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*SUMMER SCHOOL ON PARTICLE PHYSICS*

*18 June - 6 July 2001*

# GRAND UNIFICATION AND SYPERSYMMETRY

## Lectures I and II

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# ICTP Summer School Lectures (2001)

## Grand Unification and Supersymmetry

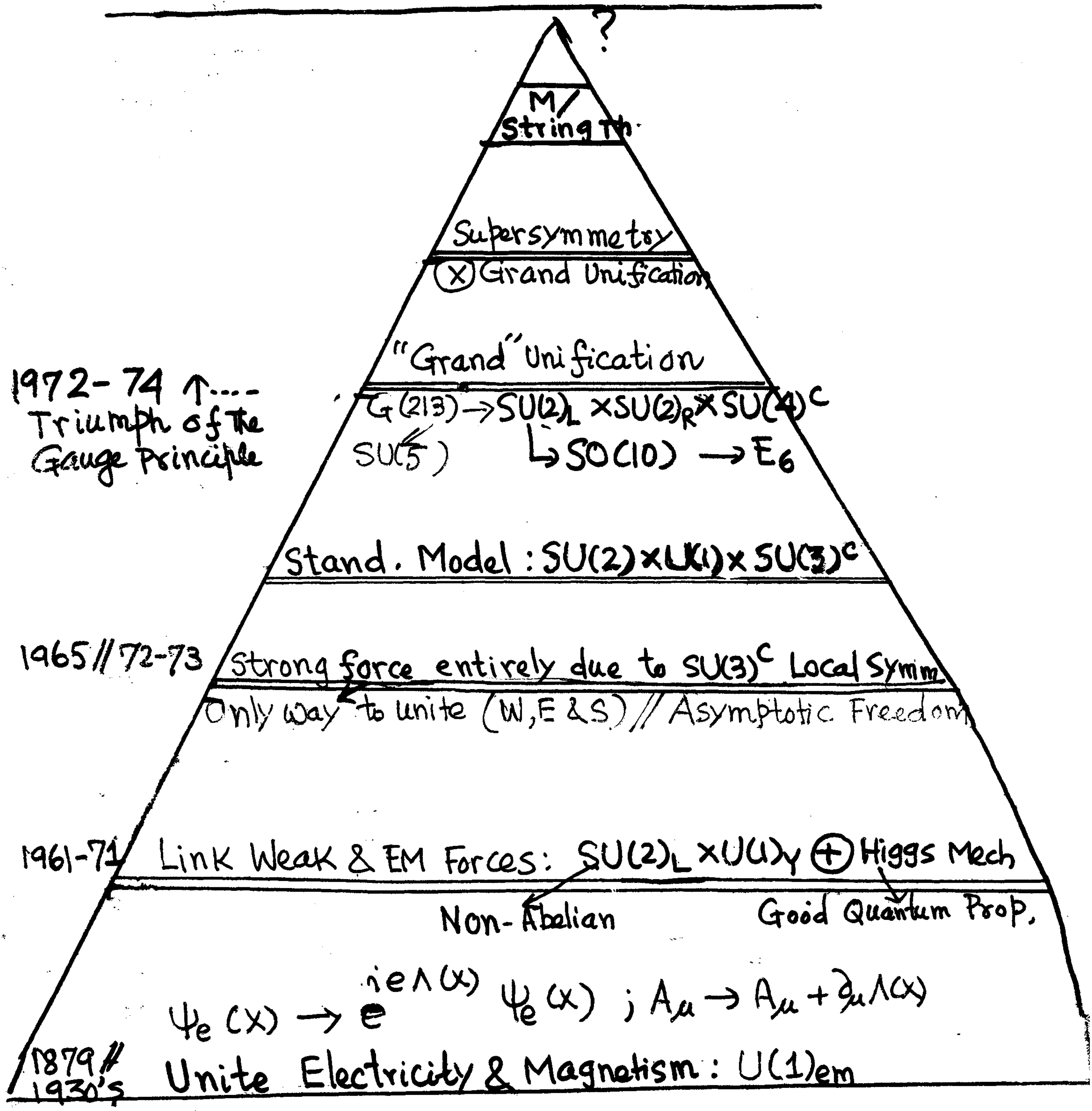
J. C. Pati

I) Introduction / Motivations / Overview

II) Fermion Masses & Mixings // Proton Decay in SUSY G(224) / SO(10)

III SUSY Breaking // SUSY CP Violation in G(224) / SO(10)

# I. Steps in the Unification Ladder



?

M/ String Th.

Supersymmetry

ⓧ Grand Unification

"Grand" Unification

1972-74 ↑....
Triumph of the Gauge Principle

$G(213) \to SU(2)_L \times SU(2)_R \times SU(4)^c$

$SU(5)$ $\quad \hookrightarrow SO(10) \to E_6$

Stand. Model : $SU(2) \times U(1) \times SU(3)^c$

1965 // 72-73 Strong force entirely due to $SU(3)^c$ Local Symm.

Only way to unite (W, E & S) // Asymptotic Freedom

1961-71 Link Weak & EM Forces: $SU(2)_L \times U(1)_Y$ ⊕ Higgs Mech

Non-Abelian $\quad$ Good Quantum Prop.

$$\psi_e(x) \to e^{ie\Lambda(x)} \psi_e(x) \; ; \; A_\mu \to A_\mu + \partial_\mu \Lambda(x)$$

1879 // 1930's Unite Electricity & Magnetism : $U(1)_{em}$

## ① Motivations for Physics Beyond The SM

- The SM based on $SU(2) \times U(1) \times SU(3)^c$ extremely successful → At least upto 100 GeV

→ A Triumph of QFT
  - Gauge Principle / Spont SB / Renormalizability.

- Next step → "Grand Unification"

Proposes a unity of $\{q \,\&\, l\}$ & of their 3 basic Forces (Weak, EM, Strong)

- 1972 This Concept introduced on purely aesthetic grounds, before the empirical successes of the SM were in place.

Conceptual Shortcomings of SM

- Five Scattered multiplets in a family
- Peculiar (Arbitrary) $Y_W$
- No Compelling reason for $Q_{em}$ quantized & $\boxed{Q_e = -Q_p}$
- Coexist. of $(q, l)$ & Coexist. of Weak, EM & Strong Forces $(g_1, g_2, g_3)$?

Idea of grand unif postulated to precisely remove these shortcomings.

$\downarrow$

A. $F = \{q, l\}$ → One Matter

$\downarrow$

B. Gauge $G$ → (Weak, EM & strong Int)

→ One Force

$G \supset SU(2)_L \times U(1) \times SU(3)^c$

→ obs. diff. between ($q$ & $l$) & the 3 gauge forces → Low-Energy Phenomena

→ Spont. Br. of $G$

$$G \xrightarrow{M \gg 1 \text{ TeV}} SU(2) \times U(1) \times SU(3)^c$$

→ at Energies > M, distinctions between ($q$, $l$) & (S, EM, Weak) forces would disappear

→ $\boxed{\text{TRUE UNITY should manifest at } E > M}$

→ Still Can See Shadows / Reflections at Low Energies.

## ④ "Grand" Unification

$$G(213) = SU(2)_L \times U(1)_{Y_W} \times SU(3)^c$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3} ; \ (u_{r,y,b})_R^{4/3} ; \ (d_{r,y,b})_R^{-2/3} , \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} ; (e^-_R)^{-2}$$

$$Q_{em} = I_{3L} + Y_W/2$$

5 disconnected multiplets in 1 Family // $Y_W$ ? //

$Q_{em}$ ? // $Q_{e^-} = -Q_p$ // Co-exist of $(q, l)$ // $g_1, g_2, g_3$ ?

$$\Downarrow$$

JCP & Salam 72-73

$$\boxed{G(224) = [SU(2)_L \times SU(2)_R \times SU(4)^c] \otimes (L \leftrightarrow R)}$$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R}$$

All 16 in one L-R Conj. mult / $Y_W$ / $Q_{em}$ quantized /

$Q_{e^-} = -Q_p$ / $\{q, l\}$ unif / (W, E, S) / $\boxed{\nu_R}$ / $\boxed{B-L}$ Generator

$$\boxed{Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}}$$

$$\Downarrow$$

$G(213) \to SU(5)$

$\bar{5} + 10$

Georgi & Glashow (74)

NO $\nu_R$, NO B-L

$SO(10)$ : $\underset{\sim}{16}$ (1 gauge coupling / $g_1 = g_2 = g_3$ at $M_U$)

(Georgi // Fritzsch, Minkowski (74/75)

# Understanding $Q_p = -Q_{e^-}$

In Stand. Model : $SU(2)_L \times U(1)_Y \times SU(3)^C$

$$Q_{em} = I_{3L} + Y/2$$

↑ Not Quantized

In $G(224)/SO(10)$

$$Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}$$

$$\left.\begin{array}{l} p = u_r\, u_y\, d_b \\ e^- = d_\ell \end{array}\right\} \therefore\ pe^- = \boxed{u_r\, u_y\, d_b\, e^-}$$

$$\Downarrow$$

$$\sum_{r,y,b,\ell} Q_{em} = 0 \ ; \ \sum_{u,d} Q_{em} = 0$$

$$\Downarrow$$

$$Q_{pe^-} = 0$$

$$\Downarrow$$

$$\boxed{Q_p = -Q_{e^-}}$$

# Worth Noting Dirac's Explanation of Quantization of Electric charge

**Dirac** $\vec{E} \leftrightarrow \vec{B}$

Existence of a magnetic monopole "$g$"

$$\Rightarrow \quad \boxed{e = \frac{2\pi n}{g}} \quad , n = 0, \pm 1, \pm 2, \cdot$$

Quantized

Interesting that one of the motivations for grand unification is charge quantization

Realistic Symmetries

$$\begin{tikzcd} \boxed{SU(2)_L \times SU(2)_R \times SU(4)^c} \arrow[d, "SSB"] \\ U(1)_{em} \times SU(3)^c \end{tikzcd}$$

, SU(5), SO(10)

$$\left.\begin{array}{l} \text{Quantization of} \\ \text{electric charge within} \\ \text{a Spont'ly broken realistic} \\ \text{Gauge Theory} \end{array}\right\} \Rightarrow \text{Topological Magnetic Monopoles}$$

't Hooft/Polyakov (1974)

$$\boxed{g_{smallest} = \left(\frac{2\pi}{e}\right) \cdot K}$$

## Spontaneous Breaking of Parity

J.C. Pati, A. Salam //
Mohapatra / J.C.P. //
Senjanovic ,
Mohapatra

$$SU(2)_L \times SU(2)_R \times SU(4)^c$$

$$F_L^e \leftrightarrow F_R^e$$

$$\vec{W}_L \leftrightarrow \vec{W}_R$$

$$"\phi_L" \leftrightarrow "\phi_R"$$

$$g_L \leftrightarrow g_R$$

Parity Can Break Spont'ly in this theory if

$$\langle \phi_L \rangle = 0 , \ \langle \phi_R \rangle \neq 0$$

$$\Downarrow$$

$m_{W_R} \gg m_{W_L} \Rightarrow$ Parity Violation a Low Energy Effect, would disappear at higher energies.

Note that the gauge symmetry $SU(2)_L \times SU(2)_R$ is <u>L ↔ R symmetric</u>, <u>Yet chiral</u>.

G(213)

| SU(5) | G(224)/SO(10) |
|---|---|
| $\underset{\sim}{\bar{5}}+\underset{\sim}{10}$ | $\underset{\sim}{16}$ |
| $\boxed{\nu_R}$ × | $\boxed{\nu_R}$ ✓ |
| $\boxed{B-L}$ × | $\boxed{B-L}$ ✓ |
| $\boxed{m_t = m_{\nu_\tau}^{Dirac}}$ × | $\boxed{m_t = m_{\nu_\tau}^{Dirac}}$ ✓ |
| L-R × | L-R ✓ |
| $\cancel{P}, \cancel{C}$ Explicit | $\cancel{P}, \cancel{C}$ spont. |
| Proton Decay $\to \bar{\nu}K^+$ ✓ | $p \to \bar{\nu}K^+$ ✓ |
| $p \to \mu^+K^0$ Negligible | $p \to \mu^+K^0$ prominent |

# Advantages of The G(224) - Symmetry

(i) Unification of all 16 members of a family within One left-right Self-Conjugate multiplet.

(ii) Quantization of Electric Charge // With $Q_{e^-} = -Q_p$

(iii) Quark-Lepton Unif. (SU(4)-Color)

(iv) Conservation of Parity at a fund. level

(V) Right-Handed ($\nu_R$'s) → A Compelling feature.

(Vi) B-L A Local Symmetry <
  - LEPTO/BARYOGENESIS
  - Autom. R-Parity.

$$G(224) \sim SO(4) \times SO(6) \longrightarrow SO(10).$$

# ⑤ Supersymmetry

Golfand, Likhtman (71)
Wess & Zumino (74)

$$\underset{\text{spin } 1/2}{\text{Fermions}} \xleftrightarrow{\;Q\;} \underset{\text{spin 0 or 1}}{\text{Bosons}}$$

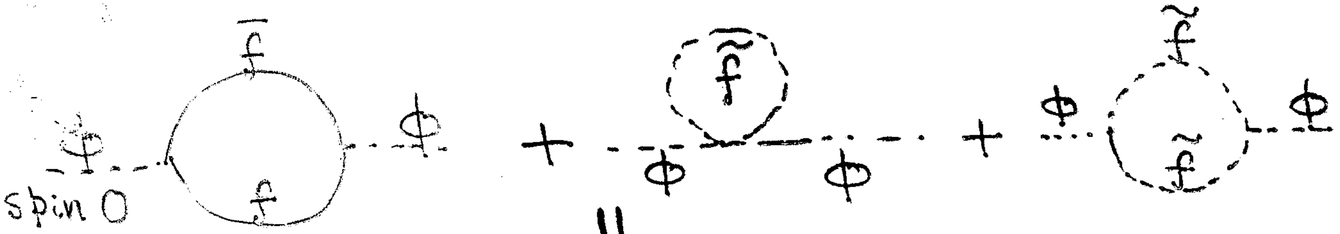
$$\{ Q, \bar{Q} \} = 2 \sigma^{\mu} P_{\mu}$$

$$(\text{quark})_{1/2} \Longleftrightarrow \text{squark } (\tilde{q})_0$$

$$(\text{lepton})_{1/2} \Longleftrightarrow \text{slepton } (\tilde{l})_0$$

$$(\text{gluon})_1 \Longleftrightarrow \text{gluino } (\tilde{g})_{1/2}$$

Remarkable quantum Properties



$$\Downarrow$$

$$\delta m^2_{Higgs} \sim \frac{\text{"}\alpha_y\text{"}}{\pi} \Big| \underset{\text{Fermion}}{m^2_f} - \underset{\text{Boson}}{m^2_{\tilde{f}}} \Big|$$

$$\Downarrow$$

Expect $\boxed{(m_{\tilde{q}}, m_{\tilde{l}}, m_{\tilde{g}}) \lesssim 1\,\text{TeV}}$ (LHC 2006)

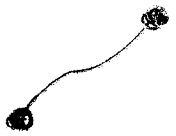
- Supersymmetry goes extremely well with the idea of grand unification.
- Lightest SUSY Particle → A Natural Candidate For Cold Dark matter.

# ⑤ String / M Theory   D = 10/11   / Green, Schwarz (1984) / Witten (95) .....

Point Particle ⟶  + Vibrational Modes

One Dim. string

$\left\{ \begin{array}{l} (q, \ell)_{1/2} + (\text{gluons}, W, Z, \gamma)_1 + (\text{Higgs})_0 \\ + \underline{(\text{graviton})}_2 + \text{SUSY Partners} + \ldots \end{array} \right.$

 ⟶ 

1 Dim World line   2D World sheet

- The Most Unifying Theory of All — Different Elementary entities Correspond to different Vibrational Modes of the Same String.
- Contains the ideas of Grand Unification & Supersymmetry
- Provides Scope For
  - a) Unification of all forces → including gravity
  - b) Good Quantum Th. of gravity
  - c) Rationale For origin of 3 families ?
  - d) Yielding All Parameters of Stand. Model / Hierarchical Yukawa Couplings / Masses & Mixings of all Fermions ?

BUT NO RELIABLE CONTACT YET With Observation

## ⑦ Viewing From Top to Bottom

**Scale**

Superstrings/M Th
$D = 10/11$

$\geq 10^{18}$ GeV → dist $\leq 10^{-32}$ cm

origin of flavor symm. & their breaking ↓ 3 families

$10^{18}$ ↓ $10^{16}$ GeV → $10^{-30}$ cm

Supersymmetric
G(224)/SO(10)/SU(5)
$d = 4$

Breaks ↓ at $10^{16}$ GeV

Standard Model G(213)
with effly SUSY spectrum
MSSM

$10^{16}$ GeV ↓ 1 TeV

q, l, W, Z, γ, gluons Massless

EW symm. Breaks $\langle \Phi \rangle \sim 300$ GeV
SUSY Ple ↓ Masses Felt

$U(1)_{em} \times SU(3)^c$

q, l, W, Z massive

String Th has not yet made reliable Contact with observation/ But Clear Suggestion for flavor Symm & their Breaking.

SUSY Unif G(224)/SO(10) Explains a no. of Obs. features/ Testable Predictions.

# Thermal History of the Universe

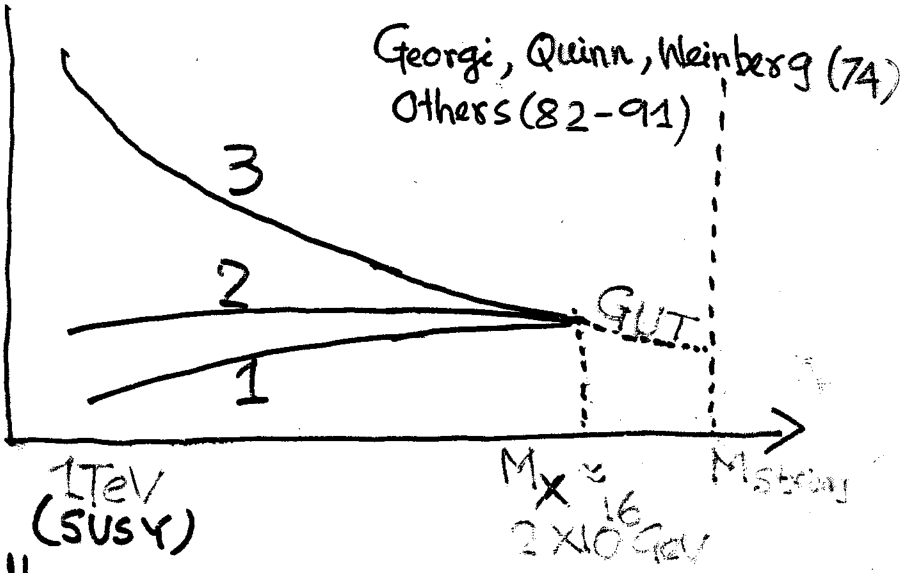
| Time | Event | Temp. | Energy(KT) |
|---|---|---|---|
| $10^{-44}$ sec. | Quantum gravity Imp.<br>($q, \bar{q}, e^-, e^+, \nu, \bar{\nu}, \gamma, \cdots$)<br>soup | $10^{32}$ K | $10^{19}$ GeV |
| $10^{-38}$ to $10^{-24}$ sec | Grand Unif Era/Inflation?<br>→Baryon or Lepton Excess Generated | $10^{29}$ to $10^{22}$ K | $10^{16}$ to $10^{9}$ GeV |
| $10^{-11}$ sec. | Electroweak Symm Breaking | $10^{16}$ K | 300 GeV |
| $10^{-5}$ Sec | Hadronization<br>$3q \to p, n$<br>$3\bar{q} \to \bar{p}, \bar{n}$<br>$q\bar{q} \to$ Mesons<br>$p\bar{p} \to$ Mesons | $10^{13}$ K | 300 MeV |
| 1/5 sec | Neutrinos decouple | $10^{11}$ K | 2 MeV |
| 100 sec. | Deuterium & Helium Synthesis | $10^{9}$ K | 1/10 MeV |
| $10^{6}$ yrs | Photons decouple Background Rad | $10^{3}$ K | 1/10 eV |
| $10^{10}$ yrs (Now) | Galaxies, Stars & We form | 3 K | $10^{-3.5}$ eV |
| $10^{34}$ Yrs | All Matter Decays? | | |

# III Evidence in Favor of Grand Unification/ Selecting The Route To Higher Unification

## ① Gauge Coupling Unification

$\sin^2\theta_W(m_Z)_{Th}$
$= 0.2315 \pm .003$
$\sin^2\theta_W(m_Z)_{obs}$
$= 0.2314 \pm .00017$

**LEP**



Georgi, Quinn, Weinberg (74)
Others (82-91)

⇓

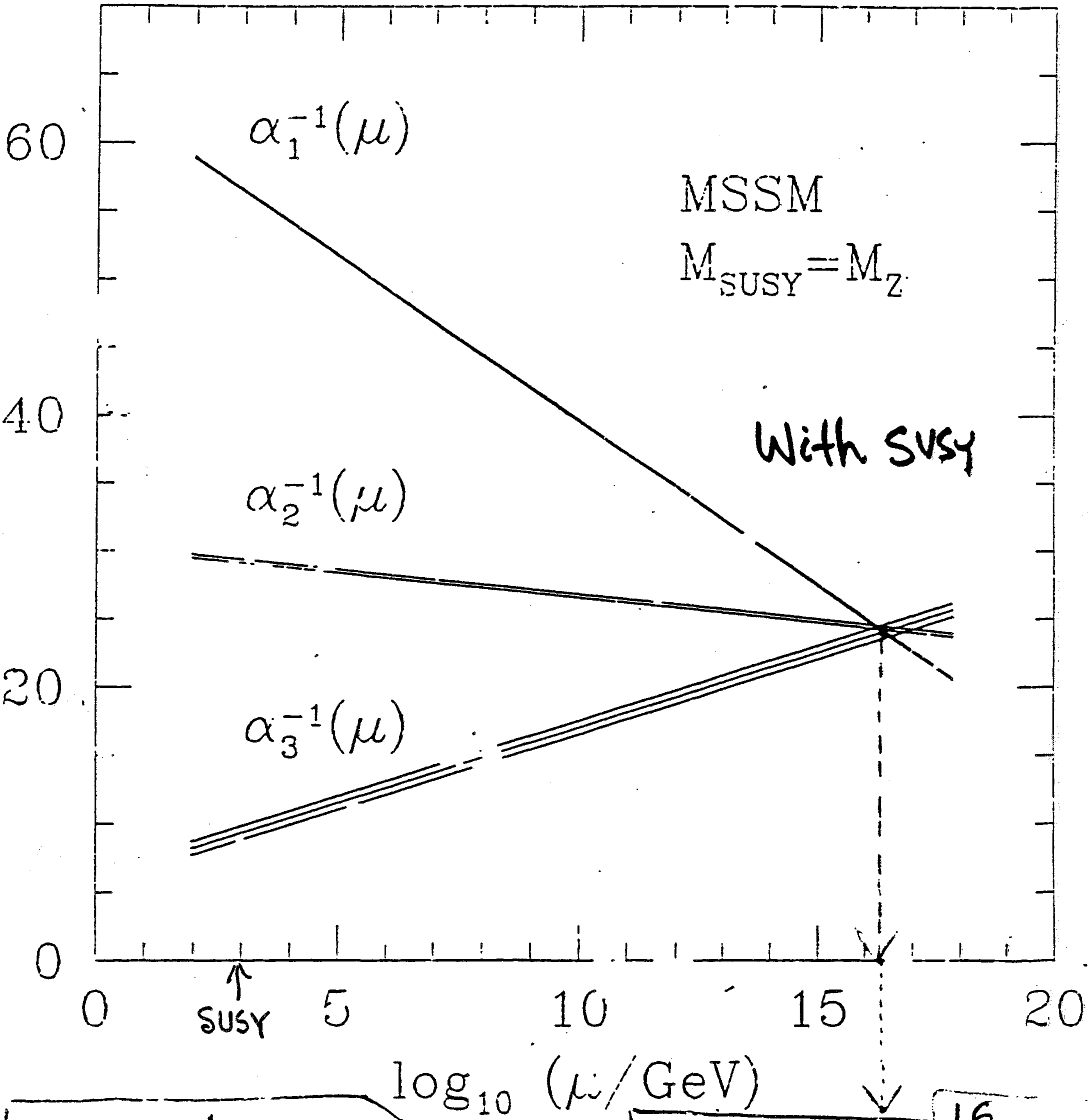
(a) Unity at $M_X \simeq 2\times10^{16}$ GeV

(b) Supports Grand Unif. & Supersymmetry

SUSY Masses ~100 GeV - 1 TeV ⟺ $\delta m_H^2 \lesssim 1\,\text{TeV}^2$

(c) Stand. Model G(213) Very likely has its origin in

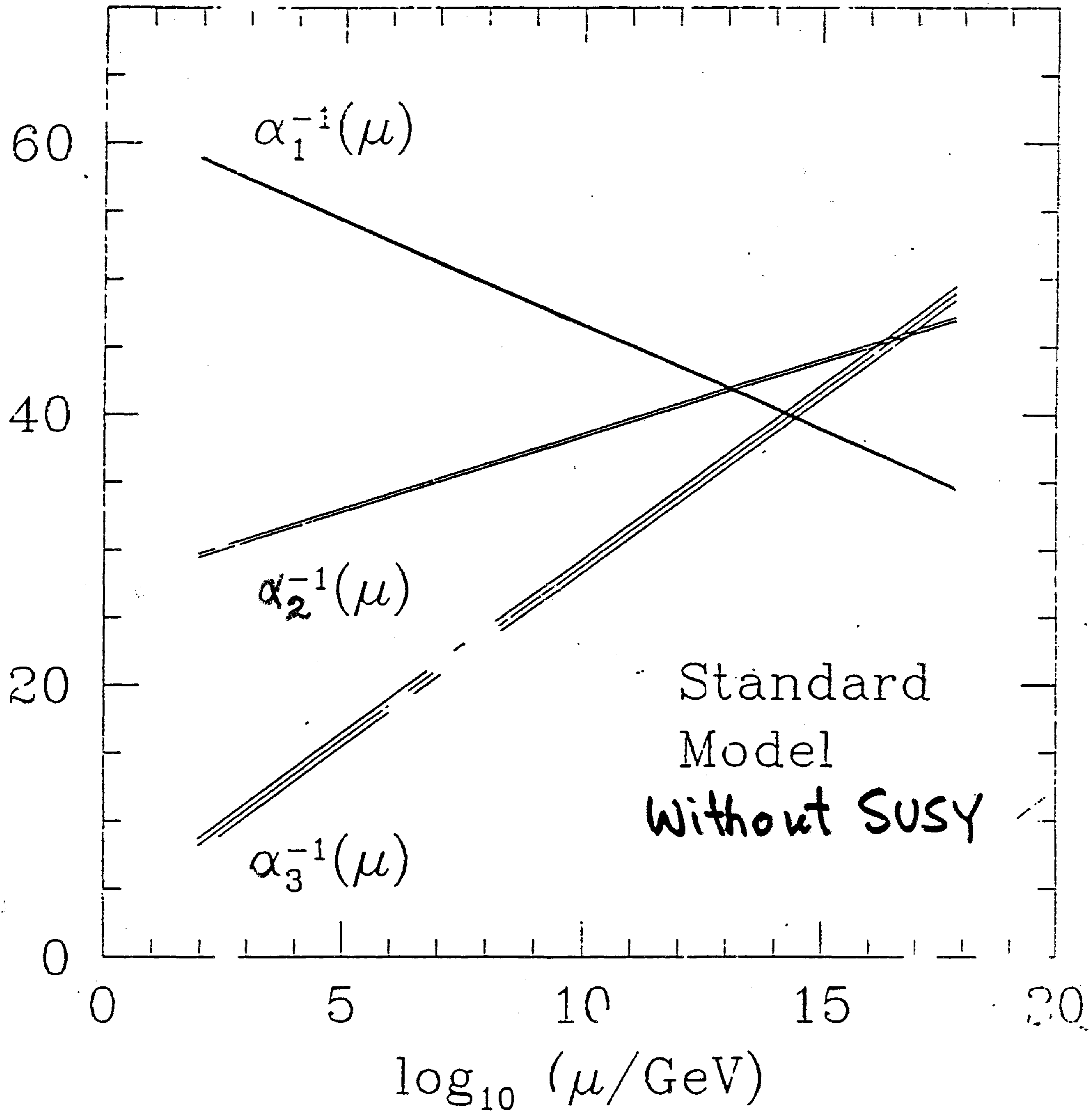
| | SU(5) | String G(224)/SO(10) | $[SU(3)]^3$ |
|---|---|---|---|
| Coupling Unif. | Yes | Yes | Yes |
| Quark-lepton Unif | Yes | Yes | NO |
| Proton decay | Yes | Yes | NO. |



Supports SUSY Unification

$M_{GUT} \approx 2 \times 10^{16}$ Gev

14.1



$\alpha_1^{-1}(\mu)$
$\alpha_2^{-1}(\mu)$
$\alpha_3^{-1}(\mu)$
Standard
Model
Without SUSY
60
40
20
0
0
5
10
15
20
$\log_{10}(\mu/\mathrm{GeV})$

# RGE for Gauge Coupling Evolution (SUSY)

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 + \sum_{j=1}^{3}\frac{b_{ij}}{8\pi^2}\alpha_i^2\alpha_j - \frac{\alpha_i^2}{32\pi^3}b_i^{Yuk}$$

$$\boxed{\equiv \ln \mu^2/M^2} \qquad b_i = \begin{pmatrix} 2n_g + \frac{3}{5} n_H \\ -6 + 2n_g + n_H \\ -9 + 2n_g \end{pmatrix}$$

→ no. of pairs of Higgs doublets $n_H = 1$ for MSSM.

MSSM ⇒ $b_3 = -3$ ; $b_2 = +1$ , $b_1 = 6\,\frac{3}{5}$

In one loop : $d\alpha_i/\alpha_i^2 = (b_i/2\pi)\,dt$

$$\Rightarrow \boxed{\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) - (b_i/2\pi)\ln(\mu^2/M^2)}$$

$$\alpha_1(M) = \alpha_2(M) = \alpha_3(M) \longleftrightarrow \text{Coupling Unification.}$$

$$\updownarrow$$

Predict

$$\boxed{\begin{aligned} \sin^2\theta_W(m_Z)_{\text{Theory}} &\simeq 0.2315 \pm .003 \\ \sin^2\theta_W(m_Z)_{\text{Expt}} &= 0.23124 \pm .00017 \end{aligned}}$$

$M_X \approx 2\times10^{16}$ GeV (MSSM or SUSY SU(5))

**Equivalently**: using measured $\sin^2\theta_W(m_Z)$, extrapolate $g_1, g_2$ → Find $M_X$, Extrapolate back $g_3$ to predict

Without GUT Scale Threshold Correction →

$$\boxed{\alpha_3(m_Z)^{0}_{MSSM\to GUT} = 0.125 - 0.13}$$

Langacker, Polonsky (Review) / Bagger et al // Bastero-Gil, Perez-Mercader // others.

$$\boxed{\alpha_3(m_Z)_{Expt} = 0.118 \pm .003}$$

Ple Data Book

⇒ 6 to 10 % discrepancy at $m_Z$

→ i.e. ≈ $\boxed{\text{2 to 3\% discrepancy at } M_X}$

MSSM ↓ GUT

$$\boxed{\alpha_3(m_Z)_{Net} = \alpha_3(m_Z)^{0}_{MSSM\to GUT} + \Delta_3}$$

THRESHOLD Corr. From GUT Scale Physics

Need $(\Delta_3/\alpha_3) \approx -(2 \text{ to } 3)\%$ at $M_X$

↓

$\approx -(6 \text{ to } 10)\%$ at $m_Z$.

# Interpreting Meeting Of The Gauge Couplings

## Most straightforward Interpretation

Have SUSY Grand Unification above $M_X \sim 2\times10^{16}$ GeV

$$\begin{array}{c} \text{SUSY } SU(5)/SO(10) \\ \langle\Sigma_i\rangle \Big\downarrow \sim M_X \sim 2\times10^{16}\text{ GeV (MSSM)} \\ SU(2)\times U(1)\times SU(3)^c \text{ (MSSM)} \end{array}$$

## Assuming String/M Theory (D=10) Fundamental

An alternative interp. is possible/String Th
→ G(213) or G(224) Can still give Coupling unif. at string scale:

$$M_{st} \approx g_{st}\times(5.2\times10^{17}\,\text{GeV}) \approx 3.6\times10^{17}\,\text{GeV}.$$

$$\approx 20\,(M_X)_{MSSM} \qquad \searrow \left(\alpha_{st}\approx\alpha_{GUT}^{MSSM}\approx0.04\right)$$

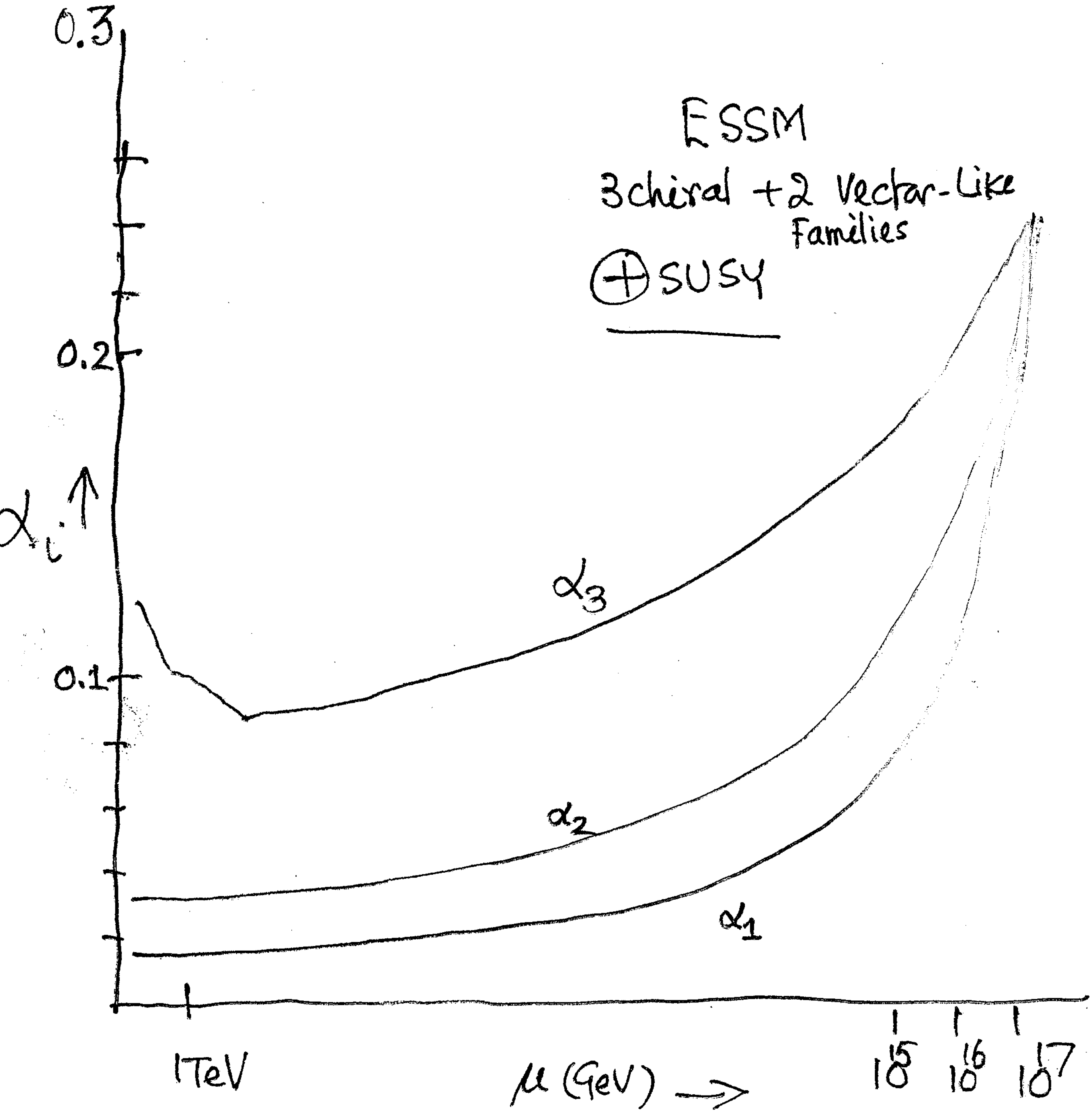
Possible Resolutions of Mismatch between $M_{st}$ & $M_X$ :

**(A)** String Duality : $M_{st}$ lowered to $M_X$ (Witten)

**(B)** ESSM : 3 chiral + A pair of Vector-like Ferm ($16_V+\overline{16}_V$) at TeV scale

Motivations
Babu, JCP (96)

(i) $\alpha_X \approx 0.25-0.3$ (Dilaton stabiliz)
(ii) $M_X \sim (\frac{1}{2}-2)\times10^{17}$ GeV
(iii) $\alpha_3(M_Z)^{\circ}_{ESSM} \approx 0.113-0.118$



$$\alpha_3(m_Z)^{\circ}_{ESSM} = 0.113 - 0.118$$

$$M_X \approx 10^{17}\,\text{GeV}\,(1/2 - 2)$$

Babu, Pati. (Phys. Lett. 96)

# String/M Th $(D=10)$

| | |
|---|---|
| $=4$ [SO(10) or SU(5)] × (G<sub></sub>Flav) × [$G_H$] | $D=4$ [G(213) or G(2213) or G(224)] × $G_{Flav}$ × $G_H$. |
| lewellen // Font et al // .... ) | (Antoniadis et al // Faraggi ...) |
| ① Coupling Unif. at $M_X$ easily understood // No mismatch ($M_X \leftrightarrow M_{st}$) | ① Mismatch ($M_X \leftrightarrow M_{st}$) ↓ Rely upon string-Duality and/or ESSM ..... |
| ② No promising (semi-realistic) 3 Family soln yet | ② ∃ promising semi-realistic 3 Family string solns |
| ③ Doublet-Triplet splitting soln → Not Yet | ③ Doublet-Triplet splitting easily solved because triplets naturally projected out |

Seems prudent to keep an open mind towards either possibility.

String - G(224) / SO(10)

Many Predictions identical /

But some differences - e.g. in proton decay

# NEUTRINO Masses

## DIRAC MASS

Like the electron, ν's Can have Dirac Mass if have both $\nu_L$ & $\nu_R$

$$m_D(\bar{\nu}_L\nu_R+\bar{\nu}_R\nu_L)$$

$\Delta Q=0$ ↑, $\Delta L=0$

$$h_D(\bar{\nu}_L\nu_R\langle\phi_H\rangle)+hc$$

↑ 247 GeV

## MAJORANA MASS

Unlike Electron, ν's can have Majorana masses With only $\nu_L$ or only $\nu_R$,

$$m_L\,\nu_L\nu_L+hc$$

$(\Delta Q=0$ ↑ $\Delta L=\mp 2)$

$$\lambda\,\nu_L\nu_L\langle\phi_H\rangle\langle\phi_H\rangle/M_{Pl}$$

⇓

$m_L\sim10^{-5}eV$

Too small for SuperK

If have $\nu_R$'s → Singlets of stand. Model

→ Can have superheavy GUT scale Majorana Masses preserving SM symmetry

$$\boxed{m_R\,\nu_R\nu_R+hc}\quad \Delta Q=0,\ \Delta L=\mp 2$$

Thus ν's are special

$\nu_R$'s KNOW ABOUT GUT SCALE PHYSICS //

$m_{Dirac}$ Knows about ELECTROWEAK PHYSICS.

## ③. $m(\nu_\tau) \approx 1/20\,eV$ (From SuperK, Assuming $m(\nu_\tau) \gg m(\nu_\mu) > m(\nu_e)$)

See-Saw Gell-Mann etal (79)/Yanagida (79)
Mohapatra, Senjanovic (80)

Dirac Mass

$$\begin{array}{c|cc} & \nu_L^\tau & \overline{\nu_R^\tau} \\ \hline \nu_L^\tau & 0 & m(\nu_D^\tau) \\ \overline{\nu_R^\tau} & m(\nu_D^\tau) & M_R \end{array} \longrightarrow \begin{bmatrix} -\frac{m(\nu_D^\tau)^2}{M_R} & 0 \\ 0 & M_R \end{bmatrix}$$

Majorana Mass
Superheavy

$$\Rightarrow \quad \boxed{m(\nu_L^\tau) \approx m(\nu_D^\tau)^2/M_R}$$

### (i) Get $M_R$ From SUSY Unif Scale $M_X \sim 2\times10^{16}$ GeV

$$\underset{(\approx 1)}{f_{33}}\, \frac{16_3\, 16_3 \langle\overline{16_H}\rangle\langle\overline{16_H}\rangle}{\underset{(2\times10^{18} GeV)}{M_{Pl}}} \Rightarrow M_R \approx \frac{(2\times10^{16} GeV)^2}{2\times10^{18} GeV} \approx \boxed{2\times10^{14} GeV}$$

JCP (98)

### (ii) Get $m(\nu_D^\tau)$ Using SU(4)-Color

$$\boxed{m_b^0 \approx m_\tau^0}$$

$$m(\nu_D^\tau) = m_t(m_X) \approx \boxed{100\ GeV}$$

$$\Rightarrow \boxed{m(\nu_L^\tau) \approx \frac{(100\ GeV)^2}{2\times10^{14} GeV} \sim (1/20\ eV)\ (1/2 \text{ to } 2 \text{ say})}$$

Thus the SuperK Result brings to light the

- Existence of $\nu_R$ (A NEW FORM OF MATTER)
- Reinforces the ideas of (i) SU(4)-Color (ii) L-R Gauge Symmetry & (iii) SUSY UNIFICATION.

In short, just this single piece of information ( $m(\nu_L^\tau) \sim 1/20\,\text{eV}$ )

**Disfavors**

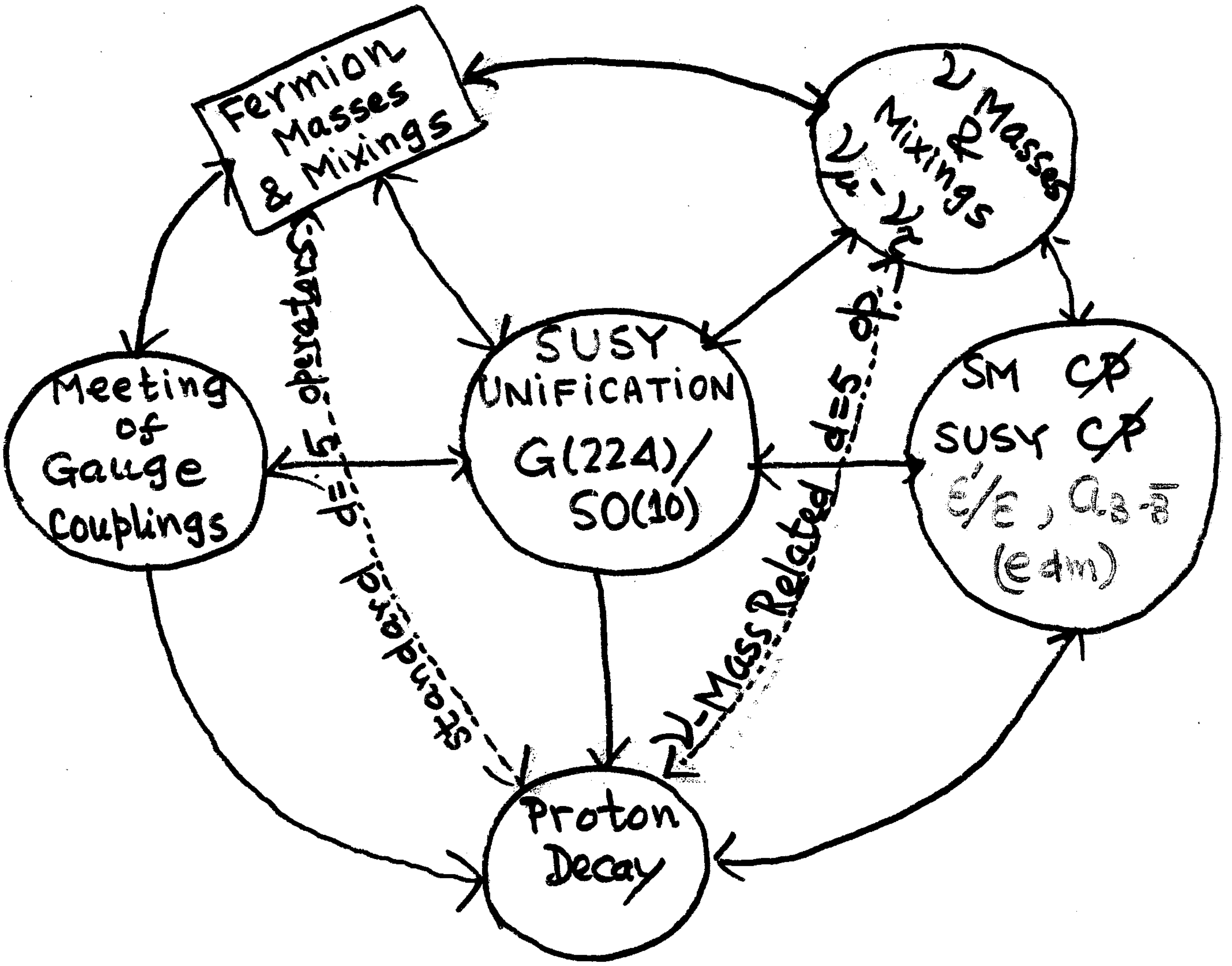
- $\cancel{SU(5)}$, $[\cancel{SU(3)}]^3$
- ~~Intermediate Scale ($\lesssim 10^{12}$ GeV) Breaking of B-L~~
- ~~Large Extra Dim.~~

**Favors**

$SU(2)_L \times SU(2)_R \times SU(4)^c$ / SO(10) Route To Higher Unification with SUSY //

Single step Breaking of G(224)/SO(10) at $M_X \sim 2 \times 10^{16}$ GeV.

# ② UNIFICATION LINKS



1) K.S. Babu, JCP - Linking $\epsilon'/\epsilon$ & $B-\bar{B}$ Asym. with ν-oscillation Through SUSY Unif. (To appear // SUSY2K)

2) K.S. Babu, JCP & F. Wilczek "Fermion Masses, ν-oscill & Proton decay in the Light of SuperK" - hep-ph/981538 / Nucl. Phys. B (2000)

3) A. Faraggi & JCP "Anomalous U(1) in String Models as the Origin of SUSY Breaking & Squark-Deg." - hep-ph 9712596 // Nucl. Phy B (98).

# III Understanding Masses & Mixings of all fermions $(q, l, \nu)$ : A Predictive SO(10)- Framework

Babu, Pati , Wilczek (-ph/981538/Nuc. Phys. 2000)

## ① Lighter Eigenvalue primarily Through off-diag mixing with heavier one : (Flavor Symm)

$$\begin{array}{cc} d & s \\ \end{array}\begin{bmatrix} 0 & \epsilon \\ \epsilon & 1 \end{bmatrix} m_s^{(0)} \xrightarrow[\epsilon \ll 1]{} \begin{bmatrix} -\epsilon^2 & 0 \\ 0 & 1+\epsilon^2 \end{bmatrix}$$

$$\rightarrow \theta_c^d \approx \epsilon \approx \sqrt{m_d/m_s} \longleftrightarrow \text{SYMMETRIC Matrix}$$

$$\underset{0.21}{\theta_c} \approx \left| \underset{0.22}{\sqrt{m_d/m_s}} - e^{i\phi} \underset{0.06}{\sqrt{m_u/m_c}} \right|$$

Works pretty Well to 25% for all $\phi$.

## ② Need For Antisymm. Component

SYMMETRIC MATRIX $\Rightarrow$ $$\underset{.04}{V_{cb}} = \left| \underset{.17}{\sqrt{m_s/m_b}} - e^{i\chi} \underset{.06}{\sqrt{m_c/m_t}} \right|$$ **Fails**

Clue To the need for antisym. Comp. $m_{ij} \neq m_{ji}$

$$\theta \rightarrow \sqrt{m_1/m_2} \sqrt{m_{ij}/m_{ji}}$$

## ③ Need Contrib $\propto$ B-L

$q \neq l$

$V_{bc} \neq \theta^{osc}_{\nu_\mu \nu_\tau}$

$m_\mu^0 \sim 3 m_s^0$

$m_e \sim 1/3\, m_d^0$

## ④ Need Up-Down Asymmetry

$$V_{CKM} \neq \mathbb{1} \Rightarrow M_{up} \text{ not } \propto M_{down}$$

## B. Minimal Higgs For SO(10) Breaking

$45_H$, $16_H$, $\overline{16}_H$, $10_H$ , $\cancel{126}_H$ , $\cancel{120}_H$

Possible large Thresh. Corrections // Seems Not allowed by Strings

Minimal set // Also allowed by String Soln.

SO(10)

$\langle 45_H \rangle \propto B-L$ $\sim M_{GUT}$

$\langle 16_H \rangle^{\tilde{\nu}_R} = \langle \overline{16}_H \rangle \longrightarrow$ Break $SU(2)_R \times U(1)_{B-L}$

$\downarrow \sim M_{GUT}$

$SU(2)_L \times U(1)_Y \times SU(3)^c$

$\downarrow \langle 10_H \rangle \sim v_{EW} \sim 246$ GeV

$U(1)_{em} \times SU(3)^c$

$16_i\, 16_j\, 10_H \longrightarrow$ Symm. in $(i \leftrightarrow j)$ , indep. of B-L

$16_i\, 16_j\, \cancel{126}_H \longrightarrow$ Symm. // $\propto B-L$

$16_i\, 16_j\, \cancel{120}_H \longrightarrow$ Antisym // $\propto B-L$

# Minimal Higgs $(45_H, 16_H, \overline{16}_H, 10_H \,\|\, S, S')$

$$
\begin{array}{lll}
 & \mathcal{L}_{yuk} = h_{33}\, 16_3\, 16_3\, 10_H & \Rightarrow \begin{array}{l} m_b^0 = m_\tau^0 \\ m_t^0 = (m_{\nu_\tau}^0)_{Dirac} \end{array} \\
1 \dashleftarrow & & \\
\sigma \dashleftarrow & + h_{23}\, 16_2\, 16_3\, 10_H\, (S/M) & \rightarrow \text{SYMM. Mixing} \\
\epsilon \dashleftarrow & + a_{23}\, 16_2\, 16_3\, 10_H \cdot \underset{\propto B-L}{45_H/M} & \Rightarrow \begin{array}{l} \text{ANTI SYM} \\ m_\mu^0 \neq m_s^0 \end{array} \\
\hat{\eta} \dashleftarrow & + g_{23}\, 16_2\, 16_3\, 16_H^{(d)} \cdot 16_H/M & \Rightarrow \text{CKM} \neq \underline{1}
\end{array}
$$

$$
U = \begin{array}{c} \begin{array}{cc} c & t \end{array} \\ \begin{pmatrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{pmatrix} \end{array} m_U^{(0)} \qquad D = \begin{array}{c} \begin{array}{cc} s & b \end{array} \\ \begin{pmatrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0
$$

$$
N_{Dirac} = \begin{array}{c} \begin{array}{cc} \nu_\mu & \nu_\tau \end{array} \\ \begin{pmatrix} 0 & -3\epsilon+\sigma \\ 3\epsilon+\sigma & 1 \end{pmatrix} \end{array} m_U^0 \qquad L = \begin{array}{c} \begin{array}{cc} \mu & \tau \end{array} \\ \begin{pmatrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{pmatrix} \end{array} m_D^0
$$

Note quark ⟶ Lepton Correlation

$$\epsilon \to -3\epsilon\ ;\ \sigma \to \sigma\ ,\ \eta \to \eta\ ,\ 1 \to 1$$

up ⟶ down correlation

$$\epsilon \to \epsilon\ ,\ 1 \to 1\ ,\ \sigma \to \eta = \hat{\eta} + \sigma$$

$$m_U^0 \to m_D^0$$

$$U = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} c & t \\ \end{array} \left(\begin{array}{cc} 0 & \varepsilon+\sigma \\ -\varepsilon+\sigma & 1 \end{array}\right) m_U^0 \quad ; \quad D = \begin{array}{cc} s & b \\ \end{array}\left(\begin{array}{cc} 0 & \varepsilon+\eta \\ -\varepsilon+\eta & 1 \end{array}\right) m_D^0$$

---

$$N_{Dirac} = \begin{array}{cc} \nu_\mu & \nu_\tau \\ \end{array}\left(\begin{array}{cc} 0 & -3\varepsilon+\sigma \\ 3\varepsilon+\sigma & 1 \end{array}\right) m_U^0 \quad ; \quad L = \begin{array}{cc} \mu & \tau \\ \end{array}\left(\begin{array}{cc} 0 & -3\varepsilon+\eta \\ 3\varepsilon+\eta & 1 \end{array}\right) m_D^0$$

---

**Input: Assume all param. real for a moment**

INPUT: $m_t^{Phys} = 174\ \text{GeV}$, $m_c(m_c) = 1.37\ \text{GeV}$,

$m_s(1\text{GeV}) \approx 110\ \text{MeV}$, $(m_\mu \,\&\, m_\tau)$ observed

$$\Downarrow$$

$\sigma \approx 0.110$, $\eta \approx 0.151$, $\boxed{\varepsilon \approx 0.095}$

$m_U^0 \approx m_t(M_X) \approx 100\text{-}120\ \text{GeV}$ ; $m_D^0 \approx 1.5\ \text{GeV}$

---

$$\Downarrow$$

$$m_b^0 \approx m_\tau^0 (1 - 8\varepsilon^2) \Rightarrow m_b(m_b) \approx 4.7\ \text{GeV}$$

$$|V_{cb}| = \left| \underbrace{\sqrt{\frac{m_s}{m_b}}}_{(0.156)} \underbrace{\left(\frac{\eta+\varepsilon}{\eta-\varepsilon}\right)^{1/2}}_{(1/2.2)} - \sqrt{\frac{m_c}{m_t}} \left(\frac{\sigma+\varepsilon}{\sigma-\varepsilon}\right)^{1/2} \right| \approx |\sigma - \eta|$$

suppression

$$\approx 0.042$$

ENHANCEMENT $\approx 1.8$

$$\Theta^{osc}_{\nu_\mu\nu_\tau} = |\theta^{l}_{\mu\tau} - \theta^{\nu}_{\mu\tau}| = \left| \sqrt{\frac{m_\mu}{m_\tau}} \left(\frac{\eta-3\varepsilon}{\eta+3\varepsilon}\right)^{1/2} + \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_\tau}}} \right|$$

$$\Downarrow$$

$$\boxed{\sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} \approx 0.85 - 0.96} \Longleftrightarrow \boxed{\text{EXPT} : 0.9 - 1}$$

$$\left.\begin{array}{l}\text{In above,} \quad \Theta^{l}_{\mu\tau} \approx \cdot 44\,\text{rad} \approx 24^{\circ} \\ \dfrac{m(\nu_2)}{m(\nu_3)} \approx 1/20, \quad \Theta^{\nu}_{\mu\tau} \approx \cdot 20\,\text{rad} \approx 12^{\circ}\end{array}\right\}.$$

Yet $\Theta_{oscil}\,(\nu_\mu - \nu_\tau)$ is large, while $\nu$-masses are hierarchical.

## Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix} \times m_D^0$$

2 New param $(\epsilon', \eta')$, but 5 new Observables just in $(q, \ell)$ System $\Rightarrow$ 3 New predictions for $(q,\ell)$ // With $\epsilon' = 0$ $\rightarrow m_u \rightarrow 0$

## $\nu$ Majorana Masses

$$M_R^\nu = \begin{pmatrix} Z & 0 & \omega \\ 0 & 0 & Y \\ \omega & Y & 1 \end{pmatrix} M_R$$

Saw before,

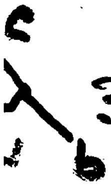
$M_R \approx (1/2 - 1) \times 10^{15}\,\text{GeV}$

Expect $y \sim 1/10$

# ④ Fermion Masses & Mixings Within a G(224) / SO(10) Framework

Babu, JCP, Wilczek (99)

| Predictions | Obs. Values |
|---|---|
| 1) $m_b(M_X) \approx m_\tau(M_X) \Rightarrow m_b^{\text{Phys}} \approx 4.7\,\text{GeV}$ | 4.5 GeV |
| 2) $(m_{\nu_\tau})^{\text{Dirac}}(M_X) \approx m_t(M_X) \Rightarrow \boxed{m(\nu_L^\tau) \sim 1/20\,\text{eV}}$ <br> SU(4)-Color | $(1/15 - 1/30)\,\text{eV}$ |
| 3) $V_{cb} \approx 0.043$ <br> $\Updownarrow$ | $0.040 \pm .002$ |
| 4) $\sin^2 2\Theta^{osc}_{\nu_\mu \nu_\tau} \approx \boxed{0.85-0.96}$ | $(0.9-1.0)$ |
| 5) $\Theta_{\text{cabibbo}} \approx 0.22$ | 0.21 |
| 6) $m_d \approx 8\,\text{MeV}$ | $9 \pm 2\,\text{MeV}$ |
| 7) $V_{ub}/V_{cb} \approx 0.07$ | $0.09 \pm .02$ |
| 8) $\nu_e - \nu_\mu$ — Small Angle MSW (Generic) | Disfavored (2.5) |
| — Large " " Possible | Somewhat Favored |



All Seven Good to Within 10% !

All Correlated with the Group Theory of G(224) / SO(10)

# Summary on Fermion Masses & Mixings

Given the bizarre pattern of masses & mixings of quarks, charged leptons, and neutrinos, it seems remarkable that this simple pattern of fermion mass matrices, motivated by ~~Group~~ Th of G(224)/SO(10) makes 7 predictions in agreement with observation to within 10 %.

→ Reason to take this framework seriously
↗ Proton Decay
↘ CP Violation

---

Do need to understand the origin of the flavor symmetries → hierarchical entries.

# B-L $\subset$ SU(4)-Color Suggested by

$$D = \begin{array}{c} \\ \\ \end{array}\left[\begin{array}{cc} s & b \\ 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{array}\right] m_D^{(0)} \quad , \quad L = \left[\begin{array}{cc} \mu & \tau \\ 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{array}\right] m_D^{(0)}$$

(1) $V_{cb} \longleftrightarrow \Theta^{osc}_{\nu_\mu \nu_\tau}$

$$\Theta_{sb} = \sqrt{\frac{m_s}{m_b}} \underbrace{\left(\frac{\eta+\epsilon}{\eta-\epsilon}\right)^{1/2}}_{\Downarrow} \qquad \Theta^{\ell}_{\mu\tau} = \sqrt{\frac{m_\mu}{m_\tau}} \underbrace{\left(\frac{\eta-3\epsilon}{\eta+3\epsilon}\right)^{1/2}}_{\Downarrow}$$

$$\underbrace{\underline{\text{suppression}} \approx 1/2.2 \longleftrightarrow \underline{\text{ENHANCEMENT}}}_{\text{CONSEQ. OF B-L \& ASYMM.}} \approx 1.8$$

---

(2) $m_b^{(0)} \approx m_\tau^{(0)}\,[1-8\epsilon^2] \longleftrightarrow$ Due to B-L

---

(3) $m_\mu^{(0)} \neq m_s^{(0)} \longleftrightarrow$ B-L

---

(4) B-L Local Symm $\longleftrightarrow$ Avoid Erasing of Baryon Asym by EW sphalerons (Kuzmin, Rubakov, Shaposhnikov

---

(5) $\left.\begin{array}{l} m_b^{(0)} \approx m_\tau^{(0)} \\ \underline{\text{and}}\ m_t^{(0)} \approx m^D_{\nu_\tau} \end{array}\right\} \longleftrightarrow$ SU(4) - Color

---

$\Downarrow$

$\therefore$ SU(4)-Color $\supset$ B-L strongly Sugg. by all <u>five features</u> $\Leftrightarrow$ G(224).

## ⑤ Proton Decay : The Hallmark of Grand Unification

One can argue that a gauge unification of {quarks & leptons} will inevitably

$\Longrightarrow$ (B, L) Non-Conservation

$\Downarrow$ Simplest

$$\boxed{\text{Proton Decay}}$$

(JCP, Salam 73
Georgi, Glashow 74)

| | proton $\rightarrow$ | lepton (anti-lepton) | + Mesons |
|---|---|---|---|
| B $\rightarrow$ | +1 | 0 | 0 |
| L $\rightarrow$ | 0 | ±1 | 0 |

**EXPT.**

Super K
$$\left\{\begin{array}{l} \Gamma^{-1}(p \rightarrow e^{+}\pi^{0}) \gtrsim 3.8 \times 10^{33} \text{ yrs} \\ \Gamma^{-1}(p \rightarrow \bar{\nu} K+) \gtrsim 1.6 \times 10^{33} \text{ yrs} \\ \Gamma^{-1}(p \rightarrow \mu^{+} K^{0}) \gtrsim 1 \times 10^{33} \text{ yrs.} \end{array}\right.$$

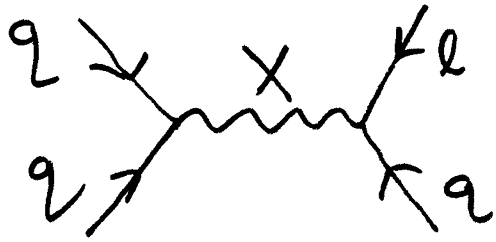
Minimal <u>Non-SUSY</u> SU(5)
$$\left\{\begin{array}{l} \Gamma^{-1}(p \rightarrow e^{+}\pi^{0}) < 10^{31} \text{ yrs} \\ \text{Excluded by expt//} \end{array}\right.$$

of course also excluded by measurement of $\sin^2\theta_W$ $\longleftrightarrow$ Lack of coupling unification.

# Mechanisms For Proton Decay in SUSY Unified Theories

## A) Mechanisms

### 1) d=6 Gauge Mediated : SU(5)/SO(10)



$$\rightarrow \frac{\text{Amp}}{g^2/M_X^2} \Rightarrow \Gamma^{-1}(e^+\pi^0) \approx 10^{35.3\pm1.5}\ \text{yrs.}$$

$$\Gamma(e^+\pi^0)^{-1}_{\text{expt}} > 3.8\times10^{33}\ \text{yrs.}$$

---

### 2) d=5, Color Triplet Higgsino Mediated

$$SO(10):\ 16_e = \{(u,d)_L,\ (\nu,e^-)_L \mid (u^c,d^c)_L,\ (\underset{\underset{(\nu_R)}{\downarrow}}{\nu^c},e^c)_L$$

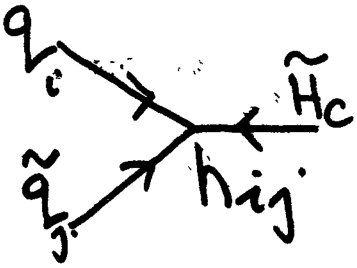
$$SO(10) \longrightarrow SU(2)_L\times SU(2)_R\times SU(4)^c$$

$$h_{ij}\ 16_i\, 16_j\, 10_H \rightarrow \text{Fermion Masses}$$

$$10_H = \underbrace{(2,2,1)}_{\substack{\text{2 Higgs doublets}\\ (H_u + H_d)}} + \boxed{(1,1,6)_H}$$

$$(1,1,6)_H \rightarrow \underset{\substack{(3^c)\\ \updownarrow \\ \tilde{H}_C}}{H_C} + \underset{\substack{(3^{*c})\\ \updownarrow \\ \tilde{H}_C'}}{H_C'}$$

$\boxed{\text{Spin } 1/2 \text{ Color triplet Higgsinos}}$



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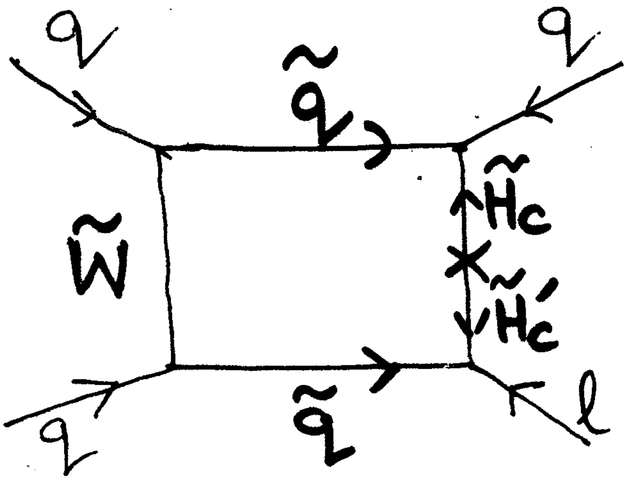
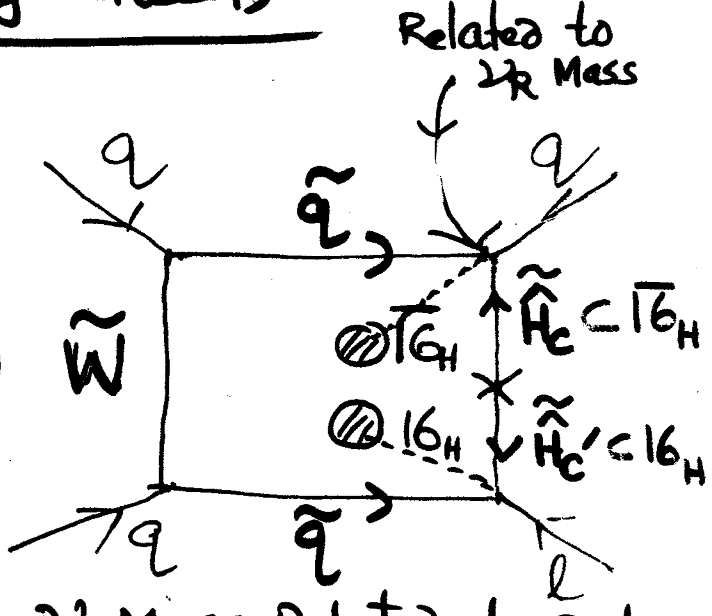
# ③ Features Which Enter Proton Decay Calculations in SO(10) or String-G(224)



Standard $d=5$ op.

Sakai, Yanagida // Weinberg

$\oplus$



$\nu$-Mass Related $d=5$ op

Babu, JCP, Wilczek (97)

## Amplitude depends upon

(1) $\left(m_{\tilde{W}}/m_{\tilde{q}}\right)\left(1/m_{\tilde{q}}\right)$ $\Big\}$ $\left(m_{\tilde{W}}/m_{\tilde{q}}\right) \approx 1/6\,(1/2 - 2)$ $\boxed{m_{\tilde{q}} \approx 1\,\text{TeV}\,(1/\sqrt{2} - \sqrt{2})}$

BPW // JCP (NNN Talk)

(2) $\beta_H\, u_L(\vec{k}) \equiv \langle 0 | d_L u_L u_L | p, \vec{k} \rangle$

[redacted] $\beta_H = \boxed{0.006\ \text{GeV}^3\ (\text{Lattice})}$ (JLQCD - 98)

$\downarrow$

In BPW-paper and in JCP (NNN-Talk)

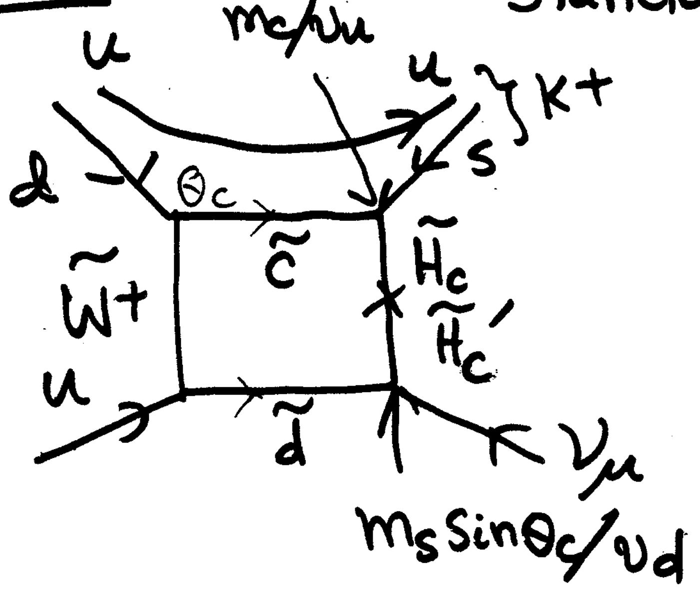
Took $\boxed{\beta_H = \left(0.006\ \text{GeV}^3\right)\left(\frac{1}{2} - 2\right)}$

Recent Improved Value $\beta_H = \left(0.014\ \text{GeV}^3\right)$ Lattice (JLQCD-2000)

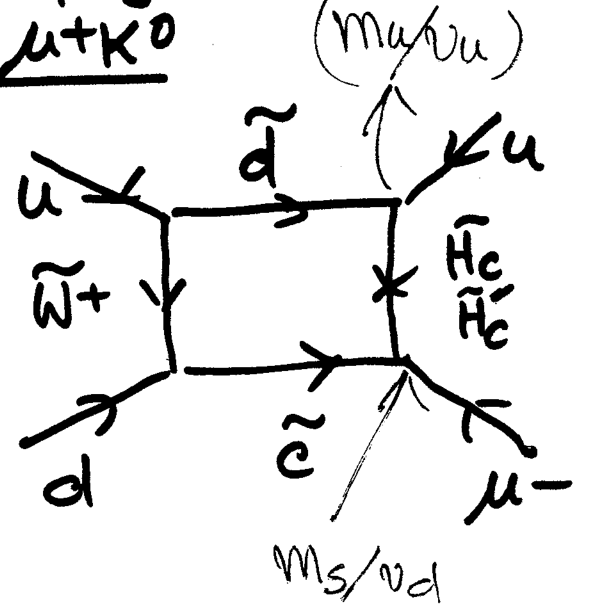
# Bose Symm + Color-Antisymm + Hier. Yukawas

"Standard" d=5

$\bar{\nu} K^+$



$\mu^+ K^0$



$$A(p\to\bar{\nu}_\mu K^+) \propto \frac{m_c m_s \sin^2\theta_c}{(v_u v_d)\, M_{H_c}}$$

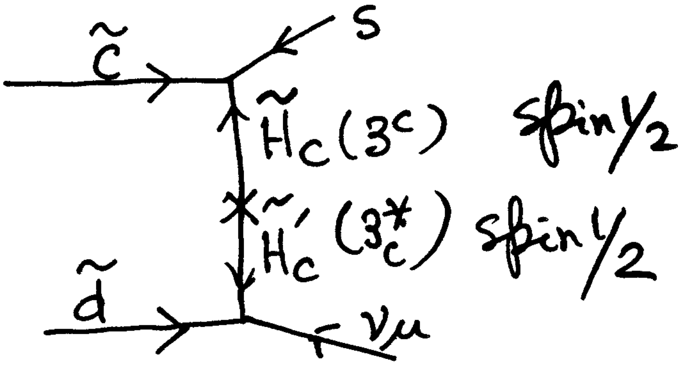
Amp
$$\propto \frac{m_u m_s \sin\theta_c}{v_u v_d\, m_{H_c}}$$

$$\left.\frac{\Gamma(\mu^+\pi^0)}{\Gamma(\bar{\nu}K^+)}\right|^{\text{Std. } d=5}_{\substack{SU(5)\\(\tilde{W})}} \approx \left(\frac{m_u}{m_c}\right)^2\left(\frac{R}{\sin^2\theta_c}\right) \approx 5\times10^{-4}$$

∴ Std. d=5 → $\mu^+\pi^0$ strongly suppressed

Holds for SUSY SU(5)

## dim 5



$$\hat{A}\,(SU(5))_{\tilde{c}\,\tilde{d}} = \frac{h^u_{22}\, h^d_{12}}{M_{H_c}} \sin\theta_c$$

$$\simeq \frac{m_c m_s \sin^2\theta_c}{v_u^2} \left(\tan\beta / m_{H_c}\right)$$

$$\simeq (1.9\times10^{-8})\left(\tan\beta / m_{H_c}\right)$$

$$\boxed{\simeq (2\times10^{-24}\,GeV^{-1})\left(\frac{\tan\beta}{2}\right)\left(\frac{2\times10^{16}\,GeV}{M_{H_c}}\right)}$$

$\tan\beta = v_u / v_d$ ; $v_u \simeq 174$ GeV.

$m_c \simeq 300$ MeV , $m_s \simeq 40$ MeV.

## MSSM ⟶ SUSY SU(5) // String G(224) or SO(10)

{3 chiral Families + $(H_u, H_d)$} ⊕ SUSY

**SUSY SU(5)** <u>d = 5 operator</u> $(p \to \bar{\nu}_\mu K^+)$,

$$\boxed{\hat{A}\,(SU(5)) \approx (1.9 \times 10^{-8})\tan\beta / M_{H_c}}$$

$M_{H_c} \lesssim 3 \times 10^{16}$ GeV (Coupling Unif) // $\tan\beta \gtrsim 3$

General

$$\boxed{\begin{aligned}\Gamma^{-1}(p \to \bar{\nu} K^+) &\approx (0.56 \times 10^{31}\,\text{yrs})\left[2 \times 10^{-24}\,\text{GeV}^{-1} / \hat{A}(\bar{\nu})\right]^2 \\ &\times \left(\frac{.67}{A_S}\right)^2 \left[\frac{.014\,\text{GeV}^3}{\beta_H}\right]^2 \left[\frac{1/6}{m_{\tilde{W}}/m_{\tilde{q}}}\right]^2 \left[\frac{m_{\tilde{q}}}{1.2\,\text{TeV}}\right]^2\end{aligned}}$$

$\Downarrow$

$$\boxed{\Gamma^{-1}(p \to \bar{\nu} K^+)_{SU(5)} \lesssim 0.5 \times 10^{33}\,\text{yrs} \times \left(\frac{\tan\beta}{3}\right)^{-2}}$$

If Stretch
All uncert
To extreme

$\Downarrow$

∴ Minimal SUSY SU(5) In Conflict with SuperK lower limit $\Gamma^{-1}(p \to \bar{\nu} K^+) \gtrsim 1.6 \times 10^{33}$ yrs.

SUSY SU(5) also disfavored by $\nu$-oscil. with $m(\nu_\tau) \approx 1/20$ eV.

# SUSY SO(10) Has some special features

$$W_H = \lambda\, 10_H \cdot 45_H \cdot 10'_H + M_{10'}\, 10'_H \cdot 10'_H$$
$$+ M_{16}\, 16_H \cdot \overline{16}_H + \lambda'\, \overline{16}_H\, \overline{16}_H\, 10_H$$

$$\langle 45_H \rangle \propto B-L$$

① "Standard" d=5 operators

$$\hat{A}(SO(10))_{std} \approx h_{33}^2 \left(\frac{M_{10'}}{M_{H_c}^2}\right)(2\times 10^{-5})$$
$$\approx \left(h_{33}^2 / M_{eff}\right)(2\times 10^{-5})$$

$$\boxed{M_{eff} \equiv M_{H_c}^2 / M_{10'} \approx M_X^2 / M_{10'}}$$

Note, $M_{eff} \neq M_{color\ triplet}$

$M_{10'}$ can be suppressed by 2 to 3 orders of magnitude compared to $M_{unif}$

∴ $\boxed{M_{eff} \text{ Typically Much larger than } M_U}$

Note also at this stage, No explicit tanβ dep.

# Doublet-Triplet splitting in SO(10)

$$W_H = \lambda\, 10_H\, 45_H\, 10'_H + M_{10}\, 10'^{\,2}_H + M_{16}\, 16_H\, \overline{16}_H$$
$$+ \lambda'\, \overline{16}_H\, \overline{16}_H\, 10_H$$

$$\langle 45_H\rangle = (a, a, a, 0, 0) \times \tau_2 \qquad \boxed{a \sim M_X}$$

$$(\bar 5_{10_H}\ \ \bar 5_{10'_H}\ \ \bar 5_{16_H}) \begin{bmatrix} 0 & \lambda\langle 45\rangle_H & \lambda'\langle\overline{16}_H\rangle \\ -\lambda\langle 45_H\rangle & M_{10'} & 0 \\ 0 & 0 & M_{16} \end{bmatrix} \begin{pmatrix} 5_{10} \\ 5_{10'} \\ 5_{\overline{16}} \end{pmatrix}$$

$\langle 45_H\rangle \propto B-L \rightarrow$ does not contribute to doublet masses

$\Rightarrow$ Light MSSM Doublets / Heavy Triplets

$$H_u = 10_u\,, \quad H_d = \cos\gamma\, H_d + \sin\gamma\, 16_d$$

$$\tan\gamma \equiv \lambda' \langle \overline{16}_H\rangle / M_{16}$$

$$\tan\beta / \cos\gamma \simeq m_t/m_b \approx 60$$

Small $\cos\gamma \Rightarrow$ small $\tan\beta$.

$$\boxed{A^{\text{proton}}_{d=5} \propto 1/M_{\text{eff}}} \qquad SO(10)$$

$M_{\text{eff}} = M_U^2/M_{10'}$ can be quite bit heavier than $M_U$ if $M_{10'} \ll M_U$.

## MSSM → SUSY G(224) / SO(10)

BPW Paper

Standard d=5 (For SO(10)) ⊕ New d=5 (For G(224) or SO(10))

$$\boxed{\hat{A}(SO(10))_{Std} \approx \frac{h_{33}^2}{M_{eff}}\left[2\times10^{-5}\right]}$$

D - T splitting

Note No explicit tanβ dep.

$$\left.\begin{array}{l} M_{eff} \approx M_X^2/M_{10'} \neq M_{H_C} \\ M_{10'} \sim (1/10 - 1/1000)M_X \end{array}\right\} \quad \boxed{M_{eff} \gg M_X}$$

Meff bounded above by Threshold Corr & demand of natural Coupling unif.

$$\boxed{\Delta\alpha_3(m_Z)_{DT} = \frac{\alpha_3(m_Z)^2(9/7)}{2\pi}\ln\frac{M_{eff}\cos\gamma}{M_X}}$$

$$\left.\begin{array}{l} \text{Natural Coupling unif} \to \text{Th. Corr} \lesssim 10\% \\ \quad \text{Match with right sign} \end{array}\right\}.$$

$$\Rightarrow \quad M_{eff}\tan\beta \lesssim 8\times10^{18}\ \text{GeV} \quad (\text{MSSM}/\text{SO(10)})$$

$$\Downarrow$$

$$\boxed{\begin{array}{c} M_{eff} \lesssim (4, 2.66, 1)\times10^{18}\ \text{GeV} \\ \tan\beta = 2, 3, 8 \end{array}}$$

This gives ⇒ upper limit on Proton lifetime.

$$\Rightarrow \Gamma^{-1}(p\to\bar{\nu}K^{+})_{\text{Std}} \approx \left[\begin{array}{ll} 0.7\times 10^{31\,^{+2.0}_{-1.7}} & (\text{I}) \\ 1.5\times 10^{31\,^{+2.4}_{-1.9}} & (\text{II}) \end{array}\right] \begin{array}{c} \text{SO(10)/} \\ \text{MSSM} \\ \tan\beta \gtrsim 3 \end{array}$$

$$\Rightarrow \boxed{\Gamma^{-1}(p\to\bar{\nu}K+)_{\text{std}} \lesssim \left[\begin{array}{ll} 0.7\times 10^{33}\ \text{yrs} & (\text{I}) \\ 3.7\times 10^{33}\ \text{yrs.} & (\text{II}) \end{array}\right]} \begin{array}{c} \text{SO(10)/} \\ \text{MSSM} \\ \tan\beta \gtrsim 3 \end{array}$$

This upper limit obtained by stretching all uncert to extreme ends.

$\beta_H = 0.007\ \text{GeV}^3$, $m_{\tilde{q}} \approx 2.4\ \text{TeV}$, $m_{\tilde{W}}/m_{\tilde{q}} \approx 1/2$

Thus MSSM embedded in SO(10) already tightly constrained, to the point of being rather disfavored, by empirical lower limit $\gtrsim 1.6\times 10^{33}$ yrs.

On the positive side, improvement by a factor of 2 or so should reveal proton decay, otherwise MSSM embedded in SO(10) would be excluded.

# ② The Case of Ext. Supersymmetric Standard Model (ESSM) → SO(10)

Babu, Pati (96, 97)
Babu, Pati, Stremnitzer (92, 93)

$3\,\text{chiral} + (H_u + H_d) + \left(\begin{array}{c}\text{One pair of Vector-Like Fam.}\\ 16 + \overline{16} \text{ at TeV-Scale}\end{array}\right)$

Consistent with (i) $\nu$-Counting // (ii) Precision EW Test

Apriori Motivations (i) Removal of Unif. MisMatch
(ii) Dilaton stabilization.

| MSSM → SO(10) | ESSM → SO(10) |
|---|---|
| $\alpha_{unif} \approx 0.04$ | $\alpha_{unif} \approx 0.25 - 0.3$<br>(semi-Perturbative) |
| $M_{unif} \approx 2 \times 10^{16}$ GeV | $M_{unif} \approx (1-2) \times 10^{17}$ GeV |
| $M_{eff} = M_{H_c}^2 / M_{10}'$<br>$\lesssim (2.7 \times 10^{18} \text{GeV}) \left(\frac{3}{\tan\beta}\right)$ | $M_{eff} \lesssim 1.8 \times 10^{19}$ GeV |
| $\alpha_3(m_Z)^0_{MSSM} \approx \cdot 125 - \cdot 13$ | $\alpha_3(m_Z)^0_{ESSM} \approx \cdot 112 - \cdot 118$ |

$$\boxed{\Gamma^{-1}(\bar{\nu}K^{+})^{\text{"Median"}}_{\text{Std}} \approx [S]_{\text{Med}}\underset{\substack{\downarrow\\ \text{Spectrum/Matrix element}}}{\{C\}_{\text{Med}}}(2.5\times10^{33}\,\text{yrs})(4/\tan\beta)^{2} \quad \text{SO(10)/ESSM.}}$$

$\beta_H = (0.014\,\text{GeV}^3)(1/2 - 2)$, $m_{\tilde{q}} \approx (1.2\,\text{TeV})(\frac{1}{\sqrt{2}} - 2)$

$m_{\tilde{W}}/m_{\tilde{q}} \approx 1/6\,(1/2 - 2) \Rightarrow \{C\} = \{64 - 1/32\}$

Take $\{C\}_{\text{med}} = 1/6$ to 6

Phase dep → $[S]_{\text{med}} = 2$ to 6.

**Contribs From Std. op only for ESSM → SO(10) With Param in Median Range: $\Gamma^{-1}(\bar{\nu}K^{+})_{\text{Std}}$**

| | | | |
|---|---|---|---|
| $\tan\beta = 4$<br>$[S] = 2.7$<br>$\{C\} = 1/2$ to 2 | $\tan\beta = 4$<br>$[S] = 6$<br>$\boxed{\{C\} = 1/6 \text{ to } 1}$ | $\tan\beta = 10$<br>$[S] = 5.4$<br>$\{C\} = 1$ to 6 | $\tan\beta = 10$<br>$[S] = 6$<br>$\{C\} = 1$ to 4 |
| $(2.5 \text{ to } 10)\times10^{33}$ yrs | $(1.8 \text{ to } 11)\times10^{33}$ yrs. | $(1.6 \text{ to } 10)\times10^{33}$ yrs. | $(1.8 \text{ to } 7)\times10^{33}$ yrs. |

**Note these are actual values of lifetimes, not upper limits.**

$$\Rightarrow \boxed{\left[\Gamma^{-1}(p\to\bar{\nu}K^{+})^{\text{ESSM}}_{\text{Std}}\right]_{\text{plausible}} \approx 10^{33} - 10^{34}\,\text{yrs}}$$

Even $m_{\tilde{q}} \approx 600\,\text{GeV}$, $\beta_H = 0.014\,\text{GeV}^3$, $m_{\tilde{W}}/m_{\tilde{q}} = 1/5$

$\Rightarrow \{C\} = 1/6$ Consistent with exptl. limit.

# ③ The New $d=5$ operators $\leftrightarrow$ $\nu$ Masses

$\left\{ \begin{array}{l} \text{MSSM or} \\ \text{ESSM} \end{array} \right\} \longrightarrow G(224)$ or $SO(10)$

$$\boxed{\Gamma^{-1}(\bar{\nu}K^+)^{\text{"nearly central"}}_{\text{New}} \approx (1-6)\times 10^{33}\,\text{Yrs.}}$$

Indep. of $\tan\beta$ $\uparrow$ SO(10) or String G(224)

Note for String-G(224) this is the only contribution to proton decay, because the Standard Color-triplets are projected out.

The new operators yield

$$B(\mu^+K^0)_{\text{Std+new}} \approx (1\% \text{ to } 50\%)$$

---

$\left.\begin{array}{l} \text{In SUSY} \\ \text{SU(5)} \end{array}\right\}$ $B(\mu^+K^0) \lesssim 10^{-3}$

# Summary of Results

1) $\Gamma^{-1}(\bar\nu K^+) \lesssim 0.5\times10^{33}$ yrs $\left\{\begin{matrix}\text{MSSM}\\ \downarrow\\ \text{SU(5)}\end{matrix}\right\} \to$ Excluded.

2) $\Gamma^{-1}(\bar\nu K^+) \lesssim 3.7\times10^{33}$ yrs $\{\text{MSSM} \to \text{SO(10)}\}$. Tightly Constrained ↓ Disfavored.

3) $\Gamma^{-1}(\bar\nu K^+) \approx 10^{33}-10^{34}$ yrs $\left\{\begin{matrix}\text{Most plausible}\\ \text{range For}\\ \text{ESSM} \to \text{SO(10)}\end{matrix}\right\}$

↑

Can realize even for nearly Central values of param // & also with $m_{\tilde q} < 1$ TeV // $\tan\beta \approx 3-20$ //

④ MSSM or ESSM → String G(224)   ONLY New op. Contribute.

$$\Gamma^{-1}(\bar\nu K^+)^{\text{Nearly Central}}_{\text{New}} \approx (1-6)\times10^{33}\ \text{yrs.}$$

$$\Gamma^{-1}(\bar\nu K^+) \lesssim 10^{34}\ \text{Yrs.}$$

⇓

Thus Conclude,

$$\boxed{\Gamma^{-1}(\bar\nu K^+) \lesssim (\tfrac{1}{2}-1)\times10^{34}\ \text{yrs.}}$$

## Section Summary

But for One missing piece – Proton Decay – evidence in Support of Grand Unification is Strong.

- Family Multiplet structure / charges
- Gauge Coupling Unification
- Neutrino oscillation
- $m_b^0 \approx m_\tau^0$ ; $\boxed{m(\nu^\tau_{Dirac}) \approx m^0_{top} \Rightarrow m(\nu^\tau_L) \sim \frac{1}{20}\,eV}$
- $V_{bc} \approx 0.04 \longleftrightarrow \sin^2 2\Theta_{\nu_\mu \nu_\tau} \approx 0.9 \leftrightarrow 1.0$
- B-L a generator ⟷ Lepto/Baryogenesis

$$\Downarrow$$

Not only support Grand Unification, but select out String G(224)/SO(10) Route To Such Unification

Unless the fitting of all the pieces a mere Coincidence, improvement in present limits by a factor of 5-10 Should turn up real events of proton decay!

$$\downarrow$$

$\boxed{\text{Need Next-Generation Detector.}}$

# ⑥ Concluding Remarks

- Our generation seems to have made remarkable progress on the road to unification.
- Many clues / signals in favor of the path taken so far.
- Exciting clues await:
  - (i) Large set of $\nu$-oscil. expts.
  - (ii) CP Violation ($B-\bar{B}$, $K-\bar{K}$, edm)
  - (iii) $(g-2)_\mu$ ? (BNL expt)
  - (iv) Dark Matter Searches
  - (v) Supersymmetry ↔ Tevatron (?), LHC.
  - (vi) Cosmolog. Parameters ($\Lambda, \Omega, \cdots$).
  - (vii) Does Proton Decay? (Next Gen. Detector)
- **Theoretical Challenges / Problems**

- Origin of 3 families / Flavor Symm / $\not{C}P$
- Origin of Supersymmetry Breaking

↕

- Choosing Ground state (Vacuum) in String / M Th
- Cosmological Constant ?