

**SUMMER SCHOOL ON PARTICLE PHYSICS**

*18 June - 6 July 2001*

**NEUTRINO PHYSICS**

**Lecture IV**

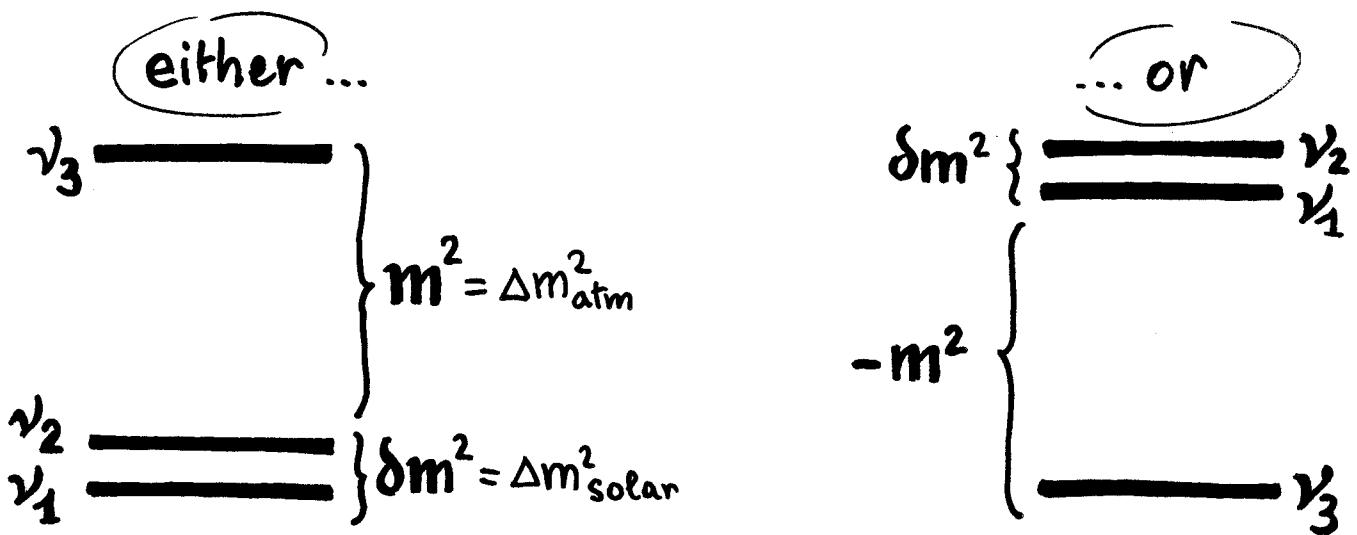
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Please note: These are preliminary notes intended for internal distribution only.



# ATM. + SOLAR $3\nu$ OSCILLATIONS

Phenomenology favors spectra with hierarchical  $\Delta m^2$ 's :



with ...  $\delta m^2 \ll m^2$

N.B.: HIERARCHY OF  $\Delta m^2$ 's DOES NOT IMPLY  
HIERARCHY OF  $m^2(\nu_i)$

mass<sup>2</sup>  
matrix :  $M^2 = \begin{pmatrix} 0 & \delta m^2 \\ \delta m^2 & \pm m^2 \end{pmatrix} + \mu^2 \cdot \mathbf{1}$

↑  
absolute mass<sup>2</sup> scale  
not defined in  $\nu$ -osc.  
experiments; must  
be inferred by other  
means.

## PARAMETER SPACE (REDUCTION)

In general, the  $3\nu$ -oscillation parameter space is spanned by:  $(\underbrace{\delta m^2, m^2}_{\text{two } \Delta m^2}, \underbrace{\omega, \varphi, \psi, \delta}_{\text{3 mixing angles}}) \rightsquigarrow$   $\uparrow_1 \text{CP phase}$

Luckily, for  $\delta m^2 \ll m^2$  the parameter space can be reduced considerably at zeroth order in  $\delta m^2/m^2$  (one mass scale dominance). Physical motivation:

### FOR TERRESTRIAL $\nu$ EXPERIMENTS,

(accelerator, reactor, atmospheric),  $\delta m^2$  is too small to be probed  $\rightarrow$  can take  $\delta m^2 \approx 0$  and  $m_{\text{terrestrial}}^2 \simeq \begin{pmatrix} 0 & 0 \\ 0 & \pm m^2 \end{pmatrix} + \mu^2 \cdot \mathbf{1}$

### FOR SOLAR $\nu$ EXPERIMENTS,

$m^2 = \Delta m_{\text{atm}}^2$  is so large that its oscillations can be taken as averaged (equivalent to take  $\pm m^2 = \infty$ ),  $m_{\text{solar}}^2 \simeq \begin{pmatrix} 0 & \delta m^2 \\ \delta m^2 & \infty \end{pmatrix} \text{ mod. 1}$

### FOR BOTH SOLAR AND TERRESTRIAL EXPTS.,

$\delta m^2/m^2 \rightarrow 0$  implies that CP effects (if any) vanish  $\Rightarrow$  can take  $\mathbf{U}$  real

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U \text{ real}$$

$$UU^T = I$$

In "CKM-like" (Maiani) parametrization:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} c_\varphi & 0 & s_\varphi \\ 0 & 1 & 0 \\ -s_\varphi & 0 & c_\varphi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\psi = \theta_{23}$$

$$\varphi = \theta_{13}$$

$$\omega = \theta_{12}$$

$$= U(\theta_{23}) \cdot U(\theta_{13}) \cdot U(\theta_{12})$$

Such a conventional ordering of Euler rotations turns out to be (accidentally) very useful in the context of one-mass scale dominance



- TERRESTRIAL EXPTS :  $\mu^2 \sim (0, 0_{\pm m^2}) \leftarrow (\nu_1, \nu_2)$    
 ~degenerate

→ any rotation in  $(\nu_1, \nu_2)$  subspace  
unobservable in terrestrial experiments :

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ -c_\psi s_\psi & 1 & \\ -s_\psi c_\psi & & 1 \end{pmatrix} \begin{pmatrix} c_\varphi & s_\varphi \\ -s_\varphi & c_\varphi \end{pmatrix} \begin{pmatrix} c_w s_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

acts on  
 $(\nu_1, \nu_2)$

→ terrestrial experiments probe only the  
mixing angles  $\psi = \theta_{23}$  and  $\varphi = \theta_{13}$

---

- SOLAR EXPTS : cannot distinguish  $\nu_\mu$  from  $\nu_\tau$   
in final state (energy so low that  $\nu_\mu, \nu_\tau$   
interactions identical and  $\mu, \tau$  production impossible)

→ any rotation in  $(\nu_\mu, \nu_\tau)$  subspace  
unobservable in solar ν experiments :

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} c_w s_w & & \\ -s_w c_w & 1 & \\ -s_\psi & & 1 \end{pmatrix}^T \begin{pmatrix} c_\varphi & s_\varphi \\ -s_\varphi & c_\varphi \end{pmatrix}^T \begin{pmatrix} 1 & & \\ c_\omega s_\omega & 1 & \\ s_\omega c_\omega & & 1 \end{pmatrix}^T \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

acts on  
 $(\nu_\mu, \nu_\tau)$

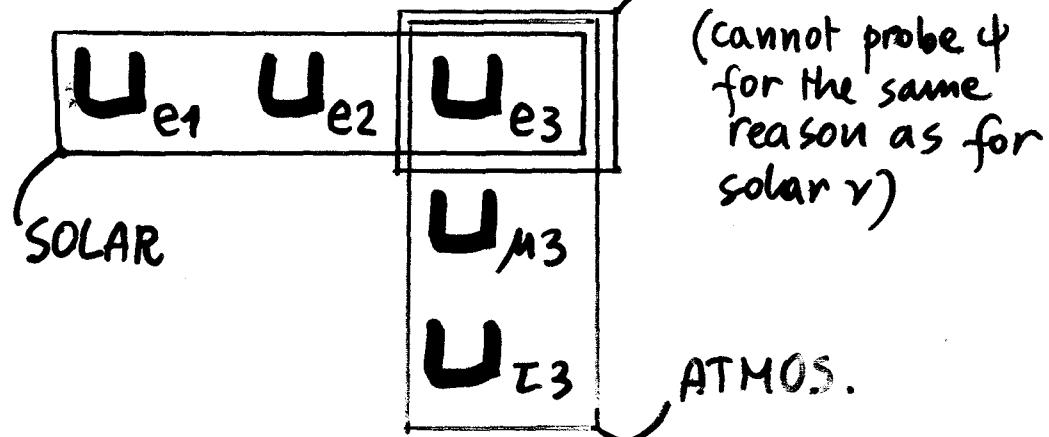
→ solar ν experiments probe only the  
mixing angles  $\omega = \theta_{12}$  and  $\varphi = \theta_{13}$

TERRESTRIAL  $\nu$   
PARAM. SPACE :  $(m^2, \psi, \varphi)$

SOLAR  $\nu$   
PARAM. SPACE :  $(\delta m^2, \omega, \varphi)$

$\varphi = \theta_{13}$  probed  
in both cases

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ & \ddots & \\ & & U_{\mu 3} \\ & & \ddots & \\ & & & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c\varphi c\omega & c\varphi s\omega & s\varphi \\ & \ddots & \\ & & c\varphi s\varphi \\ & & \ddots & \\ & & & c\varphi c\varphi \end{pmatrix}$$



## SUMMARY OF $3\nu$ PARAMETRIZATION AT ZEROTH ORDER IN $\delta m^2/m^2$ :

### TERRESTRIAL $\nu$

PROBE  $m^2$  (largest  $\Delta m^2$ ),  
AND THE FLAVOR COMPOSITION OF  $\nu_3$ :

$$\begin{aligned}\nu_3 &= U_{e3} \nu_e + U_{\mu 3} \nu_\mu + U_{\tau 3} \nu_\tau \\ &= C_\varphi (C_\varphi \nu_\tau + S_\varphi \nu_\mu) + S_\varphi \nu_e\end{aligned}$$

with  $U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1$

Reduces to  $2\nu (\nu_\mu \leftrightarrow \nu_\tau)$  for  $U_{e3} = S_\varphi = 0$

### SOLAR $\nu$

PROBE  $\delta m^2$  (smallest  $\Delta m^2$ )  
AND THE MASS COMPOSITION OF  $\nu_e$ :

$$\begin{aligned}\nu_e &= U_{e1} \nu_1 + U_{e2} \nu_2 + U_{e3} \nu_3 \\ &= C_\varphi (C_w \nu_1 + S_w \nu_2) + S_\varphi \nu_3\end{aligned}$$

with  $U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$

Reduces to pure  $2\nu (\nu_1 \leftrightarrow \nu_2)$  for  $S_\varphi = 0$

no observable ~~CP~~ effects

Relevant unitarity constraints:

$$U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$$

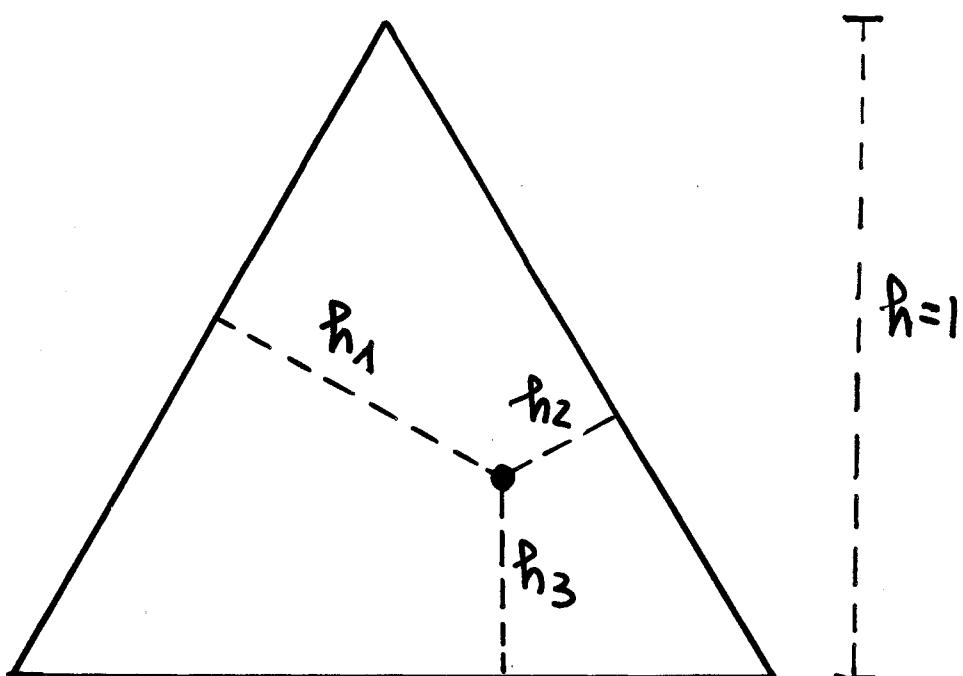
$$U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1$$

---

Constraints of the generic form

$$h_1 + h_2 + h_3 = 1$$

can be embedded in triangle graphs



$$h_1 + h_2 + h_3 = 1$$

FOR ANY POINT INSIDE THE  $\triangle$

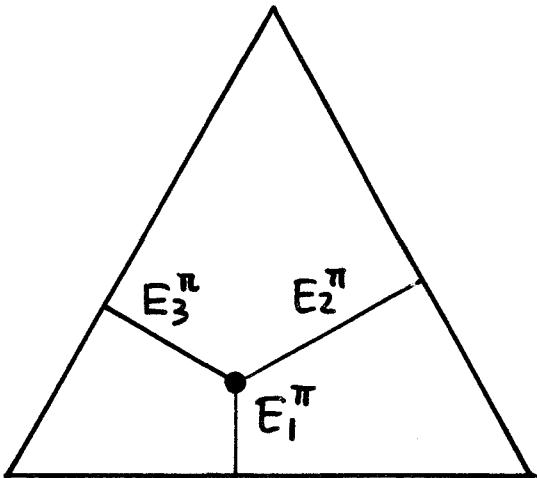
## EXAMPLES :

Dalitz, 1953

$K \rightarrow \pi\pi\pi$  decay

$$E_1^\pi + E_2^\pi + E_3^\pi = E_{\text{TOT}}$$

"Dalitz plot"

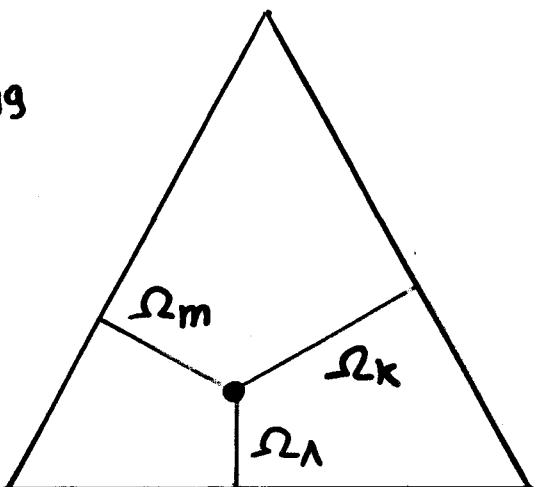


N. Bahcall, Ostriker,  
Perlmutter, Steinhardt, 1999

$\Omega_i$  bounds

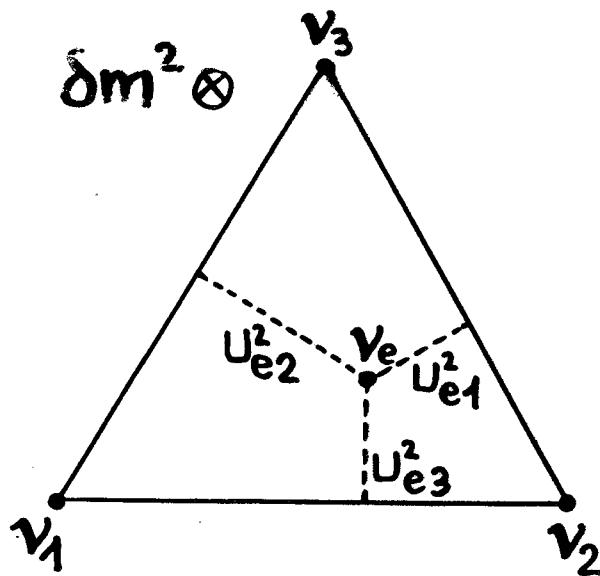
$$\Omega_m + \Omega_k + \Omega_\Lambda = 1$$

"Cosmic triangle"



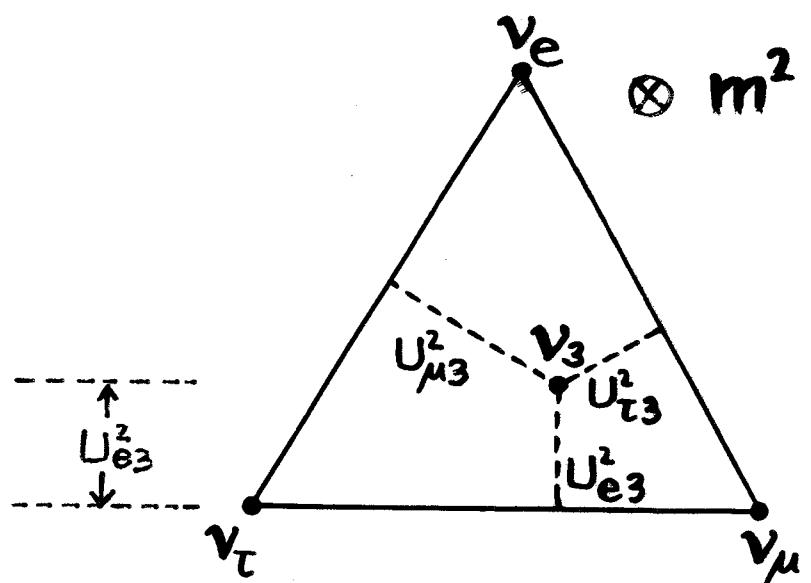
→ Unitarity suggests the following representations:

**SOLAR  $\nu$**   
parameter space:



$$U_{e1}^2 + U_{e2}^2 + U_{e3}^2 = 1$$

**TERRESTRIAL  $\nu$**   
parameter space:

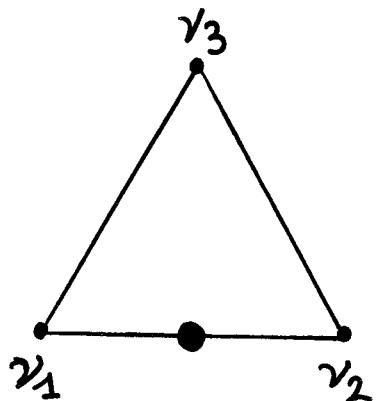


$$U_{e3}^2 + U_{\mu 3}^2 + U_{\tau 3}^2 = 1$$

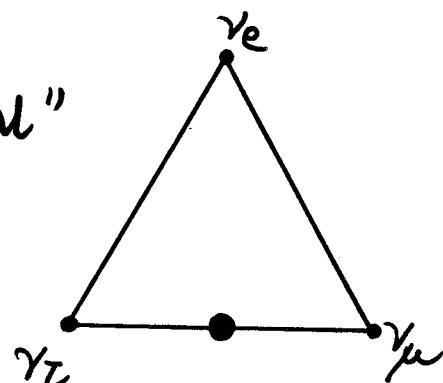
(Lisi, Fogli, Montanino, Scioscia)

# EXAMPLES

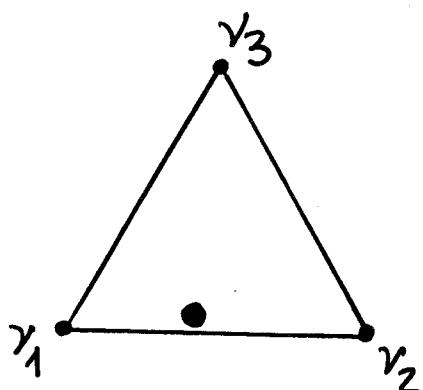
SOLAR



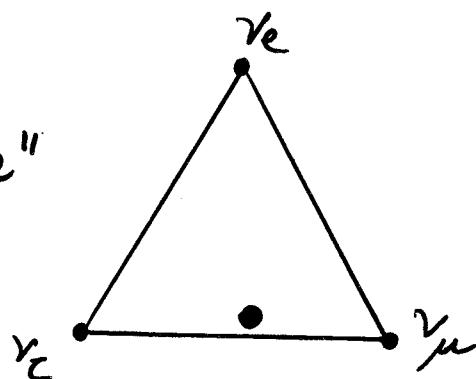
ATMOSPH.



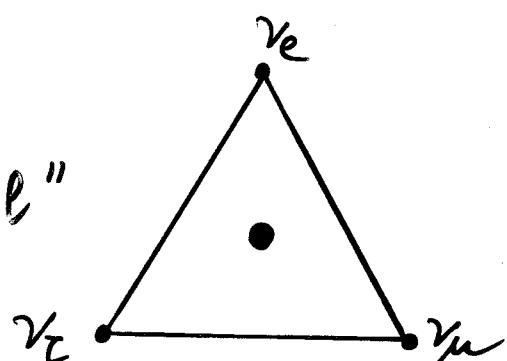
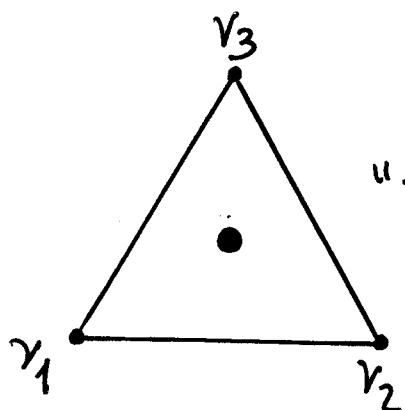
"Bi-maximal" mixing



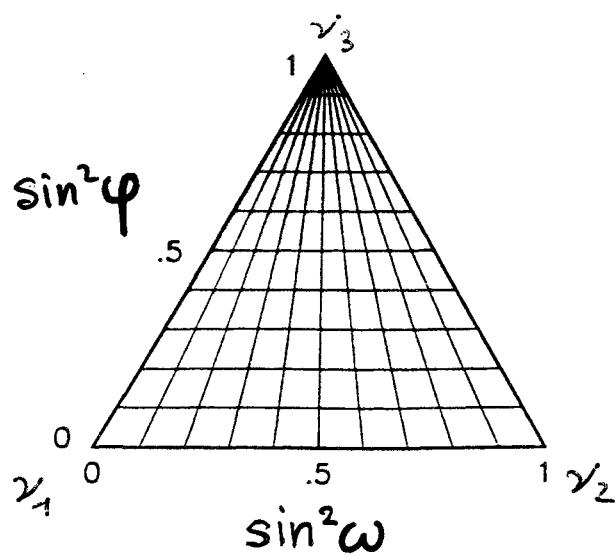
"Bi-large" mixing



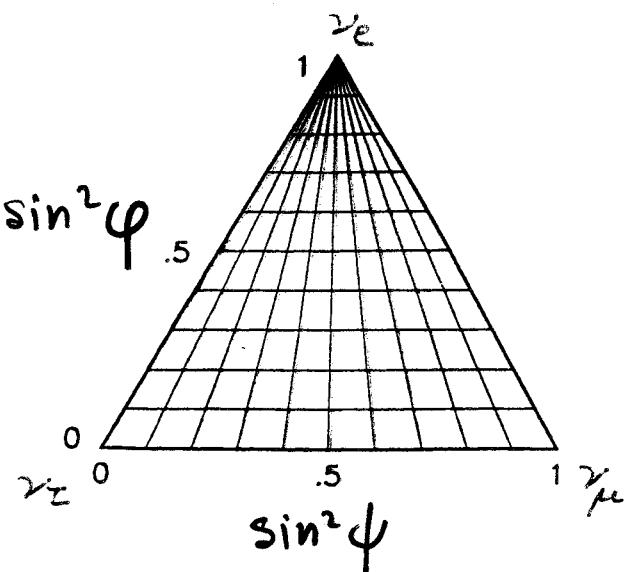
"Trimaximal" mixing



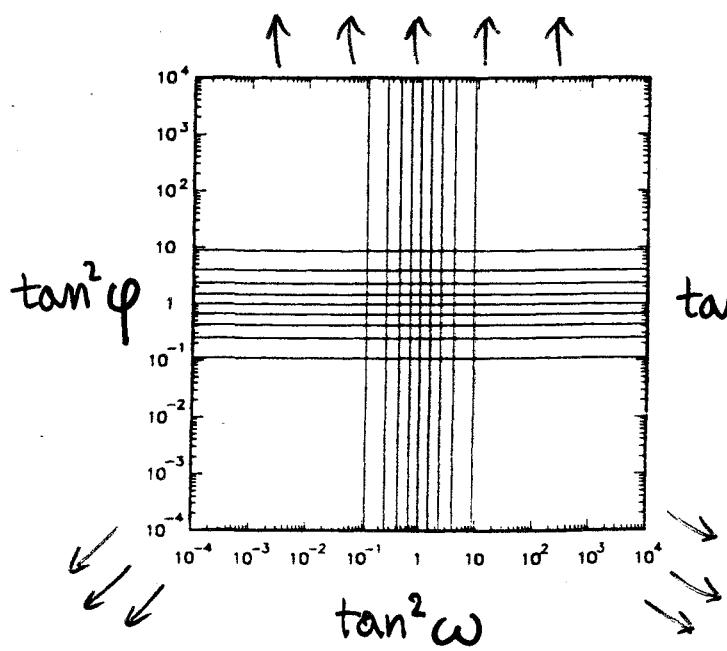
# SOLAR



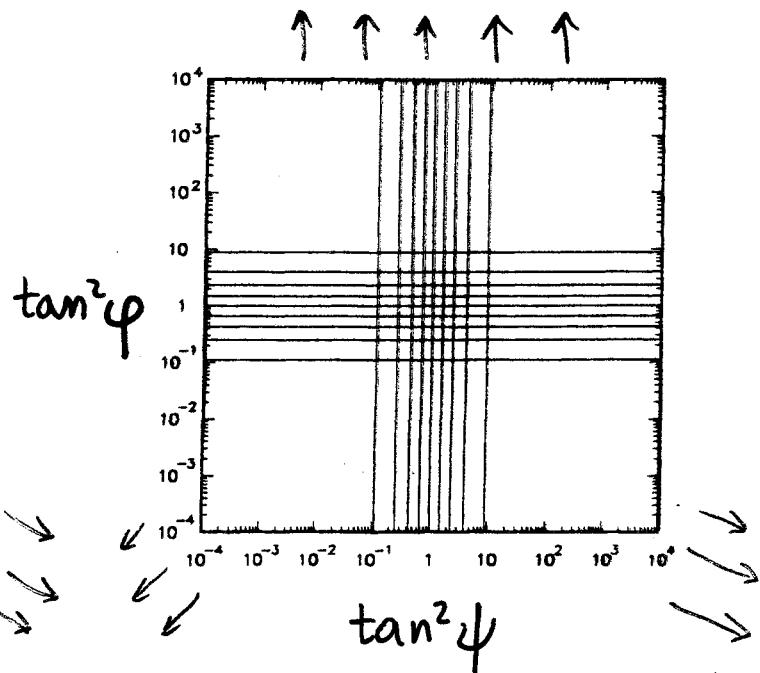
# TERRESTRIAL



$\delta m^2$  fixed



$m^2$  fixed

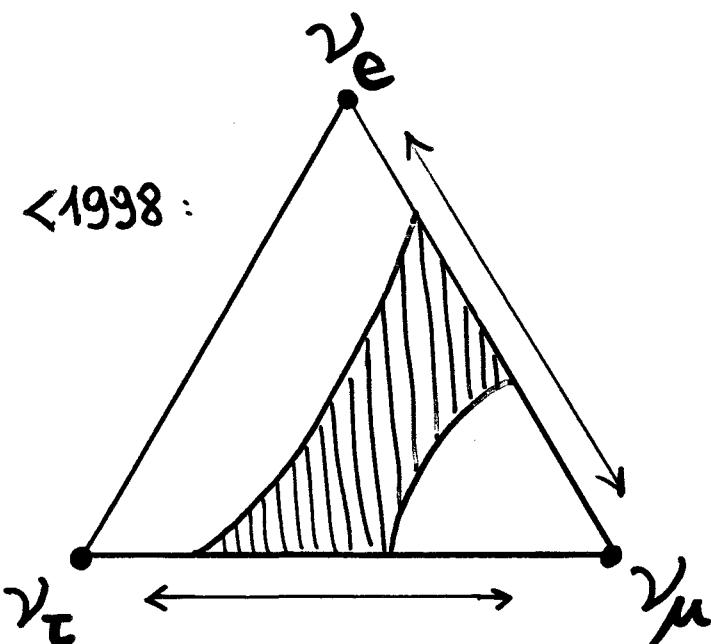


**BILOGARITHMIC MIXING/MIXING**

**REPRESENTATION**

(useful to expand zones at small mixing)

# ATMOSPHERIC 3V OSCILLATIONS

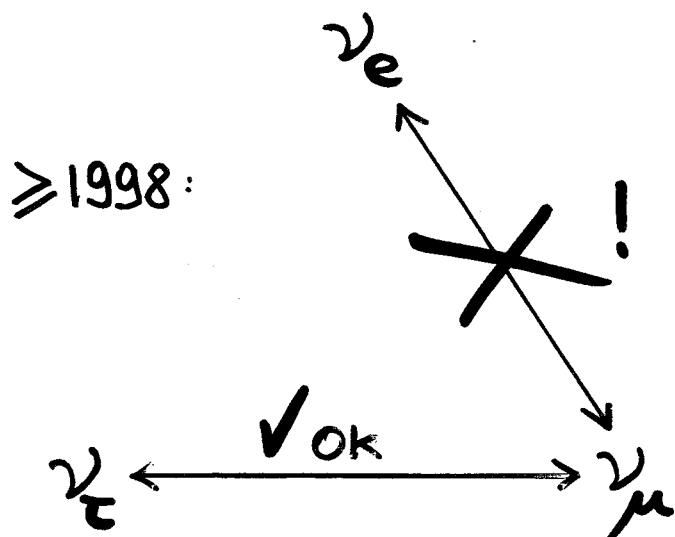


pre-SK situation:

could not distinguish  
pure  $\nu_\mu \leftrightarrow \nu_\tau$  and  
pure  $\nu_\mu \leftrightarrow \nu_e$ , as well  
as intermediate cases

■ = ALLOWED

Sk = Super-Kamiokande

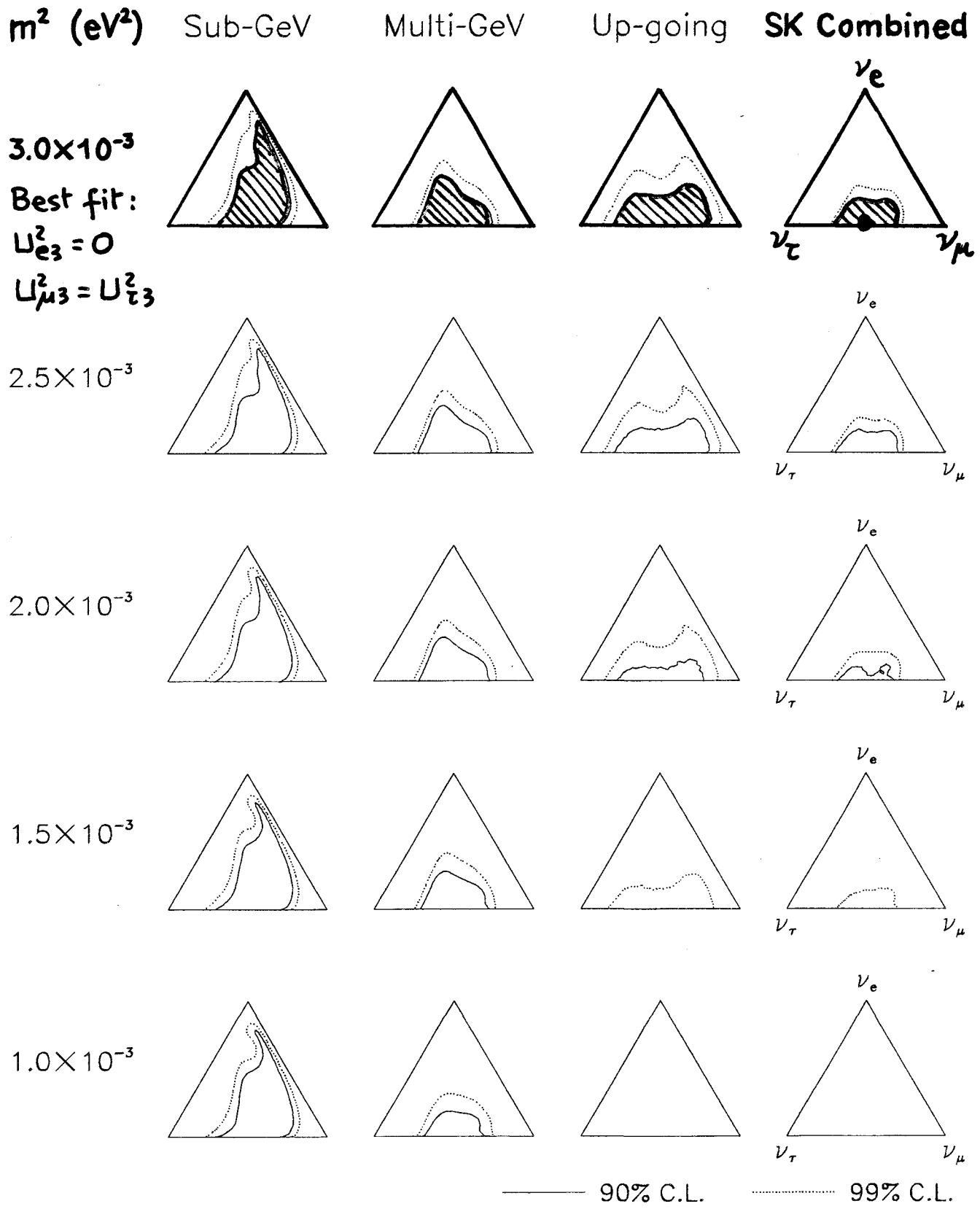


post-SK :

we expect  $\nu_\mu \leftrightarrow \nu_e$   
to be forbidden.

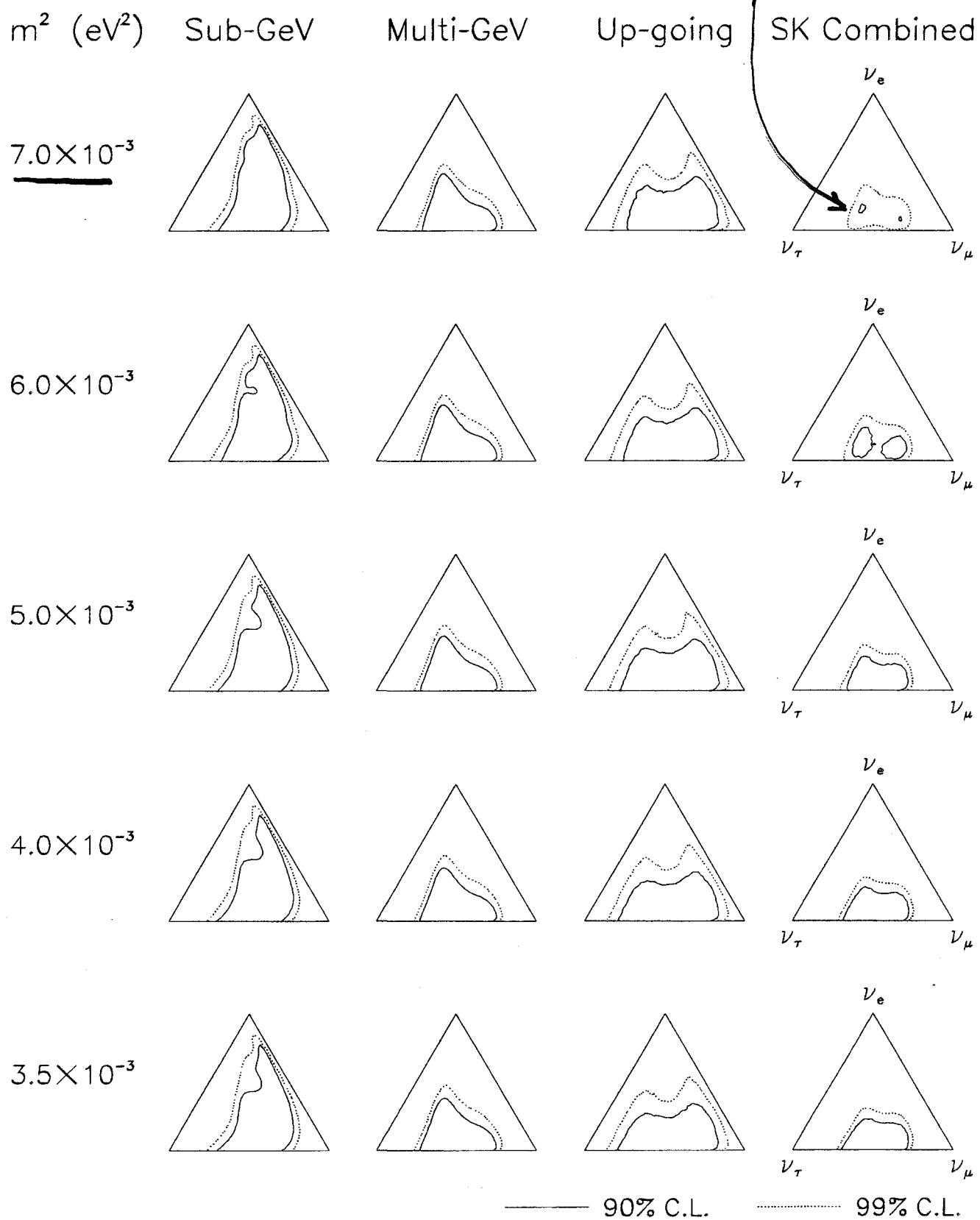
Confirmed by quantitative data analysis  
→

## SITUATION NOW (2001)



Fogli,  
E.L.,  
Marrone

SLIGHT PREFERENCE FOR  
 $U_{e3}^2 > 0$  at "HIGH"  $m^2$  only



In general, atmospheric  $\nu$  prefer  
 small or zero values for  $U_{e3}^2 = S_\varphi^2 = \sin^2 \theta_{13}$

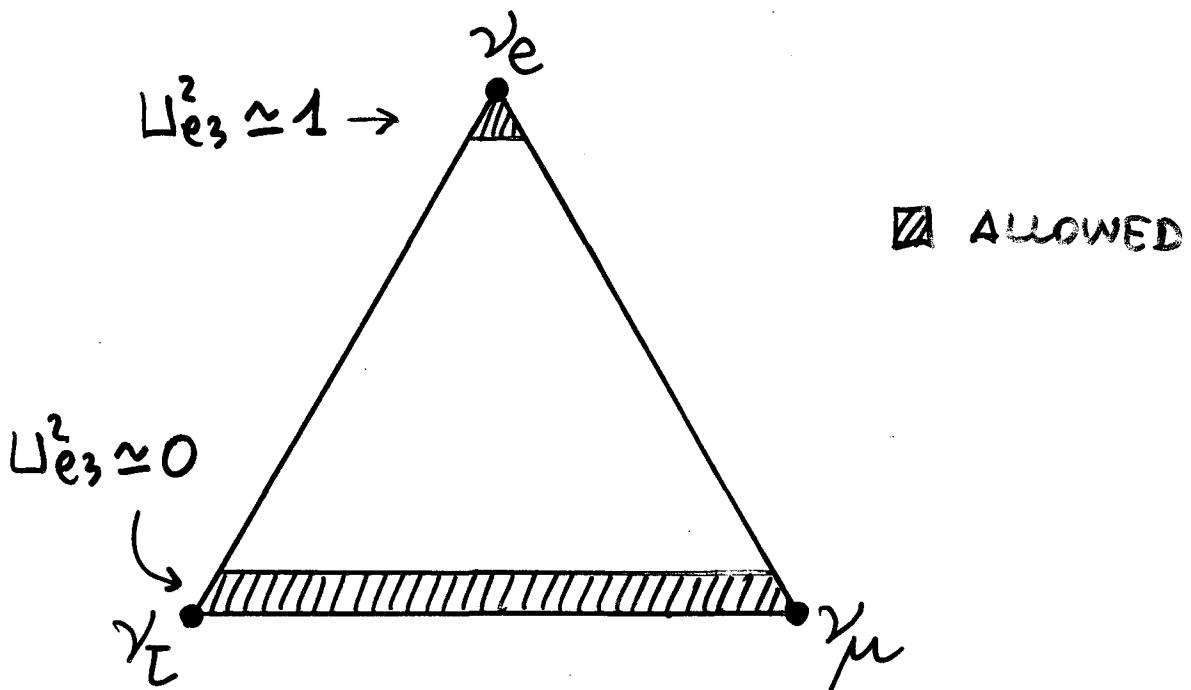
# Impact of CHOOZ

$$P_{ee}^{(CHOOZ)} = 1 - 4 U_{e3}^2 (1 - U_{e3}^2) \sin^2\left(\frac{m^2 L}{4E}\right) \leftarrow \text{theor. (3v)}$$

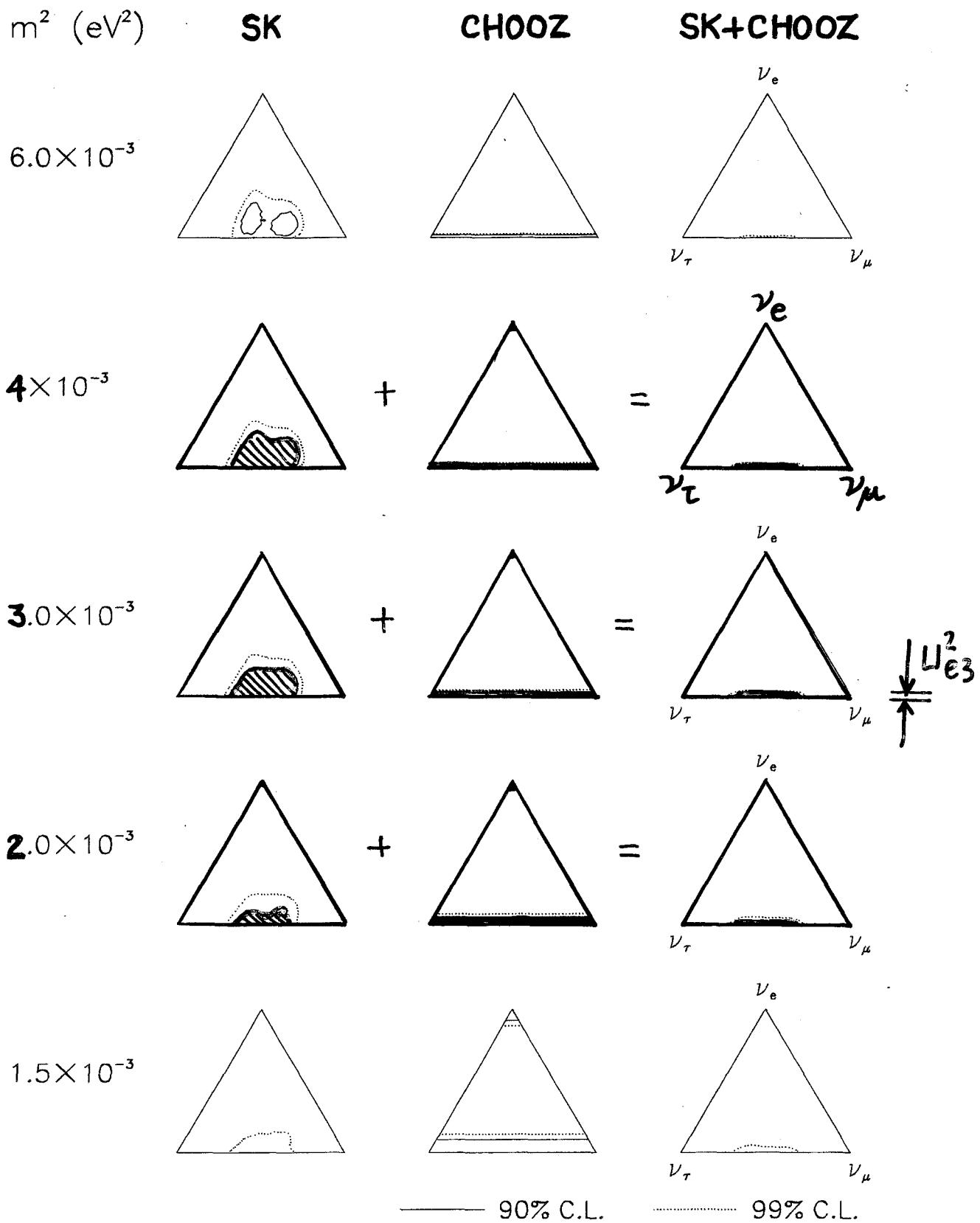
$$\simeq 1 \quad \text{for } \Delta m^2 \gtrsim 0.7 \times 10^{-3} \text{ eV}^2 \quad \leftarrow \text{expt}$$

Therefore,  $4 U_{e3}^2 (1 - U_{e3}^2) \simeq 0$

$$\rightarrow \begin{cases} \text{either } U_{e3}^2 \simeq 0 \\ \text{or } U_{e3}^2 \simeq 1 \end{cases}$$



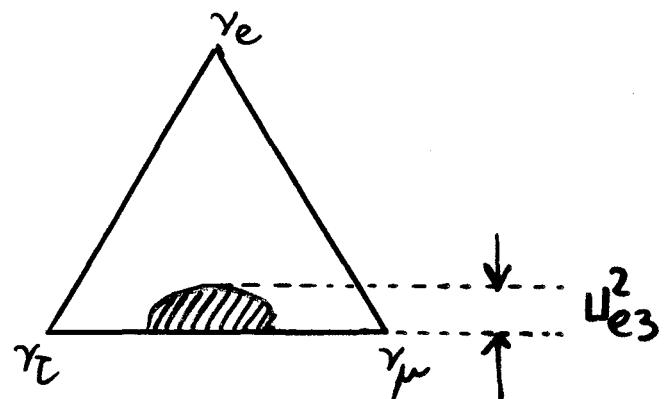
Quantitative analysis →



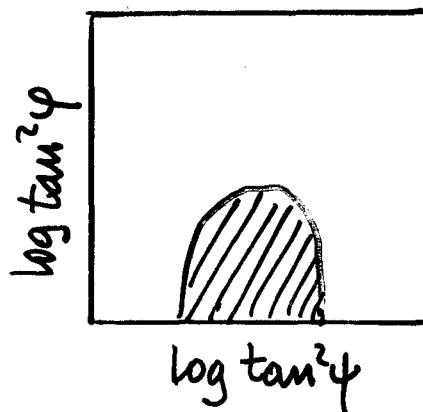
- SK + CHOOZ:  $U_{e3}^2 \lesssim \text{few \%}$
- CHALLENGE FOR FUTURE EXP. TO PROVE  $U_{e3}^2 \neq 0$  (no theoretical motivation for  $U_{e3}^2 \equiv 0$ )

# Alternative representation

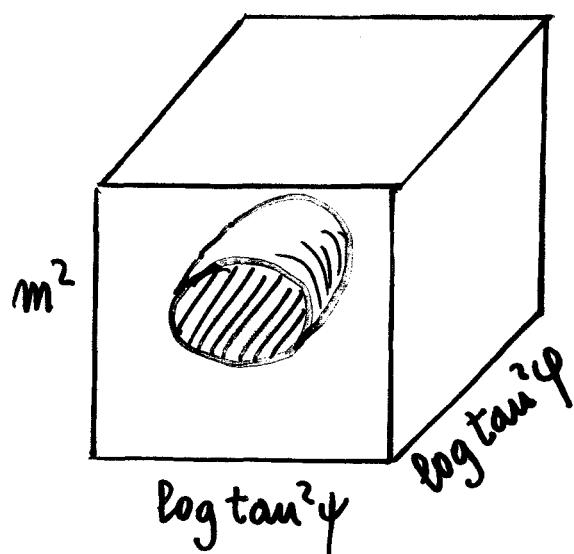
Since regions at small  $U_{e3}^2$  are difficult to draw...



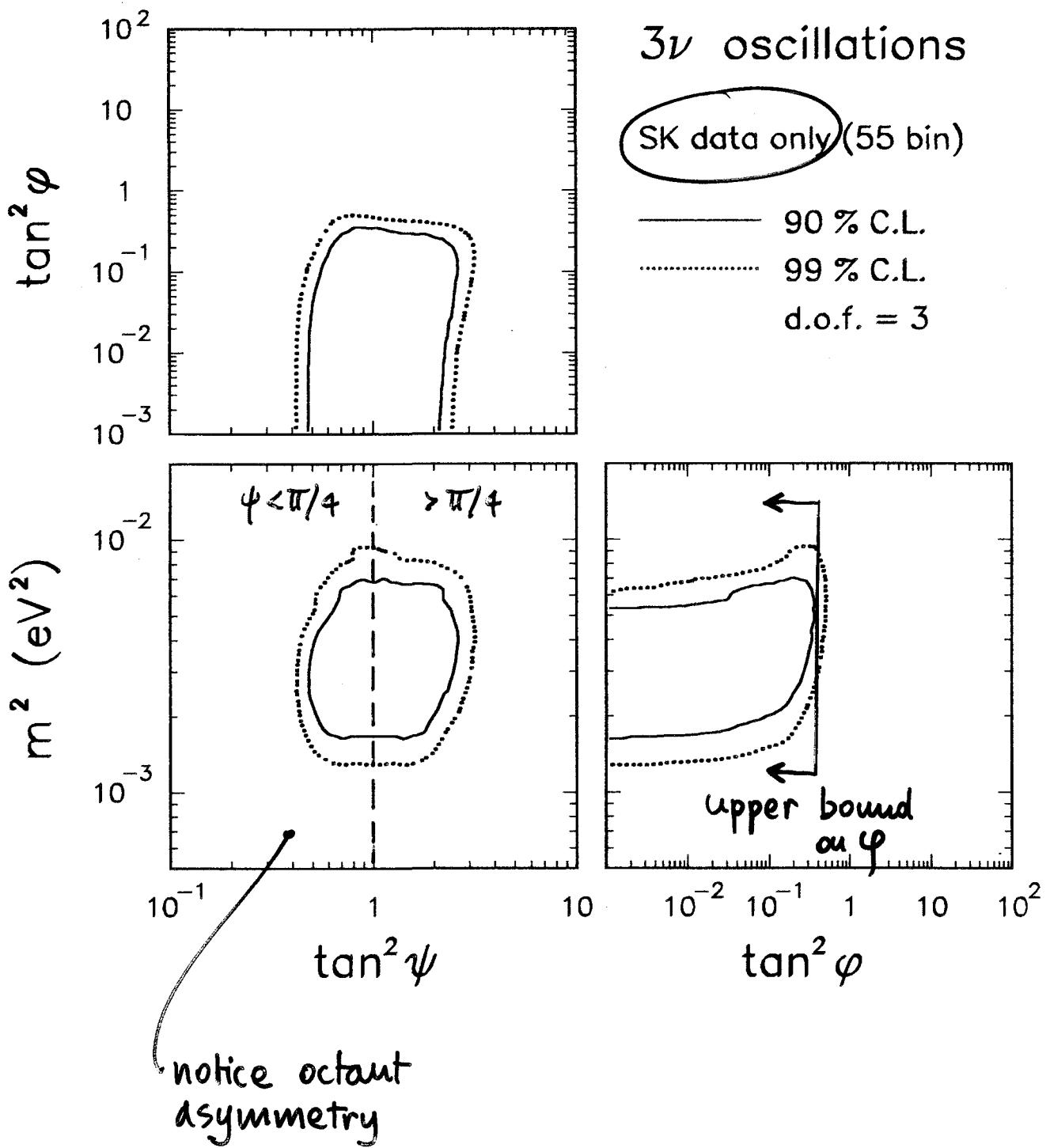
... expand in log-log plot ...

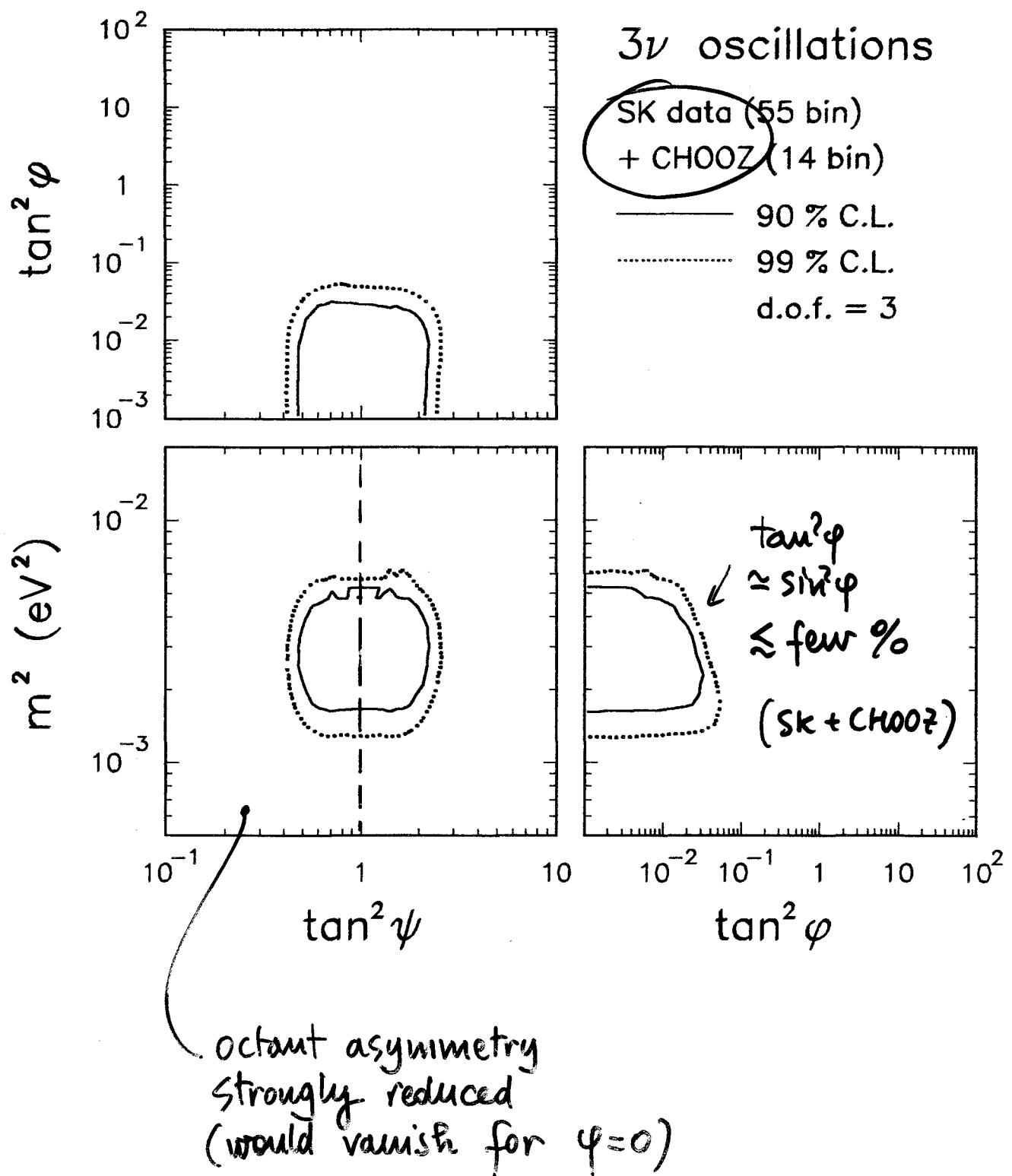


... add third dimension ( $m^2$ )



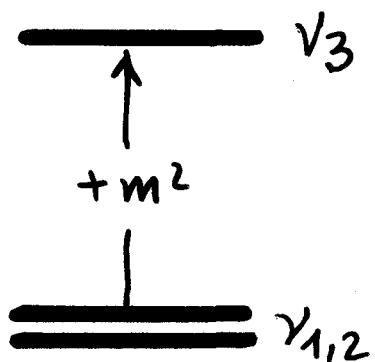
... AND PROJECT ALLOWED REGION →



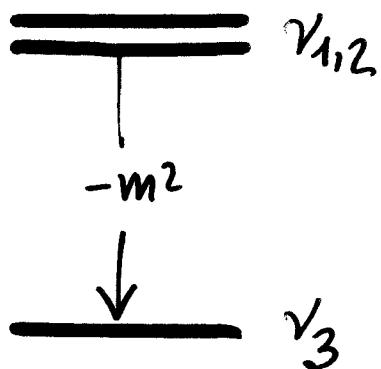


# Remark on mass spectrum:

Previous bounds  
derived for  
spectrum of the kind:  
("direct hierarchy")



Bounds ~ identical  
for alternative  
spectrum (not shown)  
("inverse hierarchy")



→ present phenomenology does not  
discriminate the two spectra  
(as far as solar+terrestrial osc.  
experiments are concerned)

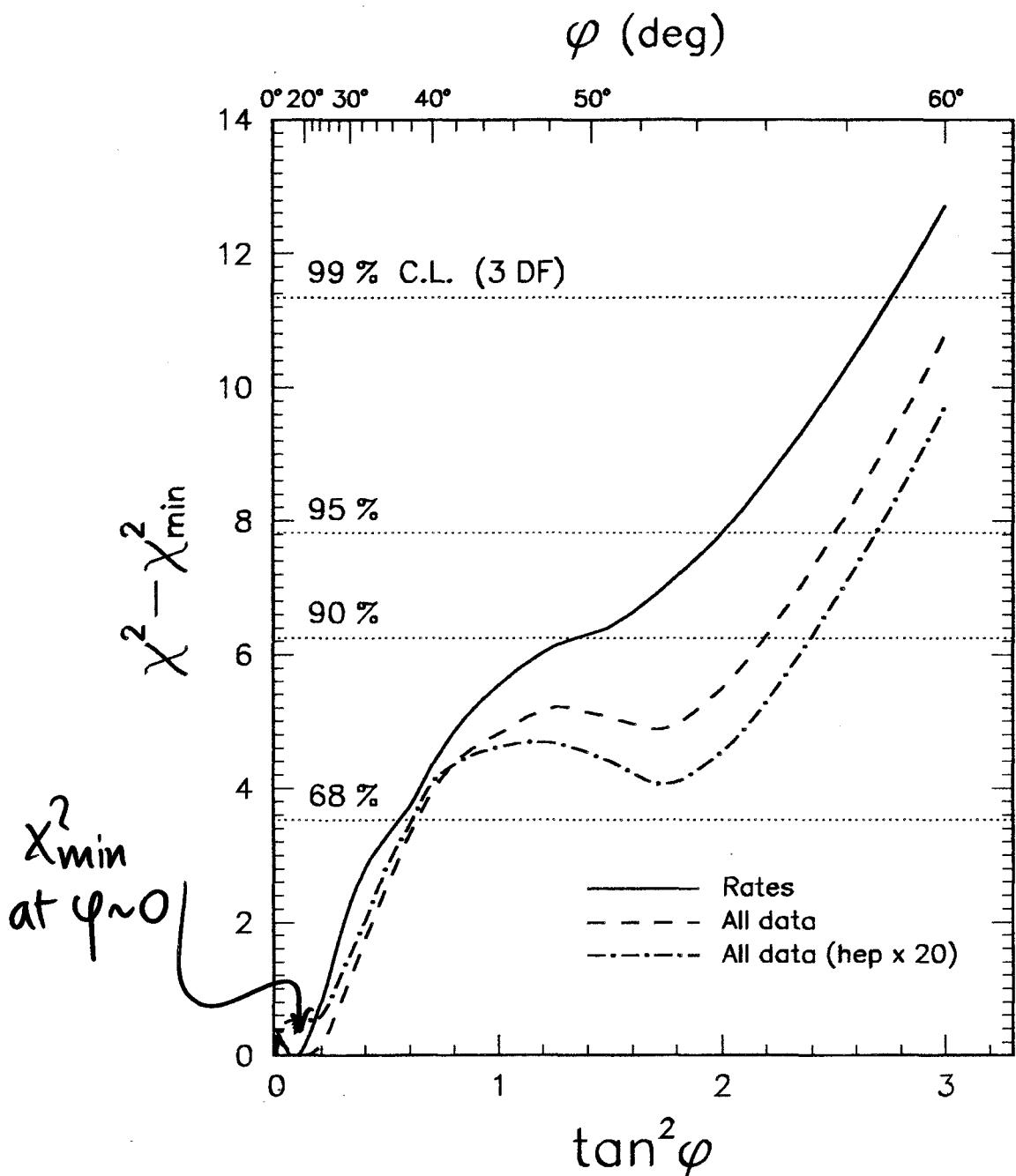
# SOLAR $3\nu$ OSCILLATIONS

$$P_{ee}^{3\nu}(\text{solar}) = \cos^4\varphi \left( P_{ee}^{2\nu} \right) + \sin^4\varphi$$

$N_e \rightarrow N_e C_\varphi^2$

Generic feature : energy dependence  
(embedded in  $P^{2\nu}$ )  
SUPPRESSED  
at large  $\varphi$

Although evidence for energy dependence  
is weak in solar neutrino data,  
 $\varphi \approx 0$  preferred  $\rightarrow$  remarkable  
convergence  
with ATM + CHOOZ  
data !



$\delta m^2$  and  $\tan^2 \omega$  unconstrained

Small  $\varphi$  preferred by solar  
 $\gamma$  data alone.

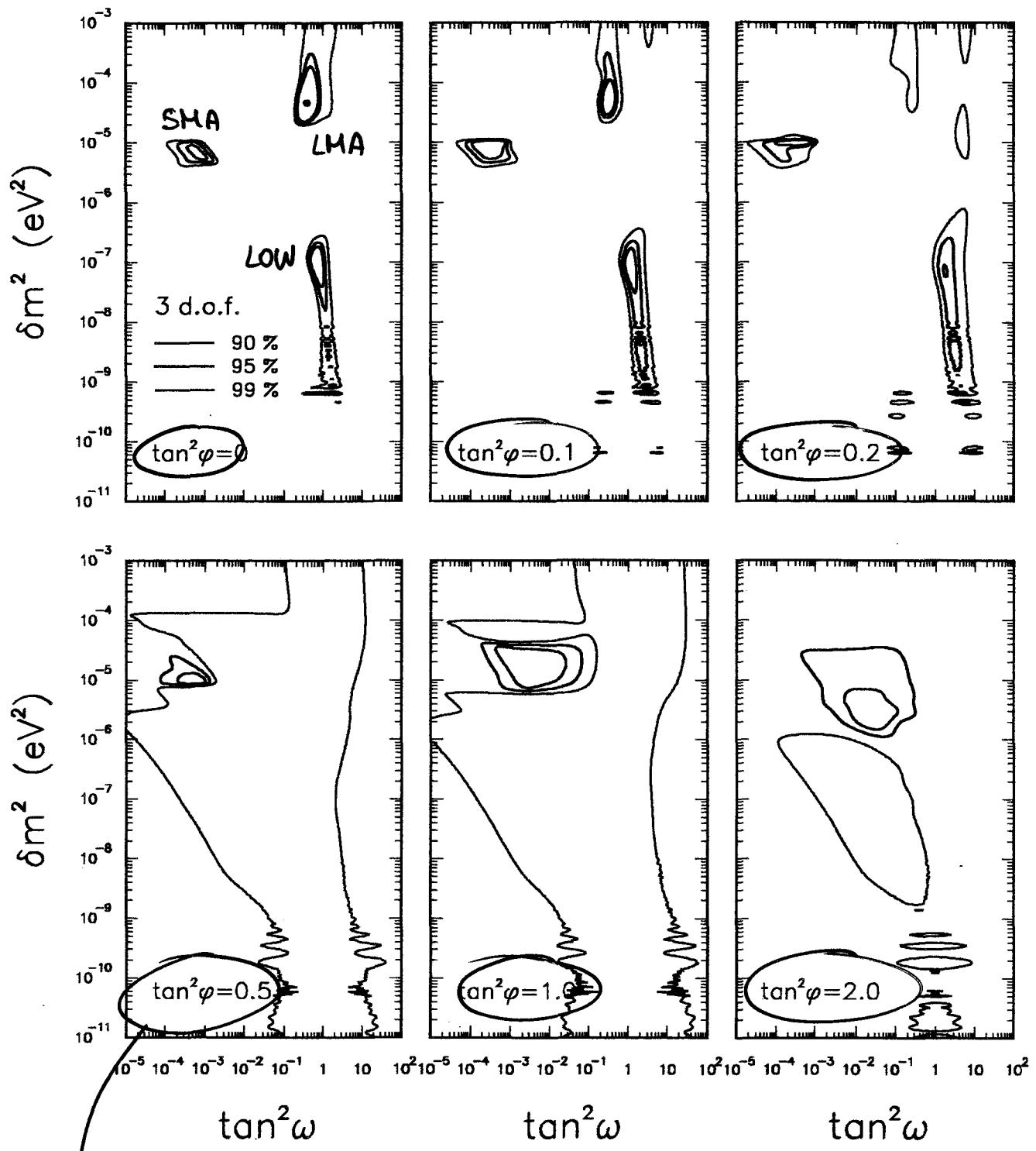
→ consistent with atm. data

→ consistent with He3,

NONTRIVIAL!

# $3\nu$ oscillations:

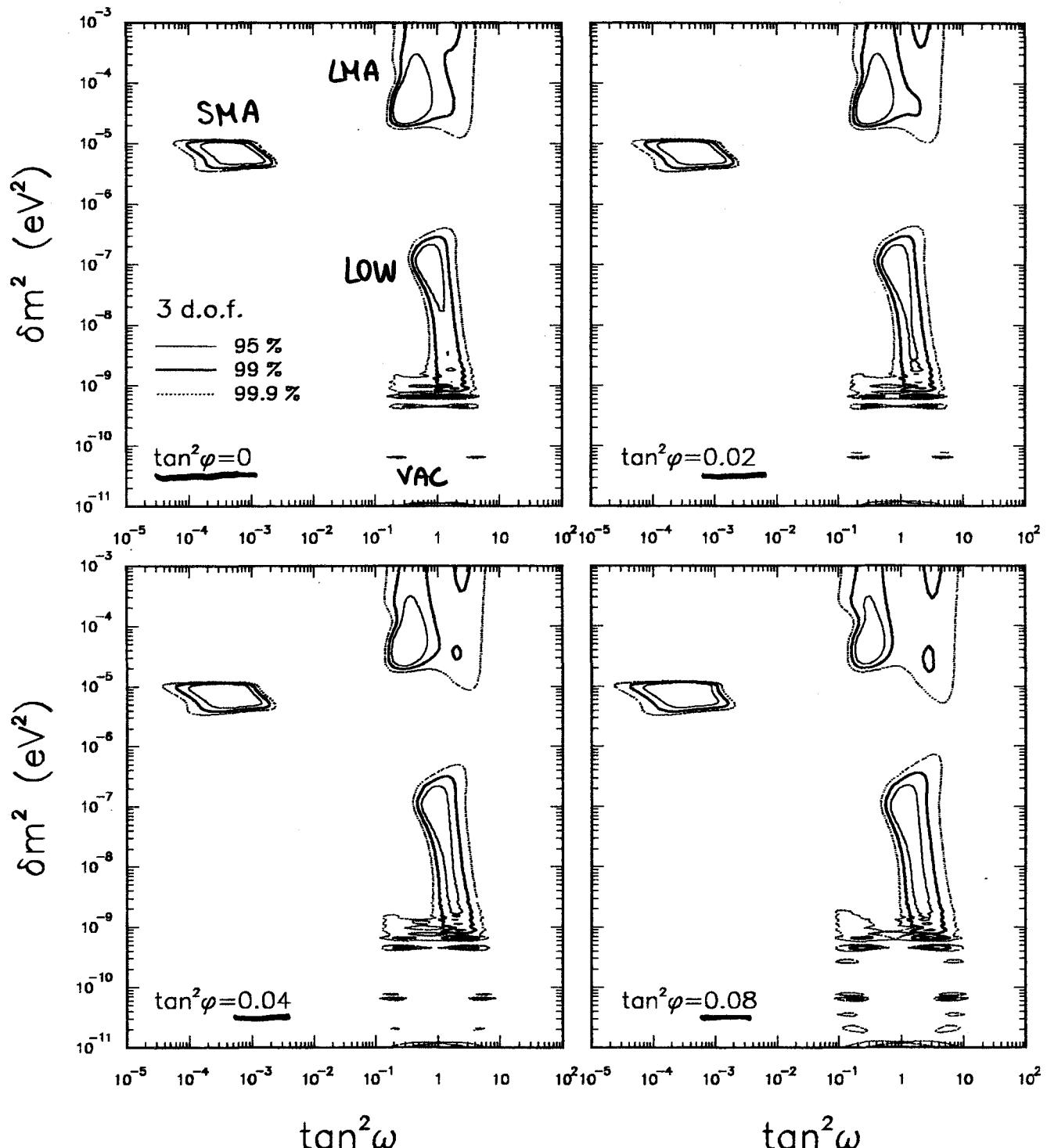
## Rates(Cl+Ga+SK) + SP(D) + SP(N)



AT RELATIVELY LARGE VALUES OF  $\varphi$ ,  
USUAL MSW SOLUTIONS PROFOUNDLY  
MODIFIED, BUT...

# $3\nu$ solar oscillations

## Rates(Cl+Ga+SK) + SP(D) + SP(N)

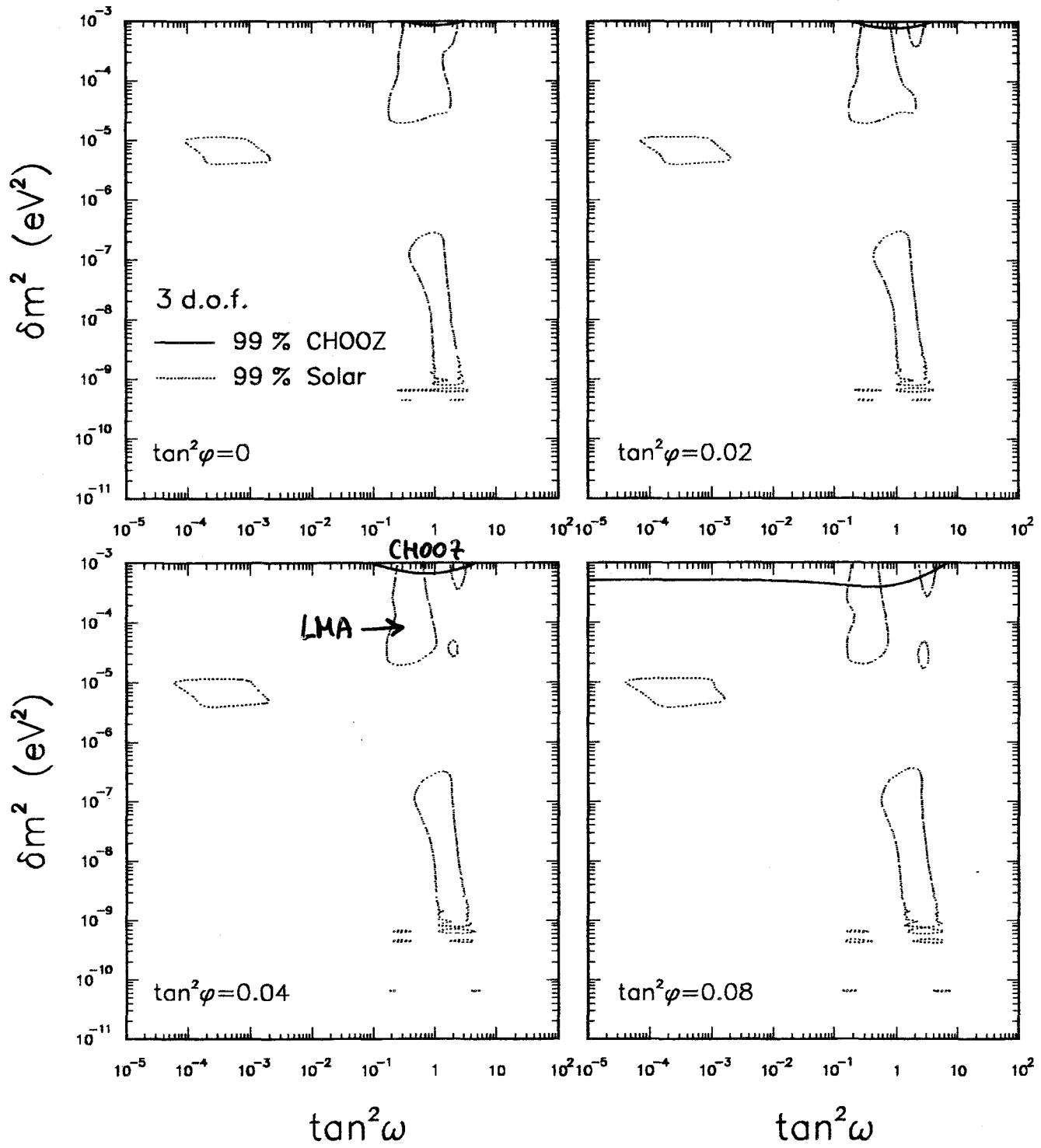


...milder modifications for  $|U_{e3}|^2 \lesssim \text{few \%}$   
(Chooz bound)

N.B.: CHOOZ also cuts upper part of LMA region

### $3\nu$ oscillations:

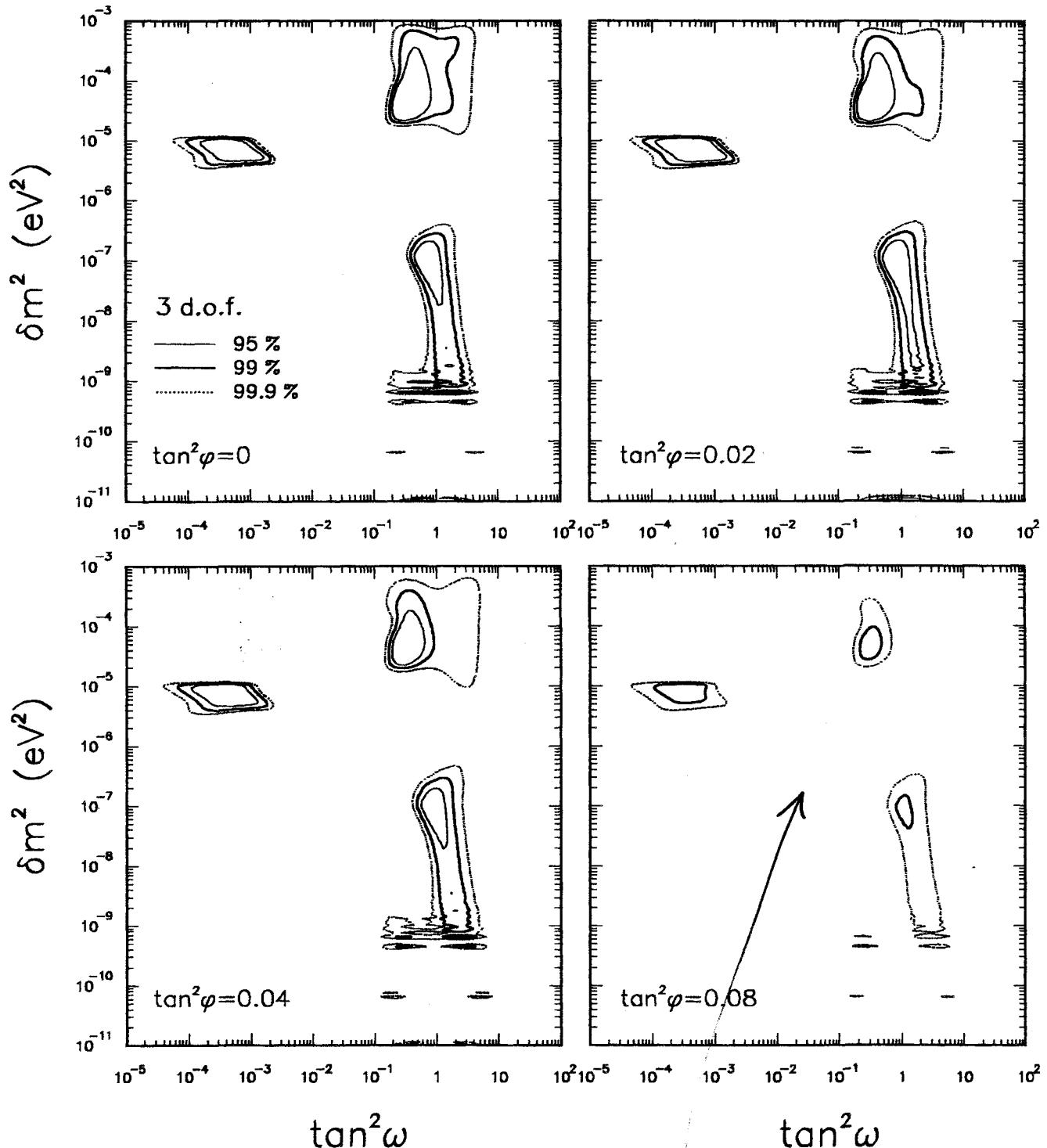
CHOOZ excluded regions ( $m^2 = +1.5 \times 10^{-3} \text{ eV}^2$ )



Final combination :

$3\nu$  oscillations:

SOLAR + CHOOZ (  $m^2 = 1.5 \times 10^{-3} \text{ eV}^2$  )



non-negligible effect  
only at the largest possible  
values of  $\varphi$  (allowed  
by Chooz)

# 3ν : GRAND TOTAL

$$\delta m^2$$

$$U_{e1}^2$$

$$U_{e2}^2$$

$$U_{e3}^2$$

MUST BE  
≤ few %

Depend on chosen  
Solar ν Solution:

LMA:  $U_{e2}^2 \lesssim U_{e1}^2$

SMA:  $U_{e1}^2 \sim 1$

LOW VAC:  $U_{e1}^2 \sim U_{e2}^2$

$$U_{\mu 3}^2$$

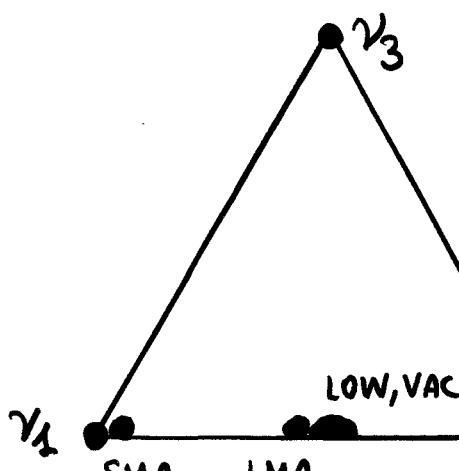
$$U_{\tau 3}^2$$

$$m^2$$

must be  
 $U_{\mu 3}^2 \sim U_{\tau 3}^2$   
within factor  
of two

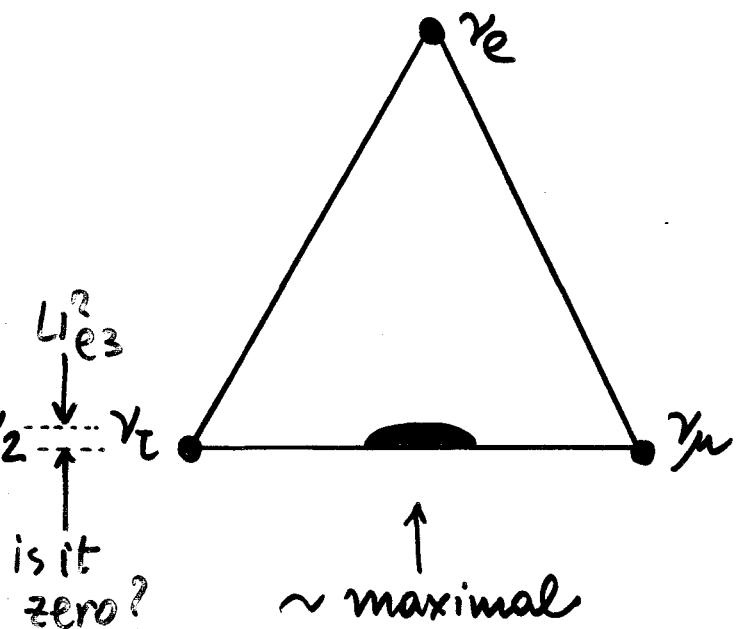
must be  
 $\sim 3 \times 10^{-3} \text{ eV}^2$   
within  
factor of two

SOLAR



must select one!

TERRESTRIAL



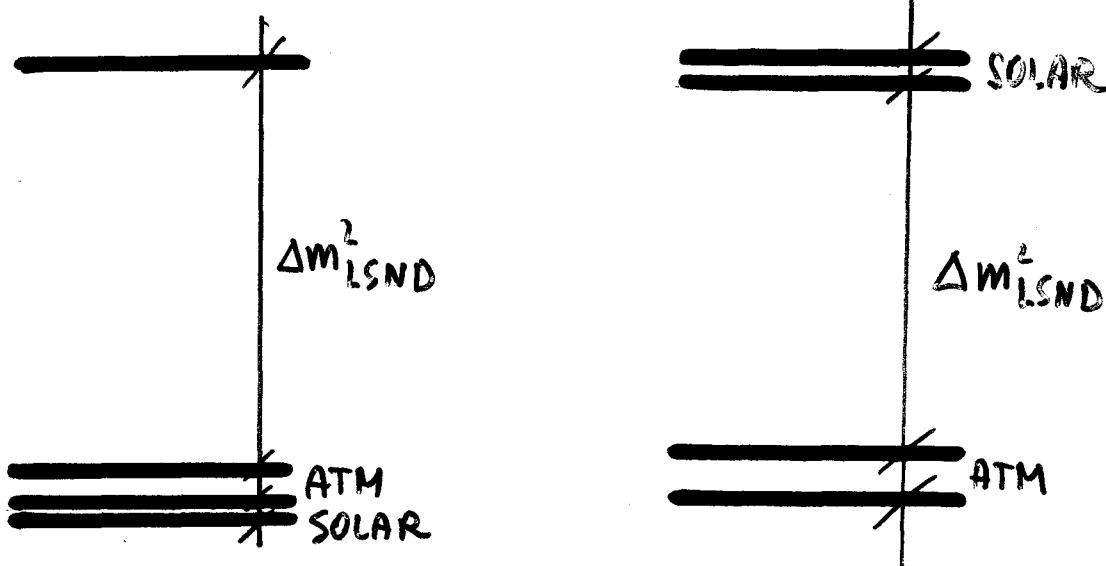
# $4\nu$ oscillations

SOLAR  
+ATM  
+LSND

$$\Delta m_{\text{solar}}^2 \ll \Delta m_{\text{atmos.}}^2 \ll \Delta m_{\text{LSND}}^2$$

$$< 10^{-3} \text{ eV}^2 \quad \sim 3 \times 10^{-3} \text{ eV}^2 \quad \sim \mathcal{O}(1 \text{ eV}^2)$$

→ TWO POSSIBLE FAMILIES  
OF SPECTRA:

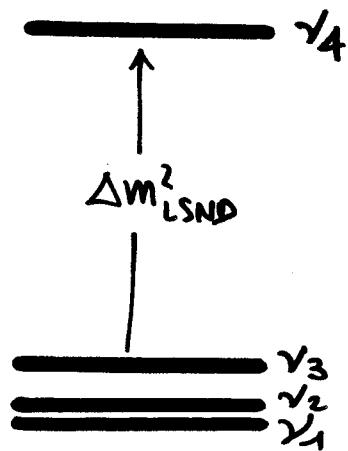


(+ spectra with  $\Delta m_{ij}^2 \rightarrow -\Delta m_{ij}^2$  etc.)

↑  
"3+1"

↑  
"2+2"

# 3+1 schemes



← The "lone" state  $\nu_4$  must be very close to a flavor eigenstate because:

- (i) LSND oscillation  $P_{\mu e}$  is small
- and (ii) all other acc/reac expt. find  $P_{\alpha\beta} \approx 0$  at  $\Delta m^2_{\text{LSND}}$  scale

$\nu_4 \approx \underline{\nu_e}$  ?

NO, otherwise  $P_{ee}(\text{solar}) \approx 1$  and no explanation of solar ν deficit

$\nu_4 \approx \underline{\nu_\mu}$  ?

NO, otherwise  $P_{\mu\mu}(\text{atm}) \approx 1$  and no explanation of atm. ν

$\nu_4 \approx \underline{\nu_\tau}$  ?

NO, otherwise for atm. ν:  
 $\nu_\mu \rightarrow \nu_e \oplus \nu_\tau$  with strong (unobserved) matter effects

$\nu_4 \approx \underline{\nu_{\text{sterile}}}$  ?

Ok, since then  $(\nu_1, \nu_2, \nu_3)$  are ~ linear combinations of  $(\nu_e, \nu_\mu, \nu_\tau) \simeq 3\nu$  osc.!  
 GOOD, BUT...

... if  $\nu_4 \simeq \nu_5$  then :

- $|U_{4S}^2| \simeq 1$
- $|U_{4e}^2$  and  $|U_{4\mu}^2$  both small  
(since  $|U_{4e}^2 + U_{4\mu}^2 + U_{4\tau}^2 + U_{4S}^2 = 1$ )

$$\rightarrow P_{\mu e}(\text{LSND}) \simeq 4 |U_{4e}^2| |U_{4\mu}^2| \underbrace{\sin^2 \left( \frac{\Delta m_{\text{LSND}}^2 \cdot L}{4E} \right)}_{\text{doubley}}.$$

Suppressed; too small!

---

Quantitative analyses show that, including  $\nu_e$  disappearance results (reactors) and  $\nu_\mu$  disappearance searches (accelerators) the value of  $|U_{4e}^2 \cdot U_{4\mu}^2|$  is too small to fit the LSND data (at  $\sim 3\sigma$  level)

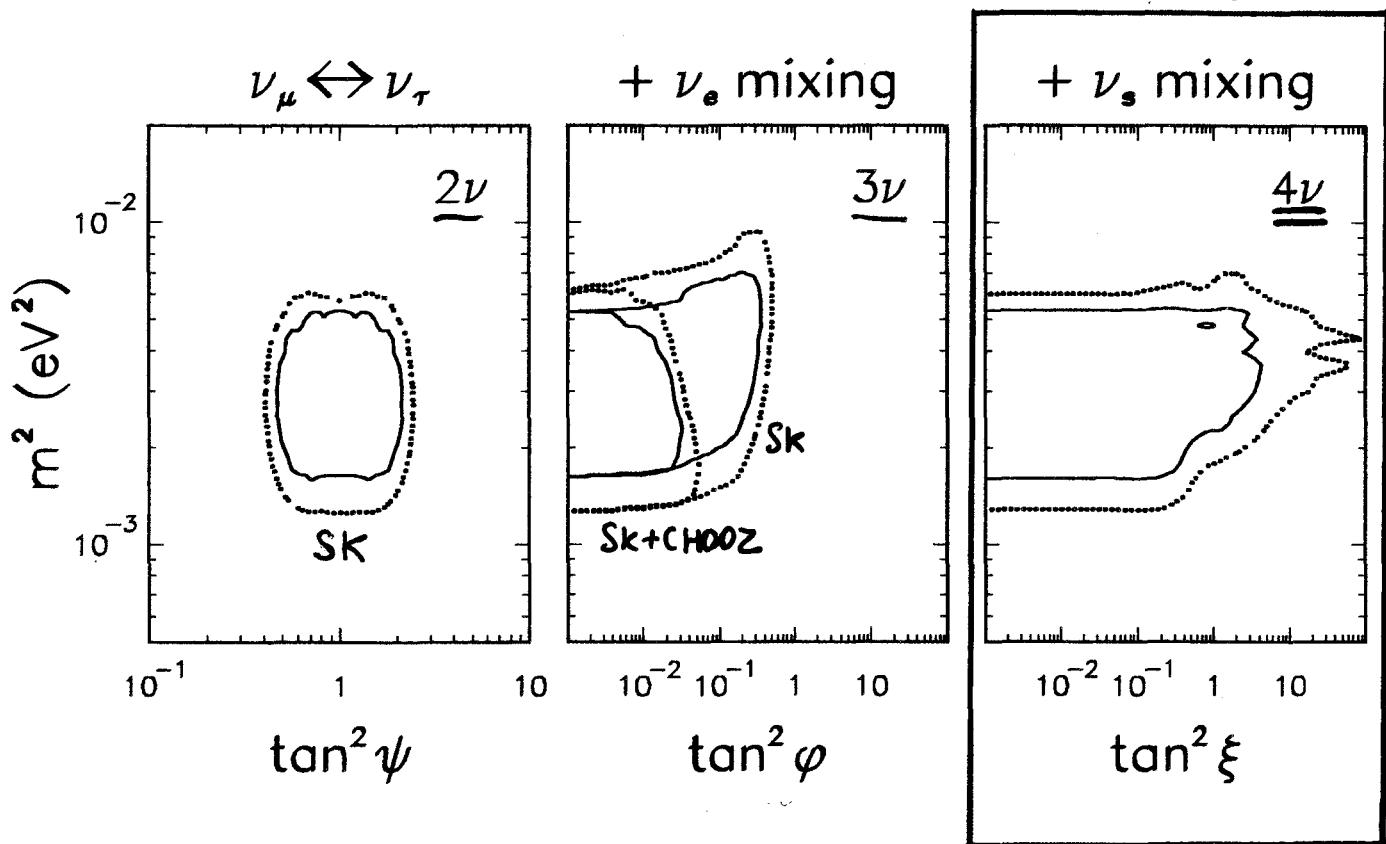
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$\rightarrow$  "TENSION" between LSND (appearance) and reactor+accelerator (disappearance) experiments at  $\sim 3\sigma$  in 3+1 SCHEMES

# ATMOSPHERIC $\nu$ :

- Best fit reached for  $\xi \approx 0$  ( $\sim$  pure  $\nu_\mu \rightarrow \nu_e$ )
- Using only zenith distrib. in SK, large  $\xi$  allowed

$4\nu$  analysis



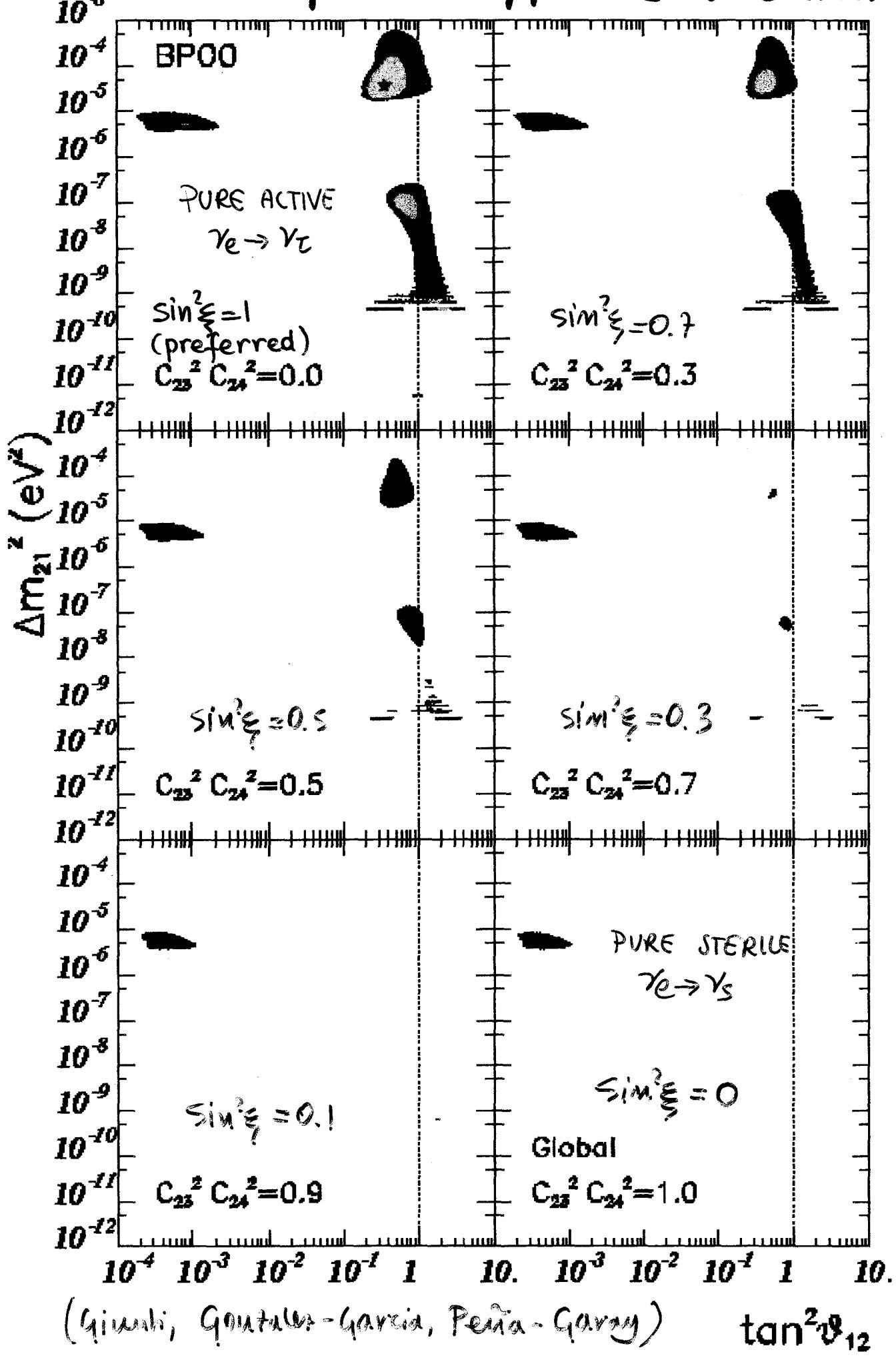
$$\begin{aligned} \nu_\mu \rightarrow 50\% \nu_s + 50\% \nu_e \\ \text{allowed } (\tan^2 \xi = 1) \end{aligned}$$

However, there are more data AGAINST additional  $\nu_\mu \rightarrow \nu_s$  (on top of  $\nu_\mu \rightarrow \nu_e$ ) :

- NC-enriched samples in SK
- "e-like" event appearance
- MACRO data

→ TOTAL COMBINATION LIKELY TO GIVE  $\sin^2 \xi \lesssim 0.3$

# Solar $\nu$ pull in opposite direction...



## 2+2 schemes

It was often assumed:

EITHER (A) ...

$$\overline{\text{ATMOS}} \quad \nu_\mu \rightarrow \nu_e \quad (\text{active})$$

... OR (B)

$$\overline{\text{ATMOS}} \quad \nu_\mu \rightarrow \nu_s \quad (\text{sterile})$$

SOLAR

$$\overline{\text{SOLAR}} \quad \nu_e \rightarrow \nu_s \quad (\text{sterile})$$

SOLAR

$$\nu_e \rightarrow \nu_\tau \quad (\text{active})$$

+ perturbations (needed to get  $P_{\mu e}(\text{LSND}) \neq 0$ )

HOWEVER, THE ABOVE CASES ARE SUB-CASES  
OF A MORE GENERAL SITUATION:

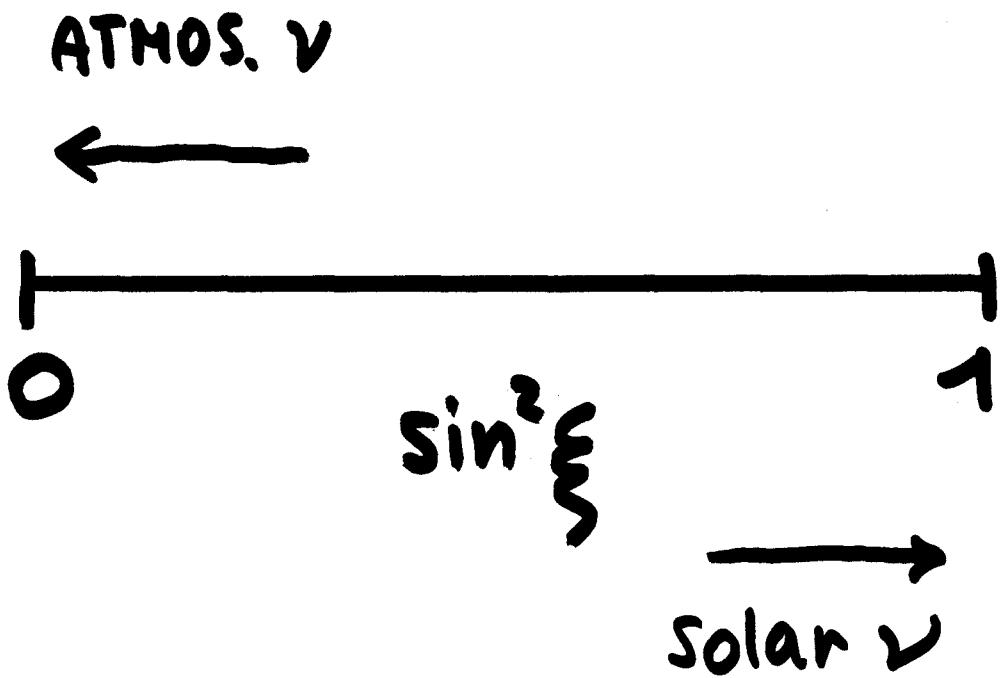
$$\begin{aligned} \text{ATMOS.: } & \nu_\mu \rightarrow \nu_+ & \begin{bmatrix} \nu_+ \\ \nu_- \end{bmatrix} = \begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix} \begin{bmatrix} \nu_e \\ \nu_\tau \end{bmatrix} \\ \text{SOLAR: } & \nu_e \rightarrow \nu_- \end{aligned}$$

→ mixed active + sterile oscillat.

get (A) for  $\sin\xi=0$

get (B) for  $\sin\xi=1$

WHAT HAPPENS FOR GENERIC  $\xi$ ?



"TENSION" BETWEEN ATMOSPHERIC  
AND SOLAR  $\nu$  DATA IN 2+2 SCHEMES  
(PULL  $\xi$  IN DIFFERENT DIRECTIONS)

MAYBE TOO EARLY TO RULE OUT 2+2,  
BUT CERTAINLY NOT A GOOD FEATURE....

## $4\nu$ summary :

- Sterile  $\nu$  introduced to accommodate LSND (with solar + atm)
- Two possible mass spectra : 3+1 & 2+2
- However, in 3+1 :  
accelerator + reactor data do not converge  
with LSND data
- ... and in 2+2 :  
solar and atm.  $\nu$  data do not converge
- Moreover, independent BBN data  
(see talk by K. Olive) prefer  $N_\nu \sim 3$   
rather than  $N_\nu \sim 4$

---

Maybe too early for a definitive "no-go theorem", but life is getting hard for the sterile neutrino...

## ABSOLUTE $\nu$ MASSES

Present  $\beta$ -decay and oscillation data already put some upper/lower bounds on the sum of  $\nu$  masses  $\sum_i m_{\nu_i}$ :

- $\sum m_\nu$  cannot be smaller than  $\sqrt{\Delta m^2_{\text{atm}}} \approx 0.05 \text{ eV}$
- Since all  $\Delta m^2_{ij} \lesssim 1 \text{ eV}^2$ , and since " $m_{\nu_e}$ "  $\lesssim$  few eV from  $\beta$ -decay, also  $\sum m_\nu \lesssim (N_\nu \cdot \text{few}) \text{ eV}$

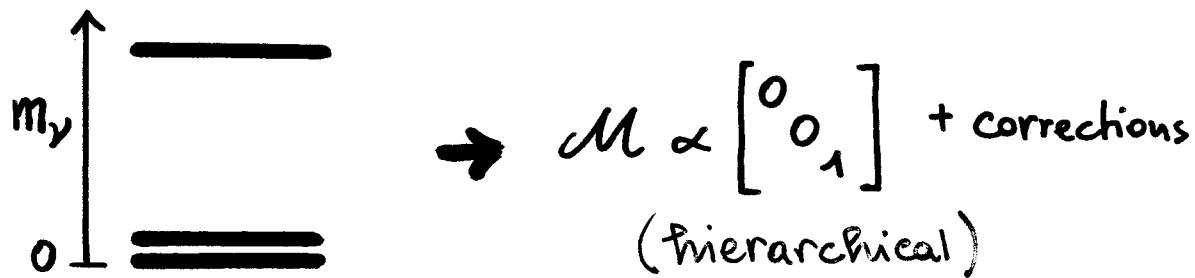


$$0.05 \text{ eV} \lesssim \sum m_\nu \lesssim \mathcal{O}(10) \text{ eV}$$

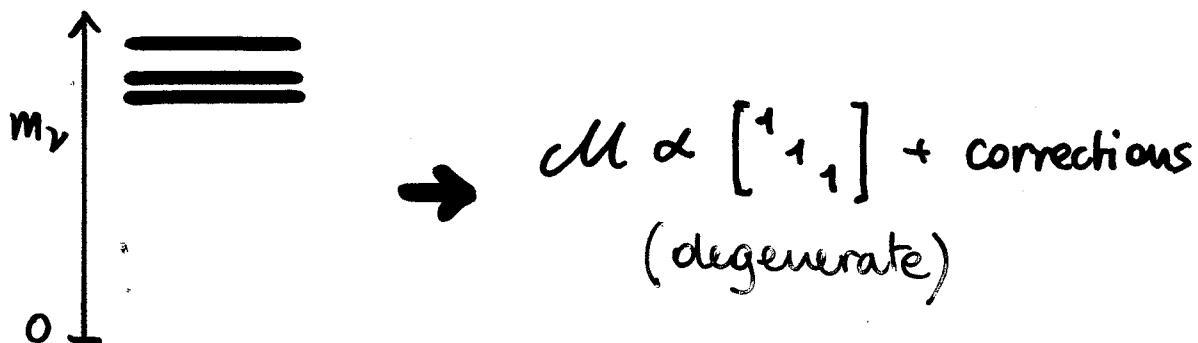
for  $N_\nu \sim 3, 4$

Still, too many possible spectra allowed in this range...

One would like to distinguish, e.g.,



from:



E.g., for bimaximal mixing:

$$U(\varphi, \omega, \psi) = U(0, \frac{\pi}{4}, \frac{\pi}{4}) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{bmatrix}$$

hierarchical  $\rightarrow U M U^+ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{bmatrix}$

degenerate  $\rightarrow U M U^+ = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

different textures

TEXTURES  $\rightarrow$  UNDERLYING SYMMETRIES (G. Ross lectures)

$\rightarrow$  need to refine info on absolute masses

# CONSTRAINTS FROM $\text{Ov}2\beta$

- Present bounds:  $\langle m_{ee} \rangle \lesssim \text{few} \times 10^{-1} \text{ eV}$
- Consider, e.g.,  $3\nu$  degenerate spectrum with bimaximal mixing; then:

$$\begin{aligned}\langle m_{ee} \rangle &= \left| \sum_i m_i e^{i\phi_i} U_{ei}^2 \right| \\ &\simeq m \left| \sum_i e^{i\phi_i} U_{ei}^2 \right| \\ &\stackrel{U_{e3}^2 \approx 0}{\simeq} m \left| U_{e1}^2 + U_{e2}^2 e^{i(\phi_2 - \phi_1)} \right|\end{aligned}$$

If  $m$  assumed to be relatively large ( $\sim \mathcal{O}(1) \text{ eV}$ ), the experimental bounds can be satisfied only if  $e^{i(\phi_2 - \phi_1)} \simeq -1$  (destructive interference of  $\nu_1$  and  $\nu_2$  contrib. in  $\text{Ov}2\beta$  decay)  $\rightarrow$  get constraints on Majorana phases

- However, if  $m$  large but solution is SMA (for solar  $\nu$  problem) then  $U_{ei}^2 \simeq 1$  and  $\langle m_{ee} \rangle \simeq m$  unavoidably  $\rightarrow$  excluded if  $m \gtrsim \text{few} \times 10^{-1} \text{ eV}$  independ. on Majorana phases (interesting!)

Similar arguments apply to the case  
of 2+2 spectrum in  $4\nu$  scenarios:

===== solar

If solar doublet  
"heavy", then solar  
 $\nu$  solution cannot be  
SMA, otherwise  
dominant contribution  
to  $\langle m_{ee} \rangle$  CANNOT be  
canceled

===== atmos.

CLEARLY, IF  $\langle m_{ee} \rangle$  UPPER LIMIT CAN  
BE REDUCED BY AN ORDER OF MAGNITUDE  
(GENIUS PROPOSAL) , THEN ONE WILL  
PROBE CONTRIBUTIONS TO  $\langle m_{ee} \rangle$  OF  
SIZE  $\sim \sqrt{\Delta m^2_{atm}}$ , AND CANCELLATIONS  
WILL BE MUCH MORE CONSTRAINED  
→ WILL THEN LEARN A LOT  
ABOUT POSSIBLE MASS SPECTRA

## 2) mass & cosmology

- Oldest cosmological bound :

$$\sum_i m_i \lesssim 92 h^2 \text{ eV}^2$$

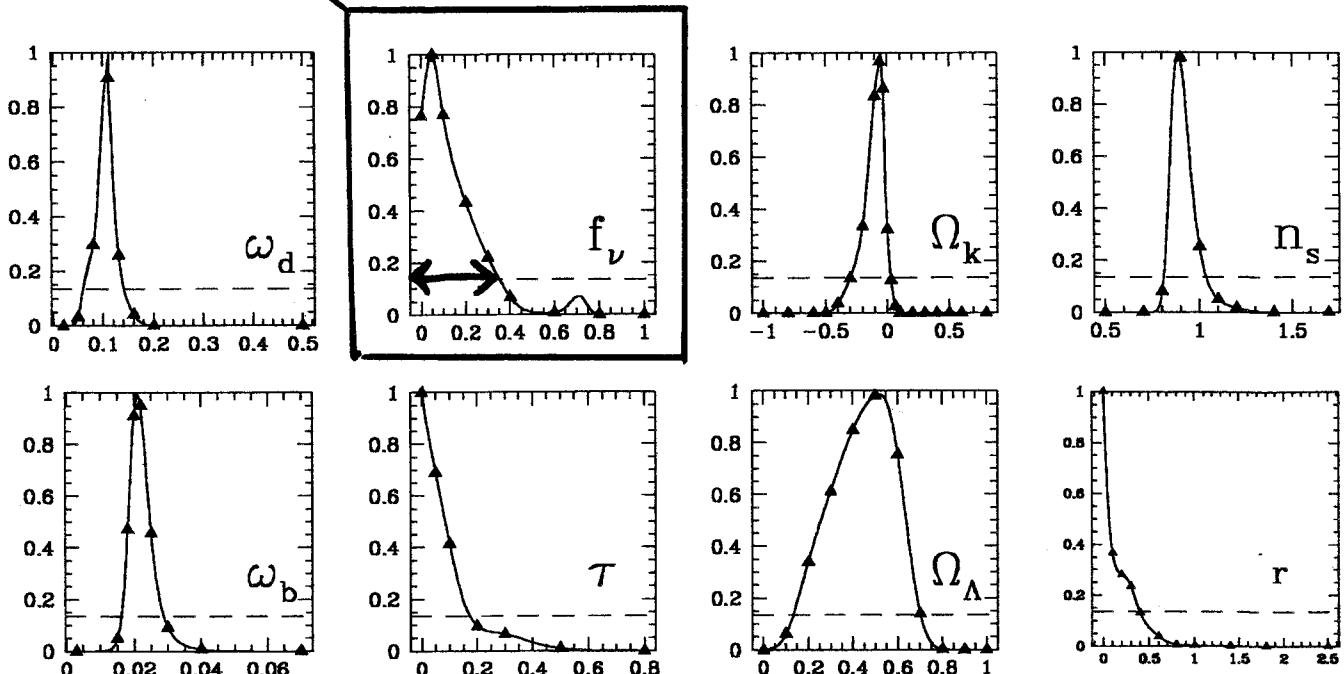
to avoid overabundance of relic  $\gamma$   
( $\Omega_\gamma = 1$  if bound is saturated)

- However, well before the above bound is saturated, neutrino masses can alter the properties of
  - DARK MATTER
  - STRUCTURE FORMATION
- Many effect contribute to dark matter and s.f. properties, so indications are somewhat indirect and come from multiparametric fits to cosmological/ astrophysical data, now made possible by "explosion" of observational data

E.g., 11-parameter fit results including

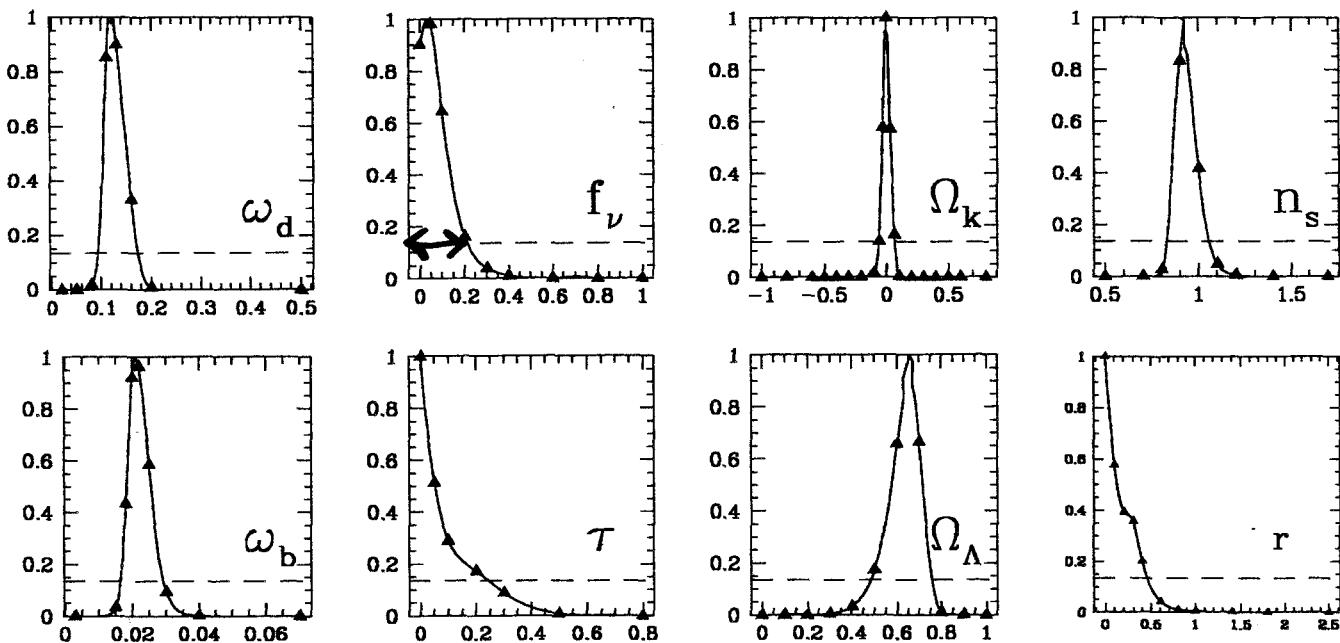
$$f_\nu = \frac{\Omega_\nu}{\Omega_{DM}} = \text{"fraction of hot D.M."}$$

$\leftrightarrow = 95\% \text{ C.L. range for } f_\nu$



- UPPER BOUND  $f_\nu \lesssim 0.35$
- No significant evidence for  $f_\nu > 0$

as before, but with prior  $\bar{h} = 0.74 \pm 0.08$



→ in any case,  $f_\nu$  preferred to be small  
 →  $\sum m_{\gamma_i} \lesssim 4.4$  eV

↑ interestingly close  
to laboratory limits

# RECAP

- Solar + atm + CHOOZ data fit nicely a 3ν oscillation scheme (convergence of all data towards  $U_{e3}^2$  small)  
Main unknowns:
  - Is  $U_{e3}^2 \neq 0$  ?
  - Solution to solar ν problem?
  - Spectrum hierarchy?
  - .....
- 4ν scheme do not accommodate easily solar+atm+LSND data , either in 3+1 or in 2+2 schemes - Need to monitor "tension" between data with future observ.
- Present absolute ν mass indications already tell us a few interesting facts ; however, decisive probes of the spectrum structure require sensitivity as low as
$$\sqrt{\Delta m_{\text{atm}}^2} \sim 5 \times 10^{-2} \text{ eV}$$
→ goal of future experiments