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SUMMER SCHOOL ON PARTICLE PHYSICS

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FERMION MASSES AND THE FLAVOUR PROBLEM

Lecture I

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Please note: These are preliminary notes intended for internal distribution only.

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Fermion Masses & the Flavour Problem

G.G. Ross, Trieste, 25/6/01.

- INTRODUCTION : Beyond the Standard Model & masses
  - SUSY (GUTs, Superstrings)
  - COMPOSITE (Technicolour, Extended T.C.)
  - LARGE NEW DIMENSIONS ("Brane" - technology)
- DETERMINATION OF THE FERMION MASS MATRICES
  - Texture zeros & all that.
- FAMILY SYMMETRIES
  - Abelian
  - Non - Abelian
  - Hierarchical breaking mechanisms
- NEUTRINO MASSES
- GRAND UNIFIED THEORIES (SUSY)
- LARGE NEW DIMENSIONS & MASSES
- STRINGS

• INTRODUCTION

The Standard Model

$SU(3) \times SU(2) \times U(1)$

$G_\mu^{a=1..8}, \; W_\mu^{b=1,2,3}, \; B_\mu$

+ 3 generations quarks & leptons

+ 1 Higgs doublet $\quad H_i = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad \varepsilon_{ij} H^{j*} \equiv H'_i = \begin{pmatrix} \bar{H}^0 \\ -H^- \end{pmatrix}$

$$\mathcal{L} = \underbrace{\underbrace{\mathcal{L}_{Gauge}}_{\substack{\| \\ \mathcal{L}_{YM} + \mathcal{L}_{WD}}}}_{U(3)^5_{Family}} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Scalar}.$$

$$\underbrace{\qquad\qquad}_{U(1)^{4\;(3)}}$$

... and beyond ..

THE FLAVOUR/FAMILY PROBLEM

- Gauge group ?
- Multiplet structure ?
- \# parameters ? $\quad M_Z, M_H, g_i, m_{q_i}, \theta_i, \delta, m_{e_i}, \Theta^{QCD}$: 19
- ν masses & mixing ?? $\quad m_{\nu_i}, \theta'_i, \delta'_{i=1,2,3}$: 9
- gravity ? $\quad$ 28

- INTRODUCTION

The Standard Model

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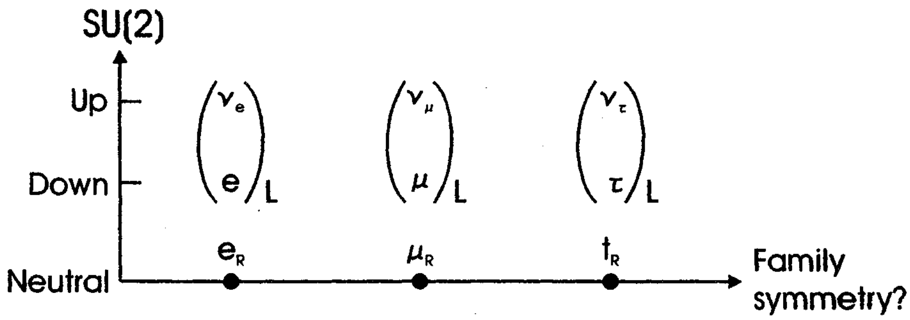
$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Scalar}.$$

$$\mathcal{L}_{Gauge} = \underbrace{\underbrace{\mathcal{L}_{YM} + \mathcal{L}_{WD}}_{U(3)^5_{\text{Family}}} \quad + \mathcal{L}_{Yukawa}}_{U(1)^{4\;(3)}}$$

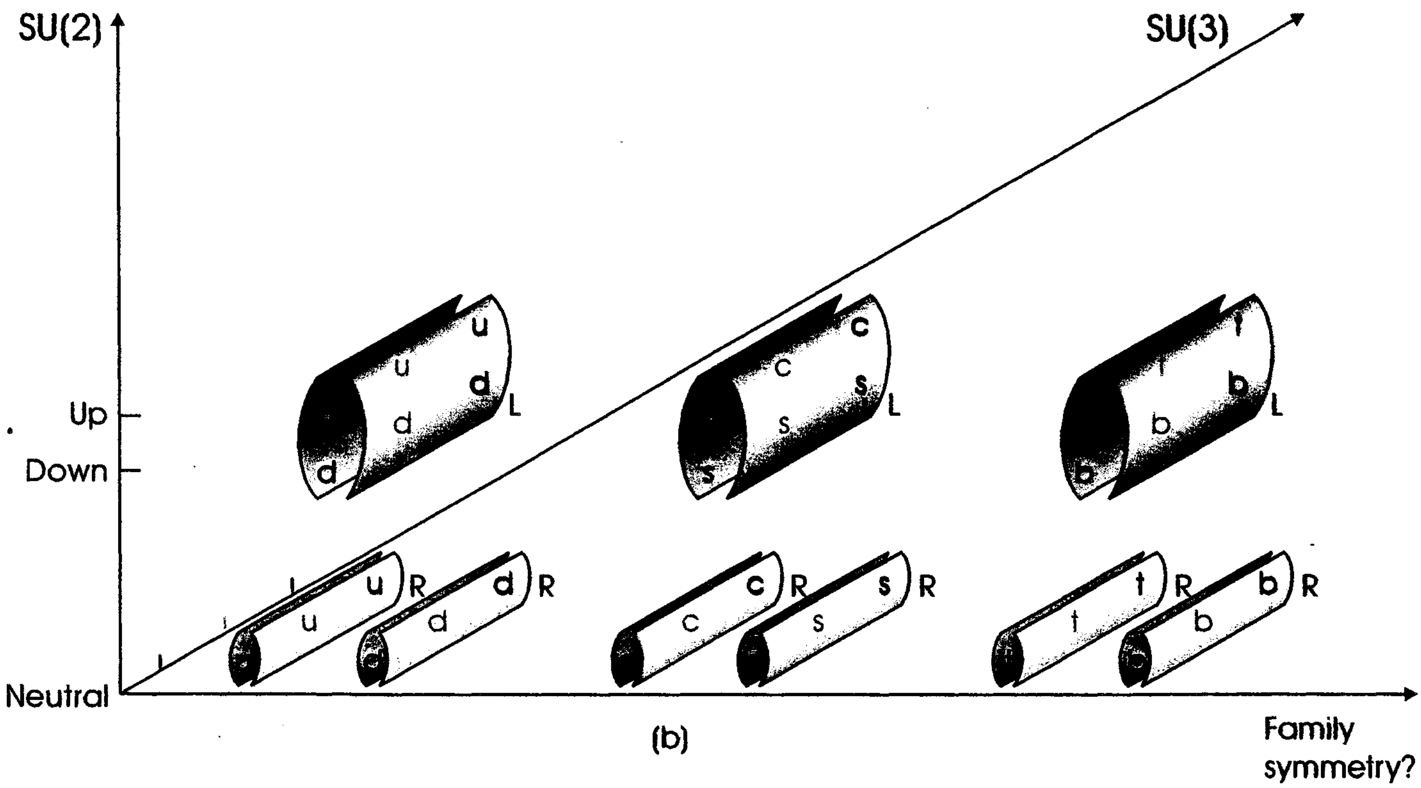
... and Beyond ..

- Gauge group?
- Multiplet structure?
- # parameters? $\quad M_Z, M_H, g_i, m_{q_i}, \theta_i, \delta, m_{\ell_i}, \Theta^{QCD}$: 19
- ν masses + mixing?? $\quad m_{\nu_i}, \Theta'_i, \delta^0_{i=1,2,3}$: 9
- gravity? $\qquad\qquad\qquad\qquad\qquad\qquad$ 28

THE BASIC MATTER STATES OF THE STANDARD MODEL.



(a)



(b)

$$\mathcal{L}_{SM} = \mathcal{L}_{YM} + \mathcal{L}_{WD} + \mathcal{L}_{Y} + \mathcal{L}_{H} \qquad (1)$$

$$\boxed{\begin{aligned}\mathcal{L}_{YM} &= \mathcal{L}_{QCD} + \mathcal{L}_{I_w} + \mathcal{L}_{Y} \ , \\ &= -\frac{1}{4g_3^2}\sum_{A=1}^{8} G^A_{\mu\nu}G^{\mu\nu A} - \frac{1}{4g_2^2}\sum_{a=1}^{3} F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4g_1^2}B_{\mu\nu}B^{\mu\nu} \ .\end{aligned}}$$

$$G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - f^{ABC}A^B_\mu A^C_\nu, \qquad A, B, C = 1, ..., 8$$

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - \epsilon^{abc}W^b_\mu W^c_\nu, \qquad a, b, c = 1, 2, 3$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\boxed{\begin{aligned}\mathcal{L}_{WD} = \sum_i^3 \Big(& L_i^\dagger \sigma^\mu \mathcal{D}_\mu L_i + \bar{\mathrm{e}}_i^\dagger \sigma^\mu \mathcal{D}_\mu \bar{\mathrm{e}}_i + \mathbf{Q}_i^\dagger \sigma^\mu \mathcal{D}_\mu \mathbf{Q}_i \\ & + \bar{\mathbf{u}}_i^\dagger \sigma^\mu \mathcal{D}_\mu \bar{\mathbf{u}}_i + \bar{\mathbf{d}}_i^\dagger \sigma^\mu \mathcal{D}_\mu \bar{\mathbf{d}}_i \Big) \ .\end{aligned}}$$

$$\mathcal{D}_\mu L_i = (\partial_\mu + i\mathbf{W}_\mu + \frac{i}{2}y_1 B_\mu)L_i \ , \qquad \mathbf{W}_\mu = \frac{1}{2}W^a_\mu(x)\tau^a \ ,$$

$$\mathcal{D}_\mu \bar{\mathrm{e}}_i = (\partial_\mu + \frac{i}{2}y_2 B_\mu)\bar{\mathrm{e}}_i \ ,$$

$$\mathcal{D}_\mu \mathbf{Q}_i = (\partial_\mu + i\mathbf{A}_\mu + i\mathbf{W}_\mu + \frac{i}{2}y_3 B_\mu)\mathbf{Q}_i \ , \qquad \mathbf{A}_\mu = \frac{1}{2}A^A_\mu(x)\lambda^A \ ,$$

$$\mathcal{D}_\mu \bar{\mathbf{u}}_i = (\partial_\mu - i\mathbf{A}^*_\mu + \frac{i}{2}y_4 B_\mu)\bar{\mathbf{u}}_i \ ,$$

$$\mathcal{D}_\mu \bar{\mathbf{d}}_i = (\partial_\mu - i\mathbf{A}^*_\mu + \frac{i}{2}y_5 B_\mu)\bar{\mathbf{d}}_i \ .$$

$$\boxed{\begin{aligned}\mathcal{L}_{\mathrm{Yu}} &= i\widehat{L}_i\bar{\mathrm{e}}_j H^* Y^{[e]}_{ij} + i\widehat{\mathbf{Q}}_i\bar{\mathbf{d}}_j H^* Y^{[d]}_{ij} + i\widehat{\mathbf{Q}}_i\bar{\mathbf{u}}_j\tau_2 H Y^{[u]}_{ij} + \text{c.c.} \\ \mathcal{L}_H &= (\mathcal{D}_\mu H)^\dagger(\mathcal{D}^\mu H) - V(H) \ ,\end{aligned}}$$

$$V = -\mu^2 H^\dagger H + \lambda(H^\dagger H)^2$$

$$\mathcal{D}_\mu H = (\partial_\mu + i\mathbf{W}_\mu + \frac{i}{2}y_h B_\mu)H$$

GAUGE INTERACTIONS

Leptons

$$\psi_L^\dagger \sigma^\mu D_\mu \psi_L =$$

$$(\nu_e^\dagger\ e^\dagger)_L \sigma^\mu \partial_\mu \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$+ \frac{i}{2}(\nu_e^\dagger\ e^\dagger)_L \begin{pmatrix} g_2 W_\mu^3 - g_1 B_\mu & g_2 W_\mu^1 - i g_2 W_\mu^2 \\ g_2 W_\mu^1 + i g_2 W_\mu^2 & -g_1 B_\mu - g_2 W_\mu^3 \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$$

$$\Rightarrow \quad -i\, e\, A_\mu (e_L^\dagger \sigma^\mu e_L + e_R^\dagger \bar{\sigma}^\mu e_R)$$ EM. $\qquad e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$

$$+ \frac{ie}{\sqrt{2}\sin\theta_w}\left(W_\mu^- \nu_{eL}^\dagger \sigma^\mu e_L + W_\mu^+ e_L^\dagger \sigma^\mu \nu_{eL}\right)$$ Charged Weak

$$+ i\frac{eZ_\mu}{\cos\theta_w \sin\theta_w}\left(\frac{1}{2}\nu_{eL}^\dagger \sigma^\mu \nu_{eL} - \frac{1}{2} e_L^\dagger \sigma^\mu e_L + \sin^2\theta_w (e_L^\dagger \sigma^\mu e_L + e_R^\dagger \bar{\sigma}^\mu e_R)\right)$$ Neutral

Quarks $\Rightarrow$ $\displaystyle\sum_{i=1}^{3} \mathbf{Q}_i^\dagger \sigma^\mu (i g_2 \mathbf{W}_\mu + \frac{i}{6} g_1 B_\mu) \mathbf{Q}_i$

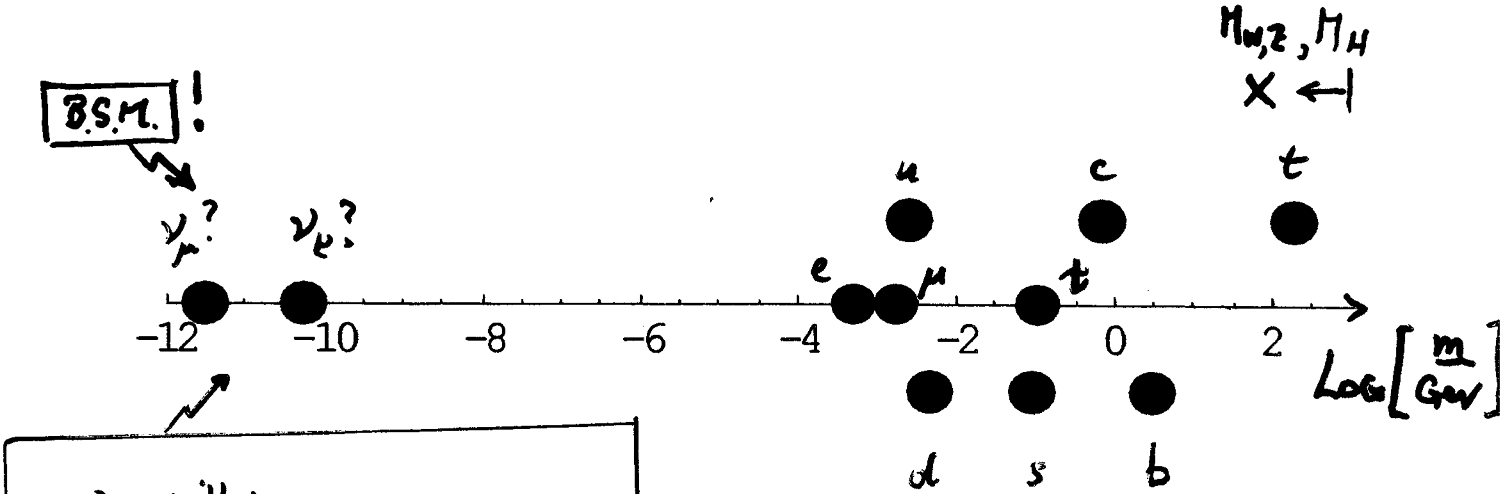
$$= \quad + ieA_\mu \left(\frac{2}{3}\mathbf{Q}_{i1}^\dagger \sigma^\mu \mathbf{Q}_{i1} - \frac{1}{3}\mathbf{Q}_{i2}^\dagger \sigma^\mu \mathbf{Q}_{i2}\right)$$

$$+ \frac{ieZ_\mu}{\cos\theta_w \sin\theta_w}\left((\frac{1}{2} - \frac{2}{3}\sin^2\theta_w)\mathbf{Q}_{1i}^\dagger \sigma^\mu \mathbf{Q}_{1i} - (\frac{1}{2} - \frac{1}{3}\sin^2\theta_w)\mathbf{Q}_{2i}^\dagger \sigma^\mu \mathbf{Q}_{2i}\right)$$

$$+ \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^+ \mathbf{Q}_{1i}^\dagger \sigma^\mu \mathbf{Q}_{2i} + \frac{ie}{\sqrt{2}\sin\theta_w} W_\mu^- \mathbf{Q}_{2i}^\dagger \sigma^\mu \mathbf{Q}_{1i}\,.$$

Unlike leptons this form does not apply to mass eigenstates

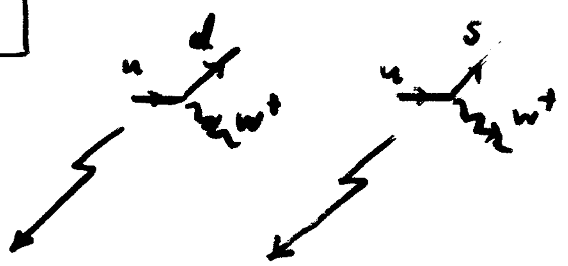
Masses and Mixing Angles



ν oscillations

Atmospheric } $\delta m^2 \sim 3.10^{-3}\,\mathrm{eV}^2$

Solar } $\delta m^2 \sim 10^{-5}\,\mathrm{eV}^2$



$$V_{CKM} = \begin{pmatrix} 0.973\text{-}0.975 & 0.217\text{-}0.222 & 0.0023\text{-}0.004 \\ 0.21\text{-}0.24 & 1.2\text{-}0.9 & 0.038\text{-}0.041 \\ 0.006\text{-}0.01 & 0.026\text{-}0.04 & 1.14\text{-}0.84 \end{pmatrix}$$

Beyond the Standard Model

- Further Unification - GUTs

e.g. $SU(3) \times SU(2) \times U(1) \subset SU(5)$ $g_1, g_2, g_3 \to g_5$

Unification scale $\boxed{M_X = (1\text{-}3)10^{16}\,\text{GeV!}}$

- Unification with gravity (strings)

$V = G_N m_1 m_2 / r, \quad G_N \mu^2 = 1,$

$$\boxed{\mu = M_{planck} = 10^{19}\,\text{GeV!}}$$

➡ Effective Field Theory

<u>but</u> $\boxed{\dfrac{M_{H,W,f_i}}{M_{X,\,Planck}} \ll 1}$?? The hierarchy problem

<u>B.S.M.</u> $\Rightarrow$

- <u>SUPERSYMMETRY</u> $m_H = 0 + O(\Lambda_{SUSY})$
- <u>COMPOSITE</u> $m_H : O(\Lambda_{COMPOSITE})$
- <u>LARGE NEW DIMENSIONS</u> $m_H : O(\Lambda_{4+d})$

Possible extensions severely constrained by <u>precision tests</u> of the Standard Model

- S, t. ... ETC disfavoured
- FCNC ... strong constraints on m_{ETC}

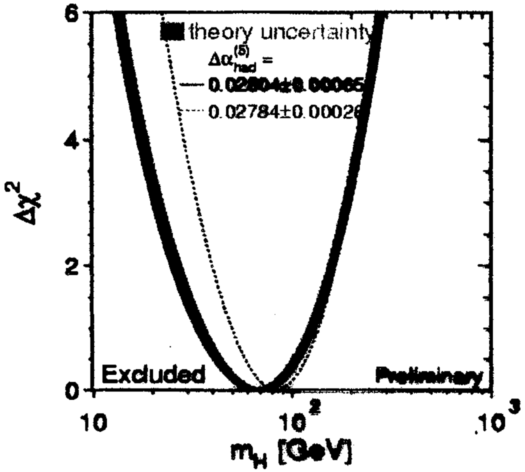
  $m_{\tilde{q}_i}^2, m_{\tilde{\ell}_i}^2$

  ...

Precision Tests

$$\Gamma_Z \Longrightarrow N_\nu = 2.984 \pm 0.008$$

Quantum corrections sensitive to new states :



Osaka 2000 (ADLO)

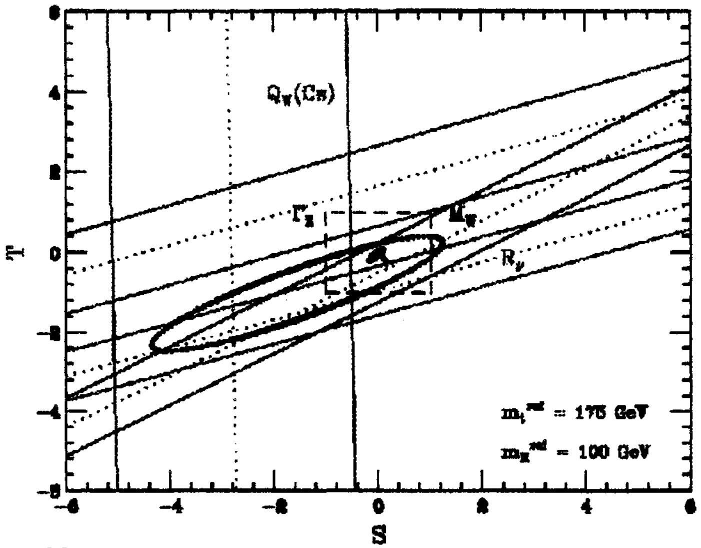
	Measurement	Pull
m_Z [GeV]	91.1875 ± 0.0021	.05
Γ_Z [GeV]	2.4952 ± 0.0023	-.42
σ^0_{hadr} [nb]	41.540 ± 0.037	1.62
R_l	20.767 ± 0.025	1.07
$A^{0,l}_{fb}$	0.01714 ± 0.00095	.75
A_e	0.1498 ± 0.0048	.38
A_τ	0.1439 ± 0.0042	-.97
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.70
m_W [GeV]	80.427 ± 0.046	.55
R_b	0.21653 ± 0.00069	1.09
R_c	0.1709 ± 0.0034	-.40
$A^{0,b}_{fb}$	0.0990 ± 0.0020	-2.38
$A^{0,c}_{fb}$	0.0689 ± 0.0035	-1.51
A_b	0.922 ± 0.023	-.55
A_c	0.631 ± 0.026	-1.43
$\sin^2\theta^{lept}_{eff}$	0.23098 ± 0.00026	-1.61
$\sin^2\theta_W$	0.2255 ± 0.0021	1.20
m_W [GeV]	80.452 ± 0.062	.81
m_t [GeV]	174.3 ± 5.1	-.01
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02804 ± 0.00065	-.29

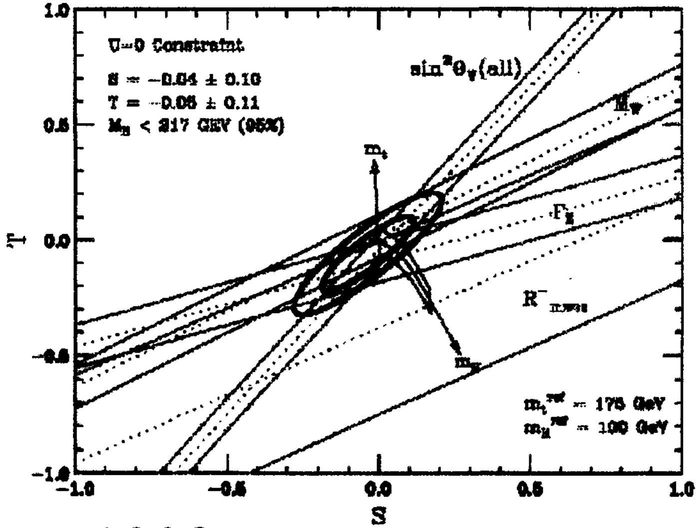
Precision measurements severely constrain possibilities

S : Weak Isospin conserving

T : Weak Isospin violating

1989





1999

Precision measurements severely constrain possibilities

Light Higgs $\Lambda \geq 5\text{TeV}$, $M_H \cong 0.3 k_{max}$???

Physics BSM

Decoupling: $S, T \sim M_Z^2/4\pi M^2$, $M \gg M_Z$ ✓

e.g. SUSY $M_H < 125$ GeV (MSSM)

Non-decoupling: Technicolour, 4th generation, q,l composite

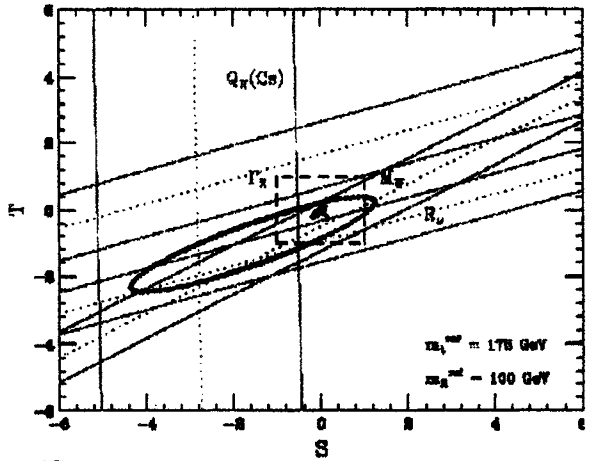
…difficult ?

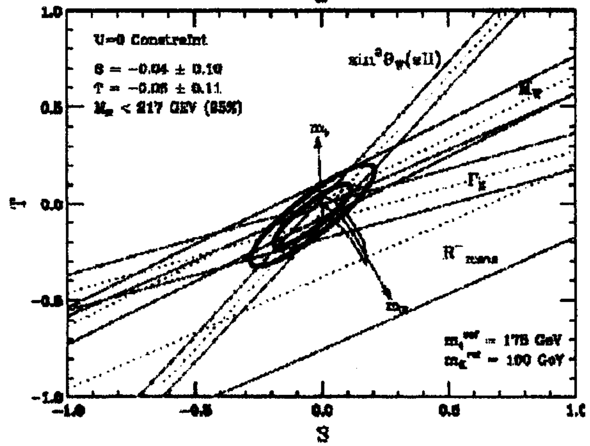
e.g. Topcolour $\langle 0|t_L t_R|0\rangle \neq 0$ .. EW breaking

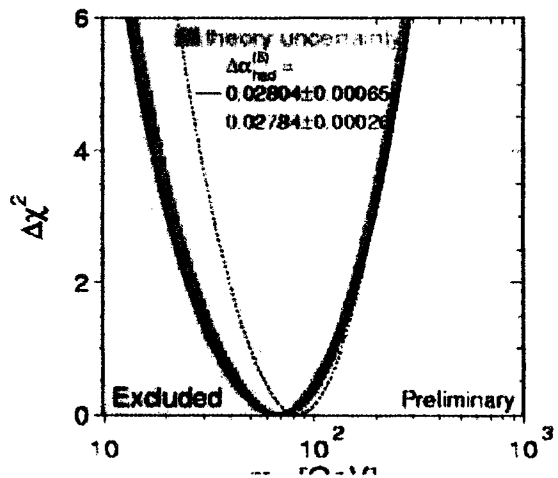
without new EW doublets

M_t too large : See saw $M_t = \langle x_L t_R\rangle\langle t_L t_{xR}\rangle / \langle x_L x_R\rangle$

X : EW singlet, Q=2/3, $\langle x_L x_R\rangle \approx 0.6\text{TeV}$







Flavour changing neutral currents (FCNC)

$SU(3)^5_{\text{family}} \Rightarrow$ GIM mechanism

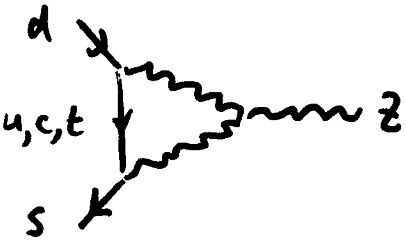
$Z_\mu J^\mu_Z$: $J^\mu_Z = \sum_{i=1}^{3} \bar\psi_i \gamma^\mu \tau_3 \psi_i$

$\psi_i^{d,m} = U_{ij}\, \psi_j^d$

$J_Z^\mu = \sum_{i=1}^{3} \bar\psi_i^m \gamma^\mu \tau_3 \psi_i$

No FCNC at tree level.

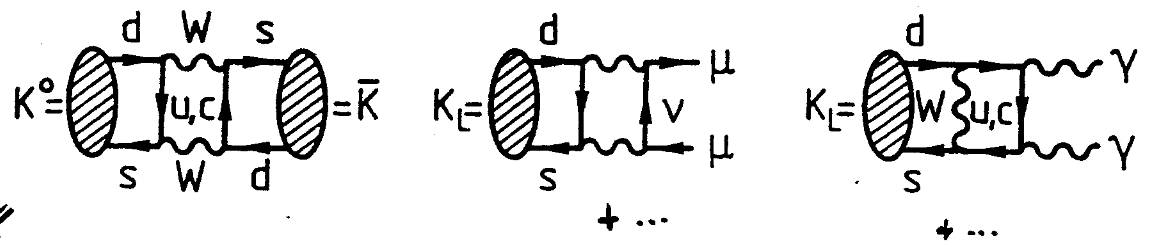
$SU(3)^5_{\text{family}}$: broken by $\mathcal{L}_{\text{Yukawa}}$ $\Rightarrow$ J^μ_Z not flavour diagonal at $O(m_i - m_j)$



$A \propto \frac{(m_u^2 - m_c^2)}{M_Z^2}$

GIM suppression

$$\boxed{\sin\theta_c \cos\theta_c \left(\frac{1}{k - m_c} - \frac{1}{k - m_u} \right) = \frac{k(m_c^2 - m_u^2) + m_u m_c (m_c - m_u)}{(k^2 - m_u^2)(k^2 - m_c^2)}}$$



$m_c^2 - m_u^2 \lesssim 1\,GeV^2$ (a) ... Gaillard, Lee (b) (c)

Fig (4.4) Quark graphs contributing to (a) $K^\circ \to \bar{K}^\circ$ (b) $K \to \mu\mu$ (c) $K \to \gamma\gamma$.

All graphs of Fig(4.4) have 4 fermion fields. A fermion field carries naive (engineering) dimension of $\frac{3}{2}$, so 4 fermion fields gives a term of dimension 6. However the graphs are contributions to the effective Lagrangian density and have dimension 4 so the coefficient of the four fermions must have dimension M^{-2}, where M is expected to be the largest mass in the loop. Thus is seems all diagrams should occur at order $G_F/\sqrt{2} \sim \frac{g_2^2}{M_W^2}$ in amplitude, whereas the experimental results of Table (4.3) indicate that the decay rates

$$\Gamma(K_L \to \mu\bar{\mu}) \simeq 2\times 10^{-5}\Gamma(K_L \to \gamma\gamma) \simeq 4\times 10^{-9}\Gamma(K^+ \to \bar{\mu}\nu) \qquad (4.65)$$

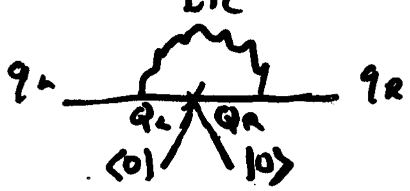
where the process $K^+ \to \bar{\mu}\nu$ has the standard weak interaction rate $\Gamma(K^+ \to \bar{\mu}\nu) = O(G_F^{\,2})$ and also the amplitude for $\bar{K}^\circ - K^\circ$ transitions is of order

$$A(K^\circ - \bar{K}^\circ) = O(G_F^{\,2}) \qquad (4.66)$$

Remarkably, this apparent discrepancy between theory and experiment is resolved in the standard model because of the GIM mechanism, which relates the coupling of the u, c and t

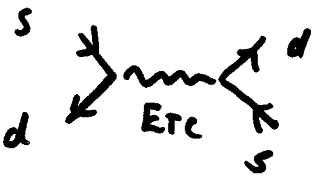
The smallness of observed FCNC (consistent with SM) is strong constraint on extensions of SM.

eg. (i) <u>Extended technicolour.</u>



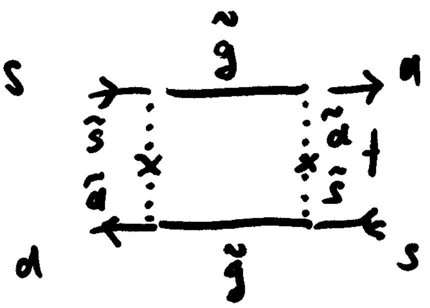
$$m_d \propto \frac{\langle 0|\bar{Q}Q|0\rangle}{m_{ETC}^2} \quad , \quad \underline{m_{ETC} < \text{TeV}}$$

FCNC

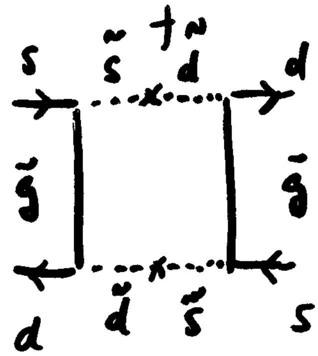


$$\underline{m_{ETC} > \text{TeV}}$$

eg (ii) <u>SUPERSYMMETRY</u>



$+$

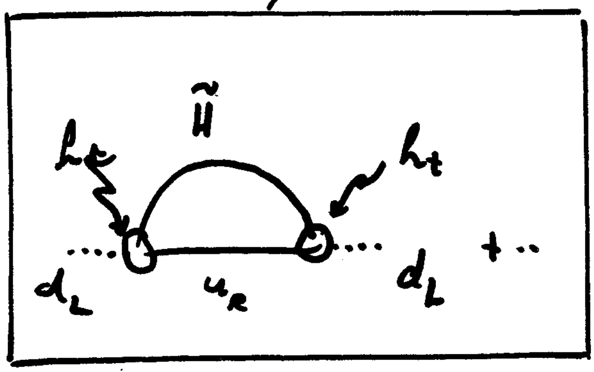


$$A \propto \frac{\Delta}{m_{\tilde{q}}^2} \equiv \delta$$

$^{+}$ $(\Delta_{LL})_{ij}$ $(\Delta_{LR})_{ij}$ $(\Delta_{RR})_{ij}$

eg In MSSM :

$$m^2_{\tilde{d}_L \tilde{d}_L} = m_d m_d^\dagger + \tilde{m}^2 \mathbb{1} + c\, m_u m_u^\dagger$$



Wilkinson, GGR
Duncan
Donoghue et al.
⋮
Gabrielli, Masiero,
Silvestrini

ie Due to last term, $m_{\tilde{d}_L \tilde{d}_L}^2$ not diagonalised when m_d is diagonalised

$$\underline{(\Delta^d_{LL})_{ij} = c\left[K (m_u^{diag})^2 K^\dagger\right]_{ij}}$$

CKM matrix

1.16

x	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{12}\right)^2_{LL}\right\|}$	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{12}\right)^2_{LR}\right\|}$	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{12}\right)_{LL}\left(\delta^d_{12}\right)_{RR}\right\|}$
0.3	1.9×10^{-2}	7.9×10^{-3}	2.5×10^{-3}
1.0	4.0×10^{-2}	4.4×10^{-3}	2.8×10^{-3}
4.0	9.3×10^{-2}	5.3×10^{-3}	4.0×10^{-3}
x	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{13}\right)^2_{LL}\right\|}$	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{13}\right)^2_{LR}\right\|}$	$\sqrt{\left\|\mathrm{Re}\left(\delta^d_{13}\right)_{LL}\left(\delta^d_{13}\right)_{RR}\right\|}$
0.3	4.6×10^{-2}	5.6×10^{-2}	1.6×10^{-2}
1.0	9.8×10^{-2}	3.3×10^{-2}	1.8×10^{-2}
4.0	2.3×10^{-1}	3.6×10^{-2}	2.5×10^{-2}
x	$\sqrt{\left\|\mathrm{Re}\,(\delta^u_{12})^2_{LL}\right\|}$	$\sqrt{\left\|\mathrm{Re}\,(\delta^u_{12})^2_{LR}\right\|}$	$\sqrt{\left\|\mathrm{Re}\,(\delta^u_{12})_{LL}(\delta^u_{12})_{RR}\right\|}$
0.3	4.7×10^{-2}	6.3×10^{-2}	1.6×10^{-2}
1.0	1.0×10^{-1}	3.1×10^{-2}	1.7×10^{-2}
4.0	2.4×10^{-1}	3.5×10^{-2}	2.5×10^{-2}

Table 1: Limits on $\mathrm{Re}\,(\delta_{ij})_{AB}\,(\delta_{ij})_{CD}$, with $A,B,C,D=(L,R)$, for a squark mass $\tilde{m}=500\mathrm{GeV}$ and for different values of $x=m^2_{\tilde{g}}/\tilde{m}^2$.

x	$\sqrt{\left\|\mathrm{Im}\left(\delta^d_{12}\right)^2_{LL}\right\|}$	$\sqrt{\left\|\mathrm{Im}\left(\delta^d_{12}\right)^2_{LR}\right\|}$	$\sqrt{\left\|\mathrm{Im}\left(\delta^d_{12}\right)_{LL}\left(\delta^d_{12}\right)_{RR}\right\|}$
0.3	1.5×10^{-3}	6.3×10^{-4}	2.0×10^{-4}
1.0	3.2×10^{-3}	3.5×10^{-4}	2.2×10^{-4}
4.0	7.5×10^{-3}	4.2×10^{-4}	3.2×10^{-4}

Table 2: Limits on $\mathrm{Im}\left(\delta^d_{12}\right)_{AB}\left(\delta^d_{12}\right)_{CD}$, with $A,B,C,D=(L,R)$, for a squark mass $\tilde{m}=500\mathrm{GeV}$ and for different values of $x=m^2_{\tilde{g}}/\tilde{m}^2$.

x	$\left\|\left(\delta^d_{23}\right)_{LL}\right\|$	$\left\|\left(\delta^d_{23}\right)_{LR}\right\|$
0.3	4.4	1.3×10^{-2}
1.0	8.2	1.6×10^{-2}
4.0	26	3.0×10^{-2}

Table 3: Limits on the $\left|\delta^d_{23}\right|$ from $b\to s\gamma$ decay for a squark mass $\tilde{m}=500\mathrm{GeV}$ and for different values of $x=m^2_{\tilde{g}}/\tilde{m}^2$.

summary: Gabrielli, Masiero, Silvestrini
hep-ph/9510215

x	$\left\|\left(\delta^l_{12}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{12}\right)_{LR}\right\|$
0.3	4.1×10^{-3}	1.4×10^{-6}
1.0	7.7×10^{-3}	1.7×10^{-6}
5.0	3.2×10^{-2}	3.8×10^{-6}
x	$\left\|\left(\delta^l_{13}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{13}\right)_{LR}\right\|$
0.3	15	8.9×10^{-2}
1.0	29	1.1×10^{-1}
5.0	1.2×10^{2}	2.4×10^{-1}
x	$\left\|\left(\delta^l_{23}\right)_{LL}\right\|$	$\left\|\left(\delta^l_{23}\right)_{LR}\right\|$
0.3	2.8	1.7×10^{-2}
1.0	5.3	2.0×10^{-2}
5.0	22	4.4×10^{-2}

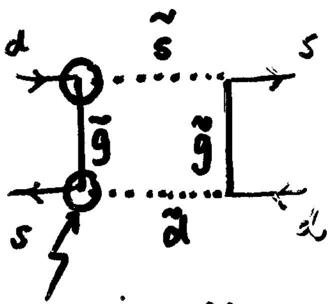
Table 4: Limits on the $\left|\delta^d_{ij}\right|$ from $l_j \to l_i\gamma$ lepton decay for a slepton mass $\tilde{m} = 100\mathrm{GeV}$ and for different values of $x = m^2_{\tilde{\gamma}}/\tilde{m}^2$.

SUSY BREAKING. : Soft susy-breaking terms.

(doesn't spoil hierarchy solution)

$$\boxed{\begin{aligned} \mathcal{L}_{SB} = & -\frac{1}{2}\sum_{\tilde{A}} M_{\tilde{A}}\lambda_{\tilde{A}}\lambda_{\tilde{A}} - m_{H_1}^2|H_1|^2 - m_{H_2}^2|H_2|^2 \\ & - m_{\tilde{q}}^2|\tilde{q}|^2 - m_{\tilde{u}}^2|\tilde{u}|^2 - m_{\tilde{d}}^2|\tilde{d}|^2 - m_{\tilde{\ell}}^2|\tilde{\ell}|^2 - m_{\tilde{E}}^2|\tilde{E}|^2 \\ & - m_0\left(h^u A^u \tilde{q}\tilde{u}^c H_2 + h^d A^d \tilde{q}\tilde{d}^c H_1 + h^\ell A^\ell \tilde{\ell}\tilde{E}^c H_1\right) \\ & - m_0\left(B\mu H_1 H_2\right) \quad + h.c. \end{aligned}}$$

F.C.N.C. constraints. eg. $\Delta S = 2$

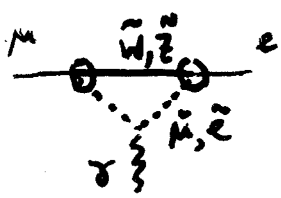


+similar bounds on up squark sector from $\tilde{W}$ exchange

$$\boxed{\begin{aligned} \frac{m_{\tilde{d}}^2 - m_{\tilde{s}}^2}{m_{\tilde{d}}^2} & \lesssim 6.10^{-3}\left(\frac{m_{\tilde{d}}}{\text{TeV}}\right)^2\left(\frac{\sin(\tilde{\theta}_c - \theta_c)}{\sin\theta_c}\right)^2 \\ \frac{m_{\tilde{e}}^2 - m_{\tilde{\mu}}^2}{m_{\tilde{e}}^2} & < 10^{-1}\left(\frac{m_{\tilde{e}}}{\text{TeV}}\right)^2\left(\frac{\sin(\tilde{\theta}_\ell - \theta_\ell)}{\sin\theta_c}\right)^2 \end{aligned}}$$

Non-zero in SU(5) ~ h_t UEH

...., Gabrielli, Masiero, Silvestrini (including $\cancel{CP}$)
Hagelin, Kelley, Tanaka
Chowdhury, Eberle ..., Konig, ..., Pokorski
Barbieri, Hall, ...
...



⇨ ORIGIN?

Determination of the mass matrices

$$M^{u,d} = V_L^{u,d\,T}\ \underline{M^{u,d}_{\text{Diagonal}}}\ V_R^{u,d} \longleftarrow ?$$

"Theory"

$$\underline{V_{CKM} = V_L^{u\,\dagger} V_L^{d}}$$

Data consistent with small m_{ij}

$$\boxed{M^{u(d)} = \begin{pmatrix} M_{11} & m_{12} & m_{13} \\ m_{21}^{\dagger} & M_{22} & m_{23} \\ m_{31}^{\dagger} & m_{32}^{\dagger} & M_{33} \end{pmatrix}}$$

† Weakly constrained by data.

.217 – .222
.208 – .24

$$V_{12} = \frac{m_{ds}}{M_s} - \frac{m_{uc}}{M_c} + \frac{m^*_{sd}}{M_s}\frac{M_d}{M_s} - \frac{m^*_{cu}}{M_c}\frac{M_u}{M_c} + \left[\frac{m_{ut}}{M_t}\frac{m^*_{sb}}{M_b}\right] \quad (28)$$

$$V_{21} = \frac{m^*_{uc}}{M_c} - \frac{m^*_{ds}}{M_s} + \frac{m_{cu}}{M_c}\frac{M_u}{M_c} - \frac{m_{sd}}{M_s}\frac{M_d}{M_s} + \left[\frac{m^*_{db}}{M_b}\frac{m_{ct}}{M_t}\right] \quad (29)$$

.038 – .041
.026 – .040

$$V_{23} = \frac{m_{sb}}{M_b} - \frac{m_{ct}}{M_t} + \frac{m^*_{bs}}{M_b}\frac{M_s}{M_b} - \frac{m^*_{tc}}{M_t}\frac{M_c}{M_t} + \quad (30)$$

$$\left[\frac{m^*_{uc}}{M_c}\left(\frac{m_{db}}{M_b} - \frac{m_{ut}}{M_t}\right) + \frac{m_{sd}}{M_b}\frac{m^*_{bd}}{M_b} - \frac{m^*_{tu}}{M_t}\frac{m_{cu}}{M_t}\right]$$

$$V_{32} = \frac{m^*_{ct}}{M_t} - \frac{m^*_{sb}}{M_b} + \frac{m_{tc}}{M_t}\frac{M_c}{M_t} - \frac{m_{bs}}{M_b}\frac{M_s}{M_b} + \quad (31)$$

$$\left[\frac{m_{ds}}{M_s}\left(\frac{m^*_{ut}}{M_t} - \frac{m^*_{db}}{M_b}\right) + \frac{m^*_{cu}}{M_t}\frac{m_{tu}}{M_t} - \frac{m_{bd}}{M_b}\frac{m^*_{sd}}{M_b}\right]$$

.0023 – .004

$$V_{13} = \left[\frac{m_{db}}{M_b} - \frac{m_{ut}}{M_t}\right] + \frac{m_{uc}}{M_c}\left(\frac{m_{ct}}{M_t} - \frac{m_{sb}}{M_b}\right) + \quad (32)$$

$$\frac{m_{ds}}{M_b}\frac{m^*_{bs}}{M_b} - \frac{m^*_{tc}}{M_t}\frac{m_{uc}}{M_t} + \left[\frac{m^*_{bd}}{M_b}\frac{M_d}{M_b} - \frac{m^*_{tu}}{M_t}\frac{M_u}{M_t}\right]$$

.0065 – .01

$$V_{31} = \left[\frac{m^*_{ut}}{M_t} - \frac{m^*_{db}}{M_b}\right] - \frac{m^*_{ds}}{M_s}\left(\frac{m^*_{ct}}{M_t} - \frac{m^*_{sb}}{M_b}\right) + \quad (33)$$

TEXTURE ZEROS

Bjorken, Krasnikov(?), GOR.

PERTURBATIVE ANALYSIS : (Bjorken)

Assume $m = \underbrace{\underset{\text{Physical mass}}{M_D} + \overset{O(m^2)}{\Delta M}}_{\text{Diagonal}} + \overset{\text{small, off diagonal}}{m}$

$$V^{L\dagger}\, m\, m^{\dagger}\, V^{L} = M_D^2$$

($V = V^{u,d}$
$m = m^{u,d}$)

$$\Rightarrow \quad (M_D + \Delta M + m)(M_D + \Delta M + m^{\dagger})\, V = V M_D^2$$

$$V_{ij} = O(m) \quad , \quad V_{ii} = 1 + O(m^2)$$

$$\Rightarrow \quad 2M_i\, \Delta M_i + \sum_j |m_{ij}|^2 + \sum_j (m_{ij} M_j + M_i m^*_{ji})\, V_{ji} = 0$$

Diagonal ii

and

$$M_i^2 V_{ij} + \sum_k (m_{ik} M_k + M_i m^*_{ki}) \underset{1 + O(m)}{V_{kj}} + \sum_k m_{ik} m^*_{jk} = V_{ij} M_j^2$$

$\neq$ ~~if~~ Diagonal

$$V_{ij} = -\frac{(m_{ij} M_j + m^*_{ji} M_i)}{(M_i^2 - M_j^2)} + \frac{(m_{ik} M_k + m^*_{ki} M_i)(m_{kj} M_j + m^*_{jk} M_k)}{(M_i^2 - M_j^2)(M_k^2 - M_j^2)}$$

$$- \frac{m_{ik}\, m^*_{jk}}{(M_i^2 - M_j^2)} + O(m^3)$$

THE FORM OF THE MASS MATRICES :

$$\underline{V_{CKM}} = \begin{array}{ccc} & \text{or } V_{ud} & \\ & \downarrow & \\ \end{array}$$

$$\underline{V_{CKM}} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \equiv V_L^{u\dagger} V_L^d$$

where
$$m^u = V_L^{u\dagger} \underline{m_D^u} V_R^u$$
$$m^d = V_L^{d\dagger} \underline{m_D^d} V_R^d$$

What do we know about the mass $\underline{\text{matrices}}$?

B. J. ..... Assume off diagonal elements small & expand

$$V_{12} = \frac{m_{D_{12}}}{M_s} - \frac{m_{u_{12}}}{M_c} + \frac{m^*_{D_{23}}\, m_{u_{13}}}{M_b\, M_t} + \frac{m^*_{D_{21}}}{M_s} \cdot \frac{M_d}{M_s} \ldots$$

$$V_{21} = -\frac{m^*_{D12}}{M_s} + \frac{m^*_{u_{12}}}{M_c} + \frac{m^*_{D_{13}}\, m_{u_{23}}}{M_b\, M_t} + \ldots$$

N.B. To 1st approx independent of $m_{D_{21}}$ etc.

$$\Rightarrow \boxed{|V_{12}| \doteq |V_{21}|}$$

$0.217 - 0.222 \doteq 0.208 - 0.24$

Simply follows from smallness of off-diagonal elements.

Similarly ...

$$V_{23} = \frac{m_{D23}}{M_b} - \frac{m_{u23}}{M_t} + \cdots$$

$$V_{32} = -\frac{m^*_{D23}}{M_b} + \frac{m^*_{u23}}{M_t} + \cdots$$

$$\boxed{|V_{23}| \doteq |V_{32}|}$$ from smallness of off-diagonal elem

$0.038 - 0.041 \doteq 0.026 - 0.040$

But $$V_{13} = \frac{m_{D13}}{M_b} - \frac{m_{u13}}{M_t} + \frac{m_{u12}}{M_c}\left(\frac{m_{u23}}{M_t} - \frac{m_{D23}}{M_b}\right) + \cdots$$

$$V_{31} = -\frac{m^*_{D13}}{M_b} + \frac{m^*_{u13}}{M_t} - \frac{m^*_{D12}}{M_s}\left(\frac{m^*_{u23}}{M_t} - \frac{m^*_{D23}}{M_b}\right) + \cdots$$

IF $m_{D13} = m_{u13} = 0$, "TEXTURE ZERO"

$0.002 - 0.005 = (0.047 - 0.07)\cdot(0.038 - 0.041)$

$$\boxed{\begin{aligned} |V_{13}| &= \sqrt{\frac{M_u}{M_c}}\,|V_{23}| \\ |V_{31}| &= \sqrt{\frac{M_d}{M_s}}\cdot\sqrt{\frac{M_c}{M_u}}\,|V_{13}| \end{aligned}}$$

$0.004 - 0.015 = (2 - 7)\cdot(0.002 - 0.004)$

TEXTURE ZERO PREDICTIONS.

$$\begin{pmatrix} \widetilde{\widetilde{Y_{11}}} & & \\ & \widetilde{\widetilde{Y_{22}}} & \\ & & \widetilde{\widetilde{Y_{33}}} \end{pmatrix} = \begin{pmatrix} 1 & -s'^Y_{12} & 0 \\ s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s'^Y_{13} \\ 0 & 1 & 0 \\ s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s'^Y_{23} \\ 0 & s'^Y_{23} & 1 \end{pmatrix} \times$$

$$\times \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s'^Y_{23} \\ 0 & -s'^Y_{23} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s'^Y_{13} \\ 0 & 1 & 0 \\ -s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & s'^Y_{12} & 0 \\ -s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\mathrm{V}_{CKM} = \begin{pmatrix} 1 & s_{12} + s^U_{13}s_{23} & s_{13} - s^U_{12}s_{23} \\ -s_{12} - s^D_{13}s_{23} & 1 & s_{23} + s^U_{12}s_{13} \\ -s_{13} + s^D_{12}s_{23} & -s_{23} - s^D_{12}s_{13} & 1 \end{pmatrix}$$

$$s_{23} = s^D_{23} - s^U_{23}, \quad s_{13} = s^D_{13} - s^U_{13} \quad s_{12} = s^D_{12} - s^U_{12}.$$

- For (1,1) Texture Zero :

(GaHo)

$Y_{11} = 0$

$|Y_{12}| = |Y_{21}|$

$$|s^U_{12}| = \sqrt{\frac{m_u}{m_c}}$$

$$|s^D_{12}| = \sqrt{\frac{m_d}{m_s}}$$

- For (1,3) texture Zero : $s_{13} = 0 \Rightarrow$

$$\boxed{\begin{aligned} \frac{|V_{ub}|}{|V_{cb}|} &= \left|\sqrt{\frac{m_u}{m_c}} - \frac{s_{13}}{s_{23}}\right| \simeq \sqrt{\frac{m_u}{m_c}} \\ \frac{|V_{td}|}{|V_{ts}|} &= \left|\sqrt{\frac{m_d}{m_s}} - \frac{s_{13}}{s_{23}}\right| \simeq \sqrt{\frac{m_d}{m_s}} \end{aligned}}$$

0

0

(Barbieri
Hall
Romanino;
Fritzsch)

Quark masses.

Current Algebra : quark mass ratios

Parameter	Value	Reference
$Q^{\dagger}$	22.7 ± 0.8	H. Leutwyler
m_u/m_d	0.533 ± 0.043	hep-ph/0102310
m_d/m_s	9.5 ± 1.7	D. Groom et al. Eur. Phys J. C15 (2000) 1

$$^{\dagger}Q = \frac{m_s/m_d}{\sqrt{1-(m_u/m_d)^2}}$$

Absolute values : QCD sum-rules (SR)
lattice QCD (LQCD)

	SR	LQCD
$m_u(\mu)$	2.4 - 3.8 MeV	2.2 - 2.7 MeV
$m_d(\mu)$	4.3 - 6.9 MeV	3.8 - 4.9 MeV
$m_s(\mu)$	83 - 130 MeV	78 - 100 MeV

$\mu = 2$ GeV : R. Gupta + K. Maltman LAUR-00-5284

$$m^{D(u)} = \begin{pmatrix} m_{11} & m_{12} & \overset{0}{\cancel{m_{13}}} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

Not strongly constrained

<u>..RTHER" TEXTURE " ZERO</u> : SUPPOSE $m^{D(u)}_{11} = 0$ (SYMMETRY?)

<u>AND</u> $m^{D(u)}_{12} = m^{D(u)}_{21}$

$$\begin{pmatrix} 0 & m^D_{12} \\ m^D_{12} & m^D_{22} \end{pmatrix} \Rightarrow \begin{array}{l} M_D + M_S = m^D_{22} \\ M_D M_S = m^{D\,2}_{12} \end{array}$$

$$\Rightarrow \boxed{V_{us} = \sqrt{\frac{M_D}{M_S}} - \sqrt{\frac{M_u}{M_c}}\, e^{i\sigma}} \leftarrow \cancel{\sigma}\text{ phase}$$

GATTO, WEINBERG, FRITZSCH ... GGR et al

cf $(0.218 - 0.224) = \left| (0.16 - 0.33) - (0.047 - 0.07)\, e^{i\sigma} \right|$

$\downarrow$ 0.22 (m_s ≈ 180 - 200 MeV)

ONLY HINT OF NEED FOR SYMMETRIC MASS MATRICES ... HERE WE WILL FOLLOW THIS HINT!