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SMR.1317 - 2

SUMMER SCHOOL ON PARTICLE PHYSICS

18 June - 6 July 2001

ASPECTS OF ASTROPARTICLE PHYSICS

Lectures I, II and III

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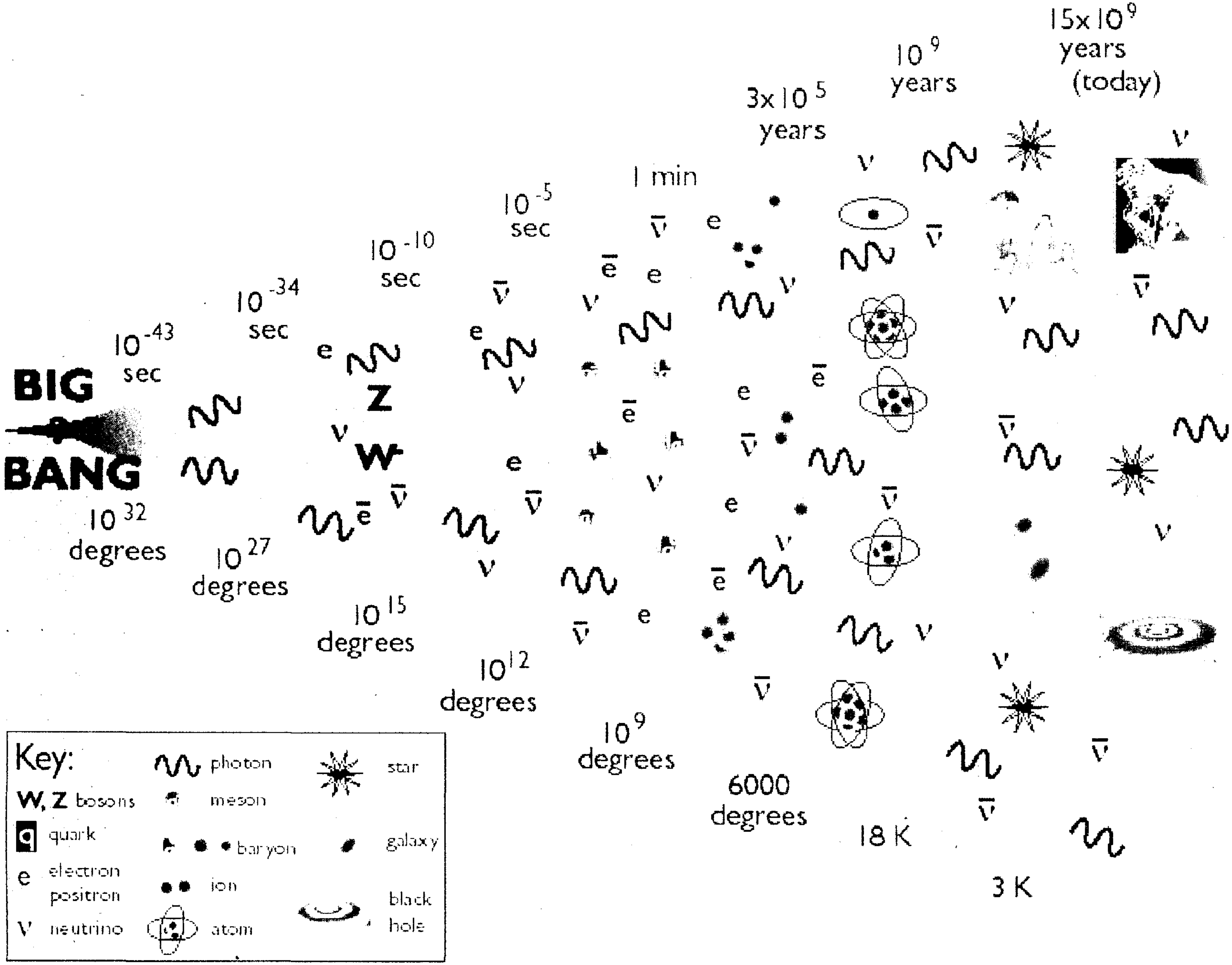
1) Standard FRW cosmology.

2) Early Universe.

3) BBN.

4) Baryo. + Inflation.

5) SUSY + Dark Matter.


BIG BANG
10^-43 sec
10^32 degrees
10^-34 sec
10^27 degrees
10^-10 sec
10^15 degrees
10^-5 sec
10^12 degrees
1 min
10^9 degrees
3x10^5 years
6000 degrees
10^9 years
18 K
15x10^9 years (today)
3 K
Key:
W, Z bosons
q quark
e electron
ē positron
ν neutrino
photon
meson
baryon
ion
atom
star
galaxy
black hole

CHRONOLOGY

t (sec)	T (GeV)	
10^{-44}	10^{18}	Planck Epoch: 1) Quantum Grav. 2) Kaluza-Klein, 3) Supergravity - Superstrings ???
10^{-35}	10^{15}	Grand Unification / Inflation Baryon Generation / Cosmological Probl Resolved.
10^{-10}	10^{2}	Weak Symmetry Breaking
10^{-5}	10^{-1}	Confinement Transition
1 -	10^{-3}	Big Bang Nucleosynthesis
10^{12} s	10^{-9}	Recombination / Galaxy Formation
0^{17} s	10^{-13}	today

Evidence for Big Bang

1) Existence of the Microwave Background Radiation at $T \approx 2.8\,°K$.

$$\frac{\delta T}{T} \lesssim \text{few} \times 10^{-5} \Rightarrow \text{isotropy and homogeneity.}$$

$$T \approx 1\,eV$$

2) Big Bang Nucleosynthesis predicts the abundances of the light elements
D, ^3He, ^4He, ^7Li
correctly!

e.g. $^4He = 25\%$ by mass

$$T \approx 1\,MeV$$

3) Baryon Generation predicts an asymmetry between baryons and antibaryons perhaps of the right order of magnitude.

Cosmology

I First Principles

a) Copernican Principle: We are not privileged observers.

\+

b) Relativity Principle: Physical laws do not depend on space-time.

= c) Cosmological Principle: $\exists$ an infinite set of observers such that the Universe isotropic in all measurable properties at all times. $\Rightarrow$ Universe is spatially homogeneous and isotropic.

Two Immediate Consequences:

1) The only true velocity fields can be expansion or contraction

2) There must exist a measure of distance independent of direction

i) besides expansion and contraction
∃ rotation, shear, combined exp. + contr.
included anisotropic Bianchi models.

\+ expansion and contraction with no center ⇒ velocity of two observers depends only on their separation

$$V_{12} = H r_{12} \qquad \text{Hubble.}$$

2) measure of distance

$$d = zc/H$$

z = redshift (blueshift) for expan. (contr.)

The Metric

A metric g is a symmetric tensor of the form

$$g = g_{\mu\nu}\, dx^{\mu}\, dx^{\nu} = ds^2$$

Vector X will be defined to be timelike, null or spacelike depending on whether

$$\begin{aligned} g(X,X) &< 0 && \text{timelike} \\ &= 0 && \text{null} \\ &> 0 && \text{spacelike} \end{aligned}$$

The Cosmological Principle $\Rightarrow$ $g_{0i} = 0$

$g_{ij} = 0$ for $i \neq j$

$$ds^2 = g_{00}\, dt^2 + g_{ii}\, dx^i dx^i$$

The most general form for a completely homogeneous + isotropic metric is the Robertson-Walker form

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

$R(t)$ is called the scale factor and k is a constant for which the geometry is determined by

$$k = +1 \quad \text{closed}$$
$$k = -1 \quad \text{open}$$
$$k = 0 \quad \text{critical.}$$

The evolution and dynamics of the spacetime are all derived by applying Einsteins eq's to the metric.

The covariant derivative in G.R. is defined by

$$D_\mu X^\nu = \partial_\mu X^\nu + \Gamma^\nu_{\mu\sigma} X^\sigma = X^\nu_{;\mu}$$

$$D_\mu X_\nu = \partial_\mu X_\nu - \Gamma^\sigma_{\mu\nu} X_\sigma = X_{\nu;\mu}.$$

$$\partial_\mu X^\nu = \frac{\partial X^\nu}{\partial x^\mu} = X^\nu_{,\mu}$$

The connection Γ is given in terms of the metric by

$$\Gamma^\mu_{\nu\sigma} = \frac{1}{2}\left\{\frac{\partial g_{\rho\sigma}}{\partial x^\nu} + \frac{\partial g_{\nu\rho}}{\partial x^\sigma} - \frac{\partial g_{\nu\sigma}}{\partial x^\rho}\right\} g^{\rho\mu}$$

The Ricci Tensor is defined by

$$R_{\mu\nu} = -\Gamma^{\alpha}_{\alpha\mu,\nu} + \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\rho\mu}\Gamma^{\rho}_{\alpha\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{\rho}_{\alpha\rho}$$

Einstein's Equation

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = +8\pi G_N T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$$R = R^{\mu}_{\mu} = R_{\mu\nu} g^{\mu\nu}$$ – Ricci scala

$$T_{\mu\nu} = P g_{\mu\nu} + (P+\rho) U_{\mu} U_{\nu}$$ – Energy Momentum Tensor

P = pressure
ρ = energy density
$U_{\mu} = (1, 0, 0, 0)$ velocity vector for isotropic fluid
Λ = cosmological constant

For the Robertson Walker Metric the non-vanishing connections are

$$\Gamma^0_{11} = \frac{R\dot{R}}{1-kr^2}, \quad \Gamma^0_{22} = R\dot{R}r^2, \quad \Gamma^0_{33} = R\dot{R}r^2\sin^2\theta$$

$$\Gamma^1_{11} = \frac{kr}{1-kr^2}$$

$$\Gamma^1_{01} = \Gamma^2_{02} = \Gamma^3_{03} = \frac{\dot{R}}{R}$$

$$\Gamma^1_{22} = -(1-kr^2)r \; ; \quad \Gamma^1_{33} = -(1-kr^2)r\sin^2\theta$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} \; ; \quad \Gamma^2_{33} = -\sin\theta\cos\theta$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \; ; \quad \Gamma^3_{23} = \Gamma^3_{32} = \frac{\cos\theta}{\sin\theta}$$

The non-vanishing Ricci coefficients are

$$R_{00} = \frac{-3\ddot{R}}{R} \qquad\qquad R^0_0 = +\frac{3\ddot{R}}{R}$$

$$R_{11} = +\left(2\dot{R}^2 + R\ddot{R} + 2k\right)\frac{1}{1-kr^2} \; ; \quad R^1_1 = +\left(\frac{2\dot{R}^2}{R^2} + \frac{\ddot{R}}{R} + \frac{2k}{R^2}\right)$$

$$R_{22} = R_{11}(1-kr^2)r^2 \qquad\qquad = R^2_2 = R^3_3$$

$$R_{33} = R_{11}(1-kr^2)r^2\sin^2\theta$$

$$R = R^{\mu}_{\mu} = +\frac{6\ddot{R}}{R} + \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2}$$

<u>00</u> term from field eqs.

$$R_{00} - \tfrac{1}{2} g_{00} R = +8\pi G_N T_{00} \mp \Lambda g_{00}$$

or $$\frac{3\ddot{R}}{R} + \frac{1}{2}\left(-\frac{6\ddot{R}}{R} - \frac{6\dot{R}^2}{R^2} - \frac{6k}{R^2}\right) = -8\pi G_N \rho - \Lambda$$

$$\boxed{\frac{\dot{R}^2}{R^2} = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}} = H^2$$

<u>11, 22 and 33</u> terms all give

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \Lambda - 8\pi G_N p$$

or substituting for $\left(\frac{\dot{R}}{R}\right)^2$

$$\boxed{\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3p)}$$

Energy Conservation

$$T^{\mu\nu}{}_{;\nu} = 0$$

$$T^{\mu\nu}{}_{;\nu} = T^{\mu\nu}{}_{,\nu} + \Gamma^{\mu}_{\nu\rho} T^{\nu\rho} + \Gamma^{\nu}_{\nu\rho} T^{\mu\rho} = 0$$

$\mu = 0$ gives

$$\dot{\rho} + \frac{3\dot{R}}{R} p + \frac{3\dot{R}}{R} \rho = 0$$

$$\boxed{\dot{\rho} = -\frac{3\dot{R}}{R}(\rho + P)}$$

k determines geometry

$k > 0$ closed Universe
$k = 0$ spatially flat
$k < 0$ open

discuss geometry of space-time

proper distance at $dt = d\theta = d\varphi = 0$

$$dl = \frac{a\,dr}{\sqrt{1-kr^2}} \qquad (ds^2 = dt^2 - dl^2$$

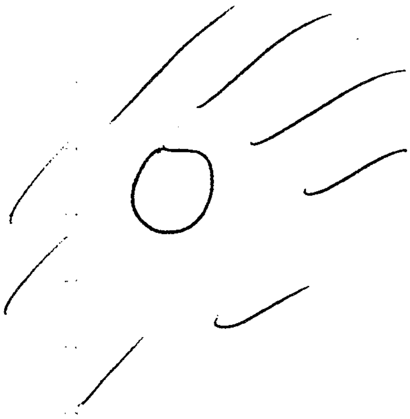
from here (the milky way = origin)
to short distances $kr^2 \ll 1$

$$dl = a\,dr \Rightarrow l(t) = a(t)\,r.$$

$r =$ comoving coordinate ie position of other galaxy
fixed

$$v = \dot{l} = \dot{a}r = \frac{\dot{a}}{a}l$$

$$\Rightarrow \quad H = \frac{\dot{a}}{a}$$



Newtonian Matter

$$\frac{d^2 l}{dt^2} = -\frac{GM}{l^2}$$

$$\dot{l}\,\ddot{l} = \frac{d}{dt}\frac{\dot{l}^2}{2} = -\frac{GM\dot{l}}{l^2} = \frac{d}{dt}\left(\frac{GM}{l}\right)$$

$$\Rightarrow \quad \dot{l}^2 = \frac{2GM}{l} + K$$

if we write $\quad l = l_0 R(t)$

$$\dot{l} = l_0 \dot{R}$$

$$\frac{\dot{R}^2}{R^2} = \frac{2GM}{l_0^3 R^3} + \frac{K}{l^2}$$

$$= \frac{8\pi G\rho}{3} + \frac{K}{l_0^2 R^2} \qquad k = \frac{-K}{l_0^2}$$

$$\frac{d}{dt}M = -p\frac{dV}{dt} \qquad V = \frac{4\pi}{3}\ell^3$$

$$= -p(4\pi\ell^2)\dot{\ell} \qquad M = \frac{4\pi}{3}\rho\ell^3$$

$$\frac{4\pi}{3}\dot{\rho}\ell^3 + 4\pi\rho\ell^2\dot{\ell} + 4\pi p\ell^2\dot{\ell} = 0$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{R}}{R} = 0$$

$$\frac{\ddot{R}}{R} = -4\pi G(\rho + P) + \underbrace{\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}}_{+\frac{8\pi G\rho}{3}}$$

$$\boxed{\frac{\ddot{R}}{R} = \frac{-4\pi G}{3}(\rho + 3P) \quad \left(+\frac{\Lambda}{3}\right)}$$

also if P is not neglected

G.R.

$$\ddot{\ell} = -\frac{4\pi G}{3}\overline{(\rho + 3P)}\,\ell$$

$$\downarrow$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P)$$

use $3P = -\left[\dot{\rho} + 3\rho\frac{\dot{R}}{R}\right]\frac{R}{\dot{R}}$

$$-\frac{\ddot{R}}{R} = \frac{4\pi G}{3}\left[\rho - \frac{\dot{\rho}R}{\dot{R}} - 3\rho\right]$$

$$\ddot{R}\dot{R} = \frac{4\pi G}{3}\left[2\rho\dot{R}R + \dot{\rho}R^2\right]$$

$$\frac{d}{dt}\left(\dot{R}^2/2\right) = \frac{4\pi G}{3}\frac{d}{dt}\left(\rho R^2\right)$$

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{K}{R^2}$$

Hubble $h_0 = .5$

$\rho_c \sim 5 \times 10^{-30}\ g\ cm^{-3}$

$\frac{2 \times 10^{-29}}{4}$

$\Lambda = 0$

$k = 0$

$H^2 = \frac{\dot{R}^2}{R^2} = \frac{8\pi G \rho}{3}$

defines ρ_c

$\rho_c = \frac{3H^2}{8\pi G}$

$= 1.88 \times 10^{-29}\ g\ cm^{-3}\ h^2$

1) recall geometry and k

$$\text{"}E\text{"} = \frac{-k}{R^2} = \left(\frac{\dot{R}}{a}\right)^2 - \frac{8\pi G \rho}{3} + \left(\frac{\Lambda}{3}\right)$$

$$K + V + V'$$

$k > 0$, $E < 0$ bound system. closed.

$$V > K \qquad V < 0$$

$k < 0$ $E > 0$ free system open

$$K > V$$

$k = 0$ $E = 0$ critical

$$K = V$$

Standard Models

1) Radiation Dominated $\quad k=0, \; p=\rho/3, \; \Lambda=0$

$$\dot{\rho} = -\frac{4\dot{R}}{R}\rho \quad \text{or} \quad \rho \sim R^{-4}$$

$$\frac{\dot{R}^2}{R^2} \sim \frac{1}{R^4} \Rightarrow R\dot{R} \sim \text{const} \quad \text{or} \quad \boxed{R \sim t^{1/2}}$$

using both

$$\frac{\dot{\rho}}{\rho} = -4\left(\frac{8\pi G\rho}{3}\right)^{1/2}$$

$$\boxed{t = \left(\frac{3}{32\pi G\rho}\right)^{1/2} + \text{constant.}}$$

) Matter dominated $\quad k=0 \quad p=0 \quad \Lambda=0$

$$\dot{\rho} = -\frac{3\dot{R}}{R}\rho \qquad \rho \sim R^{-3}$$

$$\frac{\dot{R}^2}{R^2} \sim \frac{1}{R^3} \Rightarrow R^{1/2}\dot{R} \sim \text{const} \quad \text{or} \quad \boxed{R \sim t^{2/3}}$$

$k=0=p=\rho=\Lambda \;=\;$ Minkowski

$$ds^2 = dt^2 - d\vec{x}^2$$

Models with a Cosmological Constant.

Define $\quad Q = \dfrac{3k}{R^2} - 8\pi G_N \rho$

$$\frac{\dot{R}}{R} = \pm \left[\frac{\Lambda - Q}{3} \right]^{1/2} \Rightarrow \quad Q \leq \Lambda$$

$p = (\gamma - 1)\rho$ — equation of state

$$\Rightarrow \quad \rho \sim R^{-3\gamma}$$

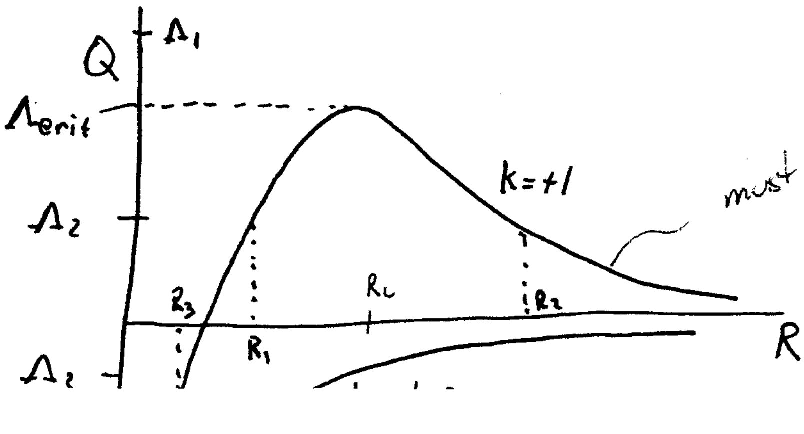
$$\frac{dQ}{dR} = \frac{-6k}{R^3} + \frac{24\pi G_N A \gamma}{R^{3\gamma+1}} = 0$$

if $k = -1, 0$ no extrema for Q

if $k = +1$ extrema $Q = 4\pi G (3\gamma - 2)\rho = \frac{4}{9} \frac{1}{H_0^4 R_0^6 \Omega_0^2}$

$$\frac{d^2Q}{dR^2} = \frac{24\pi G A}{R^{3\gamma+2}} \left[2 - 3\gamma \right] \gamma < 0 \quad \text{for} \quad 1 \leq \gamma \leq 2$$

so is a max.

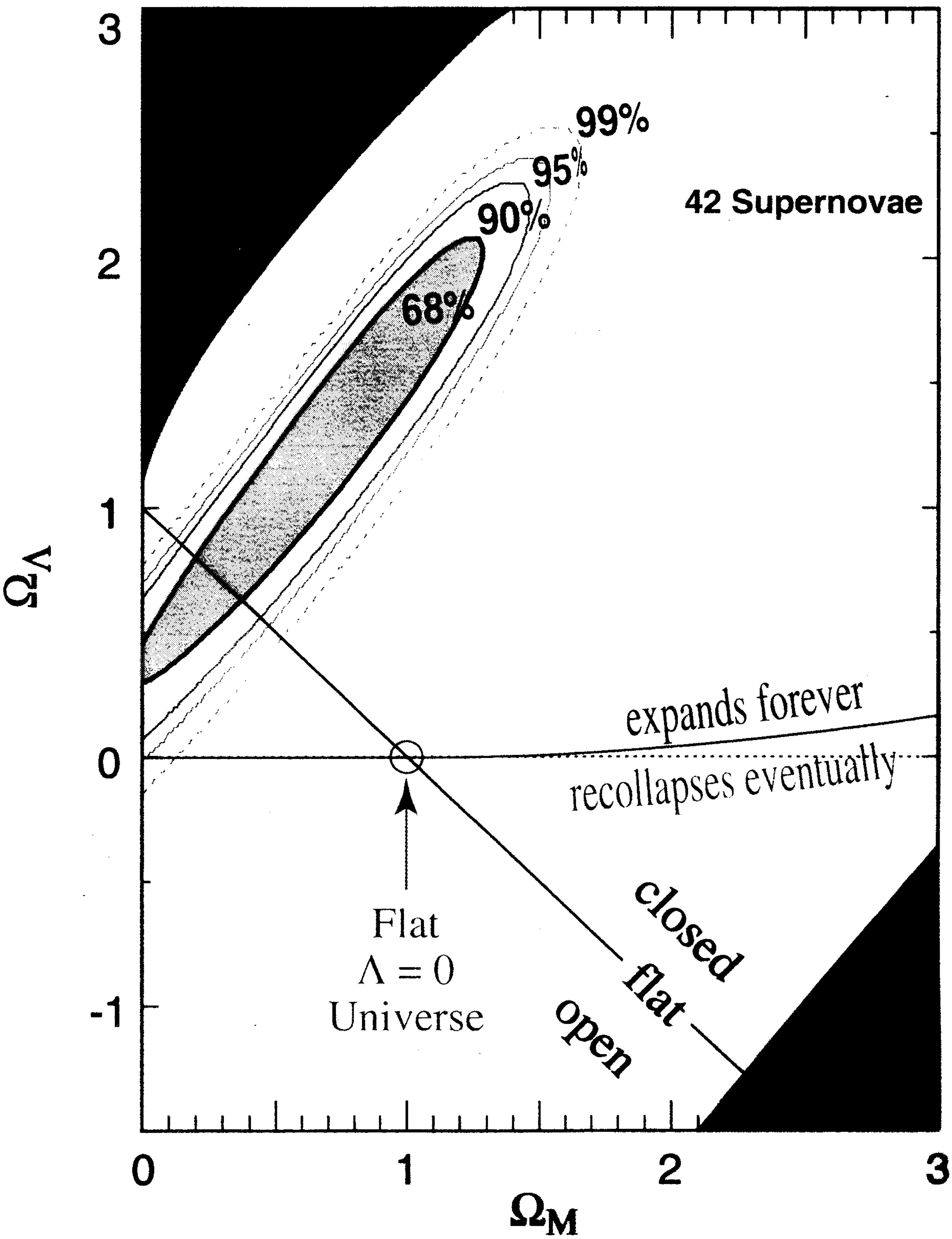


must have

$$\Omega_\Lambda > \frac{4}{27} \frac{(\Omega_0 + \Omega_\Lambda - 1)^3}{\Omega_0^2}$$

uses $\frac{k}{H_0^2 R_0^2} = (\Omega_0 + \Omega_\Lambda - 1)^2$

Supernova Cosmology Project
Perlmutter *et al.* (1998)



Ap.J.
astro-ph/9812133

$k=+1$

$\Lambda = \Lambda_1 > Q_{max}$ two solutions

1) start with singularity at $R=0$
expand to infinity ; $\frac{\ddot{R}}{R}$ has min when $Q = Q_{max}$

2) start at $R=\infty$ and contract to singularity

$\Lambda = \Lambda_{crit} = Q_{max}$ five solutions

1) Einstein Static $R=$constant $Q = Q_{max} = \Lambda_{crit}$
2) start with singularity at $R=0$ and asymptotically approach Einstein Static
3) start with (very nearly) Einstein Static and collapse to singularity at $R=0$
4) start with (very nearly) Einstein Static and expand to $R=\infty$
5) start at $R=\infty$ and collapse to ~~...~~ an asymptotically Einstein Static Universe.

$\Lambda = \Lambda_2 \geq 0$ two solutions

1) start at singularity and expand until $Q=\Lambda_2$ at R_1 and bounce back to singularity.
2) start at $R=\infty$ and collapse until $Q=\Lambda_2$ at R_2 and bounce back to $R=\infty$

$k=-1, 0$

$\underline{\Lambda \geq 0}$ two solutions

1) start at $R=0$ and expand to $R=\infty$

(our Universe appears to be of this type with $\Lambda=0$).

2) start at $R=\infty$ at collapse to singularity.

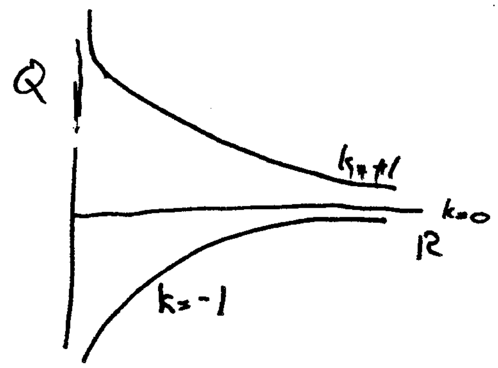
$\Lambda < 0$ one solution.

1) same as $\Lambda=\Lambda_3$ for $k=+1$.

A Universe with no matter

$\rho=0$

$$Q = \frac{3k}{R^2}$$



$k=+1$

$\underline{\Lambda > 0}$ solutions

1) same as solution 2) for $\Lambda=\Lambda_2$ for $k=+1$

$\underline{\Lambda \leq 0}$ forbidden

$k=0$

$\underline{\Lambda > 0}$ - De Sitter Space

$k=-1$ same as with matter

The Universe Today

$\Lambda \approx 0$

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{-k}{R^2} + \frac{8\pi G}{3}\rho$$

Define ρ_c the critical density such that $\rho = \rho_c$ for $k=0$

$$\rho_c = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29}\ \mathrm{g\,cm^{-3}}\ h_0^2$$

$$h_0 = \frac{H_0}{100\ \mathrm{km\,Mpc^{-1}\,s^{-1}}} \qquad \boxed{\Omega \equiv \rho/\rho_c}$$

$$q_0 \equiv -\frac{R\ddot{R}}{\dot{R}^2} \Rightarrow q_0 H_0^2 = +\frac{4\pi G_N}{3}(\rho + 3p) - \frac{\Lambda}{3}$$

$$\text{or } 2q_0 = +(3\gamma - 2)\Omega - 2\Omega_\Lambda$$

For $k \neq 0$ ~~...~~ we have

$$\frac{k}{R_0^2} = (2q_0 - 1)H_0^2 \quad \text{or} \quad \frac{8\pi G \rho_0 R_0^2}{3} = \frac{2q_0}{|2q_0 - 1|} \gtrsim 0.03 \text{ today}$$

$$(\Omega + \Omega_\Lambda - 1)H^2$$

Evolution of Ω

$\Lambda = 0$

$$\Omega = \frac{k}{R^2 H^2} + 1$$

if $k = 0$ $\quad \Omega = 1$ always.

$$R^2 H^2 = \frac{8\pi G A}{3 R^{3\gamma - 2}} - k$$

$$\Omega = \frac{k}{\frac{8\pi G A}{3 R^{3\gamma-2}} - k} + 1$$

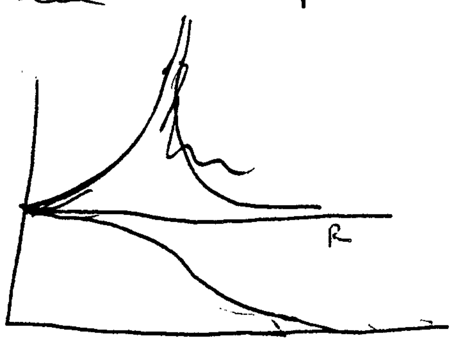
as $R \to 0$ then

$$\Omega \simeq \frac{3R^{3\gamma-2}}{8\pi G A} + 1 \to \underline{1}$$

as $R \to R_{max}$, ∞

$k = +1 \qquad \Omega \to \infty$ then $\to 1$

$k = -1 \qquad \Omega \to 0$



Age of the Universe.

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2}$$

$$\left(\frac{\dot{R}}{R_0}\right)^2 = \frac{8\pi G}{3}\rho_0\left(\frac{R_0}{R}\right)^{3\gamma-2} - \frac{k}{R_0^2} \quad + \frac{\Lambda R^2}{3R_0^2} \quad \rho_c = \frac{8\pi G}{3H^2}$$

$$\boxed{\rho = \rho_0\left(\frac{R_0}{R}\right)^{3\gamma}}$$

$$\frac{k}{R_0^2} = (\Omega_0 - 1)H_0^2 \qquad \Omega_0 = \frac{\rho_0}{\rho_c} = \frac{8\pi G \rho_0}{3H^2}$$

$$x = \frac{R}{R_0} \qquad \dot{x} = \frac{\dot{R}}{R_0}$$

$$\dot{x}^2 = \Omega_0 H_0^2 \left(\frac{R_0}{R}\right)^{3\gamma-2} - (\Omega_0 - 1)H_0^2 \quad + H^2\Omega_\Lambda x^2$$

$$\dot{x}^2 = \Omega_0 H_0^2 x^{-(3\gamma-2)} - (\Omega_0 - 1 \overset{+\Omega_\Lambda}{}) H_0^2 \quad + H_0^2\Omega_\Lambda x^2$$

$$\dot{x} = H_0\left[1 - \Omega_0 \overset{-\Omega_\Lambda}{} + \Omega_0 x^{2-3\gamma} \underset{+\Omega_\Lambda x^2}{}\right]^{1/2}$$

$$\boxed{H_0 t = \int_0^1 \frac{dx}{\left[1 - \Omega_0 + \Omega_0 x^{2-3\gamma}\right]^{1/2}}}$$

if $\Omega_0 = 1$

$$H_0 t = \int_0^1 \frac{dx}{\underset{\substack{\shortparallel \\ 1}}{\Omega^{1/2}} x^{1-\frac{3\gamma}{2}}}$$

for $\gamma = 1$ $\qquad H_0 t = \int_0^1 \frac{dx}{\cancel{}} x^{1/2}$

$$= \frac{2}{3} x^{3/2} = \frac{2}{3}$$

$H = \frac{2}{3t}$

$$\boxed{t = \frac{2}{3H}}$$

at early times. $\quad (\Omega = 1) \quad \gamma = \frac{4}{3}$

$$H_0 t = \int_0^1 x\,dx = \frac{1}{2}$$

$$\boxed{t = \frac{1}{2H}}$$

Proper distance

$$d_p = a(t) \int_0^{r_1} \frac{dr'}{\sqrt{1-kr'^2}}$$

For light paths $ds^2 = 0$ so

$$\int_{t_1}^{t} \frac{dt'}{a(t')} = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}}$$

to receive signals at $r=0$ at t from $r=r_1$ at $t=t_1$

if we let $t_1 \to 0$ then r_1 is the max distance to which we can receive any signal.

$\Rightarrow$ particle horizon

$$d_H = a(t) \int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}} = a(t) \int_0^{t_0} \frac{dt'}{a(t')}$$

For $\gamma = 4/3$ radiation.

$$d_H = a_0 t_0^{1/2} \int_0^{t_0} \frac{dt'}{a_0 t'^{1/2}} = 2t_0$$

$\gamma = 1$ Matter

$$d_H = 3t_0$$

$d_H(t)$ grows faster than $a(t)$

Thus as universe expands
more and more of the Universe
becomes visible,

$d_H = 3ct_U$ $\qquad t_U = 15\times10^9$ yrs
5×10^{17}

$= 4\times10^{28}$ cm
$= 1.4\times10^{10}$ pc $= 1.4\times10^4$ Mpc.

if $\rho = 1.88\times10^{-29}\, h_0^2$ gm cm^{-3}

with $h_0 = 1/2$ $\qquad \mathcal{M} \simeq 10^{57}$ gm. ✓
$= 6\times10^{23}\, M_\odot$ ✓

$M_G = 10^{12}\, M_\odot$

$\frac{2}{4}$ $\qquad$ $\frac{2.28 \cdot 3}{84}$

5×10^{-30} gm^{-3} $4\, (4\times10^{28})^3$ cm^3
$25\cdot5\times10^{-30}\times10^{84} = 10^3\, 10^{-30}\, 10^{84}$
10^{54}

Red Shift

Consider two observers 1, 2 and a light signal is emitted by 1 and received by 2.

the red shift $$z \equiv \frac{\nu_1 - \nu_2}{\nu_2} = \frac{v_{rel}}{c}.$$

for nearby observers.

But $v_{rel} = \dot{R}\,\delta r$ where δr is the coordinate separation of 1 and 2.

so $$v_{rel} = \frac{\dot{R}}{R} R\,\delta r = H\,\delta t \quad \text{for light signals.}$$
$$= Hd$$

where d is the physical separation

Finally $$1+z = \frac{\nu_1}{\nu_2} = 1 + \frac{\dot{R}\,\delta t}{R} = \frac{R_2 - R_1}{R_1} + 1 = \frac{R_2}{R_1}$$

H_0 is determined by measurements of z and d

$$50 \text{ km Mpc}^{-1} \text{ s}^{-1} \lesssim H_0 \lesssim 100 \text{ km Mpc}^{-1} \text{ s}^{-1}$$

Angular Size of a distant galaxy.

proper diameter $D = R(t_e)\, r\, \delta = l\delta$

take $k=0$ $\qquad r = \int_{t_e}^{t_o} \frac{dt}{R(t)}$

$$l = R(t_e)\int_{t_e}^{t_o} \frac{dt}{R(t)} \qquad\qquad R(t) = R_o t^{2/3}$$

$$= t_e^{2/3}\left[3\left(t_o^{1/3} - t_e^{1/3}\right)\right]$$

$$= 3 t_e^{2/3} t_o^{1/3}\left[1 - \left(t_e/t_o\right)^{1/3}\right]. \qquad \frac{R(t_e)}{R(t_o)} = \left(\frac{t_e}{t_o}\right)^{2/3} = \frac{1}{(1+z)}$$

$$= 3 t_e^{2/3} t_o^{1/3}\left[1 - (1+z)^{-1/2}\right]$$

$$= 3 t_o \frac{t_e^{2/3}}{t_o^{2/3}}\left[1 - (1+z)^{-1/2}\right]$$

$\boxed{k=0}$

$$= \frac{2}{H_o(1+z)}\left[1 - (1+z)^{-1/2}\right]. = r_A$$

$D = r_A \delta$

see next page

~~if L = total (absolute) luminosity. of source.~~

~~$\Rightarrow$ ~~apparent luminosity~~ energy flux observed~~

~~$$f = \frac{L}{4\pi R_o^2 r^2 (1+z)^2}$$~~

$$\ell = 3 t_e^{2/3} t_0^{1/3}\left[1 - (1+z)^{-1/2}\right]$$

$$= 3t_0 \left(\frac{t_e^{2/3}}{t_0^{2/3}}\right)\left[1-(1+z)^{-1/2}\right]$$

$$= \frac{2}{H(1+z)}\left[1-(1+z)^{-1/2}\right]$$

galaxies at high redshift sit on us

$t_0 = \frac{2}{3H}$

$d = \ell\theta$ so

$$\frac{HD(1+z)}{2\left(1-\frac{1}{\sqrt{1+z}}\right)}$$

$$\theta = Hd(1+z)/2\left(1-(1+z)^{-1/2}\right)$$

for $\Omega = 1$

small z

$$\frac{HD(1+z)}{2(1-(1-\frac{z}{2}))} = \frac{HD(1+z)}{z} \sim \frac{HD}{z}$$

at large z $\theta \propto z$

For $d = 20\,\mathrm{kpc}$

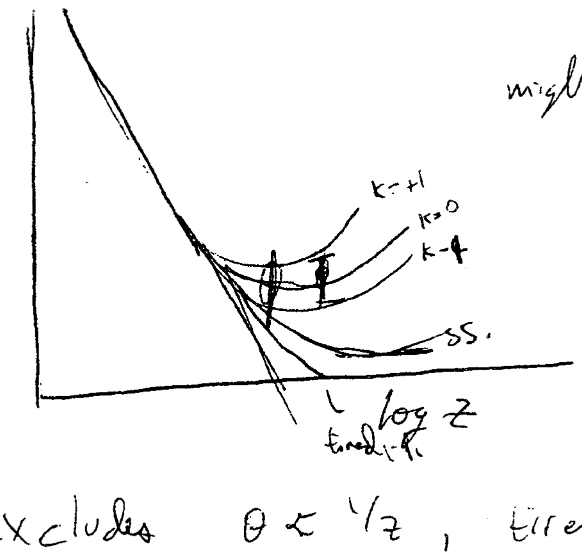
at $z = 5/4$ minimum $\theta = 4.6''\,h$

at $z = .46$ $\theta = 5.8''\,h$

$z = 3$ $\theta = 5.5''\,h$

at the time largest redshift for a galaxy now it is $z = 3.2$

might normally expect $\theta \propto \frac{1}{z}$



excludes $\theta \propto 1/z$, tired light, steady state

2. $D = R(t_e)\, r\, \delta = r_A\, \delta$ $\qquad \delta = D/r_A$

$$r = \int_{t_e}^{t_o} \frac{dt}{R(t)} \qquad \text{in steady state } R(t) = R_o\, e^{H(t-t_o)}$$

$$= \frac{1}{R_o} \int_{t_e}^{t_o} e^{-H(t-t_o)}\, dt = -\frac{1}{HR_o}\, e^{-H(t-t_o)} \Big|_{t_e}^{t_o}$$

$$= \frac{1}{HR_o}\left[e^{H(t_o-t_e)} - 1\right].$$

Now $\quad 1+z = \dfrac{R_o}{R(t_e)} = e^{H(t_o-t_e)}$

so $\quad r = \dfrac{1}{HR_o}\, z \qquad r_A = R(t_e)\, r = \dfrac{z}{H(1+z)}$

Finally $\quad \delta = D/r_A = \dfrac{HD(1+z)}{z}$

small z: $\quad \delta = HD/z$

large z: $\quad \delta \to HD$

The Density of the Universe

ρ_0 or Ω is found by ways of mass-to-light-ratios

$\rho = \left(\frac{M}{L}\right)\mathcal{L}$ - $\mathcal{L}$ is the total luminoscity density observed $\mathcal{L} \approx 2.32\times10^8 h_0 L_\odot Mpc^{-3}$

$L_\odot = 3.9\times10^{33} erg s^{-1}$

Define critical

$$\left(\frac{M}{L}\right)_c = \frac{\rho_c}{\mathcal{L}} \approx 1200 h_0$$

1) solar neighborhood

$$\frac{M}{L} \sim 2\pm1 \Rightarrow \Omega \approx (.0016 \pm .0008)/h_0$$

2) central parts of galaxies

$$\frac{M}{L} \sim (10-20)h_0 \Rightarrow \Omega \sim .008 - .017$$

3) binaries and small groups of galaxies

$$\frac{M}{L} \sim (60-180)h_0 \Rightarrow \Omega \sim .05 - .15$$

4) clusters of galaxies $(300-1000)h_0$ $.2 - .8$

Mass to Light Ratios

Luminosity density of the night sky

$$\mathcal{L} = 2\times10^{8}\, h_0\, L_\odot\, Mpc^{-3}$$

$L_\odot = 4\times10^{33}\, erg\, s^{-1}$

$$\rho = \left(\frac{M}{L}\right)\mathcal{L}$$

$\frac{L}{\mathcal{L}}$ like a volume

$$\left(\frac{M}{L}\right)_c = \rho_c/\mathcal{L} \approx 1200\, h_0$$

$$\Omega = (M/L)/(M/L)_c$$

$$M \approx \frac{v^2 d}{G}$$

$$d = \left(\frac{z}{H}\right)\delta\theta$$

$$L = 4\pi \underbrace{l}_{\text{apparent luminosity}} r^2 = 4\pi l\left(\frac{z^2}{H^2}\right)$$

The Radiation Content

The microwave background radiation is a relic of a hot phase in the early Universe.

The photon were produced when the temperature fell below $\sim 10^4\,°K$ when electrons + protons recombined to form neutral Hydrogen.

Today $T_0 \approx 2.7\,°K$ – temperature of the blackbody spectrum.

in an adiabatically expanding Universe

$$R \sim 1/T$$

so that $1 + z = T/T_0$

$$\rho_\gamma = \int \mathcal{E}_\gamma \, dn_\gamma$$

$$dn_\gamma = \frac{g_\gamma}{2\pi^2}\left[e^{\mathcal{E}/kT} - 1\right]^{-1} q^2 dq$$

$\mathcal{E}_\gamma = q$ (in general, will have

$g_\gamma = 2$ $\mathcal{E}_i = (q_i^2 + m_i^2)^{1/2}$

$\rho_\gamma = \frac{\pi^2}{15} T^4$ $(\hbar = c = k = 1)$

$$n_\gamma = \int dn_\gamma$$

$$= \frac{1}{\pi^2} \cdot 2\zeta(3)\, T^3 \quad = .244\, T^3$$

for $T = 2.7$ K $\quad \boxed{n_\gamma = 400 \text{ cm}^{-3}}$

deviations from Planck.

In general, whenever $T \sim m_i$ particle type i joins the background

$$\rho_i = \int \varepsilon_i \, dn_{q_i}$$

$$dn_{q_i} = \frac{g_i}{2\pi^2}\left[e^{(\varepsilon_{q_i}-\mu_i)/kT} \pm 1\right]^{-1} q^2 \, dq$$

$$\varepsilon_{q_i} = (m_i^2 + q_i^2)^{1/2}$$

entropy density

$$S_i = \frac{1}{T}\left[\int \varepsilon_{q_i} dn_{q_i} \mp \frac{kT}{(2\pi)^3}\int g_i \ln(1 \mp n_{q_i})\, d^3 q_i\right.$$

free energy density

$$F = \rho - Ts = \mu n - P$$

pressure

μ - is the chemical potential, $\mu_i \to -\mu_i$ for antiparticles

↳ usually taken for baryon/lepton number

but

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = \frac{n_B}{n_\gamma} \approx (2-5)\times 10^{-10}$$

Let $\rho_B = \rho_c = 1.05 \times 10^{-5} \frac{GeV}{cm^3} h_0^2$

$m_N n_N$

$n_N = 1.1 \times 10^{-5} \frac{}{cm^3} h_0^2$

$n_\gamma = 400 \ cm^{-3}$

$\frac{n_N}{n_\gamma} = 2.8 \times 10^{-8} h_0^2$

$d_H = 3ct$ $t_0 = \cancel{1.5} \times 10^9 \ yr$

$= 4 \times 10^{28} \ cm.$

$= 1.4 \times 10^4 \ Mpc$

$\frac{4\pi d_H^3}{3} = 1.15 \times 10^{13} \ Mpc^3 = 3.3 \times 10^{86} \ cm^3$

if $\rho = \rho_c$ $\Rightarrow m = 6.2 \times 10^{57} \ gm.$

$\Rightarrow 3 \times 10^{24} \ M_\odot$

large galaxy $M = 10^{12} M_\odot$

$\Rightarrow N_\gamma \simeq 10^{89} \Rightarrow N_B = 3.7 \times 10^{81}$

In the limit $T \gg m_i$

$$\rho = \left(\sum_B g_B + \frac{7}{8}\sum g_F\right)\frac{\pi^2}{30}T^4 \equiv \frac{\pi^2}{30}N(T)\,T^4$$

$$P = \frac{1}{3}\rho = \frac{\pi^2}{90}N(T)\,T^4$$

$$S = \frac{(\rho + P)}{T} = \frac{4}{3}\rho = \frac{2\pi^2}{45}N(T)\,T^3$$

thus for a radiation dominated Universe

recall

$$t = \left(\frac{3}{32\pi G\rho}\right)^{1/2} = \left(\frac{90}{32\pi^3 G N(T)}\right)^{1/2} T^{-2}$$

$$t\,T_{MeV}^2 = 2.41\,[N(T)]^{-1/2}$$

$N(T)$

		$N(T)$
$T < m_e$	only γ's + ν's	$2 + \frac{21}{4} = \frac{29}{4}$
$m_e < T < m_\mu$	add $e^\pm$s	$\frac{29}{4} + \frac{7}{2} = \frac{43}{4}$
$m_\mu < T < m_\pi$	add $\mu^\pm$'s	$\frac{43}{4} + \frac{7}{2} = \frac{57}{4}$
$m_\pi < T < T_c$ *	add π's	$\frac{57}{4} + 3 = \frac{69}{4}$
$T_c < T < m_s$	γ's + ν's, $e^\pm$ $\mu^\pm$, $u\bar{u}$ $d\bar{d}$ + gluons	$\frac{57}{4} + 16 + \frac{21}{2} + \frac{21}{2} = \frac{205}{4}$
$m_s < T < m_c$	add $s\bar{s}$	$\frac{205}{4} + \frac{21}{2} = \frac{247}{4}$
$m_c < T < m_\tau$	add $c\bar{c}$	$\frac{247}{4} + \frac{21}{2} = \frac{289}{4}$
$m_\tau < T < m_b$	add $\tau^\pm$	$\frac{289}{4} + \frac{7}{2} = \frac{303}{4}$
$m_b < T < m_t$	add $b\bar{b}$	$\frac{303}{4} + \frac{21}{2} = \frac{345}{4}$
$m_t < T < m_W$	add $t\bar{t}$	$\frac{345}{4} + \frac{21}{2} = \frac{387}{4}$

higher temperatures
need to add 9 for $W^\pm Z$
for minimal SU(5): 36 for X and Y
19 for the $\underset{\sim}{24} + \underset{\sim}{5}$

(normally $\underset{\sim}{24} + \underset{\sim}{5}$ yields 34 but 15 states were counted with the X, Y, W, Z.)

total for minimal SU(5) $N(T) = 160.75$
$T > M_X$

* quark – Ha[illegible]

Equilibrium

Particles will be said to be in equilibrium if there is a reaction whose rate is large enough.

If the Universe were not expanding eventually all particle species would come into equilibrium with each other.

For an expanding Universe we must compare the reaction rate Γ with the expansion rate or H.

Consider neutrino interactions,

$$e^+ + e^- \Leftrightarrow \nu + \bar{\nu} \quad \text{or} \quad e + \nu \Leftrightarrow e + \nu$$

the rate for these processes

$$\Gamma = \sigma n v$$

$$\sigma \sim \frac{10^{-2} T^2}{M_W^4} \qquad n \sim T^3 \qquad v \sim 1$$

$$\Gamma \sim \frac{10^{-2} T^5}{M_W^4}$$

$$H \sim \frac{30 T^2}{m} \quad \text{for} \quad T \lesssim m.$$

Recall the equation for energy conservation

$$\dot{\rho} = -\frac{3\dot{R}}{R}(\rho + p)$$

this is equivalent to

$$\dot{p}\, R^3 = \frac{d}{dt}\left(R^3(\rho + p)\right)$$

$$= \frac{d}{dt}\left(R^3 T s\right)$$

$$\dot{p} = \frac{dp}{dT}\frac{dT}{dt} = s\frac{dT}{dt}$$

so we have

$$s\frac{dT}{dt}R^3 = \frac{d}{dt}(R^3 Ts) = R^3 s\frac{dT}{dt} + T\frac{d}{dt}(R^3 s)$$

$$\Rightarrow \quad \frac{d}{dt}(R^3 s) = 0$$

entropy conservation.

Neutrinos are said to <u>decouple</u> or the weak interaction rates <u>freeze out</u> when

$$\frac{10^{-2}\,T_d^5}{M_W^4} = \frac{30\,T_d^2}{M_P}$$

$$\frac{3\times10^{3}\times10^{8}}{10^{19}} \sim 10^{-8} \qquad 30\times10^{-9}$$

$$10^{-8/3} \qquad (30)^{1/3}\times10^{-3}$$

for $M_W \sim 100\ \mathrm{GeV}$

$$\underline{T_d \sim \overset{\text{few}}{1}\ \mathrm{MeV}}$$

for $T < T_d$ neutrinos are no longer in equilibrium with the rest of the Universe

they will simply cool according to $T \sim 1/R$.

when $T < m_e$

$e^{\pm}$ pairs annihilate but can not dump the energy (or entropy) into neutrinos only photons

Because of entropy conservation photons get heated after $e^{\pm}$ annihilation

entropy of "γ"'s + ν's conserved separately for $T < T_d$

for $T \gtrsim m_e$ $\quad s_> = \frac{4}{3}\frac{\rho_>}{T} = \frac{4}{3}\left(\frac{11}{4}\right)T_>^3$

for $T \lesssim m_e$ $\quad s_< = \frac{4}{3}\frac{\rho_<}{T} = \frac{4}{3}T_<^3$

but $s_> = s_< \Rightarrow \left(\frac{T_<}{T_>}\right)^3 = \frac{11}{4}$

ie $T_<$ (after $e^{\pm}$ ann.) $= \left(\frac{11}{4}\right)^{1/3} T_>$ (before $e^{\pm}$ ann.)

and $T_\nu = T_>$ so that

today $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma \approx 1.9\ ^\circ K.$

Big Bang Baryosynthesis

Goal: To calculate η from the microphysics as Y was calculated in BBN.

The problem:

If the Universe were baryon symmetric the baryon and antibaryon density of the Universe would be determined by the freeze out of annihilations and

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma}$$

For $T \gg m_N$ $\quad \frac{n_B}{n_\gamma} \sim \mathcal{O}(1)$

For $T < m_N$ $\quad \frac{n_B}{n_\gamma} \sim \left(\frac{m_N}{T}\right)^{3/2} e^{-m_N/T}$

If the Universe were not expanding but just cooling $\frac{n_B}{n_\gamma} \to 0$.

With expansion need to compute freeze out as was the case for massive neutrinos

Again freeze out condition

$$\Gamma_A = H$$

the stronger the interaction the lower the freeze out temperature and hence a smaller abundance of particles remaining.

For nucleons, the final number would be

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \approx 10^{-19}$$

But $\eta = \frac{n_B}{n_\gamma} = (3-6)\times 10^{-10}$

nearly 10 orders of magnitude discrepency.

Furthermore no antimatter observed

in cosmic rays $\exists$ $\bar{p}$'s but generally assumed to be secondaries

no observation of $\bar{\alpha}$

Also how do we separate matter and antimatter?

statistical fluctuations $\Rightarrow \frac{n_B}{n_\gamma} \sim 10^{-40}$

causal process could only produce regions $\sim 10^{-5} M_\odot$

need $\gtrsim 10^{12} M_\odot$

$\cdot 1$ GeV

$\frac{1000}{50} = 20$ MeV

$\sigma v \sim \frac{1}{m_\pi^2}$

$\frac{M_p m_N}{m_\pi^2} = 10^{21}$ $\quad x \sim \frac{1}{50}$

$\frac{g_N}{(2\pi)^{3/2}} \langle \sigma v \rangle m^{3/2} T^{3/2} e^{-m/T} \simeq \frac{T^2}{M_p} \sqrt{N} \left(\frac{8\pi}{3}\right)^{1/2}$

$N = \frac{43}{4}$

$\frac{4}{(2\pi)^{3/2}} \frac{m_N^3}{(.14)^2} x^{3/2} e^{-1/x} = 4.5 \frac{m_N^2}{M_p} x^2$

$1.6 \times 10^{19} x^{-1/2} e^{-1/x} \simeq 1$

$x \simeq \frac{1}{43}$!

$\frac{n_B}{n_\gamma} = \frac{\frac{4}{(2\pi)^{3/2}} m^{3/2} T^{3/2} e^{-m/T}}{\frac{2\zeta(3)}{\pi^2} T^3} = \sqrt{\frac{\pi}{2}} \frac{1}{\zeta(3)} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$

$\sim \underline{10^{-19}}$!

If the Universe is

1) the asymmetry was an initial condition,

2) it was generated.

---

2) To generate an asymmetry, need 3 ingredients

a) Baryon number violating interactions

b) C and CP violation

c) Departure from thermal Equilibrium.

---

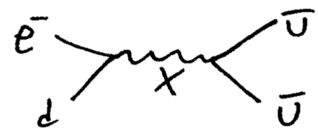
a) + b) contained is GUTs

c) from an expanding Universe

---

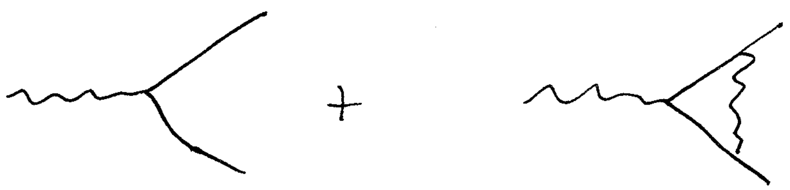
a) $\exists$ gauge + Higgs bosons whose interactions violate B

$$X \to \bar{u}\,\bar{u}$$
$$ e^{-}d$$



etc.

b) C, CP violation through interference



c) Departure from equilibrium when interaction rate do not keep up with the expansion rate.

The decay rate

$$\Gamma_D \sim \alpha M_X$$

The superheavy bosons can only begin to decay when $\Gamma_D \geq H$ or when

$$T^2 \lesssim \alpha M_X M_p (N(T))^{-1/2}$$

M_p – Planck m $= 1.22 \times 10^{19}$ GeV

In equilibrium decays should be occurring when $T \sim M_X$ (i.e. the number of X's must begin to diminish and annihilation $\sim \alpha^2$ are not effective).

$\alpha^2 T^{?} > \alpha m_X^2$

$\frac{\alpha^2}{m_X^2} T^3$ $\alpha \frac{m_X^2}{T}$

αT^3 m_X^3

Thus equilibrium is maintained

f $M_X^2 \lesssim \alpha M_X M_p (N(T))^{-1/2}$

or $M_X \lesssim \alpha M_p (N(T))^{-1/2} \sim 10^{19} \alpha$ GeV

no baryon asymmetry is produced in this case.

For $M_X \gtrsim 10^{18} \alpha$ GeV

the X lifetime is longer than the age of the Universe when $T \sim M_X$ and X's only begin to decay later when $T < M_X$ but until then $\frac{n_X}{n_\gamma} \sim 1$ and should be in equilibrium $e^{-M/T} \ll 1$. Hence are baryon asymmetry is produced.

When a pair $X, \bar{X}$ decay:

$$X \xrightarrow[r]{} \bar{u}, \bar{u} \qquad \Delta B = -2/3$$

$$\xrightarrow[1-r]{} e^-, d \qquad \Delta B = +1/3$$

$$\bar{X} \xrightarrow[\bar{r}]{} u,\ u \qquad \Delta = +2/3$$
$$\xrightarrow[1-\bar{r}]{} e^+,\ \bar{d} \qquad \Delta = -1/3$$

Because C, CP are violated

$r \neq \bar{r}$

and
$$(\Delta B)_{total} = -(2/3)r + (1/3)(1-r) + 2/3\,\bar{r} - (1/3)(1-\bar{r})$$
$$= -r + 1/3 + \bar{r} - 1/3 = (\bar{r} - r).$$

$\bar{r} - r$ depends on the degree of CP violation.

call ΔB the total baryon number generated per $X, \bar{X}$ decay.

then
$$\frac{n_B}{n_\gamma} = \frac{(\Delta B)\, n_X}{n_\gamma} = \frac{(\Delta B)\, n_X}{N(T)\, n_\gamma} \approx \underline{10^{-2}\, \Delta B}$$

The maximal asymmetry produced in the Out of Equilibrium decay scenario.

To realistically calculate the baryon asymmetry, must go through a complete set of Boltzmann equations for each particle species of interest.

Define the number of particles in phase space element $dV\, d\pi_i$

$$dN_i = u_\alpha p_i^\alpha N_i(p_i^\mu, x_i^\mu)\, dV\, d\pi_i$$

where N_i is the phase space density

$$d\pi_i = \frac{1}{(2\pi)^3} g_i \frac{d^3 p_i}{p_i^0} \qquad u^\alpha = (1,0,0,0)$$

$$n_i = \int u_\alpha p_i^\alpha N_i\, d\pi_i = \frac{g_i}{2\pi^2} \int N_i\, p_i^2\, dp_i$$

Boltzmann eq.

$$\frac{\partial N_i}{\partial t} = \left(\frac{\dot{R}}{R}\right) p \frac{\partial N_i}{\partial p} + 1/p_i^0 \sum_{\substack{j \ldots \\ l, m \ldots}} \int d\pi_j \cdots \int d\pi_l \int d\pi_m$$

$$[N_l N_m \cdots (1 \pm N_i)(1 \pm N_j) \cdots W(p_l p_m \ldots \to p_i p_j \ldots)$$

W's are the invarient transition rate.

A full set set of these coupled differential equations must then be integrated (numerically)

Must also put in all interaction.

Decays
Annihilations
Scatterings.

Boson Decay $\Gamma_{N.R.} \sim \alpha M$ $\Gamma_R \sim \alpha M^2/T$

Annihilations ~~[illegible]~~ $\Gamma_A \sim \alpha^2 n/T^2 \sim \alpha^2 T$

Scatterings $\Gamma \sim \alpha^2 n T^2/(T^2+M^2)^2$

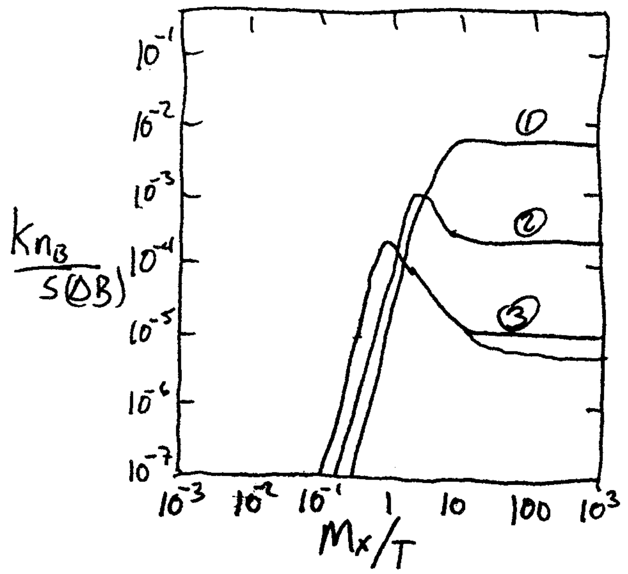
$T \lesssim M$ $\Gamma \sim \alpha^2 T^5/M^4$

$T \gtrsim M$ $\Gamma \sim \alpha^2 T$ <u>but</u> also

have t-channel $\Gamma \sim \alpha^2 T^3/M^2$

but must be screened.

Results

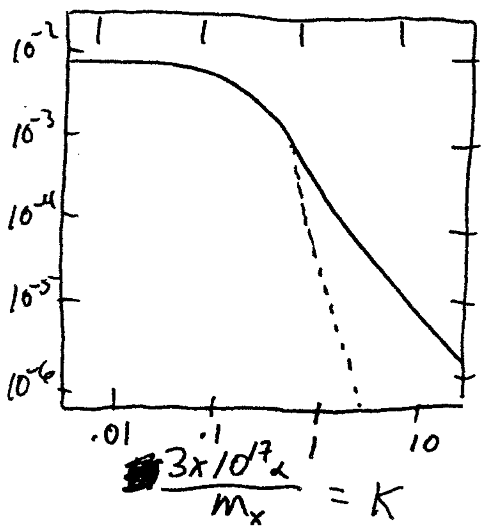


$$\frac{k n_B}{s} \approx \left(1/7\right) \frac{n_B}{n_\gamma}$$

① $M_X \approx 3\times10^{18}\alpha$ — O of Eq.-scenario reproduced (max asy

② $M_X \approx 3\times10^{17}\alpha$ — inverse decays push down the asymm

③ $M_X \approx 3\times10^{16}\alpha$ — scatterings will drive things ~~forwards~~ towards equil. further.



$$\left(\frac{k n_B}{s}\right)_{final} \approx 2\times10^{-3}(\Delta B) \times\left[(3K)^{1.2}+1\right]^{-1}$$

the dotted curve if scatterings are very importa

The Monopole Problem

Monopoles are produced whenever a simple gauge group (like $SU(5)$) is broken to one containing an explicit $U(1)$ factor (like $SU(3) \times SU(2) \times U(1)$)

At the time of the transition, about one monopole is produced per causally connected region with volume $(2t)^3$

$$\Rightarrow \quad n_m \sim (2t_c)^{-3}$$

$$t_c \sim 10^{-2} M_p / T_c^2 \qquad T_c = \text{transition temperature}$$

$$\frac{n_m}{n_\gamma} \sim \left(\frac{10 T_c}{M_p}\right)^3$$

Cosmological Limit on monopoles

$$\rho_m = m_m n_m$$

$$\Omega_m = \frac{\rho_m}{\rho_c} \quad ; \quad \Omega_m h_0^2 = 9.5\times10^4\, m_m(\mathrm{GeV})\, n_m(\mathrm{cm}^{-3})$$

for $m_m = 10^{16}$ GeV

we have $n_m <$ 10^{-21} cm^{-3} $\Omega h_0^2 < 1$

1.5×10^{-22} cm^{-3} $\Omega h_0^2 < .15$

and $\frac{n_m}{n_\gamma} <$ 2.6×10^{-24} $\Omega h^2 < 1$

4×10^{-25} $\Omega h^2 < .15$

But if $T_c = 10^{15}$ GeV

$$\frac{n_m}{n_\gamma} = 10^{-9}$$

and overproducing monopoles..

two solutions: 1) inflation

2) $T_c \lesssim 10^{10}$ GeV

Inflation

The curvature Problem:

Because we are not curvature dominated today.

$$\frac{k}{R^2} < \frac{8\pi G}{3}\rho$$

or because $R \sim 1/T$

$$\hat{k} \equiv \frac{k}{R^2 T^2} < \frac{8\pi G}{3}\frac{\Omega \rho_c}{T_0^2} < 2\times 10^{-58}\,!$$

for $\Omega < 2$, $h_0 < 1$ $T_0 > 2.7$

But $\hat{k}$ is dimensionless and constant throughout the evolution of the Universe and hence represents and initial condition.

Normally one would expect $\hat{k} \sim O(1)$ but then the Universe would have be curvature dominated when $T \sim M_p$

$$\hat{k} = \frac{k}{\ } = (\Omega-1) H^2 = (\Omega-1)\frac{8\pi^3 G}{\ } N(T)\, T^2$$

$$V_0 \propto t_0^3$$

$$V_d \propto t_d^3$$

$$V_0' = V_0 \left(\frac{T_0}{T_d}\right)^3$$

the size of our horizon scaled back to T_d

$$\frac{V_0'}{V_d} = \frac{t_0^3}{t_d^3}\left(\frac{T_0}{T_d}\right)^3 \sim 10^5$$

if $\hat{k} = 10^{-40}$

$$\frac{8\pi G}{3}\frac{\rho}{T^2} = 10^{-40}$$

$$\frac{8\pi}{3M_p^2} N \frac{\pi^2}{30} T^2$$

$$\frac{T^2}{M_p^2} = 10^{-40}\frac{90}{8\pi^3 N} \sim \frac{10^{-40}}{300} \sim 10^{-42} - 10^{-43}$$

$$\frac{T}{M_p} \sim 10^{-21}$$

$$T \sim 10^{-21} M_p \sim 10\,\mathrm{MeV}$$

$$|\Omega - 1| = \frac{90}{8\pi^3 N}\left(\frac{M_p^2}{T^2}\right)\hat{k} \Rightarrow \quad T \sim 1\,\mathrm{MeV} \quad |\Omega - 1| \leq 10^{-45}$$
$$10^{15}\,\mathrm{GeV} \quad |\Omega - 1| \leq 10^{-52}$$

~~total~~ $S^t = R^3 s = \left(\frac{k}{\hat{k}T^2}\right)^{3/2} s > 10^{87}$.

The Horizon Problem
(or isotropy problem)

The horizon volume today or causal volume $\propto t_0^3$. $t_0 \sim 3\times10^{17}$

The microwave background radiation was last scattered at $t_D \sim 3\times10^{12}$ sec when

$e^- + p \to H$

$\Rightarrow$ The horizon volume today contains

$\left(\frac{t_0}{t_D}\right)^3 \sim 10^{15}$ horizon volumes

when the photons were last scattered.

Why is $\frac{\Delta T}{T} < 10^{-4}$?

Measurements at wide angles today show that regions which were separated by 10^5 acausal regions have precisely the temperature.

$T_d = \cancel{4000}\ K$

$V_d \propto t_d \qquad t_0 = 1.5 \times 10^{10}$

$t_d = t_0 \left(T_0 / T_d\right)^{3/2} \sim 3 \times 10^5$ yrs

$V_0 \propto t_0^3$

$V_0(t_d) \propto V_0 \left(T_0 / T_d\right)^3$

$$\frac{V_0(t_d)}{V_d} = \frac{V_0}{V_d}\left(\frac{T_0}{T_d}\right)^3 = \frac{t_0^3}{t_d^3}\left(\frac{T_0}{T_d}\right)^3 \sim 4 \times 10^9$$

but $\frac{\Delta T}{T} < 10^{-4}$

Equations of Motion for

$$T_{\mu\nu} = +\partial_\mu \varphi \partial_\nu \varphi + \mathcal{L} g_{\mu\nu}$$

$g_{00} = -1$

$$\mathcal{L} = + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V$$

$$T_{00} = + \frac{1}{2} \dot{\varphi}^2 + \quad + V$$

$$T_{ii} = + \frac{1}{2} \dot{\varphi}^2 - V$$

$$T^{\mu\nu}{}_{;\nu} = T^{\mu\nu}{}_{,\nu} + \Gamma^\mu_{\nu\rho} T^{\nu\rho} + \Gamma^\nu_{\nu\rho} T^{\mu\rho} = 0$$

$\mu = 0$

$$\dot{\rho} + \frac{3\dot{R}}{R}(\rho + P) = 0$$

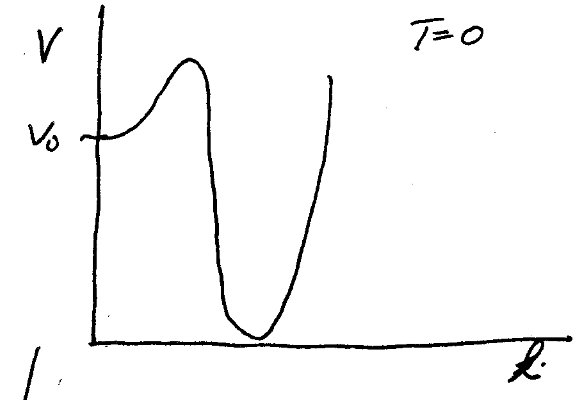
$$\dot{\varphi}\ddot{\varphi} + \frac{\partial V}{\partial \varphi}\dot{\varphi} + 3H\dot{\varphi}^2 = 0$$

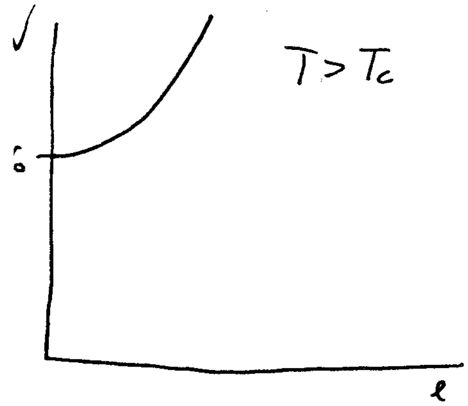
$$\boxed{\Rightarrow \ddot{\varphi} + 3H\dot{\varphi} \overset{+\Gamma\dot{\varphi}}{=} -\partial V / \partial \varphi}$$

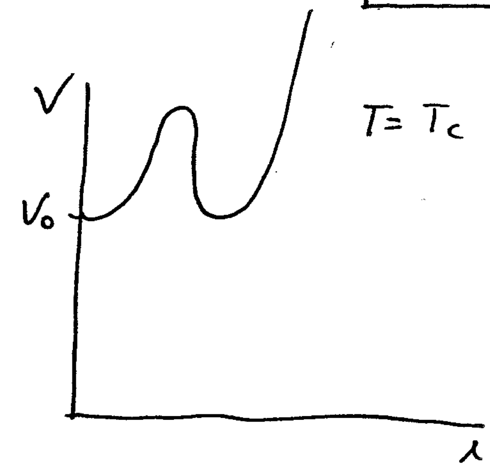
All of these problems always <u>assume</u> that the Universe throughout its history expanded adiabatically. $R \sim 1/T$.

During a phase transition this is not so

Consider a potential







l is some order parameter.

$T > T_c$ finite temperature effects keep the system in the symmetric state

$T = T_c$ the transition first become possible

If the barrier between minima is large enough then transition is delayed until $T \ll T_c$

But when $\rho \lesssim V_0$ dynamics change

V_0 – the vacuum energy density is constant and takes the form of a cosmological constant

$$\Lambda = 8\pi G V_0$$

For $\rho \ll V_0$,

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 \approx \frac{8\pi V_0}{3 M_p^2} = \frac{\Lambda}{3}$$

$$\text{or} \quad \frac{\dot{R}}{R} \sim \sqrt{\frac{\Lambda}{3}} \quad \text{and} \quad R \sim e^{Ht}$$

exponential expansion.

When the transition is over the Universe reheats to

$$T \sim V_0^{1/4}$$

$$\left(\frac{\pi^2}{30} N(T)\, T^4 = V_0\right)$$

Transition takes place through the formation of bubbles of the broken phase with a formation rate per volume

$$P \sim A e^{-B}$$

where $A^{1/4}$ is some mass a temperature scale.

B is the tunnelling action.

In general transition will take place in such a way so as to minimize B.

If $P \sim H^4$, bubbles will form fast enough to complete the transition

Einstein action

$$B = -\frac{1}{4\pi}\int d^4x \sqrt{g}\,(R - 2\Lambda)$$

$$R = R^{\mu}$$

Guth's Inflation

If the barrier were large enough and the Universe supercooled to $T << T_c$ and the transition timescale

$$\tau > \frac{65}{H}$$

then $R_f/R_i \sim \mathcal{O}(10^{28})$ while $T_f/T_i \sim \mathcal{O}(1)$

Then $\hat{k}$, $\hat{\omega}$ no longer constant and $\hat{k} = \hat{\omega} = \mathcal{O}(1)$ initially would fall to $\mathcal{O}(10^{-28})$ ie. acceptable values.

Horizon problem also disappears

Monopole problem will disappear if

$$T_H = \frac{H}{2\pi} = \sqrt{\frac{\Lambda}{12\pi^2}} = \sqrt{\frac{2V_0}{3\pi M_P^2}} < \mathcal{O}(10^{10})\ \mathrm{GeV}$$

T_H - the Hawking temperature is the lowest temperature possible during an exponentially expanding phase.

<u>Problem:</u>

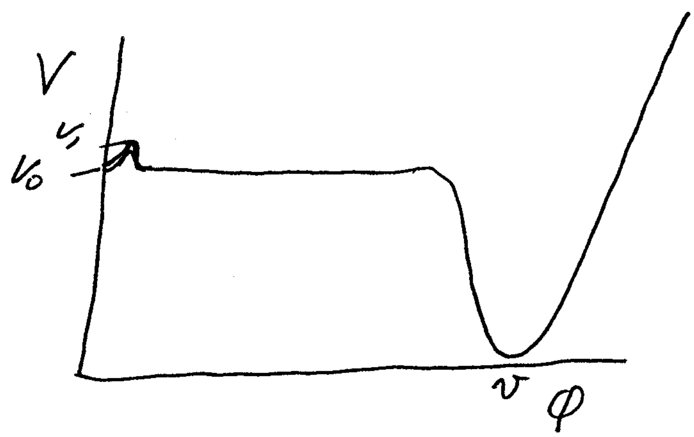
To have enough supercooling P must very small i.e. B, the action very large.

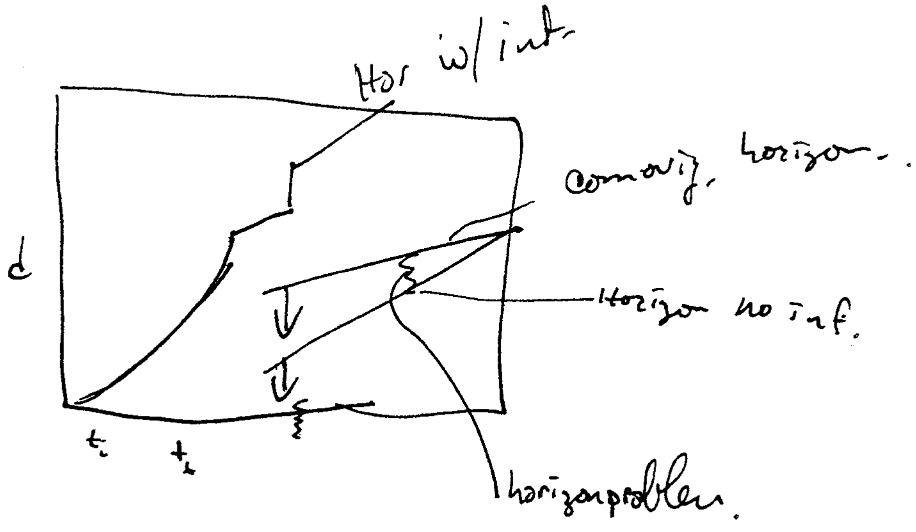
This is incompatible with "finishing the transition"

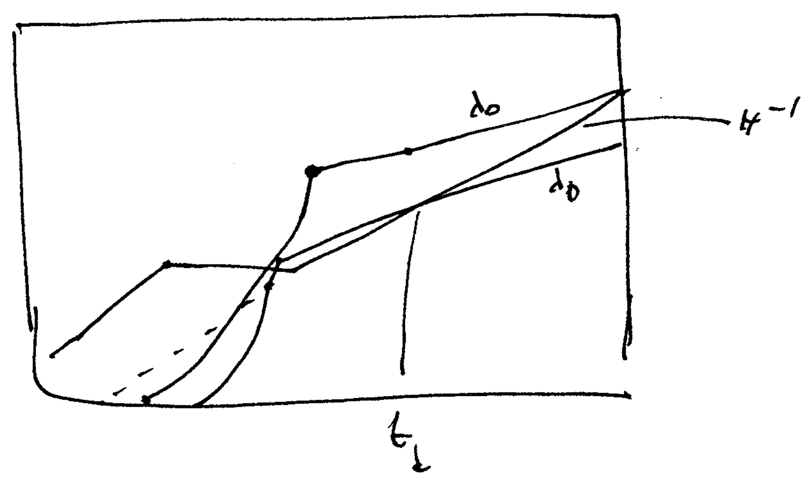
The Universe remains in the De Sitter State with a few isolated bubbles.

Linde Solution: New Inflationary Scenario

Tunnel first, Inflate Later




Hor w/ inf.
comov. horizon.
d
Horizon no inf.
t_i
t_*
horizonproblem.

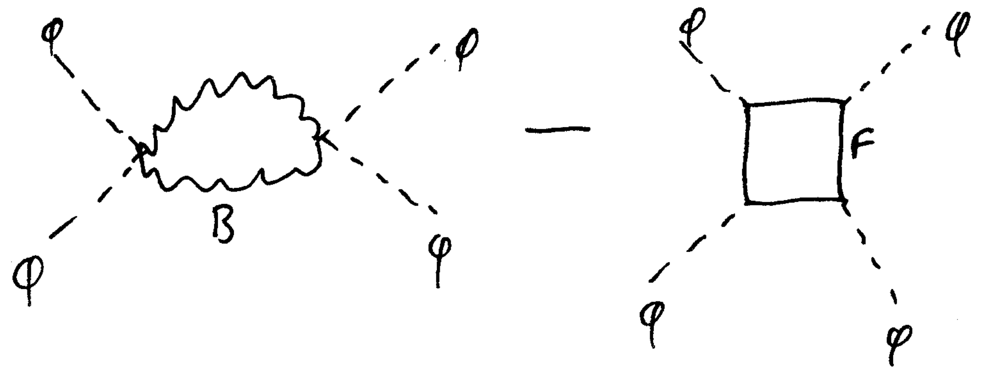

d_0
H^{-1}
d_0
t_*

Consider SU(5) transition using the Coleman-Weinberg potential

$$V(\varphi) = A\varphi^4\left(\ln\frac{\varphi^2}{v^2} - \tfrac{1}{2}\right) + D\varphi^2 + \tfrac{1}{2}Av^4$$

$$A = \frac{1}{64\pi^2 \cancel{v}^4}\left(\sum_B g_B m_B^4 - \sum_F g_F m_F^4\right)$$

$$D = \tfrac{1}{2}\left(m_0^2 + cT^2 + bR - 3\lambda\langle\varphi^2\rangle\right)$$



in SU(5) X and Y gauge bosons dominate the loop and $g_B = 36$ $m_B^2 = \frac{25}{8}g^2v^2$

$$A = \frac{5625}{1024\pi^2}g^4 \qquad g^2/4\pi = 1/41 \text{ SU(5) gauge coupling.}$$

$v = \langle 0|\varphi|0\rangle \qquad \varphi \sim \underset{\sim}{24}$

$c \simeq \frac{75}{8} g^2$ – finite temperature correction

m_0 – bare mass

$R = R^\mu_\mu$ b – unknown (but $b = 1/6$ for conformal couplings)

$-\lambda/4$ – φ^4 self coupling $\propto A$

$\langle \varphi^2 \rangle$ – quantum expectation value of φ^2 in curved space.

The idea is that if B is large after a long time eventually one bubble will form. Afterwards the potential is very flat and $V(\varphi)$ is almost constant and inflation begins after tunnelling.

The one bubble inflates so as to encompass entire visible Universe.

We live in a single bubble.

Positive Aspects

Eq. of Motion

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} \simeq \ddot{\varphi} + 3H\dot{\varphi} + m^2(\varphi)\varphi = 0$$

For $|m^2| \ll H^2$

$$\varphi \sim e^{|m^2| t / 3H}$$

Field moves very little for a period $\tau \sim 3H/|m^2|$

$$R \sim e^{H\tau} \qquad H\tau \sim \frac{H^2}{|m|^2} \sim \frac{v^4}{M_p^2 |m^2|} \sim 10^4$$

for $m \sim 10^8$ GeV.

Plenty of Inflation possible

Reheating:

No longer by bubble collision but by decay of scalar field oscillations

$$\varphi(t) \sim \frac{v}{mt} \sin mt$$

$$T_R \sim (\Gamma_D M_p)^{1/2} \qquad \Gamma_D < H_I$$

Density Fluctuations:

approximately "no scale" - scale invariant

Problems for New Inflation

Quantum fluctuations Cl...

$$V(\varphi) = V(0) + \tfrac{1}{2} m^2 \varphi^2 - \tfrac{1}{4} \lambda \varphi^4$$

$\lambda \sim 8A \ln M$

Contributions to m^2 from $3\lambda \langle \varphi^2 \rangle$

Inflationary time scale (roll-over)

$$H\tau \sim 3H^2/m^2 \gtrsim 65$$

or $m^2 \lessapprox 3\lambda\langle\varphi^2\rangle \simeq 3\lambda \frac{H^3 t}{4\pi^2} \leq \frac{3H^2}{65}$

$$\Rightarrow \lambda < O(10^{-2})$$

$$\lambda_{GUT} \geq O(1)$$

2. Density fluctuations

$$\frac{\delta\rho}{\rho} \simeq 4H\delta\tau = \frac{H^2}{\pi^{3/2}\dot{\varphi}} \simeq \left(\frac{8\lambda}{3\pi^2}\right)^{1/2} \ln^{3/2}(Hk^{-1})$$

$$\Rightarrow \frac{\delta\rho}{\rho} \sim O(10^2)$$

$\delta\rho)$ $\quad -5$

Mexican Hat

$$V(\varphi) = \lambda(\varphi^2 - v^2)^2$$

$$H^2 = \frac{8\pi}{3}\frac{\lambda v^4}{M_p^2} \qquad |m^2| = 4\lambda v^2$$

Sufficient inflation requires

$$(v/M_p)^2 \gtrsim 65/2\pi$$

Steinhardt
Turner

or $v > M_p$

Problems with $\varphi > M_P$

New

$V(\varphi)\big|_{\varphi \simeq 0}$

Chaotic

$V(\varphi)\big|_{\varphi > M_P}$

What about $V_{N.R.(\text{non})} \Rightarrow \frac{\varphi^{n+4}}{M_P^n} \lambda_n$?

$V_{N.R}$ negligible

$V_{N.R}$ dominant.

[SUGRA: $V \propto e^{\varphi^2/M_P^2}$!]

Constraint on λ_n: $\lambda_n \lesssim 10^{-12}/2^n n^{n/2}$

from $\frac{\delta\rho}{\rho}$

Reheating $\Rightarrow \lambda > 10^{-22} \Rightarrow \lambda_{n \geq 12} = 0$!

Enqvist

Malampi

$c \simeq \frac{75}{8} g^2$ — finite temperature correction

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