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SMR.1317 - 26

SUMMER SCHOOL ON PARTICLE PHYSICS

18 June - 6 July 2001

INTRODUCTION TO NONCOMMUTATIVE FIELD THEORY

Lectures III & IV

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CERN
Geneva, SWITZERLAND

Please note: These are preliminary notes intended for internal distribution only.

QUANTIZATION (Perturbativ

TAKE A REAL SCALAR FIELD.
IN ORDER TO FIND THE FEYNMAN RULES, GO TO MOMENTUM SPACE

FREE ACTION

$$S^{(2)} = \frac{1}{2}\int_x \partial\varphi * \partial\varphi + m^2 \varphi * \varphi =$$

$$= \frac{1}{2}\int_x \partial\varphi\,\partial\varphi + m^2 \varphi\varphi =$$

$$= \frac{1}{2}\int_{p,p'} (2\pi)^4 \delta(p+p')\, \varphi_p \left(-p_\mu p'^\mu + m^2\right) \varphi_{p'}$$

PROPAGATOR UNCHANGED

$$\boxed{\varphi \text{ ——— } \varphi = \frac{i}{p^2 - m^2 + i0}}$$

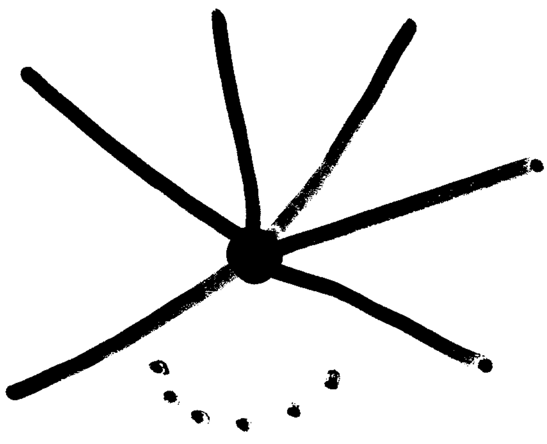
WHAT ABOUT VERTICES ?

$$\int d^d x \; \overbrace{\varphi(x) * \varphi(x) * \cdots * \varphi(x)}^{n} =$$

$$= \int dp^{(1)} \dots dp^{(n)} \; \tilde{\varphi}(p^{(1)}) \dots \tilde{\varphi}(p^{(n)}) \times$$

$$\times (2\pi)^4 \, \delta\left(\sum_{i=1}^{n} p^{(i)} \right) \times \underbrace{e^{-i\,\theta(p^{(1)} \dots p^{(n)})}}_{\text{THE MOYAL PHASE}}$$

$$\boxed{\theta(p^{(1)} \dots p^{(n)}) = \frac{1}{2} \sum_{j<k}^{n} p^{(j)}_{\alpha} \, \theta^{\alpha\beta} \, p^{(k)}_{\beta}}$$



$$= -i\lambda_n \, e^{-i\,\theta(\{p\})}$$

CYCLICALLY SYMMETRIC ONLY !!

PERTURBATIVE STUDIES

T. FILK (96)

M. CHAICHIAN, DEMINCHEV & PRESNAJDER (98)

C.P MARTIN & D. SANCHEZ-RUIZ (99)

M.M. SEIKH-JABBARI (99)

T. KRAJEWSKI & R. WULKENHAAR (99)

CHO, HINTERDING, MADORE & STEINACKER (99)

HAWKINGS (99)

VARILLY & GRACIA-BONDIA (99)

BIGATTI & SUSKIND (99)

AREF'EVA, BELOV & KOSHELEV (99)

HAYAKAWA (99)

MATUTSIS, SUSSKIND & TOUMBAS (99)

ARCIONI & VAZQUEZ-MOZO (99)

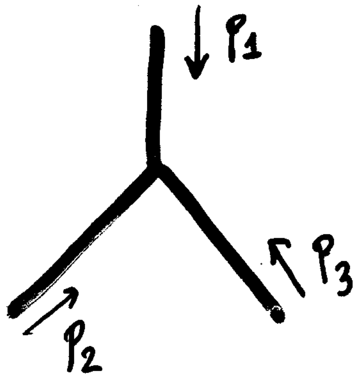
⋮

↳ MOYAL PHASES $\boxed{e^{i\theta^{ij}p_i q_j}}$ TEND TO IMPROVE U.V. BEHAVIOUR

$\boxed{\sqrt{|\theta|}}$

$\boxed{|p_{ext}\theta|}$ act as effective U.V. cutoffs

PLANARITY



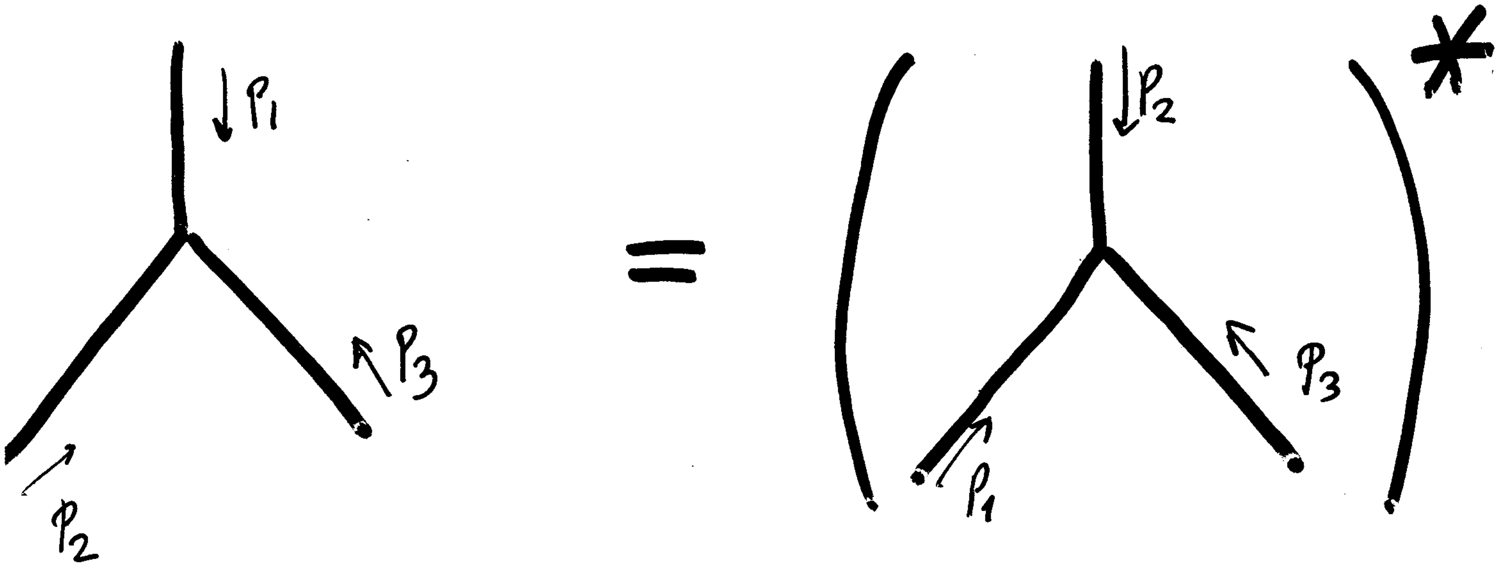
$$\sim \int_x e^{ip_1 x} * e^{ip_2 x} * e^{ip_3 x}$$

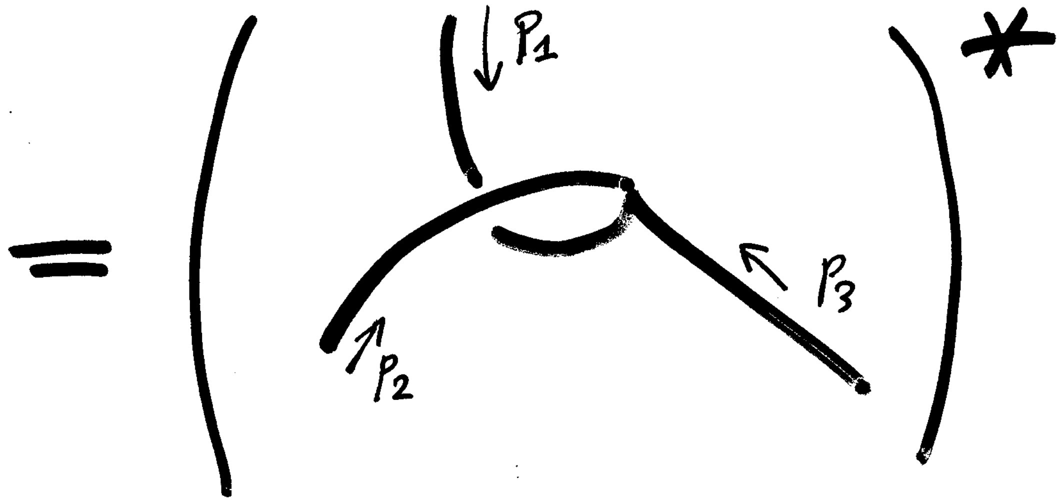
$$= \int_x e^{ip_1 x} * \left(e^{-\frac{i}{2} p_2 \times p_3} e^{i(p_2+p_3)x} \right)$$

$$= e^{-\frac{i}{2} p_2 \times p_3} e^{-\frac{i}{2} p_1 \times (p_2+p_3)} \underbrace{\int_x e^{i(p_1+p_2+p_3)x}}_{\delta(p_1+p_2+p_3)}$$

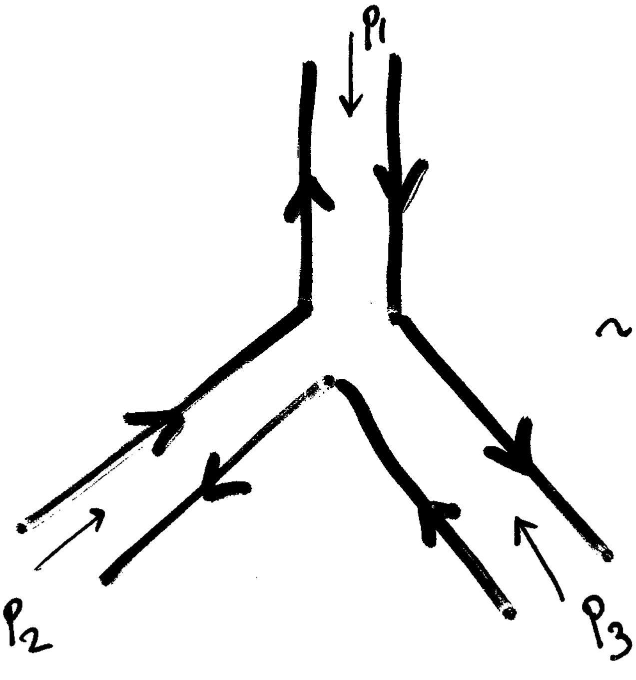
$$= e^{-\frac{i}{2} p_1 \times p_2} \delta(p_1+p_2+p_3) =$$

$$= \left(e^{-\frac{i}{2} p_2 \times p_1} \right)^{*} \delta(p_1+p_2+p_3)$$





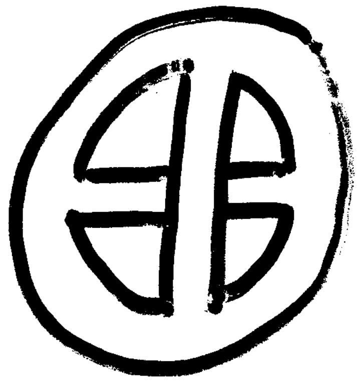
KEEP TRACK OF TWISTS BY DRAWING LINES AS ORIENTED **RIBBONS**
('t Hooft's double line notation)



$\sim e^{-\frac{i}{2} p_1 \times p_2}$

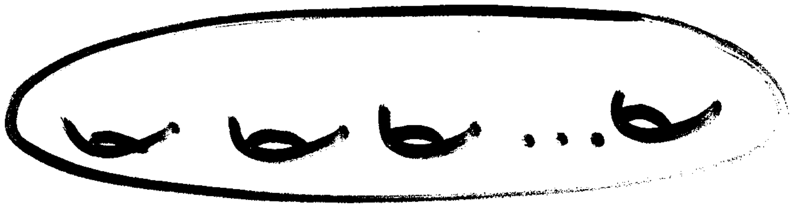
DIAGRAMS ARE THEN CLASSIFIED TOPOLOGICALLY

PLANAR



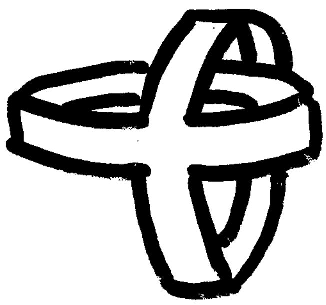
NON-PLANAR

BY THE GENUS OF THE RIEMANN SURFACE THAT THEY TESELATE.



EXAMPLE:

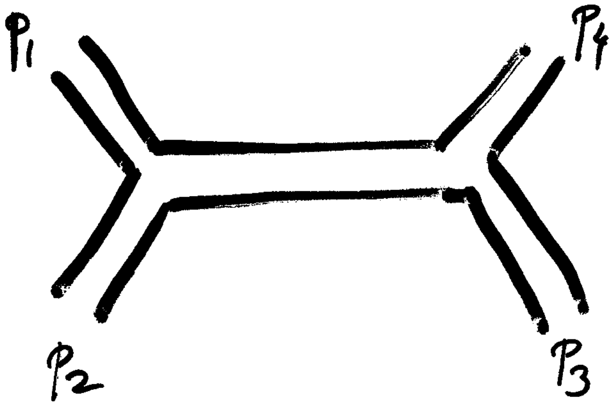
TOROIDAL →



NCFT AND LARGE-N GAUGE THEORY LOOK "STRINGY"

A CRUCIAL PROPERTY

PLANAR PROPAGATORS GIVE NO NET CONTRIBUTION TO OVERALL MOYAL PHASE

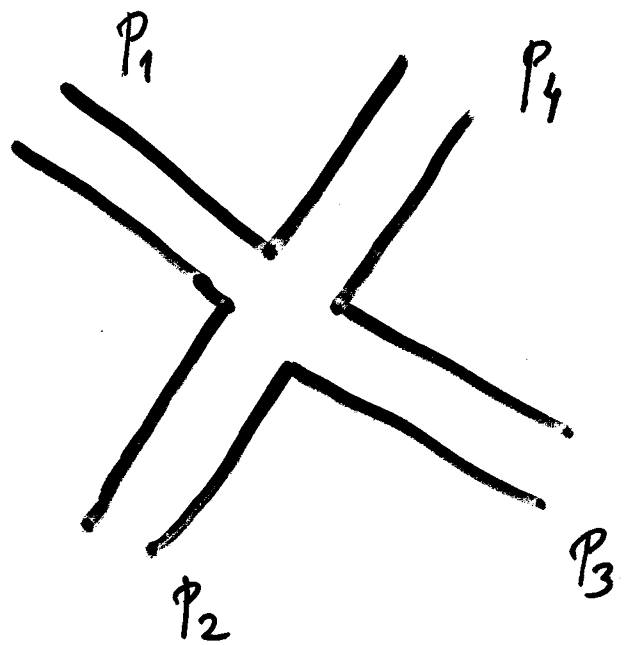


$$\sim e^{-\frac{i}{2}\left(p_1 \times p_2 + p_3 \times p_4\right)}$$

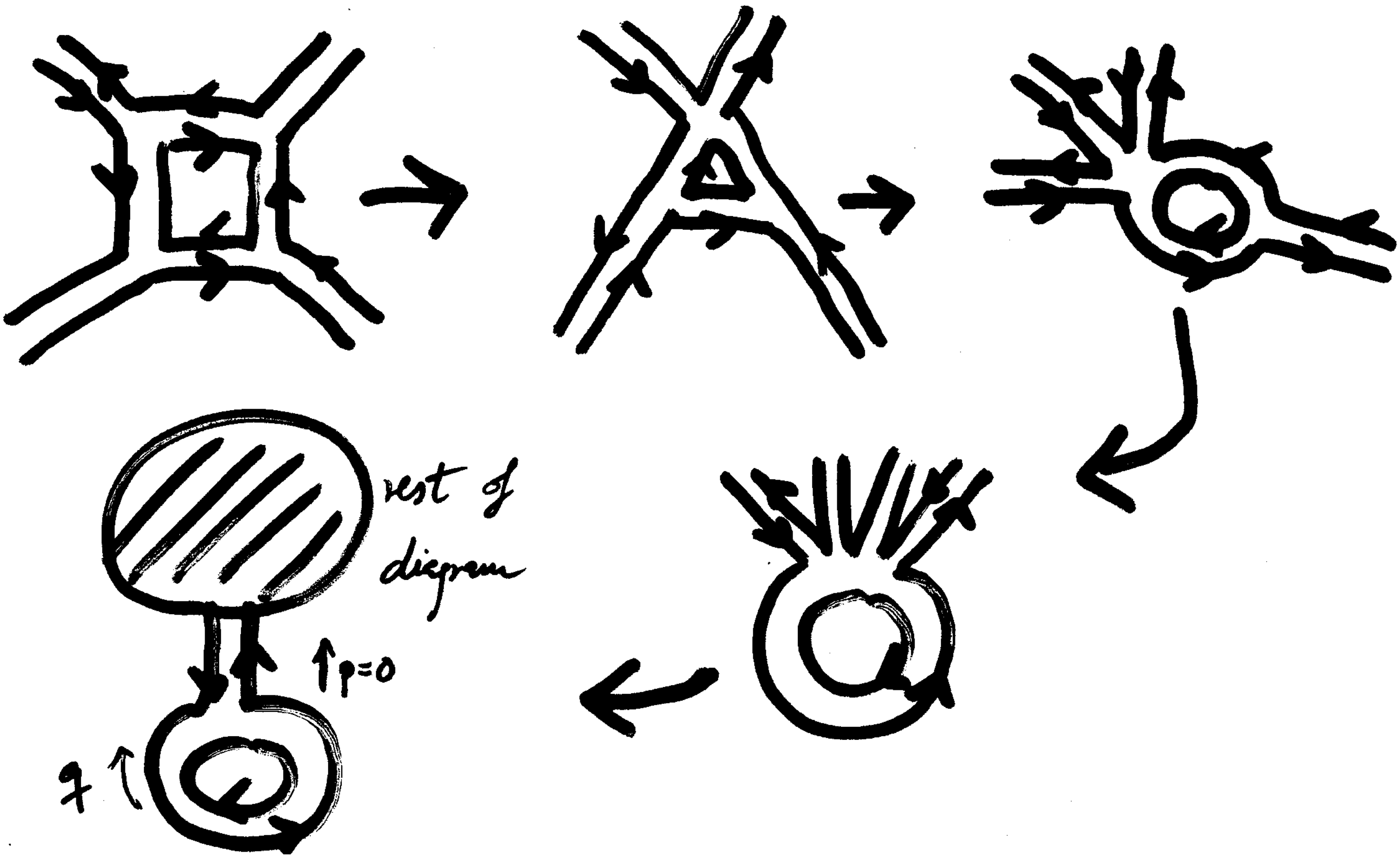
USE $\underbrace{p_1 + p_2 + p_3 + p_4 = 0}$

$$= e^{-\frac{i}{2}\left(p_1 \times p_2 + p_3 \times p_4 + p_2 \times p_3 + p_1 \times p_4 + p_1 \times p_3\right)} =$$

$=$



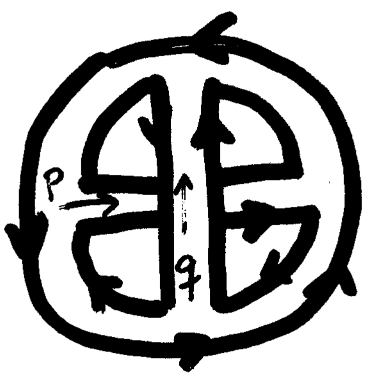
* REDUCTION OF PHASES IN PLANAR LOOPS



$$e^{i\theta(p=0,q)} = 1$$

$$\boxed{\begin{array}{c}\text{NC PHASE OF PLANAR DIAGRAMS} \\ \parallel \\ \text{NC PHASE OF EXTERNAL LEGS}\end{array}}$$

* NON-PLANAR DIAGRAMS HAVE UNCANCELLED NC PHASES

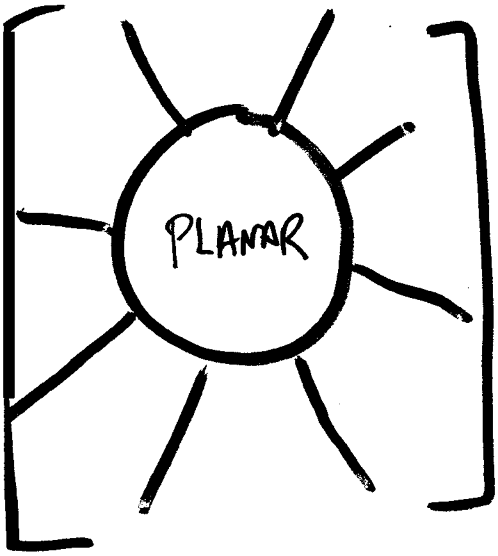


$$\sim e^{2i\theta(p,q)}$$

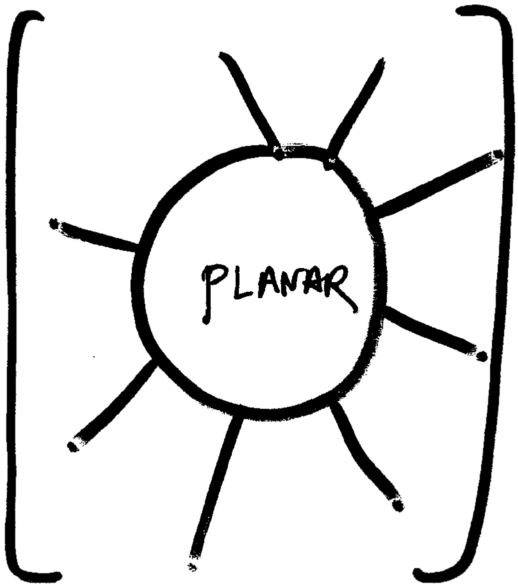
Same combinatorics as in TEK models (González-Arroyo and Okawa (83)

"θ - KINEMATICS"

González-Arroyo & Korthals-Altes 82
Filk 96

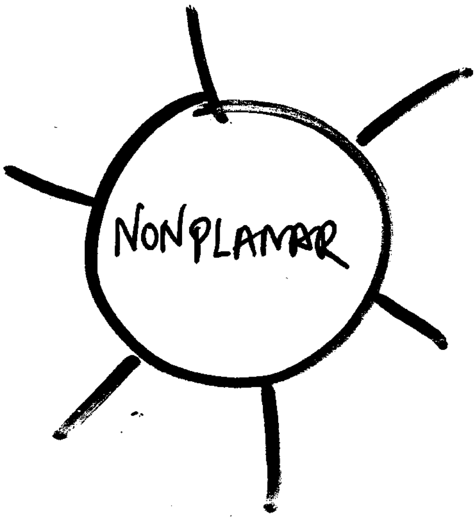


$$_{\theta} = W_{ext}$$



$$_{\theta=0}$$

$$W_{ext} = e^{-i\theta(\text{external momenta})}$$



HAS "NONTRIVIAL" θ-DEPENDENCE COMING FROM INTEGRALS OF INTERNAL LOOPS

$$\Downarrow$$

UV/IR

(Minwalla
Van Raamsdonk
Seiberg)

» INITIAL HOPES THAT

$$|\Delta X_\mu \Delta X_\nu| \gtrsim |\theta_{\mu\nu}|$$

WOULD YIELD ONE D.O.F. PER

θ- AREA ARE NOT FULFILLED,

AT LEAST IN PERTURBATION THEORY,

SINCE PLANAR DIAGRAMS HAVE

STANDARD U.V. BEHAVIOUR.

ONLY NONPLANAR DIAGRAMS ARE

CUT-OFF

* ACTUALLY, THIS IS NOT

THE END OF THE STORY...

U(N) GAUGE THEORY

$$A_\mu = A_\mu^a(p)\, T^a\, e^{ipx}$$

$T^a \in \mathrm{Lie}(\mathcal{U}(N)) \qquad \mathrm{tr}\, T^a T^b = \delta^{ab}$

NON-ABELIAN STRUCTURE "CONSTANTS"

$$A_\mu * A_\nu - A_\nu * A_\mu =$$

$$= A_\mu^a * A_\nu^b\, T^a T^b - A_\nu^b * A_\mu^a\, T^b T^a$$

$$= A_\mu^a * A_\nu^b \tfrac{1}{2}\{T^a, T^b\} + A_\mu^a * A_\nu^b \tfrac{1}{2}[T^a, T^b]$$

$$- A_\nu^b * A_\mu^a \tfrac{1}{2}\{T^a, T^b\} + A_\nu^b * A_\mu^a \tfrac{1}{2}[T^a, T^b]$$

$$= \tfrac{1}{2}\{T^a, T^b\}\,[A_\mu^a, A_\nu^b]_* + \tfrac{1}{2}[T^a, T^b]\,\{A_\mu^a, A_\nu^b\}_*$$

$$= \left(i d^{abc} T^c \sin\left(\frac{p_1 \times p_2}{2}\right) + i f^{abc} T^c \cos\left(\frac{p_1 \times p_2}{2}\right)\right) \times$$

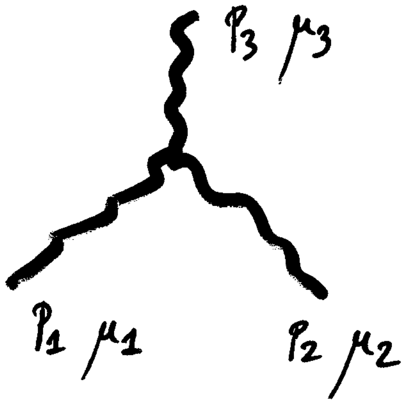
$$\times\, A_\mu^a(p_1)\, A_\nu^b(p_2)\, e^{i(p_1+p_2)x}$$

THE NONCOMMUTATIVE FEYNMAN RULES FOLLOW FROM THE ORDINARY ONES ON REPLACING:

$$\boxed{f^{abc} \rightarrow f^{abc} \cos\left(\frac{p_a \times p_b}{2}\right) + d^{abc} \sin\left(\frac{p_a \times p_b}{2}\right)}$$

SO, NONCOMMUTATIVE PHOTONS ($U(1)$ theory) GET NONLINEAR INTERACTIONS

$f^{abc} = 0 \qquad d^{abc} = 2$



$$= -g \cdot 2 \sin\left(\frac{p_1 \times p_2}{2}\right) \times$$

$$\left[(p_1 - p_2)^{\mu_3} \eta^{\mu_1 \mu_2} + (p_2 - p_3)^{\mu_1} \eta^{\mu_2 \mu_3} + (p_3 - p_1)^{\mu_2} \eta \right]$$

ONE-LOOP BETA FUNCTION

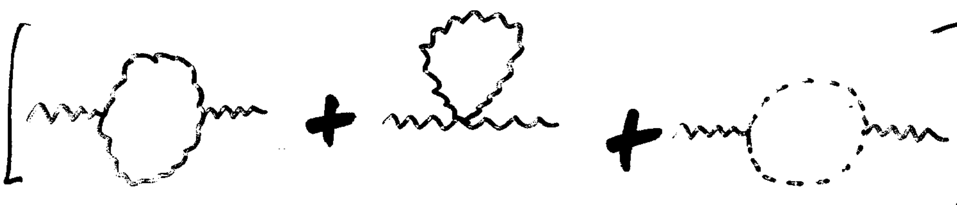
CONSIDER <u>ORDINARY</u> $\theta = 0$ $SU(N)$

$$S^{\Lambda_0}_{SU(N)} = \frac{1}{4g_0^2} \int_{|q|<\Lambda_0} \mathrm{tr}\, |F_{\mu\nu}|^2$$

INTEGRATE OUT FLUCTUATIONS ON A MOMENTUM SLICE $|k| < |q| < \Lambda_0$

$$S^{(k)}_{SU(N)} = \frac{1}{4} \int_{|q'|<|k|} dq' \, \frac{1}{g^2(k)} \, \mathrm{tr}\, F_{\mu\nu}(q') F^{\mu\nu}(-q')$$

$$\frac{1}{g^2(k)} = \frac{1}{g_0^2} + \Big[\quad \Big]^{\Lambda_0}_{|k|}$$



$$\sim \frac{1}{g_0^2} + N \int_{|k|}^{\Lambda_0} d^4q \, \frac{1}{(p-q)^2 q^2} + \cdots =$$

$$= \boxed{\frac{1}{g_0^2} + \frac{\beta_0 N}{(4\pi)^2} \log\left(k^2/\Lambda_0^2\right) + \text{finite}}$$

SAME IN ORDINARY $U(N)$

$$\boxed{\frac{1}{g^2(k)} = \frac{1}{g_0^2} + \frac{N}{(4\pi)^2}\beta_0 \log\frac{k^2}{\Lambda_0^2}}$$

$$\boxed{\beta_0 = \frac{2}{3} \times 11}$$

EXCEPT THAT WE MUST SUBTRACT THE RUNNING OF THE FREE $U(1)$ SUBGROUP

$$\mathrm{tr}\, F^2\Big|_{U(N)} = \frac{1}{N}\left(\mathrm{tr}\, F\right)^2 + \mathrm{tr}\, F^2\Big|_{SU(N)}$$

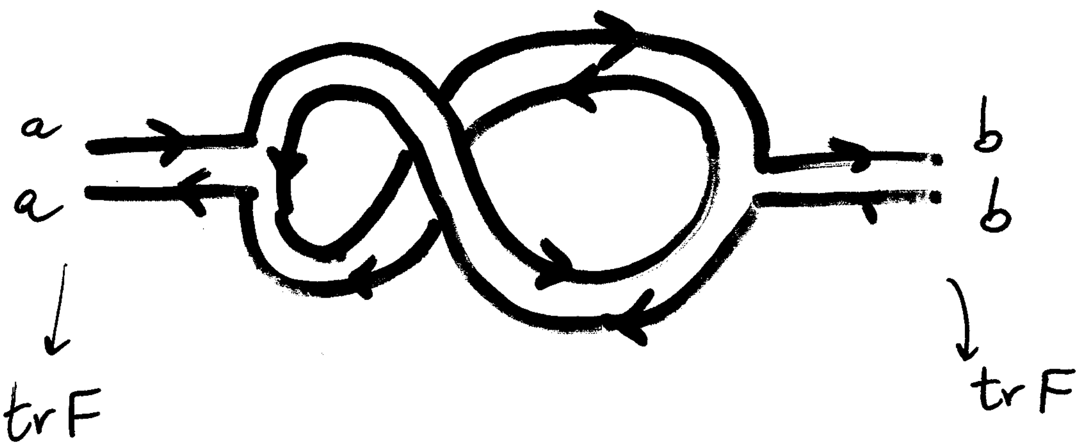
picks $\mathcal{U}(1)$ part

WE END UP WITH

$$S^{(k)}_{U(N)} \simeq \frac{1}{4} \int \left(\frac{1}{g_0^2} + \frac{\beta_0 N}{(4\pi)^2} \log \frac{k^2}{\Lambda_0^2} \right) \mathrm{tr} \left| F(k) \right|^2$$

$$\underbrace{- \frac{1}{N} \frac{\beta_0 N}{(4\pi)^2} \log \left(k^2 / \Lambda_0^2 \right) \left| \mathrm{tr}\, F(k) \right|^2}_{\text{SUBTRACTED } U(1) \text{ PART}}$$

COMES FROM A NON PLANAR DIAGRAM



ENTER $\theta \neq 0$

* PLANAR DIAGRAM GIVES THE SAME U.V. BEHAVIOUR, SINCE θ-DEPENDENCE ONLY AFFECTS EXTERNAL LEGS

* NONPLANAR DIAGRAM HAS A SURVIVING FACTOR OF

$$\sin^2 \frac{p \times q}{2}$$

FOR LARGE LOOP MOMENTUM q FAST OSCILLATION OF PHASE KILLS THE INTEGRAL

THUS, THE NONPLANAR DIAGRAM IS EFFECTIVELY CUT-OFF AT

$$|q|^2 \sim \frac{1}{|p \cdot \theta|^2} \equiv \frac{1}{\tilde{p}^2}$$

FOR $|k|^2 \theta \gg 1$ <u>THE EFFECTIVE</u>

<u>COUPLING RUNS ONLY AT THE PLANAR</u>

<u>LEVEL</u>

$$S_{U(N)} \underset{k^2\theta \gg 1}{\approx} \frac{1}{4} \int \left(\frac{1}{g_0^2} + \frac{\beta_0 N}{(4\pi)^2} \log \frac{k^2}{\Lambda_0^2} \right) \mathrm{tr}\, |F(k)|^2$$

THIS STILL MAKES SENSE FOR $N = 1$

SO, NC $U(1)$ IS ASYMPTOTICALLY FREE

$$\beta(g^2)_{U(N)_*} = \frac{d g_0^2}{d \log \Lambda_0} = \beta(g^2)_{SU(N)} = -\frac{11 g^4 N^2}{12\pi^2}$$

$$\boxed{\beta(g^2)_{U(1)_*} = \lim_{N \to 1} \beta(g^2)_{SU(N)} = -\frac{11 g^4}{12\pi^2}}$$

Martin & Ruiz

Sheikh-Jabbari

Krajewski & Wulkenhaar

NOTICE THAT THE EFFECTIVE U.V. CUTOFF OF THE NONPLANAR DIAGRAM IS $1/\tilde{k}^2$

SO, FOR $k^2\theta_2 \lesssim 1$ THE LOGARITHMIC DIVERGENCE IN THE NONPLANAR DIAGRAM GIVES

$$\log k^2/\Lambda_0^2 \longrightarrow \log\left(k^2\tilde{k}^2\right)$$

WE GET

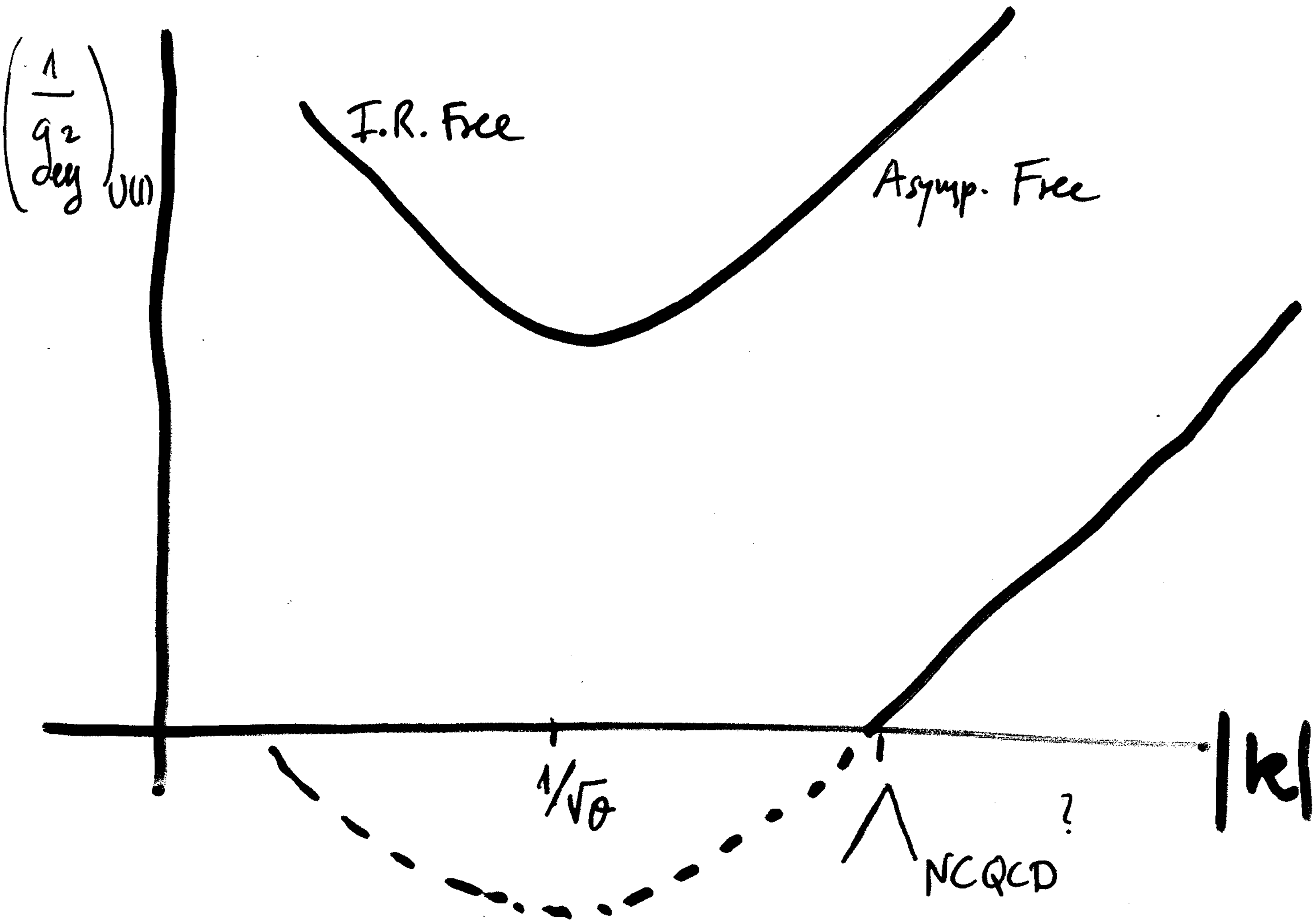
$$S_{U(N)} \underset{k^2\theta_2 \lesssim 1}{\approx} \frac{1}{4}\int\left(\frac{1}{g_0^2}+\frac{\beta_0 N}{(4\pi)^2}\log\frac{k^2}{\Lambda_0^2}\right)\mathrm{tr}\,|F(k)|^2$$

$$-\underbrace{\frac{\beta_0}{(4\pi)^2}\log\left(k^2\tilde{k}^2\right)}\,|\mathrm{tr}\,F(k)|^2$$

THIS TERM GROWS AT LOW MOMENTUM AND PRODUCES SCREENING

$$\boxed{\left(\frac{1}{g_{eff}^2}\right)_{U(1)} \underset{k^2\theta \lesssim 1}{\approx} \frac{1}{g_0^2} - \frac{\beta_0}{(4\pi)^2} \log\left(\tilde{k}^2 \lambda_0^2\right)}$$

THUS, THE $\mathcal{U}(1)$ COUPLING IS I.R.- FREE, PROVIDED WE CAN TRUST THIS LOGARITHMIC APPROXIMATION. THIS SHOULD BE THE CASE FOR ENOUGH SUSY



UV/IR PHENOMENON

LACK OF (WILSONIAN) DECOUPLING BETWEEN U.V. SCALES AND IR SCALES, EVEN IN THE PRESENCE OF EXPLICIT MASSES

IT IS BASED ON A TECHNICAL FACT. NONPLANAR DIAGRAMS HAVE IMPROVED CONVERGENCE BECAUSE OF MOYAL PHASES DEPENDING ON LOOP MOMENTA

* $e^{-\frac{i}{2} p_\mu \theta^{\mu\nu} q_\nu}$ CUTOFF $\sim \Lambda_{eff} \sim \frac{1}{\sqrt{\theta}}$

loop momenta

* $e^{-\frac{i}{2} q_\mu \theta^{\mu\nu} p_{ext}^\nu}$ $\Lambda_{eff} \sim \frac{1}{|p_{ext} \cdot \theta|}$

MINWALLA, VAN RAAMSDONK & SEIBERG MRS

FIND GENUINE SURPRISE:

NC. effects do not decouple in Pert. TH

IF $\Lambda_{eff}^{-1} = |\theta^{\mu\nu} p_\nu|$ CUTS OFF SOME U.V. DIVERGENCE, THEN THE RENORMALIZATION PROCEDURE IS "DISCONTINUOUSLY" DIFFERENT AT $\theta \neq 0$

$\downarrow$

AS $|\theta p| \longrightarrow 0$ THE DIV. IS BACK IN THE RENORMALIZED THEORY

AT FINITE θ this is a $p \to 0$ IR SINGULARITY

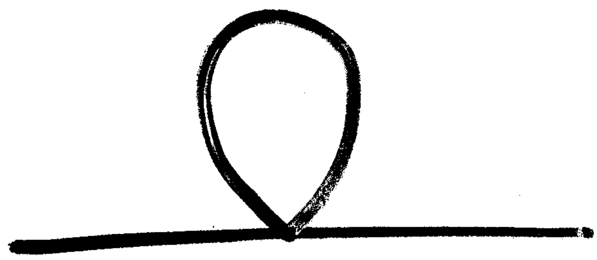
At FINITE p this is non-analytic behaviour around $\theta = 0$

SIMPLEST EXAMPLE: φ^4 TADPOLE

$$S = \int \left(\frac{1}{2} (\partial\varphi)^2 + \frac{m^2}{2} \varphi^2 + \frac{\lambda}{4!} \varphi * \varphi * \varphi * \varphi \right)$$

WE HAVE TWO ONE-LOOP NORMAL-ORDERING "TADPOLES"

PLANAR Work in euclidean signature

$\Sigma_P =$  $=$

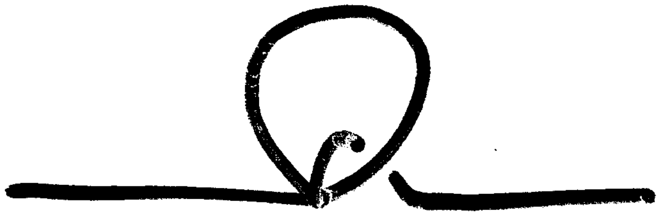
$$= \frac{\lambda}{3} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+m^2} \approx$$

$$\approx \frac{\lambda}{48\pi^2} \left(\Lambda^2 - m^2 \log \frac{\Lambda^2}{m^2} + \text{finite} \right)$$

It contributes a quadratic + logarithmic renormalization of the bare mass m

NON PLANAR

$$\Sigma_{NP} =$$



$$= \frac{\lambda}{6} \int \frac{d^4k}{(2\pi)^4} \frac{e^{-i\, k \times p}}{k^2+m^2} =$$

$$\underset{k\times p = k\cdot\tilde{p}}{=} \frac{\lambda}{24\pi^2} \frac{m^2}{\sqrt{m^2\tilde{p}^2}} K_1\left[\sqrt{m^2\tilde{p}^2}\right]$$

$\tilde{p} = p\cdot\theta$

FINITE !!

IN ORDER TO COMPARE, INTRODUCE U.V. CUTOFF. FOR EXAMPLE, REGULATE THE PROPAGATOR

$$\left[\frac{1}{p^2+m^2}\right]_\Lambda = \int_0^\infty ds\, e^{-s(p^2+m^2)} \underbrace{e^{-1/\Lambda^2 s}}_{\boxed{\text{Regulates the U.V. } s\to 0}}$$

$$\Sigma_{NP}^{\Lambda} = \frac{\lambda}{6}\int \frac{d^4k}{(2\pi)^4} \frac{e^{-i\tilde{p}\cdot k}}{[k^2+m^2]_\Lambda} =$$

$$= \frac{\lambda}{96\pi^2}\left(\Lambda_{eff}^2 - m^2 \log\frac{\Lambda_{eff}^2}{m^2} + \dots\right)$$

$$\boxed{\Lambda_{eff}^2 = \frac{1}{\tilde{p}^2 + 1/\Lambda^2}}$$

$$\begin{cases} |p| \ll \left|1/\Lambda\theta\right| \longrightarrow \Lambda_{eff} \simeq \Lambda \\ |p| \gg \left|1/\Lambda\theta\right| \longrightarrow \Lambda_{eff} \simeq \frac{1}{\sqrt{\tilde{p}^2}} \end{cases}$$

SO, IF WE SUBTRACT THE PLANAR DIVERGENCE BY RENORMALIZING THE MASS:

$$m^2 \rightarrow M^2 = m^2 + \frac{\lambda}{48\pi^2}\left(\Lambda^2 - m^2 \log\frac{\Lambda^2}{m^2}\right) + \text{const.}$$

WE HAVE A 1PI EFFECTIVE ACTION

$$\Gamma_{1PI} = \int d^4p \; \varphi(-p) \, \Gamma^{(2)}(p) \, \varphi(p) + \cdots$$

$$\boxed{\Gamma^{(2)}(p) = p^2 + M^2 + \frac{\lambda}{96\pi^2 \tilde{p}^2} - \frac{\lambda M^2}{96\pi^2} \log\left(\frac{1}{M^2 \tilde{p}^2}\right) + \cdots}$$

Since $\quad \Sigma^{1}_{NP} \approx \dfrac{\lambda}{96\pi^2 \tilde{p}^2} + \cdots \quad$ as $\Lambda \to \infty$

THUS, THE EFFECTIVE ACTION HAS A SINGULARITY AT $\tilde{p} = 0$ THAT CAN BE INTERPRETED AS

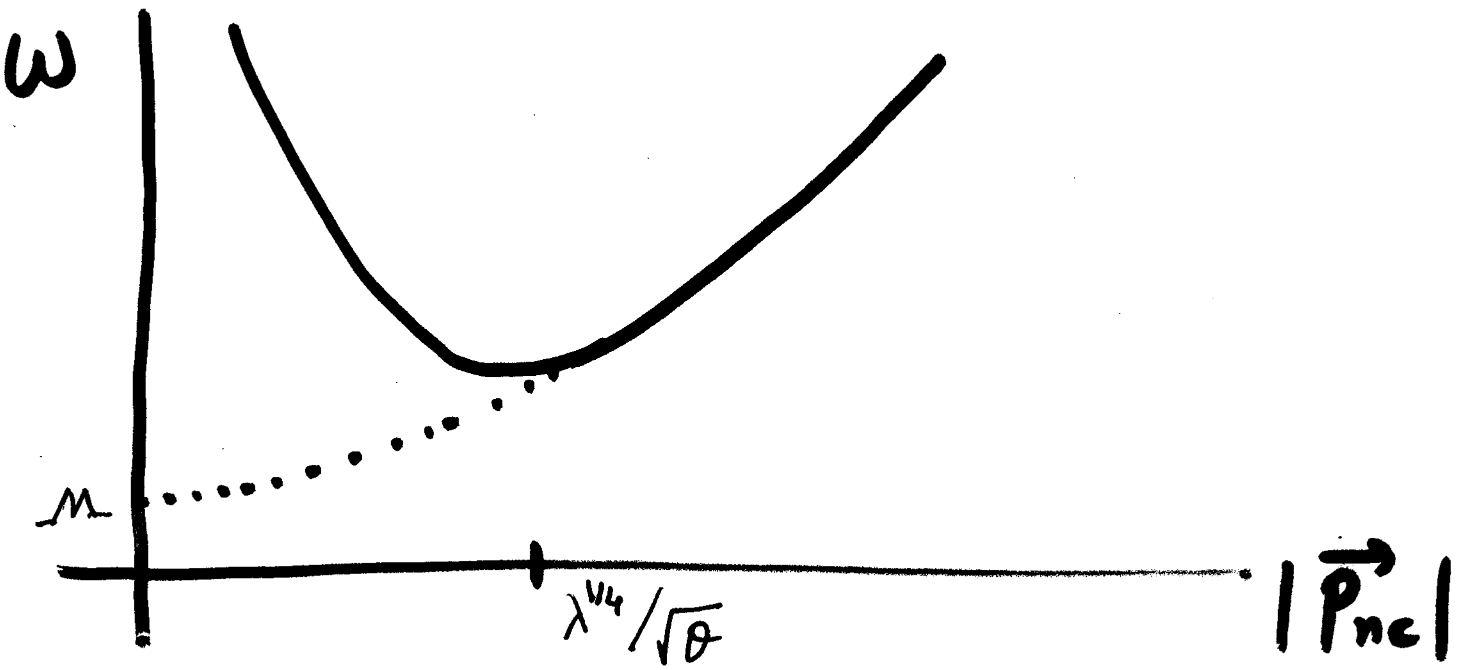
$$\tilde{p} = 0 = p_\mu \theta^{\mu\nu} \rightarrow \begin{cases} \text{IR SINGULARITY AT FIXED } \theta \\ \text{NON-ANALITICITY AT } \theta \approx 0 \\ \text{FOR FIXED } p \end{cases}$$

MODIFIED DISPERSION RELATION

$$p^2 + M^2 = 0 \quad \Longrightarrow \quad p^2 + M^2 + \frac{\lambda}{96\pi^2 \tilde{p}^2} \simeq 0$$

$$\boxed{\omega = \sqrt{\vec{p}^{\,2} + M^2 + \frac{c}{\theta^2 \vec{p}_{nc}^{\,2}}}} \qquad c = \lambda/96\pi^2$$

$\vec{p}_{nc}$ → noncommutative momentum



HUGE EFFECTS AT LOW MOMENTA!

ENTIRE LOW-ENERGY SPECTRUM REMOVED!

RELIABILITY OF ONE-LOOP

IN PERTURBATION THEORY, HIGHER ORDER CORRECTIONS TO $1/\tilde{p}^2$ ARE

$$\sim \frac{\lambda}{\tilde{p}^2}\left[\lambda \log\left(M^2 \tilde{p}^2\right)\right]^n$$

SIGNIFICANT ONLY FOR

$$\lambda \log\left(M^2 \tilde{p}^2\right) = \mathcal{O}(1)$$

SINCE $\tilde{p}^2 \sim |p\theta|^2$ WE SEE THAT PERTURBATION THEORY WILL BREAK DOWN AT NONPERTURBATIVELY SMALL MOMENTA:

$$\boxed{|p| \lesssim \frac{1}{|M\theta|} e^{-\frac{\text{const}}{\sqrt{\lambda}}}}$$

UV / IR IN GAUGE THEORY

CONSIDER THE POLARIZATION TENSOR IN NC U(1) AT ONE LOOP

$$S^{(2)} = \frac{1}{2} \int A_\mu(k) \, \Pi_{\mu\nu}(k) \, A_\nu(k)$$

IN ORDINARY (OR PLANAR) CASE, GAUGE INVARIANCE FORBIDS A QUADRATIC DIVERGENCE IN $\Pi_{\mu\nu} \sim \eta_{\mu\nu} \Lambda^2$. IT WOULD VIOLATE TRANSVERSALITY. IN FACT:

$$\Pi_{\mu\nu}(k) = \left(k_\mu k_\nu - \eta_{\mu\nu} k^2\right) \Pi(k)$$

$$\Pi(k) \sim \log \frac{k^2}{\Lambda^2} + \text{finite}$$

→ IN THE NONPLANAR CASE, THE EFFECTIVE U.V. CUTOFF OF THE DIAGRAM IS

$$\Lambda_{eff} = \min\left(\Lambda, \frac{1}{|\tilde{k}|}\right)$$

BECAUSE OF GAUGE INVARIANCE AT $\theta = 0$ WE WOULD EXPECT THAT UV/IR PHENOMENA WOULD ONLY APPEAR AT log LEVEL

$$UV/IR \sim \left(k_\mu k_\nu - \eta_{\mu\nu} k^2\right) \log\left(k^2 \tilde{k}^2\right)$$

WE FOUND SUCH TERM IN THE DISCUSSION OF THE β-FUNCTION

* $\boxed{\text{HOWEVER}}$ LORENTZ BREAKING ALLOWS ANOTHER KINEMATICAL STRUCTURE WHICH IS :

$$\left\{ \begin{array}{l} \text{PROPORTIONAL TO } \sim 1/\tilde{k}^2 \\ \text{TRANSVERSE} \end{array} \right.$$

$$\boxed{\Pi^{(NP)}_{\mu\nu} = -g^2 C \frac{\tilde{k}^\mu \tilde{k}^\nu}{\tilde{k}^4}}$$

It is transverse because

$$k_\mu \tilde{k}^\mu = k_\mu \theta^{\mu\nu} k_\nu = 0$$

THIS TERM IS GENERATED AT ONE loop with

$$C = \frac{1}{4\pi^2} \times 8 \times N \left(2 + n_s - 2 n_f \right)$$

U(N) group

complex scalars

Majorana fermions

IN PARTICULAR $C=0$ FOR SOFTLY BROKEN SUSY THEORIES.

SO, UV/IR RESPONDS TO THE NAIVE POWER COUNTING !! Matusis, Susskind and Toumbas (99)

$C \neq 0$ GIVES HUGE EFFECTS AT LOW MOMENTA. IN PARTICULAR, INSTABILITIES FOR $C < 0$

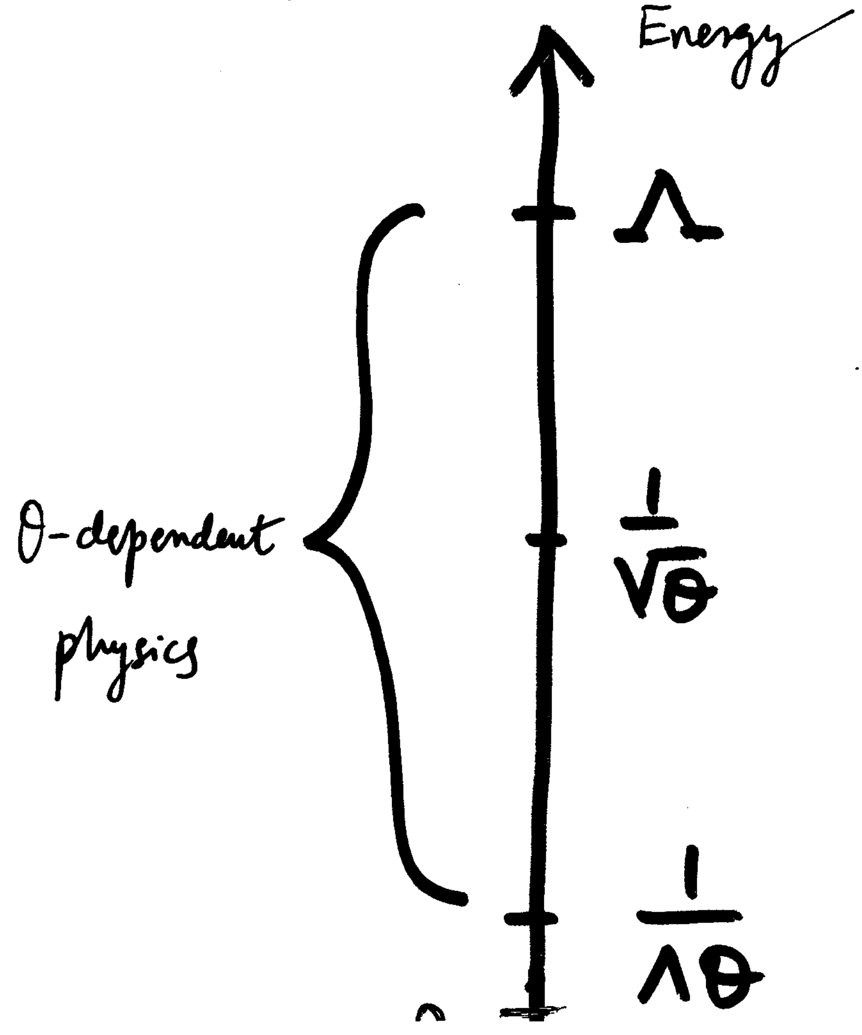
UV/IR HEURISTICS

A VIRTUAL LOOP OF HIGH MOMENTUM $|p|$ CARRIES A "DIPOLE" OF TRANSVERSE LENGTH $\approx |\theta p|$

SO, LOOP-CORRECTED QUANTITIES HAVE SIGNIFICANT θ-DEPENDENCE DOWN TO SCALES OF ORDER $|p_{low}| \sim 1/|\Lambda\theta|$ WITH

Λ THE MAXIMUM (CUTOFF) MOMENTUM CIRCULATING IN THE NONPLANAR LOOP

WE HAVE A HIERARCHY



UV / IR AND NC TIME

Consider

$$(\theta^{\mu\nu}) = \left(\begin{array}{cc|c} 0 & \theta_e & \\ -\theta_e & 0 & \mathcal{O} \\ \hline & & \begin{array}{cc} 0 & \theta_m \\ -\theta_m & 0 \end{array} \\ \mathcal{O} & & \end{array}\right)$$

$\eta^{\mu\nu} = (1, -1, -1, -1)$ $\qquad [t, x] = i\theta_e \qquad [y, z] = i\theta_m$

Then:

$$\tilde{p}^2 = (p_\mu \theta^{\mu\nu})^2 = -\theta_e^2 p_e^2 - \theta_m^2 p_m^2$$

$$p^2 = p_\mu p^\mu = p_e^2 - p_m^2$$

LOOK AT THE TADPOLE DIAGRAM AS A $1 \to 1$ SCATTERING AMPLITUDE

$$i\mathcal{M}(p \to p) = -i\lambda \frac{1}{6} \int \frac{d^4 q}{(2\pi)^4} e^{-i\tilde{p}\cdot q} \frac{i}{q^2 + i0}$$

THIS IS A STANDARD DISTRIBUTION FOURIER TRANSF

$$i\mathcal{M}(p \to p) = -i\frac{\lambda}{24\pi^2} \qquad \frac{1}{-\tilde{p}^2 + i0}$$

A STRIKING FACT:

$$\boxed{2\,\mathrm{Im}\,\mathcal{M} = \frac{\lambda}{12\pi}\,\delta(-\tilde{p}^2)}$$

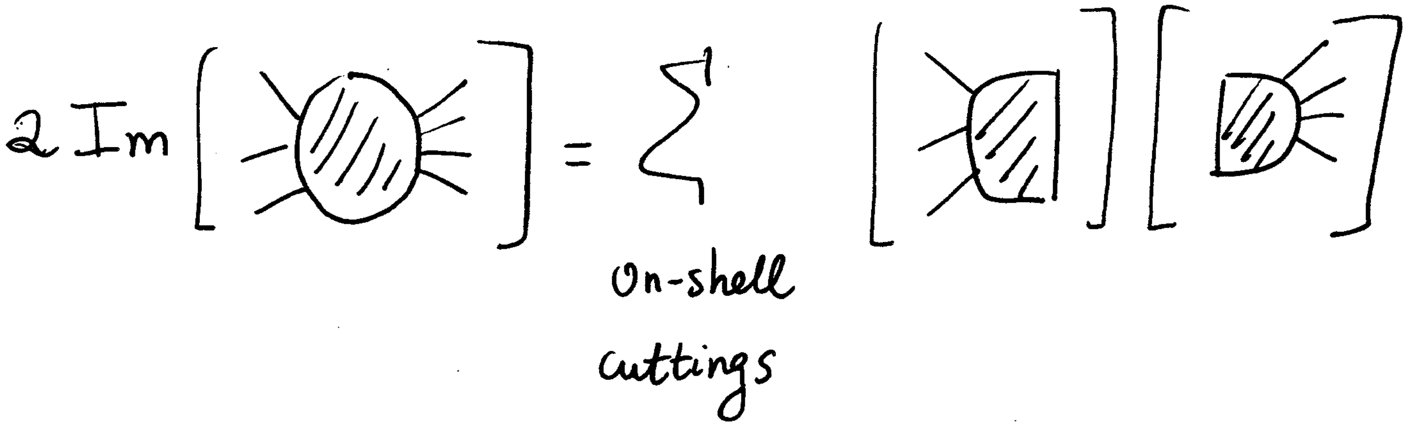
The imaginary part is a non-trivial distribution

UNLIKE ORDINARY THEORY, OR PLANAR DIAGRAM

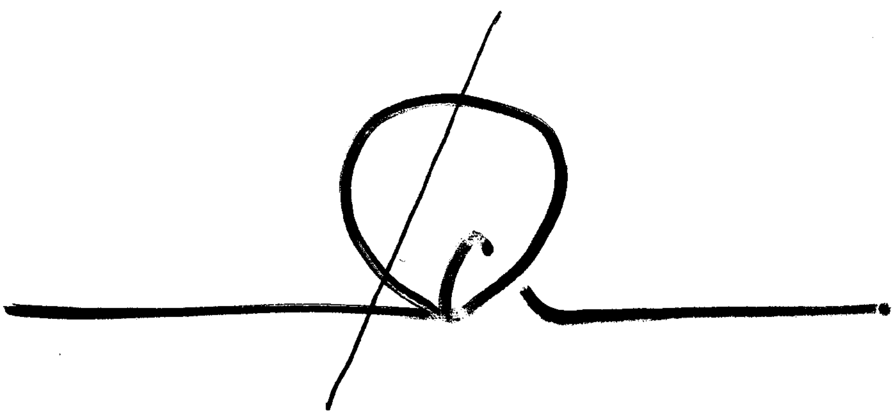
$$i\mathcal{M}_{\text{planar}} = -ic\Lambda^2 + \cdots$$

NO IMAGINARY PART, IN ACCORD TO OPTICAL THEOREM

Optical theorem



THE TADPOLE HAS NO ON-SHELL CUTTINGS



WE HAVE A FAILURE OF CUTKOSKY RULES AT THE NONPLANAR LEVEL

A FAILURE OF UNITARITY ?

Gomis & Mehen 00

STILL, CAN MANIPULATE $\mathrm{Im}\,\mathcal{M}$ SO THAT IT LOOKS LIKE AN OPTICAL THEOREM.

take $\theta_m = 0$ $\quad \theta_e \neq 0$ AND

INTRODUCE

$$1 = \int d^4k \; \delta(p-k)$$

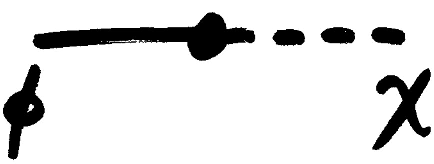
$$2\,\mathrm{Im}\,\mathcal{M} = \frac{\lambda}{12\pi}\,\delta(-\tilde{p}^2) = \frac{\lambda}{12\pi}\int d^4k\;\delta(k-p)\,\delta(-\tilde{p}^2)$$

$$\underset{\text{do } k^0 \text{ integral}}{=} \underbrace{\int \frac{d^3\vec{k}}{2(2\pi)^3\,|k_1|}}_{} \left(\frac{(2\pi)^2\lambda}{6\,\theta_e^2}\right)\delta(p-k)$$

Standard Fock-space measure for a particle with dispersion relation

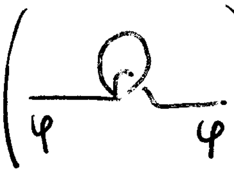
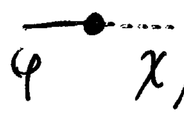
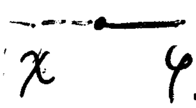
$$|k^0| = |k^1|$$

Introduce a particle $|\chi\rangle$ THAT MIXES WITH $|\phi\rangle$:



$= \lambda_{\phi\chi} = \sqrt{\dfrac{(2\pi)^2\lambda}{6\,\theta_e^2}}$

We have found:

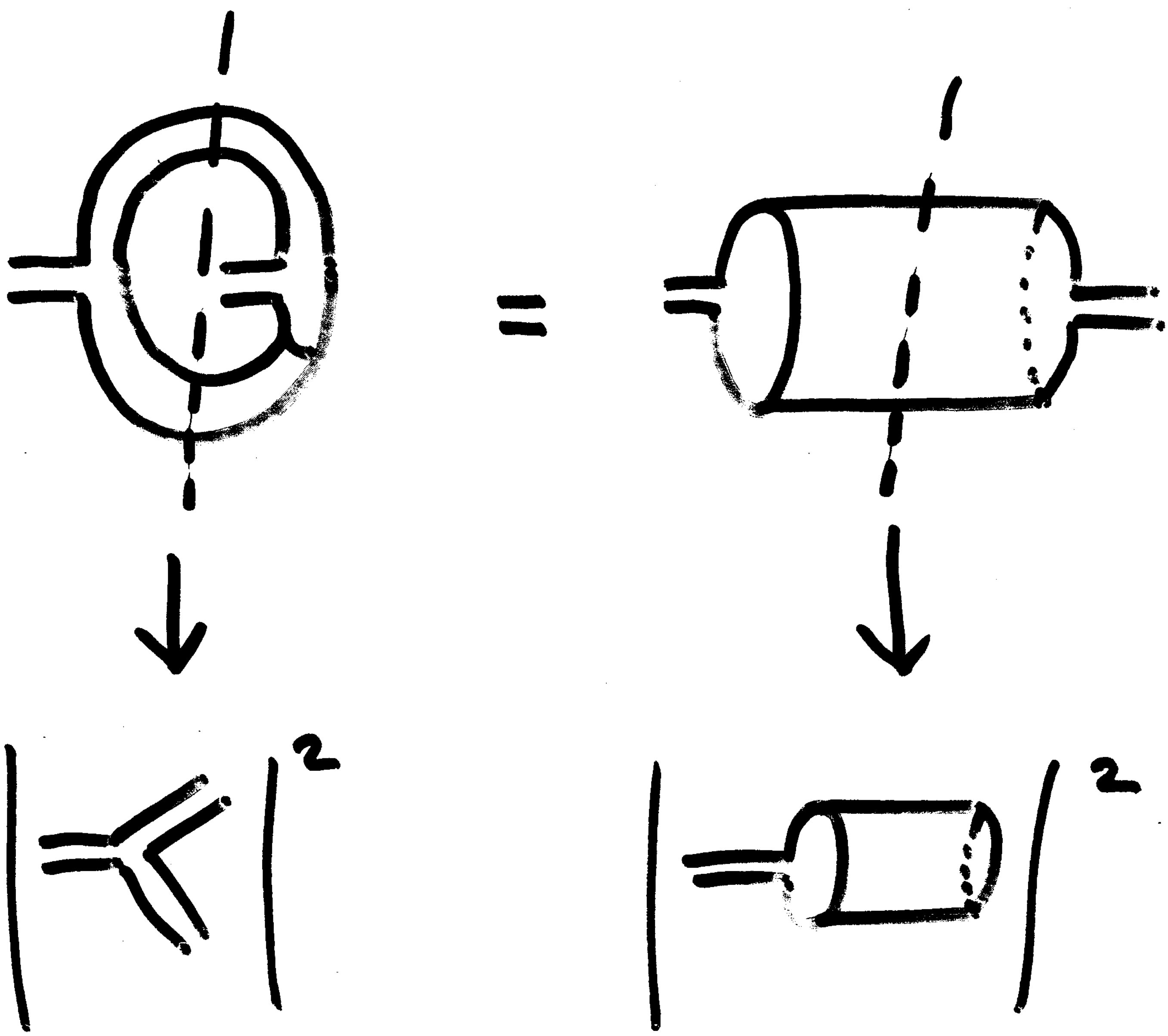
$2\,\mathrm{Im}\Big($  $\Big) = \sum'_{\mathcal{H}_\chi} \Big($  $\Big)\Big($  $\Big)$

* Notice $\lim\limits_{\theta_e \to 0} \lambda_{\phi\chi} = \infty$

* WHAT IS THE INTERPRETATION OF χ?

* WAS UNITARITY FAILURE A CONSEQUENCE OF OUR LOOKING AT A SUBSET OF HILBERT SPACE?

In dual Models:



S only unitary after adding the closed strings as asymptotic states

* THIS GENERALIZES. CAN FORMALY RESTORE UNITARITY BY ADDING $\mathcal{H}_\chi$ TO THE ASYMPTOTIC HILBERT SPACE

$$S = \begin{pmatrix} \chi \to \chi & \phi \to \chi \\ \chi \to \phi & \phi \to \phi \end{pmatrix} = (S^\dagger)^{-1}$$

---

* NOT GOOD ENOUGH SINCE $\mathcal{H}_\chi$ is sick:

(Alvarez-Gaumé, Barbón & Zwiky 01)

$\left\{ \begin{array}{l} \text{continuous spectrum} \\ \text{tachyonic} \\ \text{negative norm states.} \end{array} \right.$

In our example, if $\theta_m \neq 0$

$$2\, \mathrm{Im}\, \mathcal{M} = \int \frac{d^3\vec{k}}{2(2\pi)^3 \omega_\theta} \left(\frac{(2\pi)^2 \lambda}{6\, \theta_e^2} \right) \delta(p-k)$$

$$\omega_\theta = \sqrt{\vec{p}_e^{\,2} - \frac{\theta_m^2}{\theta_e^2} \vec{p}_m^{\,2}}$$

$\longrightarrow$ tachyonic unbounded below

* SO $\theta^{0i} \neq 0$ IS SICK IN PERTURBATION THEORY, BOTH FROM THE POINT OF VIEW OF DEFINING A LOW ENERGY F.T. FROM STRINGS, AND ALSO FROM INTERNAL CONSISTENCY (UNITARITY).

REMARKS ON θ-PHENOM.

NCFT BREAKS LORENTZ SYMMETRY AT TREE LEVEL. EVEN IF $\theta^{0i} = 0$

$\theta^{ij} = \varepsilon^{ijk}\,\theta_k$ DETERMINES A PRIVILEGED DIRECTION "IN VACUO" $\vec{\theta} = (\theta_k)$

SO:

$$|\theta| < (100\ \mathrm{GeV})^{-2}$$

TO BEGIN WITH.

DIFFICULT TO BE MORE SPECIFIC SINCE THE S.M. DOESN'T FIT NATURALLY INTO A NCFT WITH LORENTZ VIOLATION (RECALL PROBLEM OF U(1) CHARGES)

MOSTLY WORK IN **QED** SECTOR ONLY.

EXAMPLE OF N.C. TREE LEVEL EFFECT

BECAUSE OF THE DIPOLE PICTURE, LEADING INTERACTION OF e^- WITH THE NUCLEUS FIELD PROCEEDS THROUGH $x^\mu \to x^\mu - \frac{1}{2} p_\alpha \theta^{\alpha\mu}$

$$V(|\vec{x}|)_{\text{Coulomb}} \longrightarrow V\left(|\vec{x} - \tfrac{1}{2}(\vec{p}\cdot\theta)| \right) =$$

$$= -\frac{\alpha_{em} Z}{\sqrt{(\vec{x} - \frac{1}{2}\tilde{p})^2}} \simeq -\frac{\alpha_{em} Z}{|\vec{x}|} + \frac{\alpha_{em} Z}{|\vec{x}|^3}\left(-\tfrac{1}{2}\tilde{p}\cdot\vec{x}\right)$$

$$\simeq -\frac{\alpha_{em} Z}{|\vec{x}|} + \underbrace{\frac{1}{2}\frac{\alpha_{em} Z}{|\vec{x}|^3}\vec{\theta}\cdot\vec{L}}_{} + O(\theta^2 p^4)$$

$\vec{L} \to \vec{x} \wedge \vec{p}$

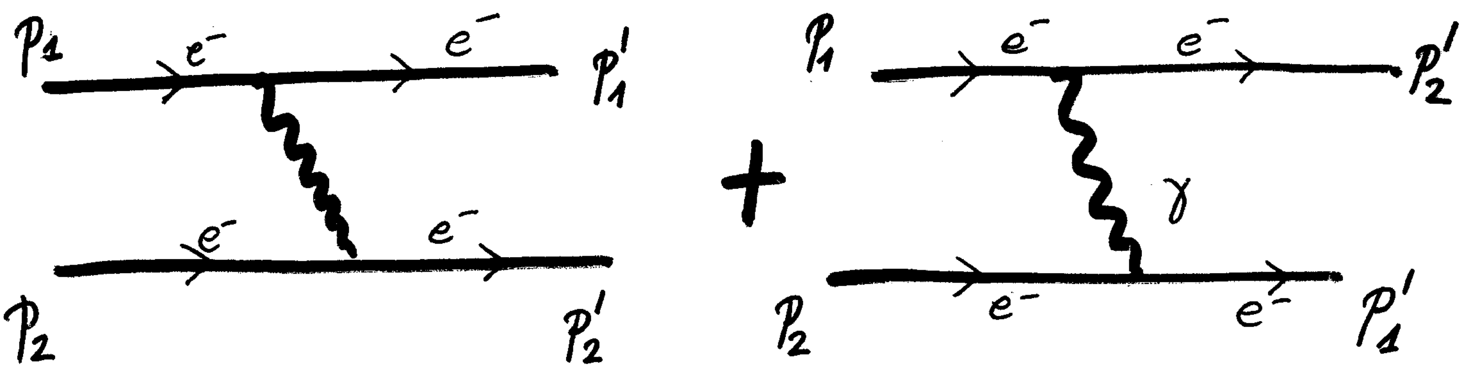
Induces "NC hyperfine splitting"

FROM LIMITS ON LAMB SHIFT:

$$|\theta| \lesssim (10\ \text{TeV})^{-2}$$

Chaichian, Sheikh-Jabbari & Tureanu (01)

COLLIDER EXAMPLE : Möller scatt.



LET US DERIVE FIRST THE $e^-\gamma e^-$ VERTEX

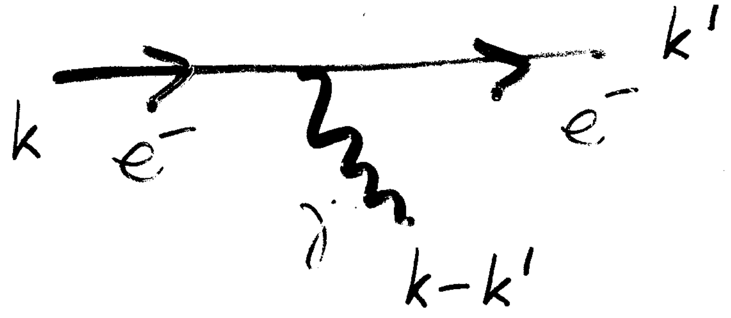
$$\boxed{e\int \bar{\Psi}\, \not{A} * \Psi}$$

$$\Psi \sim \sum b_k e^{-ikx} + d_k^+ e^{ikx}$$

$$\bar{\Psi} \sim \sum b_k^+ e^{ikx} + d_k e^{-ikx}$$

$$A \sim \sum a_k e^{-ikx} + a_k^+ e^{ikx}$$

$\lvert e^-\rangle = b^+\lvert 0\rangle$	$\lvert e^+\rangle = d^+\lvert 0\rangle$	$\lvert \gamma\rangle = a^+\lvert 0\rangle$



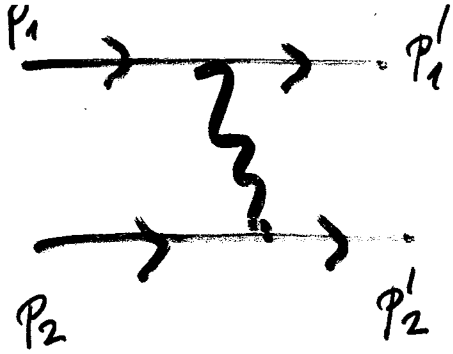
$$\sim \int_x \langle 0| b^{\dagger}_{k'} \, a^{\dagger}_{k-k'} \, b_k |0\rangle \, e^{ik'x} * e^{i(k-k')x} * e^{-ikx}$$

$$\sim \int e^{ik'x} * e^{i(k-k')x} * e^{-ikx} =$$

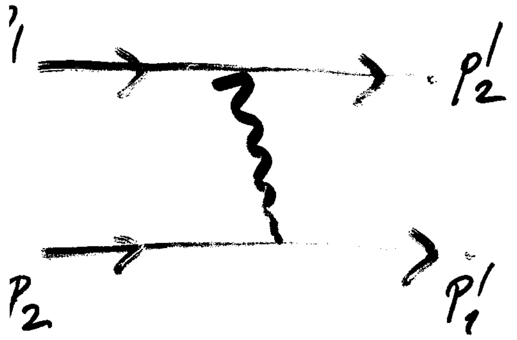
$$= \int e^{i(k-k')x} * \left(e^{-ikx} * e^{ik'x}\right) =$$

$$= \int e^{i(k-k')x} \, e^{-i(k-k')x} \, e^{-\frac{i}{2} k \times (-k')} =$$

$$= \boxed{\delta(0) \, e^{\frac{i}{2} k \times k'}}$$



$$\sim e^{i\phi_t} = e^{\frac{i}{2}\left(p_1 \times p_1' + p_2 \times p_2'\right)}$$



$$\sim e^{i\phi_u} = e^{\frac{i}{2}\left(p_1 \times p_2' + p_2 \times p_1'\right)}$$

TRANSITION PROBABILITY :

$$|M_t + M_u|^2 = |M_t|^2 + |M_u|^2 + 2\,\mathrm{Re}\left(M_t M_u^*\right)$$

θ- dependence arises only from the interference term

$$\mathrm{Re}\left(M_t M_u^*\right) \sim \mathrm{Re}\, e^{i(\phi_t - \phi_u)} \sim \boxed{\cos(\Delta\phi)}$$

$$\boxed{\Delta\phi = \frac{1}{2}\left(p_1 \times p_1' + p_2 \times p_2' - p_1 \times p_2' - p_2 \times p_1'\right)}$$

$$\boxed{\Delta\phi = \frac{1}{2}(p_1 - p_2) \times (p_1' - p_2')}$$

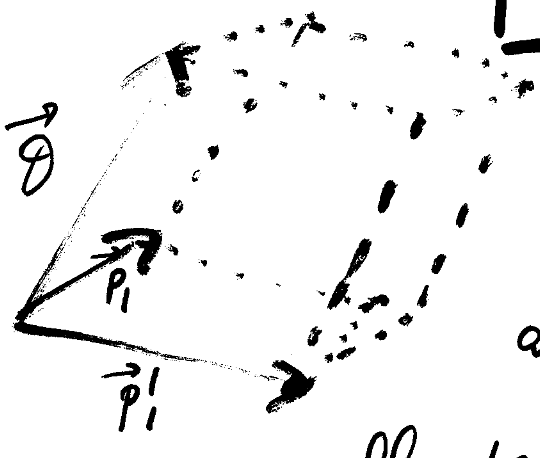
Take $\theta^{0i} = 0$ and C.o.M. frame

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' = 0 \qquad \begin{cases} \vec{p}_1 - \vec{p}_2 = 2\vec{p}_1 \\ \vec{p}_1' - \vec{p}_2' = 2\vec{p}_1' \end{cases}$$

$$\Delta\phi = 2\, \vec{p}_1 \times \vec{p}_1' = 2\, (p_1)_i\, (p_1')_j\, \theta^{ij} =$$

$$= 2\, (p_1)_i\, (p_1')_j\, \varepsilon^{ijk}\theta^k = 2\left(\vec{p}_1 \wedge \vec{p}_1'\right)\cdot\vec{\theta}$$

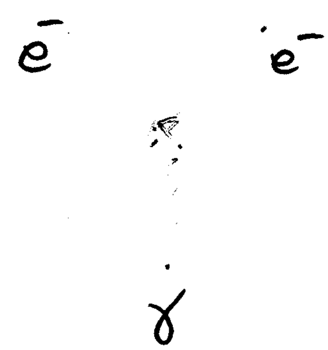
Interference $\sim$ $\boxed{\cos 2\, \mathrm{Vol}\left(\vec{p}_1, \vec{p}_1', \vec{\theta}\right)}$



Unless $\vec{p}_1, \vec{p}_1'$ and $\vec{\theta}$ are co-planar, we get effects of

$$\simeq 1 + O(E^2\theta)$$

TO LEADING ORDER THIS IS A dimension 6 operator:



$$\sim \theta^{\alpha\beta} \partial_\alpha \bar{\Psi} \not{A} \partial_\beta \Psi + \cdots$$

In general, the leading correction is:

$$(ph)_\theta = (ph)_{\theta=0} \left(1 + O(E^2\theta)\right)$$

So, 1% errors in the measurement of (ph) at $E \sim 100$ GeV gives

$$\boxed{|\theta| \lesssim \frac{1}{100\, E^2} \sim (\mathrm{TeV})^{-2}}$$

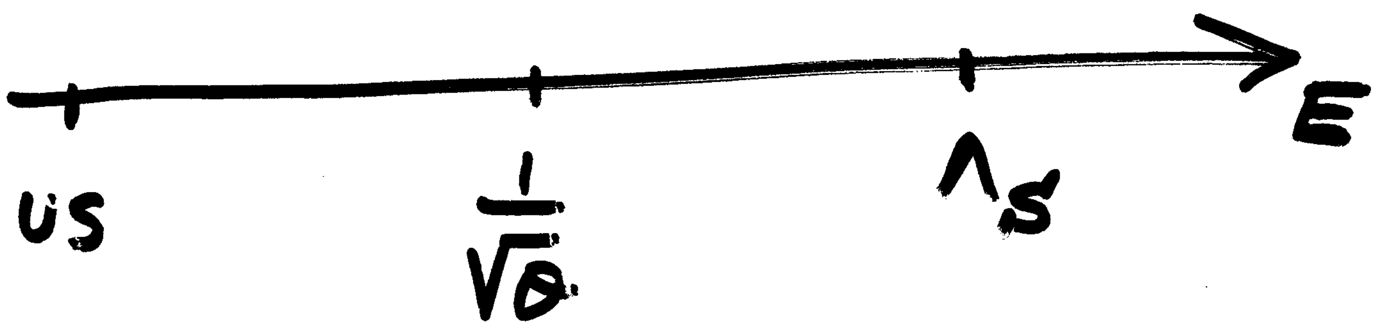
Typical effect of dim 5, 6 operators

* CAN GO LIKE THIS COMPUTING "AD NAUSEAM"

THE SITUATION IS MUCH WORSE AT 1-loop ORDER

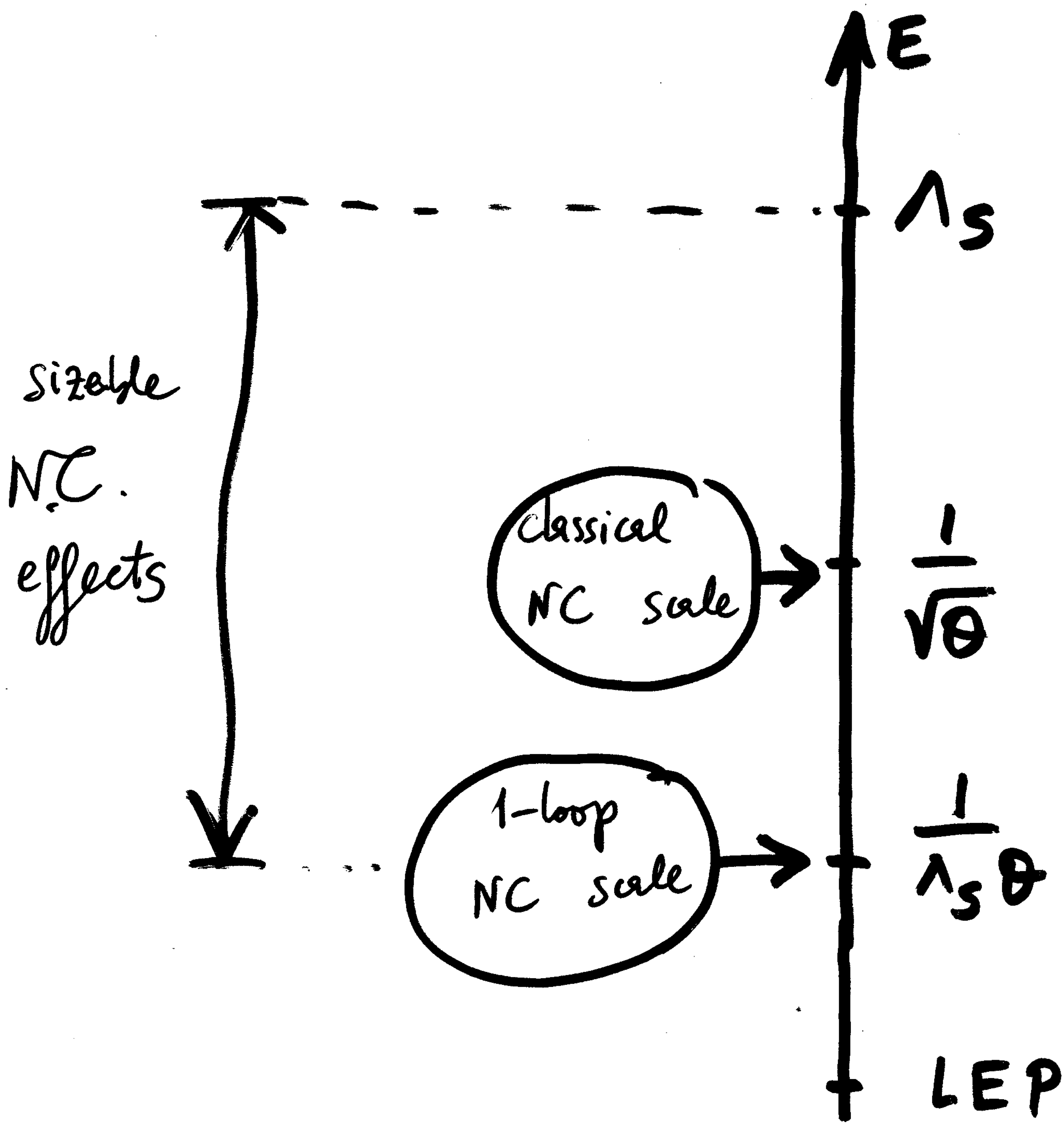
UV/IR IMPLIES THAT NC EFFECTS SHOW UP AT ENERGIES MUCH BELOW $1/\sqrt{\theta}$. IN FACT NC U(1) IS UNSTABLE AT 1-loop. We need nonperturbative input to make sense of the low energy picture

ONE POSSIBILITY TO QUENCH THE STRONG UV/IR EFFECTS IS TO HAVE SUSY ABOVE THE NC SCALE:



TYPICAL SITUATION

assuming pert. th. !



PHENOMENOLOGICAL BOUNDS REFER DIRECTLY TO $\frac{1}{\theta \Lambda_S}$ RATHER THAN $1/\sqrt{\theta}$

* THESE PROBLEMS + THE DIFFICULTIES IN EMBEDDING THE S.M. SEEM TO INDICATE THAT N.C. PHENOMENOLOGY IS JUST

PREMATURE

TO SAY THE LEAST...

BUT

OF COURSE,

you never know...

CONCLUDING REMARKS

* The UV/IR phenomenon makes NCFT interesting in itself, as a model of nonlocal QFT and as a toy model of String Field Theory

* A possible mathematical framework to study the large-N limit of ordinary gauge theories

$\ast$ **Interesting recent proposal by Susskind:**

NC - Chern - Simons as a good effective description of quantum Hall fluids

$$S = \frac{1}{4\pi\nu} \int \varepsilon^{ijk} \left(A_i * \partial_j A_k + \frac{2}{3} A_i * A_j * A_k \right)$$

ν = Filling fraction of Landau levels

$$\theta = \frac{1}{2\pi \rho_e}$$

$\hookrightarrow$ electron's density