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WITH GRAND UNIFICATION SIGNALS IN, CAN PROTON DECAY BE FAR BEHIND? *

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Abstract

It is noted that one is now in possession of a set of facts, which may be viewed as the *matching pieces of a puzzle* ; in that all of them can be resolved by just one idea - that is grand unification. These include : (i) the observed family-structure, (ii) quantization of electric charge, (iii) meeting of the three gauge couplings, (iv) neutrino oscillations; in particular the mass of ν_τ (suggested by SuperK), (v) the intricate pattern of the masses and mixings of the fermions, including the smallness of V_{cb} and the largeness of $\theta^{osc}_{\nu_\mu\nu_\tau}$, and (vi) the need for B-L to implement baryogenesis (via leptogenesis). All these pieces fit beautifully together within a single puzzle board framed by supersymmetric unification, based on SO(10) or a string-unified G(224)-symmetry. The one and the most notable piece of the puzzle still missing, however, is proton decay.

A concrete proposal is presented, within a predictive SO(10)/G(224)-framework, that successfully describes the masses and mixings of all fermions, including the neutrinos - with eight predictions, all in agreement with observation. Within this framework, a systematic study of proton decay is carried out, which pays special attention to its dependence on the fermion masses, including the superheavy Majorana masses of the right-handed neutrinos, and the threshold effects. The study (based on prior work and a recent update) shows that a conservative upper limit on the proton lifetime is about (1/2 - 1)$\times 10^{34}$ yrs, with $\overline{\nu}K^+$ being the dominant decay mode, and as a distinctive feature, μ^+K^0 being prominent. This in turn strongly suggests that an improvement in the current sensitivity by a factor of five to ten (compared to SuperK) ought to reveal proton decay. Otherwise some promising and remarkably successful ideas on unification would suffer a major setback.

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I. INTRODUCTION

The standard model of particle physics, based on the gauge symmetry $SU(2)_L \times U(1)_Y \times SU(3)_C$ [1,2] is in excellent agreement with observations, at least up to energies of order 100 GeV. Its success in turn constitutes a triumph of quantum field theory, especially of the notions of gauge invariance, spontaneous symmetry breaking, and renormalizability. The next step in the unification-ladder is associated with the concept of "grand unification", which proposes a unity of quarks and leptons, and simultaneously of their three basic forces: weak, electromagnetic and strong [3–5]. This concept was introduced on purely aesthetic grounds, in fact *before* any of the empirical successes of the standard model was in place. It was realized in 1972 that the standard model judged on aesthetic merits has some major shortcomings [3,4]. For example, it puts members of a family into five scattered multiplets, assigning rather peculiar hypercharge quantum numbers to each of them, without however providing a compelling reason for doing so. It also does not provide a fundamental reason for the quantization of electric charge, and it does not explain why the electron and proton possess exactly equal but opposite charges. Nor does it explain the co-existence of quarks and leptons, and that of the three gauge forces - weak, electromagnetic and strong - with their differing strengths.

The idea of grand unification was postulated precisely to remove these shortcomings. It introduces the notion that quarks and leptons are members of one family, linked together by a symmetry group G, and that the weak, electromagnetic and strong interactions are aspects of one force, generated by gauging this symmetry G. The group G of course inevitably contains the standard model symmetry $G(213) = SU(2)_L \times U(1)_Y \times SU(3)_C$ as a subgroup. Within this picture, the observed differences between quarks and leptons and those between the three gauge forces are assumed to be low-energy phenomena that arise through a spontaneous breaking of the unification symmetry G to the standard model symmetry G(213), at a very high energy scale $M \gg 1TeV$. As a *prediction* of the hypothesis, such differences must then disappear and the true unity of quarks and leptons and of the three gauge forces should manifest at energies exceeding the scale M.

The second and perhaps the most dramatic prediction of grand unification is proton decay. This important process, which would provide the window to view physics at truly short distances ($< 10^{-30}$ cm), is yet to be seen. Nevertheless, as I will stress in this talk, there has appeared over the years an impressive set of facts, favoring the hypothesis of grand unification. These include:

(a) The observed family structure : The five scattered multiplets of the standard model, belonging to a family, neatly become parts of a whole (*a single multiplet*), with their weak hypercharges precisely predicted by grand unification. Realization of this feature calls for an extension of the standard model symmetry $\mathrm{G(213)} = \mathrm{SU(2)}_L \times \mathrm{U(1)}_Y \times \mathrm{SU(3)}^C$ *minimally* to the symmetry group $\mathrm{G(224)} = \mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \times \mathrm{SU(4)}^C$ [3], which can be extended further into the simple group SO(10) [6], but not SU(5) [4]. The G(224) symmetry in turn introduces some additional attractive features (see Sec.II), including especially the right-handed (RH) neutrinos (ν_R's) accompanying the left-handed ones (ν_L's), and *B-L* as a local symmetry. As we will see, both of these features now seem to be needed on empirical grounds.

(b) Meeting of the gauge couplings : Such a meeting is found to occur at a scale $M_X \approx 2 \times 10^{16}$ GeV, when the three gauge couplings are extrapolated from their values measured at LEP to higher energies, in the context of supersymmetry [7]. This dramatic phenomenon supports the ideas of both grand unification and supersymmetry [8]. These in turn may well emerge from a string theory [9] or M-theory [10] (see discussion in Sec.III).

(c) Mass of $\nu_\tau \sim 1/20$ eV : Subject to the well-motivated assumption of hierarchical neutrino masses, the recent discovery of atmospheric neutrino-oscillation at SuperKamiokande [11] suggests a value for $m(\nu_\tau) \sim 1/20eV$. It has been argued (see e.g. Ref. [12]) that a mass of ν_τ of this magnitude can be understood very simply by utilizing the SU(4)-color relation $m(\nu_\tau)_{\text{Dirac}} \approx m_{\text{top}}$ and the SUSY unification scale M_X, noted above (See Sec.IV).

(d) Some intriguing features of fermion masses and mixings: These include: (i) the "observed" near equality of the masses of the b-quark and the τ-lepton at the unification-scale (i.e. $m_b^0 \approx m_\tau^0$) and (ii) the observed largeness of the ν_μ-ν_τ oscillation angle ($\sin^2 2\theta^{\text{osc}}_{\nu_\mu\nu_\tau} \geq 0.83$) [11], together with the smallness of the corresponding quark mixing parameter $V_{cb}(\approx 0.04)$ [13]. As shown in recent work by Babu, Wilczek and me [14], it turns out that these features and more can be understood remarkably well (see discussion in Sec.V) within an economical and predictive SO(10)-framework based on a minimal Higgs system. The success of this framework is in large part due simply to the group-structure of SO(10). For most purposes, that of G(224) suffices.

(e) Baryogenesis : To implement baryogenesis [15] successfully, in the presence of electroweak sphaleron effects [16], which wipe out any baryon excess generated at high temperatures in the (B-L)-conserving mode, it has become apparent that one would need B-L as a generator of the underlying symmetry, whose spontaneous violation at high temperatures would yield, for example, lepton asymmetry (leptogenesis). The latter in turn is converted to baryon-excess at lower temperatures by electroweak sphalerons. This mechanism, it turns out, yields even quantitatively the right magnitude for baryon excess [17]. The need for B-L, which is a generator of SU(4)-color, again points to the need for G(224) or SO(10) as an effective symmetry near the unification-scale M_X.

The success of each of these five features (a)-(e) seems to be non-trivial. Together they make a strong case for both supersymmetric grand unification and simultaneously for the G(224)/SO(10)-route to such unification, as being relevant to nature at short distances. However, despite these successes, as long as proton decay remains undiscovered, the hallmark of grand unification - that is *quark-lepton transformability* - would remain unrevealed.

The relevant questions in this regard then are : What is the predicted range for the lifetime of the proton - in particular an upper limit - within the empirically favored route to unification mentioned above? What are the expected dominant decay modes within this route? Are these predictions compatible with current lower limits on proton lifetime mentioned above, and if so, can they still be tested at the existing or possible near-future detectors for proton decay?

Fortunately, we are in a much better position to answer these questions now, compared to a few years ago, because meanwhile we have learnt more about the nature of grand unification. As noted above (see also Sec.II and Sec.IV), the neutrino masses and the meeting of the gauge couplings together seem to select out the supersymmetric G(224)/SO(10)-route to higher unification. The main purpose of my talk here will therefore be to address the questions raised above, in the context of this route. For the sake of comparison, however, I will state the corresponding results for the case of supersymmetric SU(5) as well.

My discussion will be based on a recent study of proton decay by Babu, Wilczek and me [14] and an update of the same as presented here. Relative to other analysis, this study has three distinctive features:

(a) It systematically takes into account the link that exists between proton decay and the masses and mixings of all fermions, including the neutrinos.

(b) In particular, in addition to the contributions from the so-called "standard" $d = 5$ operators [18] (see Sec.VI), it includes those from a *new* set of $d = 5$ operators, related to the Majorana masses of the RH neutrinos [19]. These latter are found to be as important as the standard ones.

(c) The work also incorporates GUT-scale threshold effects, which arise because of mass-splittings between the components of the SO(10)-multiplets, and lead to differences between the three gauge couplings.

Each of these features turn out to be *crucial* to gaining a reliable insight into the nature of proton decay. Our study shows that the inverse decay rate for the $\overline{\nu}K^+$-mode, which is dominant, is less than about 5×10^{33} yrs for the case of MSSM embedded in SO(10). This upper bound is obtained by making generous allowance for uncertainties in the matrix element and the SUSY-spectrum. Typically, the lifetime should of course be less than this bound.

Proton decay is studied also for the case of the extended supersymmetric standard model (ESSM), that has been proposed a few years ago [20] on theoretical grounds, pertaining to the issues of string-unification and dilaton stabilization (see Sec.VI and the appendix). This case adds an extra pair of vector-like families at the TeV-scale, transforming as $16+\overline{16}$ of SO(10), to the MSSM spectrum. While the case of ESSM is fully compatible with both neutrino-counting at LEP and precision electroweak tests, it can of course be tested directly at the LHC. Our study shows that, with the inclusion of only the standard d=5 operators (defined in Sec.VI), ESSM, embedded in SO(10), can quite plausibly lead to proton lifetimes in the range of $10^{33} - 10^{34}$ years, for nearly central values of the parameters pertaining to the SUSY-spectrum and the matrix element. Allowing for a wide variation of the parameters, owing to the contributions from both the standard and the neutrino mass-related d=5 operators (discussed in Sec.VI), proton lifetime still gets bounded above by about 10^{34} years, even for the case of ESSM, embedded in SO(10) or a string - G(224).

For either MSSM and ESSM, due to contributions from the new operators, the μ^+K^0-mode is found to be prominent, with a branching ratio typically in the range of 10-50%. By contrast, minimal SUSY SU(5), for which the new operators are absent, would lead to branching ratios $\leq 10^{-3}$ for this mode.

Thus our study of proton decay, correlated with fermion masses, strongly suggests that discovery of proton decay should be imminent. In fact,one expects that at least candidate events should be observed in the near future already at SuperK. However, allowing for the possibility that the proton lifetime may well be closer to the upper bound stated above, a next-generation detector providing a net gain in sensitivity in proton decay-searches by a factor of 5-10, compared to SuperK, would certainly be needed not just to produce proton-decay events, but also to clearly distinguish them from the background. It would of course also be essential to study the branching ratios of certain sub-dominant but crucial decay modes, such as the $\mu^+ K^0$. The importance of such improved sensitivity, in the light of the successes of supersymmetric grand unification, is emphasized at the end.

II. ADVANTAGES OF THE SYMMETRY G(224) AS A STEP TO HIGHER UNIFICATION

As mentioned in the introduction, the hypothesis of grand unification was introduced to remove some of the conceptual shortcomings of the standard model (SM). To illustrate the advantages of an early suggestion in this regard, consider the five standard model multiplets belonging to the electron-family as shown :

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{\frac{1}{3}} ; \ \left(u_r \ \ u_y \ \ u_b \right)_R^{\frac{4}{3}} ; \ \left(d_r \ \ d_y \ \ d_b \right)_R^{-\frac{2}{3}} ; \ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} ; \ \left(e^- \right)_R^{-2} . \tag{1}$$

Here the superscripts denote the respective weak hypercharges Y_W (where $Q_{em} = I_{3L} + Y_W/2$) and the subscripts L and R denote the chiralities of the respective fields. If one asks : how one can put these five multiplets into just one multiplet, the answer turns out to be simple and unique. As mentioned in the introduction, the minimal extension of the SM symmetry G(213) needed, to achieve this goal, is given by the gauge symmetry [3] :

$$\mathrm{G}(224) \ = \ \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)^C . \tag{2}$$

Subject to left-right discrete symmetry ($L \leftrightarrow R$), which is natural to G(224), all members of the electron family fall into the neat pattern :

$$F^e_{L,R} \ = \ \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R} \tag{3}$$

The multiplets F^e_L and F^e_R are left-right conjugates of each other and transform respectively as (2,1,4) and (1,2,4) of G(224); likewise for the muon and the tau families. Note that the symmetries $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ are just like the familiar isospin symmetry, except that they operate on quarks and well as leptons, and distinguish between left and right chiralities. The left weak-isospin $\mathrm{SU}(2)_L$ treats each column of F^e_L as a doublet; likewise $\mathrm{SU}(2)_R$ for F^e_R. The symmetry SU(4)-color treats each row of F^e_L *and* F^e_R as a quartet; *thus lepton number is treated as the fourth color.* Note also that postulating either SU(4)-color or $\mathrm{SU}(2)_R$ forces one to introduce a right-handed neutrino (ν_R) for each family as a singlet of the SM symmetry. *This requires that there be sixteen two-component fermions in each family, as opposed to fifteen for the SM.* The symmetry G(224) introduces an elegant charge formula :

$$Q_{em} \ = \ I_{3L} \ + \ I_{3R} \ + \ \frac{B \ - \ L}{2} \tag{4}$$

expressed in terms of familiar quantum numbers I_{3L}, I_{3R} and B-L, which applies to all forms of matter (including quarks and leptons of all six flavors, gauge and Higgs bosons). Note that the weak hypercharge given by $Y_W/2 = I_{3R} + \frac{B-L}{2}$ is now completely determined for all members of the family. The values of Y_W thus obtained precisely match the assignments shown in Eq. (1). Quite clearly, the charges I_{3L}, I_{3R} and B-L, being generators respectively of $SU(2)_L$, $SU(2)_R$ and $SU(4)^c$, are quantized; so also then is the electric charge Q_{em}.

In brief, the symmetry G(224) brings some attractive features to particle physics. These include :

(i) Unification of all 16 members of a family within one left-right self-conjugate multiplet;
(ii) Quantization of electric charge, with a reason for the fact that $Q_{\text{electron}} = -Q_{\text{proton}}$
(iii) Quark-lepton unification (through SU(4) color);
(iv) Conservation of parity at a fundamental level [3,21];
(v) Right-handed neutrinos ($\nu_R' s$) as a compelling feature; and
(vi) B-L as a local symmetry.

As mentioned in the introduction, the two distinguishing features of G(224) - i.e. the existence of the RH neutrinos and B-L as a local symmetry - now seem to be needed on empirical grounds. Furthermore, SU(4)-color provides simple relations between the masses of quarks and leptons, especially of those in the third family. As we will see in Secs.IV and V, these are in good accord with observations.

Believing in a complete unification, one is led to view the G(224) symmetry as part of a bigger symmetry, which itself may have its origin in an underlying theory, such as string theory. In this context, one may ask : Could the effective symmetry below the string scale in four dimensions (see Sec.III) be as small as just the SM symmetry G(213), even though the latter may have its origin in a bigger symmetry, which lives only in higher dimensions? I will argue in Sec.IV that the data on neutrino masses and the need for baryogenesis provide an answer to the contrary, suggesting that it is the effective symmetry in four dimensions, below the string scale, which must *minimally* contain either G(224) or a close relative $G(214) = SU(2)_L \times I_{3R} \times SU(4)^C$.

One may also ask : does the effective four dimensional symmetry have to be any bigger than G(224) near the string scale? In preparation for an answer to this question, let us recall that the smallest simple group that contains the SM symmetry G(213) is SU(5) [4]. It has the virtue of demonstrating how the main ideas of grand unification, including unification of the gauge couplings, can be realized. However, SU(5) does not contain G(224) as a subgroup. As such, it does not possess some of the advantages listed above. In particular, it does not contain the RH neutrinos as a compelling feature, and B-L as a local symmetry. Furthermore, it splits members of a family into two multiplets : $\overline{5} + 10$.

By contrast, the symmetry SO(10) has the merit, relative to SU(5), that it contains G(224) as a subgroup, and thereby retains all the advantages of G(224) listed above. (As a historical note, it is worth mentioning that these advantages had been motivated on aesthetic grounds through the symmetry G(224) [3], and *all* the ideas of higher unification were in place [3–5], before it was noted that G(224)(isomorphic to SO(4)×SO(6)) embeds nicely into SO(10) [6]). Now, *SO(10) even preserves the 16-plet family-structure of G(224) without a need for any extension.* By contrast, if one extends G(224) to the still higher symmetry E_6 [22], the advantages (i)-(vi) are retained, but in this case, one must extend the family-structure from a 16 to a 27-plet, by postulating additional fermions. In this sense, there seems to be some advantage in having the effective symmetry below the string scale to be minimally G(224) (or G(214)) and maximally no more than SO(10). I will compare the relative advantage of having either a string-derived G(224) or a string-SO(10), in the next section. First, I discuss the implications of the data on coupling unification.

III. THE NEED FOR SUPERSYMMETRY : MSSM VERSUS STRING UNIFICATIONS

It has been known for some time that the precision measurements of the standard model coupling constants (in particular $\sin^2\theta_W$) at LEP put severe constraints on the idea of grand unification. Owing to these constraints, the non-supersymmetric minimal SU(5), and for similar reasons, the one-step breaking minimal non-supersymmetric SO(10)-model as well, are now excluded [23]. But the situation changes radically if one assumes that the standard model is replaced by the minimal supersymmetric standard model (MSSM), above a threshold of about 1 TeV. In this case, the three gauge couplings are found to meet [7], to a very good approximation, barring a few percent discrepancy which can be attributed to threshold corrections (see Appendix). Their scale of meeting is given by

$$M_X \approx 2\times 10^{16}\,\text{GeV} \quad (\text{MSSM or SUSY SU(5)}) \tag{5}$$

This dramatic meeting of the three gauge couplings, or equivalently the agreement of the MSSM-based prediction of $\sin^2\theta_W(m_Z)_{\text{Th}} = 0.2315 \pm 0.003$ [24] with the observed value of $\sin^2\theta_W(m_Z) = 0.23124 \pm 0.00017$ [13], provides a strong support for the ideas of both grand unification and supersymmetry, as being relevant to physics at short distances.

In addition to being needed for achieving coupling unification there is of course an independent motivation for low-energy supersymmetry - i.e. for the existence of SUSY partners of the standard model particles with masses of order 1 TeV. This is because it protects the Higgs boson mass from getting large quantum corrections, which would (otherwise) arise from grand unification and Planck scale physics. It thereby provides at least a technical resolution of the so-called gauge-hierarchy problem. In this sense low-energy supersymmetry seems to be needed for the consistency of the hypothesis of grand unification. Supersymmetry is of course also needed for the consistency of string theory. And most important, low-energy supersymmetry can be tested at the LHC, and possibly at the Tevatron.

The most straightforward interpretation of the observed meeting of the three gauge couplings and of the scale M_X, is that a supersymmetric grand unification symmetry (often called GUT symmetry), like SU(5) or SO(10), breaks spontaneously at M_X into the standard model symmetry G(213).

Even if supersymmetric grand unification may well be a good effective theory below a certain scale $M \gtrsim M_X$, it ought to have its origin within an underlying theory like string/M theory. Such a theory is needed to unify all the forces of nature including gravity, and to provide a good quantum theory of gravity. It is also needed to provide a rationale for the existence of flavor symmetries (not available within grand unification), which distinguish between the three families and can resolve certain naturalness problems including those associated with inter-family mass hierarchy.

In the context of string or M theory, an alternative interpretation of the observed meeting of the gauge couplings is however possible. This is because, even if the effective symmetry in four dimensions emerging from a higher dimensional string theory is non-simple, like G(224) or G(213), string theory can still ensure familiar unification of the gauge couplings at the string scale. In this case, however, one needs to account for the small mismatch between the MSSM unification scale M_X (given above), and the string unification scale, given by $M_{st} \approx g_{st} \times 5.2 \times 10^{17}$ GeV $\approx 3.6 \times 10^{17}$ GeV (Here we have put $\alpha_{st} = \alpha_{GUT}$(MSSM) ≈ 0.04) [25]. Possible resolutions of this mismatch have been proposed. These include : (i) utilizing the idea of *string-duality* [26] which allows a lowering of M_{st} compared to the value shown above, or alternatively (ii) the idea of a *semi-perturbative* unification that assumes the existence of two vector-like families, transforming as $(16 + \overline{16})$ of SO(10), with masses of order one TeV [20]. The latter raises α_{GUT} to about 0.25-0.3 and simultaneously M_X, in two loop, to about $(1/2 - 2) \times 10^{17}$ GeV. (Other mechanisms resolving the mismatch are reviewed in Ref. [27]). In practice, a combination of the two mechanisms mentioned above may well be relevant. [1]

While the mismatch can thus quite plausibly be removed for a non-GUT string-derived symmetry like G(224) or G(213), a GUT symmetry like SU(5) or SO(10) would have an advantage in this regard because it would keep the gauge couplings together between M_{st} and M_X (even if $M_X \sim M_{st}/20$), and thus not even encounter the problem of a mismatch between the two scales. A supersymmetric GUT-solution (like SU(5) or SO(10)), however, has a possible disadvantage as well, because it needs certain color triplets to become superheavy by the so-called doublet-triplet splitting mechanism (see Sec.VI and Appendix), in order to avoid the problem of rapid proton decay. However, no such mechanism has emerged yet, in string theory, for the GUT-like solutions [28].

Non-GUT string solutions, based on symmetries like G(224) or G(2113) for example, have a distinct advantage in this regard, in that the dangerous color triplets, which would induce rapid proton decay, are often naturally projected out for such solutions [29,30]. Furthermore, the non-GUT solutions invariably possess new "flavor" gauge symmetries, which distinguish between families. These symmetries are immensely helpful in explaining qualitatively the observed fermion mass-hierarchy (see e.g. Ref. [30]) and resolving the so-called naturalness problems of supersymmetry such as those pertaining to the issues of squark-degeneracy [31], CP violation [32] and quantum gravity-induced rapid proton decay [33].

Weighing the advantages and possible disadvantages of both, it seems hard at present to make a priori a clear choice between a GUT versus a non-GUT string-solution. As expressed elsewhere [34], it therefore seems prudent to keep both options open and pursue their phenomenological consequences. Given the advantages of G(224) or SO(10) in the light

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[1]I have in mind the possibility of string-duality [26] lowering M_{st} for the case of semi-perturbative unification (for which $\alpha_{st} \approx 0.25$, and thus, without the use of string-duality, M_{st} would be about 10^{18} GeV) to a value of about (1-2)$\times 10^{17}$ GeV (say), and semi-perturbative unification [20] raising the MSSM value of M_X to about 5×10^{16} GeV$\approx M_{st}(1/2$ to $1/4)$ (say). In this case, an intermediate symmetry like G(224) emerging at M_{st} would be effective only within the short gap between M_{st} and M_X, where it would break into G(213). Despite this short gap, one would still have the benefits of SU(4)-color that are needed to understand neutrino masses (see sec.4). At the same time, since the gap is so small, the couplings of G(224), unified at M_{st} would remain essentially so at M_X, so as to match with the "observed" coupling unification, of the type suggested in Ref. [20].

of the neutrino masses (see Secs.II and IV), I will thus proceed by assuming that either a suitable G(224)-solution with a mechanism of the sort mentioned above, or a realistic SO(10)-solution with the needed doublet-triplet mechanism, will emerge from string theory. We will see that with this broad assumption, an economical and predictive framework emerges, which successfully accounts for a host of observed phenomena, and makes some crucial testable predictions. Fortunately, it will turn out that there are many similarities between the predictions of a string-unified G(224) and SO(10) frameworks, not only for the neutrino and the charged fermion masses, but also for proton decay. I next discuss the implications of the mass of ν_τ suggested by the SuperK data.

IV. MASS OF ν_τ: EVIDENCE IN FAVOR OF THE G(224) ROUTE

One can obtain an estimate for the mass of ν_L^τ in the context of G(224) or SO(10) by using the following three steps (see e.g.Ref. [12]):

(i) Assume that B−L and I_{3R}, contained in a string-derived G(224) or SO(10), break near the unification-scale:

$$M_X \sim 2 \times 10^{16}\,\mathrm{GeV}\,, \tag{6}$$

through VEVs of Higgs multiplets of the type suggested by string-solutions - i.e. $\langle(1,2,4)_H\rangle$ for G(224) or $\langle\overline{16}_H\rangle$ for SO(10), as opposed to 126_H which seems to be unobtainable (at least) in weakly interacting string theory [35]. In the process, the RH neutrinos (ν_R^i), which are singlets of the standard model, can and generically will acquire superheavy Majorana masses of the type $M_R^{ij}\,\nu_R^{iT}\,C^{-1}\,\nu_R^j$, by utilizing the VEV of $\langle\overline{16}_H\rangle$ and effective couplings of the form:

$$\mathcal{L}_M\,(SO(10)) \;=\; f_{ij}\;16_i \cdot 16_j\;\overline{16}_H\;\cdot \overline{16}_H/M + h.c. \tag{7}$$

A similar expression holds for G(224). Here $i,j\;=\;1,2,3$, correspond respectively to e, μ and τ families. Such gauge-invariant non-renormalizable couplings might be expected to be induced by Planck-scale physics, involving quantum gravity or stringy effects and/or tree-level exchange of superheavy states, such as those in the string tower. With f_{ij} (at least the largest among them) being of order unity, we would thus expect M to lie between $M_{Planck} \approx 2 \times 10^{18}$ GeV and $M_{string} \approx 4 \times 10^{17}$ GeV. Ignoring for the present off-diagonal mixings (for simplicity), one thus obtains [2]:

$$M_{3R} \;\approx\; \frac{f_{33}\langle\overline{16}_H\rangle^2}{M} \;\approx\; f_{33}\,(2 \times 10^{14}\,\mathrm{GeV})\,\rho^2\,(M_{Planck}/M) \tag{8}$$

This is the Majorana mass of the RH tau neturino. Guided by the value of M_X, we have substituted $\langle\overline{16}_H\rangle = (2 \times 10^{16}\,\mathrm{GeV})\,\rho$,with $\rho \approx 1/2$ to 2(say).

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[2]The effects of neutrino-mixing and of possible choice of $M = M_{string} \approx 4 \times 10^{17}$ GeV (instead of $M = M_{Planck}$) on M_{3R} are considered in Ref. [14].

(ii) Now using SU(4)-color and the Higgs multiplet $(2,2,1)_H$ of G(224) or equivalently 10_H of SO(10), one obtains the relation $m_\tau(M_X) = m_b(M_X)$, which is known to be successful. Thus, there is a good reason to believe that the third family gets its masses primarily from the 10_H or equivalently $(2,2,1)_H$ (see sec.5). In turn, this implies:

$$m(\nu^\tau_{Dirac}) \approx m_{top}(M_X) \approx (100\text{-}120)\,\text{GeV} \tag{9}$$

Note that this relationship between the Dirac mass of the tau-neutrino and the top-mass is special to SU(4)-color. It does not emerge in SU(5).

(iii) Given the superheavy Majorana masses of the RH neutrinos as well as the Dirac masses as above, the see-saw mechanism [36] yields naturally light masses for the LH neutrinos. For ν^τ_L (ignoring flavor-mixing), one thus obtains, using Eqs.(8) and (9),

$$m(\nu^\tau_L) \approx \frac{m(\nu^\tau_{Dirac})^2}{M_{3R}} \approx [(1/20)\,\text{eV}\,(1\text{-}1.44)/f_{33}\,\rho^2]\,(M/M_{Planck}) \tag{10}$$

Now, assuming the hierarchical pattern $m(\nu^e_L) \ll m(\nu^\mu_L) \ll m(\nu^\tau_L)$, which is suggested by the see-saw mechanism, and further that the SuperK observation represents $\nu^\mu_L - \nu^\tau_L$ (rather than $\nu^\mu_L - \nu_X$) oscillation, the observed $\delta m^2 \approx 1/2(10^{-2}\text{-}10^{-3})\,\text{eV}^2$ corresponds to $m(\nu^\tau_L) \approx (1/15\text{ - }1/40)$ eV. It seems *truly remarkable* that the expected magnitude of $m(\nu^\tau_L)$, given by Eq.(10), is just about what is suggested by the SuperK data, if $f_{33}\,\rho^2(M_{Planck}/M) \approx 1.3$ to 1/2. Such a range for $f_{33}\,\rho^2(M_{Planck}/M)$ seems most plausible and natural (see discussion in Ref. [12]). Note that the estimate (10) crucially depends upon the supersymmetric unification scale, which provides a value for M_{3R}, as well as on SU(4)-color that yields $m(\nu^\tau_{Dirac})$. *The agreement between the expected and the SuperK results thus clearly favors supersymmetric unification, and in the string theory context, it suggests that the effective symmetry below the string-scale should contain SU(4)-color.* Thus, minimally this effective symmetry should be either G(214) or G(224), and maximally as big as SO(10), if not E_6.

By contrast, if SU(5) is regarded as either a fundamental symmetry or as the effective symmetry below the string scale, there would be no compelling reason based on symmetry alone, to introduce a ν_R, because it is a singlet of SU(5). Second, even if one did introduce ν^i_R by hand, their Dirac masses, arising from the coupling $h^i\,\overline{5}_i\langle 5_H\rangle\nu^i_R$, would be unrelated to the up-flavor masses and thus rather arbitrary (contrast with Eq. (9)). So also would be the Majorana masses of the ν^i_R's, which are SU(5)-invariant, and thus can be even of order string scale . This would give $m(\nu^\tau_L)$ in gross conflict with the observed value.

Before passing to the next section, it is worth noting that the mass of ν_τ suggested by SuperK, as well as the observed value of $\sin^2\theta_W$ (see Sec.III), provide valuable insight into the nature of GUT symmetry breaking. They both favor the case of a *single-step breaking* (SSB) of SO(10) or a string-unified G(224) symmetry at a scale of order M_X, into the standard model symmetry G(213), as opposed to that of a multi-step breaking (MSB). The latter would correspond, for example, to SO(10) (or G(224)) breaking at a scale M_1 into G(2213), which in turn breaks at a scale $M_2 << M_1$ into G(213). One reason why the case of single-step breaking is favored over that of multi-step breaking is that the latter can accommodate but not really predict $\sin^2\theta_W$, whereas the former predicts the same successfully. Furthermore, since the Majorana mass of ν^τ_R arises arises only after $B-L$ and I_{3R} break, it would be given, for the case of MSB, by $M_{3R} \sim f_{33}(M_2^2/M)$, where $M \sim M_{st}$

(say). If $M_2 \ll M_X \sim 2\times10^{16}$ GeV, and M > M_X, one would obtain too low a value ($<< 10^{14}$ GeV) for M_{3R} (compare with Eq.(8)), and thereby too large a value for $m(\nu_L^\tau)$, compared to that suggested by SuperK. By contrast, the case of SSB yields the right magnitude for $m(\nu_\tau)$ (see Eq. (10)).

Thus the success of the result on $m(\nu_\tau)$ discussed above not only favors the symmetry G(224) or SO(10), but also clearly suggests that $B-L$ and I_{3R} break near the conventional GUT scale $M_X \sim 2\times10^{16}$ GeV, rather than at an intermediate scale $<< M_X$. In other words, the observed values of both $\sin^2\theta_W$ and $m(\nu_\tau)$ favor only *the simplest pattern of symmetry-breaking*, for which SO(10) or a string-derived G(224) symmetry breaks in one step to the standard model symmetry, rather than in multiple steps. It is of course only this simple pattern of symmetry breaking that would be rather restrictive as regards its predictions for proton decay (to be discussed in Sec.VI). I next discuss the problem of understanding the masses and mixings of all fermions.

V. UNDERSTANDING FERMION MASSES AND NEUTRINO OSCILLATIONS IN SO(10)

Understanding the masses and mixings of all quarks and charged leptons, in conjunction with those of the neutrinos, is a goal worth achieving by itself. It also turns out to be essential for the study of proton decay. I therefore present first a recent attempt in this direction, which seems most promising [14]. A few guidelines would prove to be helpful in this regard. The first of these is motivated by the desire for economy and the rest by data.

1) Hierarchy Through Off-diagonal Mixings : Recall earlier attempts [37] that attribute hierarchical masses of the first two families to mass matrices of the form :

$$M = \begin{pmatrix} 0 & \epsilon \\ \epsilon & 1 \end{pmatrix} m_s^{(0)}, \tag{11}$$

for the (d,s) quarks, and likewise for the (u,c) quarks. Here $\epsilon \sim 1/10$. The hierarchical patterns in Eq. (11) can be ensured by imposing a suitable flavor symmetry which distinguishes between the two families (that in turn may have its origin in string theory (see e.g. Ref [30]). Such a pattern has the virtues that (a) it yields a hierarchy that is much larger than the input parameter ϵ : $(m_d/m_s) \approx \epsilon^2 \ll \epsilon$, and (b) it leads to an expression for the cabibbo angle :

$$\theta_c \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi}\sqrt{\frac{m_u}{m_c}} \right|, \tag{12}$$

which is rather successful. Using $\sqrt{m_d/m_s} \approx 0.22$ and $\sqrt{m_u/m_c} \approx 0.06$, we see that Eq. (12) works to within about 25% for any value of the phase ϕ. Note that the square root formula (like $\sqrt{m_d/m_s}$) for the relevant mixing angle arises because of the symmetric form of M in Eq. (11), which in turn is ensured if the contributing Higgs is a 10 of SO(10). A generalization of the pattern in Eq. (11) would suggest that the first two families (i.e. the e and the μ) receive masses primarily through their mixing with the third family (τ), with

$(1,3)$ and $(1,2)$ elements being smaller than the $(2,3)$; while $(2,3)$ is smaller than the $(3,3)$. We will follow this guideline, except for the modification noted below.

2) The Need for an Antisymmetric Component : Although the symmetric hierarchical matrix in Eq. (11) works well for the first two families, a matrix of the same form fails altogether to reproduce V_{cb}, for which it yields :

$$V_{cb} \approx \left| \sqrt{\frac{m_s}{m_b}} - e^{i\chi} \sqrt{\frac{m_c}{m_t}} \right| . \tag{13}$$

Given that $\sqrt{m_s/m_b} \approx 0.17$ and $\sqrt{m_c/m_t} \approx 0.0.06$, we see that Eq. (13) would yield V_{cb} varying between 0.11 and 0.23, depending upon the phase χ. This is too big, compared to the observed value of $V_{cb} \approx 0.04 \pm 0.003$, by at least a factor of 3. We interpret this failure as a *clue* to the presence of an antisymmetric component in M, together with symmetrical ones (so that $m_{ij} \neq m_{ji}$), which would modify the relevant mixing angle to $\sqrt{\frac{m_i}{m_j}}\sqrt{\frac{m_{ij}}{m_{ji}}}$, where m_i and m_j denote the respective eigenvalues.

3) The Need for a Contribution Proportional to *B-L* : The success of the relations $m_b^0 \approx m_\tau^0$, and $m_t^0 \approx m(\nu_\tau)^0_{Dirac}$ (see Sec.IV), suggests that the members of the third family get their masses primarily from the VEV of a SU(4)-color singlet Higgs field that is independent of *B-L*. This is in fact ensured if the Higgs is a 10 of SO(10). However, the empirical observations of $m_s^0 \sim m_\mu^0/3$ and $m_d^0 \sim 3m_e^0$ [38] clearly call for a contribution proportional to *B-L* as well. Further, one can in fact argue that the suppression of V_{cb} (in the quark-sector) together with an enhancement of $\theta^{osc}_{\nu_\mu \nu_\tau}$ (in the lepton sector) calls for a contribution that is not only proportional to *B-L*, but also antisymmetric in the family space (as suggested above in item (2)). We show below how both of these requirements can be met, rather easily, in SO(10), even for a minimal Higgs system.

4) Up-Down Asymmetry: Finally, the up and the down-sector mass matrices must not be proportional to each other, as otherwise the CKM angles would all vanish. Note that the cubic couplings of a single 10_H will not serve the purpose in this regard.

Following Ref. [14], I now present a simple and predictive mass-matrix, based on SO(10), that satisfies *all four* requirements (1), (2), (3) and (4). The interesting point is that one can obtain such a mass-matrix for the fermions by utilizing only the minimal Higgs system, that is needed anyway to break the gauge symmetry SO(10). It consists of the set :

$$H_{minimal} = \{45_H, 16_H, \overline{16}_H, 10_H\} . \tag{14}$$

Of these, the VEV of $\langle 45_H \rangle \sim M_X$ breaks SO(10) into G(2213), and those of $\langle 16_H \rangle = \langle \overline{16}_H \rangle \sim M_X$ break G(2213) to G(213), at the unification-scale M_X. Now G(213) breaks at the electroweak scale by the VEV of $\langle 10_H \rangle$ to $U(1)_{em} \times SU(3)^c$.

One might have introduced large-dimensional tensorial multiplets of SO(10) like $\overline{126}_H$ and 120_H, both of which possess cubic level Yukawa couplings with the fermions. In particular, the coupling $16_i 16_j (120_H)$ would give the desired family-antisymmetric as well as (*B-L*)-dependent contribution. We do not however introduce these multiplets in part because they do not seem to arise in string solutions [35], and in part also because mass-splittings within such large-dimensional multiplets could give excessive threshold corrections to $\alpha_3(m_z)$

(typically exceeding 20%), rendering observed coupling unification fortuitous. By contrast, the multiplets in the minimal set (shown above) do arise in string solutions leading to SO(10). Furthermore, the threshold corrections for the minimal set are found to be naturally small, and even to have the right sign, to go with the observed coupling unification [14] (see Appendix).

The question is: can the minimal set of Higgs multiplets (see Eq.(14)) meet all the requirements listed above? Now 10_H (even several 10's) can not meet the requirements of antisymmetry and (B-L)-dependence. Furthermore, a single 10_H cannot generate CKM-mixings. This impasse disappears, however, as soon as one allows for not only cubic, but also effective non-renormalizable quartic couplings of the minimal set of Higgs fields with the fermions. These latter couplings could of course well arise through exchanges of superheavy states (e.g. those in the string tower) involving renormalizable couplings, and/or through quantum gravity.

Allowing for such cubic and quartic couplings and adopting the guideline (1) of hierarchical Yukawa couplings, as well as that of economy, we are led to suggest the following effective lagrangian for generating Dirac masses and mixings of the three families [14] (for a related but different pattern, involving a non-minimal Higgs system, see Ref [39]).

$$\begin{aligned}\mathcal{L}_{\rm Yuk} = &\; h_{33}\,\mathbf{16_3}\,\mathbf{16_3}\,\mathbf{10_H} + \left[h_{23}\,\mathbf{16_2}\,\mathbf{16_3}\,\mathbf{10_H} + a_{23}\,\mathbf{16_2}\,\mathbf{16_3}\,\mathbf{10_H}\,\mathbf{45_H}/M\right. \\ &\left. + \, g_{23}\,\mathbf{16_2}\,\mathbf{16_3}\,\mathbf{16_H}\,\mathbf{16_H}/M\right] + \{a_{12}\,\mathbf{16_1}\,\mathbf{16_2}\,\mathbf{10_H}\,\mathbf{45_H}/M \\ &+ g_{12}\,\mathbf{16_1}\,\mathbf{16_2}\,\mathbf{16_H}\,\mathbf{16_H}/M\}\,. \end{aligned} \tag{15}$$

Here, M could plausibly be of order string scale. Note that a mass matrix having essentially the form of Eq. (11) results if the first term $h_{33}\langle 10_H\rangle$ is dominant. This ensures $m_b^0 \approx m_\tau^0$ and $m_t^0 \approx m(\nu_{Dirac})^0$. Following the assumption of progressive hierarchy (equivalently appropriate flavor symmetries [3]),we presume that $h_{23} \sim h_{33}/10$, while h_{22} and h_{11}, which are not shown, are assumed to be progressively much smaller than h_{23}. Since $\langle 45_H\rangle \sim \langle 16_H\rangle \sim M_X$, while $M \sim M_{st} \sim 10M_X$, the terms $a_{23}\langle 45_H\rangle/M$ and $g_{23}\langle 16_H\rangle/M$ can quite plausibly be of order $h_{33}/10$, if $a_{23} \sim g_{23} \sim h_{33}$. By the assumption of hierarchy, we presume that $a_{12} \ll a_{23}$, and $g_{12} \ll g_{23}$

It is interesting to observe the symmetry properties of the a_{23} and g_{23}-terms. Although $10_H \times 45_H = 10 + 120 + 320$, given that $\langle 45_H\rangle$ is along B-L, which is needed to implement

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[3]Although no explicit string solution with the hierarchy in all the Yukawa couplings in Eq.(15) - i.e. in h_{ij}, a_{ij} and g_{ij} - exists as yet, one can postulate flavor symmetries of the type alluded to (e.g. two abelian U(1) symmetries), which assign flavor charges not only to the fermion families and the Higgs multiplets, but also to a few (postulated) SM singlets that acquire VEVs of order M_X. The flavor symmetry - allowed effective couplings such as $16_2 16_3 10_H < S > /M$ would lead to $h_{23} \sim < S > /M \sim 1/10$. One can verify that the full set of hierarchical couplings shown in Eq.(15) can in fact arise in the presence of two such U(1) symmetries. String theory (at least) offers the scope (as indicated by the solutions of Refs. [30] and [29]) for providing a rationale for the existence of such flavor symmetries, together with that of the SM singlets. For example, there exist solutions with the top Yukawa coupling being leading and others being hierarchical (as in Ref. [30]).

doublet-triplet splitting (see Appendix), only 120 in the decomposition contributes to the mass-matrices. This contribution is, however, antisymmetric in the family-index and, at the same time, proportional to B-L. *Thus the a_{23} term fulfills the requirements of both antisymmetry and (B-L)-dependence, simultaneously* [4]. With only h_{ij} and a_{ij}-terms, however, the up and down quark mass-matrices will be proportional to each other, which would yield $V_{CKM} = 1$. This is remedied by the g_{ij} coupling, because, the 16_H can have a VEV not only along its SM singlet component (transforming as $\tilde{\bar{\nu}}_R$) which is of GUT-scale, but also along its electroweak doublet component - call it 16_d - of the electroweak scale. The latter can arise by the the mixing of 16_d with the corresponding doublet (call it 10_d) in the 10_H. The MSSM doublet H_d, which is light, is then a mixture of 10_d and 16_d, while the orthogonal combination is superheavy (see Appendix). Since $\langle 16_d \rangle$ contributes only to the down-flavor mass matrices, but not to the up-flavor, the g_{23} and g_{12} couplings generate non-trivial CKM-mixings. *We thus see that the minimal Higgs system (as shown in Eq.(14)) satisfies apriori all the qualitative requirements (1)-(4), including the condition of $V_{CKM} \neq 1$.* I now discuss that this system works well even quantitatively.

With these six effective Yukawa couplings, the Dirac mass matrices of quarks and leptons of the three families at the unification scale take the form :

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & -\epsilon+\eta & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U, \; L = \begin{pmatrix} 0 & -3\epsilon'+\eta' & 0 \\ 3\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & 3\epsilon+\eta & 1 \end{pmatrix} m_D. \quad (16)$$

Here the matrices are multiplied by left-handed fermion fields from the left and by anti–fermion fields from the right. (U, D) stand for the mass matrices of up and down quarks, while (N, L) are the Dirac mass matrices of the neutrinos and the charged leptons. The entries $1, \epsilon$,and σ arise respectively from the h_{33}, a_{23} and h_{23} terms in Eq. (15), while η entering into D and L receives contributions from both g_{23} and h_{23}; thus $\eta \neq \sigma$. Similarly η' and ϵ' arise from g_{12} and a_{12} terms respectively. Note the quark-lepton correlations between U and N as well as D and L, and the up-down correlations between U and D as well as N and L. These correlations arise because of the symmetry property of G(224). The relative factor of -3 between quarks and leptons involving the ϵ entry reflects the fact that $\langle \mathbf{45_H} \rangle \sim$ to(B-L), while the antisymmetry in this entry arises from the group structure of SO(10), as explained above[4]. As we will see, this ϵ-entry helps to account for (a) the differences between m_s and m_μ, (b) that between m_d and m_e, and also, (c) the suppression of V_{cb} together with the enhancement of the ν_μ-ν_τ oscillation angle.

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[4]The analog of $10_H \cdot 45_H$ for the case of G(224) would be $\chi_H \equiv (2,2,1)_H \cdot (1,1,15)_H$. Although in general, the coupling of χ_H to the fermions need not be antisymmetric, for a string-derived G(224), the multiplet $(1,1,15)_H$ is most likely to arise from an underlying 45 of SO(10) (rather than 210); in this case, the couplings of χ_H must be antisymmetric like that of $10_H \cdot 45_H$.

The mass matrices in Eq.(16) contain 7 parameters [5]: ϵ, σ, η, $m_D = h_{33}\langle 10_d\rangle$, $m_U = h_{33}\langle 10_U\rangle$, η' and ϵ'. These may be determined by using, for example, the following input values: $m_t^{phys} = 174$ GeV, $m_c(m_c) = 1.37$ GeV, $m_s(1 \text{ GeV}) = 110\text{-}116$ MeV [40], $m_u(1 \text{ GeV}) \approx 6$ MeV and the observed masses of e, μ and τ, which lead to (see Ref. [14], for details):

$$\sigma \simeq 0.110\,,\ \eta \simeq 0.151\,,\ \epsilon \simeq -0.095\,,\ |\eta'| \approx 4.4 \times 10^{-3} \text{ and } \epsilon' \approx 2 \times 10^{-4}$$

$$m_U \simeq m_t(M_U) \simeq (100\text{-}120)\,\text{GeV}\,,\ m_D \simeq m_b(M_U) \simeq 1.5\,\text{GeV}\,. \tag{17}$$

Here, I will assume, only for the sake of simplicity, as in Ref. [14], that the parameters are real [6]. Note that in accord with our general expectations discussed above, each of the parameters σ, η and ϵ are found to be of order 1/10, as opposed to being [7] $O(1)$ or $O(10^{-2})$, compared to the leading (3,3)-element in Eq. (16). Having determined these parameters, we are led to a total of five predictions involving only the quarks (those for the leptons are listed separately) :

$$m_b^0 \approx m_\tau^0(1 - 8\epsilon^2)\,;\ \text{thus}\ m_b(m_b) \simeq (4.6\text{-}4.9)\,\text{GeV} \tag{18}$$

$$|V_{cb}| \simeq |\sigma - \eta| \approx \left| \sqrt{m_s/m_b}\left|\frac{\eta+\epsilon}{\eta-\epsilon}\right|^{1/2} - \sqrt{m_c/m_t}\left|\frac{\sigma+\epsilon}{\sigma-\epsilon}\right|^{1/2} \right| \simeq 0.045 \tag{19}$$

$$m_d\,(1\text{GeV}) \simeq 8\,\text{MeV} \tag{20}$$

$$\theta_C \simeq \left|\sqrt{m_d/m_s} - e^{i\phi}\sqrt{m_u/m_c}\right| \tag{21}$$

$$|V_{ub}/V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.07\,. \tag{22}$$

In making these predictions, we have extrapolated the GUT-scale values down to low energies using $\alpha_3(m_Z) = 0.118$, a SUSY threshold of 500 GeV and $\tan\beta = 5$. The results depend weakly on these choices, assuming $\tan\beta \approx 2\text{-}30$. Further, the Dirac masses and mixings of the neutrinos and the mixings of the charged leptons also get determined. We obtain :

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[5]Of these, $m_U^0 \approx m_t^0$ can in fact be estimated to within 20% accuracy by either using the argument of radiative electroweak symmetry breaking, or some promising string solutions (see e.g. Ref. [30]).

[6]Babu and I have recently studied supersymmetric CP violation within the G(224)/SO(10) framework, by using precisely the fermion mass-matrices as in Eq.(16). We have observed [32] that complexification of the parameters can lead to observed CP violation, without upsetting in the least the success of Ref. [14] (i.e. of the fermion mass-matrices of Eq.(16)) in describing the masses and mixings of all fermions, including neutrinos. Even with complexification the relative signs and the approximate magnitudes of the real parts of the parameters must be the same as in Eq.(17), to retain the success.

[7]This is one characteristic difference between our work and that of Ref. [39], where the (2,3)-element is even bigger than the (3,3).

$$m^D_{\nu_\tau}(M_U) \approx 100\text{-}120\,\mathrm{GeV};\; m^D_{\nu_\mu}(M_U) \simeq 8\,\mathrm{GeV}, \tag{23}$$

$$\theta^\ell_{\mu\tau} \approx -3\epsilon + \eta \approx \sqrt{m_\mu/m_\tau}\left|\frac{-3\epsilon+\eta}{3\epsilon+\eta}\right|^{1/2} \simeq 0.437 \tag{24}$$

$$m^D_{\nu_e} \simeq [9\epsilon'^2/(9\epsilon^2-\sigma^2)]\,m_U \simeq 0.4\,\mathrm{MeV} \tag{25}$$

$$\theta^\ell_{e\mu} \simeq \left|\frac{\eta'-3\epsilon'}{\eta'+3\epsilon'}\right|^{1/2}\sqrt{m_e/m_\mu} \simeq 0.85\sqrt{m_e/m_\mu} \simeq 0.06 \tag{26}$$

$$\theta^\ell_{e\tau} \simeq \frac{1}{0.85}\sqrt{m_e/m_\tau}\,(m_\mu/m_\tau) \simeq 0.0012\,. \tag{27}$$

In evaluating $\theta^\ell_{e\mu}$, we have assumed ϵ' and η' to be relatively positive.

Given the bizarre pattern of quark and lepton masses and mixings, it seems remarkable that the simple pattern of fermion mass-matrices, motivated by the group theory of G(224)/SO(10), gives an overall fit to all of them (Eqs.(18) through (22)) which is good to within 10%. This includes the two successful predictions on m_b and V_{cb} (Eqs.(18) and (19)). Note that in supersymmetric unified theories, the "observed" value of $m_b(m_b)$ and renormalization-group studies suggest that, for a wide range of the parameter $\tan\beta$, m_b^0 should in fact be about 10-20% *lower* than m_τ^0 [42]. This is neatly explained by the relation: $m_b^0 \approx m_\tau^0(1-8\epsilon^2)$ (Eq. (18)), where exact equality holds in the limit $\epsilon \to 0$ (due to SU(4)-color), while the decrease of m_b^0 compared to m_τ^0 by $8\epsilon^2 \sim 10\%$ is precisely because the off-diagonal ϵ-entry is proportional to B-L (see Eq. (16)).

Specially intriguing is the result on $V_{cb} \approx 0.045$ which compares well with the observed value of $\simeq 0.04$. The suppression of V_{cb}, compared to the value of 0.17 ± 0.06 obtained from Eq. (13), is now possible because the mass matrices (Eq. (16)) contain an antisymmetric component $\propto \epsilon$. That corrects the square-root formula $\theta_{sb} = \sqrt{m_s/m_b}$ (appropriate for symmetric matrices, see Eq. (11)) by the asymmetry factor $|(\eta+\epsilon)/(\eta-\epsilon)|^{1/2}$ (see Eq. (19)), and similarly for the angle θ_{ct}. This factor suppresses V_{cb} if η and ϵ have opposite signs. The interesting point is that, *the same feature necessarily enhances the corresponding mixing angle $\theta^\ell_{\mu\tau}$ in the leptonic sector*, since the asymmetry factor in this case is given by $[(-3\epsilon+\eta)/(3\epsilon+\eta)]^{1/2}$ (see Eq. (24)). This enhancement of $\theta^\ell_{\mu\tau}$ helps to account for the nearly maximal oscillation angle observed at SuperK (as discussed below). This intriguing correlation between the mixing angles in the quark versus leptonic sectors – *that is suppression of one implying enhancement of the other* – has become possible only because of the ϵ-contribution, which is simultaneously antisymmetric and is proportional to B-L. That in turn becomes possible because of the group-property of SO(10) or a string-derived G(224)[4].

Taking stock, we see an overwhelming set of facts in favor of B-L and in fact for the full SU(4)-color-symmetry. These include: (i) the suppression of V_{cb}, together with the enhancement of $\theta^\ell_{\mu\tau}$, just mentioned above, (ii) the successful relation $m_b^0 \approx m_\tau^0(1-8\epsilon^2)$, (iii) the usefulness again of the SU(4)-color-relation $m(\nu^\tau_{Dirac})^0 \approx m_t^0$ in accounting for $m(\nu^\tau_L)$(see Sec. 4), and (iv) the agreement of the relation $|m_s^0/m_\mu^0| = |(\epsilon^2-\eta^2)/(9\epsilon^2-\eta^2)|$ with the data, in that the ratio is naturally *less than* 1, if $\eta \sim \epsilon$. The presence of $9\epsilon^2$ in the denominator is because the off-diagonal entry is proportional to B-L. Finally, the need for (B-L)- as a local symmetry, to implement baryogenesis, has been noted in Sec.1.

Turning to neutrino masses, while all the entries in the Dirac mass matrix N are now fixed, to obtain the parameters for the light neutrinos, one needs to specify those of the Majorana mass matrix of the RH neutrinos ($\nu_R^{e,\mu,\tau}$). Guided by economy and the assumption of hierarchy, we consider the following pattern :

$$M_\nu^R = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R . \tag{28}$$

As discussed in Sec.IV, the magnitude of $M_R \approx$ (5-15) $\times 10^{14}$ GeV can quite plausibly be justified in the context of supersymmetric unificaton[8] (e.g. by using $M \approx M_{st} \approx 4 \times 10^{17}$ GeV in Eq. (8)). To the same extent, the magnitude of $m(\nu_\tau) \approx$ (1/10-1/30) eV, which is consistent with the SuperK value, can also be anticipated. Thus there are effectively three new parameters: x, y, and z. Since there are six observables for the three light neutrinos, one can expect three predictions. These may be taken to be $\theta^{osc}_{\nu_\mu\nu_\tau}$, m_{ν_τ} (see Eq. (10)), and for example $\theta^{osc}_{\nu_e\nu_\mu}$.

Assuming successively hierarchical entries as for the Dirac mass matrices, we presume that $|y| \sim 1/10, |z| \leq |y|/10$ and $|x| \leq z^2$. Now given that $m(\nu_\tau) \sim 1/20$ eV (as estimated in Eq. (10)), the MSW solution for the solar neutrino puzzle [43] suggests that $m(\nu_\mu)/m(\nu_\tau) \approx$ 1/10-1/30. The latter in turn yields : $|y| \approx$ (1/18 to 1/23.6), with y having the same sign as ϵ (see Eq. (17)). This solution for y obtains only by assuming that y is $O(1/10)$ rather than $O(1)$. Combining now with the mixing in the μ-τ sector determined above (see Eq. (24)), one can then determine the ν_μ-ν_τ oscillation angle. The two predictions of the model for the neutrino-system are then :

$$m(\nu_\tau) \approx (1/10 - 1/30)\,\text{eV} \tag{29}$$

$$\theta^{osc}_{\nu_\mu\nu_\tau} \simeq \theta^\ell_{\mu\tau} - \theta^\nu_{\mu\tau} \simeq \left(0.437 + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}}\right) . \tag{30}$$

$$\text{Thus, } \sin^2 2\theta^{osc}_{\nu_\mu\nu_\tau} = (0.96, 0.91, 0.86, 0.83, 0.81) \tag{31}$$

$$\text{for } m_{\nu_2}/m_{\nu_3} = (1/10, 1/15, 1/20, 1/25, 1/30) . \tag{32}$$

Both of these predictions are extremely successful.

Note the interesting point that the MSW solution, together with the requirement that $|y|$ should have a natural hierarchical value (as mentioned above), lead to y having the same sign as ϵ; that (it turns out) implies that the two contributions in Eq.(30) must *add* rather than subtract, leading to an *almost maximal oscillation angle* [14]. The other factor contributing to the enhancement of $\theta^{osc}_{\nu_\mu\nu_\tau}$ is, of course, also the asymmetry-ratio which increases $|\theta^\ell_{\mu\tau}|$ from 0.25 to 0.437 (see Eq. (24)). We see that one can derive rather plausibly a large ν_μ-ν_τ oscillation angle $\sin^2 2\theta^{osc}_{\nu_\mu\nu_\tau} \geq 0.8$, together with an understanding of hierarchical masses and mixings of the quarks and the charged leptons, while maintaining a large hierarchy in

[8]This estimate for M_R is retained even if one allows for ν_μ-ν_τ mixing (see Ref. [14]).

the seesaw derived neutrino masses ($m_{\nu_2}/m_{\nu_3} = 1/10\text{-}1/30$), all within a unified framework including both quarks and leptons. In the example exhibited here, the mixing angles for the mass eigenstates of neither the neutrinos nor the charged leptons are really large, in that $\theta^{\ell}_{\mu\tau} \simeq 0.437 \simeq 23^\circ$ and $\theta^{\nu}_{\mu\tau} \simeq (0.18\text{-}0.31) \approx (10\text{-}18)^\circ$, *yet the oscillation angle obtained by combining the two is near-maximal.* This contrasts with most works in the literature in which a large oscillation angle is obtained either entirely from the neutrino sector (with nearly degenerate neutrinos) or almost entirely from the charged lepton sector.

While $M_R \approx (5\text{-}15) \times 10^{14}$ GeV and $y \approx -1/20$ are better determined, the parameters x and z can not be obtained reliably at present because very little is known about observables involving ν_e. Taking, for concreteness, $m_{\nu_e} \approx (10^{-5}\text{-}10^{-4}$ (1 to few)) eV and $\theta^{osc}_{e\tau} \approx \theta^{\ell}_{e\tau} - \theta^{\nu}_{e\tau} \approx 10^{-3} \pm 0.03$ as inputs, we obtain : $z \sim (1\text{-}5) \times 10^{-3}$ and $x \sim$ (1 to few)$(10^{-6}\text{-}10^{-5})$, in accord with the guidelines of $|z| \sim |y|/10$ and $|x| \sim z^2$. This in turn yields : $\theta^{osc}_{e\mu} \approx \theta^{\ell}_{e\mu} - \theta^{\nu}_{e\mu} \approx 0.06 \pm 0.015$. Note that the mass of $m_{\nu_\mu} \sim 3 \times 10^{-3}$ eV, that follows from a natural hierarchical value for $y \sim -(1/20)$, and $\theta_{e\mu}$ as above, go well with the small angle MSW explanation[9] of the solar neutrinos puzzle.

It is worthnoting that although the superheavy Majorana masses of the RH neutrinos cannot be observed directly, they can be of cosmological significance. The pattern given above and the arguments given in Sec.III and in this section suggests that $M(\nu^{\tau}_R) \approx (5\text{-}15) \times 10^{14}$ GeV, $M(\nu^{\mu}_R) \approx (1\text{-}4) \times 10^{12}$ GeV (for $x \approx 1/20$); and $M(\nu^{e}_R) \sim (1/2\text{-}10) \times 10^{9}$ GeV (for $x \sim (1/2\text{-}10)10^{-6} > z^2$). A mass of $\nu^e_R \sim 10^9$ GeV is of the right magnitude for producing ν^e_R following reheating and inducing lepton asymmetry in ν^e_R decay into $H^0 + \nu^i_L$, that is subsequently converted into baryon asymmetry by the electroweak sphalerons [16,17].

In summary, we have proposed an economical and predictive pattern for the Dirac mass matrices, within the SO(10)/G(224)-framework, which is remarkably successful in describing the observed masses and mixings of *all* the quarks and charged leptons. It leads to five predictions for just the quark- system, all of which agree with observation to within 10%. The same pattern, supplemented with a similar structure for the Majorana mass matrix, accounts for both the large ν_μ-ν_τ oscillation angle and a mass of $\nu_\tau \sim 1/20$ eV, suggested by the SuperK data. Given this degree of success, it makes good sense to study proton decay concretely within this SO(10)/G(224)-framework. The results of this study [14] are presented in the next section.

Before turning to proton decay, it is worth noting that much of our discussion of fermion masses and mixings, including those of the neutrinos, is essentially unaltered if we go to the limit $\epsilon' \to 0$ of Eq. (28). This limit clearly involves:

$$m_u = 0\,,\;\; \theta_C \simeq \sqrt{m_d/m_s}\,,\;\; m_{\nu_e} = 0\,,\;\; \theta^{\nu}_{e\mu} = \theta^{\nu}_{e\tau} = 0\,.$$

$$|V_{ub}| \simeq \sqrt{\frac{\eta - \epsilon}{\eta + \epsilon}}\,\sqrt{m_d/m_b}\,(m_s/m_b) \simeq (2.1)(0.039)(0.023) \simeq 0.0019 \qquad (33)$$

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[9]Although the small angle MSW solution appears to be more generic within the approach outlined above, we have found that the large angle solution can still plausibly emerge in a limited region of parameter space, without affecting our results on fermion masses.

All other predictions remain unaltered. Now, among the observed quantities in the list above, $\theta_C \simeq \sqrt{m_d/m_s}$ is a good result. Considering that $m_u/m_t \approx 10^{-5}$, $m_u = 0$ is also a pretty good result. There are of course plausible small corrections which could arise through Planck scale physics; these could induce a small value for m_u through the (1,1)-entry $\delta \approx 10^{-5}$. For considerations of proton decay, it is worth distinguishing between these two *extreme* variants which we will refer to as cases I and II respectively.

$$\text{Case I}: \epsilon' \approx 2 \times 10^{-4}, \ \delta = 0$$

$$\text{Case II}: \delta \approx 10^{-5}, \ \epsilon' = 0. \tag{34}$$

It is worth noting that the observed value of $|V_{ub}| \approx 0.003$ favors a non-zero value of $\epsilon'(\approx (1-2) \times 10^{-4})$. Thus, in reality, ϵ' may not be zero, but it may lie in between the two extreme values listed above. In this case, the predicted proton lifetime for the standard $d = 5$ operators would be intermediate between those for the two cases, presented in Sec.VI.

VI. EXPECTATIONS FOR PROTON DECAY IN SUPERSYMMETRIC UNIFIED THEORIES

A. Preliminaries

Turning to the main purpose of this talk, I present now the reason why the unification framework based on SUSY SO(10) or G(224), together with the understanding of fermion masses and mixings discussed above, strongly suggest that proton decay should be imminent.

Recall that supersymmetric unified theories (GUTs) introduce two new features to proton decay : (i) First, by raising M_X to a higher value of about 2×10^{16} GeV (contrast with the non-supersymmetric case of nearly 3×10^{14} GeV), they strongly suppress the gauge-boson-mediated $d = 6$ proton decay operators, for which $e^+\pi^0$ would have been the dominant mode (for this case, one typically obtains : $\Gamma^{-1}(p \to e^+\pi^0)|_{d=6} \approx 10^{35.3\pm1.5}$ yrs). (ii) Second, they generate $d = 5$ proton decay operators [18] of the form $Q_iQ_jQ_kQ_l/M$ in the superpotential, through the exchange of color triplet Higginos, which are the GUT partners of the standard Higgs(ino) doublets, such as those in the $5+\overline{5}$ of SU(5) or the 10 of SO(10). Assuming that a suitable doublet-triplet splitting mechanism provides heavy GUT-scale masses to these color triplets and at the same time light masses to the doublets, these "standard" $d = 5$ operators, suppressed by just one power of the heavy mass and the small Yukawa couplings, are found to provide the dominant mechanism for proton decay in supersymmetric GUT [44–47].

Now, owing to (a) Bose symmetry of the superfields in $QQQL/M$, (b) color antisymmetry, and especially (c) the hierarchical Yukawa couplings of the Higgs doublets, it turns out that these standard $d = 5$ operators lead to dominant $\overline{\nu}K^+$ and comparable $\overline{\nu}\pi^+$ modes, but in all cases to highly suppressed $e^+\pi^0$, e^+K^0 and even μ^+K^0 modes. For instance, for minimal SUSY SU(5), one obtains (with $\tan\beta \leq 20$, say) :

$$[\,\Gamma(\mu^+K^0)/\Gamma(\overline{\nu}K^+)\,]^{SU(5)}_{std} \sim [m_u/(m_c \sin^2\theta_c)]^2\, R \approx 10^{-3}, \tag{35}$$

where $R \approx 0.1$ is the ratio of the relevant $|\text{matrix element}|^2 \times$(phase space), for the two modes.

It was recently pointed out that in SUSY unified theories based on SO(10) or G(224), which assign heavy Majorana masses to the RH neutrinos, there exists a new set of color triplets and thereby very likely a *new source* of $d = 5$ proton decay operators [19]. For instance, in the context of the minimal set of Higgs multiplets[10] $\{45_H, 16_H, \overline{16}_H$ and $10_H\}$ (see Sec.V), these new $d = 5$ operators arise by combining three effective couplings introduced before :- i.e., (a) the couplings $f_{ij}16_i16_j\overline{16}_H\overline{16}_H/M$ (see Eq.(7)) that are required to assign Majorana masses to the RH neutrinos, (b) the couplings $g_{ij}16_i16_j16_H16_H/M$, which are needed to generate non-trivial CKM mixings (see Eq.(15)), and (c) the mass term $M_{16}16_H\overline{16}_H$. For the f_{ij} couplings, there are two possible SO(10)-contractions (leading to a 45 or a 1) for the pair $16_i\overline{16}_H$, both of which contribute to the Majorana masses of the RH neutrinos, but only the non-singlet contraction (leading to 45), would contribute to d=5 proton decay operator. In the presence of non-perturbative quantum gravity, one would in general expect the two contractions to have comparable strength. Furthermore, the couplings of 45's lying in the string-tower or possibly below the string-scale, and likewise of singlets, to the $16_i \cdot \overline{16}_H$-pair, would respectively generate the two contractions. It thus seems most likely that both contractions are present, having comparable strength. Allowing for a difference between the relevant projection factors for ν_R masses versus proton decay, and also for the fact that both contractions contribute to the former, but only the non-singlet one (i.e. 45) to the latter, we would set the relevant f_{ij} coupling for proton decay to be $(f_{ij})_p \equiv (f_{ij})_\nu \cdot K$, where $(f_{ij})_\nu$ defined in Sec.IV directly yields ν_R - masses (see Eq.(8)); and K is a relative factor of order unity. As a plausible range, we will take $K \approx 1/3$ to 2 (say). In the presence of the non-singlet contraction, the color-triplet Higginos in $\overline{16}_H$ and 16_H of mass M_{16} can be exchanged between $\tilde{q}_i q_j$ and $\tilde{q}_k q_l$-pairs (correspondingly, for G(224), the color triplets would arise from $(1, 2, 4)_H$ and $(1, 2, \bar{4})_H$). This exchange generates a new set of $d = 5$ operators in the superpotential of the form

$$W_{new} \propto (f_{ij})_\nu \, g_{kl} K \, (16_i \, 16_j) \, (16_k \, 16_l) \, \langle \overline{16}_H \rangle \, \langle 16_H \rangle / M^2 \, \times \, (1/M_{16}), \tag{36}$$

which induce proton decay. Note that these operators depend, through the couplings f_{ij} and g_{kl}, both on the Majorana and on the Dirac masses of the respective fermions. *This is why within SUSY SO(10) or G(224), proton decay gets intimately linked to the masses and mixings of all fermions, including neutrinos.*

B. Framework for Calculating Proton Decay Rate

To establish notations, consider the case of minimal SUSY SU(5) and, as an example, the process $\tilde{c}\tilde{d} \to \bar{s}\bar{\nu}_\mu$, which induces $p \to \bar{\nu}_\mu K^+$. Let the strength of the corresponding $d = 5$ operator, multiplied by the product of the CKM mixing elements entering into wino-exchange vertices, (which in this case is $\sin\theta_C \cos\theta_C$) be denoted by $\hat{A}$. Thus (putting $\cos\theta_C = 1$), one obtains:

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[10]The origin of the new $d = 5$ operators in the context of other Higgs multiplets, in particular in the cases where 126_H and $\overline{126}_H$ are used to break *B-L*, has been discussed in Ref. [19].

$$\hat{A}_{\tilde{c}\tilde{d}}(SU(5)) = (h^u_{22}\, h^d_{12}/M_{H_C})\, \sin\theta_c \simeq (m_c m_s \sin^2\theta_C/v_u^2)\,(\tan\beta/M_{H_C})$$
$$\simeq (1.9\times 10^{-8})\,(\tan\beta/M_{H_C}) \approx (2\times 10^{-24}\,\mathrm{GeV}^{-1})\,(\tan\beta/2)\,(2\times 10^{16}\,\mathrm{GeV}/M_{H_C})\,, \qquad (37)$$

where $\tan\beta \equiv v_u/v_d$, and we have put $v_u = 174$ GeV and the fermion masses extrapolated to the unification-scale – i.e. $m_c \simeq 300$ MeV and $m_s \simeq 40$ MeV. The amplitude for the associated four-fermion process $dus \to \overline{\nu}_\mu$ is given by:

$$A_5(dus \to \overline{\nu}_\mu) = \hat{A}_{\tilde{c}\tilde{d}} \times (2f) \qquad (38)$$

where f is the loop-factor associated with wino-dressing. Assuming $m_{\tilde{w}} \ll m_{\tilde{q}} \sim m_{\tilde{l}}$, one gets: $f \simeq (m_{\tilde{w}}/m_{\tilde{q}}^2)(\alpha_2/4\pi)$. Using the amplitude for $(du)(s\nu_\ell)$, as in Eq. (38), ($\ell = \mu$ or τ), one then obtains [45–47,14]:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+) \approx (0.6\times 10^{31})\,\mathrm{yrs}\,\times$$
$$\left(\frac{0.67}{A_S}\right)^2 \left[\frac{0.014\,\mathrm{GeV}^3}{\beta_H}\right]^2 \left[\frac{(1/6)}{(m_{\tilde{W}}/m_{\tilde{q}})}\right]^2 \left[\frac{m_{\tilde{q}}}{1.2\,\mathrm{TeV}}\right]^2 \left[\frac{2\times 10^{-24}\,\mathrm{GeV}^{-1}}{\hat{A}(\overline{\nu})}\right]^2 . \qquad (39)$$

Here β_H denotes the hadronic matrix element defined by $\beta_H u_L(\vec{k}) \equiv \epsilon_{\alpha\beta\gamma}\langle 0|(d^\alpha_L u^\beta_L)u^\gamma_L|p,\vec{k}\rangle$. While the range $\beta_H = (0.003\text{-}0.03)$ GeV^3 has been used in the past [46], given that one lattice calculation yields $\beta_H = (5.6\pm0.5)\times10^{-3}$ GeV^3 [48], and a recent improved calculation yields $\beta_H \approx 0.014$GeV^3 [49] (whose systematic errors that may arise from scaling violations and quenching are hard to estimate [49]), we will take as a conservative, but plausible, range for β_H to be given by $(0.014\mathrm{GeV}^3)(1/2-2)$. [Compare this with the range for $\beta_H = (0.006\mathrm{GeV}^3)(1/2-2)$ as used in Ref. [14]]. Here, $A_S \approx 0.67$ stands for the short distance renormalization factor of the $d = 5$ operator. Note that the familiar factors that appear in the expression for proton lifetime – i.e., M_{H_C}, $(1+y_{tc})$ representing the interference between the $\tilde{t}$ and $\tilde{c}$ contributions, and $\tan\beta$ (see e.g. Ref. [46] and discussion in the Appendix of Ref. [14]) – are all effectively contained in $\hat{A}(\overline{\nu})$. In Ref. [14], guided by the demand of naturalness (i.e. absence of excessive fine tunning) in obtaining the Higgs boson mass, squark masses were assumed to lie in the range of 1 TeV$(1/\sqrt{2}-\sqrt{2})$, so that $m_{\tilde{q}} \lesssim 1.4$TeV. Recent work, based on the notion of focus point supersymmetry however suggests that squarks may be considerably heavier without conflicting with the demands of naturalness [50]. In the interest of obtaining a conservative upper limit on proton lifetime, we will therefore allow squark masses to be as heavy as about 2.5 TeV and as light as perhaps 600 GeV. [11]

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[11]We remark that if the recently reported (g-2) - anomaly for the muon [51] is attributed to supersymmetry [52], one would need to have extremely light s-fermions (i.e. $m_{\tilde{l}} \approx 200-400$ GeV (say) and correspondingly (for promising mechanisms of SUSY-breaking) $m_{\tilde{q}} \lesssim 300-600$ TeV (say)), and simultaneously large or very large $\tan\beta(\approx 25-50)$. However, not worring about grand unification, such light s-fermions, together with large or very large $\tan\beta$ would typically be in gross conflict with the limits on the edm's of the neutron and the electron, unless on can explain naturally the occurence of minuscule phases ($\lesssim 1/300$ to $1/1000$) and/or large cancellation. Thus,

Allowing for plausible and rather generous uncertainties in the matrix element and the spectrum we take:

$$\beta_H = (0.014\,\mathrm{GeV}^3)\,(1/2\text{-}2)$$

$$m_{\tilde{w}}/m_{\tilde{q}} = 1/6\,(1/2\text{-}2)\,, \quad \text{and} \quad m_{\tilde{q}} \approx m_{\tilde{\ell}} \approx 1.2\,\mathrm{TeV}\,(1/2\text{-}2)\,. \tag{40}$$

Using Eqs.(39-40), we get:

$$\Gamma^{-1}(p \to \overline{\nu}_\tau K^+) \approx (0.6 \times 10^{31}\,\mathrm{yrs})\,[2 \times 10^{-24}\,\mathrm{GeV}^{-1}/\hat{A}(\overline{\nu}_\ell)\,]^2\,\{64\text{-}1/64\}\,. \tag{41}$$

Note that the curly bracket would acquire its upper-end value of 64, which would serve towards maximizing proton lifetime, only provided all the uncertainties in Eq.(41) are stretched to the extreme so that $\beta_H = 0.007$ GeV^3, $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/12$ and $m_{\tilde{q}} \approx 2.4$ TeV. This relation, as well as Eq. (39) are general, depending only on $\hat{A}(\overline{\nu}_\ell)$ and on the range of parameters given in Eq. (40). They can thus be used for both SU(5) and SO(10).

The experimental lower limit on the inverse rate for the $\overline{\nu}K^+$ modes is given by [55],

$$[\sum_\ell \Gamma(p \to \overline{\nu}_\ell K^+)]^{-1}_{expt} \geq 1.6 \times 10^{33}\,\mathrm{yrs}\,. \tag{42}$$

Allowing for all the uncertainties to stretch in the same direction (in this case, the curly bracket = 64), and assuming that just one neutrino flavor (e.g. ν_μ for SU(5)) dominates, the observed limit (Eq.(42)) provides an upper bound on the amplitude[12]:

$$\hat{A}(\overline{\nu}_\ell) \leq 1 \times 10^{-24}\,\mathrm{GeV}^{-1} \tag{43}$$

which holds for both SU(5) and SO(10). Recent theoretical analyses based on LEP-limit on Higgs mass ($\gtrsim$ 114 GeV), together with certain assumptions about MSSM parameters (as

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if the $(g-2)_\mu$ - anomaly turns out to be real, it may quite possibly need a non-supersymmetric explanation, in accord with the edm-constraints which ordinarily seem to suggest that squarks are (at least) moderately heavy ($m_{\tilde{q}} \gtrsim 0.6-1$ TeV, say), and $\tan\beta$ is not too large ($\lesssim$ 3 to 10, say). We mention in passing that the extra vector - like matter - specially a $16 + \overline{16}$ of SO(10) - as proposed in the so-called extended supersymmetric standard model (ESSM) [20,53], with the heavy lepton mass being of order (150-200) hundred GeV, can provide such an explanation [54]. Motivations for the case of ESSM, based on the need for (a) removing the mismatch between MSSM and string unification scales, and (b) dilaton-stabilization, have been noted in Ref. [20]. Since ESSM is an interesting and viable variant of MSSM, and would have important implications for proton decay, we will present the results for expected proton decay rates for the cases of both MSSM and ESSM in the discussion to follow.

[12]If there are sub-dominant $\overline{\nu}_i K^+$ modes with branching ratio R, the right side of Eq. (43) should be divided by $\sqrt{1+R}$.

in CMSSM) and/or constaint from muon g-2 anomaly [51] suggest that $\tan\beta \gtrsim 3$ to 5 [56]. In the interest of getting a conservative upper limit on proton lifetime, we will therefore use, as a conservative lower limit, $\tan\beta \geq 3$. We will however exhibit relevant results often as a function of $\tan\beta$ and exhibit proton lifetimes corresponding to higher values of $\tan\beta$ as well. For minimal SU(5), using Eq.(37) and, conservatively $\tan\beta \geq 3$, one obtains a lower limit on M_{HC} given by:

$$M_{HC} \geq 5.5 \times 10^{16}\,\text{GeV} \quad (\text{SU}(5)) \tag{44}$$

At the same time, higher values of $M_{HC} > 3 \times 10^{16}$ GeV do not go very well with gauge coupling unification. Thus we already see a conflict, in the case of minimal SUSY SU(5), between the experimental limit on proton lifetime on the one hand, and coupling unification and constraint on $\tan\beta$ on the other hand. To see this conflict another way, if we keep $M_{HC} \leq 3 \times 10^{16}$ GeV (for the sake of coupling unification) we obtain from Eq.(37): $\hat{A}(\text{SU}(5)) \geq 1.9 \times 10^{-24}\,\text{GeV}^{-1}(\tan\beta/3)$. Using Eq. (41), this in turn implies that

$$\Gamma^{-1}(p \to \overline{\nu}K^+) \leq 0.6 \times 10^{33}\,\text{yrs} \times (3/\tan\beta)^2 \quad (\text{SU}(5)) \tag{45}$$

For $\tan\beta \geq 3$, a lifetime of 0.7×10^{33} years is thus a conservative upper limit. In practice, it is unlikely that all the uncertainties, including these in M_{HC} and $\tan\beta$, would stretch in the same direction to nearly extreme values so as to prolong proton lifetime. A more reasonable upper limit, for minimal SU(5), thus seems to be: $\Gamma^{-1}(p \to \overline{\nu}K^+)(SU(5)) \leq (0.3) \times 10^{33}$ yrs. Given the experimental lower limit (Eq.(42)), we see that minimal SUSY SU(5) is already excluded (or strongly disfavored) by proton decay-searches. We have of course noted in Sec.IV that SUSY SU(5) does not go well with neutrino oscillations observed at SuperK.

Now, to discuss proton decay in the context of supersymmetric SO(10), it is necessary to discuss first the mechanism for doublet-triplet splitting. Details of this discussion may be found in Ref. [14]. A synopsis is presented in the Appendix.

C. Proton Decay in Supersymmetric SO(10)

The calculation of the amplitudes $\hat{A}_{std}$ and $\hat{A}_{new}$ for the standard and the new operators for the SO(10) model, are given in detail in Ref. [14]. Here, I will present only the results. It is found that the four amplitudes $\hat{A}_{std}(\overline{\nu}_\tau K^+)$, $\hat{A}_{std}(\overline{\nu}_\mu K^+)$, $\hat{A}_{new}(\overline{\nu}_\tau K^+)$ and $\hat{A}_{new}(\overline{\nu}_\mu K^+)$ are in fact very comparable to each other, within about a factor of two to five, either way. Since there is no reason to expect a near cancellation between the standard and the new operators, especially for both $\overline{\nu}_\tau K^+$ and $\overline{\nu}_\mu K^+$ modes, we expect the net amplitude (standard + new) to be in the range exhibited by either one. Following Ref. [14], I therefore present the contributions from the standard and the new operators separately.

One important consequence of the doublet-triplet splitting mechanism for SO(10) outlined briefly in the appendix and in more detail in Ref. [14] is that the standard d=5 proton decay operators become inversely proportional to $M_{eff} \equiv [\lambda < 45_H >]^2/M_{10'} \sim M_X^2/M_{10'}$, rather than to M_{H_c}. Here, $M_{10'}$ represents the mass of $10'_H$, that enters into the D-T splitting mechanism through effective coupling $\lambda 10_H 45_H 10'_H$ in the superpotential (see Appendix, Eq.(A1)). As noted in Ref. [14], $M_{10'}$ can be naturally suppressed (due to flavor symmetries) compared to M_X, and thus M_{eff} correspondingly larger than M_X by even one to three orders of magnitude. It should be stressed that M_{eff} does not represent the physical masses of the color triplets or of the other particles in the theory. It is simply a parameter of order $M_X^2/M_{10'}$. *Thus larger values of M_{eff}, close to or even exceeding the Plank scale, do not in any way imply large corrections from quantum gravity.* Now accompanying the suppression due to M_{eff}, the standard proton decay amplitudes for SO(10) possess an intrinsic enhancement as well, compared to those for SU(5), owing primarily due to differences in their Yukawa couplings for the up sector (see Appendix C of Ref. [14]). As a result of this enhancement, combined with the suppression due to higher values of M_{eff}, a typical standard $d = 5$ amplitude for SO(10) is given by (see Appendix C of Ref. [14])

$$\hat{A}(\bar{\nu}_\mu K^+)_{std}^{SO(10)} \approx (h_{33}^2/M_{eff})(2 \times 10^{-5}),$$

which should be compared with $\hat{A}(\bar{\nu}_\mu K^+)_{std}^{SU(5)} \approx (1.9 \times 10^{-8})(\tan\beta/M_{H_c})$ (see Eq.(37)). Note, taking $h_{33}^2 \approx 1/4$, the ratio of a typical SO(10) over SU(5) amplitude is given by $(M_{H_c}/M_{eff})(88)(3/\tan\beta)$. Thus the enhancement by a factor of about 88 (for $\tan\beta = 3$), of the SO(10) compared to the SU(5) amplitude, is compensated in part by the suppression that arises from M_{eff} being larger than M_{H_c}.

In addition, note that in contrast to the case of SU(5), the SO(10) amplitude does not depend *explicitly* on $\tan\beta$. The reason is this: if the fermions acquire masses only through the 10_H in SO(10), as is well known, the up and down quark Yukawa couplings will be equal. By itself, it would lead to a large value of $\tan\beta = m_t/m_b \approx 60$ and thereby to a large enhancement in proton decay amplitude. Furthermore, it would also lead to the bad relations: $m_c/m_s = m_t/m_b$ and $V_{CKM} = 1$. However, in the presence of additional Higgs multiplets, in particular with the mixing of $(16_H)_d$ with 10_H (see Appendix and Sec.V), (a) $\tan\beta$ can get lowered to values like 3-20, (b) fermion masses get contributions from both $< 16_H >_d$ and $< 10_H >$, which correct all the bad relations stated above, and simultaneously (c) the explicit dependence of $\hat{A}$ on $\tan\beta$ disappears. It reappears, however, through restriction on threshold corrections, discussed below.

Although M_{eff} can far exceed M_X, it still gets bounded from above by demanding that coupling unification, as observed [13], should emerge as a natural prediction of the theory as opposed to being fortuitous. That in turn requires that there be no large (unpredicted) cancellation between GUT-scale threshold corrections to the gauge couplings that arise from splittings within different multiplets as well as from Plank scale physics. Following this point of view, we have argued (see Appendix) that the net "other" threshold corrections to $\alpha_3(m_Z)$

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[13] For instance, in the absence of GUT-scale threshold corrections, the MSSM value of $\alpha_3(m_Z)_{MSSM}$, assuming coupling unification, is given by $\alpha_3(m_Z)^\circ_{MSSM} = 0.125 - 0.13$ [7], which is about 5-8% higher than the observed value: $\alpha_3(m_Z)^\circ_{MSSM} = 0.118 - 0.003$ [13]. We demand that this discrepancy should be accounted for accurately by a net *negative* contribution from D-T splitting and from "other" threshold corrections (see Appendix, Eq.(A4)), without involving large cancellations. That in fact does happen for the minimal Higgs system $(45, 16, \overline{16})$ [see Ref. [14]].

arising from the Higgs (in our case 45_H, 16_H and $\overline{16}_H$) and the gauge multiplets should be negative, but conservatively and quite plausibly no more than about 10%. This in turn restricts how big can be the threshold corrections to $\alpha_3(m_Z)$ that arise from (D-T) splitting (which is positive). Since the latter is proportional to $\ln(M_{eff}\cos\gamma/M_X)$ (see Appendix), we thus obtain an upper limit on $M_{eff}\cos\gamma$. For the simplest model of D-T splitting presented in Ref. [14] and in the Appendix (Eq.(A1)), one obtains: $\cos\gamma \approx (\tan\beta)/(m_t/m_b)$. An upper limit on $M_{eff}\cos\gamma$ thus provides an upper limit on M_{eff} which is inversely proportional to $\tan\beta$. In short, our demand of natural coupling unification, together with the simplest model of D-T splitting, introduces an implicit dependence on $\tan\beta$ into the lower limit of the SO(10) - amplitude - i.e. $\hat{A}(SO(10)) \propto 1/M_{eff} \geq$ (a quantity) $\propto \tan\beta$. These considerations are reflected in the results given below.

Assuming $\tan\beta \geq 3$ and accurate coupling unification (as described above), one obtains for the case of MSSM, a conservative upper limit on $M_{eff} \leq 2.7 \times 10^{18}$ GeV $(3/\tan\beta)$ (see Appendix and Ref. [14]). Using this upper limit, we obtain a lower limit for the standard proton decay amplitude given by

$$\hat{A}(\overline{\nu}_\tau K^+)_{std} \geq \begin{bmatrix} (7.8\times 10^{-24}\,\mathrm{GeV}^{-1})\,(1/6\text{-}1/4) & \mathrm{caseI} \\ (3.3\times 10^{-24}\,\mathrm{GeV}^{-1})\,(1/6\text{-}1/2) & \mathrm{case\ II} \end{bmatrix} \begin{pmatrix} \mathrm{SO(10)/MSSM,\ with} \\ \tan\beta \geq 3 \end{pmatrix}. \quad (46)$$

Substituting into Eq.(41) and adding the contribution from the second competing mode $\overline{\nu}_\mu K^+$, with a typical branching ratio $R \approx 0.3$, we obtain

$$\Gamma^{-1}(\overline{\nu}K^+)_{std} \leq \begin{bmatrix} (0.7\times 10^{31}\,\mathrm{yrs.})\,(1.6\text{-}0.7) \\ (1.5\times 10^{31}\,\mathrm{yrs.})\,(4\text{-}0.44) \end{bmatrix} \{64\text{-}1/64\} \begin{pmatrix} \mathrm{SO(10)/MSSM,\ with} \\ \tan\beta \geq 3 \end{pmatrix}. \quad (47)$$

The upper and lower entries in Eqs.(46) and (47) correspond to the cases I and II of the fermion mass-matrix with the *extreme values* of ϵ' - i.e. $\epsilon' = 2\times 10^{-4}$ and $\epsilon' = 0$ - respectively, (see Eq.(34)). The uncertainty shown inside the square brackets correspond to that in the relative phases of the different contributions. The uncertainty of {64 to 1/32} arises from that in β_H, $(m_{\tilde{W}}/m_{\tilde{q}})$ and $m_{\tilde{q}}$ (see Eq.(40)). Thus we find that for MSSM embedded in SO(10), for the two extreme values of ϵ' (cases I and II) as mentioned above, the inverse partial proton decay rate should satisfy:

$$\Gamma^{-1}(p \to \overline{\nu}K^+)_{std} \leq \begin{bmatrix} 0.7\times 10^{31^{+2.0}_{-1.7}}\,\mathrm{yrs.} \\ 1.3\times 10^{31^{+2.4}_{-1.86}}\,yrs. \end{bmatrix} \leq \begin{bmatrix} 0.7\times 10^{33}\,\mathrm{yrs.} \\ 3.7\times 10^{33}\,\mathrm{yrs.} \end{bmatrix} \begin{pmatrix} \mathrm{SO(10)/MSSM,\ with} \\ \tan\beta \geq 3 \end{pmatrix}. \quad (48)$$

The central value of the upper limit in Eq.(48) corresponds to taking the upper limit on $M_{eff} \leq 2.7 \times 10^{18}$ GeV, which is obtained by restricting threshold corrections as described above (and in the Appendix) and by setting (conservatively) $\tan\beta \geq 3$. The uncertainties of matrix element, spectrum and choice of phases are reflected in the exponents.The uncertainty in the most sensitive entry of the fermion mass matrix - i.e. ϵ' - is fully incorporated (as regards obtaining an upper limit on the lifetime) by going from case I (with $\epsilon' = 2 \times 10^{-4}$) to case II ($\epsilon' = 0$). Note that this increases the lifetime by almost a factor of six. Any non-vanishing intermediate value of ϵ' would only shorten the lifetime compared to case II. In this sense, the larger of the two upper limits quoted above is rather conservative. We see that the predicted upper limit for case I of MSSM (with the extreme value of $\epsilon' = 2 \times 10^{-4}$)

is already in conflict with the empirical lower limit (Eq.(43)) while that for case II i.e. $\epsilon' = 0$ (with all the uncertainties stretched as mentioned above) is only about two times higher than the empirical limit.

Thus the case of MSSM embedded in SO(10) is already tightly constrained, to the point of being rather disfavored, by the limit on proton lifetime in that all the parameters need to lie near their "extreme" ends so that it may be compatible with the empirical limit (see also results for other choices of parameters listed in Table 1). The constraint is of course augmented especially by our requirement of natural coupling unification which prohibits accidental large cancellation between different threshold corrections (see Appendix); and it will be even more severe, especially within the simplest mechanism of D-T splitting (as discussed in the Appendix), if $\tan\beta$ turns out to be larger than 5 (say). On the positive side, improvement in the current limit by a factor of even 2 to 3 ought to reveal proton decay, otherwise the case of MSSM embedded in SO(10), would be clearly excluded.

D. The case of ESSM

Before discussing the contribution of the new $d = 5$ operators to proton decay, an interesting possibility, mentioned in the introduction, that would be especially relevant in the context of proton decay, if $\tan\beta$ is large, is worth noting. This is the case of the extended supersymmetric standard model (ESSM), which introduces an extra pair of vector-like families ($16 + \overline{16}$ of SO(10)), at the TeV scale [20,53]. Adding such complete SO(10)-multiplets would of course preserve coupling unification. From the point of view of adding extra families, ESSM seems to be the minimal and also the maximal extension of the MSSM, that is allowed in that it is compatible with (a) neutrino-counting, (b) precision electroweak tests, as well as (c) a semi-perturbative as opposed to non-perturbative gauge coupling unification [20,53]. [14] *The existence of two extra vector-like families can of course be tested at the LHC.*

Theoretical motivations for the case of ESSM arise because, (a) it raises $\alpha_{\rm unif}$ to a semi-perturbative value of 0.25 to 0.3, and therefore has a better chance to achieve dilaton-stabilization than the case of MSSM, for which $\alpha_{\rm unif}$ is rather weak (only 0.04); and (b) owing to increased two-loop effects [20,57], it raises the unification scale M_X to $(1/2-2)\times 10^{17}$GeV and thereby considerably reduces the problem of a mismatch [27] between the MSSM and the string unification scales (see Sec.III). A third feature relevant to proton decay is the following. In the absence of unification-scale threshold and Planck-scale effects, the ESSM value of $\alpha_3(m_Z)$ obtained by assuming gauge coupling unification, which we denote by $\alpha_3(m_Z)^\circ_{\rm ESSM}$ is lowered to about $0.112 - 0.118$ [20], compared to $\alpha_3(m_Z)^\circ_{\rm MSSM} \approx 0.125 - 0.13$.

As explained in the appendix, the net result of these two effects - i.e. a raising of M_X and a lowering of $\alpha_3(m_Z)^\circ_{\rm ESSM}$ - is that for ESSM embedded in SO(10), $\tan\beta$ can span a wide range from 3 to even 30, and simultaneously the value or the upper limit on M_{eff} can range from (60 to 6) $\times\, 10^{18}$GeV, in full accord with our criterion for accurate coupling unification

[14] For instance, addition of two pairs of vector-like families at the TeV-scale, to the three chiral families, would cause gauge couplings to become non-perturbative below the unification scale.

discussed above.

Thus, in contrast to MSSM, ESSM allows for larger values of $\tan\beta$ (like 20 or 30), without needing large threshold corrections, and simultaneously without conflicting with the limit on proton lifetime.

To be specific, consider first the case of a moderately large $\tan\beta = 20$ (say), for which one obtains $M_{eff} \approx 9 \times 10^{18}$ GeV, with the "other" threshold correction $-\delta_3'$ being about 5% (see Appendix for definition). In this case, one obtains:

$$\Gamma^{-1}(\bar{\nu}K^+)_{std} \approx \begin{bmatrix} (1.6 - 0.7) \\ (10 - 1) \end{bmatrix} \{64 - 1/64\} \, (7 \times 10^{31} \, \text{yrs}) \begin{pmatrix} \text{SO(10)/ESSM, with} \\ \tan\beta = 20 \end{pmatrix}. \quad (49)$$

As before, the upper and lower entries correspond to cases I ($\epsilon' = 2 \times 10^{-4}$) and II ($\epsilon' = 0$) of the fermion mass-matrix (see Eq.(34)). The uncertainty in the upper and lower entries in the square bracket of Eq.(49) corresponds to that in the relative phases of the different contributions for the cases I and II respectively, while the factor {64-1/64} corresponds to uncertainties in the SUSY spectrum and the matrix element (see Eq.40).

We see that by allowing for an uncertainty of a factor of $(30 - 100)$ jointly from the two brackets for Case I (and $(13 - 44)$ for Case II), proton lifetime arising from the standard operators would be expected to lie in the range of $(2.2 - 7.5) \times 10^{33}$ yrs, for the case of ESSM embedded in SO(10), with $\tan\beta = 20$. Such a range is compatible with present limits, but accessible to searches in the near future.

The other most important feature of ESSM is that, by allowing for larger values of M_{eff}, especially for smaller values of $\tan\beta \approx 3$ to 10 (say), *the contribution of the standard operators by itself can be perfectly consistent with present limit on proton lifetime even for almost central or "median" values of the parameters pertaining to the SUSY spectrum, the relevant matrix element, ϵ' and the phase-dependent factor.*

For instance, for ESSM, one obtains $M_{eff} \approx (4.5 \times 10^{19}\text{GeV})(4/tan\beta)$, with the "other" threshold correction - δ_3' being about 5% (see Appendix and Eq.(A6)). Now, *combining* cases I ($\epsilon' = 2 \times 10^{-4}$) and II ($\epsilon' = 0$), we see that the square bracket in Eq.(49) which we will denote by [S], varies from 0.7 to 10, depending upon the relative phases of the different contributions and the values of ϵ'. Thus as a "median" value, we will take $[S]_{med} \approx 2$ to 6. The curly bracket {64-1/64}, to be denoted by {C}, represents the uncertainty in the SUSY spectrum and the matrix element (see Eq.(40)). Again as a "nearly central" or "median" value, we will take $\{C\}_{med} \approx 1/6$ to 6. Setting M_{eff} as above we obtain

$$\Gamma^{-1}(\bar{\nu}K^+)_{std}^{\text{"median"}} \approx [S]_{med}\{C\}_{med}(1.8 \times 10^{33} \text{ yrs})(4/\tan\beta)^2(\text{SO(10)/ESSM}). \quad (50)$$

Choosing a few sample values of the effective parameters [S] and {C}, with low values of $\tan\beta = 4$ to 10, the corresponding values of $\Gamma^{-1}(\bar{\nu}K^+)$, following from Eq.(50), are listed below in Table 1.

Note that ignoring contributions from the new d=5 operators for a moment [15], the entries in Table 1 represent *a very plausible range of values* for the proton lifetime, for the case of

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[15] As I will discuss in the next section, we of course expect the new d=5 operators to be important

ESSM embedded in SO(10), with $\tan\beta \approx 3$ to 10 (say), *rather than upper limits for the same.* This is because they are obtained for "nearly central" or "median" values of

TABLE 1. PROTON LIFETIME, BASED ON CONTRIBUTIONS FROM ONLY THE STANDARD OPERATORS FOR THE CASE OF ESSM EMBEDDED IN SO(10), WITH PARAMETERS BEING IN THE "MEDIAN" RANGE

$\tan\beta = 4$ [S]=2.7 {C}=1/2 to 2	$\tan\beta = 4$ [S]=6 {C}=1/6 to 1	$\tan\beta = 10$ [S]=5.4 {C}=1 to 6	$\tan\beta = 10$ [S]=6 {C}=1 to 4
$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (2.5 to 10) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.8 to 11) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.6 to 10) $\times 10^{33}$ yrs	$\Gamma^{-1}(\bar{\nu}K^+)^{std}_{ESSM} \approx$ (1.8 to 7.3) $\times 10^{33}$ yrs

the parameters represented by the values of [S]$\approx$ 2 to 6 and {C}$\approx$ 1/6 to 6, as discussed above. For instance, consider the cases {C}=1 and {C}=1/6 respectively, both of which (as may be inferred from the table) can quite plausibly yield proton lifetimes in the range of 10^{33} to 10^{34} yrs. Now {C}=1 corresponds, e.g., to $\beta_H = 0.014\text{GeV}^3$ (the central value of Ref. [49]) $m_{\tilde{q}} = 1.2$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} = 1/6$ (see Eq.(40)), while that of {C}=1/6 would correspond, for example, to $\beta_H = 0.014\text{GeV}^3$, with $m_{\tilde{q}} \approx 600$GeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/5$. *In short, for the case of ESSM, with low values of* $\tan\beta \approx$ *3 to 10 (say), squark masses can be well below 1 TeV, without conflicting with present limit on proton lifetime.* This feature is not permissible within MSSM embedded in SO(10).

Thus, confining for a moment to the standard operators only, if ESSM represents low-energy physics, and if $\tan\beta$ is rather small (3 to 10, say), we do not have to stretch at all the uncertainties in the SUSY spectrum and the matrix elements to their extreme values (in contrast to the case of MSSM) in order to understand why proton decay has not been seen as yet, and still can be optimistic that it ought to be discovered in the near future, with a lifetime $\leq 10^{34}$ years. The results for a wider variation of the parameters are listed in Table 2, where contributions of the new d=5 operators are also shown.

It should also be remarked that if in the unlikely event, all the parameters (i.e. β_H, $(m_{\tilde{W}}/m_{\tilde{q}})$, $m_{\tilde{q}}$ and the phase-dependent factor) happen to be closer to their extreme values so as to extend proton lifetime, and if $\tan\beta$ is small ($\approx$ 3 to 10, say) and at the same time the value of M_{eff} is close to its allowed upper limit (see Appendix), the standard d=5 operators by themselves would tend to yield proton lifetimes exceeding even (1/3 to 1)$\times 10^{35}$ years for the case of ESSM, (see Eq.(49) and Table 2). In this case (with the parameters having nearly extreme values), however, as I will discuss shortly, the contribution of the new d=5 operators related to neutrino masses (see Eq.(36)), would dominate and quite naturally yield lifetimes bounded above in the range of $(1-10)\times 10^{33}$ years (see Sec.VI E and Table 2). *Thus in the*

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and significantly infuence proton lifetime (see e.g. Table 2). Entries in Table 1 could still represent the actual expected values of proton lifetimes, however, if the parameter K defined in VI A (also see VI E) happens to be unexpectedly small ($\ll 1$).

presence of the new operators, the range of $(10^{33} - 10^{34})$ *years for proton lifetime is not only very plausible but it also provides a conservative upper limit, for the case of ESSM embedded in SO(10).*

E. Contribution from the new d=5 operators

As mentioned in Sec.VI A, for supersymmetric G(224)/SO(10), there very likely exists a new set of d=5 operators, related to neutrino masses, which can induce proton decay (see, Eq.(42)). The decay amplitude for these operators for the leading mode (which in this case is $\bar{\nu}_\mu K^+$) becomes proportional to the quantity $P \equiv \{(f_{33})_\nu \langle\overline{16}_H\rangle/M\}h_{33}K/(M_{16}\tan\gamma)$, where $(f_{33})_\nu$ and h_{33} are the effective couplings defined in Eqs.(7) and (15) respectively, and M_{16} and $\tan\gamma$ are defined in the Appendix. The factor K, defined by $(f_{33})_p \equiv (f_{33})_\nu K$, is expected to be of order unity (see Sec.VIA for the origin of K). As a plausible range, we take $K \approx 1/3$ to 2. Using $M_{16}\tan\gamma = \lambda'\langle\overline{16}_H\rangle$ (see Appendix), and $h_{33} \approx 1/2$ (given by top mass), one gets: $P \approx ((f_{33})_\nu/M)(1/2\lambda')K$. Here M denotes the string or the Planck scale (see Sec.IV and footnote 2); thus $M \approx (1/2 - 1) \times 10^{18}$GeV; and λ' is a quartic coupling defined in the appendix. Validity of perturbative calculation suggests that λ' should not much exceed unity, while other considerations suggest that λ' should not be much less than unity either (see Ref. [14], Sec.6E). Thus, a plausible range for λ' is given by $\lambda' \approx (1/2 - \sqrt{2})$. (Note it is only the upper limit on λ' that is relevant to obtaining an upper limit on proton lifetime). Finally, from consideration of ν_τ mass, we have $(f_{33})_\nu \approx 1$ (see Sec.IV). We thus obtain: $P \approx (5 \times 10^{-19}\text{GeV}^{-1})(1/\sqrt{2}$ to 4)K. Incorporating a further uncertainty by a factor of (1/2 to 2) that arises due to choice of the relative phases of the different contributions (see Ref. [14]), the effective amplitude for the new operator is given by

$$\hat{A}(\bar{\nu}_\mu K^+)_{new} \approx (1.5 \times 10^{-24}\text{GeV}^{-1})(1/2\sqrt{2} \text{ to } 8)K \tag{51}$$

Note that this new contribution is independent of M_{eff}; *thus it is the same for ESSM as it is for MSSM, and it is independent of* $\tan\beta$. Furthermore, it turns out that the new contribution is also insensitive to ϵ'; thus it is nearly the same for cases I and II of the fermion mass-matrix. Comparing Eq.(51) with Eq.(46) we see that the new and the standard operators are typically quite comparable to one another. Since there is no reason to expect near cancellation between them (especially for both $\bar{\nu}_\mu K^+$ and $\bar{\nu}_\tau K^+$ modes), we expect the net amplitude (standard+new) to be in the range exhibited by either one. It is thus useful to obtain the inverse decay rate assuming as if the new operator dominates. Substituting Eq.(51) into Eq.(41) and allowing for the presence of the $\bar{\nu}_\tau K^+$ mode with an estimated branching ratio of nearly 0.4 (see Ref. [14]), one obtains

$$\Gamma^{-1}(\bar{\nu}K^+)_{new} \approx (1 \times 10^{31}\,\text{yrs})\,[8\text{-}1/64]\,\{64\text{-}1/64\}(K^{-2} \approx 9 \text{ to } 1/4)\,. \tag{52}$$

The square bracket represents the uncertainty reflected in Eq.(51), while the curly bracket corresponds to that in the SUSY spectrum and matrix element (Eq.(40)). Allowing for a net uncertainty at the upper end by as much as a factor of 100 to 600 (say), arising jointly from the *three brackets* in Eq.(52), which can be realized by keeping the SUSY-spectrum and the matrix element in the "nearly-central" or "intermediate" range (see below), the new operators related to neutrino masses, by themselves, lead to a proton decay lifetime given by:

$$\Gamma^{-1}(\bar{\nu}K^+)^{Median}_{new} \approx (0.7\text{-}5) \times 10^{33}\,\text{yrs. (SO(10) or string G(224))(Indep. of } \tan\beta). \quad (53)$$

For instance, taking the curly bracket in Eq.(52) to be $\approx$ 4 to 10 (say) (corresponding for example, to $\beta_H = 0.012$ GeV^3, $(m_{\tilde{W}}/m_{\tilde{q}}) \approx 1/10$ to $1/12$ and $m_{\tilde{q}} \approx$ (1 to 1.3)(1.2 TeV)), instead of its extreme value of 64, and setting the square bracket in Eq.(52) to be ≈ 6, and $K^{-2} \approx 9$, which are quite plausible, we obtain: $\Gamma^{-1}(\bar{\nu}K^+)_{new} \approx (2.2 - 4) \times 10^{33}$ yrs; independently of $\tan\beta$, for both MSSM and ESSM. Proton lifetime for other choices of parameters, which lead to similar conclusion, are listed in Table 2.

It should be stressed that the standard $d = 5$ operators (mediated by the color-triplets in the 10_H of SO(10)) may naturally be absent for a string-derived G(224)-model (see e.g. Ref. [29] and [30]), but the new $d = 5$ operators, related to the Majorana masses of the RH neutrinos and the CKM mixings, should very likely be present for such a model, as much as for SO(10). These would induce proton decay [16]. *Thus our expectations for the proton decay lifetime (as shown in Eq. (53)) and the prominence of the μ^+K^0 mode (see below) hold for a string-derived G(224)-model, just as they do for SO(10).* For a string - G(224) - model, however, the new d=5 operators would be essentially the sole source of proton decay.

Nearly the same situation emerges for the case of ESSM embedded in G(224) or SO(10), with low $\tan\beta(\approx$ 3 to 10, say), especially if the parameters (including β_H, $m_{\tilde{W}}/m_{\tilde{q}}$, $m_{\tilde{q}}$, the phase-dependent factor as well as M_{eff}) happen to be somewhat closer to their extreme values so as to extend proton lifetime. In this case, as noted in the previous sub-section, the contribution of the standard d=5 operators would be suppressed; and proton decay would proceed primarily via the new operators with a lifetime quite naturally in the range of $10^{33} - 10^{34}$ years, as exhibited above.

F. The Charged Lepton Decay Modes ($p \to \mu^+K^0$ and $p \to e^+\pi^0$)

I now note a distinguishing feature of the SO(10) or the G(224) model presented here. Allowing for uncertainties in the way the standard and the new operators can combine with each other for the three leading modes i.e. $\bar{\nu}_\tau K^+$, $\bar{\nu}_\mu K^+$ and μ^+K^0, we obtain (see Ref. [14] for details):

$$B(\mu^+K^0)_{std+new} \approx [1\% \text{ to } 50\%]\ \kappa \quad \text{(SO(10) or string G(224))} \qquad (54)$$

where κ denotes the ratio of the squares of relevant matrix elements for the μ^+K^0 and $\bar{\nu}K^+$ modes. In the absence of a reliable lattice calculation for the $\bar{\nu}K^+$ mode, one should remain open to the possibility of $\kappa \approx 1/2$ to 1 (say). We find that for a large range of parameters,

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[16] In addition, quantum gravity induced d=5 operators are also expected to be present at some level, depending upon the degree of suppression of these operators due to flavor symmetries (see e.g. Ref. [33]).

the branching ratio $B(\mu^+K^0)$ can lie in the range of 20 to 40% (if $\kappa \approx 1$). This prominence of the μ^+K^0 mode for the SO(10)/G(224) model is primarily due to contributions from the new d=5 operators. This contrasts sharply with the minimal SU(5) model, in which the μ^+K^0 mode is expected to have a branching ratio of only about 10^{-3}. In short, prominence of the μ^+K^0 mode, if seen, would clearly show the relevance of the new operators, and thereby reveal the proposed link between neutrino masses and proton decay [19].

While the d=5 operators as described here (standard and new) would lead to highly suppressed $e^+\pi^0$ mode, for MSSM or ESSM embedded in SO(10), the gauge-mediated d=6 operators, can still give proton decay into $e^+\pi^0$ with an inverse rate $\approx 10^{35.3\pm1.5}$ years, which can be as short as about 10^{34} yrs. Thus, even within supersymmetric unification, the $e^+\pi^0$ mode may well be a prominent one, competing favorably with (even) the $\bar{\nu}K^+$ mode.

G. Section Summary

In summary, our study of proton decay has been carried out within the supersymmetric SO(10) or the G(224)-framework[17], with special attention paid to its dependence on fermion masses and threshold effects. A representative set of results corresponding to different choices of parameters is presented in Tables 1 and 2. The study strongly suggests that, for either MSSM or ESSM embedded in SO(10) or G(224), an upperlimit on proton lifetime is given by

$$\tau_{proton} \leq (1/2\text{-}1) \times 10^{34}\ \text{yrs}\,, \tag{55}$$

with $\overline{\nu}K^+$ being the dominant decay mode, and μ^+K^0 being prominent. Although there are uncertainties in the matrix element, in the SUSY-spectrum, in the phase-dependent factor, $\tan\beta$ and in certain sensitive elements of the fermion mass matrix, notably ϵ' (see Eq.(48) for predictions in cases I versus II), this upper limit is obtained, for the case of MSSM embedded in SO(10), by allowing for a generous range in these parameters and stretching all of them in the same direction so as to extend proton lifetime. In this sense, while the predicted lifetime spans a wide range, the upper limit quoted above, in fact more like 3×10^{33} yrs, is most conservative, for the case of MSSM (see Eq.(48) and Table 1). It is thus tightly constrained already by the empirical lower limit on $\Gamma^{-1}(\overline{\nu}K^+)$ of 1.6×10^{33} yrs. For the case of ESSM embedded in SO(10), the standard d=5 operators are suppressed compared to the case of MSSM; as a result, by themselves they can naturally lead to lifetimes in the range of $(3-10) \times 10^{33}$ yrs., for nearly central values of the parameters pertaining to the SUSY-spectrum and the matrix element (see Eq.(50)) and Table 1. Including the contribution of the new d=5 operators, and allowing for a wide variation of the parameters mentioned above, one finds that the range of $(10^{33} - 10^{34})$ yrs for proton lifetime is not only very plausible but it also provides a rather conservative upper limit, for the case of ESSM embedded in either SO(10) or G(224) (see Sec.VI E and Table 2). Thus our study provides a clear reason to expect that the discovery of proton decay should be imminent for the case of ESSM, and

[17]As described in Secs.III, IV and V.

even more so for that of MSSM. The implication of this prediction for a next-generation detector is emphasized in the next section.

VII. CONCLUDING REMARKS

The preceding sections show that, but for one missing piece – proton decay – the evidence in support of grand unification is now strong. It includes: (i) the observed family-structure, (ii) the meeting of the gauge couplings, (iii) neutrino-oscillations, (iv) the intricate pattern of the masses and mixings of all fermions, including the neutrinos, and (v) the need for $B-L$ as a generator, to implement baryogenesis. Taken together, these not only favor grand unification but in fact select out a particular route to such unification, based on the ideas of supersymmetry, SU(4)-color and left-right symmetry. Thus they point to the relevance of an effective string-unified G(224) or SO(10)-symmetry.

Based on a systematic study of proton decay within the supersymmetric SO(10)/G(224)-framework [14], which is clearly favored by the data, and an update as presented here, I have argued that a conservative upper limit on the proton lifetime is about $(1/2 - 1)\times 10^{34}$ yrs. for the case of either MSSM or ESSM, embedded in SO(10) or a string - G(224).

So, unless the fitting of all the pieces listed above is a mere coincidence, and I believe that that is highly unlikely, discovery of proton decay should be around the corner. In particular, as mentioned in the Introduction, we expect that candidate events should very likely be observed in the near future already at SuperK. However, allowing for the possibility that proton lifetime may well be near the upper limit or value stated above, a next-generation detector providing a net gain in sensitivity by a factor five to ten, compared to SuperK, would be needed to produce real events and distinguish them unambiguously from the background. Such an improved detector would of course be essential to study the branching ratios of certain crucial though (possibly) sub-dominant decay modes such as the μ^+K^0 and $e^+\pi^0$ as mentioned in Sec.VI F.

The reason for pleading for such improved searches is that proton decay would provide us with a wealth of knowledge about physics at truly short distances ($< 10^{-30}$ cm), which cannot be gained by any other means. Specifically, the observation of proton decay, at a rate suggested above, with $\overline{\nu}K^+$ mode being dominant, would not only reveal the underlying unity of quarks and leptons but also the relevance of supersymmetry. It would also confirm a unification of the fundamental forces at a scale of order 2×10^{16} GeV. Furthermore, prominence of the μ^+K^0 mode, if seen, would have even deeper significance, in that in addition to supporting the three features mentioned above, it would also reveal the link between neutrino masses and proton decay, as discussed in Sec.VI. *In this sense, the role of proton decay in probing into physics at the most fundamental level is unique* . In view of how valuable such a probe would be and the fact that the predicted upper limit on the proton lifetime is at most a factor of three to six higher than the empirical lower limit, the argument in favor of building an improved detector seems compelling.

To conclude, the discovery of proton decay would undoubtedly constitute a landmark in the history of physics. It would provide the last, missing piece of gauge unification and would shed light on how such a unification may be extended to include gravity in the context of a deeper theory.

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APPENDIX: A NATURAL DOUBLET-TRIPLET SPLITTING MECHANISM IN SO(10)

In supersymmetric SO(10), a natural doublet–triplet splitting can be achieved by coupling the adjoint Higgs $\mathbf{45_H}$ to a $\mathbf{10_H}$ and a $\mathbf{10'_H}$, with $\mathbf{45_H}$ acquiring a unification–scale VEV in the B-L direction [58]: $\langle\mathbf{45_H}\rangle = (a,a,a,0,0)\times\tau_2$ with $a\sim M_U$. As discussed in Section V, to generate CKM mixing for fermions we require $(\mathbf{16_H})_d$ to acquire a VEV of the electroweak scale. To ensure accurate gauge coupling unification, the effective low energy theory should not contain split multiplets beyond those of MSSM. Thus the MSSM Higgs doublets must be linear combinations of the $SU(2)_L$ doublets in $\mathbf{10_H}$ and $\mathbf{16_H}$. A simple set of superpotential terms that ensures this and incorporates doublet-triplet splitting is [14]:

$$W_H = \lambda\,\mathbf{10_H}\,\mathbf{45_H}\,\mathbf{10'_H} + M_{10}\,\mathbf{10'_H}^2 + \lambda'\,\overline{\mathbf{16}}_H\,\overline{\mathbf{16}}_H\,\mathbf{10}_H + M_{16}\,\mathbf{16}_H\overline{\mathbf{16}}_H\,. \tag{A1}$$

A complete superpotential for $\mathbf{45_H}$, $\mathbf{16_H}$, $\overline{\mathbf{16}}_\mathbf{H}$, $\mathbf{10}_H$, $\mathbf{10}'_H$ and possibly other fields, which ensure that (a) $\mathbf{45_H}$, $\mathbf{16_H}$ and $\overline{\mathbf{16}}_\mathbf{H}$ acquire unification scale VEVs with $\langle\mathbf{45_H}\rangle$ being along the (B-L) direction; (b) that exactly two Higgs doublets (H_u, H_d) remain light, with H_d being a linear combination of $(\mathbf{10_H})_d$ and $(\mathbf{16_H})_d$; and (c) there are no unwanted pseudoGoldstone bosons, can be constructed. With $\langle\mathbf{45_H}\rangle$ in the B-L direction, it does not contribute to the Higgs doublet mass matrix, so one pair of Higgs doublet remains light, while all triplets acquire unification scale masses. The light MSSM Higgs doublets are [14]

$$H_u = \mathbf{10}_u\,,\quad H_d = \cos\gamma\,\mathbf{10}_d + \sin\gamma\,\mathbf{16}_d\,, \tag{A2}$$

with $\tan\gamma \equiv \lambda'\langle\overline{\mathbf{16}}_\mathbf{H}\rangle/M_{16}$. Consequently, $\langle\mathbf{10}\rangle_d = (\cos\gamma)\,v_d$, $\langle\mathbf{16}_d\rangle = (\sin\gamma)\,v_d$, with $\langle H_d\rangle = v_d$ and $\langle\mathbf{16}_d\rangle$ and $\langle\mathbf{10}_d\rangle$ denoting the electroweak VEVs of those multiplets. Note that H_u is purely in $\mathbf{10_H}$ and that $\langle\mathbf{10}_d\rangle^2 + \langle\mathbf{16}_d\rangle^2 = v_d^2$. This mechanism of doublet-triplet (DT) splitting is the simplest for the minimal Higgs systems. It has the advantage that meets the requirements of both D-T splitting and CKM-mixing. In turn, it has three special consequences:

(i) It modifies the familiar SO(10)-relation $\tan\beta \equiv v_u/v_d = m_t/m_b \approx 60$ to [18]:

[18]It is worth noting that the simple relationship between $\cos\gamma$ and $\tan\beta$ - i.e. $\cos\gamma \approx \tan\beta/(m_t/m_b)$ - would be modified if the superpotential contains an additional term like $\lambda''16_H\cdot 16_H\cdot 10'_H$, which would induce a mixing between the doublets in $10'_d$, 16_d and 10_d. That in turn will mean that the upper limit on $M_{eff}\cos\gamma$ following from considerations of threshold corrections (see below) will not be strictly proportional to $\tan\beta$. I thank Kaladi Babu for making this observation.

$$\tan\beta/\cos\gamma \approx m_t/m_b \approx 60 \tag{A3}$$

As a result, even low to moderate values of $\tan\beta \approx 3$ to 10 (say) are perfectly allowed in SO(10) (corresponding to $\cos\gamma \approx 1/20$ to $1/6$).

(ii) The most important consequence of the DT-splitting mechanism outlined above is this: In contrast to SU(5), for which the strengths of the standard d=5 operators are proportional to $(M_{H_c})^{-1}$ (where $M_{H_C} \sim few \times 10^{16}$ GeV (see Eq. (44)), for the SO(10)-model, they become proportional to M_{eff}^{-1}, where $M_{eff} = (\lambda a)^2/M_{10'} \sim M_X^2/M_{10'}$. As noted in Ref. [14], $M_{10'}$ can be naturally smaller (due to flavor symmetries) than M_X and thus M_{eff} correspondingly larger than M_X by even one to three orders of magnitude. Now the proton decay amplitudes for SO(10) in fact possess an intrinsic enhancement compared to those for SU(5), owing primarily due to differences in their Yukawa couplings for the up sector (see Appendix C in Ref. [14]). As a result, these larger values of $M_{eff} \sim (10^{18} - 10^{19})$ GeV are in fact needed for the SO(10)-model to be compatible with the observed limit on the proton lifetime. At the same time, being bounded above by considerations of threshold effects (see below), they allow optimism as regards future observation of proton decay.

(iii) M_{eff} gets bounded above by considerations of coupling unification and GUT-scale threshold effects as follows. Let us recall that in the absence of unification-scale threshold and Planck-scale effects, the MSSM value of $\alpha_3(m_Z)$ in the $\overline{\text{MS}}$ scheme, obtained by assuming gauge coupling unification, is given by $\alpha_3(m_Z)^{\circ}_{\text{MSSM}} = 0.125 - 0.13$ [7]. This is about 5 to 8% *higher* than the observed value: $\alpha_3(m_Z) = 0.118 \pm 0.003$ [13]. Now, assuming coupling unification, the net (observed) value of α_3, for the case of MSSM embedded in SU(5) or SO(10), is given by:

$$\alpha_3(m_Z)_{\text{net}} = \alpha_3(m_Z)^{\circ}_{\text{MSSM}} + \Delta\alpha_3(m_Z)^{\text{MSSM}}_{\text{DT}} + \Delta_3' \tag{A4}$$

where $\Delta\alpha_3(m_Z)_{\text{DT}}$ and Δ_3' represent GUT-scale threshold corrections respectively due to doublet-triplet splitting and the splittings in the other multiplets (like the gauge and the Higgs multiplets), all of which are evaluated at m_Z. Now, owing to mixing between 10_d and 16_d (see Eq. (A2)), one finds that $\Delta\alpha_3(m_Z)_{\text{DT}}$ is given by $(\alpha_3(m_Z)^2/2\pi)(9/7)\ln(M_{\text{eff}}\cos\gamma/M_X)$ [14].

As mentioned above, constraint from proton lifetime sets a lower limit on M_{eff} given by $M_{\text{eff}} > (1-6) \times 10^{18}$GeV. Thus, even for small $\tan\beta \approx 2$ (i.e. $\cos\gamma \approx \tan(\beta/60) \approx 1/30$), $\Delta\alpha_3(m_Z)_{\text{DT}}$ is positive; and it increases logarithmically with M_{eff}. Since $\alpha_3(m_Z)^{\circ}_{\text{MSSM}}$ is higher than $\alpha_3(m_Z)_{\text{obs}}$, and as we saw, $\Delta\alpha_3(m_Z)_{\text{DT}}$ is positive, it follows that the corrections due to *other* multiplets denoted by $\delta_3' = \Delta_3'/\alpha_3(m_Z)$ should be appropriately negative so that $\alpha_3(m_Z)_{\text{net}}$ would agree with the observed value.

In order that coupling unification may be regarded as a natural prediction of SUSY unification, as opposed to being a mere coincidence, it is important that the magnitude of the net other threshold corrections, denoted by δ_3', be negative but not any more than about 8 to 10% in magnitude (i.e. $-\delta_3' \leqslant (8-10)\%$). It was shown in Ref. [14] that the contributions from the gauge and the minimal set of Higgs multiplets (i.e. $45_H, 16_H, \overline{16}_H$ and 10_H) leads to threshold correction, denoted by δ_3', which has in fact a negative sign and quite naturally a magnitude of 4 to 8%, as needed to account for the observed coupling unification. The correction to $\alpha_3(m_Z)$ due to Planck scale physics through the effective operator $F_{\mu\nu}F^{\mu\nu}45_H/M$

does not alter the estimate of δ_3' because it vanishes due to antisymmetry in the SO(10)-contraction.

Imposing that δ_3' (evaluated at m_Z)be negative and not any more than about 10-11% in magnitude in turn provides a restriction on how big the correction due to doublet-triplet splitting - i.e. $\Delta\alpha_3(m_Z)_{\mathrm{D}\bar{T}}$ - can be. That in turn sets an upper limit on $M_{eff}\cos\gamma$, and thereby on M_{eff} for a given $\tan\beta$. For instance, for MSSM, with $\tan\beta = (2,3,8)$, one obtains (see Ref. [14]): $M_{eff} \leq (4, 2.66, 1) \times 10^{18}$GeV. Thus, conservatively, taking $\tan\beta \geq 3$, one obtains:

$$M_{eff} \lesssim 2.7 \times 10^{18}\mathrm{GeV}\ (\mathrm{MSSM}). \tag{A5}$$

Limit on M_{eff} For The case of ESSM

Next consider the restriction on M_{eff} that would arise for the case of the extended supersymmetric standard model (ESSM), which introduces an extra pair of vector-like families ($16 + \overline{16}$) of SO(10)) at the TeV scale [20](see also footnote 11). In this case, α_{unif} is raised to 0.25 to 0.3, compared to 0.04 in MSSM. Owing to increased two-loop effects the scale of unification M_X is raised to $(1/2 - 2) \times 10^{17}$GeV, while $\alpha_3(m_Z)^{\circ}_{\mathrm{ESSM}}$ is lowered to about 0.112-0.118 [20,57].

With raised M_X, the product $M_{eff}\cos\gamma \approx M_{eff}(\tan\beta)/60$ can be higher by almost a factor of five compared to that for MSSM, without altering $\Delta\alpha_3(m_Z)_{\mathrm{DT}}$. Furthermore, since $\alpha_3(m_Z)^{\circ}_{\mathrm{MSSM}}$ is typically lower than the observed value of $\alpha_3(m_Z)$ (contrast this with the case of ESSM), for ESSM, M_{eff} can be higher than that for MSSM by as much as a factor of 2 to 3, without requiring an enhancement of δ_3'. The net result is that for ESSM embedded in SO(10), $\tan\beta$ can span a wide range from 3 to even 30 (say) and simultaneously the upper limit on M_{eff} can vary over the range (60 to 6)$\times 10^{18} GeV$, satisfying

$$M_{eff} \lesssim (6 \times 10^{18}\mathrm{GeV})(30/\tan\beta)\ (\mathrm{ESSM}), \tag{A6}$$

with the unification-scale threshold corrections from "other" sources denoted by $\delta_3' = \Delta_3'/\alpha_3(m_Z)$ being negative, but no more than about 5% in magnitude. As noted above, such values of δ_3' emerge quite naturally for the minimal Higgs system. Thus, one important consequence of ESSM is that by allowing for larger values of M_{eff} (compared to MSSM), without entailing larger values of δ_3', it can be perfectly compatible with the limit on proton lifetime for almost *central values* of the parameters pertaining to the SUSY spectrum and the relevant matrix elements (see Eq.(40)). Further, larger values of $\tan\beta$ (10 to 30, say) can be compatible with proton lifetime only for the case of ESSM, but not for MSSM. These features are discussed in the text, and also exhibited in Table 2.

TABLE 2. VALUES OF PROTON LIFETIME ($\Gamma^{-1}(p \to \bar{\nu} K^+)$) FOR A WIDE RANGE OF PARAMETERS

Parameters (spectrum/Matrix element)	MSSM → SO(10) **Std. d=5** Intermed. ϵ' & phase†		ESSM → SO(10) **Std. d=5** Intermed. ϵ' & phase†		{MSSM or ESSM} → G(224)/SO(10) **New d=5††**
	$\tan\beta$=3	$\tan\beta$=20	$\tan\beta$=5	$\tan\beta$=20	Independent of $\tan\beta$
Nearly "central" { }=2	0.7×10^{32} yrs	1.6×10^{30} yrs	1.1×10^{34} yrs*	0.7×10^{33} yrs	0.7×10^{33} yrs††
Intermediate { }=8	2.8×10^{32} yrs	0.6×10^{31} yrs	0.4×10^{35} yrs*	2.8×10^{33} yrs	2.8×10^{33} yrs††
Nearly Extreme { }=32	1.1×10^{33} yrs	2.6×10^{31} yrs	1.7×10^{35} yrs*	1.1×10^{34} yrs	1.1×10^{34} yrs††

***In this case, lifetime is given by the last column.**

• Since we are interested in exhibiting expected proton lifetime near the upper end, we are not showing entries corresponding to values of the parameters for the SUSY spectrum and the matrix element (see Eq.(40), for which the curly bracket appearing in Eqs.(47), (49), (52)) would be less than one (see however Table 1). In this context, we have chosen here "nearly central", "intermediate" and "nearly extreme" values of the parameters such that the said curly bracket is given by 2, 8 and 32 respectively, instead of its extreme upper-end value of 64. For instance, the curly bracket would be 2 if β_H = (0.0117) GeV^3, $m_{\tilde{q}} \approx 1.2$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx (1/7.2)$, while it would be 8 if $\beta_H = 0.010$ GeV^3, $m_{\tilde{q}} \approx 1.44$ TeV and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/10$; and it would be 32 if, for example, $\beta_H = 0.007$ GeV^3, $m_{\tilde{q}} \approx \sqrt{2}(1.2$ TeV) and $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/12$.

† All the entries for the standard d=5 operators correspond to taking an intermediate value of $\epsilon' \approx (1 \text{ to } 1.4) \times 10^{-4}$ (as opposed to the extreme values of 2×10^{-4} and zero for cases I and II, see Eq.(34)) and an intermediate phase-dependent factor such that the uncertainty factor in the square bracket appearing in Eqs.(47) and (49) is given by 5, instead of its extreme values of $2 \times 4 = 8$ and $2.5 \times 4 = 10$, respectively.

†† For the new operators, the factor [8-1/64] appearing in Eq.(52) is taken to be 6, and K^{-2}, defined in Sec.VIA, is taken to be 9, which are quite plausible, in so far as we wish to obtain reasonable values for proton lifetime at the upper end.

• The standard d=5 operators for both MSSM and ESSM are evaluated by taking the upper limit on M_{eff} (defined in the text) that is allowed by the requirement of natural coupling unification. This requirement restricts threshold corrections and thereby sets an upper limit on M_{eff}, for a given $\tan\beta$ (see Sec.VI and Appendix).

* For all cases, the standard and the new d=5 operators must be combined to obtain the net amplitude. For the three cases of ESSM marked with an asterisk, and other similar cases which arise for low $\tan\beta \approx 3$ to 6 (say), the standard d=5 operators by themselves would lead to proton lifetimes typically exceeding $(0.1 - 0.7) \times 10^{35}$ years. For these cases, however, the contribution from the new d=5 operators would dominate, which quite naturally lead to lifetimes in the range of $(10^{33} - 10^{34})$ years (see last column).

• As shown above, the case of MSSM embedded in SO(10) is tightly constrained by present empirical lower limit on proton lifetime (Eq.(42)). In this case, only low values of $\tan\beta \leq 3$, with the parameters (pertaining to the SUSY spectrum, matrix element and phase-dependent factor) having their "nearly extreme" or extreme values (as in Eq.(40)) can lead to lifetimes in the range of $(1-3) \times 10^{33}$yrs (see Table and Eq.(48)), compatible with present empirical limit. Other cases of MSSM - especially with $\tan\beta \geq 5$ and/or "nearly central" or even "intermediate" range of parameters - seem to be excluded, subject (of course) to our requirement for natural coupling unification (see Sec.VI and Appendix).

• Including contributions from the standard and the new operators, the case of ESSM, embedded in either G(224) or SO(10), is, however, fully consistent with present limits on proton lifetime for a wide range of parameters; at the same time it provides optimism that proton decay will be discovered in the near future, with a lifetime $\leq 10^{34}$ years.

• The lower limits on proton lifetime are not exhibited. In the presence of the new operators, these can typically be as low as about 10^{29} years (even for the case of ESSM embedded in SO(10)). Such limits and even higher are of course long excluded by experiments.

• Allowing for a wide variation in the relevant parameters, we thus see that a *conservative upper limit* on proton lifetime is given by the range of $(1/2-1) \times 10^{34}$ years for ESSM and (of course) MSSM, embedded in SO(10) or string-G(224).

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A family-universal anomalous $U(1)$ in string models as the origin of supersymmetry breaking and squark degeneracy

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Abstract

Recently a promising mechanism for supersymmetry breaking that utilizes both an anomalous $U(1)$ gauge symmetry and an effective mass term $m \sim 1$ TeV of certain relevant fields has been proposed. In this paper we examine whether such a mechanism can emerge in superstring-derived free fermionic models. We observe that certain three-generation string solutions, though not all, lead to an anomalous $U(1)$ which couples universally to all three families. The advantages of this three-family universality of $U(1)_A$, compared to the two-family case, proposed in earlier works, in yielding squark degeneracy, while avoiding radiative breaking of color and charge, are noted. The root cause of the flavor universality of $U(1)_A$ is the cyclic permutation symmetry that characterizes the $Z_2 \times Z_2$ orbifold compactification with standard embedding, realized in the free fermionic models by the NAHE set. It is shown that non-renormalizable terms which contain hidden-sector condensates, generate the required suppression of the relevant mass term m, compared to the Planck scale. While the D-term of the family-universal $U(1)_A$ leads to squark degeneracy, those of the family-dependent $U(1)$'s, remarkably enough, are found to vanish for the solutions considered, owing to minimization of the potential. Motivations are provided for the combined $U(1)_A$-Dilaton SUSY breaking. © 1998 Elsevier Science B.V.

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Keywords: Realistic superstring models; Supersymmetry breaking; Squark degeneracy

1. Introduction

Understanding the origin of (i) supersymmetry breaking and simultaneously of (ii) the extreme degeneracy in the masses of the squarks in at least the first two families, as inferred from the minuscule strengths of the K^0–$\bar{K}^0$ transition, is still among the

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important unsettled issues in particle physics. Equally important is understanding the large hierarchy between the Planck scale and the SUSY breaking mass splitting δm_s, reflected by the ratio $\delta m_s/M_{\rm Pl} \sim 10^{-15}$.

Several mechanisms have been proposed to implement SUSY breaking. These include the ideas of: (i) gaugino condensation in the hidden sector [1]; (ii) dilaton dominated SUSY breaking (DDSB) [2]; (iii) gauge mediated SUSY breaking (GMSB) [3]; whose intrinsic origin is delegated to an unknown mechanism involving an effective singlet field, which couples to a set of messenger particles; and (iv) SUSY breaking, induced through joint effects of an anomalous $U(1)$ gauge interaction and effective mass terms of certain relevant fields, which carry the anomalous $U(1)$ charge [4–6]. The mass terms in case (iv) represent the scale of SUSY breaking mass splitting δm_s, and thus, on phenomenological grounds, they must be of order 1 TeV.

Among these, the mechanism of DDSB automatically yields squark degeneracy at the tree level. It however has the problem of possible color- and charge-breaking (see e.g. the last paper in Ref. [2]). The GMSB yields squark degeneracy, provided that the superpotential Yukawa interactions of the messenger fields with the Standard Model fields are suppressed. The existence of such messenger fields in superstring-derived models was proposed [8]. That of the anomalous $U(1)$ can yield the desired degeneracy provided that it couples universally to at least the first two families, which is assumed in Refs. [5,7]. Short of deriving any of these from an underlying theory, such as superstring theory, however, the scale of SUSY breaking mass splittings δm_s as well as the choice of fields and of their quantum numbers are rather arbitrary. They must therefore be put in by hand.

It is thus of great interest to examine whether any of these mechanisms could in fact emerge from within a superstring theory. Now, phenomenologically viable, solutions of string theory invariably do indeed contain an anomalous $U(1)$ as a generic feature (see e.g. Refs. [9–15], as examples of models based on the free fermionic construction [16]. We wish therefore to explore in this note the viability of supersymmetry breaking through an anomalous $U(1)$ in the context of such string-derived solutions. In particular, we examine whether they can yield either a two- or a three-family-universal anomalous $U(1)$ that would lead to squark mass degeneracy (following SUSY breaking) on the one hand, and yet would not conflict with the observed hierarchy in the masses of the fermions on the other hand; and whether these solutions can also yield non-vanishing but strongly suppressed mass terms $m \ll M_{\rm Pl}$ of certain relevant fields, which are essential to trigger SUSY breaking. The smallness of m compared to the Planck mass would then account for the large hierarchy between δm_s and $M_{\rm Pl}$.

Given that string theory yields a vast set of solutions at the tree level and that no guiding principle is available to choose between them, it is, of course, still premature to take too seriously any specific solution. Yet certain generic features of a class of solutions, related especially to their *symmetry properties*, may well survive in the final picture. With this in mind and for concreteness, we examine the issues noted above within a specific class of string-derived solutions, which are obtained in the free fermionic formulation [16], and yield non-GUT standard model-like gauge symmetries with three generations [13,14]. Later, we will comment on the issue of flavor universality of the anomalous $U(1)$ in some other solutions such as those of Refs. [9,11]. A priori motivations for considering the class of solutions obtained in Refs. [13,14] are that (a) they seem capable of generating qualitatively the right texture for fermion masses and mixings; (b) they provide a natural doublet–triplet splitting mechanism because of their non-GUT character and (c) they also possess extra gauge symmetries, beyond SUSY GUTs, which, together with the allowed pattern of VEVs, safeguard proton stability from all potential dangers [17,18], including those which may arise from higher dimensional operators and from the exchange of color triplets in the heavy tower of string states. These extra symmetries also turn out to be helpful in suppressing ν_L–$\tilde{H}$ mixing operator [19]. Last but not the least, having their origin in a string theory, they of course satisfy gauge coupling unification in spite of their non-GUT character [20]. The obvious question is whether this class of string solutions also permits supersymmetry breaking at the electroweak scale through an anomalous $U(1)$, while preserving family universality in squark masses.

In Section 2, we observe that the desired family universality of the anomalous $U(1)$ is by no means a general property of string solutions, but it holds in the class of solutions obtained in Refs. [13,14]. We point out the root cause why it holds for this class. In Sections 3 and 4, we discuss supersymmetry breaking and generation of relevant mass terms for these solutions. We show in Section 4 that there exist solutions in this class which yield operators of dimension $n \geqslant 4$, that induce highly suppressed relevant mass terms, $\sim (\frac{1}{2}$–$50)$ TeV. These mass terms, together with the anomalous $U(1)$, induce SUSY breaking. Thus these solutions have the potential for explaining (a) supersymmetry breaking, (b) gauge hierarchy and (c) squark degeneracy.

An issue of special concern is that string solutions invariably possess a host of other $U(1)$'s, which are family-dependent, and contributions from their D-terms, if non-vanishing, could potentially spoil squark degeneracy. We show in Section 3 that there exist solutions for which the contributions from the undesirable D-terms, remarkably enough, vanish owing to a minimization of the potential. In short, the class of string solutions considered here, though by no means unique, possesses three non-trivial and highly desirable features: (i) a family-universal anomalous $U(1)$, (ii) suitably suppressed mass terms of relevant fields which trigger SUSY breaking, and (iii) vanishing of family-dependent D-term contributions. In Section 5, we mention certain features of phenomenological interest. In particular string solutions of Refs. [13] or [14] lead to approximate squark degeneracy for *all three families*. This is in contrast to the case of Refs. [5,7], where the degeneracy holds (because of the choice of the anomalous charge) only for the first two families. Advantages of three- compared to two-family squark-degeneracy in avoiding radiative color- and electric charge-breaking – are noted. We also point out that the solution of Ref. [14] leads to intra-family-sfermion degeneracy (i.e. $m_{\tilde{q}_L} = m_{\tilde{u}_R} = m_{\tilde{d}_L} = m_{\tilde{L}}$, etc.), whereas that of Ref. [13] leads to considerable splitting between the members of a family. In Sections 6 and 7, we make some general remarks about the prospect of supersymmetry-breaking through $U(1)_A$, and provide motivations for the combined anomalous-$U(1)$-Dilaton SUSY breaking.

2. A family-universal anomalous $U(1)$ in a class of string solutions

We begin by recalling certain salient features of the solutions based on the free fermionic formulation [16]. They are defined by a set of boundary condition basis vectors and the associated one-loop GSO projection coefficients. The massless states are obtained by applying the generalized GSO projections. Each massless state defines a vertex operator and the cubic and higher order terms in the superpotential are obtained by calculating the correlators between the vertex operators [21,22].

The specific class of solutions [13,14] which we examine here are generated by a set of eight boundary condition basis vectors. The first five of these, denoted by $\{\mathbf{1}, S, b_1, b_2, b_3\}$, constitute the so-called NAHE set [9,23]. They are common to a large class of viable string solutions, including those of Refs. [9,11,12]. The properties of the NAHE set are crucial to understanding how the required flavor universality of the anomalous $U(1)$ may arise in certain free fermionic solutions. We therefore refer the reader to Refs. [9,23] for a definition of these basis vectors and their detailed properties. Here we note only certain salient features. The vectors $\{\mathbf{1}, S\}$ give rise to a solution with $N = 4$ space-time supersymmetry and $SO(44)$ gauge symmetry. The vectors b_1, b_2 and b_3 break $N = 4$ to $N = 1$ and $SO(44)$ to $SO(10) \times E_8 \times SO(6)^3$, where $SO(10)$ is identified with the GUT symmetry containing the Standard Model. Each of the vectors b_1, b_2 and b_3 produces 16 multiplets, each of which is a 16 of $SO(10)$; thus there are altogether 48 generations. The sixteen generations produced by each b_j are charged with respect to only one of the $SO(6)_j$-symmetries, which is why the $SO(6)_j$-symmetries provide the origin of flavor symmetries. Note that at this stage, there is a complete *permutation symmetry* between the sectors b_1, b_2 and b_3, which is reflected in the full set of gauge interactions as well as in the superpotential. It is this permutation symmetry which leads to family universality of the anomalous $U(1)$ in some models.

It is important to note that the NAHE set corresponds to a $Z_2 \times Z_2$ orbifold compactification. This seemingly apparent observation has far reaching phenomenological implications. The focus in this paper is on SUSY breaking and squark degeneracy. The correspondence of the NAHE set with $Z_2 \times Z_2$ orbifold compactification is best illustrated by adding to the NAHE set the boundary condition basis vector X with periodic boundary conditions for the world-sheet fermions $\{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$, and antiperiodic boundary conditions for all others. With a suitable choice of the generalized GSO projection coefficients the $SO(10)$ gauge group is enhanced to E_6. The $SO(6)^3$ symmetries are broken to $SO(4)^3 \times U(1)^3$. One combination of the $U(1)$ symmetries is embedded in E_6. The gauge group in this case would be $E_6 \times E_8 \times SO(4)^3 \times U(1)^2$. This extended NAHE set then corresponds to a $Z_2 \times Z_2$ orbifold with the standard embedding of the gauge connection [24]. The three sectors generated by b_1, b_2 and b_3 are the three twisted sectors of the orbifold models. The cyclic permutation symmetry associated with the NAHE set is thus simply the symmetry between the three twisted sectors of the $Z_2 \times Z_2$ orbifold, with standard embedding. The permutation symmetry also applies to the spectrum that arises from the untwisted sector, including the moduli. The phenomenological motivation for this symmetry will become apparent in the context of supersymmetry breaking and squark degeneracy. Whether this symmetry is unique to the $Z_2 \times Z_2$ orbifold compactification is an open question.

The next stage in the construction of viable solutions is the introduction of additional boundary condition basis vectors, which reduce the number of chiral generations from forty-eight to three, barring possible vector-like multiplets. These also break $SO(10)$ to one of its subgroups – e.g. $SU(5) \times U(1)$ [9], $SO(6) \times SO(4)$ [11] or $SU(3) \times SU(2) \times U(1)^2$ [10,13,14]. The hidden E_8 symmetry is typically also broken to one of its subgroups, and the horizontal $SO(6)^3$ symmetry breaks typically to Abelian factors of $U(1)^n$, where $n(\geqslant 3)$ varies between the solutions. As we discuss below, the permutation symmetry of the full set of gauge interactions with respect to the three chiral families is retained for solutions of the type presented in Refs. [13,14], in spite of the introduction of three additional boundary condition basis vectors (beyond the NAHE set). But this is not the case for the solutions of Refs. [9,11]. The reason for this difference is that in the case of Refs. [13,14] the three chiral families have their origin entirely in the sectors b_1, b_2 and b_3 respectively, and there are no additional vector-like families. By contrast, for the cases of Refs. [9,11], owing to the nature of the additional boundary condition basis vectors, there are vector-like multiplets in addition to the three chiral families, and the latter do not all arise from the sectors b_1, b_2 and b_3 respectively.

In summary, the NAHE set naturally gives rise to the permutation symmetry of the three families, or rather three groups of families, both in the gauge as well as in the superpotential sector. This symmetry need not, however, be retained in general in the presence of additional boundary condition basis vectors, beyond the NAHE set. It is intriguing that the stated symmetry is fully retained in the gauge sector (though it is partially lost in the superpotential, see below) for solutions of the type presented in Refs. [13,14]. As we will show, this permutation symmetry in the gauge sector guarantees family universality of the anomalous $U(1)$.

As concrete examples, we consider the solutions of both Ref. [13] and Ref. [14], which we will refer to as solutions I and II respectively. They are very similar for most purposes, yet they possess certain crucial differences. The gauge symmetry in both cases, arising from the NS sector, after application of all the GSO projections, has the following form, at the string scale:

$$\mathcal{G} = \left[SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3_R}}\right] \times \left[G_M = \prod_{i=1}^{6} U(1)_i\right] \times G_H. \tag{2.1}$$

Here, $U(1)_i$ denote six horizontal flavor symmetries, which descend from $SO(6)^3$, and act non-trivially on the three chiral families, Higgs multiplets as well as Hidden matter states. In both cases, $G_H = SU(5)_H \times SU(3)_H \times U(1)_H^2$, is the gauge symmetry of the hidden sector. In the model of Ref. [14] additional space-time vector bosons arise from the sector $\mathbf{1}+\alpha+2\gamma$ and enhance the $SU(3)_C$ gauge group to $SU(4)_C$ [25] (which we use in Tables 4, 5 and 6). This enhancement, however, does not affect our discussion here.

Table 1
Massless states for solution I (Ref. [13]) which transform solely under the observable gauge group

F	sector	$SU(3)\times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5)\times SU(3)$	Q_7	Q_8
L_1	b_1	(1,2)	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	(1,1)	0	0
Q_1		(3,2)	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	(1,1)	0	0
d_1		$(\bar{3},1)$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	(1,1)	0	0
N_1		(1,1)	$\frac{3}{2}$	-1	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0	(1,1)	0	0
u_1		$(\bar{3},1)$	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	(1,1)	0	0
e_1		(1,1)	$\frac{3}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	(1,1)	0	0
L_2	b_2	(1,2)	$-\frac{3}{2}$	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	(1,1)	0	0
Q_2		(3,2)	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	(1,1)	0	0
d_2		$(\bar{3},1)$	$-\frac{1}{2}$	-1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	(1,1)	0	0
N_2		(1,1)	$\frac{3}{2}$	-1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	(1,1)	0	0
u_2		$(\bar{3},1)$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	(1,1)	0	0
e_2		(1,1)	$\frac{3}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	(1,1)	0	0
L_3	b_3	(1,2)	$-\frac{3}{2}$	0	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	(1,1)	0	0
Q_3		(3,2)	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	(1,1)	0	0
d_3		$(\bar{3},1)$	$-\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	(1,1)	0	0
N_3		(1,1)	$\frac{3}{2}$	-1	0	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	(1,1)	0	0
u_3		$(\bar{3},1)$	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	(1,1)	0	0
e_3		(1,1)	$\frac{3}{2}$	1	0	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	(1,1)	0	0
h_1	NS	(1,2)	0	-1	1	0	0	0	0	0	(1,1)	0	0
h_2		(1,2)	0	-1	0	1	0	0	0	0	(1,1)	0	0
h_3		(1,2)	0	-1	0	0	1	0	0	0	(1,1)	0	0
Φ_{12}		(1,1)	0	0	1	-1	0	0	0	0	(1,1)	0	0
Φ_{13}		(1,1)	0	0	1	0	-1	0	0	0	(1,1)	0	0
Φ_{23}		(1,1)	0	0	0	1	-1	0	0	0	(1,1)	0	0
h_{45}	b_1+b_2	(1,2)	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	(1,1)	0	0
D_{45}	$+\alpha+\beta$	(3,1)	-1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	(1,1)	0	0
Φ_{45}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	0	0	0	(1,1)	0	0
$\Phi_1^{\pm}$		(1,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	±1	0	0	(1,1)	0	0
$\Phi_2^{\pm}$		(1,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	±1	0	(1,1)	0	0
$\Phi_3^{\pm}$		(1,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	±1	(1,1)	0	0

$Q_C = 3/2(B-L)$ and $Q_L = 2T_{3_R}$. The NS and the $b_1+b_2+\alpha+\beta$ sectors contain also the conjugate states ($\bar{h}_1$, etc.). The NS sector contains an additional three singlet states, $\xi_{1,2,3}$, which are neutral under all the $U(1)$ symmetries.

The massless spectrum of solution I (Ref. [13]), together with the quantum numbers of the respective states, is exhibited in Tables 1, 2 and 3. The spectrum includes the three generations that arise from the sectors b_1, b_2 and b_3, the Higgs-like multiplets $h_{1,2,3}$, h_{45} and their conjugates, the color triplets $(D_{45},\bar{D}_{45})$, the $SO(10)$ singlets $\Phi^{\pm}_{1,2,3}$, Φ_{45}, Φ_{12}, Φ_{13}, Φ_{23} and their conjugates, and the hidden sector multiplets $(V_i,\bar{V}_i,T_i\bar{T}_i)_{i=1,2,3}$. We

Table 2
Massless states for solution I (Ref. [13])

F	sector	$SU(3)\times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5)\times SU(3)$	Q_7	Q_8
V_1	$b_1+2\gamma$	$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
$\bar{V}_1$	$+(I)$	$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
T_1		$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
$\bar{T}_1$		$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$
V_2	$b_2+2\gamma$	$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
$\bar{V}_2$	$+(I)$	$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
T_2		$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
$\bar{T}_2$		$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$
V_3	$b_3+\gamma+$	$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
$\bar{V}_3$	$+(I)$	$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
T_3		$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
$\bar{T}_3$		$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$
H_1	b_1+b_2	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_2	$+\alpha+\beta$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_3	$\pm\gamma+(I)$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_4		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_5	b_1+b_3	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_6	$+\alpha+\beta$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_7	$\pm\gamma+(I)$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_8		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_9	b_2+b_3	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_{10}	$+\alpha+\beta$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_{11}	$\pm\gamma+(I)$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_{12}		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_{13}	b_1+b_3	$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,3)$	$\frac{3}{4}$	$\frac{5}{4}$
H_{14}	$+\alpha\pm\gamma$	$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,\bar{3})$	$-\frac{3}{4}$	$-\frac{5}{4}$
H_{15}	$+(I)$	$(1,2)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{16}		$(1,2)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{17}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{18}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$

see that each generation that arises from the sector b_j, is charged with respect to two of the $U(1)$'s – i.e. $U(1)_{R_j}$ and $U(1)_{R_{j+3}}$. For each right-moving gauged $U(1)$ symmetry, there is a corresponding left-moving global $U(1)$ symmetry, denoted by $U(1)_{L_j}$ and $U(1)_{L_{j+3}}$, and the states from each sector b_j are charged with respect to two of these global symmetries.

The spectrum for solution II (Ref. [14]) is very similar. The main difference is

Table 3
Massless states for solution I (Ref. [13])

F	sector	$SU(3)\times SU(2)$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5)\times SU(3)$	Q_7	Q_8
H_{19}	b_2+b_3	$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(5,1)$	$-\frac{1}{4}$	$\frac{9}{4}$
H_{20}	$+\alpha\pm\gamma$ $+(I)$	$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(\bar{5},1)$	$\frac{1}{4}$	$-\frac{9}{4}$
H_{21}		$(3,1)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{22}		$(\bar{3},1)$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{23}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{24}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{25}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{26}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{27}	b_1+b_2	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_{28}	$+b_3+\alpha$ $+\beta\pm\gamma$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_{29}	$+(I)$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_{30}		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$

that the six $SO(10)$-singlets $\Phi^{\pm}_{1,2,3}$ are replaced by only two fields $\Phi_{1,2}$, while h_{45} is accompanied by an additional doublet h'_{45} and Φ_{45} by Φ'_{45}, and the color triplets $(D_{45}, \bar{D}_{45})$ are absent. These are listed in Tables 4, 5 and 6.

While the subgroup $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3_R}}$ of $SO(10)$ treats all three families universally, it is easy to see from Table 1 that the pairs (U_1, U_4), (U_2, U_5) and (U_3, U_6), respectively couple to families 1, 2 and 3 in an *identical fashion*. Thus, on the one hand, these six $U(1)$ symmetries, having their origin in $SO(44)$, distinguish between the three families, unlike a GUT symmetry like $SO(10)$; thereby they serve as the origin of flavor symmetries, which are needed to explain the hierarchical Yukawa couplings of the three families (see below). On the other hand, as stated before, they preserve the full permutation symmetry with respect to the three families.

It is easy to check that solution I (Ref. [13]) contains six anomalous $U(1)$ symmetries: $\mathrm{Tr}\, U_1 = \mathrm{Tr}\, U_2 = \mathrm{Tr}\, U_3 = 24$, $\mathrm{Tr}\, U_4 = \mathrm{Tr}\, U_5 = \mathrm{Tr}\, U_6 = -12$. These can be expressed by one anomalous combination which is unique and five non-anomalous ones: [1]

$$U_A = \frac{1}{\sqrt{15}}\Big(2(U_1+U_2+U_3) - (U_4+U_5+U_6)\Big), \qquad \mathrm{Tr}\, Q_A = \frac{1}{\sqrt{15}}\, 180. \tag{2.2}$$

One choice for the five anomaly-free combinations is given by

$$U_{12} = \frac{1}{\sqrt{2}}\,(U_1 - U_2), \qquad U_\psi = \frac{1}{\sqrt{6}}\,(U_1 + U_2 - 2U_3), \tag{2.3}$$

$$U_{45} = \frac{1}{\sqrt{2}}\,(U_4 - U_5), \qquad U_\zeta = \frac{1}{\sqrt{6}}\,(U_4 + U_5 - 2U_6), \tag{2.4}$$

[1] The normalization of the different $U(1)$ combinations is fixed by the requirement that the conformal dimension of the massless states still gives $\bar{h} = 1$ in the new basis. We remark in advance that the proper normalization must be taken as it affects the minimization of the potential (see below).

Table 4
Massless states for solution II (Ref. [14])

F	sector	$SU(4)_C \times SU(2)_L$	$Q_{C'}$	Q_L	Q_1	Q_2	Q_3	$Q_{4'}$	$Q_{5'}$	$SU(5)_H \times SU(3)_H$	$Q_{6'}$	$Q_{8''}$
L_1	$b_1 \oplus 1$	$(1,2)$	$-\frac{3}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{8}{3}$	0
Q_1	$+\alpha+2\gamma$	$(4,2)$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{2}{3}$	0
d_1		$(\bar{4},1)$	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{2}{3}$	0
N_1		$(1,1)$	$\frac{3}{2}$	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{8}{3}$	0
e_1		$(1,1)$	$\frac{3}{2}$	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{8}{3}$	0
u_1		$(\bar{4},1)$	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{2}{3}$	0
L_2	$b_2 \oplus 1$	$(1,2)$	$-\frac{3}{2}$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{8}{3}$	0
Q_2	$+\alpha+2\gamma$	$(4,2)$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{2}{3}$	0
d_2		$(\bar{4},1)$	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{2}{3}$	0
N_2		$(1,1)$	$\frac{3}{2}$	-1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{8}{3}$	0
e_2		$(1,1)$	$\frac{3}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{8}{3}$	0
u_2		$(\bar{4},1)$	$-\frac{1}{2}$	-1	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{2}{3}$	0
L_3	$b_3 \oplus 1$	$(1,2)$	$-\frac{3}{2}$	0	0	0	$\frac{1}{2}$	0	1	$(1,1)$	$-\frac{8}{3}$	0
Q_3	$+\alpha+2\gamma$	$(4,2)$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	0	1	$(1,1)$	$-\frac{2}{3}$	0
d_3		$(\bar{4},1)$	$-\frac{1}{2}$	1	0	0	$\frac{1}{2}$	0	-1	$(1,1)$	$\frac{2}{3}$	0
N_3		$(1,1)$	$\frac{3}{2}$	-1	0	0	$\frac{1}{2}$	0	-1	$(1,1)$	$\frac{8}{3}$	0
e_3		$(1,1)$	$\frac{3}{2}$	1	0	0	$\frac{1}{2}$	0	-1	$(1,1)$	$\frac{8}{3}$	0
u_3		$(\bar{4},1)$	$-\frac{1}{2}$	-1	0	0	$\frac{1}{2}$	0	-1	$(1,1)$	$\frac{2}{3}$	0
h_1	NS	$(1,2)$	0	-1	1	0	0	0	0	$(1,1)$	0	0
h_2		$(1,2)$	0	-1	0	1	0	0	0	$(1,1)$	0	0
h_3		$(1,2)$	0	-1	0	0	1	0	0	$(1,1)$	0	0
Φ_{12}		$(1,1)$	0	0	1	-1	0	0	0	$(1,1)$	0	0
Φ_{13}		$(1,1)$	0	0	1	0	-1	0	0	$(1,1)$	0	0
Φ_{23}		$(1,1)$	0	0	0	1	-1	0	0	$(1,1)$	0	0
h_{45}	b_1+b_2	$(1,2)$	0	-1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$(1,1)$	0	0
h'_{45}	$+\alpha+\beta$	$(1,2)$	0	-1	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$(1,1)$	0	0
Φ_{45}		$(1,1)$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	0	0	$(1,1)$	$\frac{2}{3}$	0
Φ'_{45}		$(1,1)$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	0	$(1,1)$	0	0
$\Phi 1,2$		$(1,1)$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$(1,1)$	0	0

$$U_\chi = \frac{1}{\sqrt{15}} (U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6). \tag{2.5}$$

Note that the anomalous $U(1)$, containing the *sums* of $U_{1,2,3}$ and $U_{4,5,6}$ is universal with respect to all three families. *This flavor universality of the anomalous $U(1)$ is thus a consequence of the family permutation symmetry of the six $U(1)$-interactions, mentioned above.* Of the anomaly-free combinations U_{12}, U_ψ, U_{45}, and U_ζ are clearly family-dependent, but U_χ is family universal.

It is worth noting that while solution II (Ref. [14]) differs in detail from solution I as

Table 5
Massless states for solution II (Ref. [14])

F	sector	$SU(4)_C\times SU(2)_L$	$Q_{C'}$	Q_L	Q_1	Q_2	Q_3	$Q_{4'}$	$Q_{5'}$	$SU(5)_H\times SU(3)_H$	$Q_{6'}$	$Q_{8''}$
V_1	$b_1+2\gamma$	$(1,1)$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(1,3)$	$\frac{8}{3}$	$\frac{5}{2}$
$\bar{V}_1$		$(1,1)$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{8}{3}$	$-\frac{5}{2}$
T_1		$(1,1)$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$(5,1)$	$-\frac{8}{3}$	$\frac{3}{2}$
$\bar{T}_1$		$(1,1)$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(\bar{5},1)$	$\frac{8}{3}$	$-\frac{3}{2}$
V_2	$b_2+2\gamma$	$(1,1)$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,3)$	$\frac{8}{3}$	$\frac{5}{2}$
$\bar{V}_2$		$(1,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{8}{3}$	$-\frac{5}{2}$
T_2		$(1,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$(5,1)$	$-\frac{8}{3}$	$\frac{3}{2}$
$\bar{T}_2$		$(1,1)$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$(\bar{5},1)$	$\frac{8}{3}$	$-\frac{3}{2}$
V_3	$b_3+2\gamma$	$(1,1)$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-1	$(1,3)$	$\frac{8}{3}$	$\frac{5}{2}$
$\bar{V}_3$		$(1,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	$(1,\bar{3})$	$-\frac{8}{3}$	$-\frac{5}{2}$
T_3		$(1,1)$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1	$(5,1)$	$-\frac{8}{3}$	$\frac{3}{2}$
$\bar{T}_3$		$(1,1)$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-1	$(\bar{5},1)$	$\frac{8}{3}$	$-\frac{3}{2}$
l_1	b_2+b_3	$(1,2)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{l}_1$	$+\beta+\gamma+\xi$	$(1,\bar{2})$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_1		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_1$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_2		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_2$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_3		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_3$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
H_1		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(5,1)$	0	$\frac{9}{4}$
$\bar{H}_1$		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$(\bar{5},1)$	0	$-\frac{9}{4}$
l_2	b_1+b_3	$(1,2)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{l}_2$	$+\alpha+\gamma+\xi$	$(1,\bar{2})$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_4		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_4$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_5		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_5$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
S_6		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{3}{4}$	0	0	$(1,1)$	0	$-\frac{15}{4}$
$\bar{S}_6$		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	0	0	$(1,1)$	0	$\frac{15}{4}$
H_2		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	$(5,1)$	0	$\frac{9}{4}$
$\bar{H}_2$		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	$(\bar{5},1)$	0	$-\frac{9}{4}$

regards its spectrum of Higgs multiplets and the $SO(10)$ singlets, its gauge interactions nevertheless possess the full permutation symmetry with respect to the three families just like solution I. In this case, however, there are only three anomalous symmetries $U_{1,2,3}$, which can be expressed by one anomalous and two anomaly-free combinations:

Table 6
Massless states for solution II (Ref. [14])

F	sector	$SU(4)_C \times SU(2)_L$	$Q_{C'}$	Q_L	Q_1	Q_2	Q_3	$Q_{4'}$	$Q_{5'}$	$SU(5)_H \times SU(3)_H$	$Q_{6'}$	$Q_{8''}$
l_4	$1+b_1$	(1,2)	−1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	(1,1)	$\frac{16}{3}$	0
S_7	$+\alpha+2\gamma$	(1,1)	1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{16}{3}$	0
$\bar{S}_7$		(1,1)	1	−1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{16}{3}$	0
l_5	$1+b_2$	(1,2)	−1	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	(1,1)	$\frac{16}{3}$	0
S_8	$+\alpha+2\gamma$	(1,1)	1	1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{16}{3}$	0
$\bar{S}_8$		(1,1)	1	−1	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{16}{3}$	0
l_6	$1+b_3$	(1,2)	−1	0	0	0	$-\frac{1}{2}$	0	1	(1,1)	$\frac{16}{3}$	0
S_9	$+\alpha+2\gamma$	(1,1)	1	1	0	0	$-\frac{1}{2}$	0	−1	(1,1)	$-\frac{16}{3}$	0
$\bar{S}_9$		(1,1)	1	−1	0	0	$-\frac{1}{2}$	0	−1	(1,1)	$-\frac{16}{3}$	0
S_{10}	$1+s$	(1,1)	−2	0	0	0	0	−1	−1	(1,1)	$-\frac{4}{3}$	0
$\bar{S}_{10}$	$+\alpha+2\gamma$	(1,1)	−2	0	0	0	0	1	1	(1,1)	$\frac{4}{3}$	0

$$U_A = \frac{1}{\sqrt{3}}(U_1+U_2+U_3), \qquad \mathrm{Tr}\, Q_A = \frac{1}{\sqrt{3}} 72, \tag{2.6}$$

$$U_{12} = \frac{1}{\sqrt{2}}(U_1-U_2), \qquad U_\psi = \frac{1}{\sqrt{6}}(U_1+U_2-2U_3). \tag{2.7}$$

Note that the anomalous $U(1)$ is again family universal, though U_{12} and U_ψ are not.

We next examine the superpotential and the issue of generating relevant mass terms which would trigger SUSY breaking in the string solution of Ref. [13].

3. Superpotential and SUSY breaking

The relevant terms in the cubic level superpotential of solutions I and II are given by

$$\begin{aligned} W = {} & [u^c_{L_1} Q_1 \bar{h}_1 + N^c_{L_1} L_1 \bar{h}_1 + u^c_{L_2} Q_2 \bar{h}_2 + N^c_{L_2} L_2 \bar{h}_2 + u^c_{L_3} Q_3 \bar{h}_3 + N^c_{L_3} L_3 \bar{h}_3] \\ & + [h_1 \bar{h}_2 \bar{\Phi}_{12} + h_1 \bar{h}_3 \bar{\Phi}_{13} + h_2 \bar{h}_3 \bar{\Phi}_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23}] \\ & + h_3 \bar{h}_{45} \Phi_{45} + \bar{h}_3 h_{45} \bar{\Phi}_{45} + (\Phi_{23} \bar{\Phi}_{13} \Phi_{12} + \bar{\Phi}_{23} \Phi_{13} \bar{\Phi}_{12}) + \ldots \end{aligned} \tag{3.1}$$

Here a common normalization constant $\sqrt{2}g$ is not exhibited. Note that the Yukawa couplings given in the first square bracket and effectively the second bracket as well respect the family permutation symmetry, which simultaneously permutes the three families and the Higgs-multiplets $(\bar{h}_1, \bar{h}_2, \bar{h}_3)$, and likewise the Φ_{ij}'s, but the rest of the superpotential (including higher order terms), which is not shown, does not.

Note that owing to the constraints of the flavor $U(1)_i$-symmetries, which distinguish between the families, and the Higgs multiplets, $\bar{h}_1$ couples at the cubic level of the superpotential only to family 1 and not to families 2 and 3; Similarly $\bar{h}_2$ and $\bar{h}_3$ couple only to families 2 and 3 respectively. Now, for the case of solution I (Ref. [13]), where contributions of higher dimensional operators to the Higgs mass matrix have been analyzed in detail [26], it has been shown that the pair h_3 and $\bar{h}_3$ necessarily become superheavy since their masses receive contributions from the cubic level superpotential terms; and only one pair of doublets – i.e. either $(\bar{h}_1, h_{45})$ or $(\bar{h}_2, h_{45})$ – remains light, while the remaining pairs become medium heavy ($\sim 10^{12}$ GeV). It is easy to verify that for solution II (Ref. [14]) as well, h_3 and $\bar{h}_3$ become superheavy [27]. The mass pattern of the remaining Higgs doublets depend on the structure of the higher dimensional operators and the allowed pattern of VEVs. Considering the similarity of the massless spectrum in the observable sector for the two cases, however, it seems rather plausible that the Higgs spectrum for the two cases would be quite similar. Following Refs. [14,27] we will proceed by assuming that only one pair of Higgs doublets, like $(\bar{h}_1, h_{45})$ or $(\bar{h}_2, h_{45})$, remain light for both solutions I and II, and that the remaining pairs become medium or superheavy.

Since only the light Higgs scalars acquire VEVs (radiatively), it would follow, for the Higgs spectrum mentioned above, *that the up-quark member of only one family, i.e. the top and ν_τ would get masses at the level of cubic terms in W.* The masses of the other quarks and their mixings would arise through successively higher dimensional operators, which permit their couplings to the light Higgs-pair. Thus, a hierarchy in fermion masses and mixings arises in spite of the permutation symmetry of the cubic Yukawa couplings (see Ref. [13] and especially Ref. [14] for details of this discussion). Thus, ultimately, such a hierarchy has its origin in two features: (a) the family-dependent $U(1)_i$-symmetries, which force the three families to have Yukawa couplings with three distinct Higgs-multiplets, and (b) the spontaneously generated asymmetric Higgs mass-matrix.

We now turn to the pattern of symmetry breaking below the string-scale. The anomalous $U(1)_A$ is broken by the Dine–Seiberg–Witten mechanism [28] in which a potentially large Fayet–Iliopoulos D-term ξ is generated by the VEV of the dilaton field. Such a D-term would, in general, break supersymmetry, unless there is a direction in the scalar potential $\hat{\phi} = \sum \alpha_i \phi_i$ which is F-flat and also D-flat with respect to all the non-anomalous gauge symmetries and in which $\sum Q_A^i |\alpha_i|^2 < 0$. If such a direction exists, it will acquire a VEV, canceling the Fayet–Iliopoulos ξ-term, restoring supersymmetry and stabilizing the vacuum. The exception to this picture arises if there exist mass terms (m) for certain relevant fields carrying anomalous charge; in this case the anomalous D-term and the F-terms would necessarily acquire nonvanishing VEVs that are proportional to m and SUSY would be broken.

The set of D- and F-flat constraints, in the absence of such mass terms, is given by

$$\langle D_A \rangle = \langle D_\alpha \rangle = \langle D_\beta \rangle = \left\langle F_i \equiv \frac{\partial W}{\partial \eta_i} \right\rangle = 0, \tag{3.2}$$

$$D_A = \left[K_A + \sum Q_A^k |\chi_k|^2 + \xi \right], \tag{3.3}$$

$$D_\alpha = \left[K_\alpha + \sum Q_\alpha^k |\chi_k|^2 \right], \qquad \alpha \neq A, \tag{3.4}$$

$$\xi = \frac{g^2(\mathrm{Tr}\, Q_A)}{192\pi^2} M_{\mathrm{Pl}}^2. \tag{3.5}$$

Here χ_k are the fields which acquire VEVs of order $\sqrt{\xi}$, while the K-terms contain fields η_i like squarks, sleptons and Higgs bosons whose VEVs vanish, at this scale. Q_A^k and Q_A^i denote the anomalous and non-anomalous charges, which are listed in Eqs. (2.2)–(2.7) for solutions I and II, and $M_{\mathrm{Pl}} \approx 2 \times 10^{18}$ GeV denotes the reduced Planck mass. The solution (i.e. the choice of fields with non-vanishing VEVs) to the set of Eqs. (3.2)–(3.4), though nontrivial, is not unique. A few alternative solutions have been considered in Refs. [13,14,26,19].

As a general guide, note that ξ is positive and is of order $10^{-2}M_{\mathrm{Pl}}^2$. To cancel the ξ-term in $\langle D_A \rangle$, in the absence of mass terms, at least one field with negative Q_A must acquire a VEV. A large set of solutions including those of Refs. [13] and [14] assigns nonzero VEV to Φ_{45}, which is the field with the largest negative Q_A. If Φ_{45} (or a suitable alternative), acquiring a VEV, is charged with respect to one of the other symmetries, some additional fields must also acquire VEVs, so that the full set of $\langle D_{A,\alpha,j} \rangle$ must vanish.

To demonstrate how SUSY breaking could arise it is instructive to consider first a simple pattern of VEVs satisfying Eqs. (3.2)–(3.4), for the case of solution II (Ref. [14]). We will subsequently study a more complicated pattern of VEVs for solution I (Ref. [13]).

SUSY breaking in solution II

As an instructive example we consider a pattern which assigns nonzero VEVs of order $\sqrt{\xi}$ to only two fields

$$\langle \{\Phi_{45}, \Phi'_{45}\} \rangle \neq 0. \tag{3.6}$$

All other fields have zero VEV. The charges (Q_A, Q_ψ, Q_{12}) for Φ_{45} are $(-2, +1, 0)$, and those for Φ'_{45} are $(0, -3, 0)$ (see Eqs. (2.7) and Table 2). Their contributions to the respective D-terms are thus given by

$$D_A = \frac{1}{\sqrt{3}} [K_A - 2(|\Phi_{45}|^2 - |\bar{\Phi}_{45}|^2) + \hat{\xi}], \tag{3.7}$$

$$D_\psi = \frac{1}{\sqrt{6}} [K_\psi + (|\Phi_{45}|^2 - |\bar{\Phi}_{45}|^2) - 3|\Phi_{45}'|^2], \tag{3.8}$$

$$D_{12} = \frac{1}{\sqrt{2}} K_{12}, \tag{3.9}$$

where $\hat{\xi} = \sqrt{3}\,\xi$. The contribution of Φ_{45} and Φ'_{45} to all other D-terms are zero. The K-terms contain fields like squarks, sleptons and Higgs bosons which have zero VEVs. Although $\bar{\Phi}_{45}$ is assigned zero VEV, its contribution is still exhibited to demonstrate that it would be forced to have zero VEV in the presence of a mass term. All the F- and D-flat conditions are satisfied at the cubic level of the superpotential by assigning

$$|\langle\Phi_{45}\rangle_0|^2 = 3|\langle\Phi'_{45}\rangle_0|^2 = \tfrac{1}{2}\hat{\xi}. \tag{3.10}$$

All other VEVs are zero. The subscript zero signifies that the VEVs are obtained in the zero mass limit. Now introduce an effective mass term m ($\ll M_{\rm string}$) for Φ_{45} and $\bar{\Phi}_{45}$ in the superpotential:

$$W \supset m\Phi_{45}\bar{\Phi}_{45}. \tag{3.11}$$

We will discuss how such a mass term is likely to arise through higher dimensional operators in string theory. The effective potential then takes the form

$$V = \tfrac{1}{2}g^2(D_A^2 + D_\psi^2 + D_{12}^2) + m^2(|\Phi_{45}|^2 + |\bar{\Phi}_{45}|^2). \tag{3.12}$$

For simplicity of writing, we have put just one gauge coupling; in practice the various gauge couplings would differ due to the running even if they are equal at the string scale. It is now easy to verify that minimization of the potential would lead to a shift in the VEVs of Φ_{45} and Φ'_{45}.

The extremum conditions lead to the following constraints:

$$\frac{\partial V}{\partial\Phi_{45}} = 0 \quad\Rightarrow\quad (\Phi_{45})^\dagger\left[-2D_A + \frac{D_\psi}{\sqrt{2}} + \sqrt{3}\frac{m^2}{g^2}\right] = 0, \tag{3.13}$$

$$\frac{\partial V}{\partial\Phi'_{45}} = 0 \quad\Rightarrow\quad (\Phi'_{45})^\dagger[D_\psi] = 0, \tag{3.14}$$

$$\frac{\partial V}{\partial\bar{\Phi}_{45}} = 0 \quad\Rightarrow\quad (\bar{\Phi}_{45})^\dagger\left[2D_A - \frac{D_\psi}{\sqrt{2}} + \sqrt{3}\frac{m^2}{g^2}\right] = 0. \tag{3.15}$$

Here the fields and the D-terms to the right of the arrows stand for the VEVs of the respective entities. Since $\langle\Phi_{45}\rangle \neq 0$, Eq. (3.13) clearly shows that $\langle D_A\rangle$ and/or $\langle D_\psi\rangle$ must be of order m^2/g^2 and thus SUSY is broken. Since $\langle\Phi'_{45}\rangle \neq 0$, Eq. (3.14) implies $\langle D_\psi\rangle = 0$. Eq. (3.13) then yields $\langle D_A\rangle = \sqrt{3}m^2/2g^2$. Substituting this in Eq. (3.15) we see that $\langle\bar{\Phi}_{45}\rangle$ must remain zero, even with $m \neq 0$. Now, using the expressions for D_A and D_ψ given in Eqs. (3.7) and (3.8), we can determine the VEVs of Φ_{45} and Φ'_{45}. Thus, we see that, for the special choice of VEVs given by Eq. (3.10), which provides a solution to the F- and D-flat conditions (Eqs. (3.2)-(3.5)) in the massless limit, the extremum condition leads to a unique solution for the pattern of VEVs in the case of finite mass ($m(\Phi_{45}) = m(\bar{\Phi}_{45}) \neq 0$):

$$|\langle\Phi_{45}\rangle|^2 = \frac{\hat{\xi}}{2} - \frac{3}{4}\frac{m^2}{g^2}, \qquad |\langle\Phi'_{45}\rangle|^2 = \frac{\hat{\xi}}{6} - \frac{1}{4}\frac{m^2}{g^2}, \qquad |\langle\bar{\Phi}_{45}\rangle|^2 = 0; \tag{3.16}$$

$$\langle D_A\rangle = \frac{\sqrt{3}}{2}\frac{m^2}{g^2}, \qquad \langle D_\psi\rangle = \langle D_{12}\rangle = 0; \tag{3.17}$$

$$F(\Phi_{45}) = F(\Phi'_{45}) = 0, \qquad F(\bar{\Phi}_{45}) = m\sqrt{\frac{\hat{\xi}}{2} - \frac{3}{4}\frac{m^2}{g^2}}. \tag{3.18}$$

It may be verified that this VEV-pattern in fact minimizes the potential. Note that for this simple example, the D-term of only the anomalous charge Q_A, which is family universal, is non-zero, *but those of the non-universal charges Q_ψ and Q_{12} vanish owing to minimization.* This special feature arises because there are only two fields (Φ_{45} and Φ'_{45}), having non-zero VEVs, and they contribute only to two D-terms (D_A and D_ψ), but not to D_{12}. Thus, to start with, $D_{12} = 0$. Furthermore, Φ'_{45} contributes only to D_ψ, but not to D_A. Thus, extremization of V with respect to Φ'_{45} forces $D_\psi = 0$ as well (see Eq. (3.14)).

Vanishing of the non-universal D-terms has the desirable consequence that squarks of all three families receive the same contribution to their masses from the D-terms, in spite of the presence of the non-universal flavor symmetries:

$$[m^2_{\tilde{q}_i}]_{D_A} = g^2 Q^i_A \langle D_A \rangle = Q^i_A (\tfrac{1}{2}\sqrt{3}\, m^2) = \tfrac{1}{4} m^2. \tag{3.19}$$

Likewise for the sleptons. Here Q^i_A denotes the anomalous charge of the respective field. For solution II, $Q^i_A = (Q_1 + Q_2 + Q_3)^i$ is not only family universal, but it is also positive and the same ($= 1/\sqrt{12}$) for all members of a family. Thus, $\tilde{Q}_L$, $\tilde{d}_R$, $\tilde{L}$, $\tilde{u}_R$ and $\tilde{e}_R$ are degenerate (barring small F-term contributions) at the scale of $\sqrt{\xi}$. We will return to this point in Section 5.

It has been suggested in Ref. [5], that in a supergravity theory the squarks and the sleptons are expected to receive contributions to their masses from the Kähler potential through F-terms like $\lambda \int d^4\theta\, (\bar{\Phi}_{45})(\bar{\Phi}_{45})^\dagger q_i q_i^\dagger / M^2_{\rm Pl}$, where $\lambda = \mathcal{O}(1)$. Although these operators conserve all gauge symmetries, in a string theory one still needs to ascertain whether they satisfy the string-selection rules; otherwise λ would be small ($\leqslant \frac{1}{10}$) compared to unity. Deferring the study of this issue to a later work, we note that the contribution of these terms, if they are present, to squark masses is given by

$$[\Delta m^2_{\tilde{q}_i}]_F \approx \frac{\lambda |\langle F(\bar{\Phi}_{45})\rangle|^2}{M^2_{\rm Pl}} \approx \frac{\lambda m^2 \hat{\xi}}{2 M^2_{\rm Pl}} = \tfrac{1}{2}\lambda m^2 \epsilon, \tag{3.20}$$

where $\epsilon = \hat{\xi}/M^2_{pl}$. With $\hat{\xi} = \sqrt{3}\,\xi$, given by (3.5) and $\mathrm{Tr}\, Q_A = 72/\sqrt{3}$ [14], we expect $\epsilon \approx \frac{1}{60}$. In general, the F-term contributions are not expected to be universal, unless the Kähler potential possesses a certain symmetry (see remarks below). Even then, and even if $\lambda \approx (\frac{1}{2}$–$1)$ (say), these F-term contributions are suppressed compared to the D-term contribution (Eq. (3.19)) by about a factor of (60–30), for $\epsilon \approx \frac{1}{60}$. Degeneracy to this extent suffices to account for the smallness of at least the real part of the K^0–$\bar{K}^0$ transition amplitude, if $m_{\tilde{d}} \approx m_{\tilde{s}} \geqslant$ (700–1400) GeV [29].[2] Understanding the extreme smallness of the imaginary part of the K^0–$\bar{K}^0$ amplitude, which we do not address here would need additional considerations, based perhaps on symmetry properties, which may explain why the relevant phase angle is so small $\leqslant 10^{-2}$.[3]

At this stage, the following property of the string solutions under study is worth noting. We have observed that the family permutation symmetry is exact at the level of

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[2] In quoting lower limits on squark-masses, we have used a value for the product of mixing angles $(\cos\theta_d)(\sin\theta_d) \approx (\frac{1}{8}$–$\frac{1}{10})$, for the down quark-sector, which seems reasonable.

[3] The problem of SUSY CP-violation, in the context of models of SUSY breaking as proposed here, is discussed in a forthcoming paper by K.S. Babu and J.C. Pati, where a natural explanation for the extreme smallness of the ϵ-parameter is given.

the NAHE set in that it holds for the gauge interactions as well as for the super and Kähler potentials. Even after the introduction of the additional boundary condition basis vectors (α, β, γ) the permutation symmetry is still retained in the gauge interactions, as well as in the cubic Yukawa interactions of the quarks and the leptons with the Higgs fields and in the $h_i \bar{h}_j \bar{\Phi}_{ij}$-terms of the superpotential W (see Eq. (3.1)). It is lost in W only through (a) $\mathcal{O}(\Phi^3)$-terms, (b) terms involving Higgs and the exotic fields but not quarks and leptons (these are not shown in (3.1)), and (c) possibly some higher dimensional terms. As a result, the effect of this loss of the permutation symmetry on the quarks and leptons and their superpartners is extremely mild in that it is felt by them only through at most two-loop effects and higher dimensional terms (whose contributions to the masses of the (d, s)-squarks are less than or of order 200 MeV). If the Kähler potential retains the family permutation symmetry to the same extent as the superpotential W, which is plausible, but which is an issue that needs to be examined, even the F-term contributions given by (3.20) would be very nearly family universal. In this case, and/or if $\lambda \leqslant \frac{1}{10}$, the degree of squark-degeneracy would be far better (in this case, one would have $[(\tilde{m}_i^2 - \tilde{m}_j^2)/\tilde{m}^2] \ll 10^{-2}$) than that indicated above. We defer the study of the Kähler potential to later work. For the present, we will proceed by taking the squark degeneracy ratio to be no better than $\frac{1}{30}$–$\frac{1}{60}$, as obtained above.

The gauginos of the Standard Model gauge sector (i.e. gluinos, winos etc.) could, in general, receive masses through operators of the form $\lambda' \int d^2\theta\, \Phi_{45} \bar{\Phi}_{45} W_a W_a / M_{\rm Pl}^2$ $(a = 1,2,3)$ [5] which yields

$$m(\lambda_a) \approx \lambda' \langle F(\bar{\Phi}_{45}) \langle \Phi_{45} \rangle / M_{\rm Pl}^2 \approx \lambda' \epsilon m, \tag{3.21}$$

where $\lambda' \leqslant \mathcal{O}(1)$. We see the hierarchy

$$[m^2(\tilde{q}_i) \approx Q_A^i (\tfrac{1}{2}\sqrt{3}\, m^2)] > [\Delta m_{\tilde{q}_i}^2 \approx \lambda \epsilon (\tfrac{1}{2} m^2)] > [m_{\lambda_a}^2 \approx \lambda' \epsilon^2 m^2]. \tag{3.22}$$

Because of this hierarchy, it is clear that if SUSY breaking proceeds entirely through anomalous $U(1)$, the gluinos typically would be rather light. From (3.21), one obtains $m_{\tilde{g}} \approx 2\lambda' \epsilon m_{\tilde{q}} \approx \lambda'(20\text{–}60)$ GeV, for $m_{\tilde{q}} \approx (1\text{–}3)$ TeV; this may be too light, compared to the observed limit on $m_{\tilde{g}}$ of 130 GeV, unless $\lambda' \geqslant 2$ and $m_{\tilde{q}} \geqslant 3$ TeV. To make matters worse, for string solutions, as considered here, λ' vanishes at tree level and can only arise through quantum loops; thus it is expected to be small. This suggests that SUSY breaking through anomalous $U(1)$, quite plausibly, is accompanied by an additional source which provides the dominant contribution to gluino masses ($\sim$ (1–few)(100 GeV)), while preserving the squark-degeneracy, obtained through $U(1)_A$. We comment on this possibility in Section 6.

We should note that, for the sake of convenience, we have evaluated the VEVs of Φ_{45}, $\bar{\Phi}_{45}$, and $\bar{\Phi}'_{45}$ and the auxiliary fields in the flat limit. It has, however, been shown in Ref. [5], that the inclusion of supergravity effects does not restore supersymmetry, though they shift the VEVs of fields; e.g. $\langle \bar{\Phi}_{45} \rangle$ acquires a non-zero value which is typically bounded above by $\langle \Phi_{45} \rangle$. Such shifts, however, do not alter the pattern of soft masses and the hierarchy shown in (3.22).

Before discussing the origin of the mass term m and certain phenomenological issues, we first discuss SUSY breaking in solution I (Ref. [13]).

SUSY breaking in solution I

This case is more complex than the one presented above, because it has six $U(1)'s$ (in contrast to three relevant ones for solution II), four of which are non-universal, and typically several fields (not merely two) must acquire VEVs of order $\sqrt{\xi}$ to satisfy the F- and D-flat conditions. The instructive example presented above prompts us nevertheless to ask: (i) can one still find at least a local minimum of the potential V which leads to nonzero VEVs for the D-terms of only the family-universal charges – i.e. Q_A and Q_χ? (ii) If so, is that a global minimum? We find that the answer to the first question, interestingly enough is in the affirmative, and that to the second, though hard to assess in general, is also found to be the same for the limited subset of field-space, considered here.

Consider now a solution to the D- and F-flat conditions (Eqs. (3.2)–(3.5) for the case of solution I (Ref. [13])), which assigns non-zero VEVs of order $\sqrt{\xi}$ to the following set of fields:

$$\{\Phi_{45}, \bar{\Phi}_{13}, \bar{\Phi}_3^-, \bar{\Phi}_1^+, \bar{\Phi}_2^-, \xi_1\}\rangle = \mathcal{O}(\sqrt{\xi}). \tag{3.23}$$

All other fields have zero VEVs at the scale $\sqrt{\xi}$.

The contributions of these fields to the D-terms of the symmetries listed in Eqs. (2.2) and (2.3) are given by (compare with Eqs. (3.7)–(3.9))

$$D_A = \frac{1}{\sqrt{15}}\,[K_A - \sigma^2 + 4(|\bar{\Phi}_{45}|^2 - |\Phi_{45}|^2) + \hat{\xi}], \tag{3.24}$$

$$D_\psi = \frac{1}{\sqrt{6}}\,[K_\psi - 3|\bar{\Phi}_{13}|^2 + (|\Phi_{45}|^2 - |\bar{\Phi}_{45}|^2)], \tag{3.25}$$

$$D_{12} = \frac{1}{\sqrt{2}}\,[K_{12} - |\bar{\Phi}_{13}|^2 - 2|\bar{\Phi}_{12}|^2 + \tfrac{1}{3}\sigma^2 - \tfrac{1}{3}\beta^2 - \delta^2], \tag{3.26}$$

$$D_{45} = \frac{1}{\sqrt{2}}\,[K_{45} + |\delta|^2], \tag{3.27}$$

$$D_\zeta = \frac{1}{\sqrt{6}}\,[K_\zeta + \beta^2], \tag{3.28}$$

$$D_\chi = \frac{1}{\sqrt{15}}\,[K_\chi + 2\sigma^2 + 2(|\bar{\Phi}_{45}|^2 - |\Phi_{45}|^2)]. \tag{3.29}$$

Here $\hat{\xi} \equiv \sqrt{15}\,\xi$. As before, the K-terms contain fields like squarks, sleptons and Higgs-bosons which have zero VEVs. Anticipating that a field like $\bar{\Phi}_{12}$ (or Φ_{23}) which is charged under U_{12} and possibly U_ψ, but not the other $U(1)'s$, may need to acquire a VEV of order $m \ll \sqrt{\xi}$, in the presence of a mass term m, we have exhibited its contribution. The combinations σ, β and δ are defined by

$$\sigma^2 \equiv |\Phi_1^+|^2 + |\bar{\Phi}_2^-|^2 + |\bar{\Phi}_3^-|^2, \tag{3.30}$$

$$\beta^2 \equiv |\Phi_1^+|^2 + |\bar{\Phi}_2^-|^2 - 2|\bar{\Phi}_3^-|^2, \tag{3.31}$$

$$\delta^2 \equiv |\Phi_1^+|^2 - |\bar{\Phi}_2^-|^2. \tag{3.32}$$

It may be verified that all the F- and D-flat constraints ($F_i = D_A = D_\alpha = 0$) are satisfied for the $m = 0$ cubic-level superpotential, with $|\langle\Phi_{45}\rangle_0|^2 = \frac{1}{5}\hat{\xi}$, and $|\langle\bar{\Phi}_{13}\rangle_0|^2 = |\langle\Phi_1^+\rangle_0|^2 = |\langle\bar{\Phi}_2^-\rangle_0|^2 = |\langle\bar{\Phi}_3^-\rangle_0|^2 = \frac{1}{15}\hat{\xi}$, i.e.

$$|\langle\Phi_{45}\rangle_0|^2 = |\langle\sigma\rangle_0|^2 = 3|\langle\bar{\Phi}_{13}\rangle_0|^2 = \tfrac{1}{5}\hat{\xi}, \tag{3.33}$$

$$\langle\beta^2\rangle_0 = \langle\delta^2\rangle_0 = \langle\bar{\Phi}_{12}\rangle_0 = 0. \tag{3.34}$$

All other VEVs are zero. As before, the subscript zero signifies that the VEVs are obtained in the limit of zero-mass for all the fields. The VEV of the singlet ξ_1 is not determined by the F- and D-flat constraints (at least at the cubic level superpotential). Independent phenomenological considerations including quark–lepton masses and mixings, however, imply that the VEV $\langle\xi_1\rangle$ must be of order $\sqrt{\xi} \sim \mathcal{O}(g^2 M_{\rm st}/4\pi)$ [26]. In the absence of a complete solution to the vacuum selection in string theory, we will proceed by imposing this choice.

Allowing for a mass term m as in (3.11), and extremizing the potential

$$V = \tfrac{1}{2}g^2 \sum_\alpha D_\alpha^2 + m^2(|\Phi_{45}|^2 + |\bar{\Phi}_{45}|^2), \tag{3.35}$$

with respect to Φ_{45}, $\bar{\Phi}_{13}$, σ and $\bar{\Phi}_{12}$ respectively, we obtain (compare with Eqs. (3.13)–(3.15))

$$\frac{\partial V}{\partial \Phi_{45}} = 0 \quad\Rightarrow\quad (\Phi_{45})^\dagger[4(D_A + \tfrac{1}{2}D_\chi - \tfrac{1}{4}\sqrt{15}\,m'^2) - \sqrt{\tfrac{5}{2}}\,D_\psi] = 0, \tag{3.36}$$

$$\frac{\partial V}{\partial \bar{\Phi}_{13}} = 0 \quad\Rightarrow\quad (\bar{\Phi}_{13})^\dagger[D_{12} + \sqrt{3}\,D_\psi] = 0, \tag{3.37}$$

$$\frac{\partial V}{\partial \sigma} = 0 \quad\Rightarrow\quad \sigma[(D_A - 2D_\chi) - \sqrt{\tfrac{5}{6}}\,D_{12}] = 0, \tag{3.38}$$

$$\frac{\partial V}{\partial \bar{\Phi}_{12}} = 0 \quad\Rightarrow\quad (\bar{\Phi}_{12})^\dagger(D_{12}) = 0. \tag{3.39}$$

Here $m'^2 \equiv m^2/g^2$. variations with respect to β and δ are not exhibited because these can be satisfied consistently by preserving their zero mass values: $\beta = \beta_0 = 0$ and $\delta = \delta_0 = 0$. Thus, from Eqs. (3.27) and (3.28), $D_{45} = D_\zeta = 0$. As in the previous example, we see from Eq. (3.36) that $\langle D_A\rangle$, $\langle D_\chi\rangle$ and/or $\langle D_\psi\rangle$ must be of order (m^2), and thus SUSY is broken.

Unlike the previous example, however, where $\Phi'_{45} \neq 0$ uniquely led (via Eq. (3.10)) to $D_\psi = 0$, we see that Eq. (3.37), can be satisfied, given $\bar{\Phi}_{13} \neq 0$, by choosing either (a) $D_{12} = D_\psi = 0$, or (b) $D_{12} = -\sqrt{3}\,D_\psi = \mathcal{O}(m^2) \neq 0$. In short, the solutions for the D's do not appear to be unique. Case (a) would, of course be phenomenologically preferable, because it would lead to family-universal squark masses. We consider these two cases by turn and show that minimization of the potential in fact favors case (a) over case (b).

Case (a): $D_{12} = D_\psi = 0$

Given $D_{12} = 0$, Eq. (3.39) can be satisfied by choosing either $\bar{\Phi}_{12} = 0$ or $\bar{\Phi}_{12} = \mathcal{O}(m) \neq 0$. We will see that internal consistency will fix $\bar{\Phi}_{12} = \mathcal{O}(m) \neq 0$, if $D_{12} = 0$.

Given $D_{12} = D_\psi = 0$, Eqs. (3.36) and (3.38) imply: (i) $D_A + \frac{1}{2}D_\chi = \frac{1}{4}\sqrt{15}\,m'^2$ and (ii) $D_A = 2D_\chi$, which in turn imply

$$D_A = \sqrt{\tfrac{3}{5}}\,m'^2, \qquad D_\chi = \sqrt{\tfrac{3}{5}}\,(\tfrac{1}{2}m'^2). \tag{3.40}$$

Now, given $\beta = \delta = 0$, looking at the compositions of the D-terms (Eqs. (3.25)-(3.29)), $D_\psi = 0$ and $D_{12} = 0$ imply

$$|\bar{\Phi}_{13}|^2 = \tfrac{1}{3}|\Phi_{45}|^2, \tag{3.41}$$

$$\sigma^2 = |\Phi_{45}|^2 + 6|\bar{\Phi}_{12}|^2. \tag{3.42}$$

Substituting (3.42) in D_χ (see Eq. (3.29)), and putting $D_\chi = \sqrt{(3/5)}\,(\frac{1}{2}m'^2)$ (see (3.40)), we get

$$|\bar{\Phi}_{12}|^2 = \tfrac{1}{8}m'^2. \tag{3.43}$$

Thus, internal consistency for case (a) ($D_{12} = D_\psi = 0$) implies that a field like $\bar{\Phi}_{12}$ (alternatively Φ_{23} will also be adequate), which had a zero VEV to begin with (i.e. for $m = 0$), must acquire a non-zero VEV of order m.

Solving for the nonzero VEVs, and collecting the results, we obtain

$$|\langle\Phi_{45}\rangle|^2 = \tfrac{1}{5}\hat{\xi} - \tfrac{3}{4}m'^2, \qquad |\langle\bar{\Phi}_{13}\rangle|^2 = \tfrac{1}{15}\hat{\xi} - \tfrac{1}{4}m'^2, \qquad \langle\sigma^2\rangle = \tfrac{1}{5}\hat{\xi}, \tag{3.44}$$

$$\langle\bar{\Phi}_{12}\rangle^2 = \tfrac{1}{8}m'^2, \tag{3.45}$$

$$\langle\bar{\Phi}_{45}\rangle^2 = \langle\beta^2\rangle = \langle\delta^2\rangle = 0, \tag{3.46}$$

$$\langle D_A\rangle = \sqrt{\tfrac{3}{5}}\,m'^2, \qquad \langle D_\chi\rangle = \sqrt{\tfrac{3}{5}}\,(\tfrac{1}{2}m'^2), \tag{3.47}$$

$$D_\psi = D_{12} = D_\zeta = D_{45} = 0, \tag{3.48}$$

$$\langle F(\Phi_{45})\rangle = \langle F(\bar{\Phi}_{13})\rangle = \langle F(\sigma)\rangle = \langle F(\zeta)\rangle = \langle F(\delta)\rangle = 0, \tag{3.49}$$

$$\langle F(\bar{\Phi}_{45})\rangle = m\sqrt{\tfrac{1}{5}\hat{\xi} - \tfrac{3}{4}m'^2}. \tag{3.50}$$

It may be verified that the solution presented above is in fact a minimum of V. *We see that there exists at least a local minimum for which the D-terms of only the universal charges Q_A and Q_χ are non-zero.* This solution thus has the desirable feature that the D-term contributions to the squark-masses, which dominate over F-term contributions, are family universal. The rest of the phenomenological discussions (i.e. the gaugino masses and the hierarchy) are qualitatively the same as in solution II (see Eqs. 3.19)-(3.22). To be specific, now $\mathrm{Tr}\,Q_A = 180/\sqrt{15}$, but $\hat{\xi} = \sqrt{15}\,\xi$, so $\epsilon \equiv \hat{\xi}/M_{\rm Pl}^2 \approx \frac{1}{25}$. The contributions of D_A and D_χ are given by $[m^2_{\tilde{d},\tilde{s},\tilde{b}}]_{D_A,D_\chi} = g^2[Q_A D_\chi + Q_\chi D_\chi] = \frac{1}{4}m^2$, where we have put $\sqrt{15}\,Q_A = \frac{3}{2}$ and $\sqrt{15}\,Q_\chi = -\frac{1}{2}$ (see Table 1) and $D_A = \sqrt{(3/5)}\,m'^2 = 2D_\chi$. The F-term contributions (see Eq. (3.20)), which may in general be non-universal, are given by $[m^2_{\tilde{d},\tilde{s},\tilde{b}}]_F \approx \lambda|F(\bar{\Phi}_{45})|^2/M^2_{\rm Pl} \approx \lambda(\frac{1}{5}m^2)\epsilon$, which are thus suppressed by about a factor of $(\frac{1}{60}-\frac{1}{30})$ (for $\lambda \approx (\frac{1}{2}-1)$) compared to the universal D-term contributions of $g^2[Q_A D_\chi + Q_\chi D_\chi] = \frac{1}{4}m^2$. As for solution II, this is compatible with the constraints from the real part of the K^0–$\bar{K}^0$ amplitude, if $m_{\tilde{d},\tilde{s}} \gtrsim$ (700–1400) GeV.

Case (b): $D_{12} = \mathcal{O}(m^2) \neq 0$

In this case, Eqs. (3.39) and (3.37) respectively imply

$$\bar{\Phi}_{12} = 0, \quad \text{and} \quad D_\psi = -D_{12}/\sqrt{3}, \tag{3.51}$$

substituting (3.51) into (3.36) and using (3.38), we get

$$D_A = \sqrt{\tfrac{3}{5}}\,m'^2, \qquad D_\chi = \tfrac{1}{2}\left(\sqrt{\tfrac{3}{5}}\,m'^2 - \sqrt{\tfrac{5}{6}}\,D_{12}\right). \tag{3.52}$$

Given $\bar{\Phi}_{12} = \beta = \delta = 0$, the expressions for D_A, D_{12} and D_ψ and D_χ given in Eqs. (3.24)–(3.29) respectively yield

$$4|\Phi_{45}|^2 + \sigma^2 = \hat{\xi} - 3m'^2, \tag{3.53}$$

$$-|\bar{\Phi}_{13}|^2 + \tfrac{1}{3}\sigma^2 = \sqrt{2}\,D_{12}, \tag{3.54}$$

$$-3|\bar{\Phi}_{13}|^2 + |\Phi_{45}|^2 = -\sqrt{2}\,D_{12}, \tag{3.55}$$

$$\sigma^2 - |\Phi_{45}|^2 = \tfrac{3}{4}m'^2 - \frac{5}{4\sqrt{2}}\,D_{12}. \tag{3.56}$$

Combining (3.54) and (3.55), we get

$$|\Phi_{45}|^2 - \sigma^2 = -4\sqrt{2}\,D_{12}. \tag{3.57}$$

Combining (3.53)–(3.57), one obtains

$$|\Phi_{45}|^2 = \tfrac{1}{5}\hat{\xi} - \tfrac{3}{4}m'^2 + \frac{1}{4\sqrt{2}}\,D_{12}, \tag{3.58}$$

$$\sigma^2 = \tfrac{1}{5}\hat{\xi} - \frac{1}{\sqrt{2}}\,D_{12}, \tag{3.59}$$

$$|\bar{\Phi}_{13}|^2 = \tfrac{1}{15}\hat{\xi} - \tfrac{1}{4}m'^2 + \frac{3}{4\sqrt{2}}\,D_{12}. \tag{3.60}$$

Comparing (3.56) and (3.57), we get

$$D_{12} = \frac{3\sqrt{2}}{37}\,m'^2. \tag{3.61}$$

One can verify that the solution for the VEVs presented above again corresponds to a minimum of V. To compare the minimum obtained in case (a) with that of case (b) we need to study the variation of V with respect to D_{12}. To do so we evaluate the potential at the string unification scale, where the couplings of the $U(1)$'s are unified,

$$V = \tfrac{1}{2}g^2(D_A^2 + D_\chi^2 + D_\psi^2 + D_{12}^2) + m^2(|\Phi_{45}|^2 + \bar{\Phi}_{45}^2). \tag{3.62}$$

Substituting for D_A, D_χ, D_ψ, and $|\Phi_{45}|^2$ from Eqs. (3.52), (3.51), and (3.58), we get[4]

$$\frac{V}{\frac{1}{2}g^2} = \tfrac{2}{5}\hat{\xi} m'^2 + \mathcal{O}(m'^4) + \tfrac{13}{24}D_{12}^2 + m'^2\left(-\frac{1}{4\sqrt{2}} + \frac{1}{4\sqrt{2}}\right)D_{12}. \tag{3.63}$$

We see that the coefficient of the linear term in D_{12} cancels owing to contributions from D_χ^2 and $|\Phi_{45}|^2$, and thus V would increase for $D_{12} \neq 0$. This shows that the minimum of V corresponding to case (a), with $D_{12} = D_\psi = 0$, is preferred over that of case (b), with $D_{12} = -\sqrt{3}\,D_\psi \neq 0$. This in turn means that even for the more realistic, though complicated, case of solution I, there exist viable solutions for the pattern of VEVs for which only the family-universal contributions to squark masses, arising through D_A and D_χ, survive, but the non-universal D-term contributions, associated with the family-dependent $U(1)$'s, vanish owing to minimization of the potential. While this result may not hold in general, it is remarkable that it does for the solutions considered here, which are viable. The conditions for emergence of this result, which are worth studying, will thus provide an important new guideline for selecting the string solutions, and the associated patterns of zeroth order VEVs which satisfy the F- and D-flat conditions.

The degeneracy in squark-masses, obtained as above at the string-unification scale, would of course be affected, as would be the ratios of the various gauge and Yukawa couplings, when they are extrapolated to low energies through the use of the renormalization group equations. This would not, however, have a significant effect at least on the degeneracy of the squarks of the first two families, which is relevant to the K_0–$\bar{K}_0$ transition.[5]

Sign of $\langle D_A \rangle$ – Contribution to squares of scalar masses

Before discussing the origin of the mass term m, one special property of both solutions I and II is worth noting. Given that the overall sign of $U(1)_A$ is chosen such that $\mathrm{Tr}\,Q_A$ is positive, the signs of the anomalous charges of all the fields are fixed. For instance, if the sign of Q_A for squarks and/or sleptons in any solution happened to be negative, it must of course be discarded because the corresponding D_A would lead to negative

[4] The question of why the vacuum energy (cosmological constant) is so small or zero of course remains unanswered, as it is in all other analogous approaches leading to SUSY breaking.

[5] We should also add that even though $\langle D_{12}\rangle$ and $\langle D_\psi\rangle$ vanish at the level considered above, the VEVs of Higgs fields (like H_u) of electroweak scale will still induce a non-vanishing $\langle D_{12}\rangle \approx Q_{12}(H_u)|\langle H_u\rangle|^2 = (1/\sqrt{2})|\langle H_u\rangle|^2$, and likewise a non-vanishing $\langle D_\psi\rangle$. This leads to a mass splitting $|\delta m_{\tilde{d}}^2 - \delta m_{\tilde{s}}^2| \approx (\frac{1}{2}g_2^2)|(Q_{12}^{\tilde{d}} - Q_{12}^{\tilde{s}})|\langle D_{12}\rangle \approx (\frac{1}{2}g_2^2)(1/(2\sqrt{2}))(1/\sqrt{2})|\langle H_u\rangle|^2 \approx (50\ \mathrm{GeV})^2 \leqslant (\frac{1}{186})\tilde{m}^2$, for $\tilde{m} \geqslant 700$ GeV, where we have put $\langle H_u\rangle \sim 200$ GeV. As discussed in the text, lack of squark degeneracy to this extent is of course compatible with the constraint of the real part of the K_0–$\bar{K}_0$ amplitude. We thank K.S. Babu for raising this point.

contributions to the $(\text{mass})^2$ of these fields (see Eq. (3.19) and thereby to a breaking of $SU(3)$-color and/or electric charge. As may be seen from Tables 1 and 4, it is indeed remarkable that *all the squarks and the sleptons have positive Q_A for both solutions I and II.*

One must still ensure that none of the other fields carrying color and/or electric charge acquire net negative $(\text{mass})^2$. Note first of all that the hidden sector fields V_i, $\bar{V}_i$ as well as T_i and $\bar{T}_i$ (which are of course standard model singlets) have positive Q_A. The fields of possible concern for solution I are the conjugate pairs $(D_{45}, \bar{D}_{45})$ and (H_{21}, H_{22}) which carry color. Clearly one member of each such pair would have positive Q_A but the other member would have negative Q_A, as do D_{45} and H_{21}. Thus $\langle D_A \rangle$ would give negative $(\text{mass})^2$ to D_{45} and H_{21}. We have, however, checked that higher dimensional operators for solution I as well as solution II give sufficient positive contribution to the $(\text{mass})^2$ of each member of these conjugate pairs, by utilizing the VEVs of standard model singlets of order $\sqrt{\xi}$ (as in Eq. (3.23)) and hidden sector condensates. This more than compensates for the negative contribution of $\langle D_A \rangle$. Specifically, for solution I, one obtains the operators $H_{21}H_{22}\xi_1$ and $D_{45}\bar{D}_{45}H_{19}H_{20}\xi_1^3$ at $N = 3$ and $N = 7$ respectively in the superpotential W. With $\xi_1 \sim \sqrt{\xi}$ (see Eq. (3.23)), and assuming that the $(H_{21}H_{20})$-pair condenses due to the $SU(5)_H$ force which confines at a scale of 10^{13}–10^{14} GeV [30], these operators provide positive contributions to $(\text{mass})^2$ of H_{21} and H_{22} as well as of D_{45} and $\bar{D}_{45}$, that far exceed the negative contributions of $\langle D_A \rangle$, which are of order m^2 (it is worth mentioning in advance that m itself is induced only at $N = 8$ by utilizing condensate of the same type as above (see next section)).

Now each member of a conjugate pair of Higgs doublets like $(h_i, \bar{h}_i)$ would also get negative contribution to its $(\text{mass})^2 \sim (1\text{TeV})^2$ through $\langle D_A \rangle$. As mentioned in Section 3, the $(\text{mass})^2$ matrix of the Higgs-sector, including contributions from the string-generated higher dimensional operators has many entries. This matrix has been analyzed in detail in Ref. [26], which showed that only one pair of doublets remains light, while the others acquire heavy or medium heavy masses. While a reanalysis of the Higgs mass-matrix including the $\langle D_A \rangle$-contributions of order $(1\text{ TeV})^2$ deserves study in a separate work, it is clear that the latter contribution will affect only the light Higgs spectrum. (In general, it is possible that such a light Higgs may even acquire a VEV of order 1 TeV due to the $\langle D_A \rangle$-contribution at a high scale; this by itself need not, however, be objectionable.)

In summary, we note that the higher dimensional operators could not in any case have given masses to standard model *non-singlet chiral* fields like squarks and sleptons by using VEVs of only standard model singlet fields like those in Eq. (3.23) and hidden sector condensates. It is thus fortunate that these fields carry positive anomalous charges and thereby receive only positive contributions to their $(\text{mass})^2$ from $\langle D_A \rangle$ for both solutions I and II. On the other hand, higher dimensional operators can and do contribute positively to the $(\text{mass})^2$ of fields belonging to conjugate pairs, and more than compensate for negative contributions from $\langle D_A \rangle$ to such fields.

ı can arise through higher dimensional operators
sider first solution I (Ref. [13]). The allowed
:

2,

$2;$ (4.1)

$\bar{V}_2\Phi_{45}\Phi_{45}\xi_1, \qquad T_2\bar{T}_1V_1\bar{V}_2\Phi_{45}\Phi_{45}\xi_2,$ (4.2)

$\cdot\, \Phi_{13}\bar{\Phi}_{13} + \Phi_{23}\bar{\Phi}_{23})\Big],$

$\cdot\, \Phi_{13}\bar{\Phi}_{13} + \Phi_{23}\bar{\Phi}_{23})\Big],$

$_1\Phi_{45}\Phi_1^-\xi_2\left(\frac{\partial W_3}{\partial\bar{\Phi}_{12}}\right);$ (4.3)

$V_1\bar{V}_3\Phi_{45}\Phi_{45}\bar{\Phi}_{23}\xi_2,$

$V_3\bar{V}_1\Phi_{45}\Phi_{45}\bar{\Phi}_{23}\xi_2.$ (4.4)

$_{45}\bar{\Phi}_{45}T_3\bar{T}_3\Phi_{45}\bar{\Phi}_{13}\bar{\Phi}_3^-\xi_2.$ (4.5)

ıattern of VEVs listed in Eq. (3.23), no mass
ilinear mass terms like Φ_{45}^2 could arise, only if
s like $\langle T_2\bar{T}_3\rangle$ as well as $\langle V_3\bar{V}_2\rangle$ could form. For
$\langle \bar{d}s\rangle$) are forbidden. Even if they do form, the
ırder $(\Lambda_5/M_{st})^2(\Lambda_3/M_{st})^2M'$, where Λ_5 and Λ_3
ıfinement scales, respectively, and M' is of order
Vs of the singlet Φ-fields in Eq. (3.23). Taking
and $\Lambda_3 \sim 10^8$–10^{10} GeV, which are suggested
r solution I [30]. With $M_{st} \sim 10^{18}$ GeV and
erms are $\leqslant 10^{-9}$ GeV, and are thus insignificant
ıing that at least the diagonal condensates in the
ly relevant mass term is given by the $N = 8$ term
s term $m\Phi_{45}\bar{\Phi}_{45}$ which is neutral with respect to

$\left(\frac{M'}{M_{st}}\right)^3 M'.$ (4.6)

It is remarkable that m receives contributions only at $N \geqslant 8$. Since Λ_5 is 4 to 5 orders of magnitude smaller, and M' is about 10 to 30 times smaller than M_{st}, it is clear that the SUSY mass splitting m is naturally strongly suppressed compared to M_{st}. As regards its numerical value, for values of Λ_5 and M' lying in the range mentioned above, i.e. $(\Lambda_5/M_{st})^2 \sim 10^{-8}$–$10^{-10}$, $(M'/M_{st})^3 \sim 10^{-4}$ and $M' \sim \frac{1}{2}(10^{17}$ GeV), say – which are most plausible, we get

$$m \sim (\tfrac{1}{2}\text{–}50)\ \text{TeV}. \tag{4.7}$$

Since m represents the scale of supersymmetry breaking and thus the mass scale of the Higgs scalars, and in turn of m_W, we see that the string solutions under consideration do explain why the electroweak scale is so much smaller than the string scale.

5. Some phenomenological aspects

An interesting phenomenological distinction, first between the two solutions I and II, considered above, is worth noting. Although the anomalous charge Q_A for solution I [13], given by Eq. (2.2), is family universal, it distinguishes between different members of a family (see Table 1), and therefore leads to intra-family mass splittings among the scalars. Including contributions from the leading D_A-term only, these are represented, by the following relative values, at the scale of $\sqrt{\xi}$, for any given family:

$$\left[m^2(\tilde{Q}_L) : m^2(\tilde{u}_R) : m^2(\tilde{d}_R) : m^2(\tilde{L}) : m^2(\tilde{e}_R)\right]_{D_A} = 3:1:3:1:1 \quad \text{(Sol. I)}. \tag{5.1}$$

For solution II [14], on the other hand, Q_A given by Eq. (2.6), is the same for all members of a family. As a result, in so far as the leading contributions from the D_A-term, one obtains intra-family-universal scalar masses at the scale $\sqrt{\xi}$, which are given by

$$\left[m^2(\tilde{Q}_L) : m^2(\tilde{u}_R) : m^2(\tilde{d}_R) : m^2(\tilde{L}) : m^2(\tilde{e}_R)\right]_{D_A} = 1:1:1:1:1 \quad \text{(Sol. II)}. \tag{5.2}$$

Thus, eventually empirical study of the squark spectrum can in fact distinguish between the two string solutions I and II.

It is also worth noting that both string solutions I and II lead to approximately universal scalar masses (at the scale of $\sqrt{\xi}$) *for all three families*. At the same time, owing to spontaneously induced asymmetric Higgs mass spectrum (see discussion in Section 3) they lead to hierarchical fermion masses [26]. By contrast, the model of Ref. [5] assumes that $U(1)_A$ couples universally only to the first two families and thus predicts heavier squark masses (~ 5 TeV) for the first two families and lighter mass (~ 500 GeV) for the stop, while the gauginos are lighter still (~ 50–100 GeV). It has been pointed out in Ref. [31], however that models of this class [5,7] with two-family universality (of the anomalous $U(1)$) typically lead to color- and electric charge-breaking, assuming that the spectrum of the type noted above is generated near the

Planck- or the GUT-scale. This is because contributions from two-loop renormalization group evolution to the scalar masses contain terms which are proportional to the larger squark $(\text{mass})^2$ of the first two families, and are negative. This negative contribution turns the initially smaller positive stop $(\text{mass})^2 \sim (500\ \text{GeV})^2$ to negative values at the TeV-scale and thereby induces color- and charge-breaking. Models with three-family universality (of the anomalous $U(1)$), as discussed here, do not however face this problem because the squarks of all three families are nearly degenerate, with only moderately heavy masses. Several considerations suggest that they should have a mass of about 1 TeV, within a factor of two, either way, at the electroweak scale. This reduces the RGE-induced negative contribution to the squark $(\text{mass})^2$ by about an order of magnitude, while increasing the initial positive value of $m_{\tilde{t}}^2$ at the GUT or the string scale, compared to the case of two-family universality. It thereby eliminates the problem of color- and charge-breaking. Thus it seems that phenomenological considerations favor three-family universality, for SUSY breaking through anomalous $U(1)$. It is intriguing that string solutions of the type considered here yield precisely that.

6. Remarks on supersymmetry breaking through anomalous $U(1)$

Before concluding the following remarks are in order:

(1) Desirability and origin of family permutation symmetry. If supersymmetry breaking occurs entirely or dominantly through an anomalous $U(1)$, as noted in the last section, the need to avoid color- and charge-breaking suggests that the $U(1)_A$ *must be universal with respect to all three families.* At the same time, the hierarchical masses and mixings of the three families suggest that there ought to exist *flavor or horizontal gauge symmetries*, beyond GUTs in the underlying theory, which distinguish between the families and are ultimately responsible for the hierarchy in their masses. The virtues of flavor symmetries in the string context (like U_1 to U_6 for solution I and U_1 to U_5 for solution II), in this regard has been noted in previous works [34,26], and in the non-string context by several authors [35]. Furthermore, these flavor symmetries have been shown to play a crucial role in addressing certain naturalness problems of supersymmetry, such as the enormous suppressions of (a) the $d = 4$ and $d = 5$ rapid proton-decay operators [17,18], (b) ν_L–$\tilde{H}$ mixing mass [19], and (c) the mass term m of relevant fields which triggers SUSY breaking (see Section 4). We suspect that they are also responsible for the desired suppression of the μ-parameter.

As alluded to above, such family-dependent flavor symmetries, which are clearly absent in GUTs, do in fact emerge quite generically in string theory (e.g. from an underlying $SO(44)$ in the free fermionic construction). Now, typically, at least a subset of these family-dependent $U(1)$'s would appear to be anomalous in a general basis (compare with U_1 to U_6 for solution I and U_1 to U_3 for solution II); these can be grouped to give anomaly-free combinations (like U_ψ, U_{12}, U_ζ, U_{45} and U_χ in solution I (see Eqs. (2.3)–(2.5)); except for one unique combination that remains anomalous and gives the $U(1)_A$ (see Eq. (2.2)). Similar situations arise in all other semi-realistic string-derived models which exist to date; see e.g. Refs. [9-15,32,33].

The question then arises how there can be these flavor-symmetries, which distinguish between the families and are thus family-non-universal, and yet there be an anomalous $U(1)_A$, arising from linear combinations of the same flavor-symmetries, which is family universal? The only way, it appears to us, is that the flavor symmetries, although family-dependent, must still respect the *permutation symmetry* (mentioned in Sections 1 and 2) with respect to all three families. In this case $U(1)_A$ would automatically be family universal, as borne out by the examples of Eqs. (2.2) and (2.7).

Thus, SUSY breaking through $U(1)_A$, together with the presence of flavor symmetries, seem to suggest the need for the stated permutation symmetry. As stated in Section 2, such a symmetry is in fact an internal property of at least the NAHE set of boundary condition basis vectors $\{\mathbf{1}, S, b_1, b_2, b_3\}$, for which the cyclic permutation symmetry corresponds simply to the symmetry between the three twisted sectors of the $Z_2 \times Z_2$ orbifold, which arise from the sectors b_1, b_2 and b_3, respectively. In view of the importance of the permutation symmetry, as noted above, it would be interesting to know whether such a symmetry could arise without utilizing the NAHE set. While it is premature to ascertain the answer to this question at present, we note that there do in fact exist three-generation string solutions based on the free fermionic construction which utilize only a subset $\{\mathbf{1}, S, b_1, b_2\}$ of the NAHE set of basis vectors (see e.g. Ref. [15]); these, however, do not possess the cyclic permutation symmetry.

It is also worth noting that while the NAHE set (by itself) yields the permutation symmetry, it of course does not guarantee that the symmetry will be retained in the presence of additional boundary condition basis vectors, which are needed to reduce the number of generations from 48 to 3. As noted before, all four solutions exhibited respectively in Refs. [9,11,13,14], utilize the NAHE set; but only the last two retain the permutation symmetry, while the first two do not. Thus, string solutions of the type obtained in Refs. [13] and [14] appear to be particularly suited to break supersymmetry through an anomalous $U(1)$, while providing the squark degeneracy.

(2) A scenario of combined anomalous $U(1)$-dilaton SUSY breaking. It has been noted in Section 3 (see the discussion following Eq. (3.22)) that if SUSY breaking proceeds entirely through a family-universal $U(1)_A$, it would lead to the desired squark degeneracy, but it is likely to lead to unacceptably light gluinos. At this point, two apparently unrelated issues, both associated with the dilaton, are worth recalling. First, there is the well-known problem of dilaton-stabilization. Regardless of whether SUSY breaking utilizes the VEV of the dilaton-auxiliary component, F_S, or not, one needs to avoid its generic weak-coupling runaway behavior (i.e. $S \to \infty$), and obtain instead a stable minimum of its potential at a value of $S = S_0 \sim$ 10-20, rather than at infinity or 1 (for a discussion of this issue and references to various attempts for its resolution in the field-theory and string-theory/M-theory context, see e.g. Ref. [36] and references therein). Second, if SUSY breaking is dominated by the VEV of F_S, it seems that one

would encounter the problem of color- and electric charge-breakings (see e.g. last paper of Ref. [2]).

It seems to us, however, that in a mutually coupled system such as ours, supersymmetry-breaking may well proceed through multiple sources whose effects on soft masses may in general be comparable. In some cases, one of the sources may be viewed as primary and the other(s) secondary, and the former may in fact induce the latter. We have in mind the mutual couplings between (a) the dilaton superfield S and the non-perturbatively generated hidden sector $SU(5)_H$ gaugino and matter condensates (whose scale Λ_5 is proportional to $e^{(-S/2b_5)}$) on the one hand, and (b) that between S and the anomalous $U(1)_A$ gauge field via the Green–Schwarz term that generates the Fayet–Iliopoulos term ξ on the other hand (see e.g. Ref. [6]). *Because of the mutual couplings, these three components - i.e. the dilaton, the hidden sector condensates and the anomalous* $U(1)$ *- can influence each other's role significantly and thereby the nature of SUSY breaking.* The task at hand therefore is the minimization of the full effective potential for this coupled system, which receives contributions from (a) and (b), as well as possibly from additional non-perturbative terms in the Kähler potential. Such a minimization is to be carried out in the presence of the SUSY-preserving VEVs of standard model singlet fields $\{\Phi^i\}$ (see e.g. (3.23)), which are induced because of the Fayet–Iliopoulos term, and which generate the mass term m, by utilizing the hidden sector condensates, as in Section 4.

One particularly attractive possibility which we defer for further study is this. The hidden sector condensates, involving in general matter and gaugino pairs, in conjunction with the SUSY-preserving VEVs of the $\{\Phi_i\}$-fields generate the mass term m as in Section 4, which in turn triggers SUSY breaking through a family-universal $U(1)_A$, as in Section 3. This could provide at least a major contribution to squark-masses $\sim (\frac{1}{3}$–$2)$ TeV (say), which is approximately family universal. Simultaneously, the coupling of the dilaton to the hidden sector condensates as well as to the gauge field of $U(1)_A$, together perhaps with non-perturbative terms in the Kähler potential, stabilizes the dilaton at a desired value S_0, while inducing a VEV for the dilaton auxiliary field $\langle F_{S_0}\rangle \neq 0$.

Such a scenario, if it can be realized, would have the following advantages: (i) The dilaton-induced SUSY breaking ($\langle F_S\rangle \neq 0$) would not upset the squark-degeneracy that was obtained through the family-universal $U(1)_A$, because the dilaton contributes universally to the scalar masses (barring smaller loop-corrections). (ii) Since dilaton SUSY breaking assigns comparable masses to squarks and gauginos (unlike $U(1)_A$), following the relations $(\Delta m_{\tilde{g}})_{F_S} = \sqrt{3}(\delta m_{\tilde{q}})_{F_S} = \sqrt{3}(m_{3/2})_{F_S}$, however, it could provide the leading contribution to gluino and wino masses: $(\Delta m_{\tilde{g}})_{F_S} \approx \sqrt{3}(1 \text{ to few})(100)$ GeV (say) - while providing significant contributions to squark-masses.[6] This would remove the problem of light gluino for $U(1)_A$-SUSY breaking. (iii) SUSY breaking through *the combination of a universal* $U(1)_A$ *and dilaton mechanisms*, as described above, would

[6] While $\langle F_S\rangle$- and $\langle D_A\rangle$-contributions to squark-masses may be comparable, their relative proportion will be constrained by the need to avoid color- and charge-breaking (see Ref. [2]). Quite clearly, dominant contribution from $\langle F_S\rangle$ would be excluded on this ground.

of course avoid the danger of color- and charge-breaking, that confronts the scenario of purely dilaton dominated SUSY breaking (see the last paper in Ref. [2]). In short, the combined SUSY breaking mechanism involving a family-universal $U(1)_A$ and the dilaton has the advantage that each component cures the vices of the other, without upsetting any of its virtues. The feasibility of this combined source of SUSY breaking, including its effects on dilaton-stabilization, is clearly worth further study [37].

(3) Unlike the models of Refs. [5] and [7], which introduce very few fields and just the single anomalous $U(1)_A$, but without any accompanying flavor-symmetries, string solutions generically contain many fields (see e.g. Tables 1–3 and 4–6) and typically several $U(1)$'s, some of which are family-dependent. In spite of this more elaborate (though fixed) structures, it is interesting that minimization for the string solutions considered here, led to a hierarchical pattern of soft masses (see Eq. (3.22)) which is very similar to the cases of Refs. [5–7], barring of course the distinction of three- versus two-family degeneracy that arises from the differences in $U(1)_A$ (see Section 5). In particular, it is remarkable that, for the string solutions considered here, the VEVs of D_α's associated with family-dependent $U(1)_\alpha$'s turned out to vanish, owing to the requirement of a relatively global minimum of the potential. But for this feature, SUSY breaking through anomalous $U(1)$ in string models would not be viable.

(4) The necessary ingredients. From the preceding discussion and those in Sections 3 and 4, it is clear that the following set of ingredients are in fact needed in order that supersymmetry breaking through anomalous $U(1)$ can be implemented consistently, especially in the string-context: (i) family-universality of the $U(1)_A$ and therefore the family permutation symmetry of the flavor gauge symmetries as discussed above; (ii) suitably suppressed effective mass term m of relevant fields carrying the anomalous charge; (iii) positivity of the anomalous charges of the chiral squark and slepton fields; and (iv) vanishing (or adequate suppression) of the undesirable D-terms, associated with family-dependent $U(1)$'s, because of minimization of the potential. It seems truly remarkable that there do exist string solutions, as discussed here, for which all four ingredients are realized. If the anomalous $U(1)$ would turn out to provide an important source of SUSY breaking, realization of these necessary features, as well as meeting the non-trivial constraints from issues such as proton longevity [17,18] and fermion masses and mixings, together would clearly provide a very useful set of criteria in severely limiting the desired class of solutions from the vast set that is available.

(5) Gravitino mass. An important effective parameter of SUSY breaking is the mass of the gravitino. With SUSY breaking through only anomalous $U(1)$, as described in Section 3, the gravitino would receive a mass $m_{3/2} \sim \langle F(\bar{\Phi}_{45})\rangle/M_{\rm Pl} \approx m\sqrt{\epsilon} \approx (\frac{1}{10}m_{\tilde{q}}) \sim (1 \text{ to few})(100\text{ GeV})$. With additional sources of SUSY breaking, involving for example the dilaton and possibly hidden sector condensates, as motivated above, $m_{3/2}$ would get further contributions. While the relative contributions of these different

sources of SUSY breaking to $m_{3/2}$, squark and gluino masses are not easy to ascertain at present, we would still expect $m_{3/2}$ to lie in the 100 GeV- to a few TeV-range.

7. Summary

In summary, an anomalous $U(1)$ gauge symmetry, together with an effective mass term for certain relevant fields, offers a very simple mechanism to implement SUSY breaking. It is shown here that this mechanism can in fact be derived consistently, leaving aside the question of dilaton-stabilization, from an underlying string theory. While string solutions invariably possess an anomalous $U(1)$ symmetry, the requirement of three-family universality of squark masses and therefore of $U(1)_A$ is not easy to satisfy. We have shown that there do exist certain three-generation string solutions for which supersymmetry breaking through an anomalous $U(1)$ leads to both the desired three-family squark-degeneracy and the large hierarchy between the string and the electroweak scales. More specifically we have noted that these solutions, in contrast to most, possess *a cyclic permutation symmetry* between the three families, which automatically yields a set of non-anomalous but family-dependent flavor gauge symmetries on the one hand, and a *family-universal anomalous* $U(1)$ gauge symmetry on the other hand. It is the non-anomalous flavor symmetries, unavailable in GUTs, which are ultimately responsible for hierarchical fermion masses and CKM mixings as well as for the desired suppression of both the rapid proton decay operators and of the effective SUSY breaking mass parameter m. The anomalous $U(1)_A$ is, however, family universal. In other words it is not a horizontal symmetry, unlike the models of Refs. [5] and [7], and it is this feature that makes it suitable for the purposes of SUSY breaking without encountering color-and electric charge-breaking. We further note that family universality of the anomalous $U(1)$ has also been found to be desirable in recent attempts to fit the fermion mass spectrum by the use of Abelian horizontal symmetries [38].

We have remarked that the family permutation symmetry of the solutions of interest [13,14], is a joint consequence of (a) the NAHE set of boundary condition basis vectors which corresponds to a $Z_2 \times Z_2$ orbifold compactification, and (b) the special choice of additional boundary condition basis vectors, beyond the NAHE set, which serve to reduce the number of generations from 48 to 3. While suitable variations in (b) could still allow the permutation symmetry to be retained, it is far from clear whether the same can still be realized without the NAHE set.

As regards the issue of supersymmetry-breaking, we have noted that the marriage of the two sources for such a breaking – i.e. through $U(1)_A$ and through the dilaton – if it can be realized, would be most attractive because it would combine the advantages of both, while each would remove the disadvantage of the other. Realization of this combined mechanism, would thus be of major importance.

To conclude, if the D-term of the anomalous $U(1)$ makes a major contribution to squark masses, it must be family universal. In this case, if the NAHE set turns out to be a necessary ingredient to obtain a family-universal anomalous $U(1)$, it would be an indication that the string vacuum is in the vicinity of the $Z_2 \times Z_2$ orbifold, with the standard embedding of the gauge connection. Thus, the question about the necessity of the NAHE set for obtaining the family permutation symmetry is an interesting and important one, worth further study.

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After the submission of our paper, an interesting work has recently appeared [39], where the authors demonstrate the emergence of the *combined* anomalous $U(1)$-dilaton SUSY breaking picture (i.e. $\langle D_A \rangle \neq 0$ *and* $\langle F_S \rangle \neq 0$), which was motivated in Section 6 of our paper, because of the mutual coupling between the two systems and also on phenomenological grounds. While the dilaton-contribution seems to dominate over that of $\langle D_A \rangle$ in the cases studied in Ref. [39], as the authors note, the relative contributions of $\langle D_A \rangle$ and $\langle F_S \rangle$ to squark masses would of course depend upon the manner of dilaton-stabilization. Following remarks in Section 6, a desirable solution would seem to be one in which the $\langle D_A \rangle$ contribution is at least comparable to that of $\langle F_S \rangle$, so that the problem of color/hypercharge breaking is avoided. This issue needs further study.

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