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SMR.1317 - 14

SUMMER SCHOOL ON PARTICLE PHYSICS

18 June - 6 July 2001

FERMION MASSES AND THE FLAVOUR PROBLEM

Lecture II

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Please note: These are preliminary notes intended for internal distribution only.

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$$m^{D(u)} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}^{S_3^0}$$

Not strongly constrained

FURTHER "TEXTURE" ZERO : SUPPOSE $m_{11}^{D(u)} = 0$

AND $m_{12}^{D(u)} = m_{21}^{D(u)}$

$$\begin{pmatrix} 0 & m_{12}^D \\ {}^{\dagger}m_{12}^D & m_{22}^D \end{pmatrix} \Rightarrow \begin{array}{l} M_D + M_S = m_{22}^D \\ M_D M_S = m_{12}^{D\,2} \end{array} \Rightarrow \frac{m_{12}^D}{m_{22}^D} = \sqrt{\frac{M_D M_S}{(M_D + M_S)^2}}$$

$$\Rightarrow \boxed{V_{us} = \sqrt{\frac{M_D}{M_S}} - \sqrt{\frac{M_u}{M_c}}\, e^{i\phi}}$$

Gatto, Sartori, Tonin
Fritzsch
Weinberg

$\cancel{CP}$ phase : Romanino, Roberts, GGR, Velasco-Sevilla.

is $.217 \to .222$ cf $|(.206 \to .214) - (.07 \to .076)e^{i\delta}|$

$= .213 \to .223$, $\delta = 90°$

† Only hint of need for symmetric mass matrix ... we will often follow this hint!

Quark masses.

Current Algebra : quark mass ratios

Parameter	Value	Reference
$Q^{\dagger}$	22.7 ± 0.8	H. Leutwyler
m_u/m_d	0.533 ± 0.043	hep-ph/0102310
m_c/m_s	9.5 ± 1.7	D. Groom et al.
		Eur. Phys J. C15 (2000) 1

$$^{\dagger}\ Q = \frac{m_s/m_d}{\sqrt{1-(m_u/m_d)^2}}$$

Absolute values : QCD sum-rules (SR)
lattice QCD (LQCD)

	SR	LQCD
$m_u(\mu)$	2.4 - 3.8 MeV	2.2 - 2.7 MeV
$m_d(\mu)$	4.3 - 6.9 MeV	3.8 - 4.9 MeV
$m_s(\mu)$	83 - 130 MeV	78 - 100 MeV

$\mu = 2$ GeV : R. Gupta + K. Maltman LAUR-00-5284

TEXTURE ZERO PREDICTIONS.

$$\begin{pmatrix} \widetilde{\widetilde{Y_{11}}} & & \\ & \widetilde{\widetilde{Y_{22}}} & \\ & & \widetilde{\widetilde{Y_{33}}} \end{pmatrix} = \begin{pmatrix} 1 & -s'^Y_{12} & 0 \\ s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s'^Y_{13} \\ 0 & 1 & 0 \\ s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s'^Y_{23} \\ 0 & s'^Y_{23} & 1 \end{pmatrix} \times$$

$$\times \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s'^Y_{23} \\ 0 & -s'^Y_{23} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s'^Y_{13} \\ 0 & 1 & 0 \\ -s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & s'^Y_{12} & 0 \\ -s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1 & s_{12} + s^U_{13} s_{23} & s_{13} - s^U_{12} s_{23} \\ -s_{12} - s^D_{13} s_{23} & 1 & s_{23} + s^U_{12} s_{13} \\ -s_{13} + s^D_{12} s_{23} & -s_{23} - s^D_{12} s_{13} & 1 \end{pmatrix}$$

$$s_{23} = s^D_{23} - s^U_{23}, \quad s_{13} = s^D_{13} - s^U_{13} \quad s_{12} = s^D_{12} - s^U_{12}.$$

- For (1,1) Texture Zero :

(GaHo)

$Y_{11} = 0$

$|Y_{12}| = |Y_{21}|$

$$|s^U_{12}| = \sqrt{\frac{m_u}{m_c}}$$

$$|s^D_{12}| = \sqrt{\frac{m_d}{m_s}}$$

- For (1,3) texture Zero : $s_{13} = 0 \Rightarrow$

0

$$\boxed{\begin{aligned} \frac{|V_{ub}|}{|V_{cb}|} &= \left|\sqrt{\frac{m_u}{m_c}} - \frac{s_{13}}{s_{23}}\right| \approx \sqrt{\frac{m_u}{m_c}} \\ \frac{|V_{td}|}{|V_{ts}|} &= \left|\sqrt{\frac{m_d}{m_s}} - \frac{s_{13}}{s_{23}}\right| \approx \sqrt{\frac{m_d}{m_s}} \end{aligned}}$$

0

(Barbieri
Hall
Romanino ;
Fritzsch)

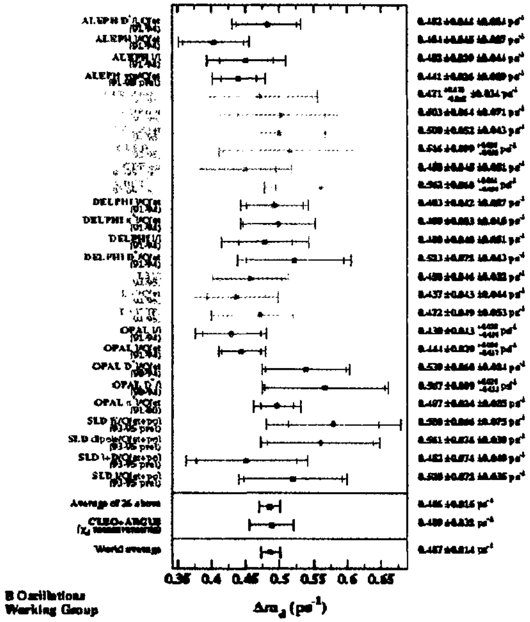
Family structure and LEP

The b-CKM† unitarity triangle

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$

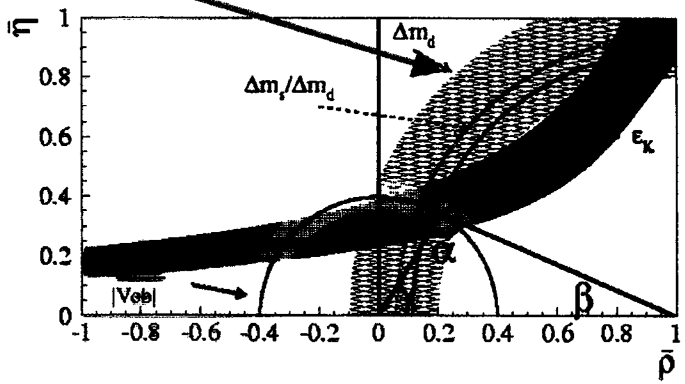
$$\overline{AC} = \frac{1-\lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \qquad \overline{AB} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$

Δm_d in 2000



- LEP weight=70 %
- Δm_d and χ_d results agree

$\Delta m_d = (0.487 \pm 0.014)$ ps^{-1} $\quad \sigma(\Delta m_d) < 3\%$



Measurement	$V_{CKM}\times$other	Constraint
$b\to u/b\to c$	$\lvert V_{ub}/V_{cb}\rvert^2$	$\bar\rho^2+\bar\eta^2$
Δm_d	$\lvert V_{td}\rvert^2 f_{B_d}^2 \hat B_{B_d} f(m_t)$	$(1-\bar\rho)^2+\bar\eta^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\lvert\frac{V_{td}}{V_{ts}}\right\rvert^2 \frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 \hat B_{B_s}}$	$(1-\bar\rho)^2+\bar\eta^2$
ϵ_K	$f(A,\bar\eta,\bar\rho,\hat B_K)$	$\propto \bar\eta(1-\bar\rho)$

$\bar\rho = \rho(1-\lambda^2/2), \; \bar\eta = \eta(1-\lambda^2/2)$

†

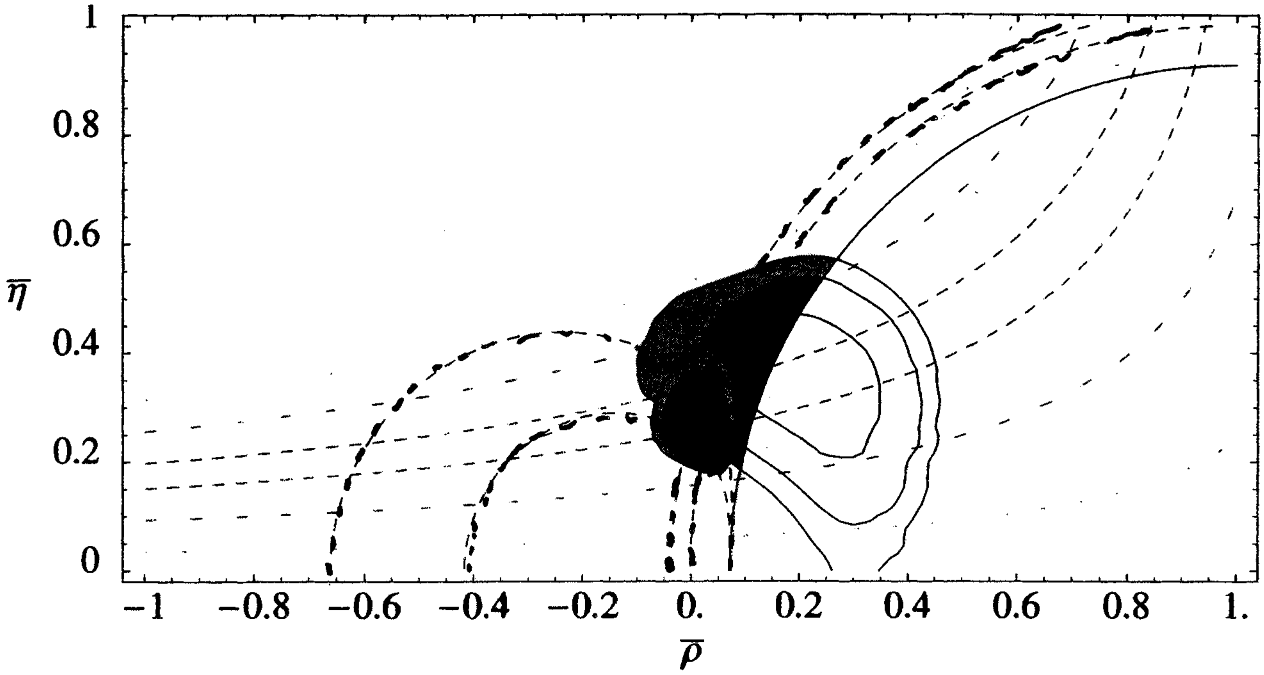
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}_{\text{Wolfenstein}}$$

PRECISE TEST OF TEXTURE ZEROS : (1,1), (1,3), (3,1) zeros.

Barbieri, Hall, Romanino

$$|V_{ub}| = \sqrt{\frac{m_u}{m_c}}\,|V_{cb}| \qquad = \frac{\lambda}{c}\sqrt{\bar\rho^2+\bar\eta^2}\;|V_{cb}|$$

$$|V_{td}| = \sqrt{\frac{m_d}{m_s}}\,|V_{ts}| \qquad = \frac{\lambda}{c}\sqrt{(1-\bar\rho)^2+\bar\eta^2}\;|V_{ts}|$$



$$\bar\rho = c\rho \;,\; \bar\eta = c\eta \qquad ; \qquad c = \sqrt{1-\lambda^2}$$

Wolfenstein parameters

$$\ldots\quad ((1-\bar\rho)^2+\bar\eta^2)^2 - \left(\frac{m_c}{m_s}\right)^2(\bar\rho^2+\bar\eta^2)^2 = \frac{c^4}{\lambda^4 Q^2}\dagger$$

$$\ldots\quad (1-\bar\rho)^2+\bar\eta^2 = \frac{c^2}{\lambda^2 Q\sqrt{1-(m_u/m_d)^2}}$$

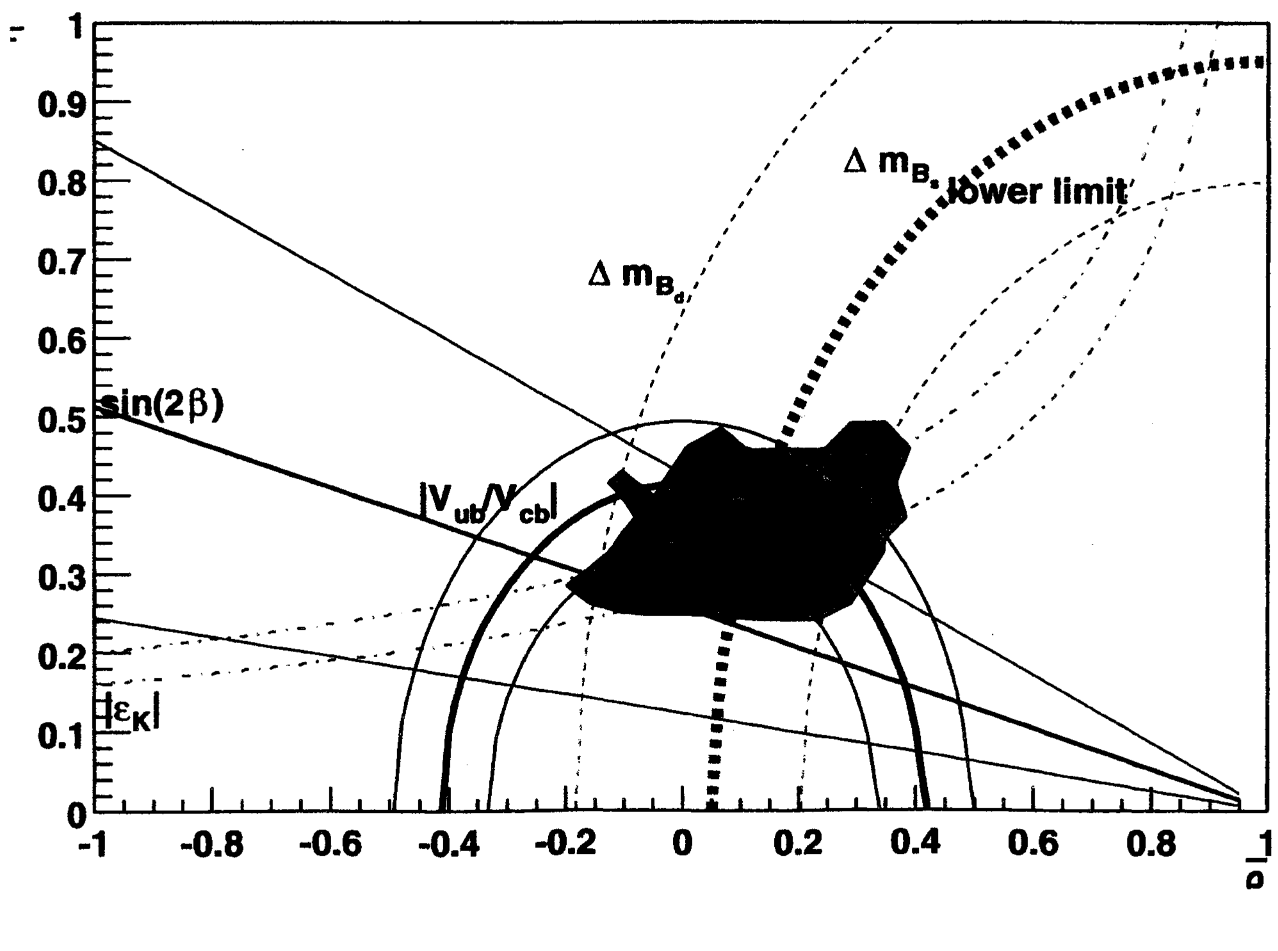
$$\dagger\; Q = \frac{m_s/m_d}{(1-(m_u/m_d)^2)^{1/2}}$$

- Updated fit of SM to expt. results. (using $\sin 2\beta = 0.48 \pm .24$)

The confidence levels are at 99%, 95% + 68% c.l.

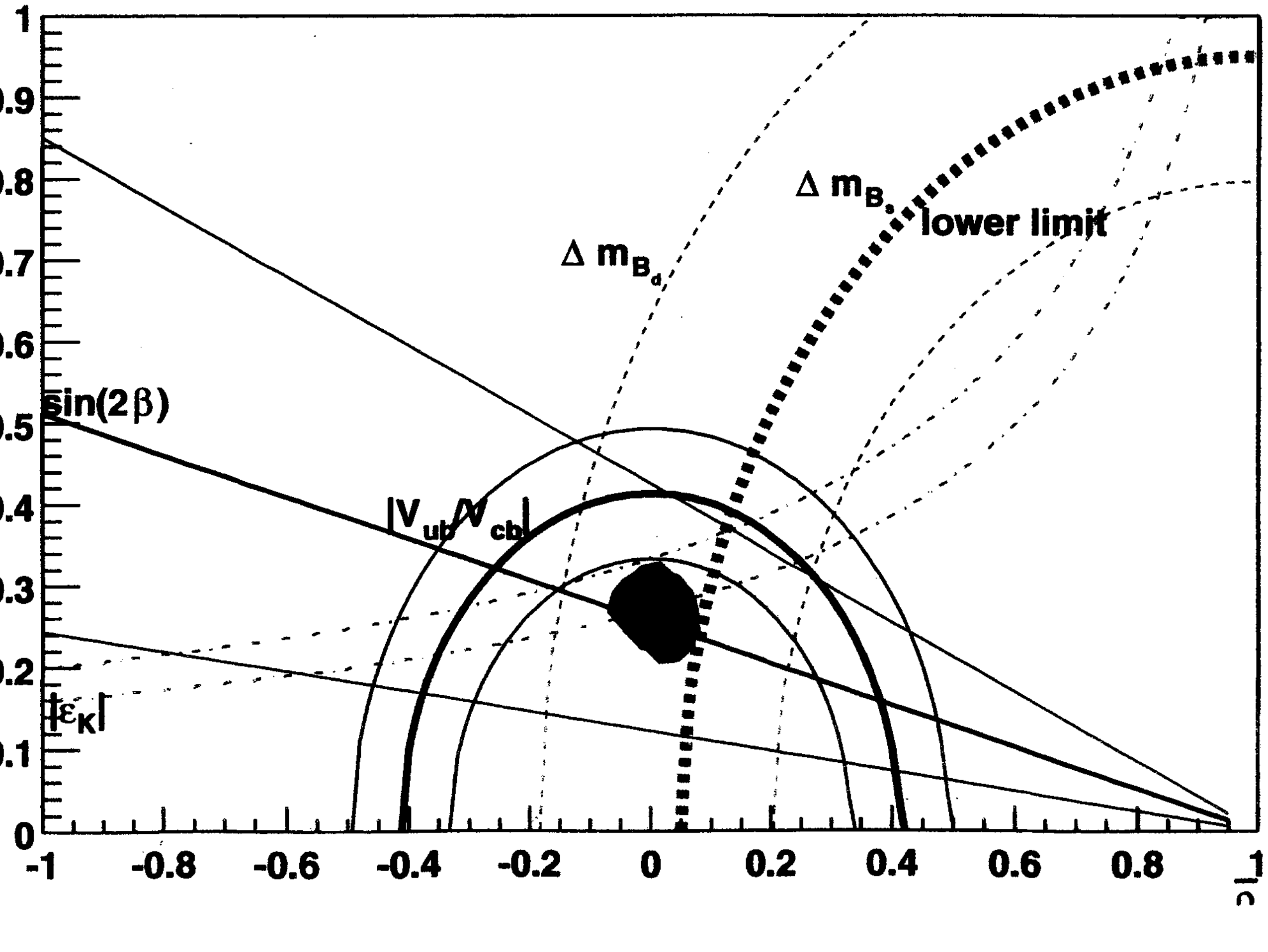
Romanino, GGR,
Velasco-Sevilla
Roberts.

(cf Caravaglios, Parodi,
Roudeau, Stocchi).

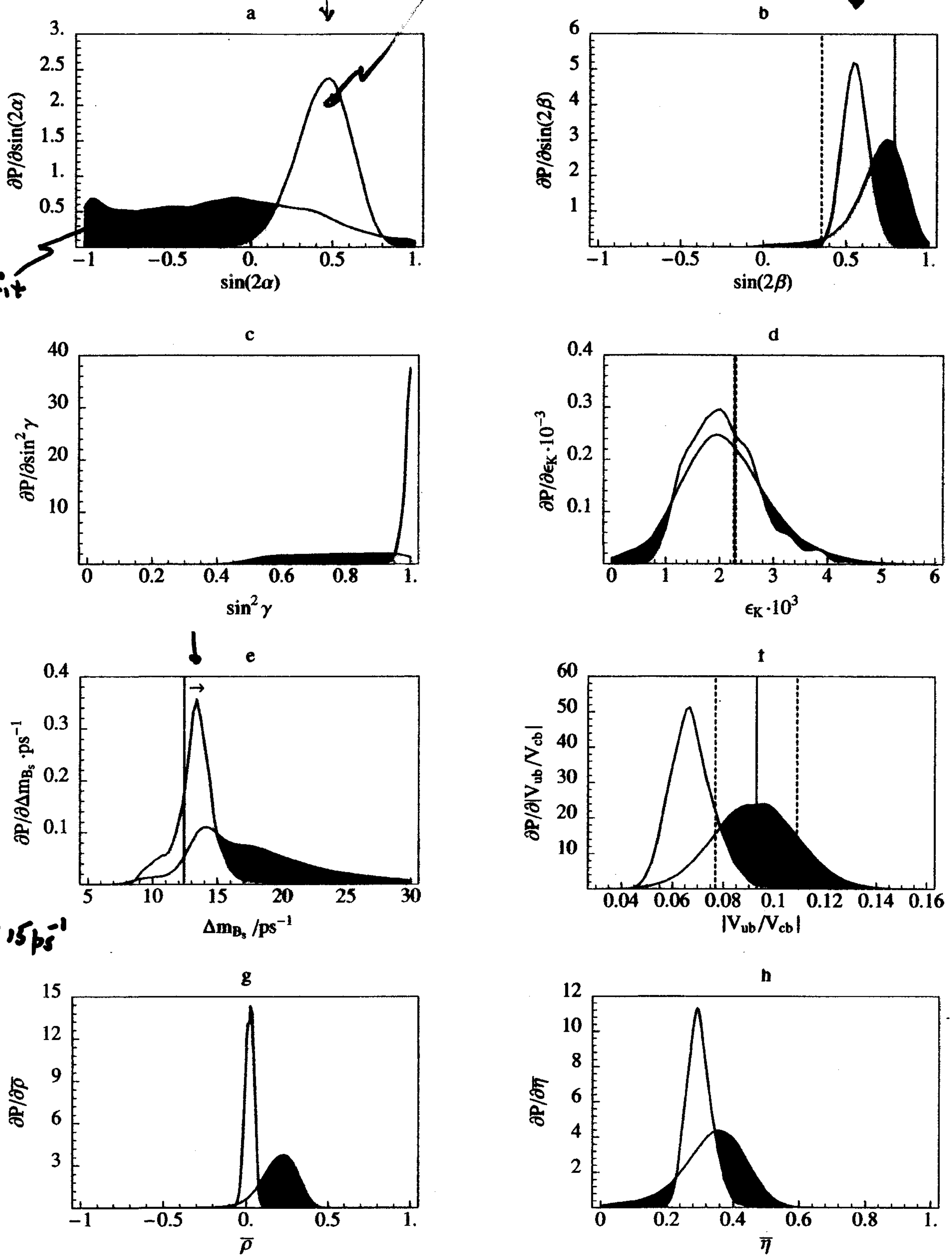


- Fit to texture zero predictions

$$V_{td}/V_{ts} = \sqrt{\frac{m_d}{m_s}} \, , \quad V_{ub}/V_{cb} = \sqrt{\frac{m_u}{m_c}} \quad + \ldots$$



TZ prediction

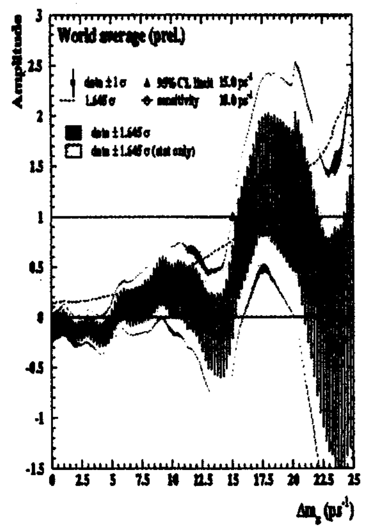


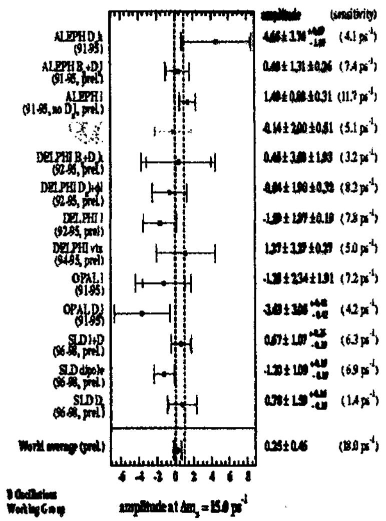
SM fit

$\Delta m_{B_s} < 15\,ps^{-1}$

Barbieri, Hall, Romanino.

Results in 2000





$\Delta m_s > 15.0$ ps^{-1} at 95% C.L.

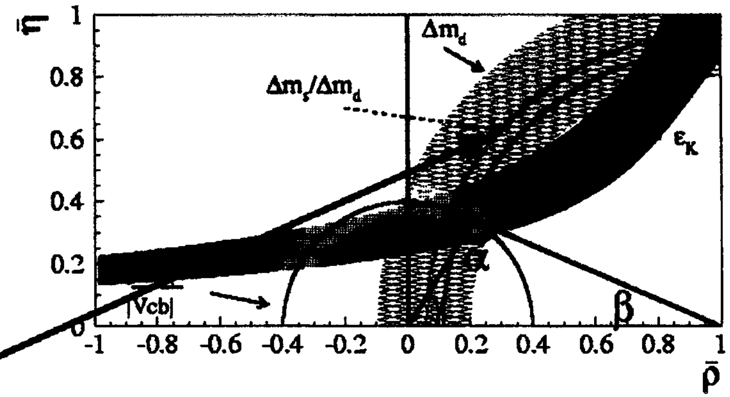
SLD has 50% weight

2.5σ effect at 17.8 ps^{-1}

The b-CKM unitarity triangle

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$

$$\overline{AC} = \frac{1-\lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \qquad \overline{AB} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$



Measurement	$V_{CKM}\times$other	Constraint
$b \to u / b \to c$	$\lvert V_{ub}/V_{cb}\rvert^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$\lvert V_{td}\rvert^2 f_{B_d}^2 \hat{B}_{B_d} f(m_t)$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\lvert\frac{V_{td}}{V_{ts}}\right\rvert^2 \frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 \hat{B}_{B_s}}$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
ϵ_K	$f(A, \bar{\eta}, \bar{\rho}, \hat{B}_K)$	$\propto \bar{\eta}(1-\bar{\rho})$

$\bar{\rho} = \rho(1-\lambda^2/2)$, $\bar{\eta} = \eta(1-\lambda^2/2)$

$$
\begin{pmatrix} \widetilde{\widetilde{\widetilde{Y_{11}}}} & & \\ & \widetilde{\widetilde{\widetilde{Y_{22}}}} & \\ & & \widetilde{\widetilde{\widetilde{Y_{33}}}} \end{pmatrix} = \begin{pmatrix} 1 & -s'^Y_{12} & 0 \\ s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s'^Y_{13} \\ 0 & 1 & 0 \\ s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s'^Y_{23} \\ 0 & s'^Y_{23} & 1 \end{pmatrix} \times
$$

$$
\times \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s'^Y_{23} \\ 0 & -s'^Y_{23} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s'^Y_{13} \\ 0 & 1 & 0 \\ -s'^Y_{13} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & s'^Y_{12} & 0 \\ -s'^Y_{12} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
$$

$$
\mathbf{V}_{CKM} = \begin{pmatrix} 1 & s_{12} + s^U_{13} s_{23} & s_{13} - s^U_{12} s_{23} \\ -s_{12} - s^D_{13} s_{23} & 1 & s_{23} + s^U_{12} s_{13} \\ -s_{13} + s^D_{12} s_{23} & -s_{23} - s^D_{12} s_{13} & 1 \end{pmatrix}
$$

$$
s_{23} = s^D_{23} - s^U_{23}, \qquad s_{13} = s^D_{13} - s^U_{13} \qquad s_{12} = s^D_{12} - s^U_{12}.
$$

For (1,1) Texture Zero :

$$
\begin{aligned}
|s^U_{12}| &= \sqrt{\frac{m_u}{m_c}} \\
|s^D_{12}| &= \sqrt{\frac{m_d}{m_s}}
\end{aligned}
$$

For (1,3) texture Zero :

0 #

$$
\begin{aligned}
(0.085-1) \quad &\simeq \frac{|V_{ub}|}{|V_{cb}|} = \left|\sqrt{\frac{m_u}{m_c}} - \frac{s_{13}}{s_{23}}\right| \simeq \left| .053 - \frac{s_{13}}{s_{23}} \right| \\
0.22 \quad &\simeq \frac{|V_{td}|}{|V_{ts}|} = \left|\sqrt{\frac{m_d}{m_s}} - \frac{s_{13}}{s_{23}}\right| \simeq \left| 0.19 - \frac{s_{13}}{s_{23}} \right|
\end{aligned}
$$

✓

SYMMETRIC MASS MATRICES.

$$m^u \propto \begin{pmatrix} 0 & 2\varepsilon^3 e^{i\phi} & ? \\ 2\varepsilon^3 e^{i\phi} & \varepsilon^2 & ? \\ ? & ? & 1 \end{pmatrix} \qquad \varepsilon = 0.05$$

$$m^d \propto \begin{pmatrix} 0 & 1.5\bar{\varepsilon}^3 & \alpha e^{i\psi} \\ 1.5\bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1.3\bar{\varepsilon}^2 \\ \alpha e^{i\psi} & 1.3\bar{\varepsilon}^2 & 1 \end{pmatrix} \qquad \bar{\varepsilon} = 0.15$$

SOLUTION I : $\alpha = 2.7\,\bar{\varepsilon}^4$, $\phi = 90^\circ$, $\psi = -24^\circ$

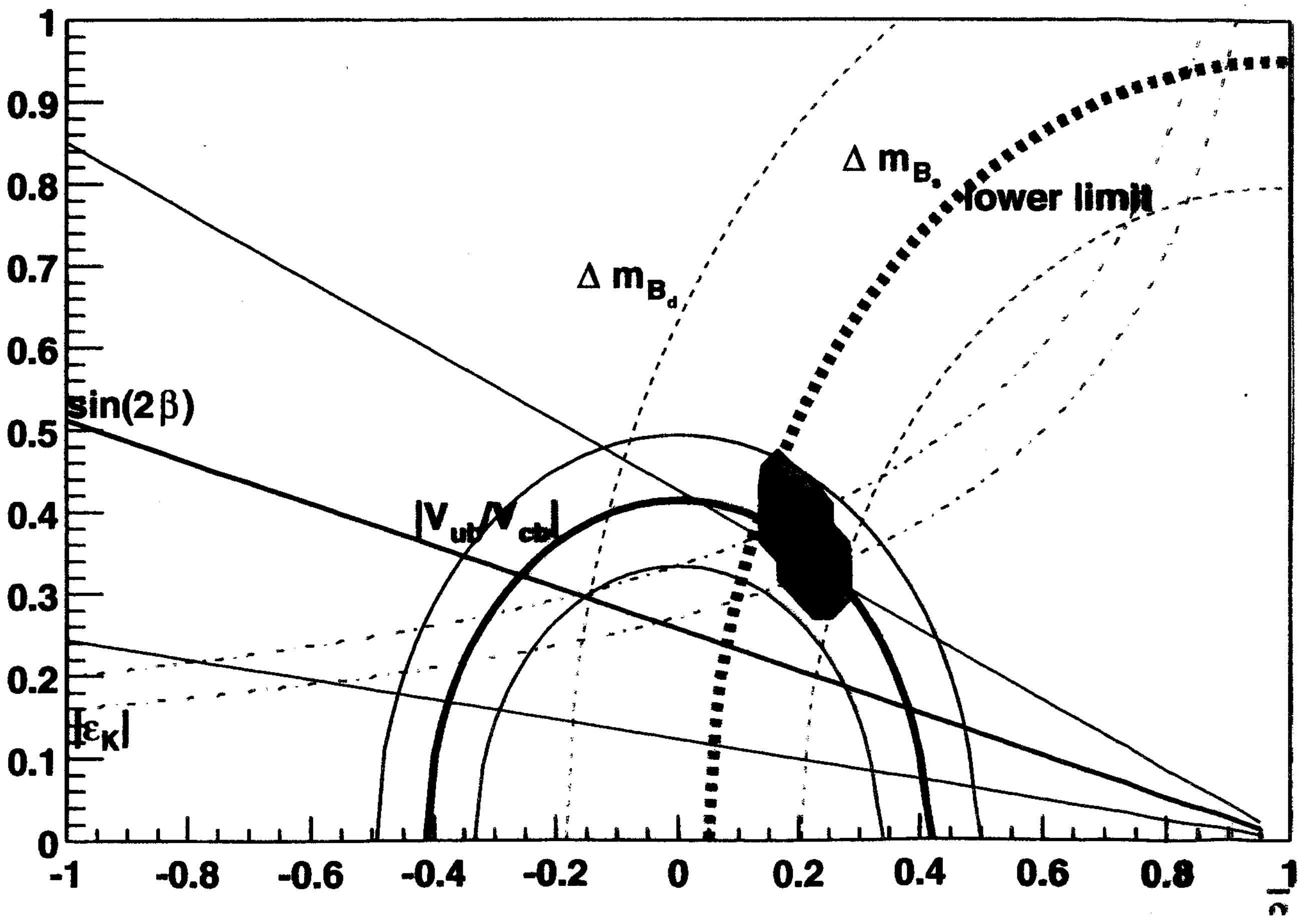
$$\boxed{\text{SOLUTION II} : \quad \alpha = 1.5\,\bar{\varepsilon}^3, \quad \phi = -90^\circ, \quad \psi = -60^\circ}$$

ASYMMETRIC MASS MATRICES.

$$m^u \propto \begin{pmatrix} 0 & 2\varepsilon^3 & 0 \\ 2\varepsilon^3 & 0 & 1.4\varepsilon \\ 0 & 0.4\varepsilon & 1 \end{pmatrix} \qquad m^d \propto \begin{pmatrix} 0 & 1.5\bar{\varepsilon}^3 & 0 \\ 1.5\bar{\varepsilon}^3 & 0 & 3.5\bar{\varepsilon}^2 \\ 0 & 0.3 & 1 \end{pmatrix}$$

• Fit allowing for a non-zero (1,3) $\left[\overset{=(3,1)}{}\right.$ entry in M_d.

$(1,3) \sim \bar{\varepsilon}^4$ or $\bar{\varepsilon}^3$ (2-solutions)



m_t, θ_t consistent with symmetric mass matrices with texture zeros : JUST 5 POSSIBILITIES †

"Texture" zero . really $\lesssim O(\lambda^9)$

Texture zero ... really $\lesssim \lambda^5$

Solution	Y_u	Y_d
1	$\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$
2	$\begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix}$
3	$\begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \sqrt{2}\lambda^4 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}$
4	$\begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \sqrt{3}\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
5	$\begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \frac{\lambda^2}{\sqrt{2}} \\ \lambda^4 & \frac{\lambda^2}{\sqrt{2}} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Poorly determined ... could be $\lesssim 1$.

$\lambda \doteq \sin\theta_c \doteq .23$

Table 1: Approximate forms for the symmetric textures of quark masses.

0.038 - 0.041

Solution	$\frac{m_c}{m_t}$	$\frac{m_u}{m_c}$	V_{cb}	$\frac{V_{ub}}{V_{cb}}$
"Experiment"	0.006-0.01	0.003-0.005	0.02-0.05	0.05-0.13
1	6.7 10^{-3}	0.0046	6 10^{-2}	0.068
2	6.7 10^{-3}	0.0023	3.8 10^{-2}	0.0484
3	6.7 10^{-3}	0.0046	6 10^{-2}	0.078
4	4.9 10^{-3}	0.0087	6.8 10^{-2}	0.040
5	6.1 10^{-3}	0.003	4.8 10^{-2}	0.068

Table 2: Predictions following from the five symmetric texture solutions using λ=0.22. All solutions give V_{us}=0.22 and $\frac{m_d}{m_s}$=0.05 and $\frac{m_s}{m_b}$=0.03, in agreement with the experimental results $\frac{m_d}{m_s}$=0.04- 0.067 and $\frac{m_s}{m_b}$=0.03-0.07[10].

Ramond, Roberts, GGR

† Analysis uses "data" extended to GUT scale where symmetries (zeros) expected ... (zeros insensitive to T.Z)

RADIATIVE CORRECTIONS

$$16\pi^2 \frac{d(h_u/h_t)}{dt} = -\frac{3}{2}\left(b h_t^2 + c h_b^2\right)\left(\frac{h_u}{h_t}\right)$$

$$16\pi^2 \frac{d(h_d/h_b)}{dt} = -\frac{3}{2}\left(c h_t^2 + b h_b^2\right)\left(\frac{h_d}{h_b}\right)$$

$$16\pi^2 \frac{d|V_{ij}|}{dt} = -\frac{3c}{2}\left(h_t^2 + h_b^2\right)|V_{ij}|\ , \quad i,j = 13, 31, 23, 32.$$

$$\frac{d|V_{12}|}{dt} = O\left(V_{12}^4\right)$$

$$16\pi^2 \frac{d|J_{CP}|}{dt} = -3c\left(h_t^2 + h_b^2\right)|J_{CP}|$$

$c = 2/3$
$b = 2$ MSSM

OLECHOWSKI, POKORSKI

WOLFENSTEIN PARAMETERISATION

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\boxed{\begin{array}{l} 16\pi^2 \frac{dA}{dt} = -\frac{3c}{2}\left(h_t^2 + h_b^2\right) A \\ \frac{d\lambda}{dt} \simeq 0 \\ \frac{d\rho}{dt} \simeq 0 \qquad \frac{d\eta}{dt} \simeq 0. \end{array}}$$

Approx:

$A \downarrow 30\%$

$\frac{h_b}{h_{d,s}} \uparrow 30\%$

$\frac{h_t}{h_{u,c}} \uparrow 2.5$

$(h_t = 1.25)$

Models of fermion masses.

$$m \sim \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \quad \underset{\text{rank } 1}{\overset{\text{rank } 1}{\rightleftarrows}} \; U \quad \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

hierarchy   democracy

⇨ Structure of mass matrices strongly suggests :

BROKEN SYMMETRY

Spontaneously broken symmetry.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\langle\theta\rangle \neq 0} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & a\lambda^2 \\ 0 & a\lambda^2 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & b\lambda^6 & 0 \\ b\lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{bmatrix}$$

$$a, b = O(1) \; , \quad \lambda = \frac{\langle\theta\rangle}{M}.$$

⇨
- SYMMETRY
- BREAKING MECHANISM ... origin of order parameter, λ.

SYMMETRIES.

GUTs : Relates single family eg $SU(5)$ $m_b = m_\tau$.

FAMILY : Non Yukawa interactions have large symmetry.

$$\sum_{i=1}^{3} \bar{\psi}_i^a \not{D} \psi_i^a \quad , a = l, Q, u^c, d^c, e^c \quad \Rightarrow \quad U(3)^5$$

$$a = \nu_R^c \quad \Rightarrow \quad U(3)^6$$

Candidate family symmetry $\subset U(3)^6$.

... continuous : $U(1)$, $U(2)$, $U(3)$ ...

$\uparrow$ generic in string compactification.

(May be broken at high scale ... if gauged† no direct low energy phenomena)

... discrete† : eg $Z(2)$ (R-parity) , Z_N

Non-Abelian eg. $Z_2 \ltimes Z_2$

	Z_2	$\ltimes\ Z_2$.
A	$\to A$	$\to B$
B	$\to iB$	$\to A$

... also common in string compactification

---

† Gauged continuous & discrete symmetries constrained by anomaly cancellation

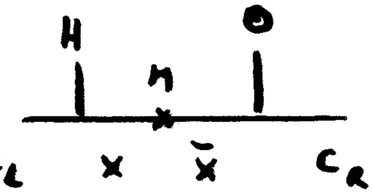
BREAKING MECHANISM

⇨ (Spontaneous) breaking in <u>massive sector</u> $\lambda = \frac{\langle\theta\rangle}{M}$

"flavon" - ugh!

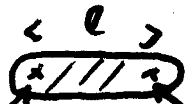
Massive propagator.

"Froggatt - Nielsen"



⇨ <u>Radiative</u> breaking ... breaking communicated to SM sector by ≳ 1-loop. graphs (cf extended technicolour) $\lambda = \frac{h^2}{4\pi^2}$

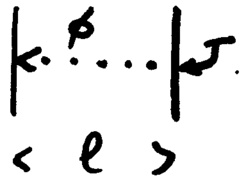
⇨ <u>String</u> suppression of Yukawa coupling $\lambda = e^{-m_s l}$



Fields at fixed points

⇨ <u>Bulk flavon propagation</u> in theories with large new dimension

SM brane

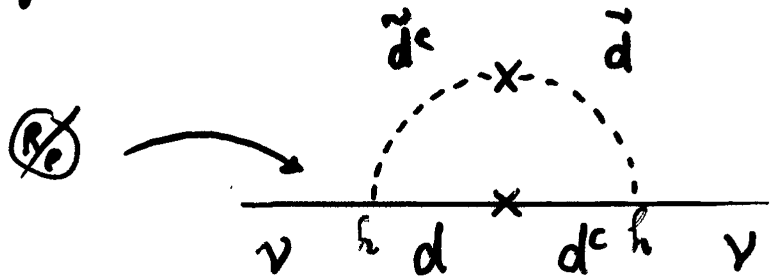


symmetry breaking on distant brane

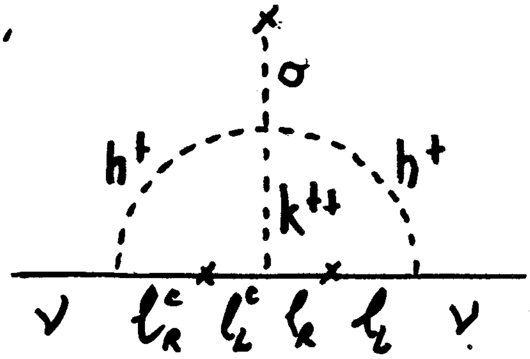
$\lambda = e^{-m_\phi l}$

RADIATIVE MASSES

eg



eg'



$$m_\nu : \frac{1}{16\pi^2} h^2 \frac{m_d^2 A_{SUSY}}{m_{\tilde{d}}^2}.$$

$$\frac{m_\nu}{m_d} \sim 10^{-5} h^2$$ ... small without GUT mass

(but loses connection with gauge unification scale)

Fig 7 : Examples of radiative mass generation in SUSY model and non - SUSY model.

<u>Symmetry</u> ? Simplest possibility $\tilde{U}(1)$ (cf strings)

<u>2-generation case</u> :

$$\begin{array}{cc} Q_{\bar{c}_R} & 0 \end{array}$$

$$(\bar{c}_R \;\; \bar{t}_R) \begin{bmatrix} 0 & 0 \\ 0 & \lambda H \end{bmatrix} \begin{pmatrix} c_L \\ t_L \end{pmatrix} \begin{array}{c} Q_{c_L} \\ 0 \end{array} \qquad \text{only } m_t \neq 0$$

$Q_H = 0$

› Break $\tilde{U}(1)$ spontaneously via $\langle \phi \rangle$ $(Q_\phi = -1)$

$$\boxed{H \left(\frac{\langle \phi \rangle}{M} \right)^{Q_{\bar{c}_R}} \equiv \varepsilon_R H}$$

$$(\bar{c}_R \;\; \bar{t}_R) \begin{bmatrix} \cdot\cdot & \\ & \lambda H \end{bmatrix} \begin{pmatrix} c_L \\ t_L \end{pmatrix}$$

$$\boxed{H \left(\frac{\langle \phi \rangle}{M} \right)^{Q_{c_L}} \equiv H \varepsilon_L}$$

($O(1)$ Yukawa couplings)

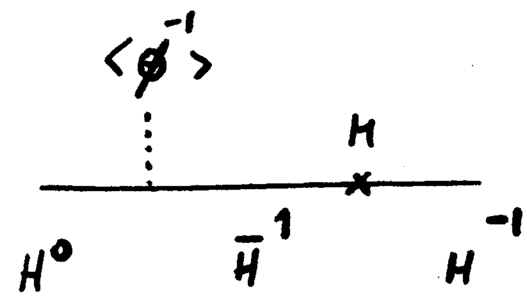
$$\frac{m^q}{m_t} = \begin{bmatrix} \lambda' \varepsilon_R \varepsilon_L & \varepsilon_R \\ \varepsilon_L & 1 \end{bmatrix}$$

Mixing L's det. by charge

$$\varepsilon_L = \left(\frac{\langle \phi \rangle}{M} \right)^{Q_{c_L}}$$

$$= R_R^\dagger \frac{m^{q,\text{Diagonal}}}{m_t} R_L \doteq \underline{\underline{\begin{pmatrix} 1 & \varepsilon_R \\ -\varepsilon_R & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_R \varepsilon_L & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -\varepsilon_L \\ +\varepsilon_L & 1 \end{pmatrix}}}$$

$$\begin{array}{c} \begin{array}{cc} 1 & \;\; 0 \end{array} \\ (\bar{c}_L \;\; \bar{t}_R) \end{array} \begin{bmatrix} -2 & -1 \\ -1 & \lambda H^0 \end{bmatrix} \begin{pmatrix} c_L \\ t_L \end{pmatrix} \begin{array}{c} 1 \\ 0 \end{array}$$



$$\mathcal{L} \subset M \underbrace{\bar{H}^1 H^{-1}}_{\text{Massive Higgs bosons}}$$

Froggatt - Nielsen mixing between H^0 and H^{-1}

$$\boxed{H_{light} \simeq H^0 - \frac{\langle \phi \rangle}{M} H^{-1}}$$

$$\Rightarrow \quad (\bar{c}_L \;\; \bar{t}_R) \begin{bmatrix} \left(\frac{\langle\phi\rangle}{M}\right)^2 & \frac{\langle\phi\rangle}{M} \\ \frac{\langle\phi\rangle}{M} & 1 \end{bmatrix} \begin{pmatrix} c_L \\ t_L \end{pmatrix} H^{light}$$

- **3-generation case**:

$$\boxed{SU(3) \times SU(2) \times U(1) \times \tilde{U}(1).}$$

Always possible ... const doesn't affect m_{ij}

$$\begin{array}{cc} & \begin{array}{ccc} \tilde{U}(1): \alpha_3 & \alpha_{2c} & \alpha_{1c} \\ u_L & c_L & t_L \end{array} \\ m_u = & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \quad \begin{array}{lll} u_L & \alpha_3 & = (-\alpha_2 - \alpha_1) = -3 \\ c_L & \alpha_2 & = 2 \\ t_L & \alpha_1 & = 1 \end{array}$$

$$\boxed{t^c \, t_L \, H_1}$$

$$\alpha_1 \quad \alpha_1 \quad (-2\alpha_1)$$

⇨ $\tilde{U}(1)$ SPONTANEOUSLY BROKEN

$$\langle \theta \rangle = \langle \bar{\theta} \rangle \neq 0$$

$$1 \qquad -1$$

(Not necessary ... assumed here for simplicity)

$$\frac{m_u}{m_t} = \begin{pmatrix} \left(\frac{\theta}{M}\right)^{2|\alpha_3 - \alpha_1|} & \left(\frac{\theta}{M}\right)^{|\alpha_2 + \alpha_3 - 2\alpha_1|} & \left(\frac{\theta}{M}\right)^{|\alpha_3 - \alpha_1|} \\ .. & \left(\frac{\theta}{M}\right)^{2|\alpha_2 - \alpha_1|} & \left(\frac{\theta}{M}\right)^{|\alpha_2 - \alpha_1|} \\ .. & .. & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon^{2|a+2|} & \varepsilon^{3} & \varepsilon^{|a+2|} \\ .. & \varepsilon^{2|a-1|} & \varepsilon^{|a-1|} \\ .. & .. & 1 \end{pmatrix}$$

$$\varepsilon = \left(\frac{\theta}{M}\right)^{\alpha_1}, \quad a = \frac{\alpha_2}{\alpha_1}$$

$\boxed{(1,1), (1,3) \text{ approximate texture zeros for } a > 1 \ !}$ ✓

HIERARCHY STRUCTURE. $a=2$

$\tilde{U}(1)$ Family :

	u,d	c,s	t,b	Θ, $\bar\Theta$
Q	-3	2	1	1 -1

Assuming $\langle\theta\rangle \simeq \langle\bar\theta\rangle$

m_d must have same structure!

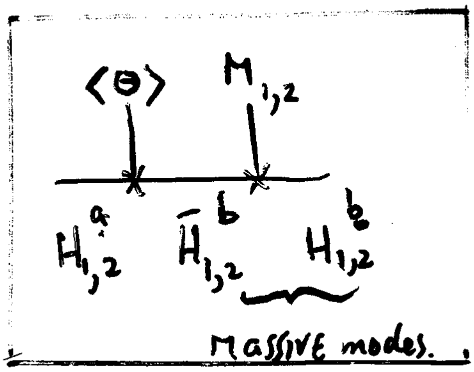
$\varepsilon_u \sim \lambda^2$ fits soln 2. ε_u^4 ✓

$\varepsilon_d \sim \lambda$ fits soln. 2. ε_d^4 ✓

$$\frac{m_u}{m_t} = \begin{pmatrix} 0 & \varepsilon_u^3 & 0 \\ \varepsilon_u^3 & \varepsilon_u^2 & \varepsilon_u \\ 0 & \varepsilon_u & 1 \end{pmatrix} ? \qquad \frac{m_d}{m_b} = \begin{pmatrix} 0 & \varepsilon_d^3 & 0 \\ \varepsilon_d^3 & \varepsilon_d^2 & \varepsilon_d \\ 0 & \varepsilon_d & 1 \end{pmatrix} ?$$

$$\boxed{\frac{\langle\Theta\rangle}{M_{H_2}}} \qquad \boxed{\frac{\langle\Theta\rangle}{M_{H_1}}}$$

$$\begin{pmatrix} \cdot & \cdot & H_1^b \\ \cdot & H_1^b & H_1^a \end{pmatrix}_c$$



TEXTURE ZEROS

$$H^{light} = H^a + \frac{\langle\Theta\rangle}{M} H^b + \dots$$

Ibanez GGR

$$\frac{V_{ub}}{V_{cb}} = \sqrt{\frac{m_u}{m_c}} \quad ; \quad |V_{us}| = \left|\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}}\, e^{i\delta}\right| + \dots$$

$$a=2 \longrightarrow \left(\frac{m_s}{m_b}\right)^3 \simeq \frac{m_d m_s}{m_b^2} \quad ; \quad \left(\frac{m_c}{m_t}\right)^3 \approx \frac{m_u m_c}{m_t^2}, \quad \epsilon \approx \bar\epsilon^2$$

$$\frac{m_d m_b}{m_s^2} = \frac{m_u m_t}{m_c^2} \qquad |V_{cb}| = \left|\sqrt{a\frac{m_s}{m_b}} - \sqrt{\alpha'\frac{m_c}{m_t}}\, e^{i\delta}\right| ???$$

* SU(2)e for couplings * FURTHER SYMMETRY

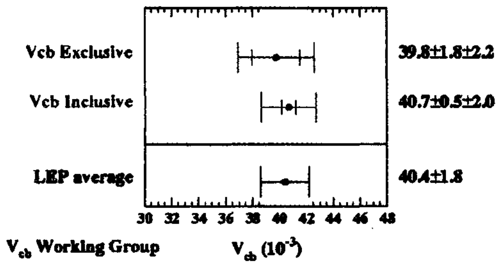
uncertain due to O(1) coeffs

Family structure and LEP

The b-CKM unitarity triangle

LEP $|V_{cb}|$ measurements

$$\frac{1}{\tau_{B_d}}\frac{dBR(\overline{B}^0_d \to D^{*+}\ell^-\overline{\nu_\ell})}{dw} = \mathcal{K}(w)\mathcal{F}^2(w)|V_{cb}|^2$$

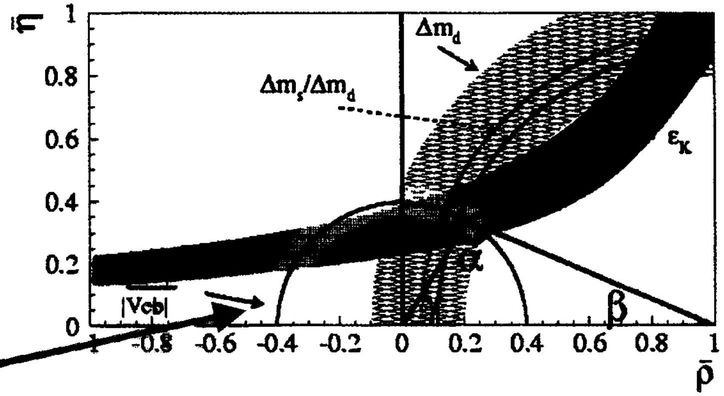


Considering only LEP measurements (uncertainties from Theory dominate):

$$|V_{cb}| = (40.4 \pm 1.8)10^{-3}$$
$$A = 0.838 \pm 0.037$$

$$V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb} = 0$$

$$\overline{AC} = \frac{1-\lambda^2/2}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| \qquad \overline{AB} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|$$



Measurement	$V_{CKM}\times$other	Constraint
$b \to u/b \to c$	$\|V_{ub}/V_{cb}\|^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$\|V_{td}\|^2 f^2_{B_d}\hat{B}_{B_d}f(m_t)$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
$\frac{\Delta m_d}{\Delta m_s}$	$\left\|\frac{V_{td}}{V_{ts}}\right\|^2 \frac{f^2_{B_d}B_{B_d}}{f^2_{B_s}\hat{B}_{B_s}}$	$(1-\bar{\rho})^2 + \bar{\eta}^2$
ϵ_K	$f(A,\bar{\eta},\bar{\rho},\hat{B}_K)$	$\propto \bar{\eta}(1-\bar{\rho})$

$\bar{\rho} = \rho(1-\lambda^2/2),\ \bar{\eta} = \eta(1-\lambda^2/2)$

<u>V_{cb}</u>

$$Y_d : \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix}$$

$$V_{cb} \simeq \frac{c}{d}$$

$$m_b \simeq d$$

$$m_s \simeq -b + \frac{c^2}{d}$$

$$m_d \simeq \frac{a^2}{m_s}$$

Then $\quad \dfrac{c^2}{d^2} = \dfrac{m_s}{m_b} + \dfrac{b}{d} \quad = O\left(\dfrac{m_s}{m_b}\right).$

But precise value sensitive to details of Yukawa couplings since $\quad b = O(\bar{\varepsilon}^2)$, $c = O(\bar{\varepsilon})$, $d = O(1)$.

$$V_{cb} = \frac{1}{2}\sqrt{\frac{m_s}{2m_b}} - \sqrt{\frac{m_u}{m_t}} \quad = 0.054 \pm 0.02$$

... limitation of Abelian symmetry predictions. ...

need theory of coefficients to predict V_{cb}.

Coefficients of O(1)

$$\frac{m_u}{m_t} = \begin{pmatrix} 0 & \varepsilon^3 & 0 \\ \varepsilon^3 & e^{i\frac{\pi}{2}}\varepsilon^2 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix}$$

$$\frac{m_d}{m_b} = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & 0 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \frac{1}{2}\bar{\varepsilon} \\ 0 & \frac{1}{2}\bar{\varepsilon} & 1 \end{pmatrix}$$

$$H_1^{light} = H_1^a + \frac{\langle\Theta\rangle}{2M_1} H^b + \dots$$

$$\varepsilon = \frac{\langle\Theta\rangle}{M_2} = 0.05 \qquad \bar{\varepsilon} = \frac{\langle\Theta\rangle}{M_1} = 0.18$$

$\Rightarrow$ $V_{cb} = 0.04$ ($V_{cb} = 0$ in $SU(2)_R$ symmetry limit , - $M_2 = M_1$)