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SUMMER SCHOOL ON PARTICLE PHYSICS

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FERMION MASSES AND THE FLAVOUR PROBLEM

Lecture III

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Please note: These are preliminary notes intended for internal distribution only.

Texture zeros for leptons ?

$b \gg 1$, texture zeros

$$
\begin{array}{cccc}
 & \beta_3 & \beta_2 & \beta_1 = \alpha_1 \\
m_e = & \multicolumn{3}{c}{\begin{pmatrix} 0 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & \cdot & \cdot \\ 0 & \cdot & 1 \end{pmatrix}}
\end{array}
\qquad
\begin{array}{ll}
e & \beta_3 = -\beta_2 - \alpha_1 \\
\mu & \beta_2 \\
\tau & \beta_1 = \alpha_1
\end{array}
$$

Indep. of β_2

... same Higgs as gives

m_{d_i} ... same $\bar{\epsilon}$.

$m_b \sim m_\tau$ by choice $\beta_1 = \alpha_1$ ✓

Texture zeros $\Rightarrow$ $\boxed{\mathrm{Det}\, m_\ell = \bar{\epsilon}^6 = \mathrm{Det}\, m_d.}$ ✓

$(m_e m_\mu m_\tau = m_d m_s m_b \underbrace{(\times 3?)}_{\text{[illegible]}})$

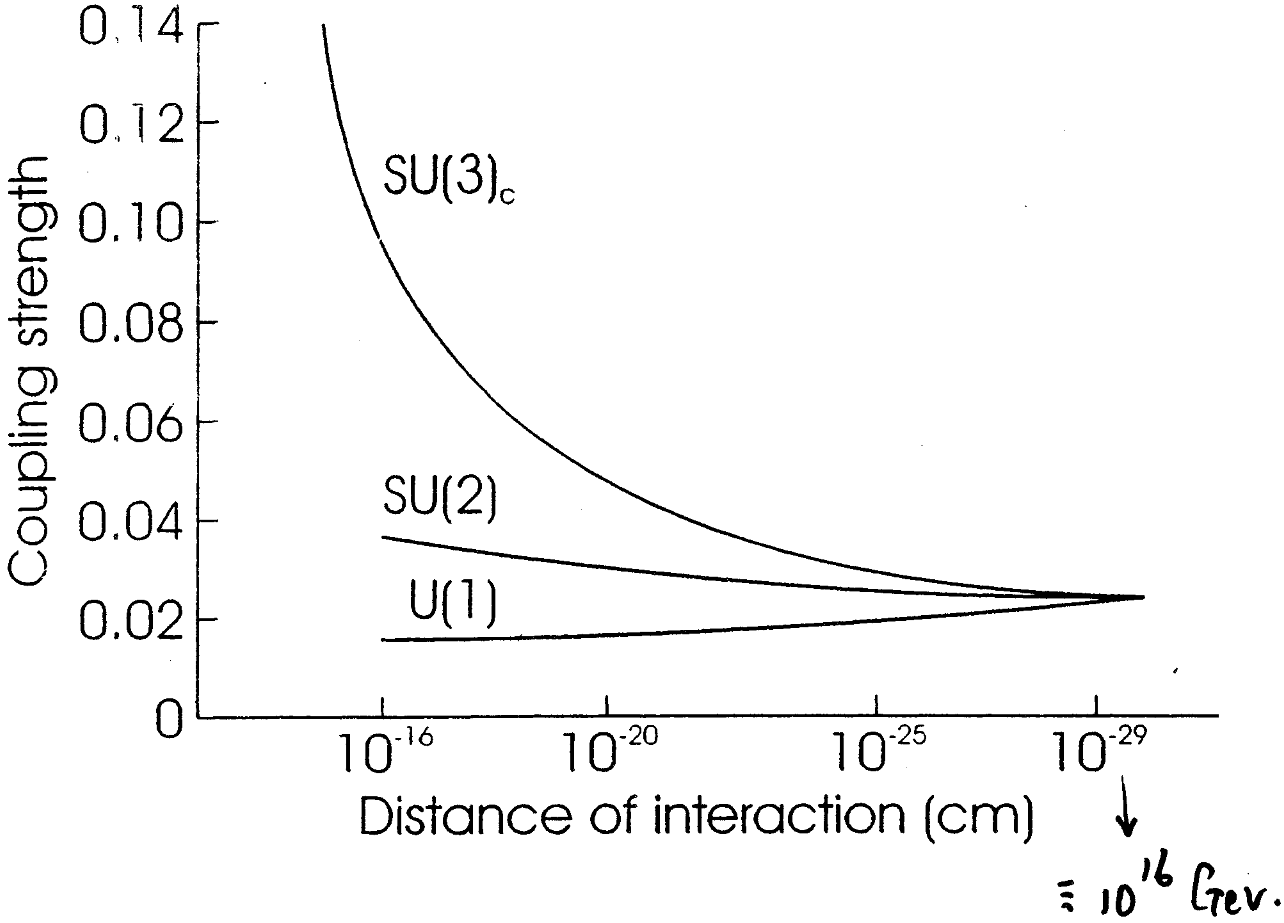
To explain $\dfrac{m_\mu}{m_\tau} \neq \dfrac{m_s}{m_b}$, need either $\beta_2 \neq \alpha_2$ <u>or</u>

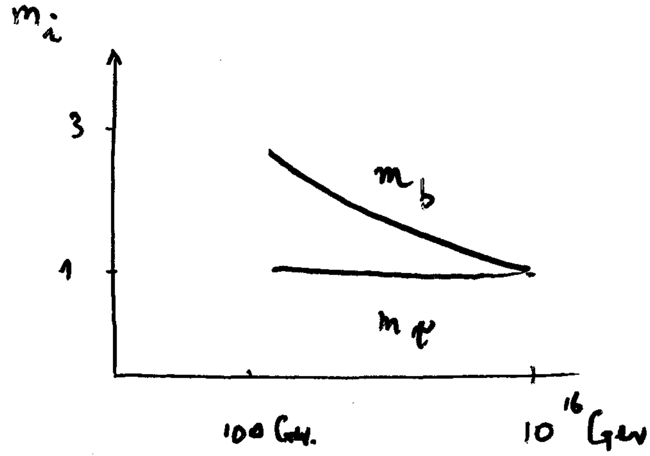
Georgi - Jarlskog. , $\beta_2 = \alpha_2$ $(SO(10), SU(5))$

$$
m_d = \begin{pmatrix} 0 & h''\bar{\epsilon}^3 & 0 \\ h''\bar{\epsilon}^3 & k\bar{\epsilon}^2 & h'\bar{\epsilon} \\ 0 & h'\bar{\epsilon} & h \end{pmatrix}
\qquad
m_e = \begin{pmatrix} 0 & h''\bar{\epsilon}^3 & 0 \\ h''\bar{\epsilon}^3 & 3k\bar{\epsilon}^2 & h'\bar{\epsilon} \\ 0 & h'\bar{\epsilon} & h \end{pmatrix}
$$

$m_\ell = m_d$ $\qquad$ $m_e = 3m_d$.

$m = h\, \bar{5}_\ell\, 10_\ell\, \bar{5}_{Higgs}$ or $k\, \bar{5}_\ell\, 10_\ell\, 45_{Higgs}$





$m_b = m_\tau \big|_{M_X} \Rightarrow m_b \simeq 3 m_\tau \big|_{M_Z}$ ✓

$\underline{SU(5)} \quad \supset \quad \underline{SU(3) \times SU(2) \times U(1)}$

$$\psi_{\bar 5} = \begin{bmatrix} d^c \\ d^c \\ d^c \\ \nu_e \\ e \end{bmatrix}_L = (\bar 3, 1) + (1, 2)$$

G^a, W^i, X

$3Q_{d^c} + Q_\nu + Q_e = 0$

$\boxed{Q_d = \frac{1}{3} Q_e}$

$$\chi^{10} = \frac{1}{\sqrt 2}\begin{bmatrix} 0 & u^c & u^c & u & d \\ -u^c & 0 & u^c & u & d \\ u^c & -u^c & 0 & u & d \\ u & u & u & 0 & e^+ \\ d & d & d & e^+ & 0 \end{bmatrix}_L$$

$\bar 5 \times \bar 5 = 15 + 10$

$\dots \; a_{jk} = \frac{1}{\sqrt 2}(a_j a_k - a_k a_j)$

$\begin{bmatrix} H_T \\ H_D \end{bmatrix}$ DOUBLET-TRIPLET SPLITTING NEEDED?

- $SU(5) \xrightarrow{\langle \Sigma^A_{24} \rangle \approx M_X} SU(3) \times SU(2) \times U(1) \xrightarrow{\langle H_5 \rangle \approx M_W} SU(3) \times U(1)$

- $\sin^2\theta_W = \left.\frac{Tr(T^2_{3L})}{Tr(Q^2)} = \frac{3}{8}\right|_{M = M_X} = \frac{g_1^2}{g_1^2 + g_2^2} \quad ; \quad g_2 = g_3$

- $\mathcal{L}_Y = h\, \bar\psi^p_{5R}\, \chi^{10}_{p,q\,L}\, H^q_5 \quad \Rightarrow \quad m_b = m_\tau \Big|_{M = M_X} = h \langle H^5_5 \rangle$

$\left(\mathcal{L}'_Y = h'\, \bar\psi^p_R\, \underset{45.}{H^{sq}_p}\, \chi_{sq\,L} \quad \Rightarrow \quad m_\ell = 3 m_q \right)$

Masses in SU(5)

$$\underline{5} \times \underline{10} = \underline{5} + \underline{\overline{45}}$$

$$10 \times 10 = \underline{\overline{5}} + \underline{45}$$

$$\mathcal{L}_f^{5} = \frac{1}{2}(\overset{10}{\overset{\downarrow}{\chi^{\dagger}}})^{\alpha\beta}\gamma^0 m_1 \left[\overset{\downarrow}{H}_\alpha \psi_\beta - \overset{\downarrow}{H}_\beta \psi_\alpha\right) - \frac{1}{4}\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\chi_{\alpha\beta} M_2 \overset{\downarrow}{H}_\gamma \chi_{\delta\varepsilon}$$

$$\langle H_\alpha \rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{bmatrix}$$

$$\underline{m_d = m_e} \quad ; \quad m_s = m_\mu \quad ; \quad m_b = m_\tau$$

$$\mathcal{L}_f^{45} = h' \psi^{+p} H_{45\,p}^{\;\;sq} \chi_{sq}$$

$$m\underline{_d = 3m_e} \quad ; \quad m_s = 3m_\mu \quad ; \quad m_b = 3m_\tau$$

Georgi-Jarlskog

$$M_d = \begin{pmatrix} 0 & a'\langle H^5\rangle & 0 \\ a\langle H^5\rangle & \underline{c}\langle H^{45}\rangle & 0 \\ 0 & 0 & b\langle H^5\rangle \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & a'\langle H^5\rangle & 0 \\ a\langle H^5\rangle & \underline{3c}\langle H^{45}\rangle & 0 \\ 0 & 0 & b\langle H^5\rangle \end{pmatrix}$$

Radiative correction

$$m_\tau = m_b \cdot \frac{1}{3}$$

$$m_\mu = 3m_s \cdot \frac{1}{3}$$

$$m_e = \frac{1}{3} m_d \cdot \frac{1}{3}$$

SO(10)

$$SO(10) \supset \begin{array}{l} SU(5) \times U(1) \dagger \\ SU(6) \times SU(4) \\ SU(4) \times SU(2)_L \times SU(2)_R \end{array}$$

Reps:

$$10 = 5 + \bar{5} \quad \dagger$$

$$\underline{16} = 10 + \bar{5} + 1$$

$$45 = 24 + 10 + \overline{10} + 1$$

$$120 = 5 + \bar{5} + 10 + \overline{10} + 45 + \overline{45}$$

Fermion masses / Yukawa couplings

$$\underline{16}.\underline{16}.\underline{10} = (10+\bar{5}+1)(10+\bar{5}+1)(5+\bar{5})$$

$$\supset \quad 10.\bar{5}.\bar{5} + 10.10.5$$

$$\Rightarrow \quad h_b = h_\tau = h_t$$

$$m_d = m_e \; , \quad m_u = m_\nu .$$

$$\underline{16}.\underline{16}.\underline{120} = (10+\bar{5}+1)(10+\bar{5}+1)(5+\bar{5}+10+\overline{10}+45+\overline{45})$$

$$\supset \quad \bar{5}.10.45$$

$$\underset{\langle 45 \rangle \neq 0}{\Rightarrow} \quad m_e = 3 m_d \; , \quad m_\nu = 0, \quad m_u \neq 0.$$

ANOMALY STRUCTURE. The $\tilde{U}(1)$ can only be made anomaly free via Green-Schwarz term^{+} (cf strings)

$$\boxed{\begin{aligned} \mathcal{L}^{\text{gauge}}_{N=1} &= \frac{\overbrace{S_R}^{\text{dilaton}}}{4} \sum_i' k_i F_i^2 + i \overbrace{\eta}^{\text{Axion}} \sum_i k_i F_i \tilde{F}_i \\ \eta &\to \eta - \Theta(x)\, \delta_{GS} \\ A_\mu^{\tilde{u}} &\to A_\mu^{\tilde{u}} + \partial_\mu \Theta(x) \end{aligned}}$$

α_i not traceless now

A_3 : $SU(3)^2\, \tilde{U}(1)$: $2 \sum_i \alpha_i$

$H_1 = -2\alpha_1$
$H_2 = w\alpha_1$

A_2 : $SU(2)^2\, \tilde{U}(1)$: $\frac{3}{2} \sum_i \alpha_i + \frac{1}{2}\sum_i \beta_i + \frac{\alpha_1}{2}(w-2)$

A_1 : $U(1)_Y^2\, \tilde{U}(1)$: $\frac{11}{6} \sum_i \alpha_i + \frac{3}{2} \sum_i \beta_i + \frac{\alpha_1}{2}(w-2)$

$^{+}$ SUSY : CANNOT USE $\sum \alpha_i = \sum \beta_i = 0$, $w = -2$ BECAUSE OF HIGGSINO

$\Rightarrow$ Anomaly cancellation if $\frac{A_3}{k_3} = \frac{A_2}{k_2} = \frac{A_1}{k_1}$

- $m_b = m_\tau \Rightarrow \beta_1 = \alpha_1$
- $\text{Det}(m_d) = \text{Det}(m_e) \Rightarrow \beta_2 + \beta_3 = \alpha_2 + \alpha_3$,

Then $A_3 = A_2$ (from $g_2 = g_3$ at unification) ie $\alpha_i = 0$

"GUTS" without GUTS!

$\Rightarrow A_3 : A_2 : A_1 = 1 : 1 : \frac{5}{3} \Rightarrow \boxed{\sin^2\Theta_W = \frac{k_2}{k_1 + k_2} = \frac{3}{8}}$

D - flatness

$$D^2 \cdot \quad \Big| \underbrace{\sum_i x_i}_{\text{U(1) charge}} |\phi_i|^2 - \underbrace{\xi^2}_{\text{Possible anomalous term.}} \Big|^2$$

SOFT susy breaking mass

Then $-m_i^2 |\phi_i|^2 + D^2$ has minimum

$\langle \phi_{+1} \rangle \simeq \langle \phi_{-1} \rangle$ if $\langle \phi \rangle \gg \xi$

$\langle \phi_{+1} \rangle \approx \xi$ if $\langle \phi \rangle \sim \xi$

Here $\xi = \frac{1}{\sqrt{192\pi}} M_{Planck}$ (or less !)
$\underbrace{\hphantom{less !}}_{\text{Ibanez}}$

- D-term contribution to squark masses?

$\langle D^2 \rangle \approx m_i^4$ from minimisation

But if squarks have u(1) charge

$$D^2 \rightarrow - \langle D \rangle x_q |\tilde{q}|^2 = \underbrace{m_i^2}_{\text{Small}} x_q |\tilde{q}|^2$$

Family symmetries?

- Abelian, $\tilde{U}(1)$

$$m^{u,d} \propto \begin{pmatrix} \sim 0 & \varepsilon_{u,d}^3 & \varepsilon_{u,d}^4 \\ \varepsilon_{u,d}^3 & \varepsilon_{u,d}^2 & \varepsilon_{u,d} \\ \varepsilon_{u,d}^8 & \varepsilon_{u,d} & 1 \end{pmatrix}$$

✓

$\dagger$ ✗

	u	c	t
	d	s	b
Q:	-3	2	1

Origin?
...strings?

$\dagger$... needs coefficient $\frac{1}{2}$ in $(m^d)_{2,3}$

- Non-Abelian, eg $SU(3)_f$

... $m_0^2(\tilde{u}^2 + \tilde{c}^2 + \tilde{t}^2 + \tilde{d}^2 + \tilde{s}^2 + \tilde{b}^2)$: FCNC ✓

$$m^{u,d} \propto \begin{pmatrix} 0 & \varepsilon_{u,d}^3 & \varepsilon_{u,d}^3 \\ \varepsilon_{u,d}^3 & \varepsilon_{u,d}^2 & \varepsilon_{u,d}^2 \\ \varepsilon_{u,d}^3 & \varepsilon_{u,d}^2 & 1 \end{pmatrix}$$

✓

non-Abelian structure?

$\langle\phi\rangle = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ vacuum alignment

An $SU(3)_f$ model. (supersymmetric)

Multiplets

		$SU(3)$
$\psi^a_{i=1,2,3}$	$: \; a = q, \ell, u^c, d^c, e^c, \nu^c$	3
$H_{1,2}\,,\, X$		1
$\phi_3^{\;i},\ \bar{\phi}_{3\,i}$	SM singlets	$\bar{3}, 3$
$\phi_2^{\;i},\ \bar{\phi}_{2\,i}$	SM singlets	$\bar{3}, 3$

Vacuum alignment:

$$m_3^2 |\phi_3|^2 + m_{\bar{3}}^2 |\phi_{\bar{3}}|^2 + m_2^2 |\phi_2|^2 + m_{\bar{2}}^2 |\phi_{\bar{2}}|^2 + D\text{-terms} + |F_X|^2$$

$$P \supset \lambda X \phi_2 \bar{\phi}_2 + Y(\phi_2 \bar{\phi}_1 - \mu^2)$$

$$\phi_3^{\;i} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} M_1 \,, \qquad \bar{\phi}_{3\,i}^{\;T} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} M_1$$

$$\phi_2^{\;i} \approx \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix} \varepsilon M_2 \,, \qquad \bar{\phi}_{2\,i}^{\;T} \approx \begin{pmatrix} 0 \\ i \\ 1 \end{pmatrix} \varepsilon M_2$$

<u>Full symmetry</u> :

	$SU(3)_f$	$\times Z_2$	$\times Z_R$
ϕ_1	$\bar{3}$	$-$	$+1$
ϕ_2	$\bar{3}$	$+$	$+1$
$\bar{\phi}_1$	3	$+$	-1
$\bar{\phi}_2$	3	$+$	0

- Then allowed superpotential terms are :

$$\frac{1}{M^2}\psi_i^c\phi_1^i\,\phi_1^j\psi_j H + \frac{1}{M^2}\psi_i^c\phi_2^i\,\phi_2^j\psi_j H$$

$$+\frac{\delta}{M^4}\psi_i^c\phi_1^i\,\phi_2^j\psi_j(\phi_1\bar{\phi}_2)H$$

$$+\frac{1}{M^5}\varepsilon^{ijk}\psi_i^c\psi_j\bar{\phi}_2(\phi_1\bar{\phi}_2)^2 H$$

$$+\frac{1}{M^8}(\varepsilon^{ijk}\psi_i^c\bar{\phi}_{1j}\bar{\phi}_{2k})(\varepsilon^{ijk}\psi_i\bar{\phi}_{1j}\bar{\phi}_{2k})(\phi_1\bar{\phi}_2)^4 H$$

+ higher order (smaller) terms.

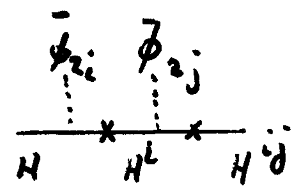
- Expansion parameter different for up + down sector ; form invariant

$$m^{u,d} \propto \begin{pmatrix} \varepsilon_{u,d}^{6\,\text{or}\,0} & \varepsilon_{u,d}^3 & \varepsilon_{u,d}^3 \\ \varepsilon_{u,d}^3 & (1+\delta)\varepsilon_{u,d}^2 & \varepsilon_{u,d}^2 \\ \varepsilon_{u,d}^3 & \varepsilon_{u,d}^2 & 1+\varepsilon_{u,d}^2 \end{pmatrix}$$

- Lepton masses ?

Charged leptons

$$m^d \propto \begin{pmatrix} 0 & \varepsilon_d^3 & \varepsilon_d^3 \\ \varepsilon_d^3 & \varepsilon_d^2 & \varepsilon_d^2 \\ \varepsilon_d^3 & \varepsilon_d^2 & 1 \end{pmatrix} \qquad m^\ell \propto \begin{pmatrix} 0 & \varepsilon_\ell^3 & \varepsilon_\ell^3 \\ \varepsilon_\ell^3 & {}^{\dagger}3\varepsilon_\ell^2 & 3\varepsilon_\ell^2 \\ \varepsilon_\ell^3 & 3\varepsilon_\ell^2 & 1 \end{pmatrix}$$

$\Rightarrow$ $\varepsilon_\ell = \varepsilon_d$ (Froggatt-Nielsen mixing in Higgs )††

$\Rightarrow$ SU(5) (SO(10)) relations between couplings†

Radiative corrns.

$$\chi^{10}\,\psi^{5}\,H^{5} \quad : \quad m_\tau = m_b \cdot \tfrac{1}{3}$$

$$\chi^{10}\,\psi^{5}\,H^{45} \quad : \quad m_\mu = 3\, m_s \cdot \tfrac{1}{3}$$

$$\mathrm{Det}(m^d) = \mathrm{Det}(m^\ell) \quad : \quad m_e = \tfrac{1}{3}\, m_d \cdot \tfrac{1}{3}$$

Neutrinos $\quad m^\nu_{ij}\, \bar\nu_{L_i} \nu_{R_j} + M^\nu_{ij}\, \nu_{R_i} \nu_{R_j}$

$$m^\nu \propto \begin{pmatrix} 0 & \varepsilon_u^3 & \varepsilon_u^3 \\ \varepsilon_u^3 & \varepsilon_u^2 & \varepsilon_u^2 \\ \varepsilon_u^3 & \varepsilon_u^2 & 1 \end{pmatrix} \qquad M^\nu \propto \begin{pmatrix} 0 & \varepsilon_\nu^3 & \varepsilon_\nu^3 \\ \varepsilon_\nu^3 & \varepsilon_\nu^2 & \varepsilon_\nu^2 \\ \varepsilon_\nu^3 & \varepsilon_\nu^2 & 1 \end{pmatrix}$$

Higgs mixing††

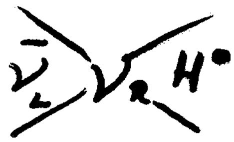
$\phi^{I_w = 0}$; New Higgs ... new expansion parameter.

• THE NEUTRINO SECTOR

Masses : $m_\nu \ll m_q, m_\ell$?

In the Standard Model ν's are massless :

⇨ No ν_R, no Dirac mass



$Q = 0 \ldots \nu$'s special,

⇨ Majorana Mass ? $M \underbrace{\nu_L \nu_L}_{}$

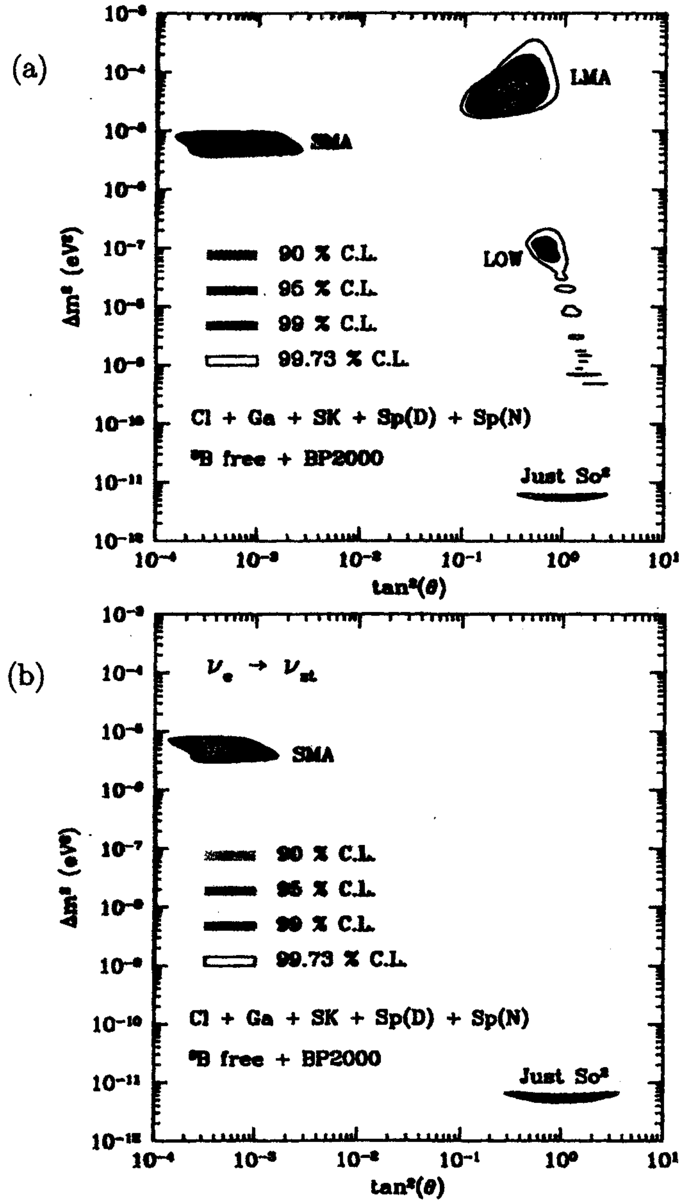
$L = 2$, $I_W = 1$ Forbidden in S.M.

ie Majorana mass requires L # violation + EW breaking

BEYOND THE STANDARD MODEL EXPECT NEUTRINO MASSES :

⇨	"See-Saw"	$\dfrac{\langle H \rangle^2}{M} \nu_L \nu_L$	; small mass if M large.
⇨	"Radiative"	$\dfrac{h^2}{16\pi^2} \lambda^2 \dfrac{\langle H \rangle^2}{M} \nu_L \nu_L$	; small without GUT mass.
⇨	Dirac	$h' \nu_L \bar{\nu}_R$	; h' small ?

Figure 3: Global solutions including Super-Kamiokande rate and with free ^{8}B and *hep* fluxes. (a) Active neutrinos. (b) Sterile neutrinos. The input data include the total rates measured in the Homestake, SAGE, GALLEX + GNO, and Super-Kamiokande experiments and the electron recoil energy spectrum measured by Super-Kamiokande during the day and also the spectrum measured at night. The best-fit points are marked by dark circles; the allowed regions are shown at 90%, 95%, 99%, and 99.73% C.L. .



over vacuum oscillations and no additional averaging is necessary when using this

Bahcall, Krastev, Smirnov

hep-ph/0103179

<u>NEUTRINOS ?</u>

<u>"SEE-SAW" MECHANISM</u> : <u>TWO</u> POSSIBLE NEUTRINO MASSES

$$m_{Dirac}\,\bar{\nu}_L\,\nu_R \;+\; M_{Majorana}\,\overbrace{\nu_R\,\nu_R}^{L=2}$$

$m^{\nu}_D = m_q$ (SO(10)) ?

SU(3)×SU(2)×U(1) invariant

$$M_M \overset{?}{\sim} \underset{10^{16}\,\text{GeV}}{M_X} \gg M_W,\, m_q$$

$$\boxed{(\bar{\nu}_L\;\; \nu_R)\begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix}\begin{pmatrix} \bar{\nu}_L \\ \nu_R \end{pmatrix}}$$

[illegible]
Gell-Mann
[illegible]
[illegible]

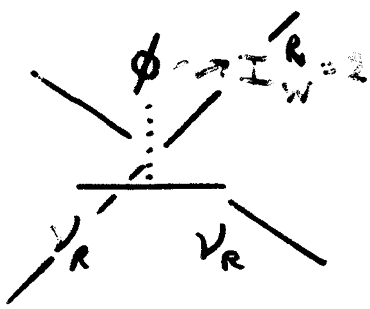
<u>Eigenvalues</u> :

$$m_{Heavy} \simeq M_M$$

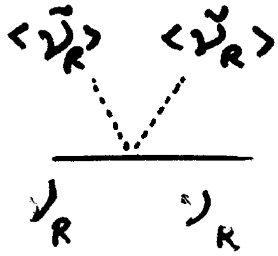
$$m_{Light} \simeq \frac{m_D^2}{M_M} \sim \frac{(175)^2}{10^{16}}\,\text{Gev} \sim 3.10^{-3}\,\text{ev}$$

"Natural" scale.

<u>BUT</u> M_M OFTEN LESS .... eg IN (<u>LEVEL 1) STRINGS</u>



$\Rightarrow$

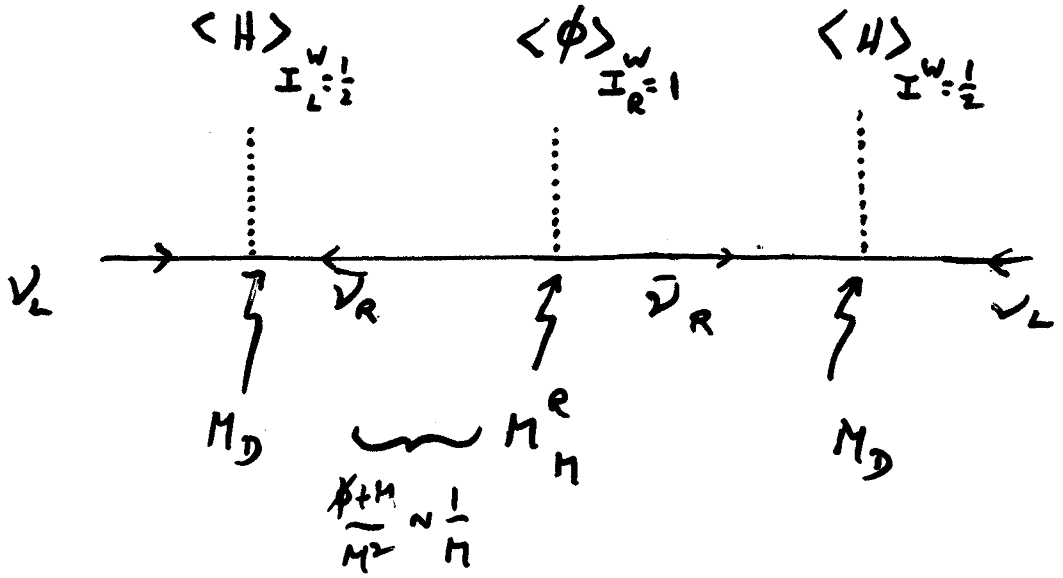


$$M_M \approx \frac{(10^{16})^2}{10^{19}}\,\text{Ge}$$

$$\Rightarrow \boxed{m_{\nu_\tau} \approx 1\,\text{ev}}$$

$m_{\nu_i} \propto m_{q_i}^2 \Rightarrow m_{\nu_e} \approx 10^{-9}\,\text{ev}$, $m_{\nu_\mu} \approx 10^{-4}\,\text{ev}$ ... large hierarchy

Graphical interpretation.



- Mass arises due to mixing of "light" states with "heavy" states

$$m_{eff}\, \nu_L \nu_L \quad : \quad m_{eff} = \frac{M_D \cdot M_M^R \cdot M_D}{M_M^{R\,2}}$$

$$\rightarrow \underline{\underline{M_D\ M_M^{-1}\ M_D.}}$$

MIXING IN THE LEPTON SECTOR.

$$M^E \quad , \quad M^\nu = M_D^\nu \, M^{-1} \, M_D^{\nu T}$$

$$U_{MNS} = V^{E\dagger} V^\nu$$

$\Rightarrow$ $\underline{V^E}$: $\quad M^E = \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix}$?

$$V^E_{12} = \sqrt{\frac{m_e}{m_\mu}} \quad ; \quad \sin^2 2\theta_{12} \sim 3.10^{-2} \quad \checkmark$$

$$V^E_{23} = \frac{c}{d} \lesssim \sqrt{\frac{m_\mu}{m_\tau}} \quad \text{TOO SMALL.}$$

$(\text{cf } V^D_{23} < 0.04)$

- $\underline{\text{Asymmetric form}}$? $\quad SU(5) : \; 5 = \begin{pmatrix} d_R^c \\ d_G^c \\ d_B^c \\ \nu \\ e \end{pmatrix}$

$$M^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 1? & 1 \end{pmatrix} \qquad M^E = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 1? \\ 0 & 2\lambda^3 & 1 \end{pmatrix}$$

cf Altarelli, Feruglio; Lola...

ν masses in $SU(3)$ model.

$$m^e = \begin{pmatrix} 0 & \varepsilon_d^3 & \varepsilon_d^3 \\ \varepsilon_d^3 & (1+\delta)\zeta\varepsilon_d^2 & \zeta\varepsilon_d^2 \\ \varepsilon_d^3 & \zeta\varepsilon_d^2 & 1+\zeta\varepsilon_d^2 \end{pmatrix}$$

$$m_D^\nu = \begin{pmatrix} 0 & \varepsilon_\nu^3 & \varepsilon_\nu^3 \\ \varepsilon_\nu^3 & \varepsilon_\nu^2 & \varepsilon_\nu^2 \\ \varepsilon_\nu^3 & \varepsilon_\nu^2 + O(\varepsilon_\nu^3) & 1+\varepsilon_\nu^2 \end{pmatrix}$$

M_M^ν ? ... need to identify $I_{W_L}=0$ field responsible for Majorana mass ... $\langle \Phi \rangle \equiv \langle \tilde{\bar{\nu}}_R^3 \rangle \langle \tilde{\bar{\nu}}_R^3 \rangle$

(vacuum alignment : $P \supset (\bar{\phi}_{3,i}\, \bar{\nu}_R^i)^2$

Then $\alpha(\nu_i\, \tilde{\bar{\nu}}^i)^2$ gives dominant (3,3) term...

$$M_M^\nu \propto \langle \Phi \rangle \begin{pmatrix} 0 & \varepsilon_\nu^4 & \varepsilon_\nu^4 \\ \varepsilon_\nu^4 & \varepsilon_\nu^2 & \varepsilon_\nu^2 \\ \varepsilon_\nu^4 & \varepsilon_\nu^2 & \alpha \end{pmatrix}$$

Evalues : $\varepsilon_\nu^6, \varepsilon_\nu^2, \alpha$

$$M^{\nu}_{eff} = m^{\nu}_{D} \cdot M^{\nu^{-1}}_{M} \, m^{\nu^{T}}_{D}$$

$$\mathcal{L} = \frac{\varepsilon_\nu^6 (\nu_\mu + \nu_\tau)^2}{\langle \Phi \rangle \varepsilon_\nu^6} + \frac{\varepsilon_\nu^4 (\nu_\mu + \nu_\tau + \varepsilon_\nu \nu_\mu + \varepsilon_\nu \nu_e)^2}{\langle \Phi \rangle \varepsilon_\nu^2}$$

dominant term

$$+ \frac{(\nu_\tau + \varepsilon_\nu^2 \nu_\mu + \varepsilon_\nu^3 \nu_e)^2}{\langle \Phi \rangle \alpha}$$

$\Rightarrow$ Near Bi-maximal mixing, $m_1/m_2 \sim \varepsilon_\nu^{-4}$ ... Low or vac oscillation solution.

SUMMARY.

- Family symmetries can generate viable hierarchy for fermion masses + mixings in terms of symmetry breaking parameters

- The symmetry can simultaneously describe quark, charged lepton + neutrino masses... the different mixing angles can arise naturally through the "see-saw" mechanism ... large neutrino mixings arise because of small quark mixing!

- Here I explored simplest + most constrained case of symmetric mass matrices. Relaxing this condition leads to other means of describing large neutrino mixing ... I will consider some examples later.