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SUMMER SCHOOL ON PARTICLE PHYSICS

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FERMION MASSES AND THE FLAVOUR PROBLEM

Lecture IV

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Please note: These are preliminary notes intended for internal distribution only.

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Fermion masses and the flavour problem

- BOTTOM-UP APPROACH   Data ⇒ Underlying Structure ... new symmetry?

- TOP-DOWN

Theory .... $\left\{ \begin{array}{l} \text{COMPOSITE ... TECHNICOLOUR / TOPCOLOUR} \\ \text{SUSY ... GUTS / STRINGS} \\ \text{LARGE NEW DIMENSIONS} \end{array} \right.$ ⇒ Low Energy Structure

$$m = \begin{pmatrix} \sim 0 & \lambda_5 \varepsilon^3 & \lambda_4 \varepsilon^3 \\ .. & \lambda_3 \varepsilon^2 & \lambda_2 \varepsilon^2 \\ .. & .. & \lambda_1 1 \end{pmatrix}$$

... in all cases radiative corrections to masses may play an important role. Even in the Standard Model radiative corrections drive λ_1^t towards an

INFRA-RED FIXED POINT

- INFRA-RED STRUCTURE AND FERMION MASSES.

eg. Standard Model.

Top Yukawa

$$\frac{d \ln(h_t)}{dt} = \frac{1}{8\pi^2}\left\{ 8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_1^2 - \frac{9}{2}h_t^2 \right\}$$

$\frac{1}{2}\ln\left(\frac{\mu_0^2}{\mu^2}\right)$

$$\boxed{\frac{d \ln g_3}{dt} = \frac{-7}{8\pi^2}g_3^2}$$

$$\boxed{\frac{d \ln\left(\frac{h_t}{g_3}\right)}{dt} = g_3^2 - \frac{9}{2}h_t^2 \;..}$$

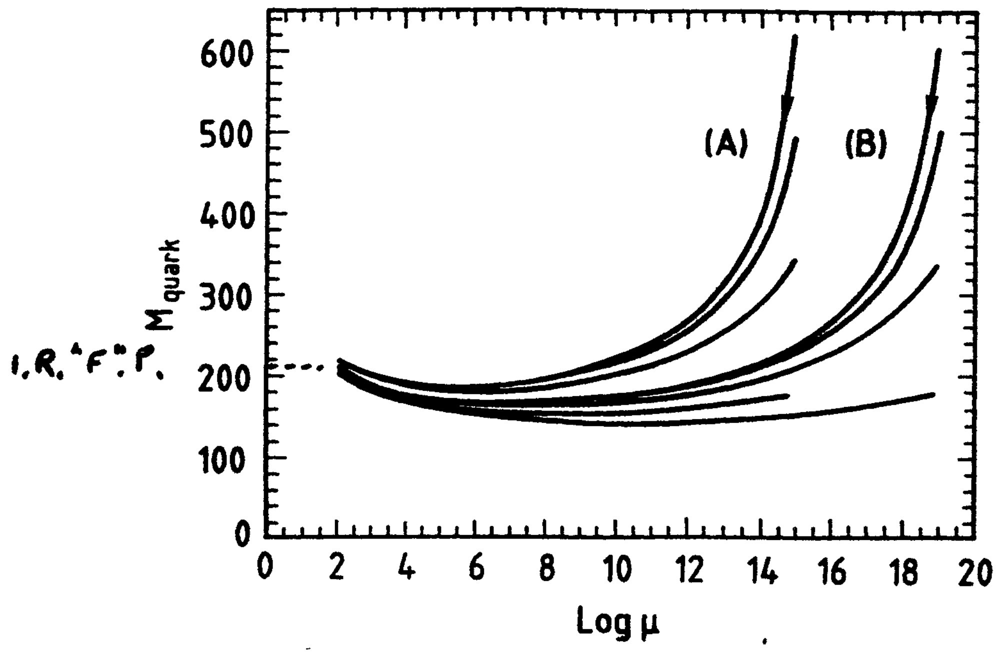
↳ INFRA-RED STABLE FIXED POINT

$$\boxed{\left(\frac{h_t^2}{g_3^2}\right)^* = \frac{2}{9}.}$$

BUT C. HILL SHOWED THAT FOR $t \sim \frac{1}{2}\ln\left(\frac{M_P^2}{M_W^2}\right)$

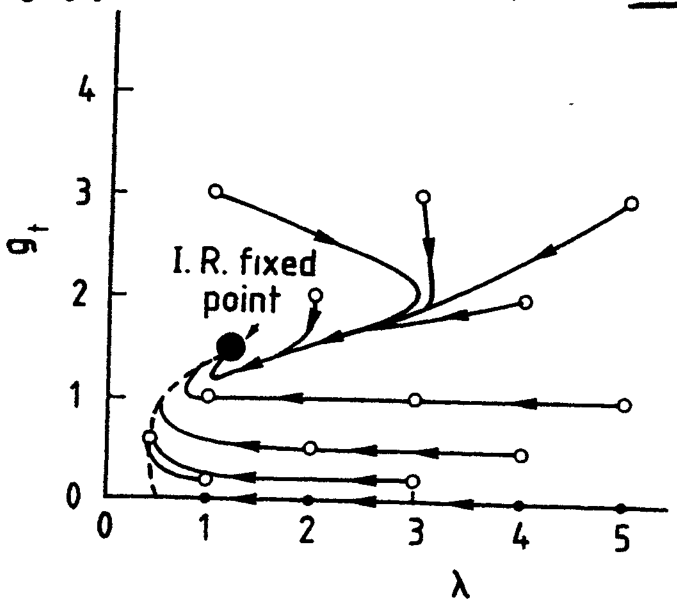
BEHAVIOUR GOVERNED BY "QUASI" FIXED POINT...

Infra-red structure and fermion masses



Pendleton, Ross;
Hill.

Figure 4: The running top quark mass as a function of the scale μ A. $\Lambda = \underline{10^{15} GeV}$ B. $\Lambda = \underline{10^{19} GeV}$



$\lambda |H|^4$ also controlled by IRFP

$(m_H = 270 \pm 30)$

S.M.

Figure 5: Full RG trajectories showing the joint evolution of the top quark coupling g_t and the quartic scalar coupling λ

$\alpha_{vis} \to$ strong.

$m_t^* = 210 \sin\beta$ GeV $\Rightarrow$

(190 $h_b = h_\tau$)

(m_{Higgs} = 120 GeV)

$\boxed{\begin{array}{l} m_t = 175 \text{ GeV} \\ \frac{m_b}{m_\tau} = 1.67 \checkmark \\ n_5 + 3n_{10} = 20 \end{array}}$

ANALYTIC SOLUTION OF RG EQNS $(g_2 = g_1 = 0)$

$$\underbrace{\frac{dY_t}{dt}}_{t\,=\,\frac{1}{2}\ln\left(\frac{\mu_0^2}{\mu^2}\right)} = \overbrace{Y_t}^{h_t^2/(4\pi)^2}\left(r_3\, \overbrace{\tilde{\alpha}_3}^{g_3^2/(4\pi)^2} - s\, Y_t \right)$$

$$\tilde{\alpha}_3(t) = \frac{\tilde{\alpha}_3(0)}{(1 + \beta_3 t)}, \qquad \beta_3 = \tilde{\alpha}_3(0)\, b_3$$

$$Y_t(t) = \frac{Y_t(0)\, E_1(t)}{1 + s\, Y_t(0)\, F_1(t)}$$

where

$$E_1(t) = (1+\beta_3 t)^{(B_3 - 1)}$$

$$F_1(t) = \frac{(1+\beta_3 t)^{B_3}}{B_3 \beta_3} - \frac{1}{B_3 \beta_3}$$

$$B_3 = \frac{r_3}{b_3} + 1$$

$$\boxed{\frac{Y_t(t)}{\tilde{\alpha}_3(t)} = \frac{\overbrace{\left(\frac{Y_t}{\tilde{\alpha}_3}\right)^{*}}^{B_3 b_3 / s}}{1 - \underbrace{\left(\frac{\alpha_3(t)}{\alpha_3(0)}\right)^{B_3}}_{\text{Not small at } M_W}\left(1 - \frac{\tilde{\alpha}_3(0)}{Y_t(0)}\left(\frac{Y_t}{\tilde{\alpha}}\right)^{*}\right)}}$$

Not small at M_W

... "Quasi"-fixed point

C. Hill

M. [illegible], G. ...

IMPLICATION FOR m_t:

- **S.M.**

$$b_3 = -7 \qquad B_3 = -\frac{1}{7}$$
$$r = 8$$
$$s = 9/2 \qquad \left(\frac{Y_t}{\tilde{\alpha}_3}\right)^* = \frac{2}{9}$$

$$\left(\frac{Y_t}{\alpha_3}\right)^{QFP} = \frac{\left(\frac{Y_t}{\tilde{\alpha}_3}\right)^*}{1 - \underbrace{\left(\frac{\alpha_3(t)}{\alpha_3(0)}\right)^{B_3}}_{0.8}}$$

$m_t^* = 110$ GeV

$m_t^{QFP} = 240$ GeV

$(M_X = 10^{16}$ GeV$)$

M. Lanzag[illegible]

- **MSSM.**

$$b_3 = -3 \qquad B_3 = -\frac{7}{9}$$
$$r = \frac{16}{3} \qquad \left(\frac{Y_t}{\tilde{\alpha}_3}\right)^* = \frac{7}{18}$$
$$s = 1$$

$$\left(\frac{\alpha_3(t)}{\alpha_3(0)}\right)^{B_3} = 0.46$$

$$\Rightarrow \quad m_t^* = 155 \sin\beta$$

small h_b $\rightarrow$ $\boxed{m_t^{QFP} = 210 \sin\beta}$

(Including g_2, g_1 corrections)

$\Rightarrow$ 190 $h_b = h_t$.

⇨ TECHNICOLOUR, TOPCOLOUR ..

- S, T constraints disfavour additional EW doublets...

top-colour $\langle \bar{t} t \rangle$ condensate breaks EW symmetry Hill

... but difficult to get $\langle t \bar{t} \rangle$ large enough while

m_t kept light

... top-condensation Seesaw — Dobrescu, Hill, Chivukula, Georgi

$$\boxed{SU(3)_1 \times SU(3)_2 \times SU(2)_W \times U(1)_Y}$$

topcolour: 3rd generation / Generations 1, 2.

$$\psi_L = (t_L \; b_L) \quad = (3, 1, 2, 1/3)$$

$$t_R, \chi_L \quad (1, 3, 1, 4/3)$$

$$\chi_R \quad (3, 1, 1, 4/3) \quad + \ldots$$

SM states + χ_L, χ_R $SU(2)_W$ singlets

$$H = \begin{pmatrix} \bar{\chi}_R \, t_L \\ \bar{\chi}_R \, b_L \end{pmatrix} \quad \text{composite}$$

$$\boxed{\begin{array}{c} h_1 \qquad h_2 \qquad \Phi^i_j = (3,\bar{3},1,0) \\ SU(3)_1 \times SU(3)_2 \longrightarrow SU(3)_{QCD} \\ \langle \phi \rangle = v\,\delta^i_j \end{array}}$$

Further dynamical scalar. (EW singlet)

$$\mathcal{L}^{int}_{eff} = -\frac{g^2_{TC}}{M^2}\left(\bar{\psi}_L \gamma^\mu \frac{\lambda^A}{2}\psi_L\right)\left(\bar{\chi}_R \gamma_\mu \frac{\lambda^a}{2}\chi_R\right) + LL + RR$$

$$\equiv -\frac{g^2_{TC}}{M^2}(\bar{\psi}_L \chi_R)(\bar{\chi}_R \psi_L) + \ldots \ldots$$

$(h_1^2 + h_2^2)\,v^2$

$^{\bar\psi}_{\chi}\times^{\bar\chi}_{\psi}$: NJL effective 4 fermi interaction ... gives composite Higgs

$\bar{t}_L \chi_R \langle H \rangle$ $\qquad \bar{\chi}_L \chi_R \Phi$ $\qquad \bar{\chi}_L t_R \Phi$

fermion mass

$$\boxed{-\mathcal{L}_{eff} = m_{t\chi}\,\bar{t}_L \chi_R + \mu_{\chi\chi}\,\bar{\chi}_L \chi_R + \mu_{\chi t_R}\,\bar{\chi}_L t_R + h.c.}$$

$$v^2 = \frac{N_c}{8\pi^2} m^2_{t\chi} \ln\left(\frac{M^2}{\mu_{\chi\chi}}\right)$$

: GAP eq $\left\{ \succ\!\cdots\!\prec = \times + \times\!\!\bigcirc\!\!\times + \times\!\!\bigcirc\!\!\bigcirc\!\!\times + \ldots \right.$

To get EW breaking scale $\qquad m_{t\chi} \sim 600$ GeV.

... see saw ..

$$(\bar{t}_L\ \bar{\chi}_L)\begin{pmatrix} 0 & m_{t\chi} \\ \mu_{\chi t} & \mu_{\chi\chi}\end{pmatrix}\begin{pmatrix} t_R \\ \chi_R \end{pmatrix} \Rightarrow m_t = \frac{m_{t\chi}\,\mu_{\chi t}}{\mu_{\chi\chi}} = \underline{m_{t\chi}\tan\theta}$$

$\tan\theta$ is small, as required, if adjust new gauge coupling strengths

Masses for light quarks. :

1st two generations : Both LH & RH quarks transform under $SU(3)_2$ ... source of 4-f. interaction

$$\langle \bar{\psi}_L^3 \chi_R \rangle \bar{c}_R \psi_L^2 \qquad \text{etc.}$$

... generate mass but no obvious structure yet.

b-quark : b_R, χ_R have same strong gauge interaction ... must introduce additional interactions to "tilt" vacuum to prevent formation of potentially large $\bar{b}_R \psi_L^3$ condensate which would give too large a b-quark mass

... no obvious pattern ... several alternatives examined

eg all quark carry topcolour Georgi, Grant

or fundamental scalars communicate mass to light generations

Aranda, Carone

Topcolour phenomena.

Higgs sector very rich :

$$\phi_{AB} \equiv \bar{B}_R A_L \qquad (A, B = b, t, \chi) \quad : \quad 18 \text{ real scalars}$$

including 3 weak doublets : $\phi_{\psi A} \equiv \bar{A}_R \psi_L$ (b, t, χ)

3 weak singlets : $\phi_{\chi A} \equiv \bar{A}_R \chi_L$

$h^0, H^0 \approx \phi_{tt}, \phi_{t\chi}$ in EW breaking sector.

- h^0 ; mass $\sim m_{t\chi} = 600\,\text{GeV}$ (only smaller if $M^2_{\chi\chi} \approx M^2_{t\chi}$)

- $H^0, H^\pm, A^0$ ... heavy mass $\sim \frac{m_{t\chi}}{\varepsilon}$

→ • $H^0_{\chi t}, A^0_{\chi t}$ CP-even, CP odd light only if $M^2_{\chi t} \approx M^2_{t\chi}$

→ • $A^0_{\chi\chi}$ CP-odd light only if $M^2_{\chi\chi} \approx M^2_{t\chi}$

→ • ϕ_{bb} ... mass M_{tb}. (arbitrary parameter)

→ • $H^{\pm}_{tb}$ ... $\gtrsim 250\,\text{GeV}$, min of M_{tb}, $M_{\chi b}$ small

- $H^0_{\chi\chi}, H^\pm_{\chi b}$ ... CP-even, large masses

Large New Dimensions. + Masses

Arkani-Hamed, Dimopoulos, Dvali
[illegible]

Compactification radius

$$V_{Gravity}(r) \sim \frac{m_1\, m_2}{M^{n+2}_{Planck\,(4+n)}} \; \frac{1}{r^{n+1}} \qquad (r \ll R)$$

$$\sim \frac{m_1 m_2}{M^{n+2}_{Planck\,(4+n)}} \; \frac{1}{R^n} \; \frac{1}{r} \qquad (r \gg R)$$

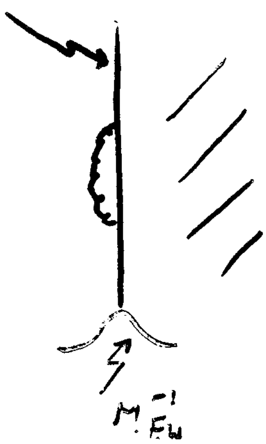
ie $$\boxed{M^2_{Planck,4} = M^{2+n}_{Planck,4+n}\, R^n.}$$

$$M^* \equiv M_{Planck,4+n} = M_{EW} \sim 1\,\text{TeV} \quad ?$$ (Hierarchy problem

$$R \sim 10^{\frac{30}{n}-17}\,\text{cm} \left(\frac{1\,\text{TeV}}{M_{EW}}\right)^{1+\frac{2}{n}}$$

$\underline{n \geq 2 \;\Rightarrow\; R \lesssim 1\,\text{mm}}$ (limit of Cavendish expts)

3-brane.



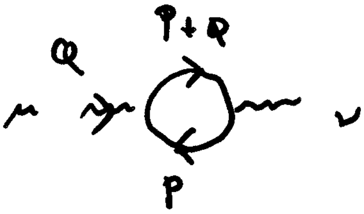
- SM states confined to 3-brane (or sub-brane)
- Gravity + moduli states propagate in all bulk

... Energy-loss (neutral particles) + possible return with displaced vertices of $O(R)$!

• POWER LAW RUNNING.

Compactified (string) models.

Consider the simple example $D=5$ with one dimension a ⊙ with radius $R = \Lambda_c^{-1}$



$$\pi^{\mu\nu} = i\left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q)$$

String scale

$$\Pi(Q) = \beta_0 \sum_{n=0}^{n=\Lambda_s R} \int \frac{d^4 P}{(2\pi)^4} \frac{1}{P^2+n^2\Lambda_c^2} \cdot \frac{1}{(P+Q)^2+n^2\Lambda_c^2}$$

Sum over Kaluza-Klein excitations.

compactification scale

For $Q \ll 1/R$

Power law change from $\Lambda_c < P < \Lambda_s$

$$\Pi(Q) \simeq \frac{\beta_0}{(4\pi)^2}\left(\ln(QR)^2 - 2(\Lambda_s R - 1)\right)$$

$$\Pi(Q) \simeq i\beta_0 \int \frac{d^5 P}{(2\pi)^4} \frac{1}{(P^2+\underbrace{n^2\Lambda_c^2}_{P_5^2})} \frac{1}{(P+Q)^2+\underbrace{n^2\Lambda_c^2}_{P_5^2}}$$

$$\simeq -2\frac{\beta_0}{(4\pi)^2}(\Lambda_s R - 1)$$

i.e. $$\alpha^{-1}(t_c) = \alpha^{-1}(0) + \frac{6}{4\pi}\left(\frac{M_p}{\Lambda_c}\right)^{D-4}$$

- <u>POWER LAW RUNNING TO IRFP</u>

Lanzagorta, GGR
Able, King.
Bando et al.

volume factor

$$\frac{d\alpha}{dt} = -\frac{\tilde{b}}{2\pi} X_{\delta_g} \left(\frac{\Lambda}{\mu}\right)^{\delta_g} \alpha^2$$

$$\frac{dY}{dt} = \frac{Y}{2\pi}\left(c_i X_{\delta_{gi}} \left(\frac{\Lambda}{\mu}\right)^{\delta_{g_i}} \alpha - \tilde{a}_i X_{\delta_i} \left(\frac{\Lambda}{\mu}\right)^{\delta_i} \alpha_y \right)$$

⇨ rapid approach to IRFP. ✓

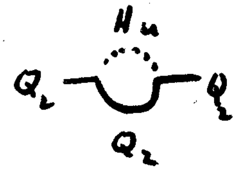
- <u>Hierarchy generation</u>

e.g. $\lambda Q_3^2 H_u$: $\delta = 0$

$Q_2^2 H_u$ $\quad \delta = 3$

$\vdots$

	5	6	7	8	9	10
Q_1	x	x	x	x	x	x
Q_2	x	x	x			
Q_3						
H_u	x	x	x	x	x	x
H_d	x		x		x	



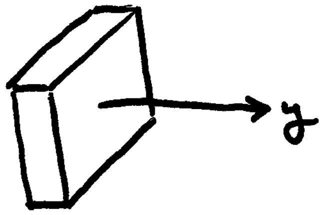
$$m^u = \begin{pmatrix} \varepsilon^6 & \varepsilon^6 & \varepsilon^6 \\ \varepsilon^3 & \varepsilon^3 & \varepsilon^3 \\ 1 & 1 & 1 \end{pmatrix} e^{-\delta_g'} g$$

$$m^d = \begin{pmatrix} \varepsilon^3 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^2 & \varepsilon^2 & \varepsilon^2 \\ 1 & 1 & 1 \end{pmatrix} e^{-\delta_g'} g .$$

- <u>Fermion Hierarchy Generation.</u>

Higgs profile thickness

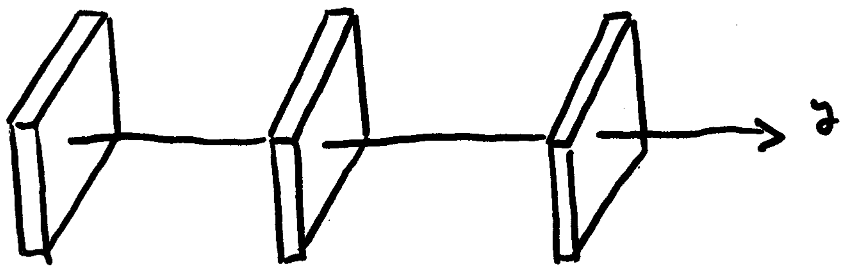
$$\psi_H(y) \propto e^{-|y|a}$$



Dvali, Shifman

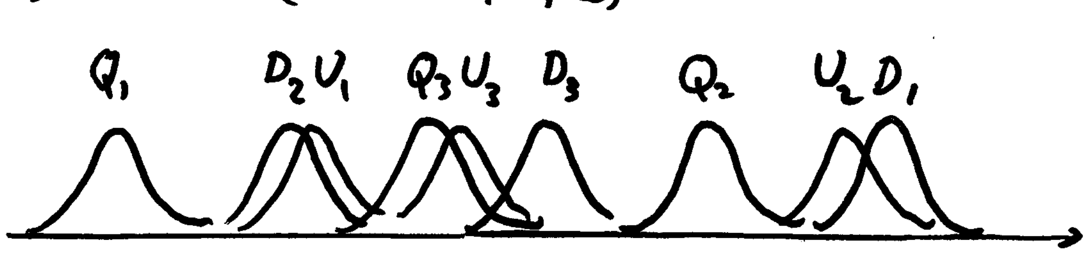
Arkani-Hamed, Schmaltz

Assign Higgs + quarks to different locations :



$$m_{ij} = \int dy \; v \; \exp - \left[|y| a + b|y-y_i| + b|y-y_j| \right]$$

Mirabelli, Schmaltz (+Gaussian profile)...



m_{q_i} , V_{CKM} : 9 measurables $\Rightarrow$ $m_s^{\overline{MS}}(2\,\mathrm{GeV}) \simeq 1.19 \left(\frac{V_{ub} V_{cb}}{V_{us}} \right)^{1/2} m_b^{\overline{MS}}(m_b)$

... 8 parameter fit $\simeq 12.0$ MeV

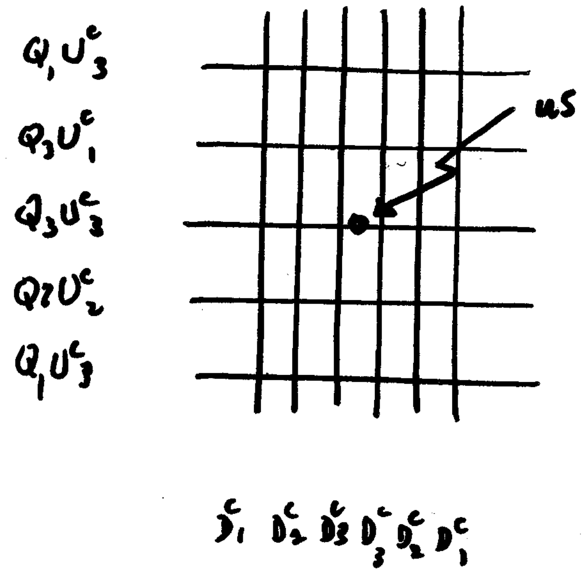
<u>Variations on the theme</u> :

Arkani-Hamed, Hall, Smith Weiner.

SM on 1 brane : $U(3)^5$ <u>family symmetry</u>.

Broken by sources on distant branes.

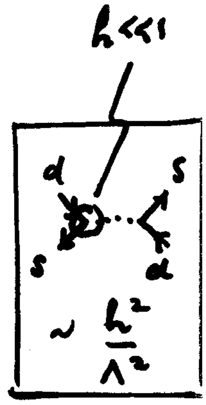
eg Quarks : $U(3)^3$



$$\lambda^u \sim \begin{pmatrix} 0 & 0 & \varepsilon' \\ 0 & \varepsilon & 0 \\ \varepsilon & 0 & 1 \end{pmatrix} \qquad V_{CKM} = \begin{pmatrix} 0.976 & .219 & .006 \\ .219 & .975 & .039 \\ .007 & .039 & .999 \end{pmatrix}$$

$m_d = 4.3$ MeV $m_u = 1.3$ MeV
$m_s = 130$ MeV $m_c = 1.3$ GeV
$m_b = 4.3$ GeV $m_t = 174$ GeV

Assuming equal grid spacing there are 5 parameters ... results agree to 50%.



NOTE : L.N.D. allow for <u>low-scale breaking</u> of family symmetry

- **Standard Model on the brane?**

... solves hierarchy problem $\qquad M \sim \text{TeV}$

... gravity in bulk (but ~localised on Planck brane)

.... gauge coupling unification?

... proton decay? $\qquad \frac{1}{M^2}\,\psi_i\psi_j\psi_k\psi_l$

... ν masses? $\qquad \frac{1}{M^2}\,\nu_L\nu_L HH$

- **Standard Model in bulk ?**

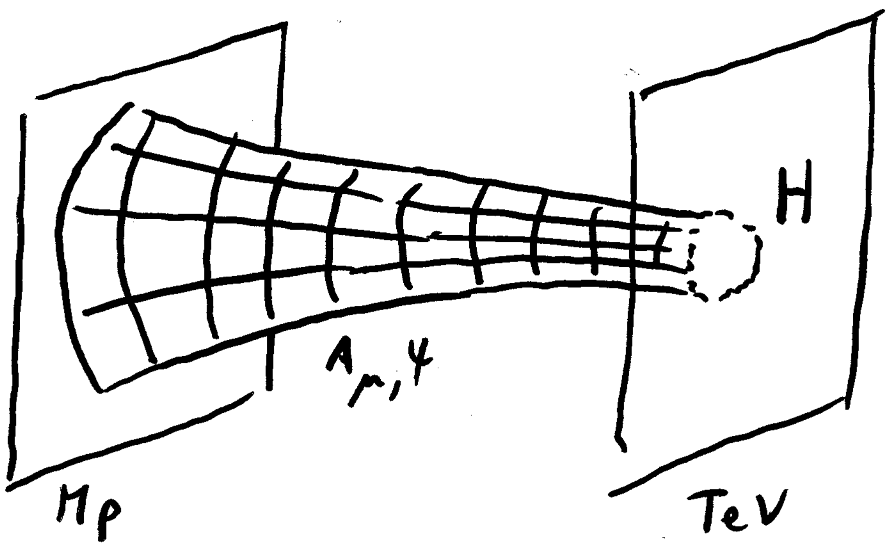
Zero modes protected by :

gauge fields : gauge invariance

fermion : chiral symmetry.

scalar : not protected

... no supersymmetry $\Rightarrow$ scalar must be on TeV-brane.

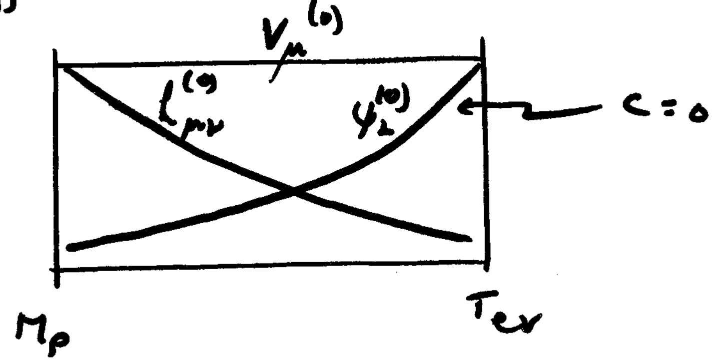


Zero modes in bulk

$$h_{\mu\nu}^{(0)}(y) \sim e^{-k|y|}$$

$$V_\mu^{(0)}(y) \sim \frac{1}{\sqrt{V}}$$

$$\psi^{(0)}(y) \sim e^{\left(\frac{1}{2}-c\right)k|y|} \qquad [m_\psi = ck\,\varepsilon(y)]$$



ie $V_\mu^{(0)}$: not localised

$h_{\mu\nu}^{(0)}$: localised on Planck brane.

$\psi_L^{(0)}$: can be localised on <u>either</u> brane, depending on bulk mass parameters

Yukawa couplings

$$\int d^4x \int dy \sqrt{-g}\, \lambda_{ij}^{5}\, H(x) \left[\bar{\psi}_{iL}(x,y)\, \psi_{jR}(x,y) + h.c.\right] \delta(y-\pi R)$$

$$\equiv \int d^4x\, \lambda_{ij}\, H(x) \left[\psi_{iL}^{(0)}(x)\, \psi_{jR}^{(0)}(x) + h.c.\right]$$

$$\boxed{\lambda_{ij} = \frac{\lambda_{ij}^{(5)} k}{N_{iL} N_{jR}}\, e^{(1-c_{iL}-c_{jR})\pi k R}}$$

$$N_{iL} \sim e^{(1-2c_{iL})\pi k R/2}$$

March-Russell, Dimopoulos, Dvali

Dienes, Dudas, Gherghetta

Dvali, Smirnov

Neutrino mass

⇨ Right-handed ν^{s} in the bulk. :

moduli - SM singlet

$$S^{int} = \lambda \int d^4x \;\; \ell(x)\, h(x)\, \nu_R(\underbrace{x, y=0}_{\sum_n \frac{1}{\sqrt{2\pi R M_*}} \nu_{Rn}^{(c)}(x)\, e^{iny/R}})$$

ex: 1 extra dimension

flux spreading.

Needed to get canonical norm in 4D

$$m_\nu = \frac{\lambda \langle h \rangle}{\sqrt{RM_*}} \longrightarrow \frac{\lambda \langle h \rangle}{\sqrt{R^n M_*^n}} \qquad (n \text{ extra-dims})$$

$$= \frac{\lambda \langle h \rangle M_*}{M_{Planck,4}} \sim \underline{\lambda\, 10^{-4} eV \cdot \left(\frac{M_*}{1 TeV}\right)}$$

.. viability depends on cosmology .... left to the reader!

⇨ Breaking lepton # on distant walls .

$$m_\nu^{Majorana} \sim \frac{\langle h \rangle^2}{M_*} \left(\frac{M_*}{M_{Planck,4}}\right)^{2-4/n}$$

$$\sim \underline{\frac{\langle h \rangle^2}{M_{Planck}}} \quad \text{for } n=4$$

USUAL SEE-SAW WITHOUT LARGE SCALE

(- BUT LOSES GAUGE UNIFICATION)

Solar ν oscillations from large extra dimensions

Lukas, Ramond, Romanino, GGR

(developing ideas of Smirnov, Dvali)

$$S^{5D.}_{bulk} = \int d^4x\, dy\, \left(\bar{\Psi}_I \gamma^A i\partial_A \Psi_I - \mu_I \bar{\Psi}_I \Psi_I\right)$$

Bulk mass term ... significant effect on phenomenology

From 4-D viewpoint

$$\xi_I(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_n \xi_{nI}(x)\, e^{-iny/R} \qquad \Psi = \begin{pmatrix} \bar{\xi} \\ \eta \end{pmatrix}$$

$$\eta_I(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_n \eta_{nI}(x)\, e^{iny/R}$$

$S_{bulk} + S_{brane}$ gives 4D mass Lagrangian

$$\boxed{-\mathcal{L}_m = \sum_n \xi_{nI}\left[\left(\mu_I - \frac{in}{R}\right)\eta_{nI} + m_{Ii}\nu_i\right]}$$

$$m_{Ii} = \lambda_{Ii}\, v\, \frac{M_*}{M_{P,4}}$$

... describes the 3 SM $\nu_{i=e,\mu,t}$ and 2 sterile or "bulk" ξ_{nI}, η_{nI} for each mode number n and each bulk fermion $I = 1 \ldots N$

For small brane-bulk mixings, LH current eigenstates ν_i given by

$$\boxed{\nu_i = U_{ik}\,\hat{\nu}_k + \frac{m^*_{Ii}}{M_{0I}} N_{0I} + \sqrt{2}\sum_{n\geq 1} \frac{m^*_{Ii}}{M_{nI}} N_{nI}}$$

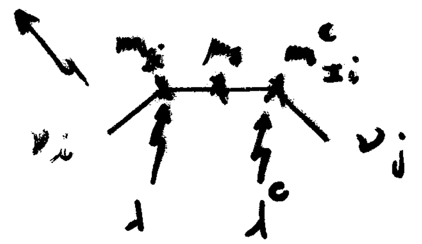
can get mass from

L violating term

$\lambda^c_{Ii}\, \bar{\nu}^c_{RIi} L_i H$

$$\boxed{\begin{aligned} N_{nI} &\sim \eta_{nI} + \eta_{-nI} \\ M_{nI} &= \sqrt{\mu_I^2 + \frac{n^2}{R^2}} \end{aligned}}$$

$$\boxed{\begin{aligned} m_{ij} &= -\sum_n \frac{(m_{Ii} m^c_{Ij} + m^c_{Ii} m_{Ij})\,\mu_I}{\mu_I^2 + n^2/R^2} \\ &= -\pi R\,(m_{Ii} m^c_{Ij} + m^c_{Ii} m_{Ij}) \coth(\pi R \mu_I)^{\dagger} \end{aligned}}$$



"see-saw"

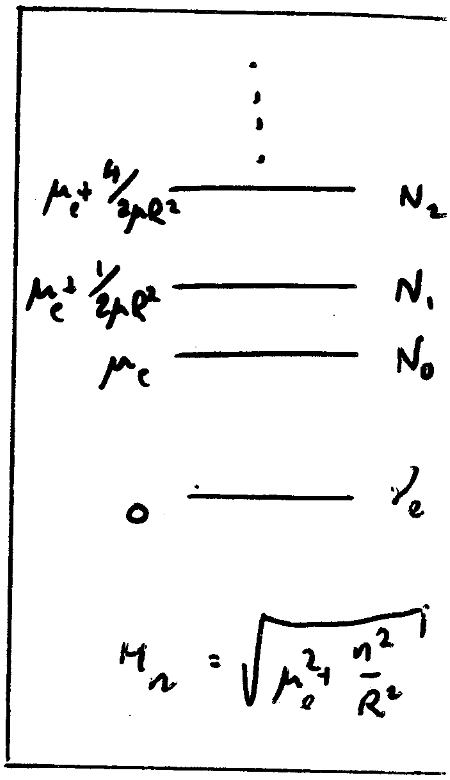
N.B. Significant difference with $\mu_I \gg m_{Ii}$ since

$M_{0I} \gg \hat{m}_k$

$\dagger$ NB Modified see saw $\sum_n \frac{1}{\mu_I} \to R$, since # of

states ...

1 generation case & solar ν oscillation



(No $L\!\!\!/$ violation)

- μ_e, R, m_e parameters
  m_e: brane-bulk mixing.

- Mixing of ν_e to N_n : $\Theta_n \simeq \sqrt{2}\,\frac{m_e}{M_n}$.

- Mass difference : $\Delta m_n^2 = \mu_e^2 + \frac{n^2}{R^2}$

- Fit to Super-K data. ↗

Best fit $\quad 1/R = 5 \cdot 10^{-3}$ eV

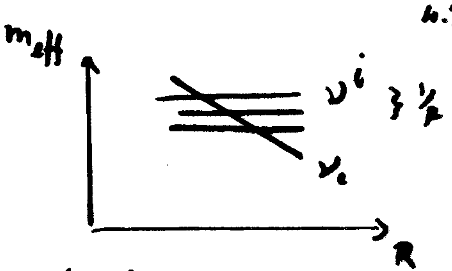
$\mu_e = (2-3) \cdot 10^{-3}$ eV $\quad$ ($E_{res} \sim \frac{\mu_e^2}{2V_e}$,

μ_e chosen to convert B_e ν', not pp)

- Fit to Energy spectra ↗

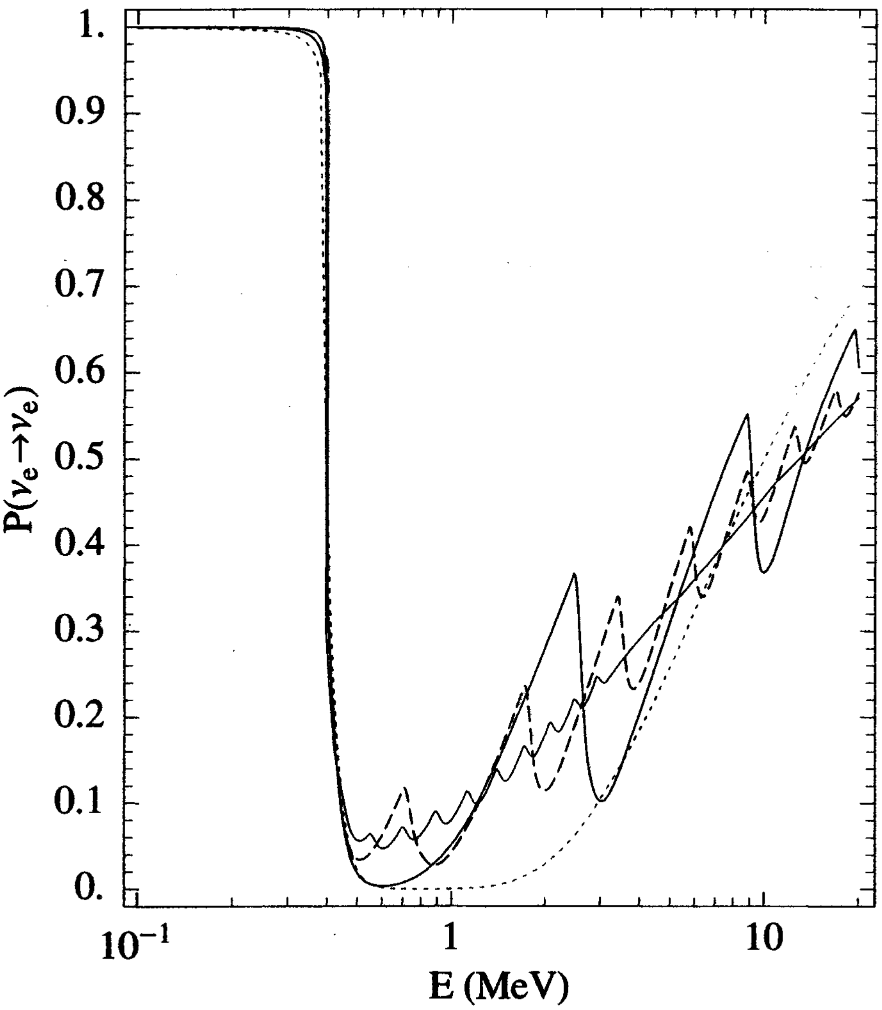
- Day-night asymmetry small

cf. $\frac{D-N}{(D+N)/2} = -0.034 \pm 0.022\,^{+0.013}_{-0.012}$



Level crossing : MSW

Survival probabilities



$\mu_e \sim 2.1 \cdot 10^{-3}$

$\left(\frac{1}{R}, m_e\right) = (20, 0.058) \cdot 10^{-3}$ eV ···· (≡ single sterile)

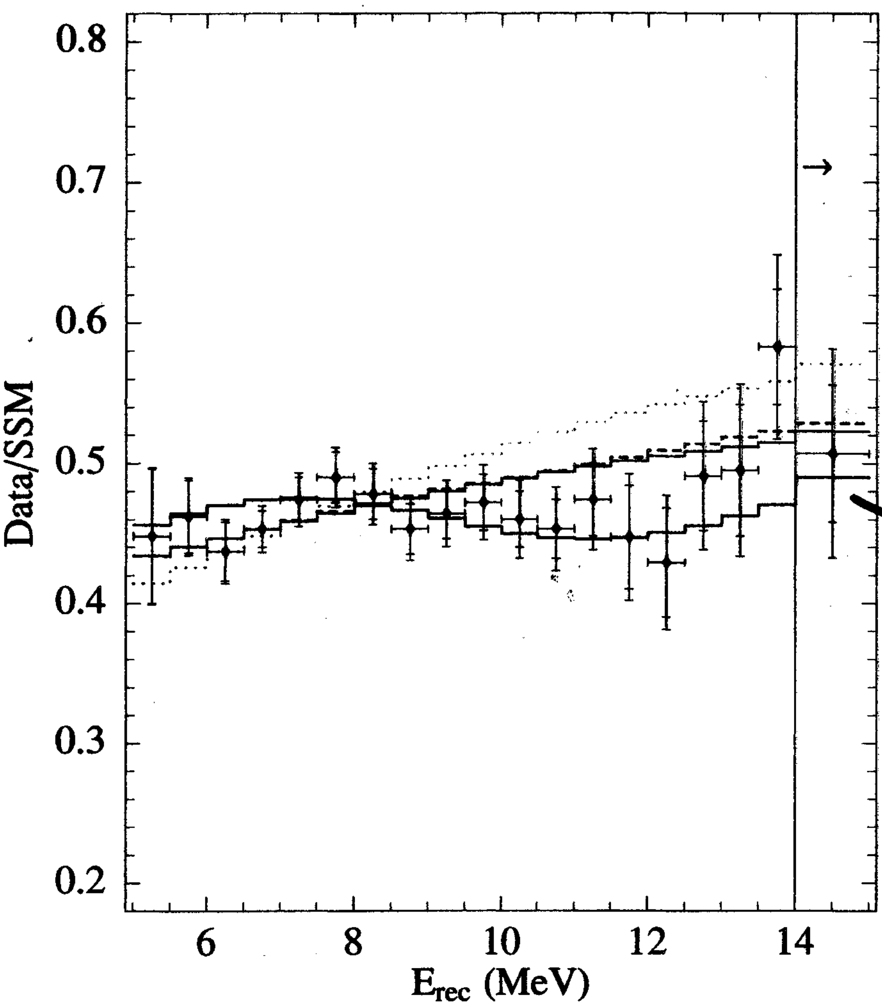
$(5, 0.035) \cdot 10^{-3}$ eV

$(2, 0.023) \cdot 10^{-3}$ eV - - - -

$(0.5, 0.012) \cdot 10^{-3}$ eV

Solar ν's

Recoil electron spectrum - SuperK.



R : 0.1 mm