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SUMMER SCHOOL ON PARTICLE PHYSICS

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NEUTRINO PHYSICS

Lecture II

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Lecture II

NEUTRINO OSCILLATIONS - THEORY -

ν oscillations :

GENERAL CONSEQUENCE OF MIXING OF FLAVOR STATES ν_α WITH MASSIVE STATES ν_i

$$\begin{array}{r} 3 \text{ active} \left\{ \vphantom{\begin{matrix} a\\b\\c \end{matrix}} \right. \\ \text{sterile } 0 \leq n_s \leq \infty \left\{ \vphantom{\begin{matrix} a\\b \end{matrix}} \right. \end{array} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \vdots \end{pmatrix} = U_{\alpha i} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix}$$

$$UU^+ = 1$$

Importance : MACROSCOPIC phenomenon

SMALLNESS OF ν MASS (WITH RESPECT TO DETECTABLE ν ENERGIES)

→ can ignore exceedingly small chirality flips during propagation

→ can use "Dirac-like" terminology:
"ν" $= \nu_L$, "$\bar{\nu}$" $= \nu_R$ ....

→ can often treat ν fields as "wavefunctions" (Q.M.-like notation)

Explore propagation hamiltonians $\mathcal{H}$ of increasing complexity

$$i \frac{\partial}{\partial t} \nu_\alpha = \mathcal{H} \nu_\alpha$$

$\mathcal{H}$ in flavor basis

3 massless ν in vacuum

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \mathcal{H}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \qquad m(\nu_\alpha)\equiv 0$$

FOR A BEAM OF MOMENTUM p:

$$\mathcal{H} = \begin{bmatrix}E_e & & \\ & E_\mu & \\ & & E_\tau\end{bmatrix} = \begin{bmatrix}p & & \\ & p & \\ & & p\end{bmatrix} = p\mathbb{1}$$

$$|\nu_\alpha\rangle_t = e^{-ipt}\,|\nu_\alpha\rangle_0$$

FLAVOR CONSERVED

NOTE: overall phase $\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \to e^{i\phi}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$ unobservable in squared amplitudes $|\langle\nu_\beta|\nu_\alpha\rangle|^2$ (true for more general $\mathcal{H}$ also)

$\rightarrow$ $\mathcal{H}$ defined modulo $\lambda\mathbb{1}$

arbitrary factor

3 massless ν in matter

$$i\frac{\partial}{\partial t}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \mathcal{H}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \qquad m(\nu_\alpha)\equiv 0$$

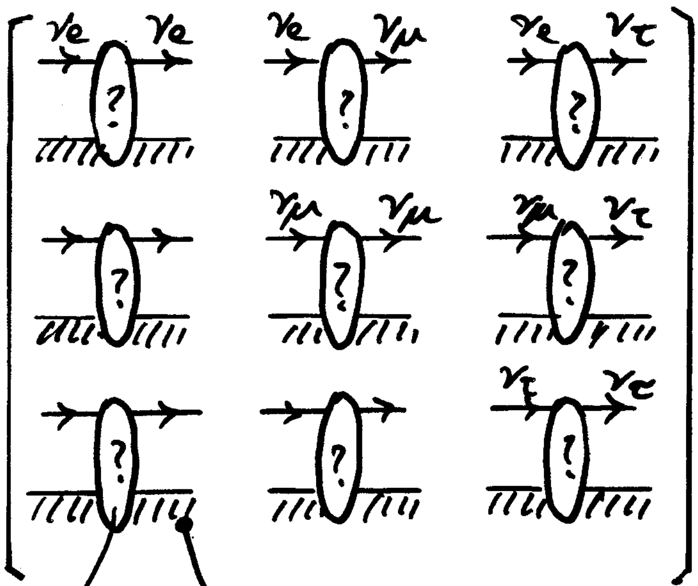
$$\mathcal{H} = \underset{\substack{\uparrow\\ \text{kinematics}}}{p\mathbb{1}} + \underset{\substack{\uparrow\\ \text{dynamics}}}{V}$$

V = interaction potential in matter (Wolfenstein)

symbolically:

$$p\mathbb{1} = \begin{bmatrix} \xrightarrow{\nu_e} & \emptyset & \emptyset \\ \emptyset & \xrightarrow{\nu_\mu} & \emptyset \\ \emptyset & \emptyset & \xrightarrow{\nu_\tau} \end{bmatrix} \leftarrow \text{free streaming}$$

V =

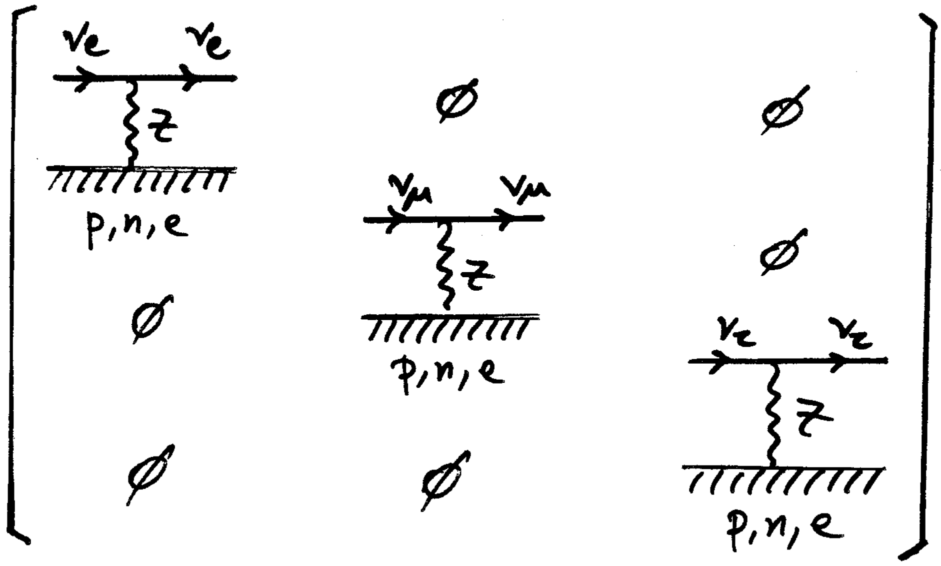


← dynamical contribution to forward scattering

interaction?   background matter

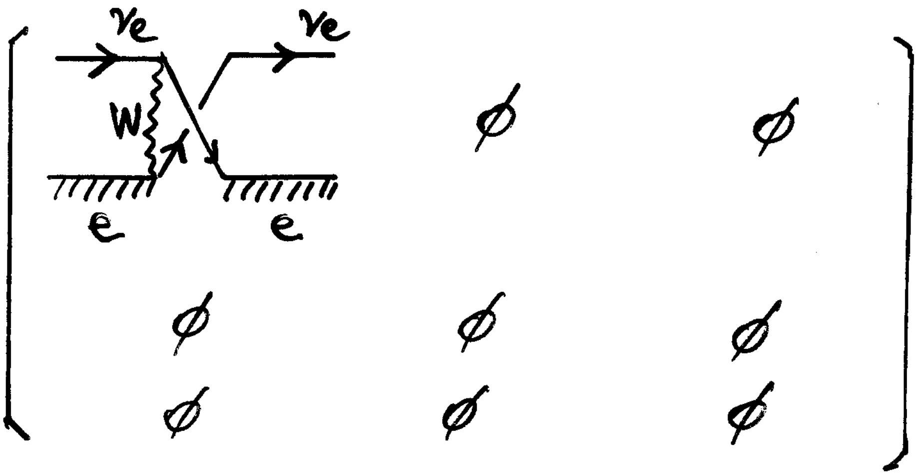
In the Standard Electroweak model, the "interaction blob"  is well-defined:

$V =$



← V_{NC}

$+$



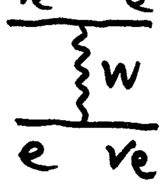
← V_{CC}

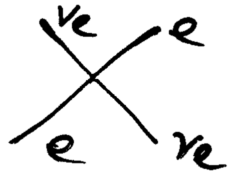
only for ν_e (in ordinary matter)

$$V = V_{NC} + V_{CC}$$

$V_{NC} \propto \mathbb{1}$ up to small higher-order corrections

→ Relevant potential is V_{ee}^{cc}

$V_{cc}^{ee} =$ 

$\simeq$ 

Evaluation of V_{CC}^{ee} :

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}} \underbrace{\bar{e}\gamma^\mu(1-\gamma_5)\nu_e}_{J^{CC}} \otimes \underbrace{\bar{\nu}_e\gamma_\mu(1-\gamma_5)e}_{J^{CC}}$$

$$\stackrel{\text{Fierz}}{=} \frac{G_F}{\sqrt{2}} \underbrace{\bar{e}\gamma^\mu(1-\gamma_5)e}_{J_e} \otimes \underbrace{\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e}_{J_\nu}$$

From the ν viewpoint, the electron is ~nonrelativistic and ~unpolarized

→ DIRAC REPRESENTATION

$$e \simeq \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \quad \bar{e}\gamma^\mu(1-\gamma_5)e \simeq (\underset{\text{density}}{\xi^+\xi}, \underset{\text{polariz.} \simeq 0}{\xi^+\vec{\sigma}\xi}) \simeq N_e\delta_{\mu 0}$$

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}} N_e \bar{\nu}_e\gamma_0(1-\gamma_5)\nu_e = \sqrt{2}\,G_F N_e\, \bar{\nu}_{eL}\gamma_0\nu_{eL}$$

$$\boxed{V_{CC}^{ee} = \sqrt{2}\,G_F N_e}$$

Units: $$2VE = \frac{2\sqrt{2}\,G_F N_e E}{[eV^2]} = 1.53 \times 10^{-4} \frac{N_e}{[mol/cm^3]} \frac{E}{[GeV]}$$

$$\mathcal{H} = \begin{pmatrix} p + V_{CC}^{ee} & & \\ & p & \\ & & p \end{pmatrix} \quad \text{mod } \mathbb{1}$$

→ no off-diagonal elements

→ flavor conserved (no $\nu_\alpha \to \nu_\beta$ with $\alpha \neq \beta$)

ν TYPE	Bkgd matter	Interaction potential V
ν_e	e	$\frac{1}{\sqrt{2}} G_F (4s_W^2 + 1)(N_e - N_{\bar{e}})$
$\nu_{\mu,\tau}$	e	$\frac{1}{\sqrt{2}} G_F (4s_W^2 - 1)(N_e - N_{\bar{e}})$
$\nu_{e,\mu,\tau}$	n	$\frac{1}{\sqrt{2}} G_F (N_{\bar{n}} - N_n)$
$\nu_{e,\mu,\tau}$	p	$\frac{1}{\sqrt{2}} G_F (1 - 4s_W^2)(N_p - N_{\bar{p}})$
ν_s	e,p,n	$\emptyset$

$\nu \to \bar{\nu}$ $\qquad\qquad$ $V \to -V$

In ordinary matter ($N_e = N_p$, $N_{\bar{e}} = N_{\bar{p}} = N_{\bar{n}} = 0$)

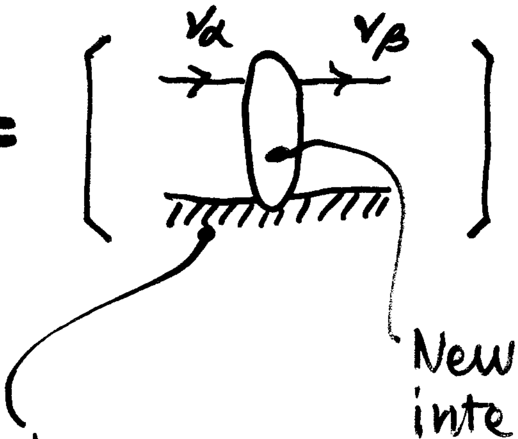
$$V_e - V_{\mu,\tau} = \sqrt{2} G_F N_e \quad \} \text{ as before}$$

$$V_\mu - V_\tau = \emptyset \quad \} \text{ vacuum-like}$$

$$\left.\begin{aligned} V_s - V_{\mu,\tau} &= \sqrt{2} G_F \frac{N_n}{2} \\ V_e - V_s &= \sqrt{2} G_F \left(N_e - \tfrac{1}{2} N_n\right) \end{aligned}\right\} \text{ important for } \nu_s \text{ phenomenology}$$

But Beyond the SM one can have

$V_{\alpha\beta} =$



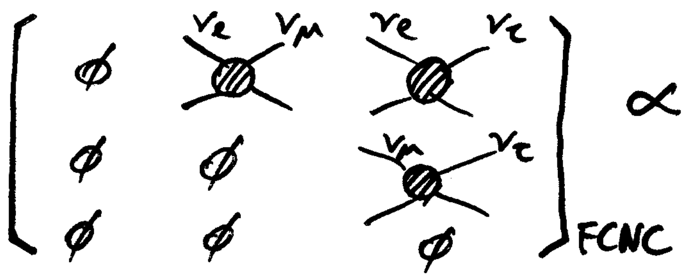
with $V_{\alpha\beta} \neq 0$ for $\alpha \neq \beta$!

New interaction

Background field (matter, "spacetime", gravity ...)

E.g., in SUSY with R-parity breaking can get FCNC $\neq 0$:

$V \ni$



$\propto \epsilon_{\alpha\beta} G_F N_f$

↑ small

Or, if Equivalence Principle violated for ν, can get $V_\mu - V_\tau \neq 0$

Case of FCNC interesting : → $\mathcal{H}$ nondiagonal even for massless ν

→ can get flavor transitions without ν mass (unlikely but possible in principle)

3 massive ν in vacuum (unmixed case)

Assume $\quad m(\nu_\alpha) = \delta_{\alpha i} \cdot m_i$

$$E_i = \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

(ultrarelativistic neutrinos, $x \simeq t$)

$$\mathcal{H} = \begin{bmatrix} E_1 & & \\ & E_2 & \\ & & E_3 \end{bmatrix} \simeq \begin{bmatrix} p & & \\ & p & \\ & & p \end{bmatrix} + \begin{bmatrix} \frac{m_1^2}{2E} & & \\ & \frac{m_2^2}{2E} & \\ & & \frac{m_3^2}{2E} \end{bmatrix}$$

$$= p\mathbb{1} + \frac{\mathcal{M}^2}{2E}$$

$$\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

- $\mathcal{H}$ diagonal
- $\rightarrow$ no flavor transitions

3 massive & mixed ν in vacuum

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$\nu_\alpha = U_{\alpha i}\nu_i \quad ; \quad m(\nu_i) = m_i$$
$$UU^+ = 1$$

- HAMILTONIAN DIAGONAL IN MASS BASIS:

$$\mathcal{H}_{mass} = \frac{\mathcal{M}^2}{2p} + p\mathbb{1}$$

$$\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$$

- TRANSFORM TO FLAVOR BASIS:

$$\mathcal{H} = U\frac{\mathcal{M}^2}{2E}U^+ + p\mathbb{1}$$

- IF NO $\cancel{CP}$, U real and usual parametriz. is:

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_\psi & s_\psi\\ 0 & -s_\psi & c_\psi\end{pmatrix}\begin{pmatrix}c_\varphi & 0 & s_\varphi\\ 0 & 1 & 0\\ -s_\varphi & 0 & c_\varphi\end{pmatrix}\begin{pmatrix}c_\omega & s_\omega & 0\\ -s_\omega & c_\omega & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\psi = \theta_{23} \qquad \varphi = \theta_{13} \qquad \omega = \theta_{12} \qquad \in [0, \pi/2]$$

$$= \begin{pmatrix} c_\omega c_\varphi & s_\omega c_\varphi & s_\varphi\\ -s_\omega c_\psi - c_\omega s_\psi s_\varphi & c_\omega c_\psi - s_\omega s_\psi s_\varphi & s_\psi c_\varphi\\ s_\omega s_\psi - c_\omega c_\psi s_\varphi & -c_\omega s_\psi - s_\omega c_\psi s_\varphi & c_\psi c_\varphi\end{pmatrix}$$

(same θ_{ij} ordering as for quarks)

- IF $\cancel{CP}$ AND MASS/mixing originate from Dirac mass terms:

$$U = \begin{pmatrix} c_\omega c_\varphi & s_\omega c_\varphi & \tilde{s}_\varphi^* \\ -s_\omega c_\psi - c_\omega s_\psi \tilde{s}_\varphi & c_\omega c_\psi - s_\omega s_\psi \tilde{s}_\varphi & s_\psi c_\varphi \\ s_\omega s_\psi - c_\omega c_\psi \tilde{s}_\varphi & -c_\omega s_\psi - s_\omega c_\psi \tilde{s}_\varphi & c_\psi c_\varphi \end{pmatrix}$$

where $\tilde{s}_\varphi = s_\varphi e^{i\delta}$ $(0 \le \delta \le 2\pi)$

1 $\cancel{CP}$ phase as for quarks

If mass/mixing originates from Majorana mass terms:

$$U \to U \cdot V \quad , \quad V = \begin{pmatrix} 1 & & \\ & e^{i\phi_2} & \\ & & e^{i(\phi_3 - \delta)} \end{pmatrix}$$

TWO NEW PHASES ϕ_2, ϕ_3

... but no effect on oscillations:

$$UV \frac{\mathcal{M}^2}{2E} (UV)^+ = U \left(V \frac{\mathcal{M}^2}{2E} V^+ \right) U^+ = U \frac{\mathcal{M}^2}{2E} U^+$$

→ **NOT POSSIBLE TO DISTINGUISH DIRAC/MAJOR. IN ν OSCILLATIONS**

HOWEVER, $\phi_{2,3}$ may show up in $0\nu 2\beta$

2ν oscillations in vacuum

- Take only one mixing angle $\theta \neq 0$; e.g.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \Delta m^2 = m_2^2 - m_1^2$$

- $$\mathcal{H} = \frac{1}{4E} \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} -\Delta m^2 & \\ & +\Delta m^2 \end{pmatrix} \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}$$

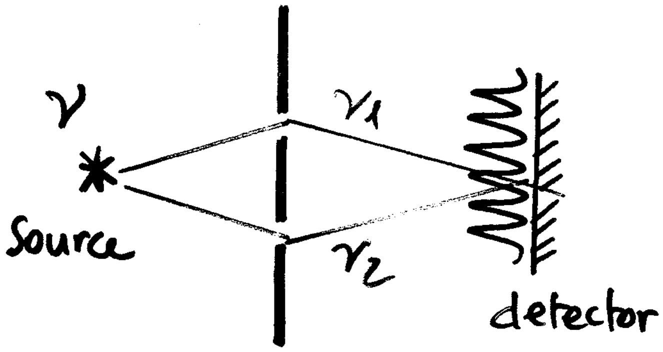
use "mod $\mathbb{1}$" to make $\mathrm{tr}(\mathcal{H}) = 0$

- Evolution:

$$A(\nu_e \to \nu_\mu) = \underbrace{(0 \;\; 1)}_{\text{final } \nu_\mu} \underbrace{\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}}_{\text{back to flavor basis}} \underbrace{\begin{pmatrix} e^{i\frac{\Delta m^2}{4E}x} & 0 \\ 0 & e^{-i\frac{\Delta m^2}{4E}x} \end{pmatrix}}_{\text{evolve in mass basis}} \underbrace{\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}}_{\text{rotate to mass basis}} \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{\text{initial } \nu_e}$$

- $$\boxed{P(\nu_e \to \nu_\mu) = |A(\nu_e \to \nu_\mu)|^2 = \sin^2 2\theta \, \sin^2\left(\underbrace{\frac{\Delta m^2 L}{4E}}_{1.27 \frac{\Delta m^2}{\mathrm{eV}^2} \frac{L}{\mathrm{km}} \frac{\mathrm{GeV}}{E}}\right)}$$

"TWO-SLIT" EXPERIMENT



length scales:

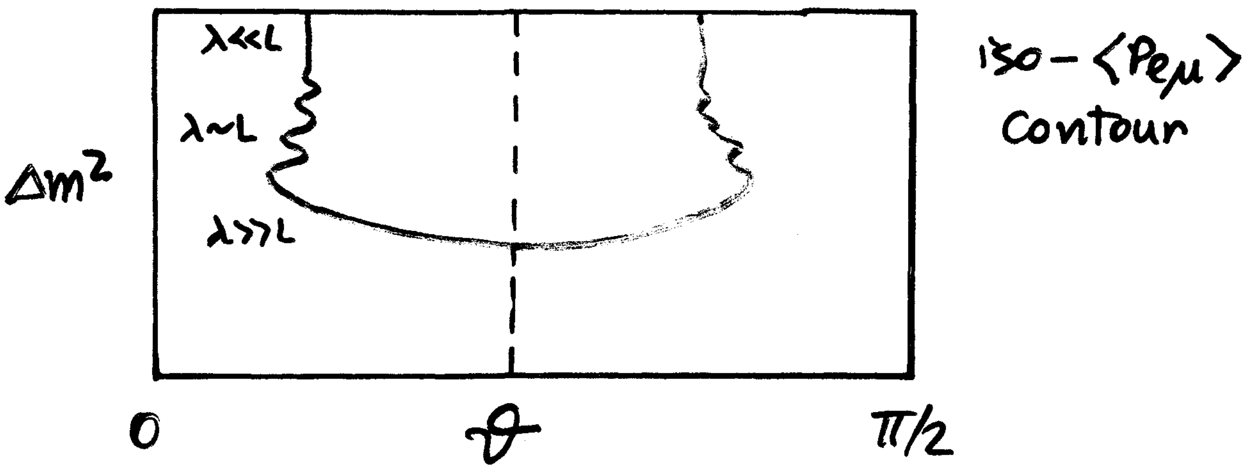
L

$$\lambda = \frac{4\pi E}{\Delta m^2} \quad \text{(osc. length)}$$

observed :

$$\langle P_{e\mu} \rangle = \sin^2 2\theta \left\langle \sin^2\left(\frac{\Delta m^2 L}{4E}\right)\right\rangle$$ "smearing"

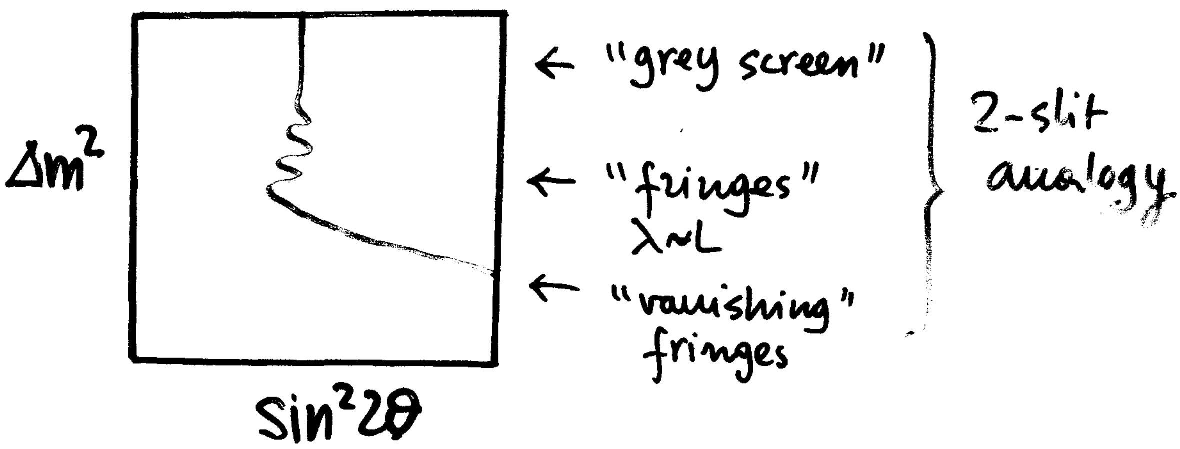
$$\rightarrow \frac{1}{2}\sin^2 2\theta \quad \text{for } \Delta m^2 \rightarrow \infty$$



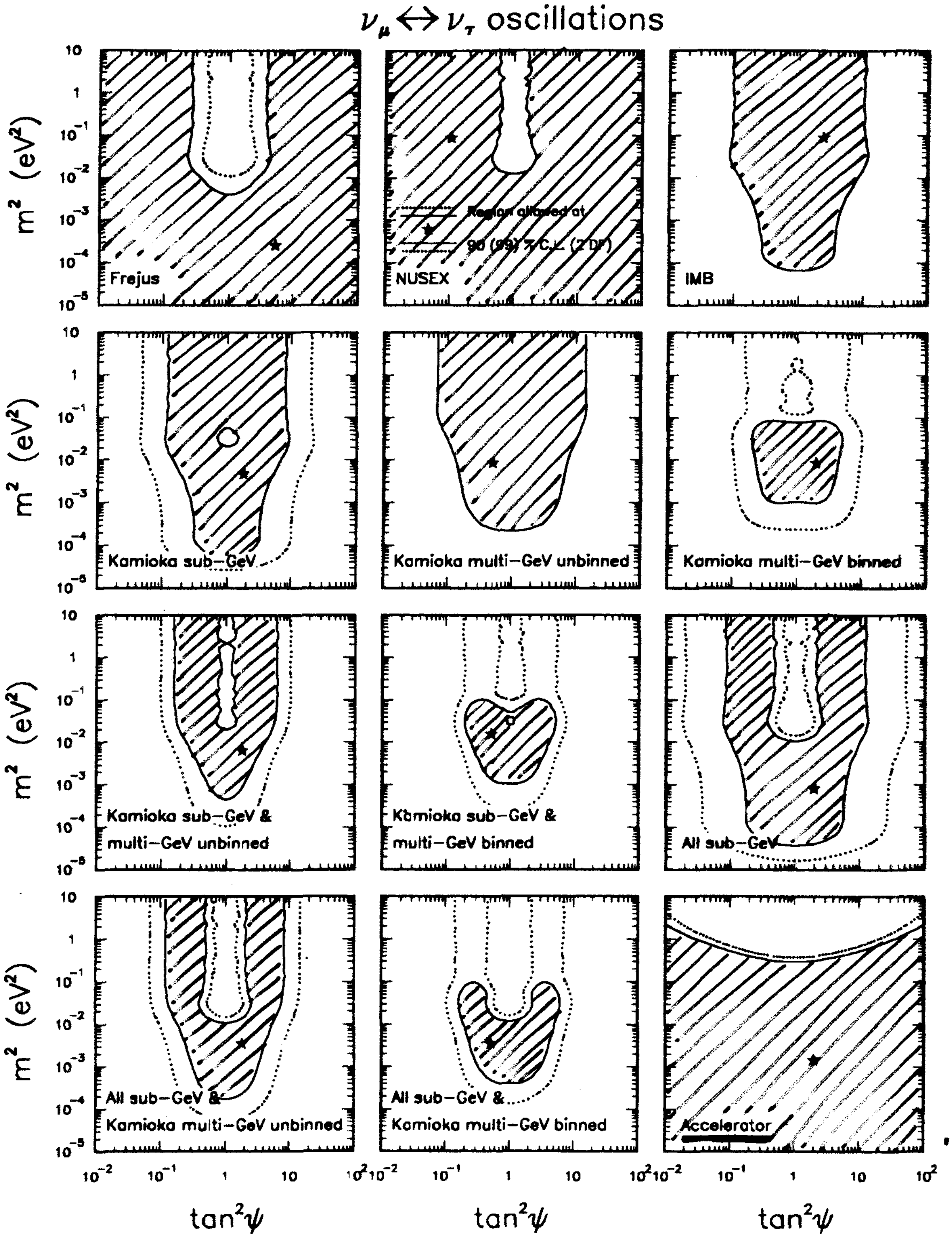
octant symmetry :

$$P_{e\mu}(\theta) = P_{e\mu}(\pi/2 - \theta)$$

→ FOLD 2nd octant onto first to get usual plot



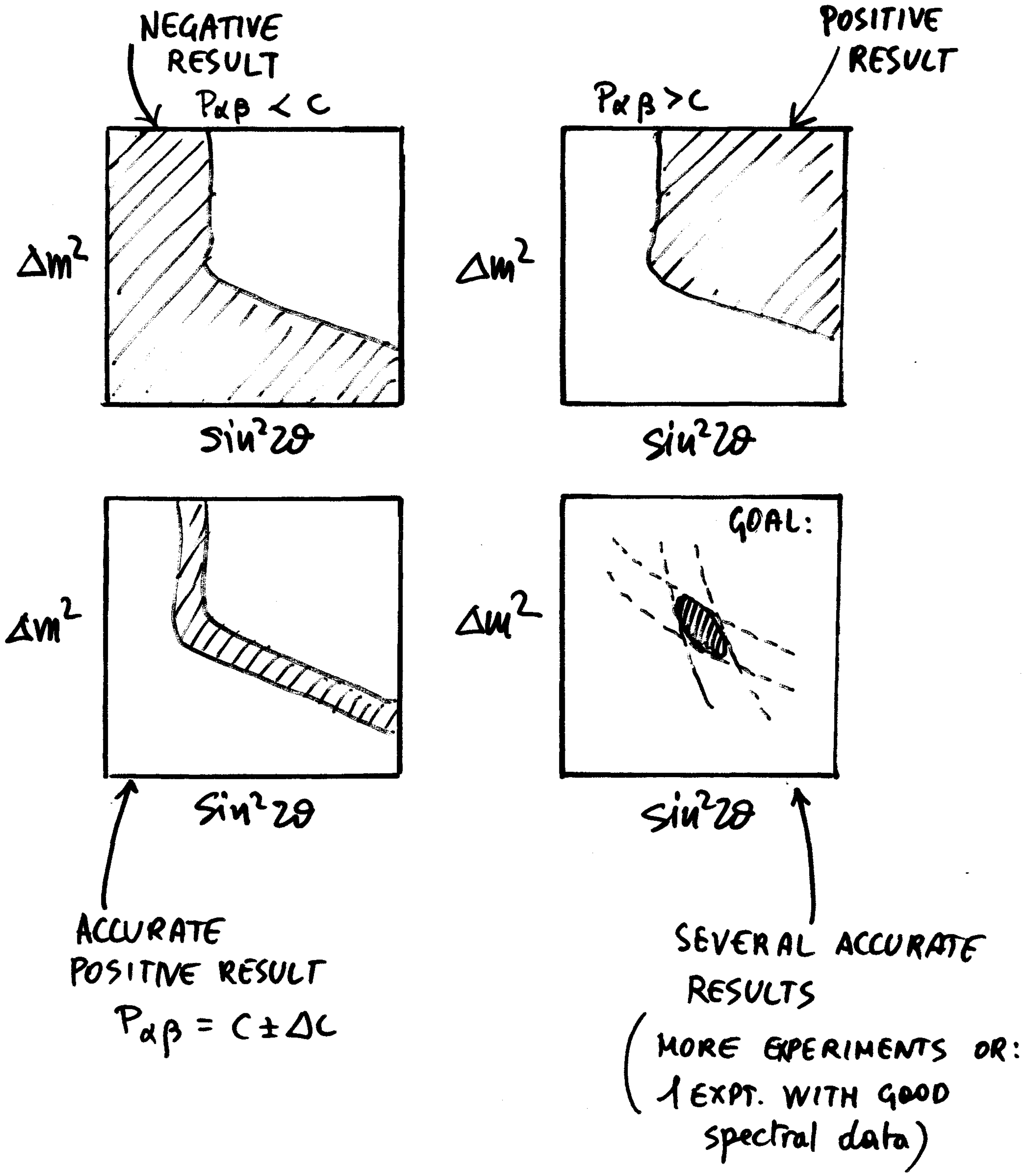
$\nu_\mu \leftrightarrow \nu_\tau$

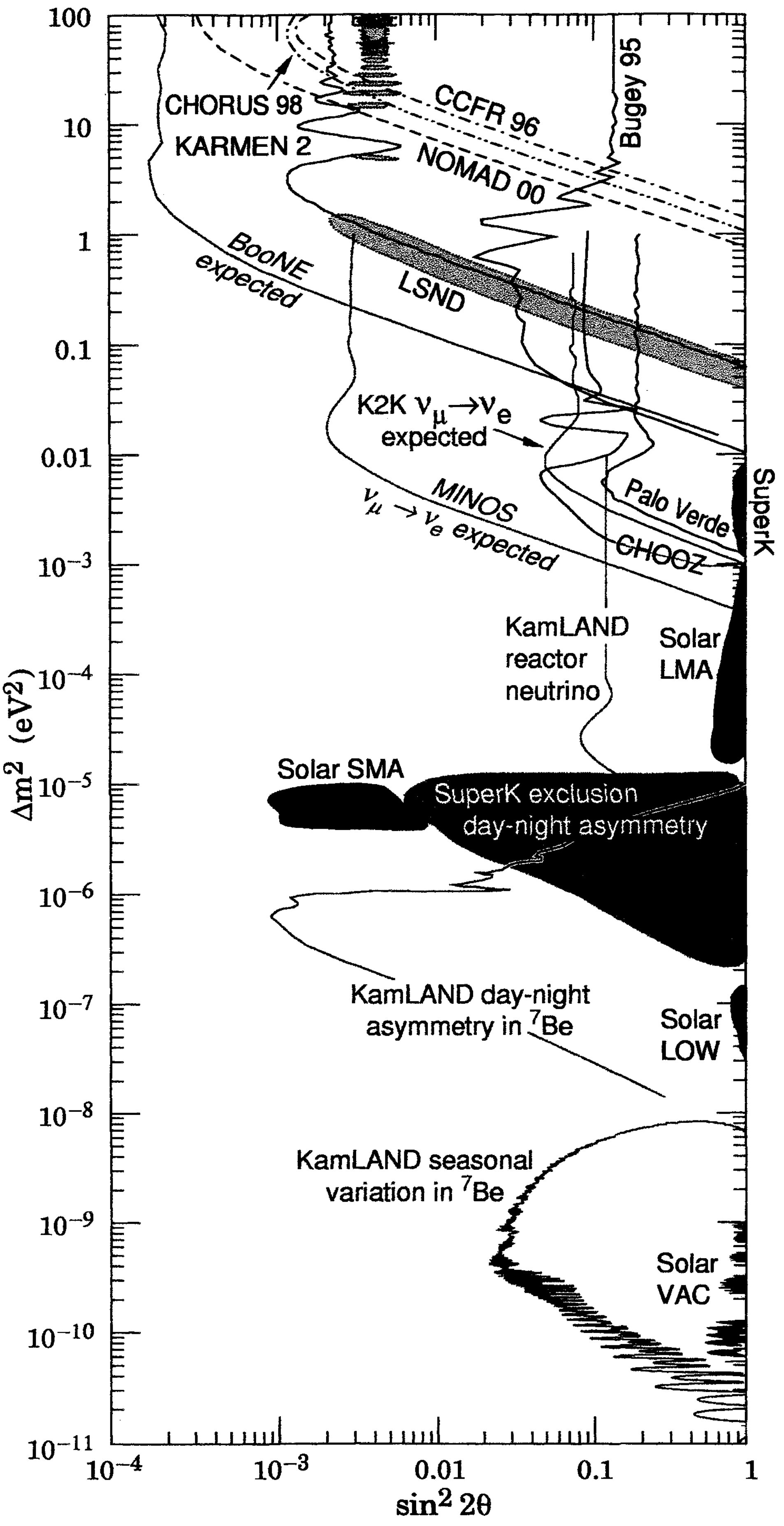


Example of octant symmetry :
pre-SK analysis of $\nu_\mu \leftrightarrow \nu_\tau$ atm. ν
(~1995)

TYPICAL EXPT. RESULTS

= ALLOWED





Review of Particle Properties (2000)

2 massive ν in constant matter

$$\mathcal{H} = \frac{1}{2E}\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$$

$V \neq 0$ for (ν_e, ν_μ), (ν_e, ν_s), (ν_μ, ν_s)

(ν_μ, ν_e) case: $V_e - V_\mu = \sqrt{2} G_F N_e$

$$\mathcal{H} = \frac{\Delta m^2}{4E}\begin{bmatrix} -c_{2\theta} + \frac{A}{\Delta m^2} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} - \frac{A}{\Delta m^2} \end{bmatrix} \quad \text{mod } \mathbb{1}$$

$A = 2\sqrt{2} G_F N_e E$ $\quad (A \to -A$ for $\bar{\nu})$

DIAGONALIZATION:

"Δm^2" in matter $\quad$ "mixing" in matter

$$\mathcal{H} = \frac{1}{4E}\begin{pmatrix} c_{\theta_m} & s_{\theta_m} \\ -s_{\theta_m} & c_{\theta_m} \end{pmatrix}\begin{pmatrix} -\Delta m_m^2 & \\ & +\Delta m_m^2 \end{pmatrix}\begin{pmatrix} c_{\theta_m} & -s_{\theta_m} \\ s_{\theta_m} & c_{\theta_m} \end{pmatrix}$$

ν_{1m}
ν_{2m}
mass eigenstates "in matter"

$$\rightarrow P(\nu_e \to \nu_\mu) = \sin^2 2\theta_m \sin^2\left(\frac{\Delta m_m^2 L}{4E}\right)$$

formally equivalent to vacuum, but...

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(c_{2\theta} - \frac{A}{\Delta m^2}\right)^2 + s_{2\theta}^2}}$$ ← "Breit-Wigner" form

$$\cos 2\theta_m = \frac{\cos 2\theta - A/\Delta m^2}{\sqrt{\left(c_{2\theta} - \frac{A}{\Delta m^2}\right) + s_{2\theta}^2}}$$

$$\Delta m_m^2 = \Delta m^2 \frac{\sin 2\theta}{\sin 2\theta_m}$$

→ can get a MSW (Mikheyev-Smirnov Wolfenstein) resonant behaviour for

$$c_{2\theta} \sim \frac{A}{\Delta m^2} \iff \Delta m^2 \cos 2\theta = 2\sqrt{2}\, G_F N_e E$$

$$\to \sin^2 2\theta_m \sim 1 \quad \text{(enhancement)}$$

$$\to \Delta m_m^2 \text{ minimized}$$

→ can get matter-suppressed oscillations for

$$A \gg \Delta m^2 \to \sin 2\theta_m \sim 0$$

MATTER CAN PROFOUNDLY MODIFY OSCILLATION AMPLITUDE (ENHANCEMEN/ SUPPRESSION) AND ENERGY DEPENDENCE

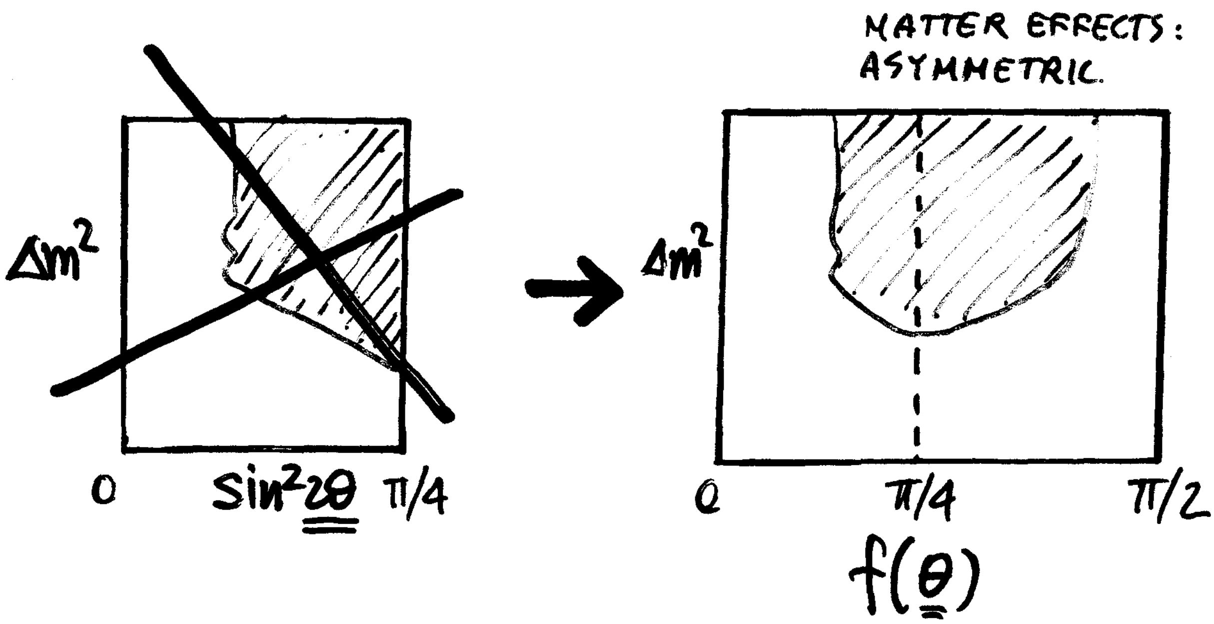
- NEW LENGTH SCALE $\lambda_0 = \frac{\sqrt{2}\pi}{G_F N_e}$
- IMPORTANT EFFECTS WHEN $\lambda \sim \lambda_0$

MATTER EFFECTS <u>NOT</u> OCTANT-SYMMETRIC!

$$X(\theta) \neq X(\pi/2 - \theta)$$

where $X = \Delta m^2_m, \theta_m, P_{e\mu}$

→ MUST UNFOLD 2nd OCTANT

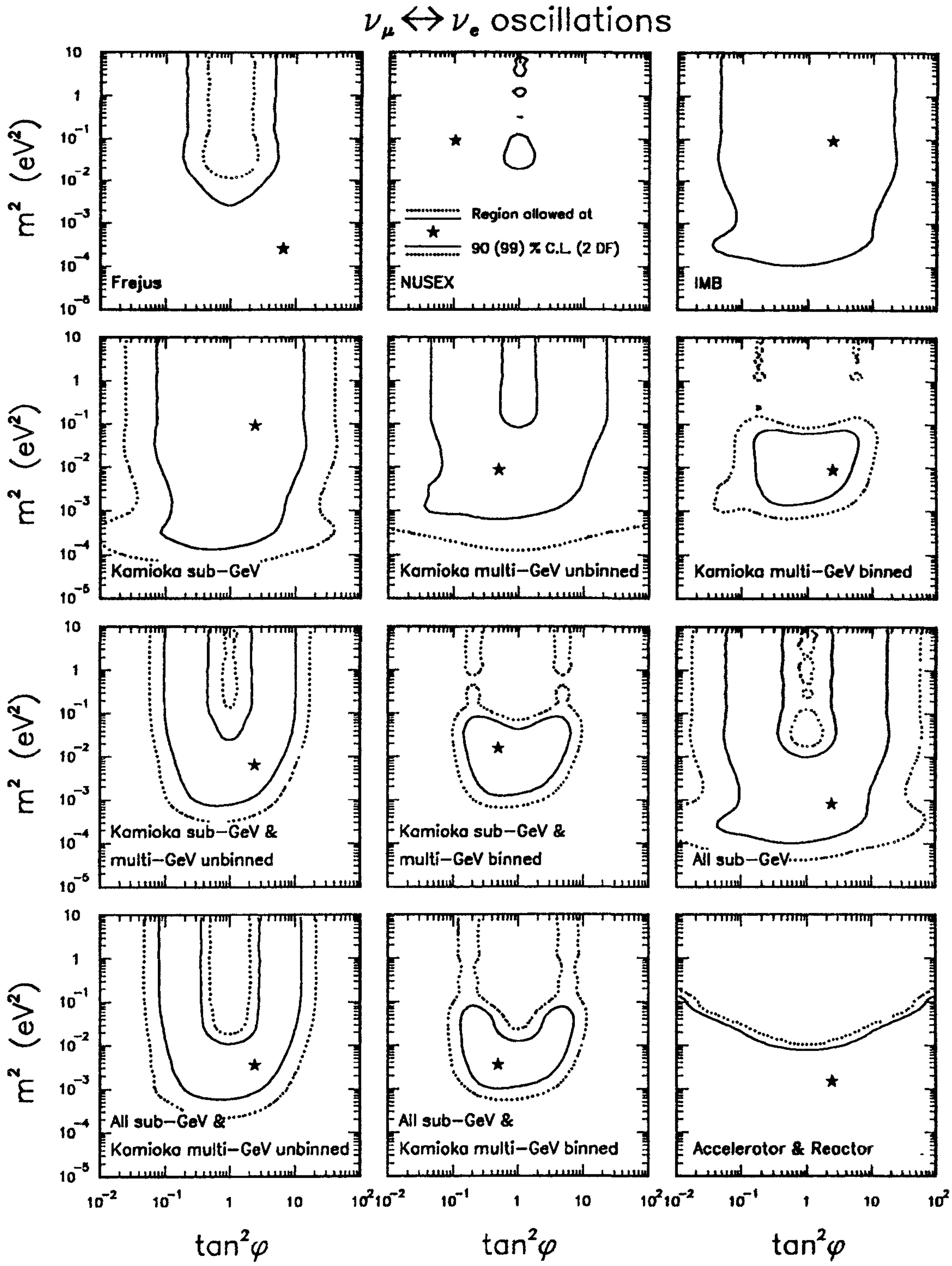


useful $f(\theta)$: $\sin^2\theta$ (LINEAR SCALE)

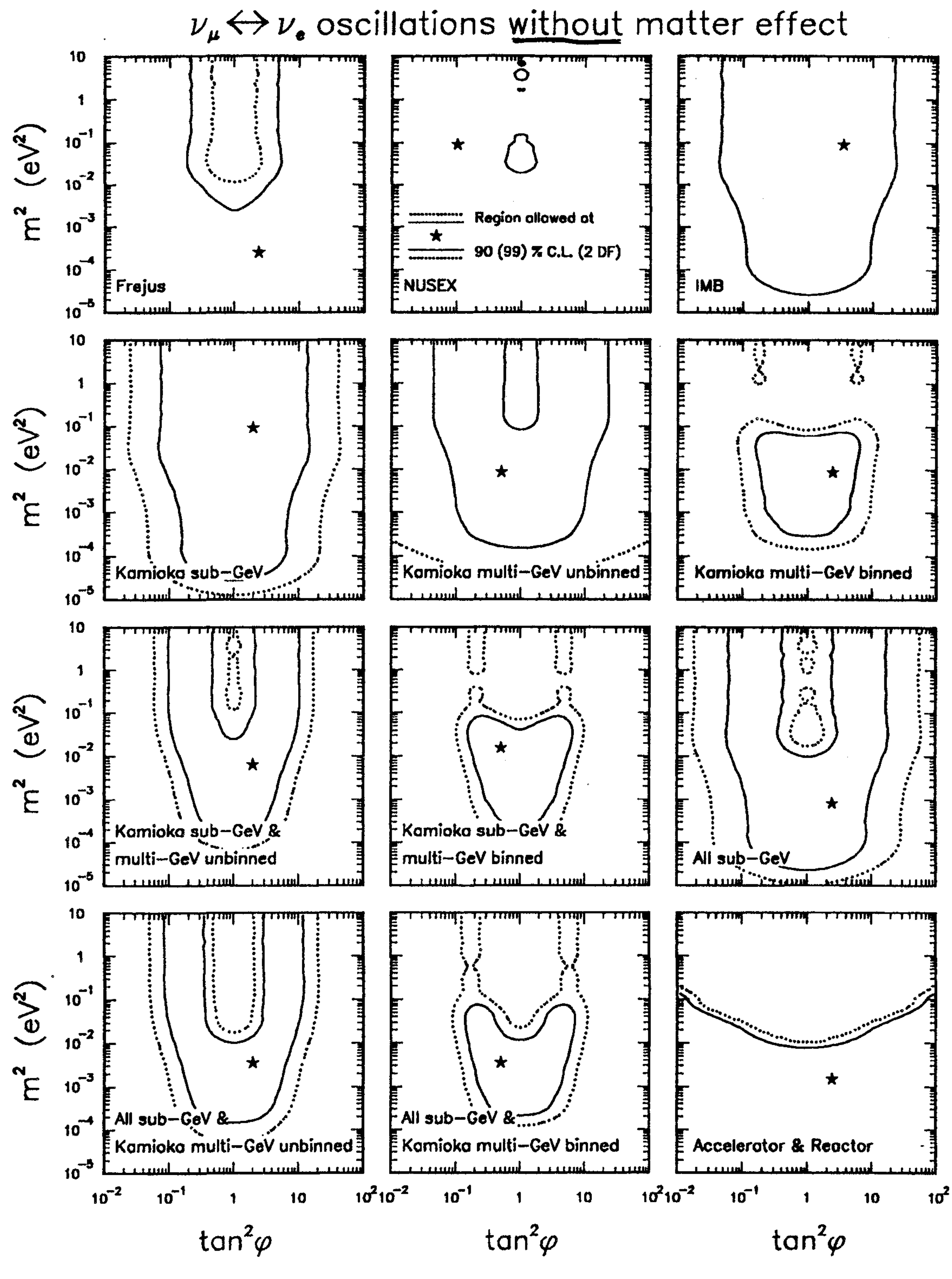
$\tan^2\theta$ (LOG SCALE)

↓

preserve octant symmetry when applicable



$\nu_\mu \to \nu_e$ asym. with matter effects

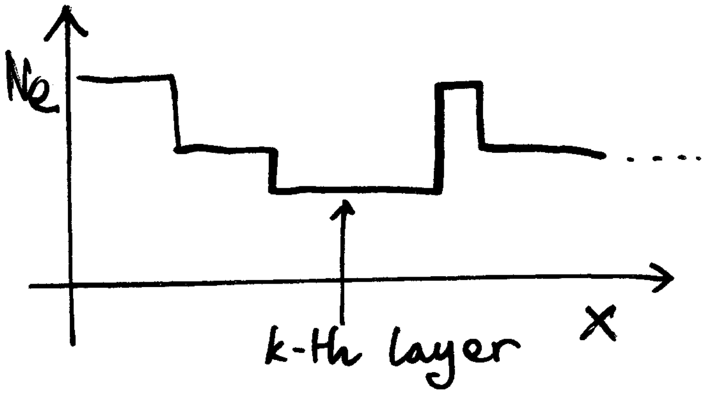


$\nu_\mu \leftrightarrow \nu_e$ oscillations without matter effect
m² (eV²)
tan²φ
Frejus
NUSEX
Region allowed at
90 (99) % C.L. (2 DF)
IMB
Kamioka sub-GeV
Kamioka multi-GeV unbinned
Kamioka multi-GeV binned
Kamioka sub-GeV & multi-GeV unbinned
Kamioka sub-GeV & multi-GeV binned
All sub-GeV
All sub-GeV & Kamioka multi-GeV unbinned
All sub-GeV & Kamioka multi-GeV binned
Accelerator & Reactor

2ν in layered matter

[e.g., Earth (mantle + core)]

Assume step-like N_e



convenient to work in flavor basis
(flavor conserved across boundary)

SINGLE LAYER EVOLUTION $i \to f$ (already seen before)

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_f = \underbrace{\begin{pmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{pmatrix}}_{\text{mixing in matter}} \underbrace{\begin{pmatrix} e^{i\phi_k} & \\ & e^{-i\phi_k} \end{pmatrix}}_{\text{phases in matter}} \begin{pmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_i$$

$$= \begin{pmatrix} A^{(k)}_{\alpha\alpha} & A^{(k)}_{\alpha\beta} \\ A^{(k)}_{\beta\alpha} & A^{(k)}_{\beta\beta} \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_i$$

$$A^{(k)}_{\alpha\alpha} = \cos\phi_k + i\sin\phi_k \cos 2\theta_k$$
$$A^{(k)}_{\alpha\beta} = A^{(k)}_{\beta\alpha} = -i\sin\phi_k \sin 2\theta_k$$
$$A^{(k)}_{\beta\beta} = \cos\phi_k - i\sin\phi_k \cos 2\theta_k$$

MAXIMAL CONVERSION $\alpha \to \beta \Leftrightarrow A^{(k)}_{\alpha\alpha} = 0$

$$\to \begin{cases} \cos\phi_k = 0 \\ \cos 2\theta_k = 0 \end{cases}$$

$\leftarrow$ usual MSW resonance condition

TWO-LAYER EVOLUTION $i \to f$:

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_f = \underbrace{\begin{pmatrix} A^{(2)}_{\alpha\alpha} & A^{(2)}_{\alpha\beta} \\ A^{(2)}_{\beta\alpha} & A^{(2)}_{\beta\beta} \end{pmatrix}}_{\text{2nd layer}} \underbrace{\begin{pmatrix} A^{(1)}_{\alpha\alpha} & A^{(1)}_{\alpha\beta} \\ A^{(1)}_{\beta\alpha} & A^{(1)}_{\beta\beta} \end{pmatrix}}_{\text{1st layer}} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_i$$

$$= \begin{pmatrix} A_{\alpha\alpha} & A_{\alpha\beta} \\ A_{\beta\alpha} & A_{\beta\beta} \end{pmatrix} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}_i$$

$$A_{\alpha\alpha} = A^{(2)}_{\alpha\alpha} A^{(1)}_{\alpha\alpha} + A^{(2)}_{\alpha\beta} A^{(1)}_{\beta\alpha}$$

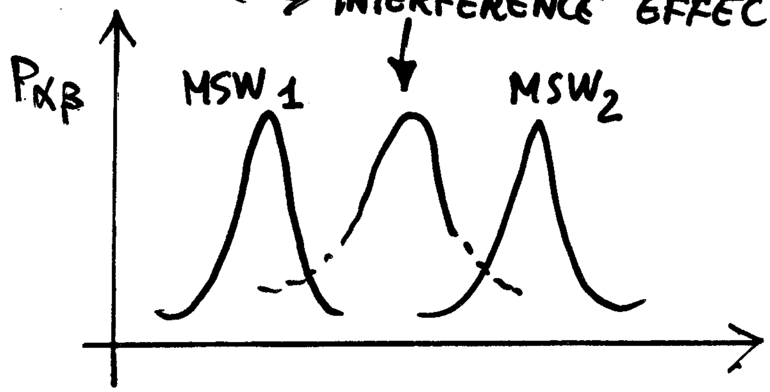
$$= \cos\varphi_1 \cos\varphi_2 - \cos(2\theta_2 - 2\theta_1)\sin\varphi_1 \sin\varphi_2$$
$$- i(\cos 2\theta_1 \sin\varphi_1 \cos\varphi_2 + \cos 2\theta_2 \cos\varphi_1 \sin\varphi_2)$$

Maximal conversion $\quad A_{\alpha\alpha} = 0 \to \begin{cases} \tan\theta_1 \tan\theta_2 \cos(2\theta_2 - 2\theta_1) = 1 \\ \tan\theta_1 \cos 2\theta_1 = -\tan\theta_2 \cos 2\theta_2 \end{cases}$

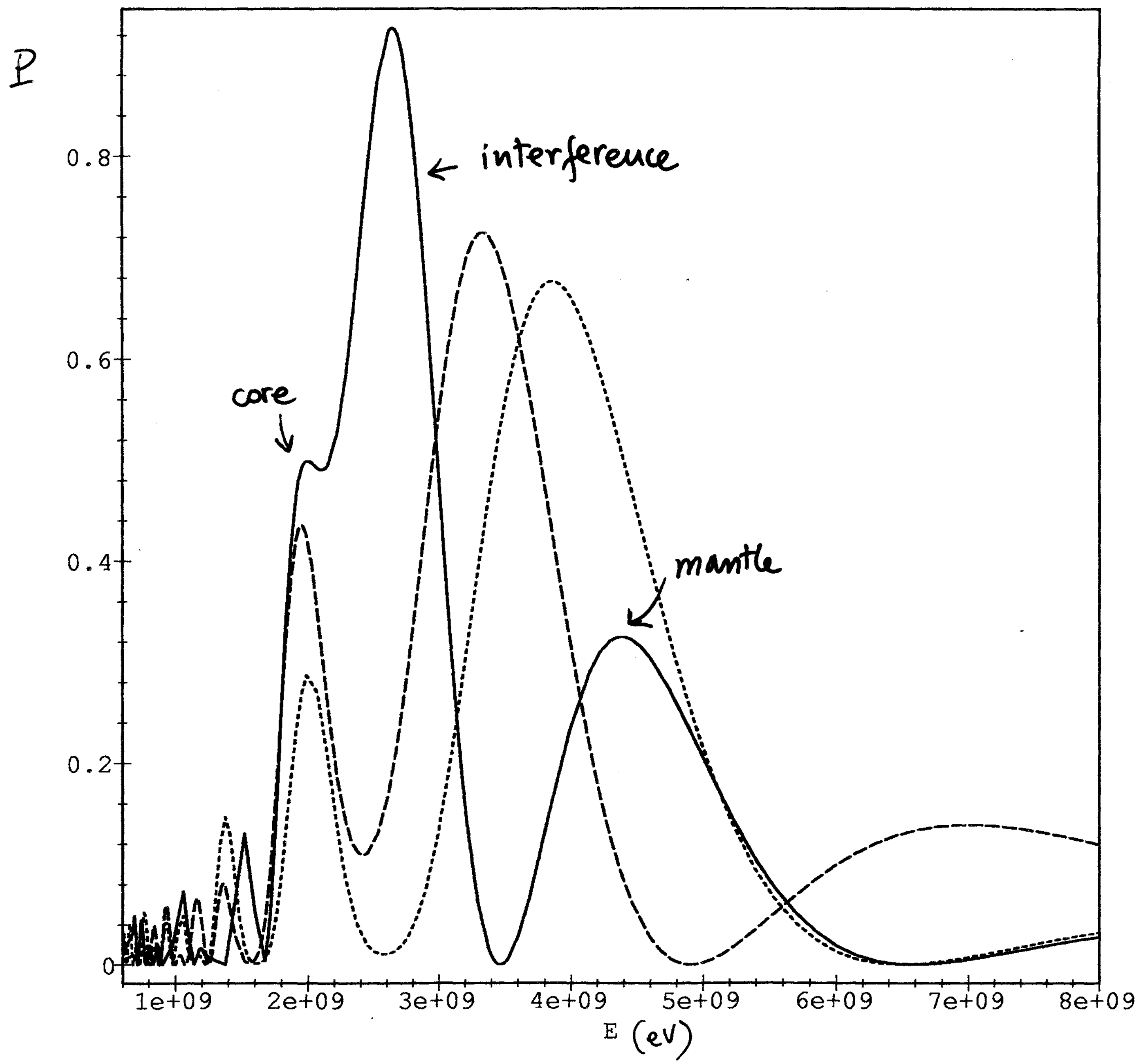
$\uparrow$

$\neq$ from usual MSW resonance

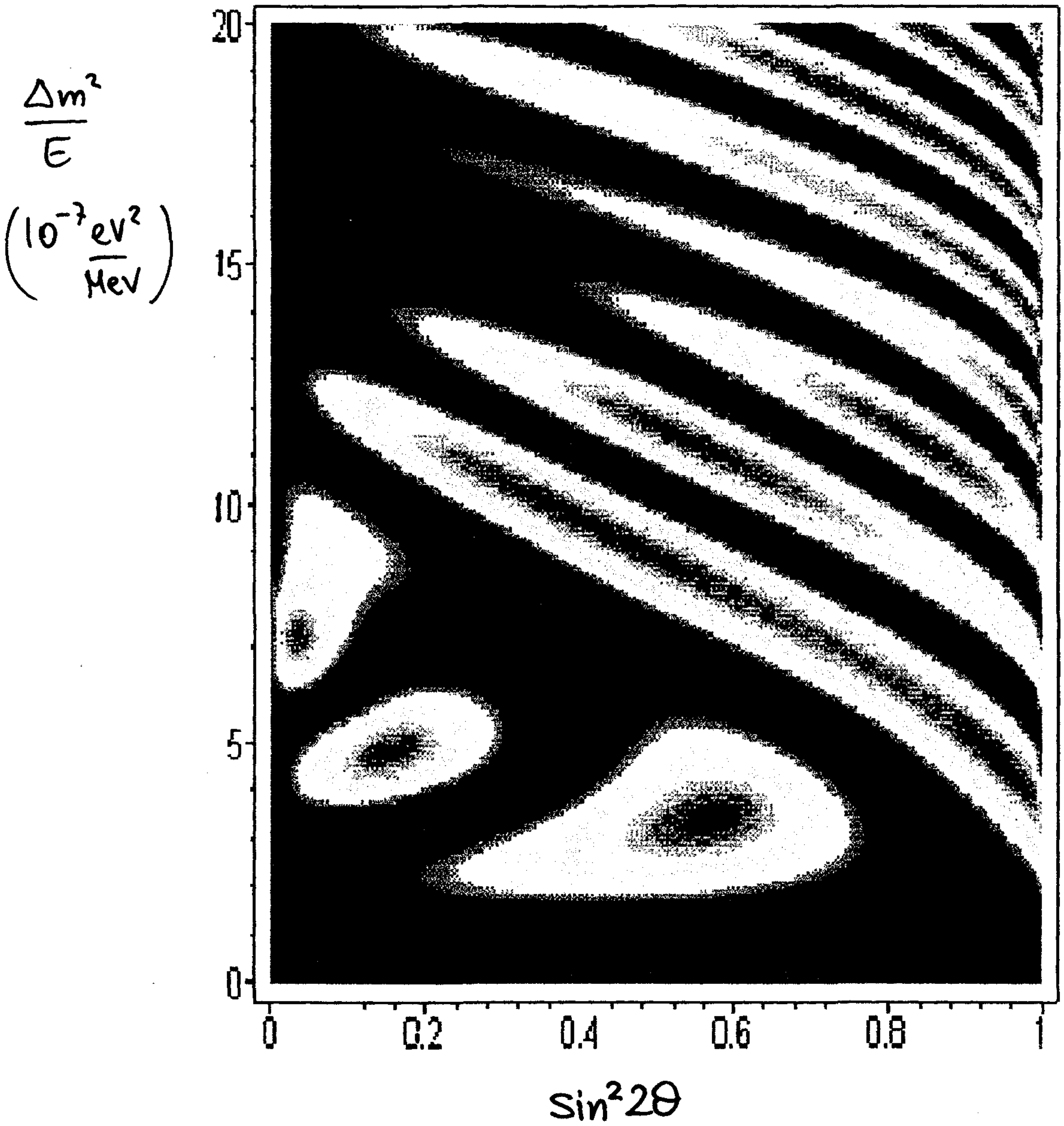
$\to$ INTERFERENCE EFFECT



("TRIPLET" OF PEAKS)


P
interference
core
mantle
0.8
0.6
0.4
0.2
0
1e+09
2e+09
3e+09
4e+09
5e+09
6e+09
7e+09
8e+09
E (eV)

$P_{\mu e}$ contours ($P_{\mu e} \sim 1$ ← red)



Earth Diameter crossing (mantle + Core + mantle)

· Possibly rich phenomenology

Unfortunately ~ below present expt. sensitivity

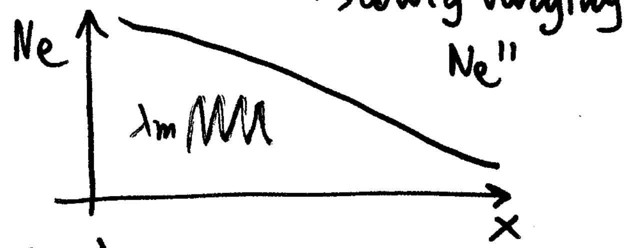
Chizhov & Petcov

2ν oscillations in variable matter density

REQUIRES, IN GENERAL, NUMERICAL SOLUTIONS.

Analytical approx. possible in several cases of phenomenological interest.

- **ADIABATIC EVOL.**



At each point,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_m(x) & \sin\theta_m(x) \\ -\sin\theta_m(x) & \cos\theta_m(x) \end{pmatrix} \begin{pmatrix} \nu_{1m}(x) \\ \nu_{2m}(x) \end{pmatrix}$$

with $P(\nu_{1m} \to \nu_{2m}) \simeq 0$ ("no crossing")

Typically, $\lambda_m \ll L \to$ phase info. lost

$\to$ can propagate "probabilities" rather than amplitudes

$$P(\nu_e \to \nu_e) = \underset{\text{final } \nu_e}{\underbrace{(1\ \ 0)}} \underset{\text{rotate back}}{\underbrace{\begin{pmatrix} \cos^2\theta_f & \sin^2\theta_f \\ \sin^2\theta_f & \cos^2\theta_f \end{pmatrix}}} \underset{\text{no crossing}}{\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}} \underset{\text{rotate to } \nu^m_{1,2}}{\underbrace{\begin{pmatrix} \cos^2\theta_i & \sin^2\theta_i \\ \sin^2\theta_i & \cos^2\theta_i \end{pmatrix}}} \underset{\text{initial } \nu_e}{\underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}}$$

$$= \frac{1}{2}\left(1 + \cos 2\theta_i \cos 2\theta_f\right)$$

If initial N_i large, then $\cos 2\theta_i \sim -1$

and $P_{ee} \simeq \frac{1}{2}\left(1 - \cos 2\theta_f\right) \ll 1$ if θ_f small

CORRECTIONS TO ADIABATICITY:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1-P_c & P_c \\ P_c & 1-P_c \end{pmatrix}$$

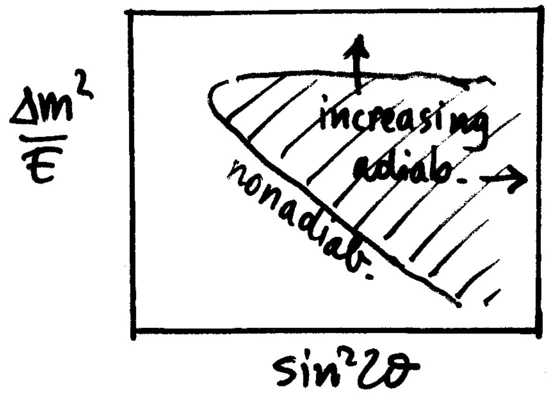
crossing $\nu_{1m} \to \nu_{2m}$ ("tunnelling")

$$P_{ee} = \frac{1}{2} + \left(\frac{1}{2} - P_c\right) \cos 2\vartheta_i \cos 2\vartheta_f$$

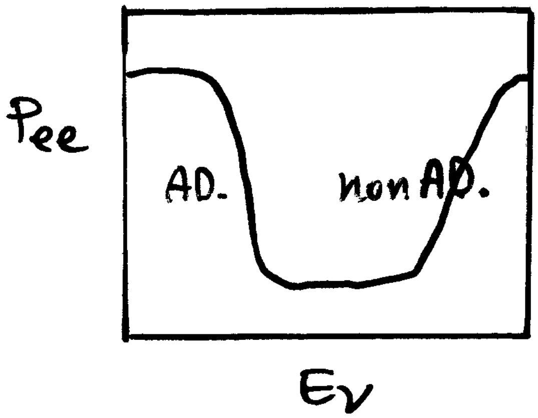
enormous literature on P_c

IN THE SUN, SOLAR MATTER CAN SUPPRESS P_{ee} THROUGH AD. / NON-AD. TRANSITIONS:

$P_{ee} \ll 1$

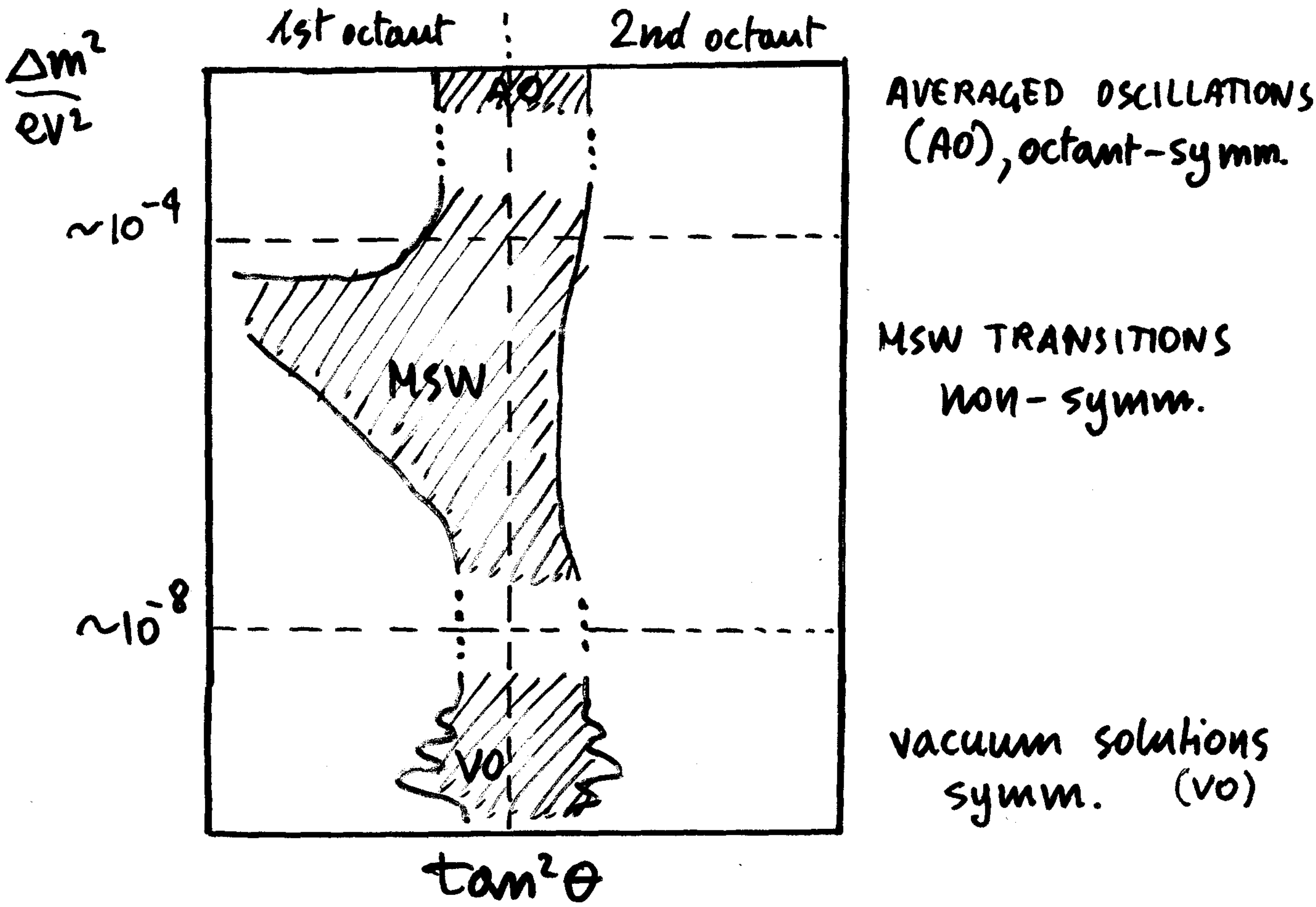


"MSW" triangle = zone of small P_{ee}



← strong differences from vacuum case

Solar ν



Recent work ("relatively" recent in some cases)

- "HORIZONTAL" IMPROVEMENTS: ENSURE SMOOTH PASSAGE FROM 1st to 2nd OCTANT AT ANY Δm^2 (max. violation of adiabaticity, etc.)
- "VERTICAL" IMPROVEMENTS: ENSURE SMOOTH PASSAGE FROM VO TO MSW (quasi-vacuum oscill.) AND FROM MSW TO AO (quasi-averaged oscill.)

→ PHYSICS VERY WELL UNDERSTOOD IN THE WHOLE PLANE; CALCULATIONS ACCURATE TO $\lesssim$ 1%.

2ν oscillat. with nonstandard $\mathcal{H}$

$$\mathcal{H} = C \cdot E^{n}$$

energy exponent

off-diagonal

GENERAL FORM FOR TRANSITION PROB.:

$$\rightarrow \quad \underline{P_{\alpha\beta} = A \cdot \sin^2(B \cdot E^{n} / L)}$$

RECOVER STANDARD CASE FOR $n = -1$

$\rightarrow L/E$

$n \neq -1 \Rightarrow$ NONSTANDARD DYNAMICS

e.g.:

$n = 0 \quad \rightarrow$ FCNC

$n = +1 \quad \rightarrow$ Violation of Lorentz invar. or of Equiv. Principle

....

$\boxed{\text{"more radical" nonstandard } \mathcal{H}}$

Standard Schrödinger eq. $\quad i\frac{\partial}{\partial t}\nu = \mathcal{H}\nu \quad , \quad \mathcal{H}$ hermitian

$\downarrow$

standard Liouville eq. $\quad \dot{\varrho} = -i[H,\varrho] \quad , \quad \varrho$ = density matrix

(i) $\frac{d}{dt}\mathrm{Tr}(\varrho) = 0 \quad \rightarrow$ conserv. of probability

(ii) $\frac{d}{dt}\mathrm{Tr}(\varrho^2) = 0 \quad \rightarrow$ conserv. of purity

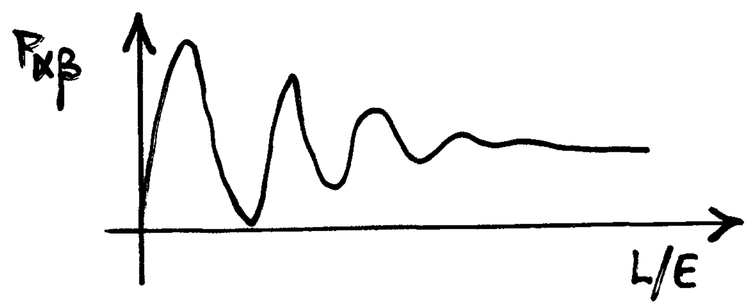
GIVE UP (i) : $\quad \mathcal{H} \rightarrow \mathcal{H} - i\Gamma \quad \leftarrow$ **ν decay**

get overall disappearance of ν with time

GIVE UP (ii) : $\quad \dot{\varrho} = -i[H,\varrho] + \mathcal{D}[\varrho] \leftarrow$ dissipative term pure $\rightarrow$ mixed states

ν decoherence

In both cases, get damped oscillations:



CP violation

requires $n \geq 3$ neutrinos to be observable in oscillations

$$\underset{\text{source}}{\nu_\alpha} \longrightarrow \underset{\text{target}}{\nu_\beta} \qquad \underset{\text{target}}{\bar{\nu}_\beta} \longleftarrow \underset{\text{source}}{\bar{\nu}_\alpha} \qquad \underset{\text{source}}{\bar{\nu}_\beta} \longrightarrow \underset{\text{target}}{\bar{\nu}_\alpha}$$

↑ **CP** ↑ **T**

$$\text{CPT}: \quad P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$$

$$\text{if CP}: \quad P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

$$\text{if } \cancel{\text{CP}}: \quad P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

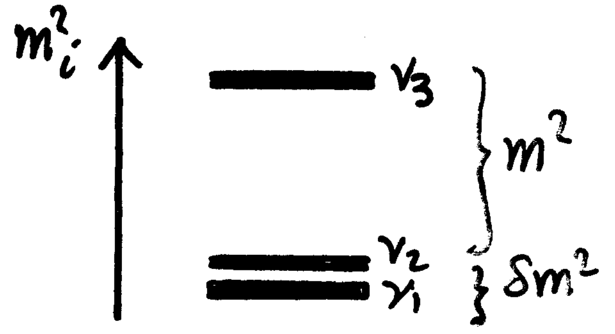
$$\Delta P_{CP} \propto \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos\theta_{13} \times$$
$$\times \sin\left(\frac{\Delta m^2_{12} L}{4E}\right) \cdot \sin\left(\frac{\Delta m^2_{23} L}{4E}\right) \cdot \sin\left(\frac{\Delta m^2_{13} L}{4E}\right)$$
$$\times \sin\delta$$

VANISHES IF : $\delta \to 0$
or: 1 mixing angle $\to 0$
or: 1 $\Delta m^2 \to 0$

VERY DIFFICULT. MOREOVER, "FAKE" $\cancel{\text{CP}}$ IN MATTER (ordinary matter does not contain antiparticles!)

MAY BE POSSIBLE AT FUTURE ν-fact

3ν with one dominant mass scale



$\delta m^2 \ll m^2$

e.g. $\uparrow$ solar $\quad \uparrow$ atmos.

$$\mathcal{M}^2 = \begin{pmatrix} 0 & & \\ & \delta m^2 & \\ & & m^2 \end{pmatrix} \quad \text{mod } \mathbb{1}$$

- FROM THE POINT OF VIEW OF "ATMOSPHERIC" ν:

$$\mathcal{M}^2 \simeq \begin{pmatrix} 0 & & \\ & 0 & \\ & & m^2 \end{pmatrix} \qquad \cancel{CP} \text{ unobservable}$$

$$\rightarrow P_{\alpha\alpha} \simeq 1 - 4U_{\alpha 3}^2 (1 - U_{\alpha 3}^2) \sin^2\left(\frac{m^2 L}{4E}\right)$$

$$P_{\alpha\beta} \simeq 4U_{\alpha 3}^2 U_{\beta 3}^2 \sin^2\left(\frac{m^2 L}{4E}\right)$$

Parameters: $(m^2, U_{e3}^2, U_{\mu 3}^2, U_{\tau 3}^2)$

- IN MATTER, PARAMETERS REMAIN THE SAME FOR ATM-ν, BUT $P_{\alpha\alpha}$ and $P_{\alpha\beta}$ RECEIVE CORRECTIONS:
  1) $\Delta m_{12}^2 = 0$ in vac. $\longrightarrow$ $\Delta m_{12,m}^2 \neq 0$ in matter
  2) case $m^2 \rightarrow -m^2$ (heavy solar doublet) distinguishable for $U_{e3} \neq 0$

- From the point of view of "solar" ν (probing $\Delta m^2_{12} \equiv \delta m^2$):

$$\mathcal{M}^2 \cong \begin{pmatrix} 0 & \delta m^2 & \\ & & \infty \end{pmatrix}$$

$\cancel{\varphi}$ unobserv. in P_{ee}

$$P_{ee} = 1 - 4U^2_{e1}U^2_{e2}\sin^2\left(\frac{\delta m^2 L}{4E}\right) - 4U^2_{e2}U^2_{e3}\sin^2(\infty) - 4U^2_{e1}U^2_{e3}\sin^2(\infty)$$

PARAMETERS: $(\delta m^2, U^2_{e1}, U^2_{e2}, U^2_{e3})$

Taking $\sin^2(\infty) \sim \frac{1}{2}$:

$$P_{ee} = (1-U^2_{e3})^2 - 4U^2_{e1}U^2_{e2}\sin^2\left(\frac{\delta m^2 L}{4E}\right) + U^4_{e3}$$

$$= c^4_\varphi \left[1 - \sin^2 2\omega \sin^2\left(\frac{\delta m^2 L}{4E}\right)\right] + s^4_\varphi$$

"2ν" probability

$$\to P^{3\nu}_{ee} = c^4_\varphi \text{ "} P^{2\nu}_{ee} \text{" } + s^4_\varphi \qquad \varphi = \theta_{13}$$

structure remains the same in matter (modulo $N_e \to N_e \cdot c^2_\varphi$)

1 active + ∞ sterile ν's

(EXTRA DIMENSIONS)

$$\mathcal{H} = \frac{MM^\dagger}{2E} + V$$

In matter, $V \neq 0$:

$V_e - V_s \neq 0$

$V_{\mu,\tau} - V_s \neq 0$

Let us focus on the vacuum case ($V=0$)

$$\mathcal{H} = \frac{MM^\dagger}{2E} \qquad M = \begin{bmatrix} m & \sqrt{2}m & \sqrt{2}m & \sqrt{2}m & \cdots \\ 0 & 1/R & 0 & 0 & \cdots \\ 0 & 0 & 2/R & 0 & \cdots \\ \vdots & \vdots & & 3/R & \ddots \end{bmatrix}$$

Define $\xi = R \cdot m$

$$R^2 MM^\dagger = \lim_{N \to \infty} \begin{bmatrix} (1+2N)\xi^2 & \sqrt{2}\xi & 2\sqrt{2}\xi & \cdots & N\sqrt{2}\xi \\ \sqrt{2}\xi & 1 & 0 & \cdots & 0 \\ 2\sqrt{2}\xi & 0 & 4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N\sqrt{2}\xi & 0 & 0 & \cdots & N^2 \end{bmatrix}$$

"almost" diagonal.

→ DIAGONALIZATION

Eigenvalue equation is of the form:

$$\det\begin{bmatrix} X_{00}-\lambda^2 & X_{01} & X_{02} & \cdots & X_{0N} \\ X_{10} & X_{11}-\lambda^2 & 0 & \cdots & 0 \\ X_{20} & 0 & X_{22}-\lambda^2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{2N} & 0 & 0 & \cdots & X_{NN}-\lambda^2 \end{bmatrix} \quad \leftarrow \text{expand 1st row}$$

$$= (X_{00}-\lambda^2)\det\begin{bmatrix} X_{11}-\lambda^2 & 0 & \cdots & 0 \\ 0 & X_{22}-\lambda^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_{NN}-\lambda^2 \end{bmatrix} \quad \leftarrow \text{trivial}$$

$$- X_{01}\det\begin{bmatrix} X_{10} & 0 & \cdots & 0 \\ X_{20} & X_{22}-\lambda^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X_{N0} & 0 & \cdots & X_{NN}-\lambda^2 \end{bmatrix} \quad \leftarrow \text{expand 1st row}$$

$$+ X_{02}\det\begin{bmatrix} X_{10} & X_{11}-\lambda^2 & \cdots & 0 \\ X_{20} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ X_{N0} & 0 & \cdots & X_{NN}-\lambda^2 \end{bmatrix} \quad \leftarrow \text{expand 2nd row}$$

$$\pm \ldots$$

$$= (X_{00}-\lambda^2)\prod_{k=1}^{N}(X_{kk}-\lambda^2) - X_{01}X_{10}\frac{\prod_{k=1}^{N}(X_{kk}-\lambda^2)}{X_{11}-\lambda^2}$$

$$- X_{02}X_{20}\frac{\prod_{k=1}^{N}(X_{kk}-\lambda^2)}{X_{22}-\lambda^2} \quad \ldots$$

$$= \left[(X_{00}-\lambda^2) - \sum_{k=1}^{N}\frac{X_{0k}X_{k0}}{X_{kk}-\lambda^2}\right]\prod_{k=1}^{N}(X_{kk}-\lambda^2) = 0$$

In our case $(V=0)$: $\begin{cases} X_{00} = (1+2N)\xi^2 \\ X_{0k} = X_{k0} = k\sqrt{2}\,\xi \\ X_{kk} = k^2 \end{cases}$

$$\lim_{N\to\infty} \left[(1+2N)\xi^2 - \lambda^2 - \sum_{k=1}^{N} \frac{2\xi^2 k^2}{k^2-\lambda^2} \right] \underbrace{\prod_{k=1}^{N} (k^2-\lambda^2)}_{\text{but } \lambda = k \text{ not eigenvalues}} = 0$$

$$\rightarrow \quad \xi^2 - \lambda^2 - 2\lambda^2\xi^2 \sum_{k=1}^{\infty} \frac{1}{k^2-\lambda^2} = 0$$

use $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2-\lambda^2} = \frac{1}{2}\left(\frac{1}{\lambda^2} - \frac{\pi \cot \pi\lambda}{\lambda}\right)$

$\rightarrow$ EQUATION FOR λ_n^2 eigenvalues of $R^2 M M^\dagger$:

$$\boxed{\lambda_n^2 - \pi\lambda_n \xi^2 \cot \pi\lambda_n = 0}$$

(VACUUM)
$\xi = mR$

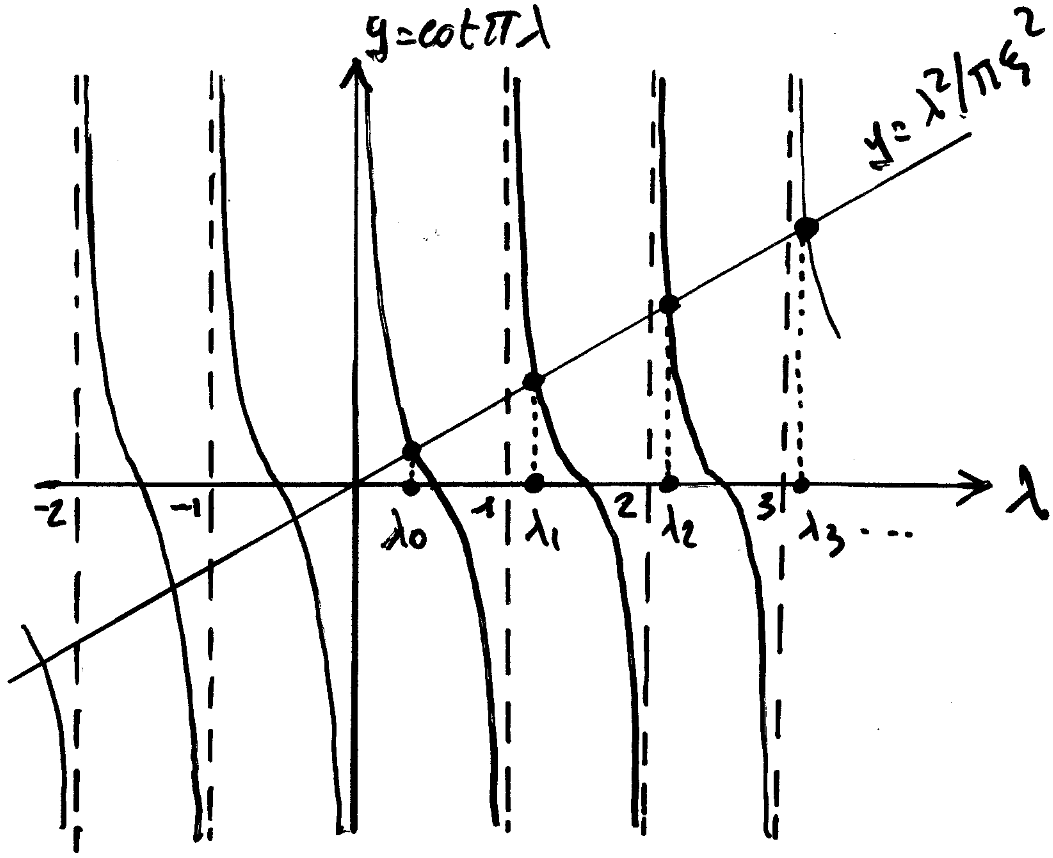
IN MATTER ONE WOULD GET

$$\boxed{\lambda_n^2 - \eta - \pi\lambda_n \xi^2 \cot \pi\lambda_n = 0}$$

$\eta = 2EVR^2$

Solutions of eigenvalue equation in vacuum determined by parametric intersection

$$\begin{cases} y = \lambda / \pi\xi^2 \\ y = \cot \pi\lambda \end{cases}$$



$\lambda_n \sim n$ asymptotically

"$-n$" solutions folded into "$+n$"
(only λ_n^2 relevant)

MIXING MATRIX: need eigenvectors

$$
R^2 M M^\dagger = \begin{array}{c} \begin{array}{cccccc} \nu_e & \nu^{(1)} & \nu^{(2)} & \cdots & \nu^{(N)} \end{array} \\ \left[\begin{array}{cccccc} x_{00} & x_{01} & x_{02} & \cdots & x_{0N} \\ x_{10} & x_{11} & 0 & \cdots & 0 \\ x_{20} & 0 & x_{22} & & \vdots \\ \vdots & \vdots & \vdots & \ddots & \\ x_{N0} & 0 & 0 & \cdots & x_{NN} \end{array}\right] \end{array} \begin{array}{l} \\ \nu_e \\ \nu^{(1)} \\ \vdots \\ \nu^{(N)} \end{array}
$$

$\leftarrow$ initial flavor basis

mass basis $\stackrel{\text{def}}{=} (\tilde{\nu}_0, \tilde{\nu}_1, \tilde{\nu}_2, \ldots, \tilde{\nu}_N)$

$$
\begin{bmatrix} \nu_e \\ \nu^{(1)} \\ \nu^{(2)} \\ \vdots \\ \nu^{(N)} \end{bmatrix} = \begin{bmatrix} U_{e0} & U_{e1} & U_{e2} & \cdots & U_{eN} \\ U_{10} & U_{11} & U_{12} & \cdots & U_{1N} \\ U_{20} & U_{21} & U_{22} & \cdots & U_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ U_{N0} & U_{N1} & U_{N2} & \cdots & U_{NN} \end{bmatrix} \begin{bmatrix} \tilde{\nu}_0 \\ \tilde{\nu}_1 \\ \tilde{\nu}_2 \\ \vdots \\ \tilde{\nu}_N \end{bmatrix}
$$

$\uparrow$ flavor $\qquad$ $\uparrow$ mixing $\qquad$ $\uparrow$ mass

(real: $U^{-1} = U^T$)

Eigenvector equation:

$$
R^2 M M^\dagger \begin{bmatrix} U_{en} \\ U_{1n} \\ \vdots \\ U_{Nn} \end{bmatrix} = \lambda_n^2 \begin{bmatrix} U_{en} \\ U_{1n} \\ \vdots \\ U_{Nn} \end{bmatrix} \qquad n = 0, 1, \ldots, N
$$

N+1 equations $\oplus$ eigenvector normalization

$$\left(\sum_{i=e,1,\ldots N} U_{in}^2 = 1 \right)$$

$\rightarrow$ N indep. eq.

$\rightarrow$ Eliminate 1st eq.

get $X_{k0} U_{en} + X_{kk} U_{kn} = \lambda_n^2 U_{kn}$ $(k=1,\dots,N)$

$\rightarrow U_{kn} = -\dfrac{X_{k0}}{X_{kk}-\lambda^2} U_{en}$

$\rightarrow \sum_{k=1}^{N} U_{kn}^2 = \left(\sum_{k=1}^{N} \dfrac{X_{k0}^2}{(X_{kk}-\lambda_k^2)^2}\right) U_{en}^2$

but $\sum_{k=1}^{N} U_{kN}^2 = 1 - U_{en}^2$ (normalization) so that

$$U_{en}^2 \left(1 + \sum_{k=1}^{N} \frac{X_{k0}^2}{(X_{kk}-\lambda_n^2)^2}\right) = 1$$

$$\boxed{\left(U_{en}^2\right)^{-1} = 1 + 2\xi^2 \sum_{k=1}^{N} \frac{k^2}{(k^2-\lambda_n^2)^2}}$$

ν_e mixing matrix elements

e or any other active state

Focus on U_{en}^2, since $\nu^{(1)}$ sterile

$\rightarrow$ only $P(\nu_e \rightarrow \nu_e)$ observable

U_{en}^2

The $\left(\lim\limits_{N\to\infty} \sum\limits_{k=1}^{N}\right)$ term can be put in closed form:

$$\sum_{k=1}^{\infty} \frac{2k^2}{(k^2-\lambda_n^2)^2} = \frac{\partial}{\partial\alpha} \sum_{k=1}^{\infty} \frac{1}{\lambda_n^2-\alpha^2 k^2}\Bigg|_{\alpha=1}$$

$$= \frac{\partial}{\partial\alpha}\frac{1}{2}\left[\frac{\pi\cot\pi\lambda_n/\alpha}{\alpha\lambda_n} - \frac{1}{\lambda_n^2}\right]_{\alpha=1}$$

$$= \frac{\pi^2}{2}(1+\cot^2\pi\lambda_n) - \frac{\pi}{2\lambda_n}\cot\pi\lambda_n$$

Using also eigenvalue equation $(\cot\pi\lambda_n = \lambda_n/\pi\xi^2)$ get:

$$\boxed{(U_{en}^2)^{-1} = 2\left[1+\pi^2\xi^2 + \lambda_n^2/\xi^2\right]}$$

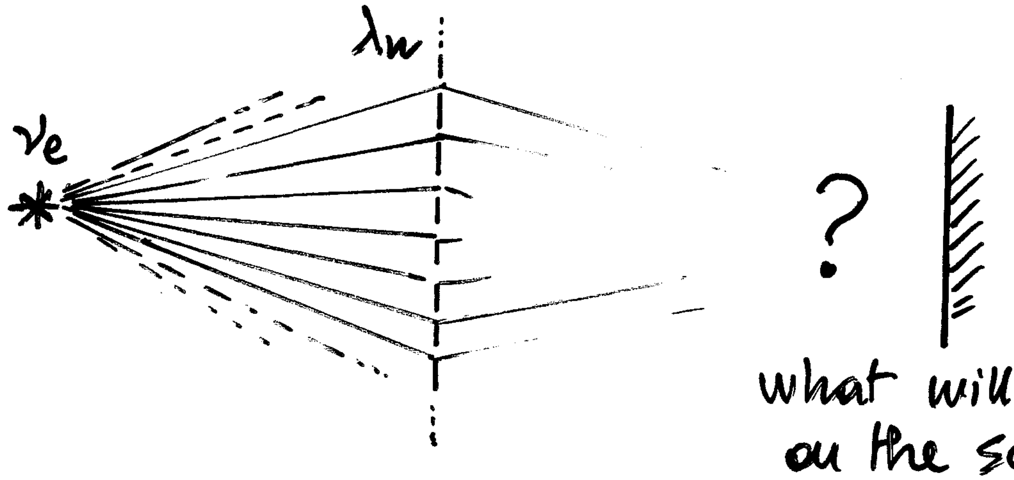
OSCILLATION AMPLITUDE:

L = length

$$A(\nu_e\to\nu_e) = \sum_{n=0}^{\infty} U_{en}^2 \, e^{-i\frac{\lambda_n^2}{2ER^2}L}$$

$$= 2\sum_{n=0}^{\infty} \frac{e^{-i\frac{\lambda_n^2}{2ER^2}L}}{1+\pi^2\xi^2+\lambda_n^2/\xi^2}$$

← Infinite interfering amplitudes



what will appear on the screen?

Interference pattern generally difficult to calculate for $\xi \sim \mathcal{O}(1)$.

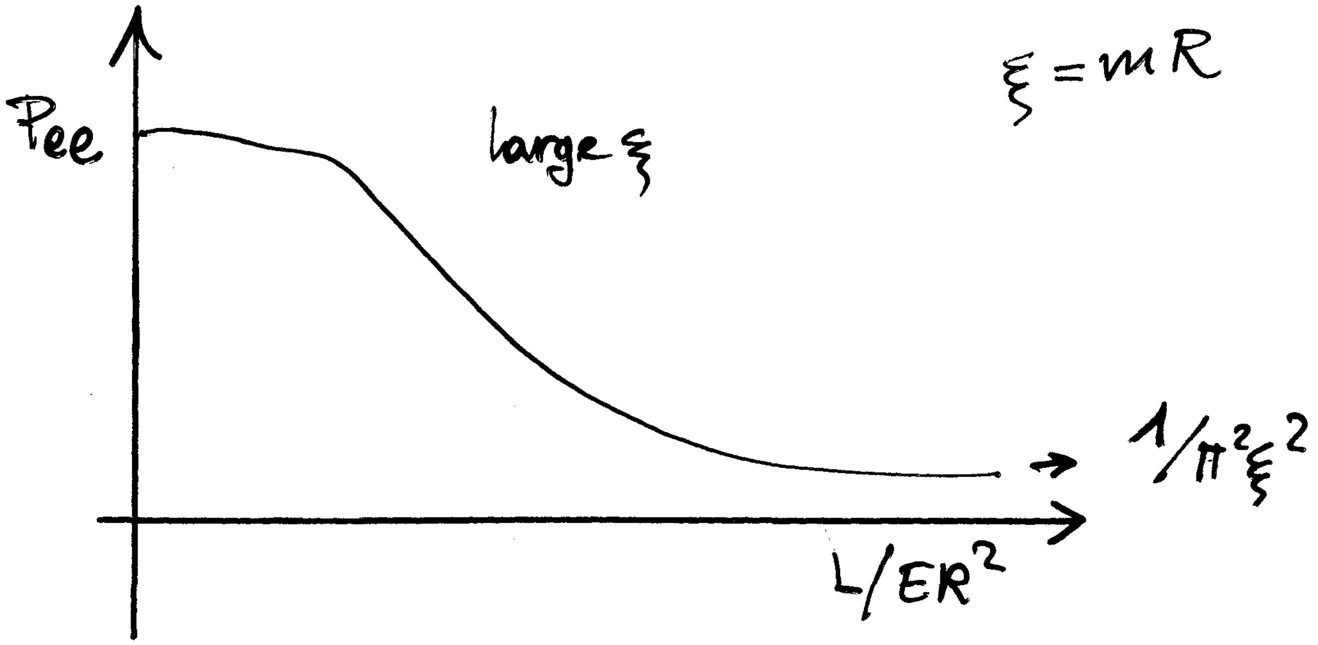
If $\xi \gg 1$ can write $\sum_n \to \int du$

$$A^*(\nu_e \to \nu_e) \simeq 2\int_0^\infty du \, \frac{1}{\pi^2\xi^2 + \frac{u^2}{\xi^2}} \, e^{-u^2\left(-i\frac{m^2 L}{2E\xi^2}\right)}$$

$$\simeq e^z\left(1 - \mathrm{erf}\sqrt{z}\right) \quad \text{where } z = -i(\pi\xi^2/R)^2(L/2$$

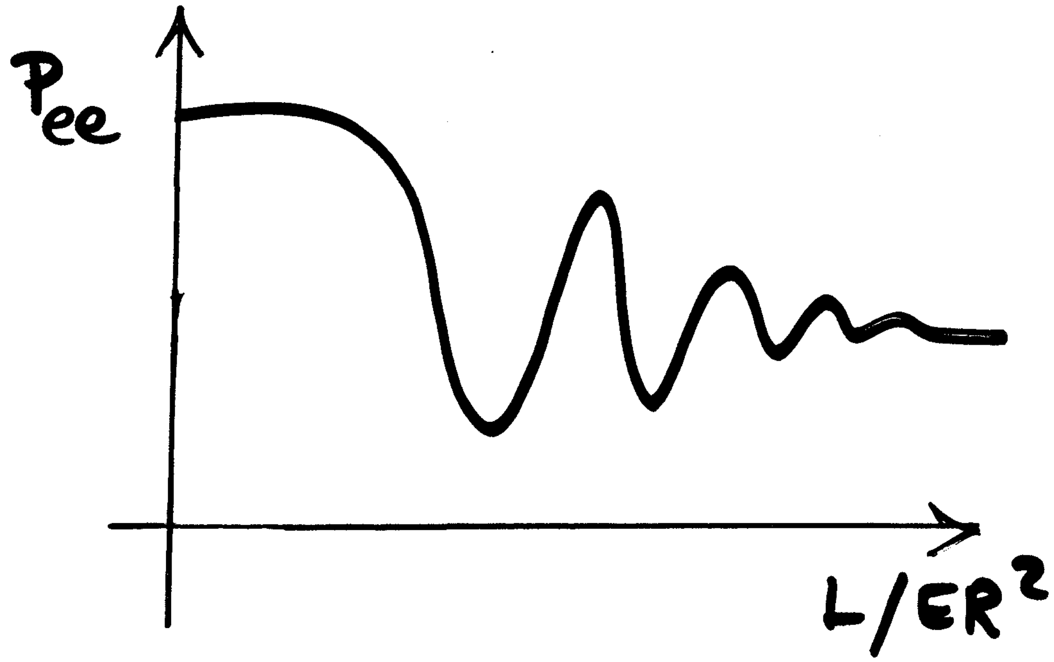
(use $\frac{1}{\pi}\int_{-\infty}^{+\infty} dt \, \frac{e^{-zt^2}}{1+t^2} = e^z\left(1 - \mathrm{erf}\sqrt{z}\right)$)

$$P(\nu_e \to \nu_e) = AA^* = \left|1 - \mathrm{erf}\,\pi\xi^2\sqrt{-i\frac{L}{2ER^2}}\right|^2$$



SUM OF INFINITE HARMONICS CAN GIVE MONOTONIC Pee
(i.e., "NON-OSCILLATING" Pee)

HOWEVER, FOR GENERIC $\xi = mR \sim \mathcal{O}(1)$, expect P_{ee} of the form:



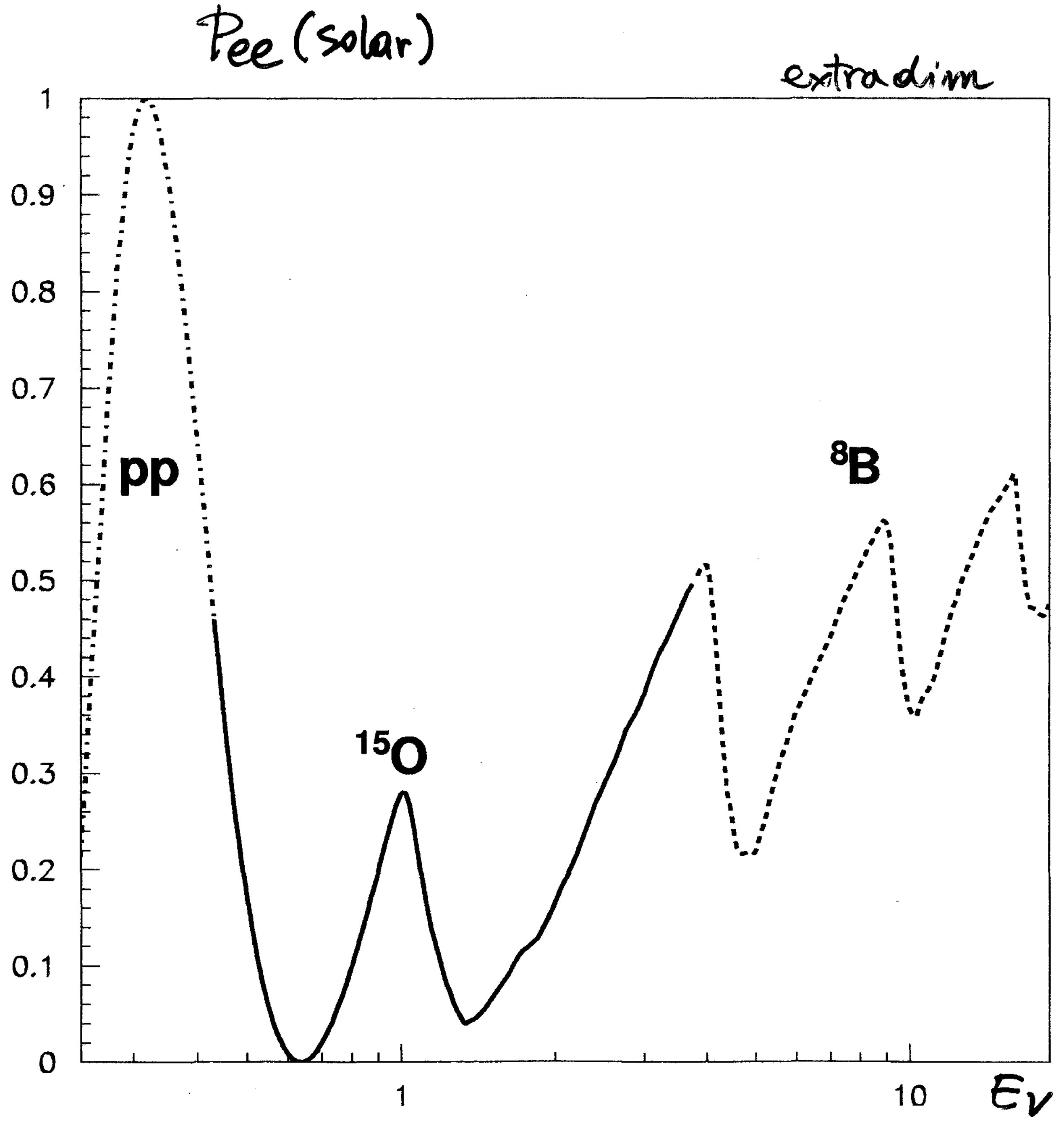
↑

DISAPPEARANCE IN STERILE STATES VIA OSCILLATIONS

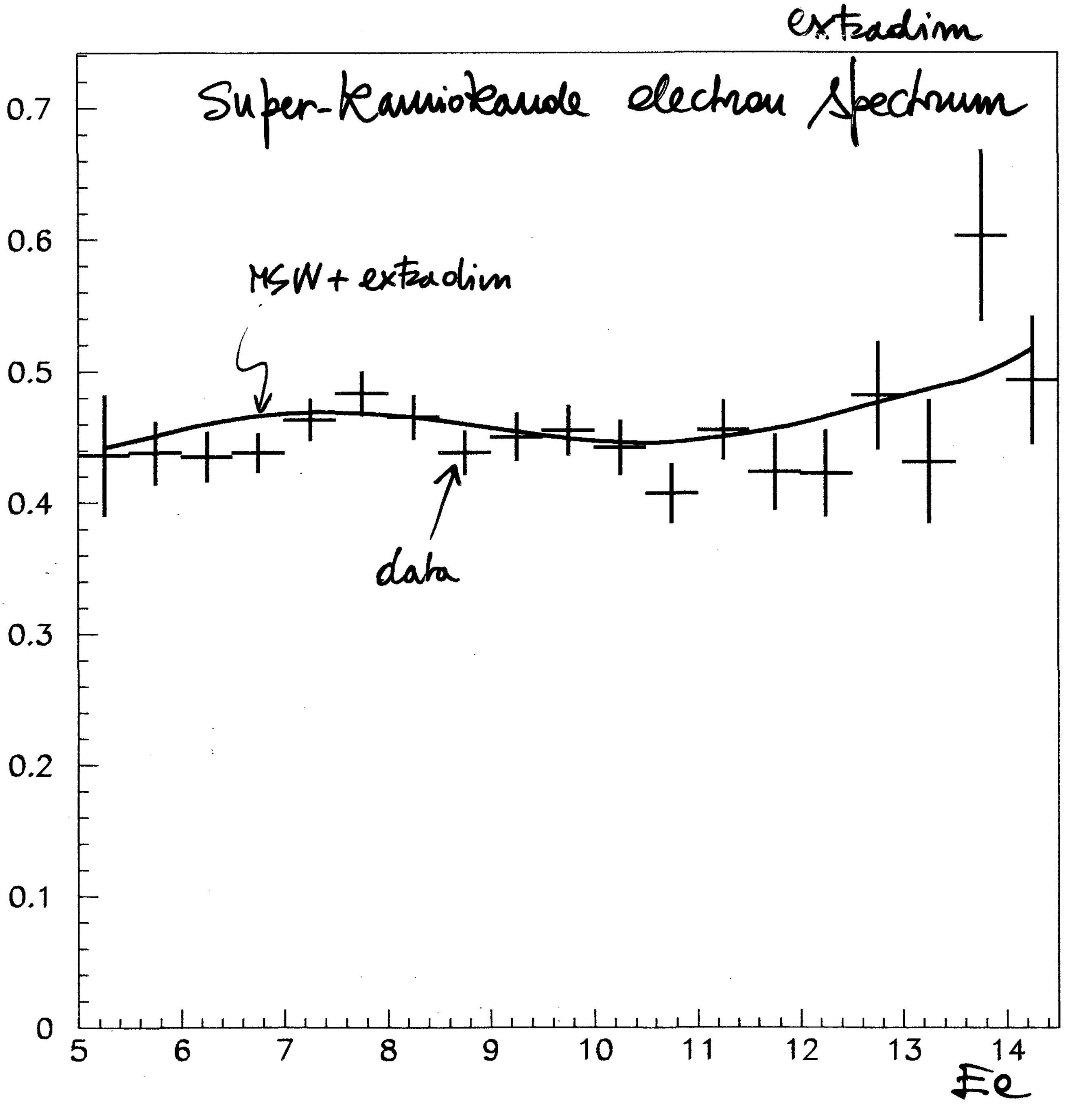
Examples →

Dvali, Smirnov ; Ross;
Caldwell
....

POSSIBLY
... STRONG MODIFICATIONS
OF USUAL MSW Pee
THROUGH ADDITIONAL
HARMONICS ...



... BUT SMEARED EFFECTS
ON OBSERVABLE QUANTITIES



Barbieri, Creminelli, Strumia

negligible interference of the KK towers. For small values of $1/R$ there is however the possibility of a transition $\nu_e \to \nu_{KK}$ using the MSW effect, which is compatible with the solar data [20]. It requires $1/R \approx 3 \times 10^{-3}$ eV and a mixing with the KK states determined by $\xi_2 \approx 0.01$, or $m_2 \approx 10^{-(4\div5)}$ eV, so that a fit of SK atmospheric data requires $\xi_3 \sim 2$. When the parameter μ of Section 4 is specified for the electron neutrino and with the solar density profile, the resonant MSW conversion mentioned there (μ positive, small ξ) takes place and suppresses the different components of the solar ν_e spectrum as possibly observed by the various solar neutrino experiments.

7. Special features of the proposed solutions

extradim

Some alternative descriptions of the atmospheric neutrinos appear possible. The crucial point, however, would be to indicate precise signatures of such solutions visible in appropriate neutrino experiments. To this purpose Fig. 4 is of interest. We give there, versus L/E_ν, the probabilities $P_{\mu\mu}$ and $P_{\mu\tau}$ that correspond to the fits of the SK results shown in Fig. 3. A few features of these plots might be relevant for an experimental discrimination of the various possibilities.

1. The absence of a first clear dip in the L/E_ν-shape of $P_{\mu\mu}$ is a characteristic of the KK fits that we have discussed at intermediate and big ξ, at clear variance with the shape of $P_{\mu\mu}$ in the standard $\nu_\mu \to \nu_\tau$ interpretation of the data.
2. The non-standard transition from unoscillated to oscillated atmospheric neutrinos requires a L/E_ν-range longer than the standard one and even the one that would be produced by neutrino decay [28–31]. Therefore, unlike what happens in the standard case, a good fit of atmospheric data significantly constrains the outcome of ν_μ disappearance experiments. For example the on-going K2K experiment [32] should observe only 65% ÷ 85% of the events with respect to the no-oscillation case,

Pμμ (atmos.)

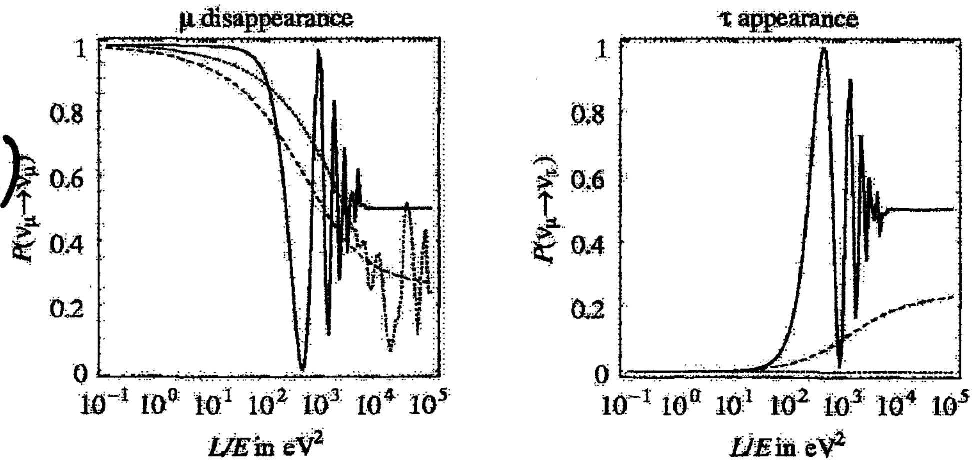


Fig. 4. The $P_{\mu\mu}$ (a) and $P_{\mu\tau}$ (b) that give the best SK fits (see caption of Fig. 1 for colour version). Continuous blue line: standard $\nu_\mu \to \nu_\tau$ fit. Dotted red line: $\nu_\mu \to \nu_{KK}$ fit with intermediate $\xi = 1/2$. Dashed green line: $\nu_\mu \to \nu_\tau, \nu_{KK}$ fit with large ξ.

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... more harmonics...

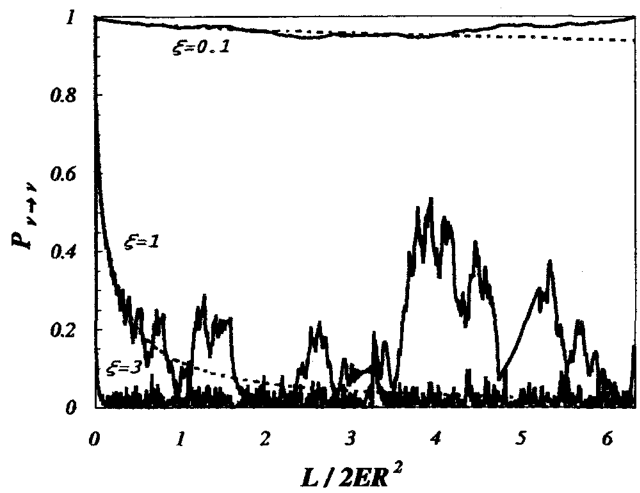

FIG. 3. Exact survival probability for ν_L versus $L/2E$ in units of R^2 (as it is explicit in the argument) for three different values of ξ. Dotted lines represent the continuos approximation discused in the main text. Note that only the low ξ limit has a periodic behaviour.

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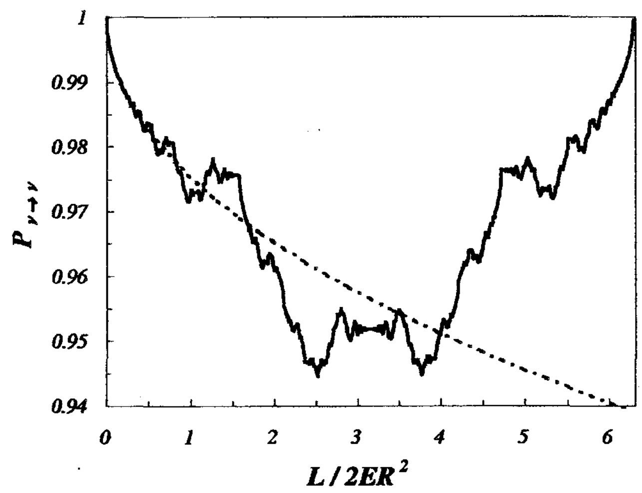
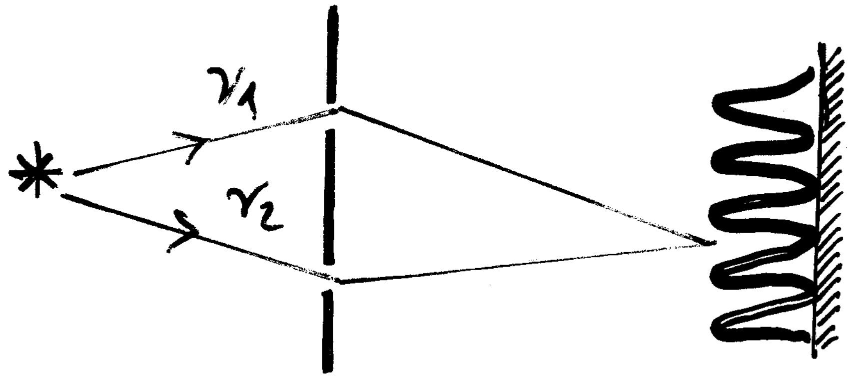


FIG. 4. Here we show an amplification of the survival probability for the $\xi = 0.1$ case showed in figure 3. Note the large number of wiggles produced by the oscillation of consecutive levels in Eq. (9). We also depict the continuos limit (dotted line) for comparison.

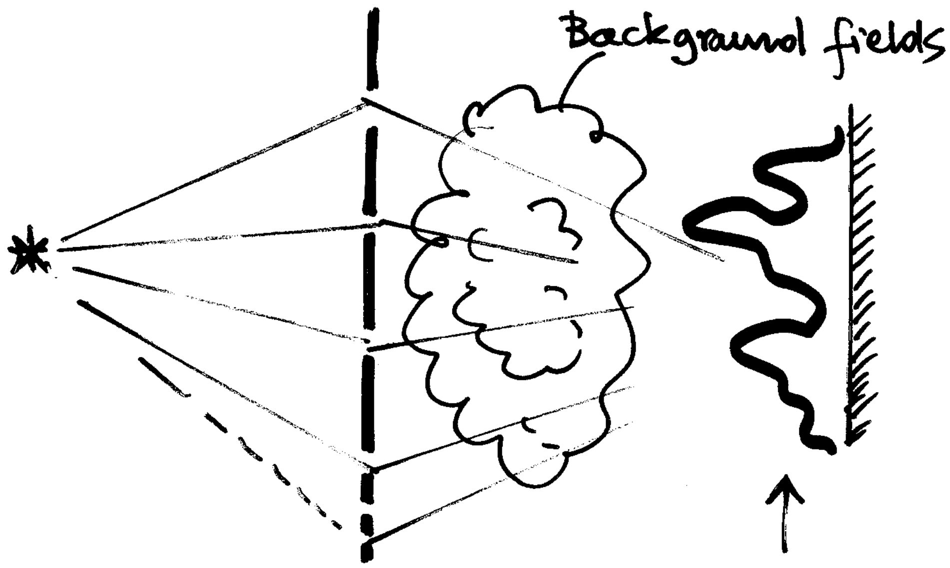
MESSAGE:

YOU CAN GET MODIFICATIONS TO THE SIMPLE "TWO-SLIT" INTERFERENCE PATTERN...



"TWO-FAMILY VACUUM OSCILLATIONS"

... BY ADDING MORE SLITS (= ν STATES) ← possibly ∞!
AND/OR ALTERING MEDIUM (= [new] ν INTERACTIONS WITH BACKGROUD FIELDS/MATTER)



Can get "anything" you like with enough imagination!

... BUT...

... EXPERIMENTS RULE OUT MANY PATTERNS AND FAVOUR ONLY A FEW ...

$\boxed{\text{ALTHOUGH}}$

IT MUST BE CLEARLY SAID THAT THERE IS NO REAL EVIDENCE FOR "OSCILLATIONS" ← Technically speaking (i.e., dips and bumps in the interference pattern) BUT ONLY FOR FLAVOR TRANSITIONS

$P(\nu_e \to \nu_e) < 1$ sol. ***

$P(\nu_\mu \to \nu_\mu) < 1$ atm. *****

$P(\nu_\mu \to \nu_e) \neq 0$ LSND *

DETAILED INTERFERENCE PATTERN (disappearance + reapparance of flavors) NOT YET OBSERVED CLEARLY/DIRECTLY