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SMR.1317 - 24

SUMMER SCHOOL ON PARTICLE PHYSICS

18 June - 6 July 2001

FERMION MASSES AND THE FLAVOUR PROBLEM

Lecture V

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Please note: These are preliminary notes intended for internal distribution only.

• GRAND UNIFICATION & STRING UNIFICATION.

⇨ Multiplet structure (except Higgs) fits elegantly in GUT AND STRING.

⇨ Fermion masses : $m_b = m_\tau \big|_{M_X}$ *

$$m_\mu = 3 m_s \big|_{M_X}$$

$$m_e = \frac{1}{3} m_d \big|_{M_X}$$

... MAY ALSO APPLY IN STRING THEORY WITH UNDERLYING GUT

⇨ Gauge coupling unification ***

... MAY PERSIST IN STRING EVEN WITHOUT GUT BELOW COMPACTIFICATION THRESHOLD : $g_i = k_i$

⇨ Multiplet structure fixed ... elegant mechanism for doublet/triplet splitting ... only fundamental representations in level-1 strings ($k_i = 1$)

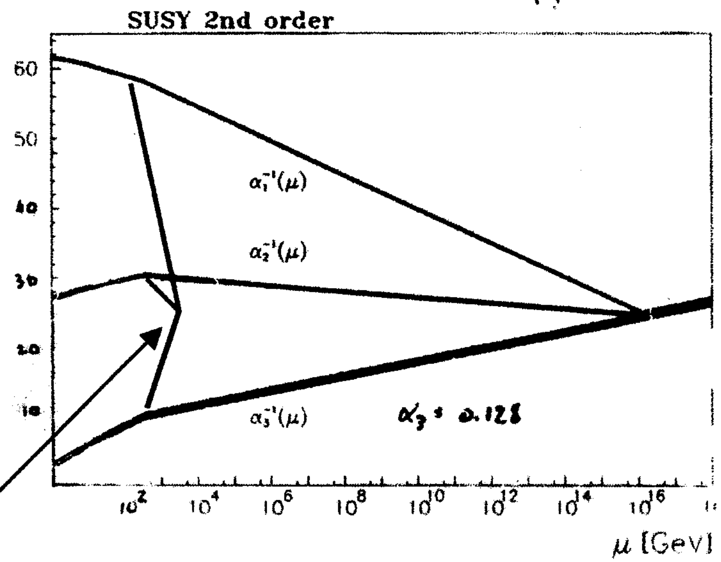
⇨ Symmetries determined ($U(1)^s$, Z_N) ... family symm? Couplings determined ... difficult unless at special (moduli

Unification Hints

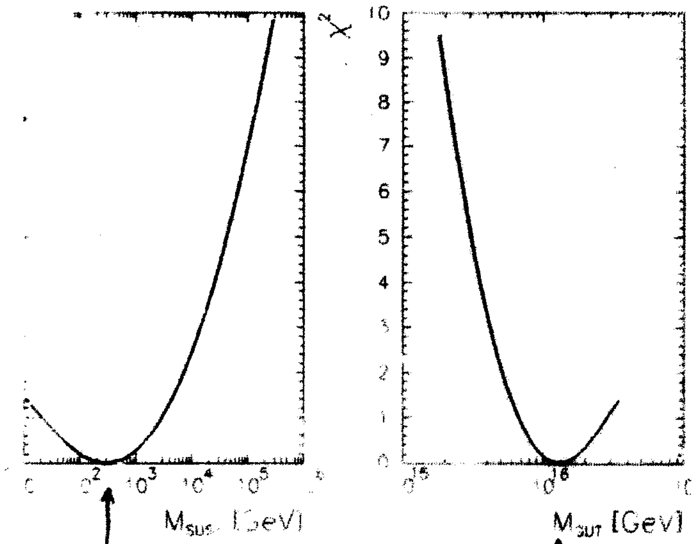
$$\alpha_i^{-1}(\mu) = \alpha_{GU}^{-1}(\mu) + \frac{b_i}{4\pi}\ln\left(\frac{M_X^2}{\mu^2}\right) + \dots$$

Amaldi, de Boer, Furstenau

MSSM



See also
Ellis, Kelley, Nanopoulos;
Langacker, Luo
Roberts, GGR



$M_{susy} = 10^{2.5\pm1.0}$

$10^{16.1\pm0.3}$

Large New Dimensions

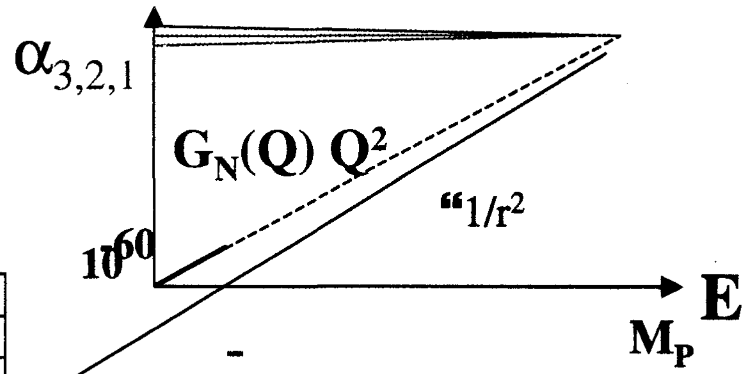
Unification at a TEV???

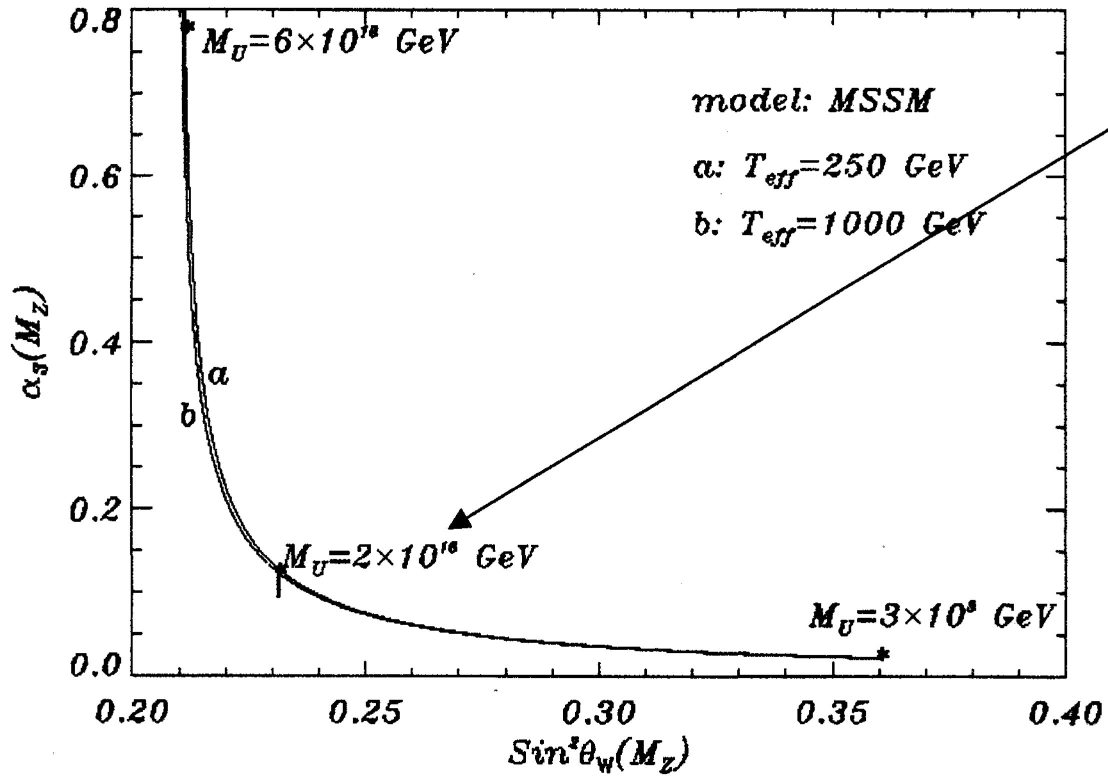
Dienes, Dudas, Ghergetta,

Dimopoulos, Raby, Wilczek
Ibanez, GGR

Unification Hints

SUSY UNIFICATION ✓





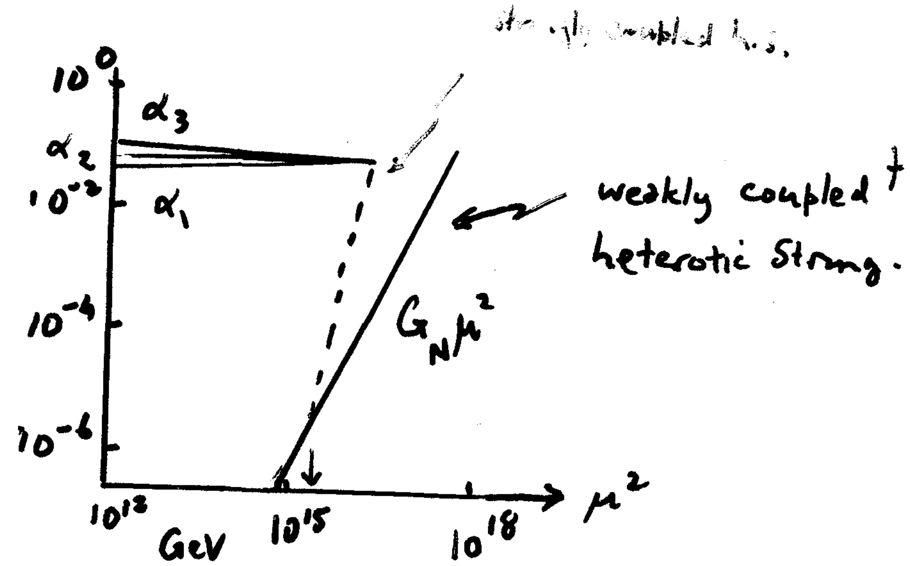
Unification with gravity!

GGR. D.Ghilencea

$$\sin^2\theta_W = 0.2337 \mp 0.0015 - 0.25(\alpha_s - 0.119)$$

c.f. 0.2312 ∓ 0.0002 Expt.

• UNIFICATION WITH GRAVITY



$$\mathcal{L}^{H.S.}_{eff} = -\underbrace{\int d^{10}x \, \sqrt{g}}_{\int d^4x \, V} \, \overbrace{e^{-2\phi}}^{\alpha_{10}^{-1}} \left(\frac{4}{\alpha'^4} R + \frac{k_i}{\alpha'^3} \mathrm{Tr}\, F_i^2 + \dots \right)$$

† α_{10}, α_{GUT} small.

$\alpha' = \frac{1}{M_{string}^2}$ only scale

$$G_N = \frac{\alpha_{10}\,\alpha'^4}{64\pi V}, \quad \alpha_{GUT} = \frac{\alpha_{10}\,\alpha'^3}{16\pi V} \Rightarrow G_N = \frac{\alpha_{GUT}\,\alpha'}{4}.$$

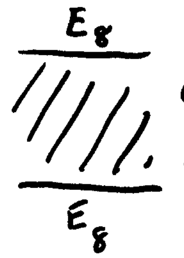
$$\boxed{\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g^2_{string}} + b_i \ln\left[\frac{M_{STRING}}{M_Z}\right] + \Delta_i}$$

$M_{string} = g_{string} \cdot M_{Planck} = 5 \cdot 10^{17}$ GeV ... close!

Strongly coupled H.S. - At low energies ≡ 11D SUGRA

M theory on $R^{10} \times S_1/Z_2$.

E. Witten, P. Horava



$$\mathcal{L} = -\frac{1}{2\pi K_{11}^2}\int_{M^{11}} d^{11}x \sqrt{g}\, R - \sum_i \frac{1}{8\pi(4\pi K_{11}^2)^{2/3}}\int_{M_i^{10}} d^{10}x \sqrt{g}\, \mathrm{Tr} F_i^2$$

$$G_N = \frac{K_{11}^2}{16\pi^2 V \underline{\underline{R_{11}}}} \qquad \alpha_G = \frac{(4\pi K_{11}^2)^{2/3}}{2V}$$

$$\Rightarrow \quad \boxed{\pi R_{11} = 12\, M_{11}^{-1} = (5.10^{15}\, \mathrm{GeV})^{-1}}$$

where $\quad M_{11} = 6.10^{16}\, \mathrm{GeV} \quad (\equiv K_{11}^{-2/9})$

However Witten showed R_{11} cannot be arbitrarily large...

$E_8 \times E_6 \quad : \quad \dfrac{1}{g^2_{6,8}} = S_R \pm \alpha R_{11} \quad (\equiv R^6_{6,8})$ also Ibanez, Nilles ..

R_{11} large $\quad : \quad g_8 \to \infty^{\dagger},\ R_8 \to 0 \quad \Rightarrow$ Max value for R_{11}.

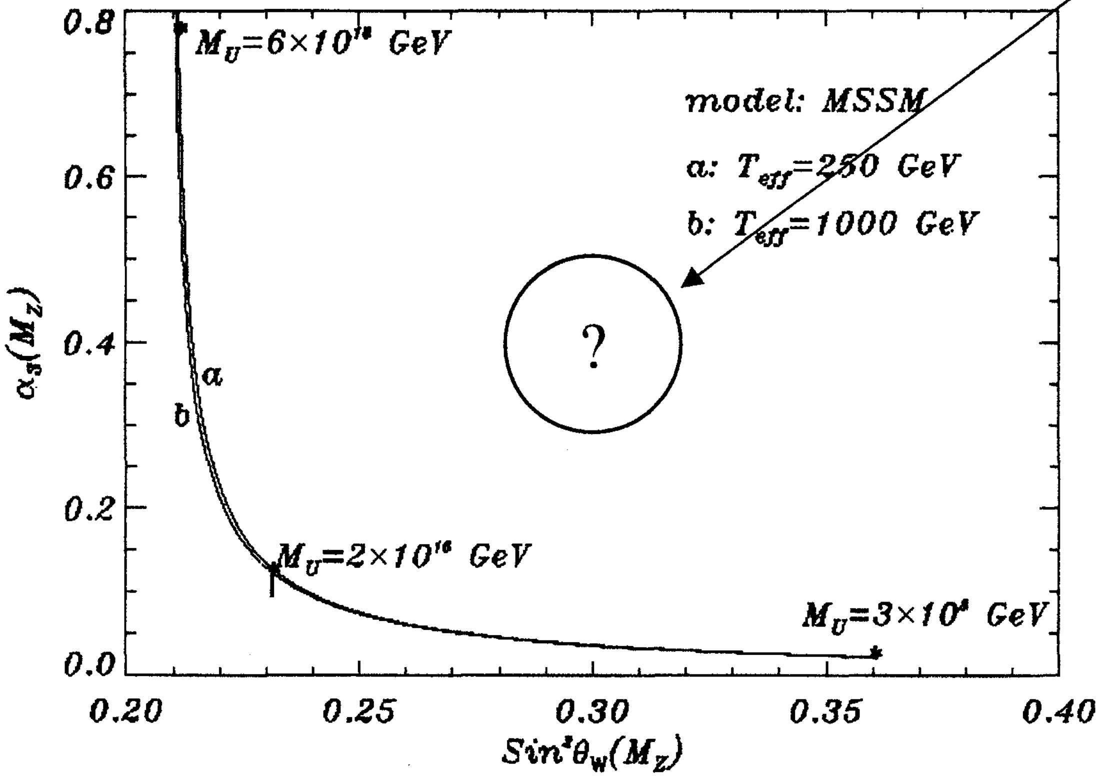
† But $\Rightarrow \Lambda_{SUSY} \simeq M_X$?? gaugino condensation

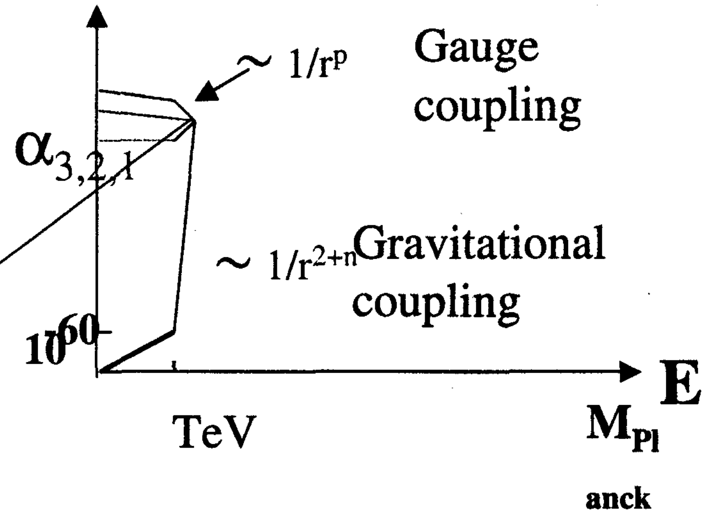
$$\Rightarrow \quad \boxed{\pi R_{11} \lesssim (5.10^{\pm 15}\, \mathrm{GeV})^{-1}}$$

α model dependent... Wilson lines? ... non-standard embedding Nilles..

Unification Hints

Unification at a TeV? ✕





Profile of a string theory.

- No low scale unification
- "SU(5)" -normalisation †
- $\mu_0 \simeq M_S$

~~for details~~

.. D. Ghilencea

Weakly coupled heterotic string ?

$$M_X = g M_S \simeq 20\, M_{GUT} \;?$$

⇨ Large string threshold corrections ... $T \sim 9$ ✗

⇨ Wilson line breaking , $T \simeq 1$, $C \simeq 1$ ✓ (near superconformal point)††

⇨ GUT $< \mu_0$ ✓ (But $\mu_0 \simeq M_S$ still) ← 2-loop.

Strongly coupled heterotic string ?

$$M_X \Rightarrow M_X (R_{11} \mu_0)^{-1} , \quad R_{11} \sim 10^{-14}\,\text{fm} \quad ✓$$

but $R \sim 10^{-14}$ fm is maximum for any new dimension to preserve high scale unification

only gravity propagating.

† cf Ibanez, Quevedo type I/I' models with no N=2 sector ...

... extreme sensitivity to new Higgs sector needed to compensate for non-"SU(5)" normalisation ($5\,H_D$, $3(d+d^c)$)

Radiative string effects

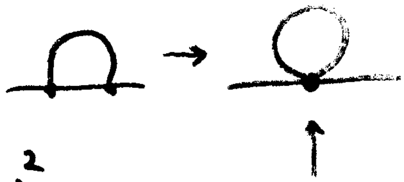
$$\boxed{\alpha_i^{-1}(\mu) = k_i\,\alpha^{-1} + \frac{b_i}{2\pi}\ln\left(\frac{M_{string}^2}{\mu^2}\right) + \Delta_i}$$

Contribution of massive string states

Kaluza-Klein Winding

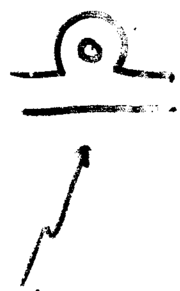
$$\Delta_i = \sum_i{}' \tilde{b}_i \ln\left(\frac{m_i^2}{M_{string}^2}\right) + \sum_{l,n\in Z} \tilde{b}_i \ln\left(\frac{\sum_j l_j^2 \frac{1}{R^2} + \sum_j n_j^2 \frac{R^2}{\alpha'}}{M_{string}^2}\right).$$

$l, n \to \infty$; Divergent ?



$$\text{F.T. :} \quad \int_0^\infty \frac{dt_2}{t_2}\, e^{-\frac{m^2 t_2}{M_{string}^2}} = \ln\left(\frac{m_i^2}{M_{string}^2}\right) + \text{divergent term}$$

$$\Delta_i^{string}{}_{orbifold} = \sum_i \tilde{b}_i \int_{-1/2}^{1/2} dt_1 \int_{\sqrt{1-t_1^2}}^{\infty} \frac{dt_2}{t_2}\, e^{-\frac{m_i^2 t_2}{M_{string}^2}}$$



REGULATES DIVERGENCE (H.S.)

$$\boxed{\Delta_i = \sum_i \tilde{b}_i \log\left[\left|\eta(T/2\alpha')\right|^4 \frac{(T+\bar{T})}{2\alpha'} \cdots\right]}$$

Dixon Kaplunovsky Louis ; Ferrara, Kounnas, Lüst, Zwirner ...

$T \sim R^2$, moduli

Features of string "threshold" corrections

- Δ_i comes only from $N=2$ sector† ($N=4$ non-ren. thm)

... dominated by KK contribution with $m_i^2 \lesssim M_{string}^2$

String cuts off massive states

$\sim e^{-2\pi \frac{m^2}{M_{string}^2}}$

- Power law running for $T \sim R^2$ large.

$$\Delta_i \sim \tilde{b}_i \log\left[|\eta(T)|^4 (T+\bar{T})\right] \quad ; \quad \eta(T) = q^{1/24} \prod_n (1-q^n)$$

$e^{-2\pi T}$

$$\sim \tilde{b}_i T \equiv \tilde{b}_i R^2$$

Corresponds to decompactification : $T^6 = T^4 \times T^2$

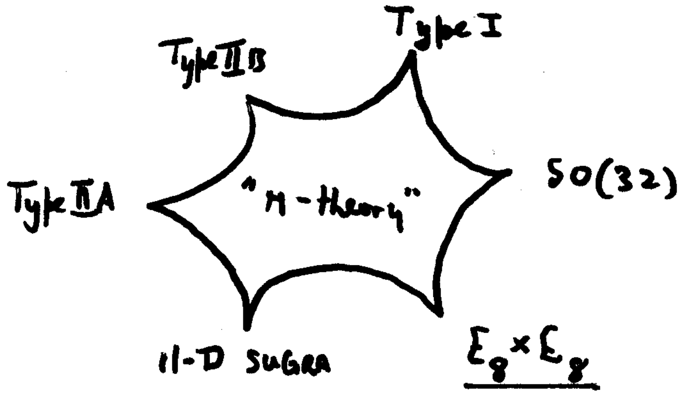
$N=4$ $\quad N=2^\dagger$

... due to susy only 2D "decompactifies"

The top-down approach

TOE $\overset{?}{\Rightarrow}$ MSSM

TOE $\equiv$ string



- Unification : Heterotic string naturally fills "profile"

- SUSY ? String needs SUSY for consistency

  ... low-energy SUSY possible ✓

- G ? eg. $E_8 \times E_8$ heterotic string, D = 10

$\Rightarrow$ COMPACTIFICATION $\longrightarrow$ $E_8 \times E_6$ D = 4, N = 1 SUSY — Low-energy SUSY constraint - "standard embedding"

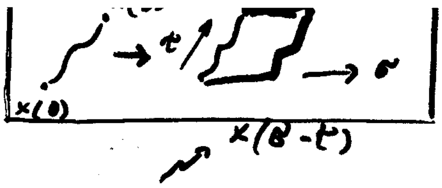
$$E_6 \supset SO(10), SU(5) \supset SU(3) \times SU(2) \times U(1) \checkmark$$

+ CHIRAL STRUCTURE ✓

- g_i ? $g_i \longrightarrow g$ (SU(5) normalisation) ✓

in addition M_X, α_G determined !

Orbifold compactification :



Heterotic string $\quad |D=10, \text{fermionic}\rangle_{RM} \otimes |D=26, \text{bosonic}\rangle_{LM}$

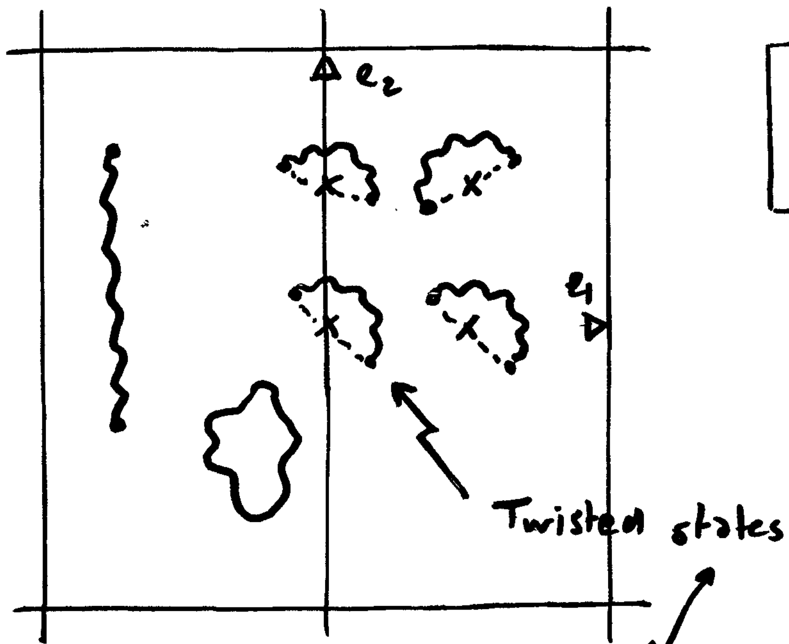
Compactified on the orbifold O_5

$$\boxed{O_5 = \frac{T_R^6}{P} \otimes \frac{T_L^6}{P} \otimes \frac{T_{E_8\times E_8}}{G}}$$

Point group $\qquad\qquad$ Gauge twisting group

Strings on the orbifold :

Toy example : $\quad T^2 = \frac{R^2}{\Lambda}$ $\quad$ lattice $e_1=(1,0)$, $e_2=(0,1)$



$$\boxed{O = \frac{T^2}{P}}$$

$\underline{PX = -X}$

$$X^i(\sigma=2\pi) = \left(\Theta X(\sigma=0)\right)^i + v^i$$

Radiative string effects

Δ_i:

$E_8 \times E_8$ Heterotic string in $D=10$.

$$\left\{ \begin{array}{l} X_R^{i}(\sigma+\tau) \quad (i = 1\ldots6) \\ X_L^{i}(\sigma-\tau),\ X_L^{I}(\sigma-\tau) \quad (I = 1\ldots16) \end{array} \right.$$

$\downarrow$ Orbifold, Calabi Yau ...

$E_8 \times E_6$ in $D=4$, $N=1$ SUSY

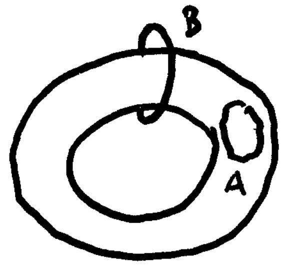
Untwisted strings. :

Quantised momentum + winding #$^{+}$

Kaluza Klein modes.

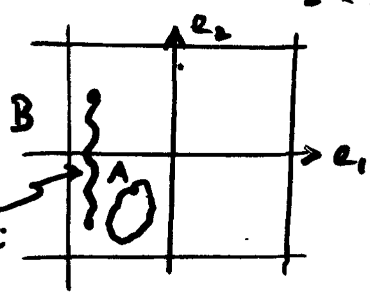
$$X_R^{i}(\sigma+\tau) = x_0^{i} + P_R^{i}(\sigma+\tau) + \frac{i}{2}\sum_n \frac{\alpha_n^{i}}{n} e^{-\frac{in(\sigma+\tau)}{2\pi R}}, \quad i = 1\ldots6$$

e.g. 2-D compactification on torus : $T^2 = R^2/\Lambda$ lattice $e_1 = (1,0)$, $e_2 = (0,1)$



Winding^{+}

$X^{i}(\sigma=2\pi) = X^{i}(0) + w^{i}$

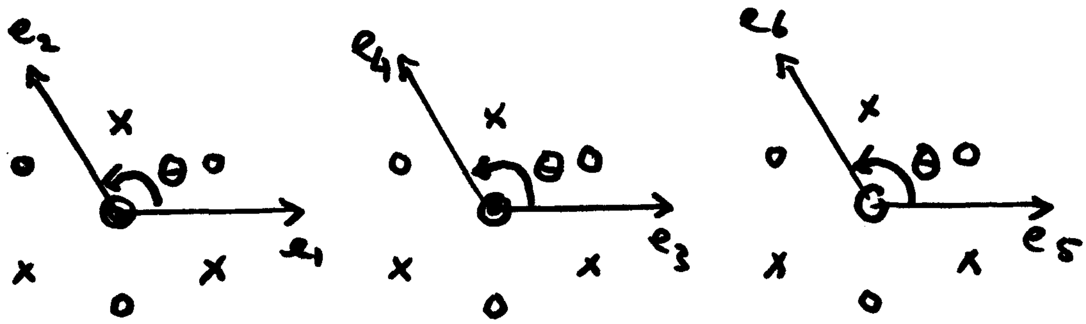


$$\frac{m_R^2}{8} = \frac{P_R^2}{2} + N \qquad \frac{m_L^2}{8} = \underbrace{\frac{P_L^2}{2}}_{m^2 R^2} + \underbrace{\tilde{N} - 1}_{n^2/R^2}$$

Invariance under $R \to 1/R$, $m \rightleftarrows n$ DUALITY

$\underline{Z_3 \text{ orbifold}}$: $T^6 = \dfrac{R^6}{SU(3)^3}$

$$O = \frac{T^6}{P} \;, \qquad PX = e^{\frac{2\pi i}{3}} X.$$



$3 \times 3 \times 3 = 27$ fixed points

Orbifold symmetries

Fixed points labeled by $(\Theta^k, e_\nu) \equiv T_k$

$$\Theta T_{k(\gamma)} = e^{2\pi i n/k} T_{k(\gamma)} \equiv \gamma T_{k(\gamma)}$$

Then non-vanishing couplings involving twisted states should satisfy

$$\boxed{\begin{array}{l} \prod_a (\Theta_a^k, e_a) = (1,0) \\ \prod_a \gamma_a = 1 \quad : \quad Z_k \text{ symmetry.} \end{array}}$$

Mass matrix structure from orbifolds

eg $\underline{Z_6 \text{ orbifold.}}$ $\equiv \dfrac{R^6}{(SO(3)\times G_2^2)\,P}$

Massless matter states in T_1, T_2, T_3, T_4 sectors

Coupling	Condition	
$T_2^{3\ell}\, T_3^{2m}$	$\ell>0,\ m>0$	Kobayashi
$T_4^{3\ell}\, T_3^{2m}$	$\ell>0,\ m>0$	Xing
$T_1^{2\ell}\, T_2^{m}\, T_4^{n}$	$\ell=2p+1,\ 2\ell+2m+n=3q,\ \ell>0,\ m>0,\ n>0$	(Casas,
$T_1^{\ell}\, T_2^{m}\, T_3^{n}\, T_4^{p}$	$\ell>0,\ n>0,\ m$ or $p>0$	Muñoz)

Not all textures possible but some realistic ones.

eg. The fields T_2, T_3, T_1 contain generations 1, 2, 3 respectively. T_4 contains H. Then (with some Θ alignment)

$$\mathcal{M}_{ud} \propto \begin{pmatrix} \varepsilon^8_{u,d} & \varepsilon^3_{u,d} & \varepsilon^3_{u,d} \\ \varepsilon^3_{u,d} & \varepsilon^2_{u,d} & \varepsilon^2_{u,d} \\ \varepsilon^3_{u,d} & \varepsilon^2_{u,d} & 1 \end{pmatrix} \qquad \varepsilon = \frac{\Theta}{M}$$

CONSTRUCTION OF 4-D STRING THEORIES

- Want compactification to preserve $N=1$ SuSy in 4D. $\Rightarrow$ $N=2$, $D=2$ WORLD SHEET SUPERSYMMETRY of RIGHT MOVERS.

... simplest scheme has also $N=2$ for LEFT MOVERS.

- String world sheet has conformal invariance.

$\Rightarrow$ Can construct viable 4D string theory requiring underlying $(2,2)$ superconformal invariance.

GEPNER CONSTRUCTION

$$\boxed{\psi = |D=4\rangle\, |N=2 \text{ SCT}\rangle_{RM} \otimes |\text{GAUGE}\rangle\, |N=2 \text{ SCT}\rangle_{LM}}$$

Conformal anomaly cancellation $\Rightarrow (c,\tilde{c}) = (9,9)$

$\equiv$ Calabi Yau, but not at large radius ✓

N=2 SUPERCONFORMAL ALGEBRA

Virasoro

$$T_B(z)\, T_F^{\pm}(0) \sim \frac{3}{2z^2} T_F^{\pm}(0) + \frac{1}{z}\partial T_F^{\pm}(0)$$

$$T_B(z)\, J(0) \sim \frac{1}{z^2} j(0) + \frac{1}{z}\partial j(0)$$

$$T_F^{+}(z)\, T_F^{-}(0) \sim \frac{2c}{3z^2} + \frac{2}{z^2} j(0) + \frac{2}{z} T_B(0) + \frac{1}{z}\partial j(0)$$

$$j(z)\, T_F^{\pm}(0) \sim \pm\frac{1}{z} T_F^{\pm}(0)$$

$$j(z)\, j(0) \sim \frac{c}{3z^2}$$

$$T_B = \sum_{n\in\mathbb{Z}} \frac{L_n}{z^{n+2}}, \quad j(z) = \sum_n \frac{J_n}{z^{n+1}}, \quad T_F^{\pm}(z) = \sum_r \frac{G_r^{\pm}}{z^{r+3/2}}.$$

E.M. Tensor

$$[L_m, G_r^{\pm}] = \left(\frac{m}{2} - r\right) G_{m+r}$$

$$[L_m, J_n] = -n J_{m+n}$$

$$\{G_r^{+}, G_s^{-}\} = 2L_{r+s} + (r-s) J_{r+s} + \frac{c}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r,-s}$$

$$\{G_r^{+}, G_s^{+}\} = \{G_r^{-}, G_s^{-}\} = 0$$

$$[J_n, G_r^{\pm}] = \pm G_{r+n}^{\pm}$$

$$[J_m, J_n] = \frac{c}{3} m\,\delta_{m,-n}.$$

mass operator.

Primary fields : $L_0 |h\rangle = h|h\rangle$

$L_m|h\rangle = 0, \; m>0., \; \tilde{G}_r^{\pm}|h\rangle = 0, \; r>0$ etc.

$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3-m)\delta_{\dots}$ Virasoro al...

N = 2 minimal superconformal theories

Unitary reps exist for $c \geq 3$ at the discrete values

$$\boxed{c = \frac{3k}{k+2}, \quad k = 0, 1..}$$

States of minimal models at level k : $|\ell, q, s\rangle$

$$\boxed{h = \frac{\ell(\ell+2) - q^2}{4(k+2)}}$$

NS sector.

$$\boxed{Q = \frac{q}{k+2}.}$$

Each minimal model : $U(1)$

Gepner construction

$$\boxed{\psi^{\text{Internal}}_{L(R)} = \prod_{i=1}^{n} \otimes |k_i, \ell_i, q_i, s_i\rangle}$$

To cancel conformal anomaly : $\sum_{i=1}^{n} \frac{k_i}{k_i+2} = 3.$

Finite set of possibilities eg $1^9, 3^5, 1.16^3$ ....

Latter GGR

3-generation model of interest here

$$\Psi_{\text{Gepner}} = \psi^{RM}_{\text{space-time}} \times \prod_{i=1}^{n} |l_i, q_i, s_i\rangle^{RM} \otimes \psi^{LM}_{\text{Gauge}} \times \prod_{i=1}^{n} |\bar{l}_i, \bar{q}_i, \bar{s}_i\rangle$$

- Modular invariance $\quad l, \bar{l}$ correlate $\quad$ A, D, E invariants

  eg A: $\quad \delta_{l_i \bar{l}_i}$

  Also $\quad \delta_{q_i \bar{q}_i}, \; \delta_{s_i \bar{s}_i}$

- GSO projection : $N=1$ supersymmetry ; $\beta_0 = (0, \overbrace{1 \ldots 1}^{q_i}, \overbrace{1 \ldots 1}^{s_i})$

  $\bar{q}_i = q_i + \frac{1}{2} \beta_0$

- Symmetry : $\quad E_8 \times E_6 \times U(1)^{n-1} \times$ Discrete Symmetries

Discrete symmetries: Level k_i SCT : $\underbrace{Z_{k_i+2} \times \tilde{Z}_{k_i+2}}_{\text{Diag } Z_{k_i+2}}$

$+ \underbrace{\Sigma l_i^J = 0, \; \Sigma \bar{l}_i^J = 0}_{\text{"isospin"}}$

⇨ Find Gepner models have same symmetry as CICY models at large radius limit ; at original point in moduli space symmetry enhanced → $U(1)^{n-1} \times$ Discrete symmetries

Couplings in Gepner model known :

Given by conventional <u>Wigner 3j - symbols</u> with underlying $su(2)$ invariance !

$$\boxed{\begin{aligned} & c\left(\begin{pmatrix} j_1 & m_1 \\ j_1 & \bar{m}_1 \end{pmatrix} \begin{pmatrix} j_2 & m_2 \\ j_2 & \bar{m}_2 \end{pmatrix} \begin{pmatrix} j_3 & m_3 \\ j_3 & \bar{m}_3 \end{pmatrix} \right) \\ & \quad = n_A(j_1, j_2, j_3) \begin{bmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{bmatrix} \begin{bmatrix} j_1 & j_2 & j_3 \\ \bar{m}_1 & \bar{m}_2 & \bar{m}_3 \end{bmatrix} \end{aligned}}$$

where $|k_i, l_i, q_i, s_i\rangle$ has $j = l_i/2$, $m = (q_i - s_i)/2$

eg. Construction of 3-generation (Tian-Yau) manifold via SC

$c = 9$: $k=1 \times (k=16)^3$. : $E_8' \times E_6 \times U(1)^3_{126} \times Z_3 \times Z_{18}^3 \times P_3$

A invariant E invariant.

Discrete symmetries $\supset$ $Z_3 \times Z_3'$

Permutations $(0,0,6,12) \equiv 6q_3 + 12q_4$

"Freely Acting" "Not-freely-acting"

doesn't matter in SC construct.

Modding out by $Z_3 \times Z_3'$ † $\Rightarrow$ 3 generation model $=$ 3gen CICY

in large radius limit.

.... Massless chiral $(27, \overline{27})$ multiplets $h = \bar{h} = \frac{1}{2}$: $9.27 + 6.\overline{27}$

+ singlets * known.

---

† $\tilde{q}_i = q_i + n\,(0,0,6,12)$

twist

$$l_1 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix}$$

$$l_2 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$$

$$l_3 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix}$$

$$l_4 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix}$$

$$l_5 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & 12 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$$

$$l_6 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix}$$

$$l_7 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix}$$

$$l_8 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & 2 & 0 \end{pmatrix}$$

$$l_9 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix},$$

Table 1: The massless fields of the three generation Gepner construction transfoming as the 27 representation of E_6. The four factors refer to the 1.16^3 superconformal factors of the construction. The quantum numbers given refer to the scalar component transforming as the 10 of the SO(10) subgroup of E_6.

$$\bar{l}_1 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & -8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\bar{l}_2 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix}$$

$$\bar{l}_3 : \begin{pmatrix} 1 & 1 & 0 \\ 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & -8 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 & 0 \\ 8 & -8 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\bar{l}_4 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix}$$

$$\bar{l}_5 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & -12 & 0 \end{pmatrix}$$

$$\bar{l}_6 : \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 12 & 12 & 0 \\ 12 & -12 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix}$$

Table 2: The massless fields of the three generation Gepner construction transfoming as the $\overline{27}$ representation of E_6. The four factors refer to the 1.16^3 superconformal factors of the construction. The quantum numbers given refer to the scalar component transforming as the 10 of the SO(10) subgroup of E_6

$\times(1+f(c))$

c Moduli $f(0)=0$

$\langle c \rangle$ can be complex...

CP phases.

Table 1. Couplings of the massless states given in the text without R_i-, C_i-singlets.

Coupling	Value	Coupling	Value	Coupling	Value
$l_4 l_5 l_3$	0.8221	$l_4 q_3 Q_1$	0.4747	$l_9 \bar{l}_2 \phi_{14}$	0.7536
$l_7 l_7 l_3$	0.2779	$l_1 q_2 Q_3$	0.5773	$l_9 \bar{l}_3 \phi_5$	0.4200
$l_1 l_5 l_6$	0.5773	$l_1 q_3 Q_2$	0.5773	$l_6 \bar{l}_2 \phi_{12}$	0.9230
$l_1 l_4 l_6$	0.5773	$l_5 \bar{l}_6 \phi_{16}$	0.5817	$l_7 \bar{l}_4 \phi_{15}$	1.0226
$l_2 l_4 l_6$	0.4747	$l_4 \bar{l}_5 \phi_{17}$	0.5817	$l_1 \bar{l}_1 \phi_3$	0.2911
$l_2 l_5 l_6$	0.4747	$l_8 \bar{l}_1 \phi_{18}$	0.4012	$\phi_3 \phi_4 \phi_{15}$	0.7401
$l_2 l_6 l_7$	0.6760	$l_8 \bar{l}_3 \phi_1$	0.4167	$\phi_3 \phi_{22} \phi_{24}$	0.1592
$l_8 l_9 l_2$	0.6351	$l_9 \bar{l}_1 \phi_{19}$	0.4012	$\phi_3 \phi_7 \phi_{20}$	0.5550
$\bar{l}_4 \bar{l}_4 \bar{l}_2$	0.2779	$l_9 \bar{l}_3 \phi_2$	0.4167	$\phi_{12} \phi_4 \phi_{24}$	0.0420
$\bar{l}_1 \bar{l}_3 \bar{l}_4$	1.1537	$l_6 \bar{l}_1 \phi_{20}$	0.6527	$\phi_{12} \phi_{11} \phi_{16}$	0.6527
$\bar{l}_5 \bar{l}_6 \bar{l}_2$	0.8222	$l_6 \bar{l}_3 \phi_4$	0.7844	$\phi_{12} \phi_{10} \phi_{17}$	0.6527
$q_3 q_3 q_1$	0.2685	$l_7 \bar{l}_3 \phi_3$	0.2180	$\phi_7 \phi_1 \phi_{19}$	0.4653
$q_1 q_2 q_3$	0.6532	$l_1 \bar{l}_5 \phi_{11}$	0.5773	$\phi_7 \phi_2 \phi_{18}$	0.4653
$Q_3 Q_3 Q_1$	0.2685	$l_1 \bar{l}_6 \phi_{10}$	0.5773	$\phi_7 \phi_6 \phi_{19}$	0.4688
$Q_1 Q_2 Q_3$	0.6532	$l_2 \bar{l}_1 \phi_3$	0.2400	$\phi_7 \phi_5 \phi_{18}$	0.4688
$l_3 q_3 Q_3$	1.0524	$l_2 \bar{l}_5 \phi_8$	0.5817	$\phi_{15} \phi_1 \phi_2$	0.8060
$l_2 q_2 Q_2$	0.5773	$l_2 \bar{l}_6 \phi_9$	0.5817	$\phi_{15} \phi_6 \phi_5$	0.8181
$l_2 q_3 Q_3$	0.3903	$l_2 \bar{l}_4 \phi_7$	1.0150	$\phi_8 \phi_6 \phi_{18}$	0.3826
$l_4 q_1 Q_2$	0.5773	$l_3 \bar{l}_1 \phi_4$	0.5329	$\phi_9 \phi_5 \phi_{19}$	0.3826
$l_5 q_2 Q_1$	0.5773	$l_5 \bar{l}_3 \phi_3$	0.3053	$\phi_6 \phi_6 \phi_{17}$	0.3146
$l_6 q_1 Q_3$	0.7390	$l_4 \bar{l}_3 \phi_3$	0.3053	$\phi_5 \phi_5 \phi_{16}$	0.3146
$l_6 q_3 Q_1$	0.7390	$l_8 \bar{l}_2 \phi_{13}$	0.7536		
$l_5 q_1 Q_3$	0.4747	$l_8 \bar{l}_3 \phi_6$	0.4200		

$$\langle \mathbf{27}^3 \tilde{R}^n \tilde{C}^m \rangle = 0\,, \tag{11}$$

$$\langle \overline{\mathbf{27}}^3 \tilde{R}^m \tilde{C}^n \rangle = 0\,, \tag{12}$$

$$\langle \mathbf{27}\,\overline{\mathbf{27}}\,\tilde{R}^n \rangle = 0 \quad \text{for } n > 0\,, \quad m \geq 0\,. \tag{13}$$

$11D \to 4D \times CY_3 \times \mathbb{Z}_2$

Calabi - Yau compactification

(cf $10D \to 4D \times R_0$)

$$\Rightarrow \quad G = \overset{\text{hidden}}{E_8} \times \overset{\text{visible}}{E_6} \times (N=1\ \text{SUSY}) \quad \text{(heterotic string)}$$

$$\Rightarrow \text{Multiplet Structure:} \quad h_{2,1}\ 27 + h_{1,1}\ \overline{27}$$

$(16+10+1)_{SO(10)} = (10+\bar{5}+1 + 5+\bar{5}+1)_{SU(5)}$

Hodge numbers

$$\frac{|X|}{2} = |h_{21} - h_{11}| = n_{\text{generations}}$$

... an explanation for replication of generations! ✓

$$\Rightarrow \quad \text{Eg.} \quad R_0 = \text{CICY} : \quad CP_3 \times CP_3 . = (x_0, x_1, x_2, x_3) \times (y_0, y_1, y_2, y_3)$$

$x_i \equiv \lambda x_i$

3 independent complex coords
$$\begin{cases} 0 = \sum_i x_i^3 + a_1 x_0 x_1 x_2 + a_2 x_0 x_1 x_3 + a_3 x_0 x_2 x_3 + a_4 x_1 x_2 x_3 \\ 0 = \sum_i y_i^3 + b_1 y_0 y_1 y_2 + b_2 y_0 y_1 y_3 + b_3 y_0 y_2 y_3 + b_4 y_1 y_2 y_3 \\ 0 = \sum_i c_{ij} x_i y_j \qquad (c_{00} = 1) \end{cases}$$

(Tian Yau)

- $\boxed{h_{2,1} = 23 \quad , \quad h_{1,1} = 14 \quad \Rightarrow \quad n_{\text{generations}} = 9}$

- $G = E_8 \times E_6 \times U(1)^3 \times (N=1\ \text{SUSY})$ $\qquad \alpha = e^{\frac{2\pi i}{3}}$

(at special point in moduli space)

- + discrete symmetries eg D : $\begin{matrix}(x_0\, x_1\, x_2\, x_3) \\ (y_0\, y_1\, y_2\, y_3)\end{matrix} \to \begin{matrix}(x_0, \alpha^2 x_1, \alpha x_2, \alpha x_3) \\ (y_0, \alpha y_1, \alpha^2 y_2, \alpha^2 y_3)\end{matrix}$

Tian-Yau : 3 gen : $R = R_0 / D$. ; $\boxed{X \to \frac{X}{3}}$ ✓

^ String Unification.

4-D THEORY.

⇨ $[SU(3) \times SU(2) \times U(1)] \times (N=1)$

+ M_X prediction ?

+ Level 1 theory - no adjoints

+ No need for Higgs triplet partners.

BUT THEN WHY DO FAMILIES FILL $SU(5)$ REPRESENTATIONS ?

$3.(\bar{5}+10) + H_1 + H_2$?

THERE IS A NATURAL EXPLANATION WITHIN CONTEXT OF WILSON LINE BREAKING: $R_0 \to R_0/D \; : U_g = P\exp\int_\gamma -A_{m,a} T^a dx^m$

Defines $D \to \bar{D} \in E_6$.

- THE UNDERLYING THEORY HAS A GUT STRUCTURE : $E_8 \times E_6$, $E_8 \times SO(10)$, $E_8 \times SU(5)$ ...

- THE BREAKING BY WILSON LINES[†] LEAVES COMPLETE GUT REPRESENTATIONS PLUS SPLIT REPRESENTATIONS

† i.e. Projection of $D\bar{D}$ singlets.

An $SU(3)\times SU(2)\times U(1)$ STRING THEORY?

(NO ADJOINTS AVAILABLE).

... MUST BREAK E_6 ON COMPACTIFICATION ... WILSON LINES

$$R_0 \to R_0/D \quad : \quad U_g = P\exp\int_\gamma -iA_{m,a}T^a\,dx^m$$

$$\left(\text{Defines } D \to \bar{D} \subset E_6 .\right)$$

States must satisfy $\phi(x) = \bar{g}\,\phi(g(x))$ ie $D\bar{D}$ singlets.

eg Tian-Yau: $[CP_3 \times CP_3]_5$, $E_8\times E_6 \times U(1)^3$

R_0: $23.\,27 + 14.\,27$ 9 generations

$\equiv 3.3.27 + 4.3.(27+\overline{27}) + 2.1.(27+\overline{27})$

Z_3: $(1,\alpha,\alpha^2)$ $(1,\alpha,\alpha^2)$ 1

R_0/D : D-singlets : $3.27 + 6(27+\overline{27})$ 3 generations

$R_0/D\big|_{W\text{-}L}$ $D\bar{D}$-singlets : $3.\text{"}27\text{"} + 4(\text{"}27\text{"}+\text{"}\overline{27}\text{"}) + 2(27+\overline{27})_1\big|_X$

$\} \to SU(3)\times SU(3)\times SU(3)$

$$\boxed{\begin{aligned} &27 \supset X_1 + Y_\alpha + Z_{\alpha^2} \\ &\text{"}27\text{"} = 27_1\big|_X + 27_{\alpha^2}\big|_Y + 27_\alpha\big|_Z \end{aligned}}$$

If $27_1\big|_X = H_1$ ... a natural source of Higgs ... $H_1 + \bar{H}_1\ (\equiv H_2)$

$$E_6 \supset SU(3)^3$$

$$27 = (1, \bar{3}, 3) + (3, 3, 1) + (\bar{3}, 1, \bar{3})$$

$$= \ell + Q + Q^c$$

$$\ell = \begin{bmatrix} H^0 & H^+ & e^c \\ H^- & \bar{H}^0 & \nu^c \\ e^- & \nu & N \end{bmatrix}_L$$

$$Q = \begin{bmatrix} u \\ d \\ D \end{bmatrix}$$

$$Q^c = \begin{bmatrix} u^c \\ d^c \\ D^c \end{bmatrix}$$

Wilson line breaking

$$Z_3^G \cdot 27 = 1\,\ell + \alpha Q + \alpha^2 Q^c$$

Wilson line breaking in 3 generation superconformal models

$$\Psi = \psi^{RM}_{\text{Space-Time}} \times \prod_{i=1}^{n} |l_i\, q_i\, s_i\rangle^{RM} \otimes \psi^{LM}_{\text{Gauge}} \times \prod_{i=1}^{n} |\bar{l}_i\, \bar{q}_i\, \bar{s}_i\rangle$$

eg. Z_3' : $(0,0,6,12)$

$$\left.\begin{array}{l} \bar{q}_i = q_i + n.(0,0,6,12) \\ \text{Gauge} = |v\rangle + n.\,|s\rangle \end{array}\right\} \text{Simultaneous twist in internal + gauge sector}$$

Discrete Symm. $\supset \underset{}{\overset{\text{Permutations}}{Z_3}} \times Z_3'^{\,(0,0,6,12)}$

W.L.   WL

3 Gen Tian-Yau : $9.\text{"}27\text{"} + 6.\text{"}\overline{27}\text{"}$

leptons

New 3 gen model. : $9(1,3,\bar{3}) + 6(1,\bar{3},3)$

$3.(3,\bar{3},1) + 3(\bar{3},1,3)$

q_L q_R^c

$$\phi_{19}: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & -6 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 0 & 0 \end{pmatrix}$$
$$\phi_{20}: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix}$$
$$\phi_{21}: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -10 & 0 \end{pmatrix}$$
$$\phi_{22}: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 6 & 2 & 0 \end{pmatrix}$$
$$\phi_{23}: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & -8 & 0 \end{pmatrix}$$
$$\phi_{24}: \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 2 & 0 \end{pmatrix}$$
$$\phi_{25}: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 6 & -2 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix}$$
$$\phi_{26}: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & -2 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 6 & -2 & 0 \end{pmatrix}$$

Table 3: The scalar components of the massless E_6 gauge singlet fields.

$$q_1: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$q_2: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix}$$
$$q_3: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix}$$

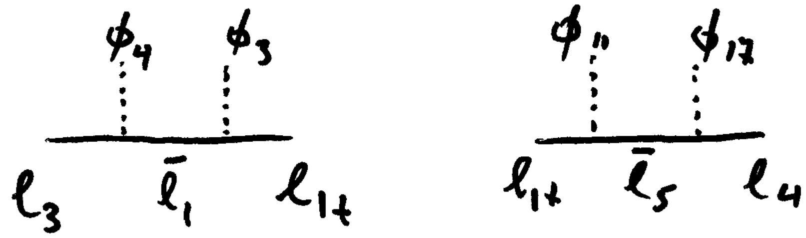
$$Q_1: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix}$$
$$Q_2: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 10 & 10 & 0 \\ 10 & 10 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$Q_3: \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 8 & 8 & 0 \\ 8 & 8 & 0 \end{pmatrix} \begin{pmatrix} 6 & 6 & 0 \\ 6 & 6 & 0 \end{pmatrix} \begin{pmatrix} 4 & 4 & 0 \\ 4 & 4 & 0 \end{pmatrix}.$$

Table 4: Quark and lepton fields for the case of Wilson line breaking with the twist generated by the (0,3,6,0) element.

Fermion masses

$$\mathcal{L}_{Yuk} = (Q_1 Q_2 Q_3) \begin{pmatrix} 0 & 0.58\,\ell_{4+} & 0.47\,\ell_{4+} \\ 0.58\,\ell_{4+} & 0.58\,\ell_{1+} & 0.58\,\ell_{1+} \\ 0.47\,\ell_{4+} & 0.58\,\ell_{1+} & 0.39\,\ell_{1+} + 1.05\,\ell_3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

Higgs mixing :



$$H = \ell_3 + \#\underbrace{\frac{\langle\phi_4\rangle\langle\phi_3\rangle}{M^2_{\bar{\ell}_1}}}_{\varepsilon_1}\ell_{1+} + \#\frac{\langle\phi_4\rangle\langle\phi_3\rangle}{M^2_{\bar{\ell}_1}} \cdot \underbrace{\frac{\langle\phi_{11}\rangle\langle\phi_{17}\rangle}{M^2_{\bar{\ell}_5}}}_{\varepsilon_2}\ell_4$$

known Yukawa couplings

$$\boxed{\mathcal{M}^{u,d} \propto \begin{pmatrix} 0 & 0.6\,\varepsilon_2^{u,d} & 0.5\,\varepsilon_2^{u,d} \\ 0.6\,\varepsilon_2^{u,d} & 0.6\,\varepsilon_1^{u,d} & 0.6\,\varepsilon_1^{u,d} \\ 0.5\,\varepsilon_2^{u,d} & 0.6\,\varepsilon_1^{u,d} & 1 + 0.4\,\varepsilon_1^{u,d} \end{pmatrix}}$$

Vacuum alignment : Need

$$0.6\,\varepsilon_1^{u,d} = \varepsilon_{u,d}^2$$

$$0.5\,\varepsilon_2^{u,d} = \varepsilon_{u,d}^3 .$$

Summary.

⇨ m_{q_i}, V_{CKM} ⇨ $m^{u,d}$

⇨ Structure consistent with simple family symmetries

... some relations emerge (TZ predictions ; magnitude of matrix elements ; possible Non-Abelian Structure)

... consistent picture with charged leptons and neutrinos

$$M_{eff} = M^D . M^{-1} . M^D .$$

⇨ Origin of structure ? Many ideas

$$\left.\begin{array}{l}\text{Composite}\\ \text{Large new dimensions}\\ \text{GUTs / Strings}\end{array}\right\} \begin{array}{l}\text{strongly constrained if}\\ \text{wish to preserve}\\ \text{gauge unification prediction}\end{array}$$

⇨ Strings as the ToE ?

+ multiplet structure, FN mixing
+ family symmetries $U(1), Z_N, SU(2)$..
+ GUT relations
+ calculable couplings

... specific examples can yield viable mass matrices

⇨ Choice of string vacuum ? Depends on SUSY breaking

...most difficult and most interesting aspect still to be understood

Fermion masses and the flavour problem

General reviews:

G. Ross	1999 TASI lectures
M. Peskin	hep-ph/0002041
J. Ellis	hep-ph/9812235
F. Zwirner	hep-ph/9601300

Specific references by subject (hep-ph numbers)

FCNC.	Gabrielli, Masiero, Silvestrini	9510215
	Masiero, Vives	0003133
Quark masses	Fusaoka, Koide	9712201
	Gupta, Meltman.	0101132
Textures	Fritzsch	9909303
	Roberts, Romanino, Ross, Velasco-Sevilla	0104088
	Barbieri, Hall, Romanino	9812384
Topcolour	Hill	9411426
	Chivukula, Dobrescu, Georgi, Hill	9809470.

Large New Dimensions ... Arkani-Hamed next week!

Yukawa couplings: Dvali-Shifman 0001072

Mirabelli Schmaltz 9912265

Arkani-Hamed, Hall, Smith, Weiner 9909326

Arkani-Hamed, Schmaltz, PRD 2000

Gherghetta, Pomarol. { 0003129, 0012378 }

Infra Red fixed points: 0002102, 0003120 } Brando et al

Abel, King 9809467

Lanzagorta, Ross 9507366

Strings

For an introduction to orbifold compactification see
Ibanez CERN-TH.4769/87 (available on the web)

Mass matrices Kobayashi Xing 9712432
Kobayashi 9507244

Gepner construction : Gepner 9301089 & refs therein

Schenk & Schmidt : Int. J. mod Phys A7 (1992) 8021

Pokorski & Ross ; 9809537 ; 9707402

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