to a novel probabilistic characterization of the set of fixed points. We develop the geometry of reparameterization updates, and use it analyze convergence. A fundamental property of relabeling updates is that they leave invariant the distribution on the full graph, which has a number of important consequences. For instance, it enables us to derive an exact relation between the true marginals on an arbitrary graph with cycles, and the approximations provided by TRP or BP. We also develop bounds on this approximation error, which illuminate the conditions that govern performance of such techniques. Our results also have natural extensions to approximations (e.g., Kikuchi) that involve clustering nodes.

A technical report is available at the web site:
http://ssg.mit.edu/group/mjwain/mjwain.shtml

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p−spin Ising spin glasses with r−spin ferromagnetic interactions; phase diagrams and relevance for error−correcting codes

Peter Gillin (1), Hidetoshi Nishimori (2), and David Sherrington(1)

(1) Physics Department, University of Oxford, Theoretical Physics, 1 Keble Road, Oxford, OX1 3NP
(2) Department of Physics, Tokyo University of Technology, Oh−Okayama, Meguro−ku, Tokyo 152−8551, Japan

Abstract

We consider the thermodynamics of an infinite−range Ising p−spin glass model with an additional r−spin ferromagnetic interaction. As Sourlas has shown, such a model with $p=r$ is relevant to the de−coding of binary code transmissions, with the ferromagnetic term describing the signal and the spin−glass term the effect of transmission noise. Also relevant to de−coding is the presence or otherwise of glassiness in the ferromagnetic phase, since it corresponds to hindering of algorithms for message retrieval. $p\to\infty$ spin glasses are known to be related to the random energy model (REM) and a recent study of a REM with an $r\to\infty$ ferromagnetic term led Dorlas and Wedagadera to conclude that the ferromagnetic state would be fully glassy in this limit and Sourlas codes consequently problematic. Our study of general p and r exposes subtleties which they overlooked and leads to the opposite conclusion for the relevant case of $r=p\to\infty$.

For $r = 2$ there is a continuous transition to a ferromagnetic phase, while for $r>2$ the transition is first order. For general p and r we find both glassy and non−glassy ferromagnetic phases, with replica symmetry breaking of both the one step and full varieties, and show how the phase diagram changes as r is varied from below to above p. In particular we obtain new results for the case where $r=p>2$, demonstrating that the stable ferromagnetic phase in regions relevant to Marginal Posterior Maximizer (MPM) de−coding is non−glassy, as desired.

In addition to general results obtainable by replica theory methods we have also derived a number of further special results for $p=r$ on the Nishimori line (which is that relevant to MPM de−coding) using exact gauge transformation methods.

Typical Properties of Turbo Codes and Low Density Parity Check Codes: The Statistical Mechanics Point of View

Andrea Montanari

Abstract

I will discuss the typical behavior of Turbo Codes and Low Density Parity Check Codes as the channel parameters (i.e. the noise level) and some decoding parameter are varied.

I will address several points such as: (i) the local stability threshold; (ii) the thresholds under maximum−likelihood and symbol−MAP decoding (and the related issue of replica

symmetry breaking); (iii) the threshold under sum−product decoding (i.e. the dynamic phase transition); (iv) when do (i) and (iii) coincide (i.e. where is the tricritical point)?

Bounds on the Performance of Belief Propagation Decoding
David Burshtein and Gadi Miller

Abstract

We consider Gallager's soft decoding (belief propagation) algorithm for decoding low density parity check (LDPC) codes, when applied to an arbitrary binary−input symmetric−output channel. By considering the expected values of the messages, we derive both lower and upper bounds on the performance of the algorithm.
We also derive various properties of the decoding algorithm, such as robustness to the initial values of the messages.
Our results apply both to regular and irregular LDPC codes.

Embedded Trees: Estimation of Gaussian Processes on Graphs with Cycles
Erik Sudderth, MIT, Joint work with Martin Wainwright and Alan Willsky, MIT

Abstract

We present the embedded trees algorithm, an iterative technique for estimation of Gaussian processes defined on arbitrary graphs.
By exactly solving a series of modified problems on embedded spanning trees, it computes the conditional means with an efficiency comparable to or better than other techniques. In addition, the spanning tree decomposition may be adapted to compute the exact error covariances for graphs with cycles. We provide conditions guaranteeing the algorithm's convergence, and an analysis of the factors controlling convergence rate. We will also briefly examine the modeling power of sparsely connected loopy graphs, for which the embedded trees inference procedure is especially efficient.

Copies of the relevant paper (in both postscript and pdf formats) are available at the following URL: http://ssg.mit.edu/group/esuddert/trieste

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