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SMR.1317 - 15

SUMMER SCHOOL ON PARTICLE PHYSICS

18 June - 6 July 2001

NEUTRINO PHYSICS

Lecture I

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June 2001

NEUTRINO PHYSICS

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ν : THE OLDEST KNOWN FUNDAMENTAL PARTICLE AFTER THE ELECTRON AND THE PHOTON

$\downarrow$

ENORMOUS EXPERIMENTAL + THEORETICAL ACTIVITY SINCE ITS INTRODUCTION IN PARTICLE PHYSICS (PAULI → kinematics ; FERMI → dynamics)

+ Boost of interest after 1998 from atmospheric ν oscillations in Superkamiokande (currently: hundreds of papers/year)

$\downarrow$

Only some old + new selected topics discussed here

OUTLINE

I. neutrino currents & mass terms

II. neutrino oscillations (theory)

III & IV. atmospheric ν oscill. (2ν)
pre-SNO solar ν oscill. (2ν)
solar + atmospheric (3ν)
Solar + atmosph. + LSND (4ν)
indications about absolute ν masses

V. special lecture on recent SNO results and implications

TOPICS IN THESE LECTURES ALSO TREATED IN MANY EXCELLENT REVIEWS

Bilenky & Petcov
Mikheyev & Smirnov
Kuo & Pantaleone
Raffelt
Haxton & Holstein
Gelmini & Roulet
Bilenky, Giunti & Grimus
Valle
Langacker
Akhmedov
Oberauer & von Feilitzsch
Barger
Klapdor
Kayser
Zuber
Sarkar
....

AND BOOKS

Kim & Pevsner
Mohapatra & Pal
Kayser
Bahcall
Boehm & Vogel
Gaisser
......

WHERE ONE CAN FIND REFERENCES TO AUTHORS / PAPERS / EXPERIMENTS / IDEAS ... (HERE QUOTED ONLY <u>OCCASIONALLY</u>)

Lecture I

FERMION CURRENTS IN THE STANDARD MODEL $SU(2)_L \otimes U(1)_Y$

- **BUILDING BLOCKS:**

$$\begin{pmatrix} U^\alpha \\ D^\alpha \end{pmatrix}_L \qquad \begin{matrix} U_R^\alpha \\ D_R^\alpha \end{matrix}$$

$\alpha = 1,2,3$
$=$ generation index

$U =$ "up" fermions
$D =$ "down" fermions
$P_{L,R} = \frac{1 \mp \gamma_5}{2}$ (more on chiral states later)

- **CHARGES:**

$(T, T_3) = SU(2)_L$ charge
$Y = 2(Q - T_3) = U(1)_Y$ charge (factor 2 conventional)
$Q =$ E.M. charge

- **GAUGE BOSONS** (after SSB)

A_μ $(m=0)$
Z_μ $(m=M_Z)$
$W_\mu^\pm$ $(m=M_W)$

- **FERMION CURRENTS**

$$A_\mu \rightarrow \quad J_\mu^{EM} = \sum_\alpha \bar{U}^\alpha Q \gamma_\mu U^\alpha + (U \to D)$$

$$W_\mu^\pm \rightarrow \quad J_\mu^+ = \sum_\alpha \bar{D}_L^\alpha \gamma_\mu U_L^\alpha$$

$$J_\mu^- = \sum_\alpha \bar{U}_L^\alpha \gamma_\mu D_L^\alpha$$

$$Z_\mu \rightarrow \quad J_\mu^Z = \sum_\alpha \bar{U}_L^\alpha (T_3 - Q \sin^2\theta_W) \gamma_\mu U_L^\alpha + U_R^\alpha (-Q \sin^2\theta_W) \gamma_\mu U_R^\alpha + (U \to D)$$

- **LOW-ENERGY LIMIT**

$$\mathcal{L}_{CC+NC} = -\frac{4 G_F}{\sqrt{2}} \left[J_\mu^+ J^{\mu -} + \rho J_\mu^Z J^{\mu Z} \right]$$

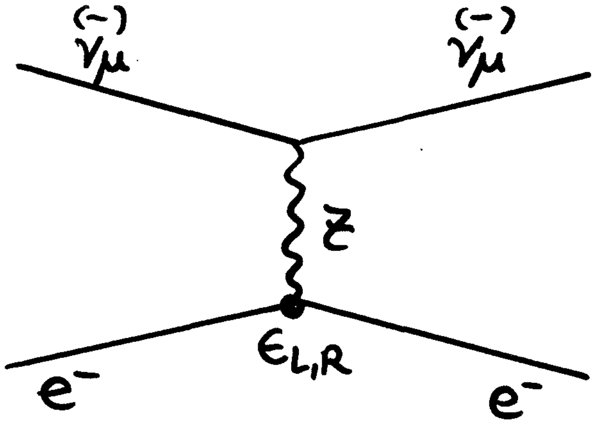
$\rho = 1$ if SSB induced by Higgs doublet

$\tan\theta_W = g'/g$

g : $SU(2)_L$ coupling
g' : $U(1)_Y$ "

θ_W = bookkeeping parameter (can be eliminated in terms of mass spectrum $+ (\alpha, G_F)$ $(+\alpha_s$ if $SU(3)_c))$

PROBING NC



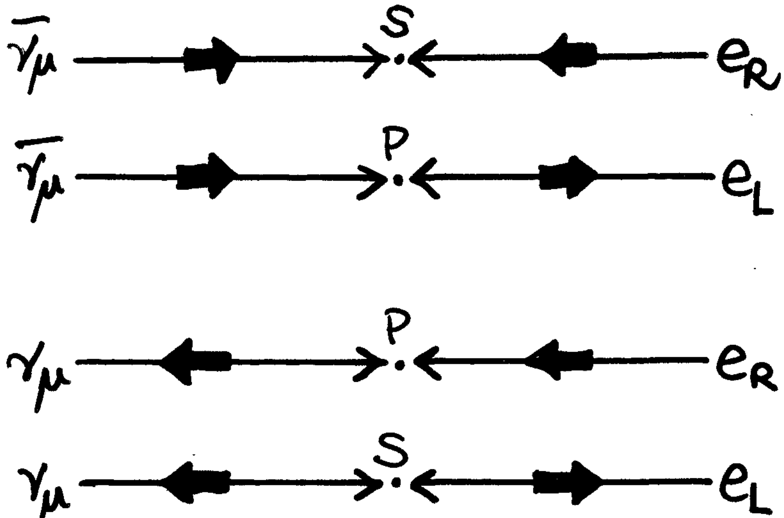
$\overset{(-)}{\nu}_\mu$ scattering on e^-

NC electron charges:

$$\epsilon_L = (T_3 - Q s_W^2) e_L = -\tfrac{1}{2} + s_W^2$$

$$\epsilon_R = (T_3 - Q s_W^2) e_R = 0 + s_W^2$$

At high energies, helicity $\sim$ chirality and total (ν, e) spin $J=0$ (S-wave) or $J=1$ (P-wave) in C.M. system



$\frac{d\sigma}{dy} \propto$

ϵ_R^2

$\epsilon_L^2 (1-y)^2$

$\epsilon_R^2 (1-y)^2$

ϵ_L^2

$y = \frac{E_e}{E_\nu}$ (fractional electron energy)

At low energies, helicity $\neq$ chirality and a further LR correction appears

$$\propto \epsilon_L \epsilon_R \frac{m_e}{E_\nu} \cdot y$$

$\rightarrow$ important for accurate calculations

PROBING FERMION CURRENTS WITH NEUTRINOS

NEUTRINOS HAVE BEEN USED TO :

1) Assess strength of weak interactions (G_F)

2) Probe V-A structure of $J_\mu^\pm$ (CC)

3) Probe ($T_3 - Q s_W^2$) charge of J_μ^Z (NC)

4) Probe CC + NC interference

....

EXAMPLES :

1) β decay , μ decay

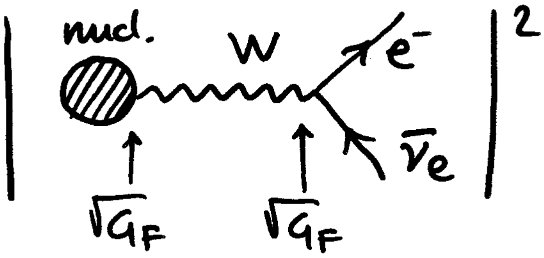
2) $\pi \rightarrow \mu\bar{\nu}, e\bar{\nu}$ decay

3) $\overset{(-)}{\nu}_\mu e$ scattering

4) $\overset{(-)}{\nu}_e e$ scattering

PROBING G_F

- **β-decay**



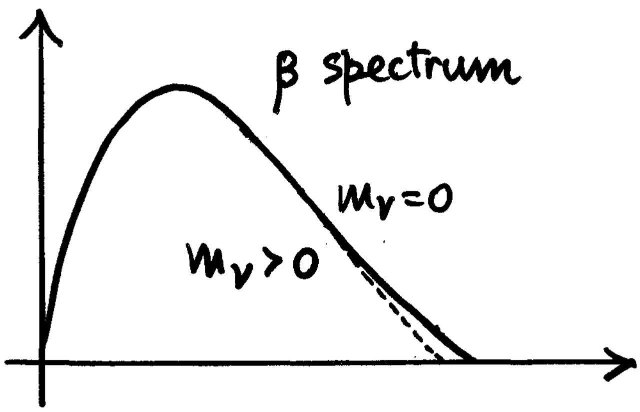
$\propto G_F^2$

DECAY RATE $\quad d\Gamma \propto G_F^2 \times (\text{phase space})$

ENERGY SPECTRUM:

$$\frac{d\Gamma}{dE_e} \propto G_F^2\, p_e E_e (Q - E_e)^2 \qquad (m_\nu = 0)$$

$$\propto G_F^2\, p_e E_e (Q - E_e)\sqrt{(Q-E_e)^2 + m_\nu^2} \qquad (m_\nu > 0)$$



current limits:

"m_{ν_e}" < few eV

- **μ-decay** $\quad \Gamma_\mu = \frac{1}{\tau_\mu} \propto G_F^2 m_\mu^5$

"defines" G_F

PROBING V-A

in π decay

Dirac eq. for free particle (Weyl repres.)

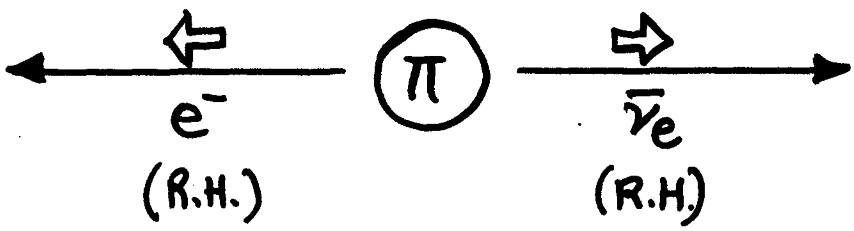
$$\begin{bmatrix} -\frac{m}{E} & 1+h \\ 1-h & -\frac{m}{E} \end{bmatrix} \begin{bmatrix} \phi_R \\ \phi_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$h = \frac{\vec{p}\cdot\vec{\sigma}}{E} \quad \rightarrow \textbf{helicity}$$

$$\phi_{R,L} = \frac{1 \pm \gamma_5}{2}\psi \quad \rightarrow \textbf{chirality}$$

For $m/E \rightarrow 0$: $\quad h\,\phi_{R,L} \simeq \pm\,\phi_{R,L} + \mathcal{O}(m/E)$

helicity $\simeq$ chirality



"wrong" chirality up to $\mathcal{O}(m_e/E)$

$\pi \rightarrow e^- \bar{\nu}_e$ forbidden by V-A for $m_e \rightarrow 0$

$$\frac{\Gamma(\pi \rightarrow e\nu)}{\Gamma(\pi \rightarrow \mu\nu)} \simeq \underbrace{\left(\frac{m_e}{m_\mu}\right)^2}_{\ll 1} \underbrace{\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2}_{> 1} \ll 1$$

chiral suppress. factor

phase space

Differential cross sections :

$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e^-) \simeq \frac{2 G_F^2 m_e E_\nu}{\pi} \left[\epsilon_R^2 + \epsilon_L^2 (1-y)^2 \right]$$

$$\frac{d\sigma}{dy}(\nu_\mu e^-) \simeq 2\frac{G_F^2 m_e E_\nu}{\pi} \left[\epsilon_L^2 + \epsilon_R^2 (1-y)^2 \right]$$

Total cross sections:

$$\int (1-y)^2 dy = \frac{1}{3} \quad \leftarrow$$ only 1/3 of $\vec{J}=1$ states allowed by J conservation

$$\sigma(\bar{\nu}_\mu e^-) \propto \left(\epsilon_R^2 + \frac{1}{3}\epsilon_L^2\right) E_\nu$$

$$\sigma(\nu_\mu e^-) \propto \left(\epsilon_L^2 + \frac{1}{3}\epsilon_R^2\right) E_\nu$$

$$\sim \mathcal{O}\left(10^{-44} \frac{E_\nu}{10\,\text{MeV}} \text{cm}^2\right)$$

"History" :

$$R = \frac{\sigma}{\bar{\sigma}} = \frac{3\epsilon_L^2 + \epsilon_R^2}{3\epsilon_R^2 + \epsilon_L^2} = 3\,\frac{1 - 4s_W^2 + \frac{16}{3}s_W^4}{1 - 4s_W^2 + 16\,s_W^4}$$

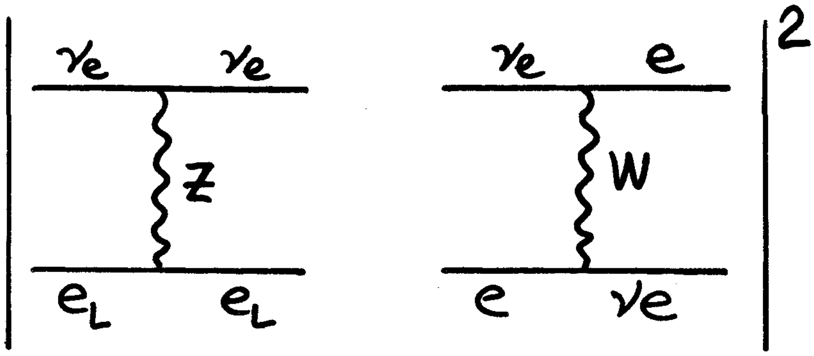
allowed early estimates of s_W^2 and of tree-level $M_{W,Z}$ masses from:

$$s_W^2 = \frac{\pi\alpha}{\sqrt{2}\,G_F M_W^2}$$

$$s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$$

PROBING W-Z INTERFERENCE

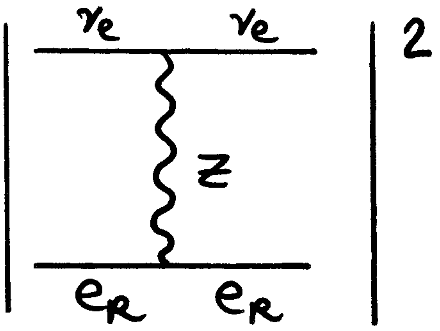
$\overset{(-)}{\nu}_e e$ scattering



$\propto (\epsilon_L + 1)^2$

$\uparrow \quad \uparrow$

Z + W



$\propto \epsilon_R^2 (1-y)^2$

$$\frac{d\sigma}{dy}(\nu_e e^-) \simeq \frac{2G_F^2 m_e E_\nu}{\pi}\left[(\epsilon_L+1)^2 + \epsilon_R^2 (1-y)^2\right]$$

$$\frac{d\sigma}{dy}(\bar{\nu}_e e) \simeq \frac{2G_F^2 m_e E_\nu}{\pi}\left[(\epsilon_R+1)^2 + \epsilon_L^2 (1-y)^2\right]$$

W Z interference

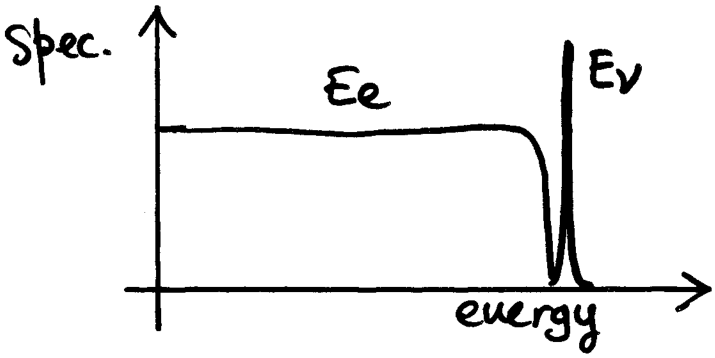
IMPORTANT EFFECT ON DIFFERENTIAL AND TOTAL $\overset{(-)}{\nu}_e e$ CROSS SECTIONS $\rightarrow$

- $\sigma(\nu_\mu) < \sigma(\nu_e)$

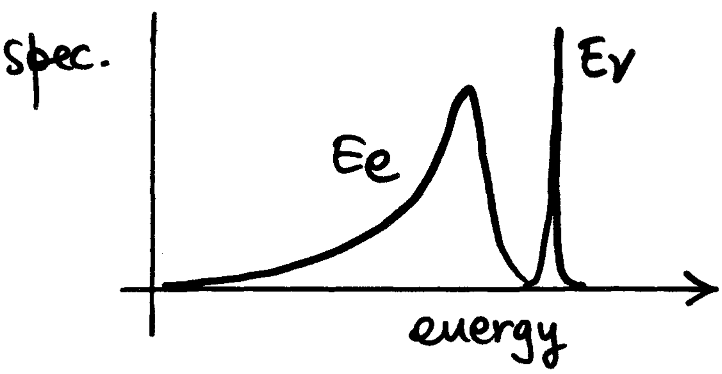
$$\frac{\sigma(\nu_\mu e)}{\sigma(\nu_e e)} \simeq \frac{\epsilon_L^2 + \epsilon_R^2/3}{(\epsilon_L+1)^2 + \epsilon_R^2/3} \sim \frac{1}{7}$$

- $\nu_e e^- \to \nu_e e^-$: flat electron spectrum

$$\frac{d\sigma}{dy}(\nu_e e^-) \propto 1 + \underbrace{\frac{\epsilon_R^2}{(\epsilon_L+1)^2}}_{\text{small}}(1-y)^2 \sim \text{const}$$



$\nu_e e^- \to \nu_e e^-$



... to be compared with

$\nu_e d \to p p e^-$

$\to$ IMPORTANT FOR SOLAR ν EXPERIMENTS (SK VS SNO)

FERMION MASSES IN THE S.M.

- $\underset{\text{HIGGS}}{\phi} = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} \xrightarrow{\text{SSB}} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$

- YUKAWA LAGRANGIAN :

$$-\mathcal{L}_Y = \sum_{\alpha\beta} f_D^{\alpha\beta} \, \overline{(U^\alpha, D^\alpha)}_L \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} D_R^\beta$$

$$+ \sum_{\alpha\beta} f_U^{\alpha\beta} \, \overline{(U^\alpha, D^\alpha)}_L \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} U_R^\beta \quad \leftarrow \begin{array}{l} \text{get mass} \\ \text{from } \tilde{\phi} \\ = i\sigma_2 \phi^* \end{array}$$

$$= \sum_{\alpha\beta} \bar{D}_L^\alpha \underbrace{M_D^{\alpha\beta}}_{} D_R^\beta + \bar{U}_L^\alpha \underbrace{M_U^{\alpha\beta}}_{} U_R^\beta$$

generic 3 x 3
complex matrices

→ **diagonalization**

THEOREM: Generic M $(N \times N)$ is diagonalizable through biunitary tranformation:

$$S^{+} M T = M_d$$
$$= \mathrm{diag}(m_1, m_2, \ldots, m_N)$$

with $SS^{+} = TT^{+} = 1$

Proof: MM^{+} is hermitian

$$S^{+}(MM^{+})S = M_d^2 = \mathrm{diag}(m_1^2, m_2^2, \ldots, m_N^2)$$

with
$$m_i^2 = (M_d^2)_{ii}$$
$$= [(S^{+}M)(S^{+}M)^{+}]_{ii}$$
$$= \sum_j (S^{+}M)_{ij} (S^{+}M)_{ij}^{*}$$
$$= \sum_j |(S^{+}M)_{ij}|^2 > 0$$

$\rightarrow MM^{+}$ has real, positive eigenvalues m_i^2

Define: $M_d = \sqrt{M_d^2} = \mathrm{diag}(m_1, m_2, \ldots, m_N)$

then:
$$H = S M_d S^{+} \rightarrow \text{hermitian}$$
$$V = H^{-1} M \rightarrow \text{unitary}$$
$$T = V^{+} S \rightarrow \text{unitary}$$

$$M_d = S^{+} H S = S^{+} M V^{+} S = S^{+} M T \quad \blacksquare$$

INVARIANCE: THE CURRENTS

$$J_\mu^- = \sum_\alpha \bar{U}_L^\alpha \gamma_\mu D_L^\alpha$$

$$J_\mu^Z = \sum_\alpha \bar{U}_L^\alpha (T_3 - Q s_W^2) \gamma_\mu U_L^\alpha + \bar{U}_R^\alpha (-Q s_W^2) \gamma_\mu U_R^\alpha + (U \to D)$$

$$J_\mu^{EM} = \sum_\alpha \bar{U}^\alpha Q \gamma_\mu U^\alpha + (U \to D)$$

ARE INVARIANT UNDER THE TRANSFORMATIONS:

(i) $U_R^\alpha \to T^{\alpha\beta} U_R^\beta$

(ii) $U_L^\alpha \to S^{\alpha\beta} U_L^\beta$

(iii) $D_L^\alpha \to S^{\alpha\beta} D_L^\beta$

same S, $SS^+ = 1$, $TT^+ = 1$, $WW^+ = 1$

(iv) $D_R^\alpha \to W^{\alpha\beta} D_R^\beta$

THIS IMPLIES THAT EITHER M_D or M_U CAN BE CHOSEN DIAGONAL WITHOUT AFFECTING THE CURRENTS

USUAL TRICK :

- **<u>FOR QUARKS</u>**, use (i),(ii),(iii) to identify T and S with the matrices diagonalizing M_U ($M_U = S^{+} M_U^{diag} T$) and use (iv) to identify W with one of the matrices diagonalizing M_D ($M_D = V^{+} M_D^{diag} W$) $\rightarrow$ ONLY ONE PHYSICAL UNITARY TRANSFORM.,

$$D_L^{\alpha} \rightarrow \mathbf{V}^{\alpha\beta} D_L^{\beta}$$

which affects $J_\mu^{\pm}$ (<u>NOT</u> J_μ^{EM}, J_μ^{Z}):

$$J_\mu^{-} \rightarrow \sum_{\alpha\beta} \bar{U}_L^{\alpha} \gamma_\mu \underset{\substack{\uparrow \\ \text{CKM}}}{\mathbf{V}^{\alpha\beta}} D_L^{\beta}$$

- **<u>FOR LEPTONS</u>**

Only one matrix to diagonalize if $m_\nu = 0$ ($\nexists \nu_R$) .... $\rightarrow$ no observable CKM matrix in the S.M. with minimal fermion content

WHAT IF ν_R^{α} INTRODUCED IN THE SAME WAY AS FOR QUARK?

...can get m_ν but :

- NO HINT FOR m_ν SMALL
- MASS TERMS FOR ν CAN BE MORE GENERAL THAN FOR QUARKS

MASSLESS AND MASSIVE NEUTRAL FERMIONS

IN DIRAC REPRESENTATION

$$\gamma^0_D = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \quad \vec{\gamma}_D = \begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix} \quad \gamma^5 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$$

"PARTICLE" Solution $\quad \psi_P \sim \begin{bmatrix} \xi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\xi \end{bmatrix} e^{-ip_\mu x^\mu} \quad \xi\xi^+ = 1$

"ANTIPARTICLE" Solution $\quad \psi_A \sim \begin{bmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \\ \chi \end{bmatrix} e^{ip_\mu x^\mu} \quad \chi\chi^+ = 1$

ξ, χ = Pauli Spinors

NONRELATIVISTIC (particle) limit

$$\psi_P \sim \begin{bmatrix} \xi \\ 0 \end{bmatrix} \quad ; \quad \overline{\psi}_P \sim \begin{bmatrix} \xi^+ \\ 0 \end{bmatrix}^T$$

$$S = \overline{\psi}\psi \simeq |\xi|^2$$

$$P = \overline{\psi}\gamma^5\psi \simeq 0$$

$$\left.\begin{aligned} V &= \overline{\psi}\gamma^\mu\psi \simeq (|\xi|^2, \vec{0}) \\ A &= \overline{\psi}\gamma^\mu\gamma^5\psi \simeq (0, \xi^+\vec{\sigma}\xi) \end{aligned}\right\} \text{useful later}$$

DIRAC REPRESENTATION USEFUL TO DEFINE PARTICLE-ANTIPARTICLE CONJUGATION OPERATOR

$$\psi^c = \mathcal{C}(\psi)$$

$$\psi_{P,A} = \mathcal{C}(\psi_{A,P})$$

$$\boxed{\begin{aligned}\mathcal{C}(\psi) &= i\gamma^2\psi^* \\ &= i\gamma^2\gamma^0\bar{\psi}^T \\ &= C\bar{\psi}^T \\ &= \psi^c\end{aligned}}$$

$$C = i\gamma^2\gamma^0$$

- E.g. prove that $\mathcal{C}(\psi_P) = \psi_A$; hint: use $\sigma_2\vec{\sigma}^* = -\vec{\sigma}\sigma_2$ and set $\chi = -i\sigma_2\xi^*$.

- Prove that if ψ is E.M. charged, i.e. if $[i\gamma^\mu(\partial_\mu - iqA_\mu) - m]\psi = 0$ then $[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi^c = 0$

CONVENTION : when operations such as $P_{L,R}$, $\overline{(\;)}$, and $\mathcal{C}$, are involved,

$P_{L,R}$ act befor C which acts before $\overline{(\cdot)}$

$$\Downarrow$$

$$\psi^c_{L,R} = (P_{L,R}\psi)^c = (\psi_{L,R})^c = P_{R,L}(\psi^c)$$

$$\overline{\psi}_{L,R} = \overline{(P_{L,R}\psi)} = \overline{(\psi_{L,R})} = \overline{\psi}\, P_{R,L}$$

$$\overline{\psi}^c = \overline{(\psi^c)}$$

$$\overline{\psi}^c_{L,R} = \overline{(P_{L,R}\psi)^c} = \overline{(\psi_{L,R})^c} = \overline{P_{R,L}(\psi^c)} = \overline{\psi}^c P_{L,R}$$

- **WEYL REPRESENTATION**

CHANGE BASIS $\quad \psi \to T\psi$

$$\gamma^\mu \to T\gamma^\mu T^{-1}$$

with $T = \frac{1}{\sqrt{2}}(\gamma^0_D + \gamma^5_D) = \frac{1}{\sqrt{2}}\begin{bmatrix} I & I \\ I & -I \end{bmatrix}$

$$\gamma^0_W = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \qquad \vec{\gamma}_W = \begin{bmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \qquad \gamma^5_W = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

$$\psi_R = \frac{1+\gamma_5}{2}\psi = \begin{bmatrix} \phi_R \\ 0 \end{bmatrix}$$

$$\psi_L = \frac{1-\gamma_5}{2}\psi = \begin{bmatrix} 0 \\ \phi_L \end{bmatrix}$$

"Fundamental" objects under Lorentz group

$$\boxed{\begin{aligned} x'^\mu &= e^{i(\vec{\omega}\cdot\vec{J} + \vec{u}\cdot\vec{k})}\, x^\mu \\ &\qquad \text{rotation} \quad \text{boost} \\ \phi'_R &= e^{i(\vec{\omega} - i\vec{u})\cdot\frac{\vec{\sigma}}{2}}\,\phi_R \\ \phi'_L &= e^{i(\vec{\omega} + i\vec{u})\cdot\frac{\vec{\sigma}}{2}}\,\phi_L \end{aligned}}$$

coupled by Dirac equation; decoupled only if $m=0$ (Weyl spinors)

THEOREM: Given ϕ_R (R.H.), $i\sigma_2\phi_R^*$ is L.H.

" ϕ_L (L.H.), $-i\sigma_2\phi_L^*$ is R.H.

(Hint: use $\sigma_2\vec{\sigma}^* = -\vec{\sigma}\sigma_2$ and infinitesimal transformations)

→ can make a Dirac spinor ψ with two R.H. spinors u and v:

$$\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix} = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

$$\boxed{\begin{array}{lll}
\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix} & \psi_L = \begin{bmatrix} 0 \\ i\sigma_2 v^* \end{bmatrix} & \psi_R = \begin{bmatrix} u \\ 0 \end{bmatrix} \\[2ex]
\overline{\psi} = \left[-iv^T\sigma_2, u^+\right] & \overline{\psi}_L = \left[-iv^T\sigma_2, 0\right] & \overline{\psi}_R = \left[0, u^+\right] \\[2ex]
\psi^c = \begin{bmatrix} v \\ i\sigma_2 u^* \end{bmatrix} & \psi_L^c = \begin{bmatrix} v \\ 0 \end{bmatrix} & \psi_R^c = \begin{bmatrix} 0 \\ i\sigma_2 u^* \end{bmatrix} \\[2ex]
\overline{\psi}^c = \left[-iu^T\sigma_2, v^+\right] & \overline{\psi}_L^c = \left[0, v^+\right] & \overline{\psi}_R^c = \left[-iu^T\sigma_2, 0\right]
\end{array}}$$

... with $\mathcal{C}$ swapping u and v

(analogously, one can make ψ with L.H. spinors)

In general, no relation between u and v

Given $\psi = \begin{bmatrix} u \\ i\sigma_2 v^* \end{bmatrix}$ $\quad$ (u, v L.H.)

$u \neq v$ $\longrightarrow$ **DIRAC** ν

$u = v$ $\longrightarrow$ **MAJORANA** ν

($\psi = \psi^c$, see previous page)

MAJORANA NEUTRINOS ARE THEIR OWN ANTIPARTICLES
→ MUST BE COMPLETELY NEUTRAL
(NO E.M. CHARGE, NO GENERALIZED CHARGE)

MORE GENERALLY, FOR A MAJORANA ν:

$$\psi_M = \psi_M^c \cdot e^{i\phi_M}$$

"Majorana creation phase"
can be different from +1
(examples later)

SUMMARY OF ν REPRESENTATIONS:

$m=0$

$$\psi = \begin{pmatrix} \nu_R \\ 0 \end{pmatrix} = \psi_R$$

$$\text{or } \psi = \begin{pmatrix} 0 \\ \nu_L \end{pmatrix} = \psi_L$$

WEYL
simplest massless case, 2 dof

$m \neq 0$

$$\psi = \begin{pmatrix} \nu_R \\ i\sigma_2 \nu_R^* \end{pmatrix} = \psi_R + \psi_R^c = \psi^c$$

$$\text{or } \psi = \begin{pmatrix} -i\sigma_2 \nu_L^* \\ \nu_L \end{pmatrix} = \psi_L + \psi_L^c = \psi^c$$

MAJORANA
simplest massive case, 2 dof

$m \neq 0$

$$\psi = \begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix} = \psi_R + \psi_L \neq \psi^c$$

DIRAC
general massive case, 4 dof

PARADOX & RESOLUTION:

Q. How can it be $\nu_e = \overline{\nu}_e$ if (antiparticle)

$$\nu_e + n \to p + e^-$$
$$\overline{\nu}_e + p \to n + e^+$$

$$\overline{\nu}_e + n \not\to p + e^-$$
$$\nu_e + p \not\to n + e^+ \quad ?$$

(Here ν_e = neutral fermion produced in β^+ decay
$\overline{\nu}_e$ = " " " " β^- decay)

A1. Indeed, $\nu_e \neq \overline{\nu}_e$ (Dirac), and lepton number is conserved : $\Delta L_e = 0$

A2. It is $\nu = \overline{\nu}$ (Majorana), and we call

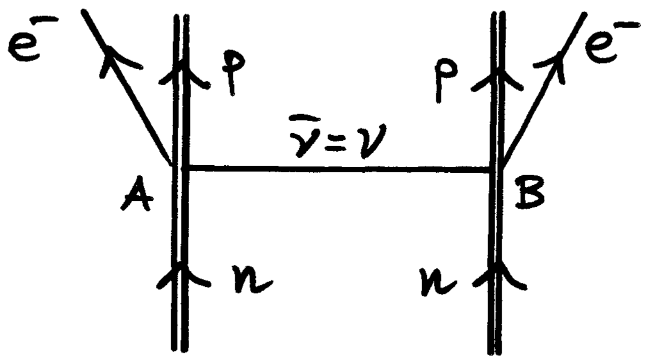
$$\text{“}\nu_e\text{”} = P_L \nu \,,$$
$$\text{“}\overline{\nu}_e\text{”} = P_R \nu \,.$$

The initial "ν_e" is produced L.H. in β^+decay and remains dominantly so due to small m_ν, so the reaction $\nu_e p \to n e^+$ is chirally suppressed by V-A (up to terms of $\mathcal{O}(m_\nu/E)$).

In principle, however, such reaction can take place for small energies : $\Delta L_e = 2$ at $\mathcal{O}(m_\nu/E)$

PHYSICAL IMPLICATIONS OF MAJORANA NEUTRINOS: $0\nu 2\beta$ DECAY

NUCLEUS CHANGES CHARGE BY TWO UNITS AND EMITS COUPLE OF ELECTRONS:



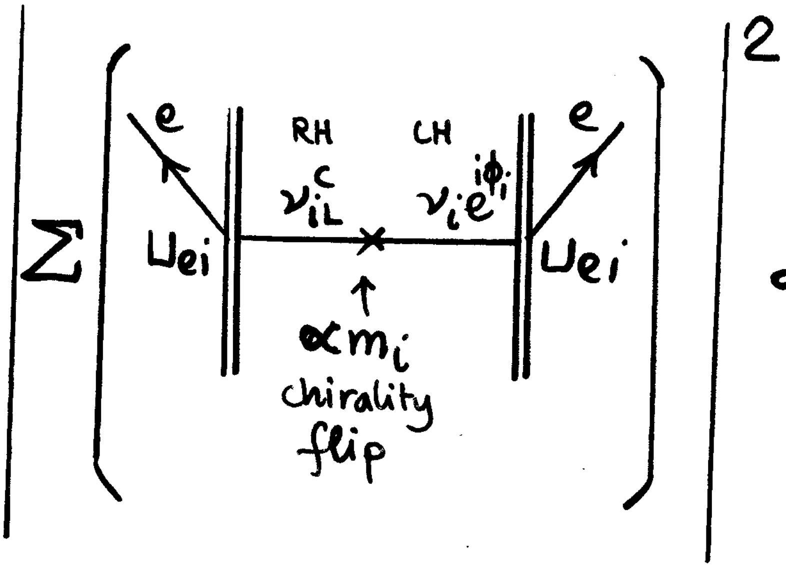
EXTREMELY RARE: $(\text{WEAK})^2$!

INTUITIVE PICTURE:

- A $\bar{\nu}_e$ (RH) is emitted in A
- If it is massive, it can develop a LH component $\propto m_\nu$ (not possible if WEYL ν)
- If $\nu = \bar{\nu}$, such component is a LH neutrino (not possible if Dirac ν)
- The ν_L is absorbed in B and transformed in e^-
- Lepton # violated (2 units)

WHAT IS OBSERVED IN $0\nu 2\beta$ decay ?

In general, ν_e = superposition of Majorana fields ν_i with coefficients U_{ei} (complex) and creation phases $e^{i\phi_i}$



$$\propto \left| \sum_i U_{ei}^2 m_i e^{i\phi_i} \right|^2$$

$$= \left| \sum_i |U_{ei}|^2 m_i e^{i\phi_i'} \right|^2$$

$$= \langle m_{ee} \rangle^2$$

"effective electron mass"

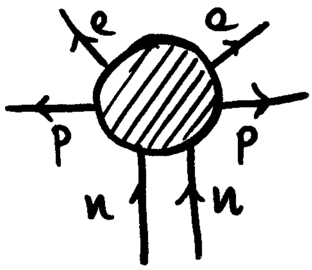
current limits : $\langle m_{ee} \rangle <$ few $\times 10^{-1}$ eV

GLOBAL PHASES ϕ_i' (mixing + Majorana phases) ARE PHYSICAL $\rightarrow$ CAN GET CONSTRUCTIVE/DESTRUCTIVE INTERFERENCE AMONG (i,j) CHANNELS

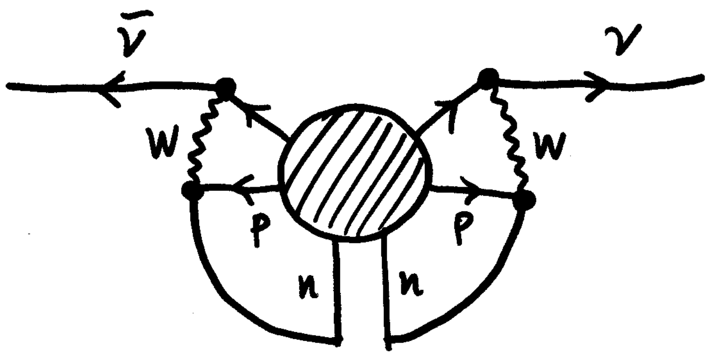
$\rightarrow$ $\langle m_{ee} \rangle$ may be small due to "cancellation"

PROFOUND LINK BETWEEN $0\nu 2\beta$ decay and Majorana ν :

Independently on the mechanism for $0\nu 2\beta$ decay,



get a Majorana mass term,



NEUTRINO MASS TERM FOR ONE FAMILY

Generate $m\bar{\psi}\psi$ from:

1) $\psi = \psi_L + \psi_R$ (DIRAC) $\rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$

2) $\psi = \psi_L + \psi_L^c$ (MAJOR.) $\rightarrow \bar{\psi}\psi = \bar{\psi}_L\psi_L^c + \bar{\psi}_L^c\psi_L$

3) $\psi = \psi_R + \psi_R^c$ (MAJOR.) $\rightarrow \bar{\psi}\psi = \bar{\psi}_R\psi_R^c + \bar{\psi}_R^c\psi_R$

HOW ? e.g. $\left\langle \begin{array}{ll} \text{doublet} & \Phi \\ \text{singlet} & \varphi \\ \text{triplet} & \vec{\varphi} \end{array} \right.$ Higgs $\Big\}$ Beyond S.M.

$$\mathcal{L} \ni h\,(\bar{\nu}_L\,\bar{e}_L)\,\Phi\,\nu_R$$ Doublet × doublet × singlet

$$+\,h'\,(\bar{\nu}_R^c\nu_R + \bar{\nu}_R\nu_R^c)\,\varphi$$ singlet × singlet × singlet

$$+\,h''\,(\bar{\nu}_L\bar{e}_L)\,\frac{\vec{\varphi}\cdot\vec{\sigma}}{2}\begin{pmatrix}\nu_L^c\\ e_L^c\end{pmatrix}$$ doublet × triplet × doublet

$\xrightarrow{SSB}$ $-\mathcal{L}_{mass} =$

$$m_D\,(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$$ DIRAC

$$+\,m_R\,(\bar{\nu}_R^c\nu_R + \bar{\nu}_R\nu_R^c)$$ MAJORANA $\Big\}$ Beyond S.M.

$$+\,m_L\,(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L)$$ MAJORANA

MAJORANA mass terms not invariant under any global $U(1)$: $\psi \rightarrow e^{i\phi}\psi$

$\rightarrow$ no additive (lepton) number conserved

- Re-write

$$-\mathcal{L}_{mass} = \left(\bar{\nu}_L + \bar{\nu}_L^c, \bar{\nu}_R + \bar{\nu}_R^c\right) \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L + \nu_L^c \\ \nu_R + \nu_R^c \end{pmatrix}$$

(Majorana basis)

→ intuitively clear that, in general, diagonalization will give Majorana ν as eigenstates

Diagonalization:

$$\mathcal{M} = \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \qquad \begin{aligned} T &= \mathrm{Tr}\,\mathcal{M} = m_L + m_R \\ D &= \det \mathcal{M} = m_L m_R - m_D^2 \end{aligned}$$

eigenvalues: $m_\pm \frac{1}{2}\left(T \pm \sqrt{T^2 - 4D}\right)$

$$\sin 2\theta = \frac{2m_D}{\sqrt{T^2-4D}} \qquad \cos 2\theta = \frac{m_L - m_R}{\sqrt{T^2-4D}}$$

$$\begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta \\ S_\theta & C_\theta \end{bmatrix}$$

$$[v_1, v_2] \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \underbrace{[v_1' \; v_2']}_{\text{eigenvect.}} \begin{bmatrix} m_+ & 0 \\ 0 & m_- \end{bmatrix} \begin{bmatrix} v_1' \\ v_2' \end{bmatrix}$$

$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} C_\theta & S_\theta \\ -S_\theta & C_\theta \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

θ NOT Cabibbo (mixing) angle (1 family only!)

"DIRAC" CASE: $\mathcal{M} = \begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix}$

Recover dirac ν.

- Eigenvectors:

$$\phi_1 = \frac{1}{\sqrt{2}}\left[(\nu_L + \nu_L^c) + (\nu_R + \nu_R^c)\right] \qquad \text{mass } m$$

$$\phi_2 = \frac{1}{\sqrt{2}}\left[-(\nu_L + \nu_L^c) + (\nu_R + \nu_R^c)\right] \qquad \text{mass } -m$$

- "Negative" mass not a problem (-1 = Majorana phase).
  Define $\tilde{\phi}_2 = \gamma_5 \phi_2 = \frac{1}{\sqrt{2}}\left[(\nu_L - \nu_L^c) + (\nu_R - \nu_R^c)\right]$
  which obeys Dirac eq. with positive m.
  Note: $\tilde{\phi}_2^c = -\tilde{\phi}_2$

- ϕ_1 and $\tilde{\phi}_2$ have both mass m. Observable (active) component is

$$\nu_L = P_L \nu = P_L(\nu_L + \nu_R) = P_L \frac{1}{\sqrt{2}}(\phi_1 + \tilde{\phi}_2) = \frac{1}{\sqrt{2}}(\phi_1 + \tilde{\phi}_2)_L$$

$\rightarrow$ get a Dirac spinor ν ($\nu \neq \nu^c$) with mass $m = m(\phi_1) = m(\tilde{\phi}_2)$

SEE-SAW CASE : $\mathcal{M} = \begin{bmatrix} 0 & m \\ m & M \end{bmatrix}$

$\exists \nu_R$ in fermion multiplets of many SM extensions; e.g.

$\underline{16}$ of SO(10) $\begin{pmatrix} u_L & u_L & u_L & \nu_L \\ d_L & d_L & d_L & e_L \\ u_R & u_R & u_R & \boldsymbol{\nu_R} \\ d_R & d_R & d_R & e_R \end{pmatrix}$

→ get Majorana mass term $\underset{\uparrow}{M} \, (\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c)$

presumably large mass scale charachteristic of SM extension

Eigenvectors: (at $O(m/M)$)

$\phi_1 = (\nu_R + \nu_R^c) + \frac{m}{M} (\nu_L + \nu_L^c)$ ← mass M

$\phi_2 = (\nu_L + \nu_L^c) + \frac{m}{M} (\nu_R + \nu_R^c)$ ← mass $-\frac{m^2}{M}$

$\tilde{\phi}_2 = \gamma_5 \phi_2 = (\nu_L - \nu_L^c) + \frac{m}{M} (\nu_R - \nu_R^c)$ ← mass $+\frac{m^2}{M}$

- THE LIGHT MAJORANA STATE IS ACTIVE ($\nu_L \in \tilde{\phi}_2$)
- THE MASS OF $\tilde{\phi}_2$ CAN BE VERY SMALL

$$m(\tilde{\phi}_2) = \frac{m^2}{M}$$

← Dirac scale (SM SSB) as for quarks

← heavy scale

(see - saw)

GENERAL DIRAC + MAJORANA CASE (>1 FAMILIES)

1) START FROM :

- 3 LH gauge doublets $\nu_{\alpha L}$ $\alpha = e, \mu, \tau$
  (active neutrinos)

- n_s RH gauge singlets ν_{sR} $s = 1, 2, \dots, n_s$

2) BUILD COLUMN OF LH fields $\nu_L = \begin{pmatrix} \nu_{\alpha L} \\ \nu_{sR}^c \end{pmatrix}$ dim $= 3 + n_s$

3) WRITE MASS TERM $\mathcal{L}_M = -\frac{1}{2}\, \overline{\nu_L^c}\, M\, \nu_L$

row of RH fields

column of LH fields

$$M = \begin{bmatrix} M^L & M^D \\ M^{D'} & M_R \end{bmatrix}$$

$M_L = 3 \times 3$ (Majorana)
$M_D^{(')} = 3 \times n_s$ (Dirac)
$M_R = n_s \times n_s$ (Majorana)

$\mathcal{C}$ antisimmety + anticommutation $\rightarrow$ $M_L = M_L^T$, $M_R = M_R^T$, $M_D' = M_D^T$

$\rightarrow M =$ symmetric

4) DIAGONALIZE

DIAGONALIZATION OF GENERAL DIRAC+MAJORANA MATRIX GIVES (AT LEAST) THREE IMPORTANT DIFFERENCES WITH RESPECT TO THE PURE DIRAC (i.e., "quark-like") CASE:

1) Eigenvectors ν_K (i.e., Mass eigenstates) are, in general, Majorana $\rightarrow \exists$ $0\nu 2\beta$ decay

2) The left-handed column $\begin{pmatrix} \nu_{\alpha L} \\ \nu_{SR}^c \end{pmatrix}$ is a linear combination of ν_{kL}:

$$\begin{array}{l} \text{act.} \left\{ \begin{array}{c} \\ \\ \\ \end{array} \right. \\ \text{ster.} \left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right. \end{array} \begin{bmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \\ \nu_{1R}^c \\ \nu_{2R}^c \\ \vdots \\ \nu_{n_S R}^c \end{bmatrix} = U \begin{bmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \\ \vdots \\ \vdots \\ \nu_{(3+n_S)L} \end{bmatrix} \rightarrow$$

inversely, massive states are superposition of active + sterile states

(active/sterile mixing)

3) Since M is symmetric, only one matrix is needed for diagonalization (<u>not</u> biunitary)
→ less freedom to reabsorb phases

E.g. for 3 generations: $U \ni \delta_{CP}$ DIRAC ("quark"-like)

$$U \ni \underbrace{\delta_{CP}, \phi', \phi''}_{\text{3 phases}} \quad \text{MAJORANA}$$

↑
can show up in $0\nu 2\beta$

$N_\nu = \infty$: the case of extra dimensions

Arkhani-Ahmed, Dimopoulos, Dvali, March-Russel
Dienes, Dudas, Gherghetta
Dvali, Smirnov
Barbieri, Creminelli, Strumia
Mohapatra, Nandi, Perez-L.
Caldwell, Mohapatra, Yellin
Ross, Romanino .....

- N extra-dim. of size R_i $\longrightarrow$ gravitational potential modified at short distances $r \lesssim R_i$; dimensionally :

$$\frac{M_P^{-2}}{r} \rightarrow \frac{M_*^{-N-2}}{r \prod_{i=1,..,N} R_i}$$ M_* = new mass scale

$$M_* \sim M_P^{\frac{2}{N+2}} \left(\prod_{i=1}^{N} R_i\right)^{-\frac{1}{N+2}}$$ can be much lower than M_P

- Expt. : $R_i < 1\ \mathrm{mm} \sim 10^{12}\ \mathrm{GeV}^{-1}$

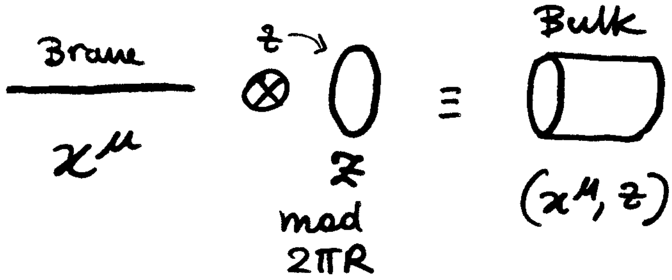
  $\rightarrow N=1$: M_* is not "interestingly small"

  $N=1$: $M_* \sim M_P^{2/3} R^{-1/3} \gtrsim 10^9\ \mathrm{GeV}$ "too large"

  $N=2$: $M_* \sim M_P^{2/4} (R^2)^{-1/4} \gtrsim 10^{3.5}\ \mathrm{GeV}$ "interesting"

- However, for simplicity let us consider $N_{eff} = 1$ $\quad (R_{i>1} \ll R_1)$

- COORDINATES :



- FREE PARTICLE IN BULK (mass M)

$$\phi(x,z) \sim e^{i(p^\mu x_\mu - p_z z)}$$

PERIODIC CONDITION

$$\phi(x, z+2\pi R) = \phi(x,z)$$

$\rightarrow p_z$ QUANTIZATION

$$p_z = \frac{n}{R} \qquad n = 0, \pm 1, \pm 2, \cdots$$

KLEIN-GORDON EQUATION

$$p_\mu p^\mu - p_z p^z = p_\mu p^\mu - \frac{n^2}{R^2} = M^2$$

- "BRANE VIEWPOINT" :

$$m^2_{brane} = p_\mu p^\mu = M^2 + \frac{n^2}{R} \qquad (\text{kk tower})$$

(also for M=0)

- IDEA : TAKE MASSLESS STERILE ψ IN BULK AND COUPLE IT TO USUAL LEPTON DOUBLET THROUGH THE HIGGS
  - $\rightarrow$ GET ∞ MASSIVE STATES AFTER SSB
  - $\rightarrow$ GET POSSIBLE HINT FOR m_ν SMALLNESS

Action $S = \int d^4x \, dz \, \overline{\Psi}(x,z) \, i \Gamma^A \partial_A \Psi(x,z)$

$A = 0, 1, 2, 3, z$

$\Gamma^A = (\gamma^\mu, -i\gamma^5)$

Fourier expansion

$$\underbrace{\Psi(x,z)}_{\text{Bulk}} \sim \sum_{n=-\infty}^{+\infty} \underbrace{\frac{\psi^{(n)}(x)}{\sqrt{2\pi R}}}_{\text{Brane}} \cdot \underbrace{\exp\left(in\frac{z}{R}\right)}_{\text{factorized } z}$$

$R^{-1/2}$ needed dimensionally

KK tower interaction on brane

Couple $\Psi(x, z=0) = \sum_{n=-\infty}^{+\infty} \frac{\psi^{(n)}(x)}{\sqrt{2\pi R}}$

to electron doublet $(\nu_e \, e) = \ell$

through Higgs H : massless

$$\text{Yuk.} \sim k \otimes H \otimes \frac{\psi^{(n)}}{\sqrt{2\pi R}} \otimes \ell$$

coupling

$[k] = \text{mass}^{-1/2}$

After SSB : $m_\nu \sim \frac{k v}{\sqrt{R}}$ S.M. $\langle H \rangle$

- NATURAL TO ASSUME $k \sim \mathcal{O}(M^{*1/2})$

$$\rightarrow m_\nu \sim \frac{k v}{\sqrt{R}} \sim \frac{v}{\sqrt{R}} \frac{1}{\sqrt{M_*}} \overset{N_{eff}=1}{\sim} v \frac{M_*}{M_P}$$

$$\boxed{\frac{m_\nu}{v} \sim \frac{M_*}{M_P}}$$

RELATES SMALLNESS OF m_ν TO SMALLNESS OF M_*/M_P

$\rightarrow$ m_ν might give info on M_*

- **MASS SPECTRUM**

At $z=0$, KK tower gives a contribution:

$$-\mathcal{L}_{mass} \ni \sum_{n=-\infty}^{+\infty} \frac{n}{R} \overline{\psi}_L^{(n)} \psi_R^{(n)} + h.c.$$

as well as Yukawa's after SSB:

$$-\mathcal{L}_{mass} \ni \underbrace{m}_{\sim v \frac{M_*}{M_P}} \cdot \sum_{n=-\infty}^{+\infty} \overline{\nu}_{eL} \psi_R^{(n)} + h.c.$$

- Separate massless (zero) mode $n=0$ and sum up $\pm n$:

$$-\mathcal{L}_{mass} \ni \sum_{n=1}^{\infty} \frac{M}{R}\left(\overline{\psi}_L^{(n)}\psi_R^{(n)} - \overline{\psi}_L^{(-n)}\psi_R^{(-n)}\right)$$
$$+ m\,\overline{\nu}_{eL}\psi_R^{(0)}$$
$$+ m\sum_{n=1}^{\infty}\overline{\nu}_{eL}\left(\psi_R^{(n)} + \psi_R^{(-n)}\right) + h.c.$$

- To get canonical $\mathcal{L}_{mass}$ define

$$\nu_R^{(n)} = \frac{1}{\sqrt{2}}\left(\psi_R^{(n)} + \psi_R^{(-n)}\right) \qquad \psi_R^{(n)} = \frac{\nu_R^{(n)} + N_R^{(n)}}{\sqrt{2}}$$
$$\nu_L^{(n)} = \frac{1}{\sqrt{2}}\left(\psi_L^{(n)} - \psi_L^{(-n)}\right) \qquad \psi_R^{(-n)} = \frac{\nu_R^{(n)} - N_R^{(n)}}{\sqrt{2}}$$
$$N_R^{(n)} = \frac{1}{\sqrt{2}}\left(\psi_R^{(n)} - \psi_R^{(-n)}\right) \quad \rightarrow \quad \psi_L^{(n)} = \frac{\nu_L^{(n)} + N_L^{(n)}}{\sqrt{2}}$$
$$N_L^{(n)} = \frac{1}{\sqrt{2}}\left(\psi_L^{(n)} + \psi_L^{(-n)}\right) \qquad \psi_L^{(-n)} = \frac{-\nu_L^{(n)} + N_L^{(n)}}{\sqrt{2}}$$
$$\nu_R^{(0)} = \psi_R^{(0)}$$

- ... and get

$$\mathcal{L}_{mass} \ni m\,\overline{\nu}_{eL}\nu_R^{(0)}$$
$$+\sqrt{2}\,m\sum_{n=1}^{\infty}\overline{\nu}_{eL}\nu_R^{(n)}$$
$$+\sum_{n=1}^{\infty}\frac{M}{R}\Big(\overline{\nu}_L^{(n)}\nu_R^{(n)} + \underbrace{\overline{N}_L^{(n)}N_R^{(n)}}_{\text{decoupled (free)}}\Big) + h.c.$$

● FINALLY, PUT $\nu_L^{(0)} = \overline{\nu_{eL}}$ AND GET

$$-\mathcal{L}_{mass} \ni \left[\bar{\nu}_L^{(0)}\, \bar{\nu}_L^{(1)}\, \bar{\nu}_L^{(2)} \ldots \bar{\nu}_L^{(N)} \ldots\right] \left[M_{ij}\right] \begin{bmatrix} \nu_R^{(0)} \\ \nu_R^{(1)} \\ \vdots \end{bmatrix}$$

where

$$M = \begin{pmatrix} m & \sqrt{2}m & \sqrt{2}m & \sqrt{2}m & \sqrt{2}m & \cdots \\ 0 & 1/R & 0 & 0 & 0 & \cdots \\ 0 & 0 & 2/R & 0 & 0 & \cdots \\ 0 & 0 & 0 & 3/R & 0 & \cdots \\ 0 & 0 & 0 & 0 & 4/R & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

mass matrix

($MM^{\dagger}$ diagonalized in the context of ν oscillations — later)

RECAP

- NEUTRINO CURRENTS WELL UNDERSTOOD AND TESTED

- NEUTRINO NATURE (WEYL ? MAJORANA ? DIRAC ?) DIFFICULT TO EXPLORE IN PRACTICE, DUE TO CHIRALITY OF INTERACTIONS + SMALLNESS OF ν MASS. HOWEVER:

  $m_\nu \neq 0 \rightarrow$ NOT WEYL

  $\exists\, 0\nu 2\beta \rightarrow$ NOT DIRAC

- NEUTRINO MASS TERMS CAN BE MORE GENERAL THAN IN THE QUARK SECTOR, AND POINT TOWARDS NEW PHYSICS IN GENERAL
  - nonstandard Higgs sector
  - heavy ν_R scale M (see-saw)
  - new space time properties (extradim ...)

  ..... hope to explain smallness of m_ν

- MASS MATRIX DIAGONALIZATION IMPLIES THAT, IN GENERAL, MASSIVE STATES CAN BE SUPERPOSITION NOT ONLY OF ACTIVE FLAVOR STATES (AS FOR QUARKS) BUT ALSO OF STERILE STATES (NOT INTERACTING UNDER $SU(2)\otimes U(1)$).