SMR.1317 - 31

SUMMER SCHOOL ON PARTICLE PHYSICS

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QCD AND QUARK-GLUON PLASMA

Lectures I, II & III

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Please note: These are preliminary notes intended for internal distribution only.

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E. Shuryak

Lecture 1

QCD Vacuum and Instantons

Recom. Review

T. Schafer, E. Shuryak RMP 70 (1998) 323

Nonperturbative Phenomena and Phases of QCD

E.V.Shuryak
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L1 • – Introduction. Theory of chiral symmetry breaking, from Nambu-Jona-Lasinio model to instantons.

L2 • –The QCD correlation functions and hadronic structure;

L3 • – The QCD phase diagram. Finite temperature transition. Properties of Quark-Gluon PLasma.

• – QCD at finite density, Color superconducting phases.

L4 • – Overview of what we have learned about hadronic matter and its properties from heavy ion collisions, mostly at CERN SPS and BNL RHIC. Quark-Gluon Plasma

L5 • - Instanton/sphaleron mechanism in hh′ and AA collisions

A small map, before we start...

1950's QED $\Rightarrow$ 1970's SM $=$ U_1, SU_2, SU_3 (WEAK, DET.) $\Rightarrow$ 20?? Beyond the SM

whatever it may be

???

QCD is not a weak coupling problem, but

$\Rightarrow$ "<u>nonperturbative</u>" one

There are 2 meanings to this term:

① 1 (few) diagrams is not enough $\rightarrow$ <u>resummations</u> are needed

(In this language the H atom is also non-pert. since solving Schredinger eqn. is a resummation)

② Phenomena which cannot be expressed by perturbative diagrams **at all**

- e.g. <u>instantons</u> (tunneling to another minimum)

  $\sim \exp\left(\frac{\text{const}}{g^2}\right) \rightarrow$ cannot be expanded in g^n

  (that would be my main topic) $\rightarrow$ Progress in 1980-1990's chiral sym. br, correlators

- confinement $\rightarrow$ still not understood, nor is it modelled well...

⊛ Strongly interacting theories, like QCD (its SUSY analogs etc) may have many different <u>phases</u> which are <u>qualitatively</u> different

QCD Phases

(another small map...)

L1+2

The QCD vacuum ← QCD ground state
(or "hadronic phase" we live in)
- chiral condensate $\langle\bar{q}q\rangle \neq 0$
- confinement

T (temperature) — L3 →

Quark-Gluon Plasma
$\langle\bar{q}q\rangle = 0$
no other condensates
"normal" phase

⇓

Studied in High Energy Heavy Ion Collisions

μ_B (density) — L3 →

Color Supercond.
$\langle q_i q_j\rangle \neq 0$ diquark condens. breaks color
$\langle\bar{q}q\rangle$ may or may not be present

⇓

Makes "neutron stars" and the end of supernova even more interesting

? — L3 → $N_f > N_f^c$ (number of flavors)

Conformal Phase
no condensates
no scale
no particles

SPS at CERN
RHIC at Brookhaven
↳ 1st run in summer 2000
lots of unexpected phenomena

L4-5

The Little Bang

Overview: scales and approximations

It is assumed that some basic facts about perturbative QCD are known

- Asymptotic freedom - the charge is running

$$\alpha_s(Q) = \frac{g^2}{4\pi} = \frac{2\pi}{b\ log(Q/\Lambda_{QCD})}$$

The 1st coeff. of Gell-Mann-Law function is $b = (11/3)N_c - (2/3)N_f$

$\alpha_s(Q)$ is small at $Q >> \Lambda_{QCD}$

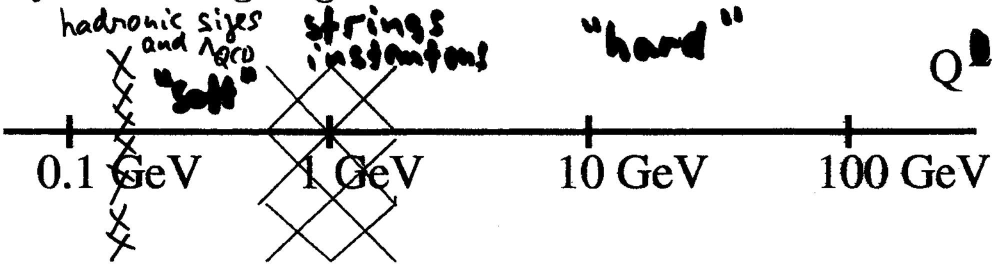
- The Landau pole prevents us from going to infrared ? $\alpha_s \to \infty$ as $Q \to \Lambda_{QCD}$?
- "Dimensional transmutation" - running charge defines a dimensional scale $\Lambda_{QCD} \sim 200 MeV$ (exact number depend on exact def.)
- Is Λ_{QCD} really the scale at which one has to abandon pQCD?

No! It is actually around $\Lambda_\chi \sim 1$ GeV

General Settings

The barriers still exist between description of perturbative and non-perturbative effects:

- – One barrier is the famous "1 GeV scale", which is simultaneously the lower boundary of pQCD and the upper bound of say chiral Lagrangians.



Effective theories
chiral Lagrangians, NJL
[redacted]

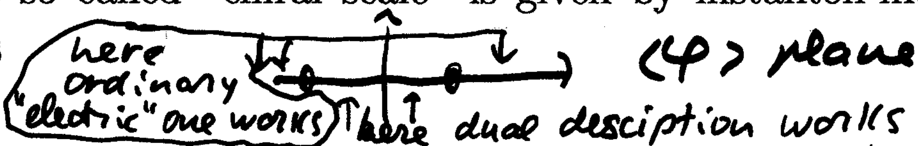
perturbative domain, parton description

instanton liquid model

the transition

in pQCD it is not seen: all logs are limited by Λ_{QCD} instead

- – The so called "chiral scale" is given by instanton-induced effects



Very amusing correspondence between N=2 SUSY (Seiberg-Witten Theory) and QCD in IMPLICATION OF EXACT SUSY GAUGE COUPLINGS FOR QCD. By L. Randall, R. Rattazzi, E. Shuryak Phys.Rev.D59:035005,1999 e-Print Archive: hep-ph/9803258)

N=2 SUSY   exact coupling (Witten-Seiberg 94)   vs   one loop + one instanton

QCD   one (or more) loops + one instanton

very similar behaviour, singularity at $\sim 3 \cdot \Lambda_{QCD}$
$\sim 2\sqrt{2}\,\Lambda_{SUSY}$

N=2 SUSY gauge theory

$$G^a_{\mu\nu} \to \lambda^a_\alpha \to \varphi^a, \qquad G^a_{\mu\nu} \to \psi^a_\alpha \to \varphi^a$$

(two Majorana fermions)

has multiple non-equivalent vacua, numerated by $u =$ [redacted] $= \frac{1}{2} Tr \langle \varphi^2 \rangle$ or a - Higgs VEV

Seiberg and Witten (94) have solved it, they have found explicitly "prepotential" $\mathcal{F}(\Phi)$

$$\mathcal{F}''(a) = \tau(a) = \frac{4\pi i}{g^2(a)} + \frac{\theta}{2\pi}$$ "running coupling"

$\mathcal{F}'''(a)$ is "anomalous magnetic moment" $\mathcal{F}'''(\lambda \sigma_{\mu\nu} \psi) G_{\mu\nu}$

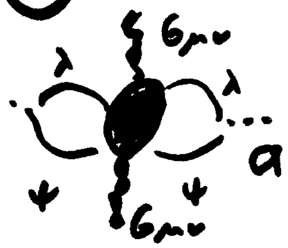
$\mathcal{F}''''(a)$ is 't Hooft vertex strength $\mathcal{F}''' \psi^2 \lambda^2$

How the result looks like in the weak coupling $(a \gg \Lambda)$?

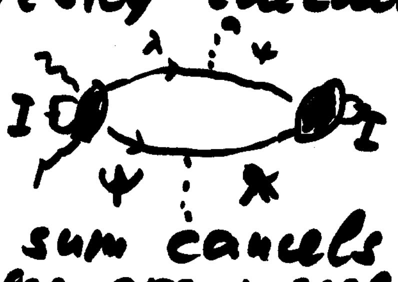
$$\frac{8\pi^2}{g^2(a)} = \underbrace{4 \log \left(\frac{2a^2}{\Lambda^2} \right)}_{\text{one loop } \beta \text{ function}} - \underbrace{12 \left(\frac{\Lambda}{a} \right)^4}_{\text{1 instanton}} - \underbrace{(3 \cdot 4 \cdot 5 \cdot 7) \left(\frac{\Lambda}{a} \right)^8}_{\text{2 instantons}} + \ldots$$

(1) Only $\left(\frac{\Lambda}{a} \right)^{4n}$ appear → instantons (!!!)

(2) These two have been directly calculated



Finnel + Pouliot 95



Dorey, Khoze Mattis 96

(3) The instanton sum cancels the log, where monopoles are massless

R. Rattazzi, L. Randall, E.S., → ~~in progress~~ hep-ph/9803258 PRD...

Effective charges (defined as coef. of $G_{\mu\nu}^2$)

$\begin{cases} \text{in } \underline{QCD} & (\text{a la Callan, Dashen, Gross-}T.) \\ \text{vs } \underline{N=2 \text{ SUSY QCD}} & (\text{a la Witten-Seiberg}) \end{cases}$

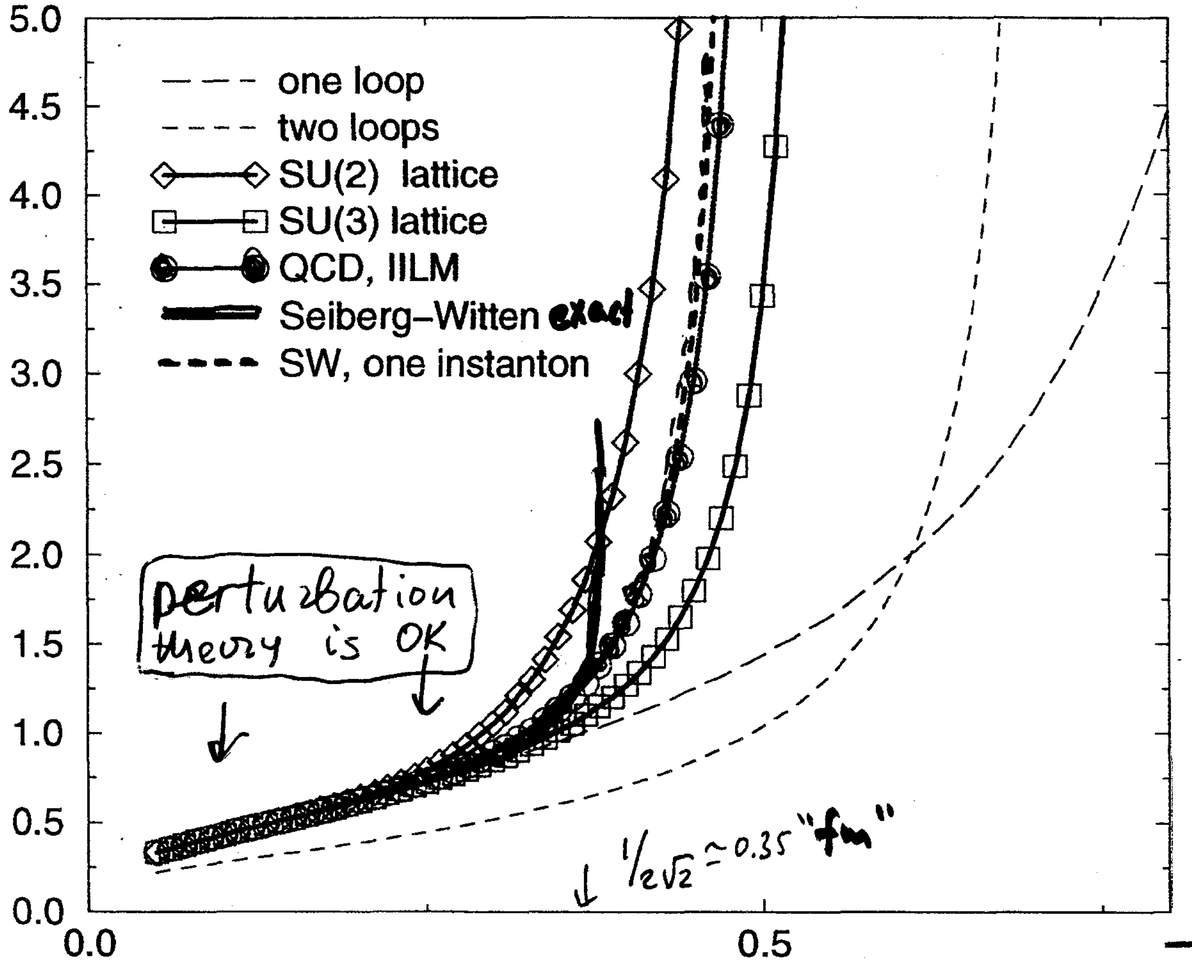
integrate $\langle d^2 \rangle$ up to instanton size $a = \rho_{max}$, $\rho < a = \rho_{max}$

very similar picture!
(although different in IR!!!)

$b = \frac{11}{3}N_c - \frac{2}{3}N_f$ QCD

$b = 4$ $N=2$ SUSY QCD

$\frac{b g_{eff}^2}{8\pi^2}$



LANDAU POLE

"1 fm"

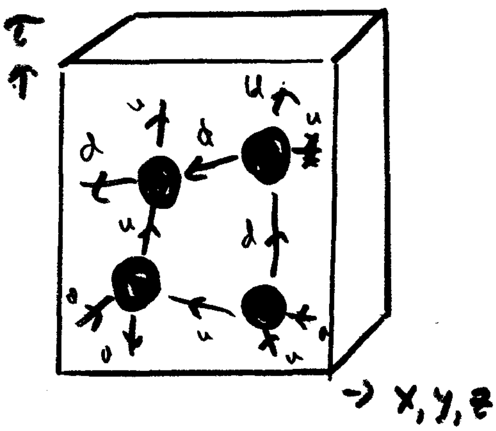
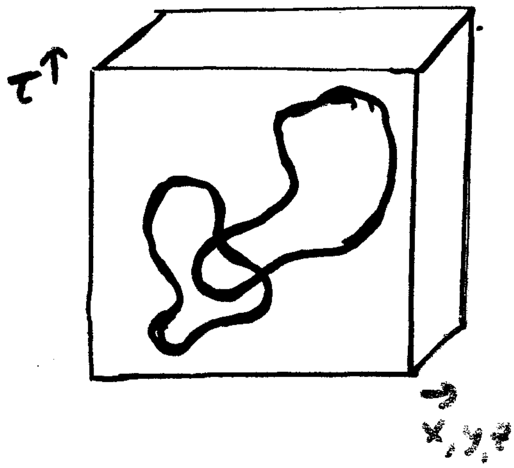
scale: a [fm], $\frac{\Lambda}{2u}$ } both blow up perturbatively at 1 - **Landau pole**

$u = \frac{1}{2}\langle \varphi^2 \rangle$
Higgs VEV

? ? ? ? 3ed picture confinement is due to vortices (2d) not monopoles (?)

Two major pictures of the QCD vacuum

E.S. Diakonov Petrov ... Negele et al

The "instanton liquid"	The "dual superconductor"
tunneling events	monopole loops
	
quark condensate $< \bar{\psi}\psi > \neq 0$ $SU(N_f)$ chiral symmetry is spontaneously broken $U_A(1)$ is dynamically broken Important for π, η', N etc	string tension $\sigma \neq 0$ color confinement Important for $\Upsilon, J/\psi$ etc
Rather complicated **statistical mechanics**	Rather complicated objects **and** unclear **quantum mechanics**

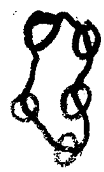
't Hooft Mandelstam ...

⊕ Suzuki et al

⊕ Recently, in SUSY Seiberg, Witten

⊖ some gluons and q's have no charge ?! σ ratios

⊗ Both topological → (like in weather forecasts...)



⊛ However, both pictures are probably much simpler than the original Quantum Field Theory language

⇒ experience of Condensed Matter (+Nuclear..) Physics can be used!

How they are related ???

→ In fact, strongly correlated Lattice-96

What instantons can do ?

↳ reproduce quantitatively

* Solve the U(1) problem 72

(why η' is heavy, unlike (π, K, η))? (S. Weinberg)

→ Single instanton leads to interaction (G 't Hooft) which violates this symmetry 76

(Note: effect is very large)

* Explain spontaneous breaking of $SU(N_f)_c$ chiral symmetry

$\langle\bar\psi\psi\rangle \neq 0$, light pions, all other consequences like f_π, low energy $\pi\pi$, πh interactions ...

Note: it is many-body effect, so one has to study ensemble of instantons

$\boxed{\text{70-80's}}$

* Explain formation of lowest states in ≈ all mesonic and baryonic channels (ρ, N, Δ ....)

$\boxed{\text{90's}}$

(without confinement!)

* Explain chiral restoration at $T \neq 0$, $\mu \neq 0$, or N_f ↗ recent

color superconductivity ...

- - - - - - - - -

* Everything (?)

→ N=2 SUSY QCD (Seiberg-Witten) (94)

Exact answers = (one pert. loop) + (nice series in instanton interactions) + nothing else!

N=4 SUSY ⇔ AdS_5 Also exact results, Mattis et al

Semi-classical approximation, instantons and anomalies

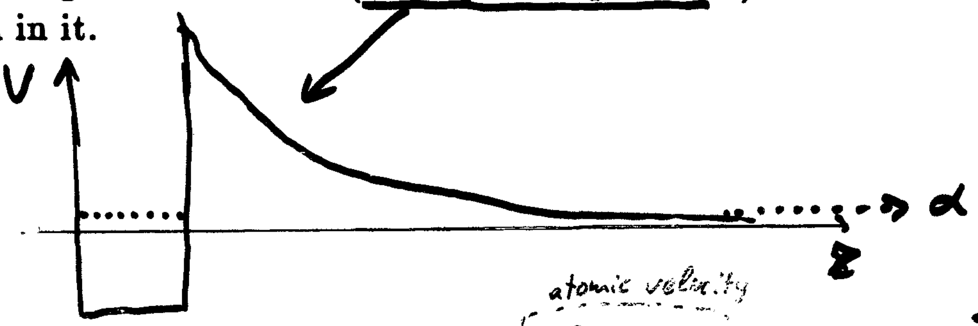
- Overview: scales and approximations
- Going from Minkowski to Euclidean space
- Tunneling in quantum mechanics: first instantons
- Instantons in Yang-Mills theories
- Fermionic zero modes and the chiral anomaly
- U(1) Chiral symmetry breaking in 1-flavor theory
- Instantons in electroweak sector, and in SUSY ~~theories~~
- SU(N_f) Chiral symmetry breaking

Tunneling in general

- – **Who was the first to discover 'tunneling' phenomena in quantum mechanics, and in what context it was first done?**

  It was George Gamow, in late 20's, and the context was alpha-decay of nuclei. He explained *the mistery of radioactivity*: why nuclei should wait sometimes for billion years to decay, in spite of the fact that typical nuclear time scale is about $10^{-22} sec$. 'Tunneling' means going through the mountain (the repulsive potential) AS IF there is a tunnel in it.

GG's popular books



atomic velocity

$$Probability \sim exp(-\frac{2\pi(2e^2)(Z-2)}{\hbar v}) \qquad 10^{-(30\div35)}$$

  Note: v is our small parameter: the α particle velocity, which is small compared to atomic one. Numerical factor 2π is important!

- – **How one can use *classical* mechanics for description of *classically forbidden* phenomena?**

- – **Hint. In quantum mechanics energy is conserved, and Schreodinger eq. also can be understood as**

$$E = \frac{<p^2>}{2m} + <V(x)> \qquad (\simeq 0)$$

  In classically allowed region, $p^2 > 0$ and the wave function is a wave $\psi \sim exp(ipx)$ with real p. However, if we are in *classically forbidden* region $E < V$, we must have *negative kinetic energy* or *imaginary* p.

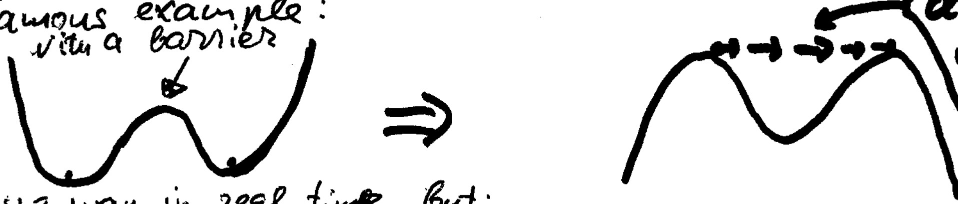
Then $\psi \sim exp(-|p|x)$, and one understands why tunneling is a very rare event, etc.

• – Trick: if p is imaginary, why do not try to interpret it as *motion in imaginary time?*
Changing t to $\tau = it$ we have new classical equation of motion:

$$m\frac{d^2x}{d\tau^2} = -F = -\frac{d(-V)}{dx}$$

It is the same as flipping the potential upside down! Then classical paths certainly exist.

Famous example: with a barrier

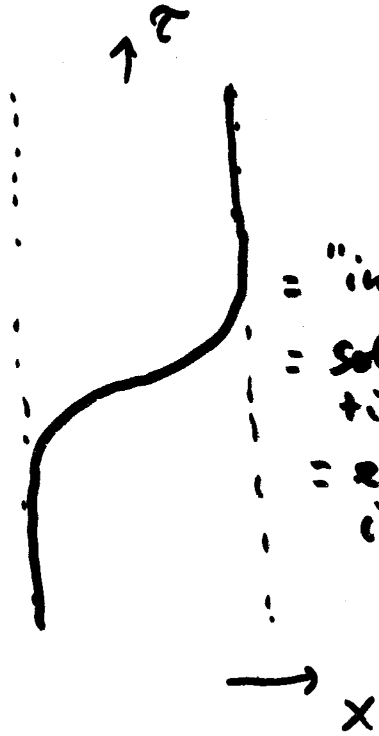


no way in real time, But:

classical path in imaginary time!

= instanton

Using Feynman path integral one can go to imaginary time easily (in fact, this is what he did to get correct continuation to real time) The weight of any path is exp(-S[x(τ)]), and this essentially gives the tunneling probablity.



= "instanton"

= soliton in time

= exist for an instant

't Hooft

3

Tunneling and instantons in gauge theories

Polyakov et al 75
t Hooft 76, Jackiw
Rebbi; Callan Dashen
Gross 78

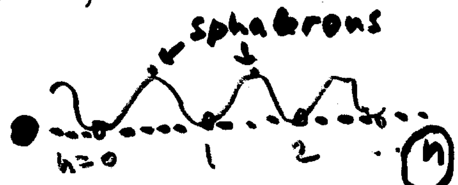
- **Topology and classical vacua**

Vacuum (classical) = configuration, minimizing the potential energy. ("configuration" is time-independent, no kinetic energy)

The gauge $A_0 = 0$. Then $E_i = \partial_0 A_i$ and its value squared is kinetic energy, while the (still non-linear) magnetic part is the potential one.

Minimum is at zero magnetic field $G^a_{ij} = 0$, so vector potential should be "pure gauge"

$$A_\mu = (2i/g) S \partial_\mu S^+$$



We have to classify gauge matrices $S(\vec{r})$: they project 3-dim space to the group. SU(2) group has 3 parameters.

$$S = exp(if(r)\tau_a r_a/r)$$

Chern Simons #

Such projections have the integer number n (called the winding number) which counts how many times the group manifold is covered.

$$n = \epsilon^{ijk}(1/24\pi^2)\int d^3x Tr[(S^+\partial_i S)(S^+\partial_j S)(S^+\partial_k S)]$$

For the particular example above

$$n = (1/2\pi)[f(0) - f(\infty) - sin(2f(0))/2 + sin(2f(\infty))/2]$$

So matrices with different n are topologically different and one cannot obtain one from another by means of CONTINUOUS gauge transformation.

The instanton is the path configuration with Q=1

$$A^a_\mu(x) = (2/g)\eta_{a\mu\nu}x_\nu/(x^2+\rho^2)$$

where eta is the so called 't Hooft symbol. It is $\epsilon_{a\mu\nu}$ if all indices are not equal to 4, $\delta_{a\mu}$ if $\nu = 4$ and $-\delta_{a\nu}$ if $\mu = 4$. There is also symbol $\bar{\eta}_{a\mu\nu}$, in which last two statements (with delta) has the opposite sign.

But A is not yet physical quantity, the action density:

$$(G^a_{\mu\nu})^2 = 192\rho^4/(x^2+\rho^2)^4$$

is finite everywhere, and concentrated in a spot of the radius ρ, the so called instanton radius.

't Hooft 76

Integral overs small fluctuations around it leads to instanton density generalized to $SU(N_c)$

$$\frac{dn_+}{d^4z} = \frac{.466exp(-1.679N_c)}{(N_c-1)!(N_c-2)!}[8\pi^2/g^2(\rho)]^{2N_c}exp[-8\pi^2/g^2(\rho)]d\rho/\rho^5$$

Now, the (one-loop)asymptotic freedom formula

$$8\pi^2/g^2(\rho) = blog(1/\rho\Lambda), b = (11/3)N_c - (2/3)N_f$$

leads to

$$\frac{dn_+}{d^4z} \sim \int \frac{d\rho}{\rho^5}(\rho\Lambda)^b$$

Is it convergent at large ρ? Yes

Where? Why?

you will see $\rho < \rho_0 \sim 1/3$ fm

still unknown

Main features of the "Instanton liquid"

In 1981 I come up with the so called 'Instanton Liquid Model', starting from the following question:

- – What are the typical instanton size ρ and separation R?

From several arguments (especially the magnitude of the gluon and quark condensate $<\bar{q}q>$) I have concluded that

$$\boxed{R \simeq 1 fm}$$

$$\boxed{\rho_c \simeq 1/3fm = (600MeV)^{-1}}$$

[redacted]
(Negele et al, 93)
$\langle\rho\rangle = 0.35\, fm$
density $= 1.3$-$1.6\ /fm^3$
$R \doteq 0.9\, fm$
Lattice

If so, some important cosequences follow:

- – DILUTENESS.

$$\rho/R \sim 1/3$$

where R is the typical distance between the pseudoparticles. It is not very small ratio, but in 4-dim space it enters in 4-th power, so only few per cent of the space-time is occupied by strong field.

- – SEMICLASSICAL FORMULAE ARE APPLICABLE. $(\rho/R)^4 \sim 10^{-2}$

The action is large enough

$$S_0 = 8\pi^2/g(\rho)^2 \sim 10 \gg 1$$

Quantum corrections go as $1/S_0$ and are presumably small enough. $\sim 10\%$

- –INTERACTION DOES NOT DESTROY INSTANTONS.

Estimated by the dipole formula, interaction was found to be typically

$$|\delta S_{int}| \sim (2-3) \ll S_0$$

$$\frac{\delta S}{S} \sim (\rho/R)^4$$

- –LIQUID, NOT GAS.

interaction is not negligible in the statistical mechanics of instantons:

$$exp|\delta S_{int}| \sim 20 \gg 1$$

$$n\rho^4 \simeq C\, S_0^{2N_c - \frac{b'}{2b}}\, e^{-S_0}\, e^{-S_{int}}$$
$$10^{-2} \simeq \frac{4}{50} \cdot 10^3 \cdot 10^{-4.5} \cdot 10$$
How diluteness works!

Instantons in QCD vacuum

$$A_\mu = \frac{2}{g}\,\frac{\eta^a_{\mu\nu}\,x^\nu}{x^2+\rho^2}$$

describes *tunneling* between topologically different vacua...

[G. t'Hooft (1976)]

[Polyakov et al (1975)]

$$\left(G^a_{\mu\nu}\right)^2 \sim \frac{1}{(x^2+\rho^2)^4} \qquad \boxed{\text{well localized in 4D}}$$

Similar (although different) from hypothetical Nambu-Jona-Lasinio interaction, which is to create

$\langle\bar\psi\psi\rangle \neq 0 \quad m_{const} \sim 350\text{-}400\,\text{Me}$

pions and all that

$\sigma, \pi =$ OOOOOO...

But also $\eta' \to$ (sign opposite!)

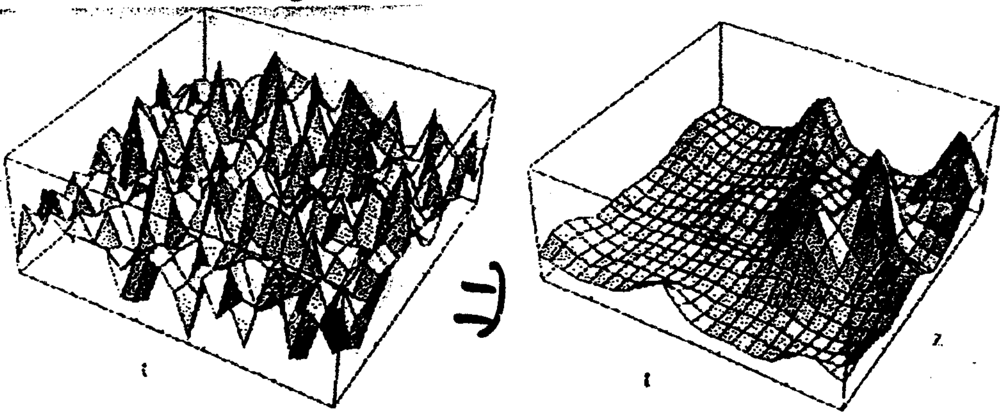
E.S. (82) $\to$ **Instantons create $\langle\bar\psi\psi\rangle$, etc** !

$$n = n_+ + n_- \simeq 1\,\text{fm}^{-4}, \quad \rho \simeq \tfrac{1}{3}\,\text{fm}$$

diluteness parameter $\quad n\rho^4 \sim \left(\tfrac{1}{3}\right)^4 \ll 1$!

Hunting for instantons on the lattice:

action drops by 2-3 orders!



Before and after (25 cooling steps) the fog goes away...

$\boxed{\text{Chu, Grandy, Negele, Huang - 94}}$

$\boxed{\bar\rho = 0.35\ \text{fm} \ :\ R \simeq 0.9\ \text{fm}}$ $\leftarrow$ Very close to my $\rho = 1/3$, $R = 1$ fm

+ many others

From lattice:

[A. Hasenfratz, C. Nieter
hep-lat/9806026]

At small ρ

$$\frac{dN}{d\rho} \simeq \frac{dN_{pert}}{d\rho} = \frac{c}{\rho^5}\left(\frac{8\pi^2}{g^2}\right)^{2N_c}(\rho\Lambda)^{11} \sim \rho^6$$

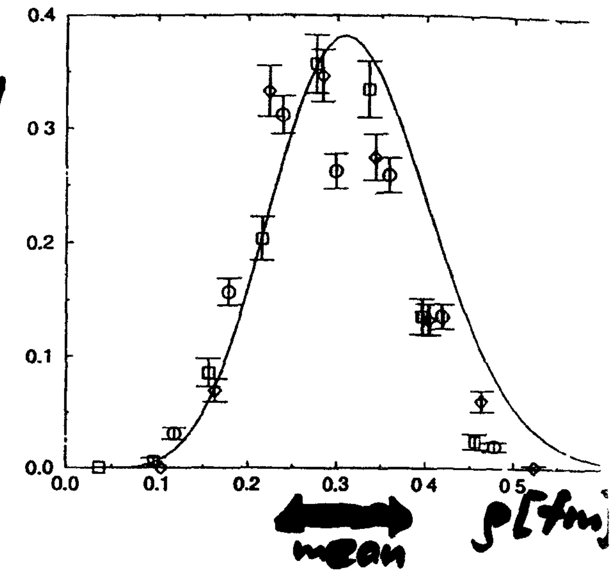
at large ρ suppressed

Let us divide pert. theory out:

$$\rho^{-6}\frac{dN}{d\rho} \longrightarrow$$

* Suppression is $O(\rho^2)$ not ρ^4 (and sign is negative)
* It is the same for small and large ρ
* The ΔS involved are not small, ΔS is up to ~ 7!

so it looks like general law not expansion...



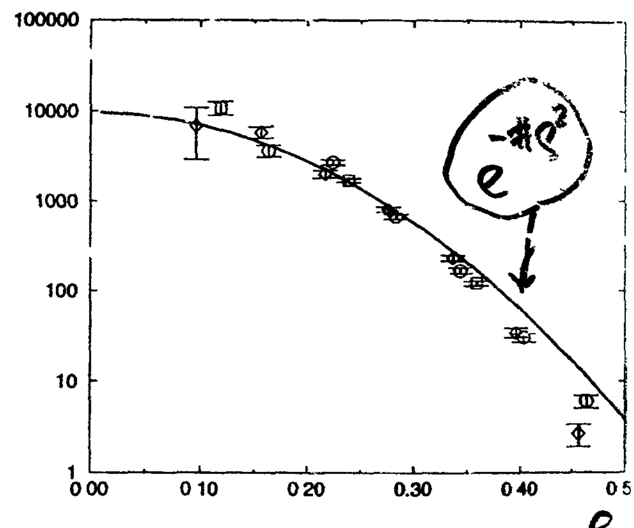


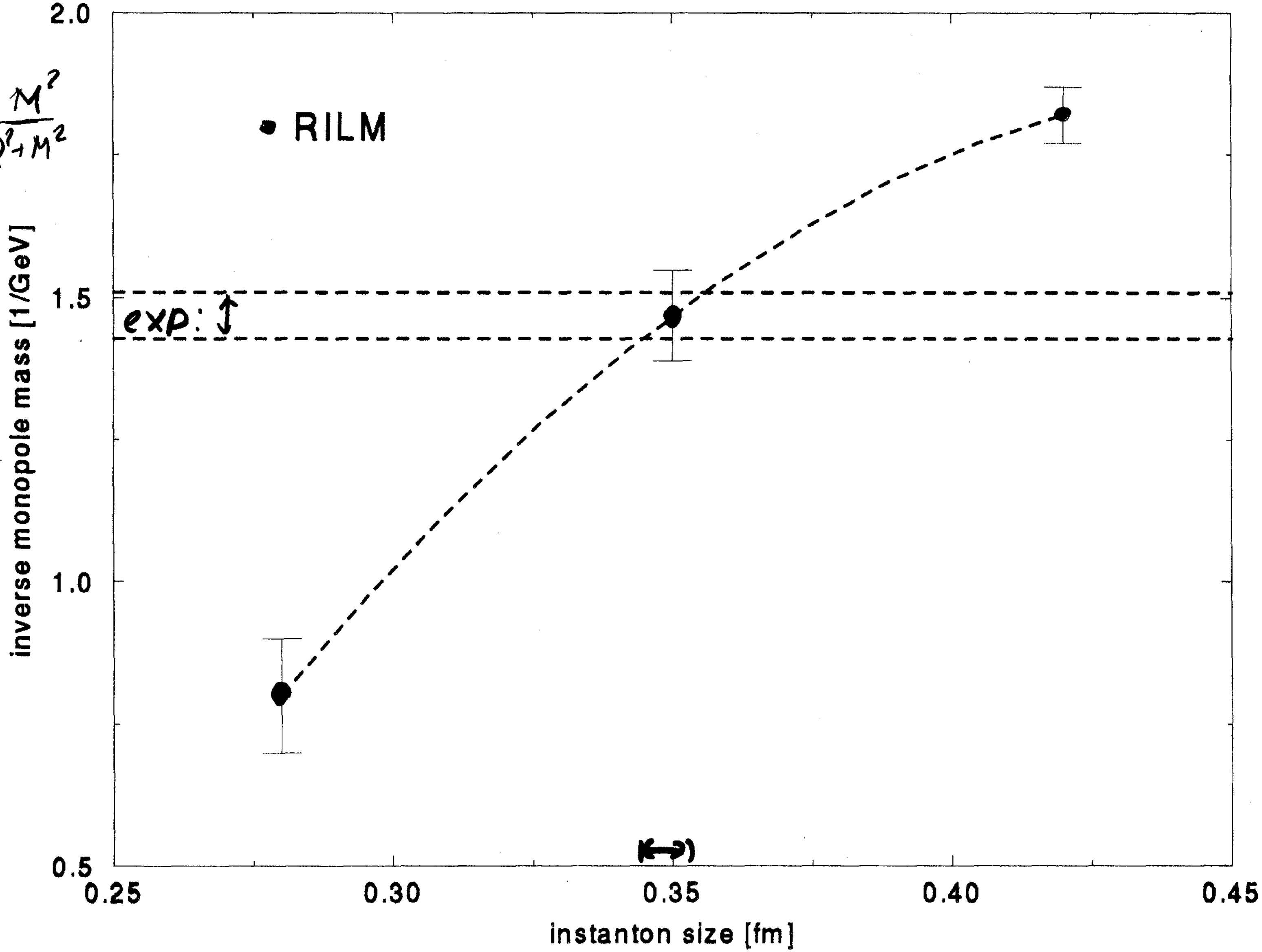
FIG 1 (a) The instanton density $dn/d\rho d^4z$, [fm^{-5}] ve its size ρ [fm]. (b) The combination $\rho^{-6}dn/d\rho d^4z$, in w the main one-loop behavior drops out for $N_c = 3, N_f$ The points are from the lattice work [8], for this theory, β=5.85 (diamonds), 6 0 (squares) and 6 1 (circles). T comparison should demonstrate that results are rather tice-independent The line corresponds to the proposed pression $\sim exp(-2\pi\sigma\rho^2)$, see text.

A. Blotz + E.S.
PL B 98

pion 3-point function

electromagnetic formfactor

$f_\pi = \frac{M^2}{Q^2+M^2}$



← not the ρ mass!

Chiral Symmetries ($m_q \to 0$)

$U(1)_A$ $\quad \psi'_f = e^{i\gamma_5 \alpha} \psi_f$ flavor ↑ no flavor ↑ flavor	$SU(N_f)$ $\quad \psi'_f = (e^{i\gamma_5 \vec{\tau}\vec{\lambda}})_{ff'} \psi_{f'}$ ↑ flavor generators $\partial_\mu j^a_{\mu 5} = O(m_q)$
is anomalous, ('tHooft 76) Broken explicitly By 1 inst $\bar{u}_R u_L \bar{d}_R d_L \bar{s}_R s_L \to e^{i6\alpha} \bar{u}u\bar{d}d\bar{s}s$ Note: in electroweak Breaks B	Broken spontaneously By the instanton ensemble (in $V \to \infty$ limit, etc Before $m \to 0$) (ES-82 Diakonov, Petrov 86) ....
α excitations → $\boxed{\eta'}$ is very massive (958 MeV)	$\boxed{\vec{\pi}}$ (N_f^2-1) are massless Goldstone modes

Dirac Operator + its eigenvectors

$$i\not{D} = \left[i\partial_\mu \delta_{ik} + g \frac{t^a_{ik}}{2} A^a_\mu \right] \gamma_\mu$$

$$i\not{D}\psi_\lambda = \underline{\underline{\lambda}}\, \psi_\lambda$$

↑ not a mass but "virtuality"

In Euclidean (lattice) form:

To understand λ spectrum is important because:

- $\det \not{D} = \prod_\lambda \lambda$
- $S(x,y) = \sum_\lambda \frac{\psi_\lambda(x)\psi^+_\lambda(y)}{\lambda + im}$ ← spectral reps of the propagator

↳ Casher-Banks relation

$$\langle \bar{\psi}\psi \rangle = Tr\, S(x,x) = \frac{1}{V_4}\int d^4x\, Tr S$$

$$\underset{m\to 0}{\Rightarrow} \pi \underbrace{\frac{dN}{d\lambda}(\lambda=0)}$$

density of states at the surface of the Dirac sea

Sets the scale for "const. quark mass" and all hadronic masses

Anomalies

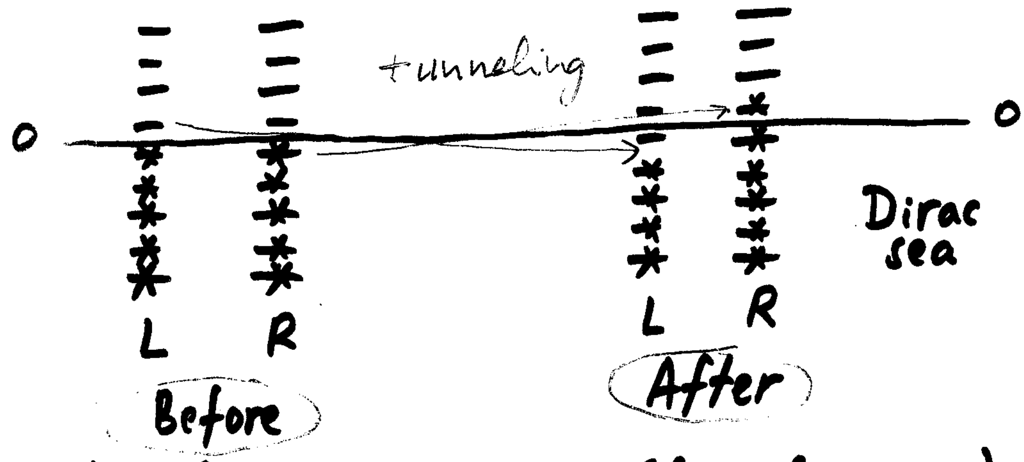
Light fermions do strange things while tunneling...

[In weak interactions Baryon number becomes violated!
e.g.: $u+d \to \bar{d}+\bar{s}+2\bar{c}+3\bar{t}+e^{+}+\tau, \mu^{+}+\tau^{+}$]

In QCD axial charge $(=N_L - N_R)$ is not conserved

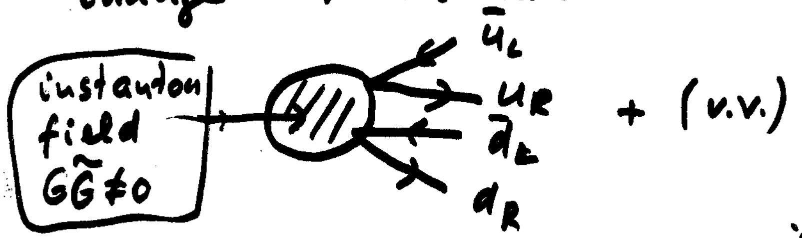
$\partial_\mu \bar{\Psi}\gamma_\mu\gamma_5\Psi \neq 0 \sim G_{\mu\nu}\tilde{G}_{\mu\nu}$ or $(\vec{E}\cdot\vec{B})$

its integral is quantized because it is $\boxed{\text{the topological charge}}$



$\boxed{\text{1 instanton creats 1 pair}}$
(for each light fermionic flavor)

Levels are the same (free fermions) but occupations change → all are moved by 1 level!



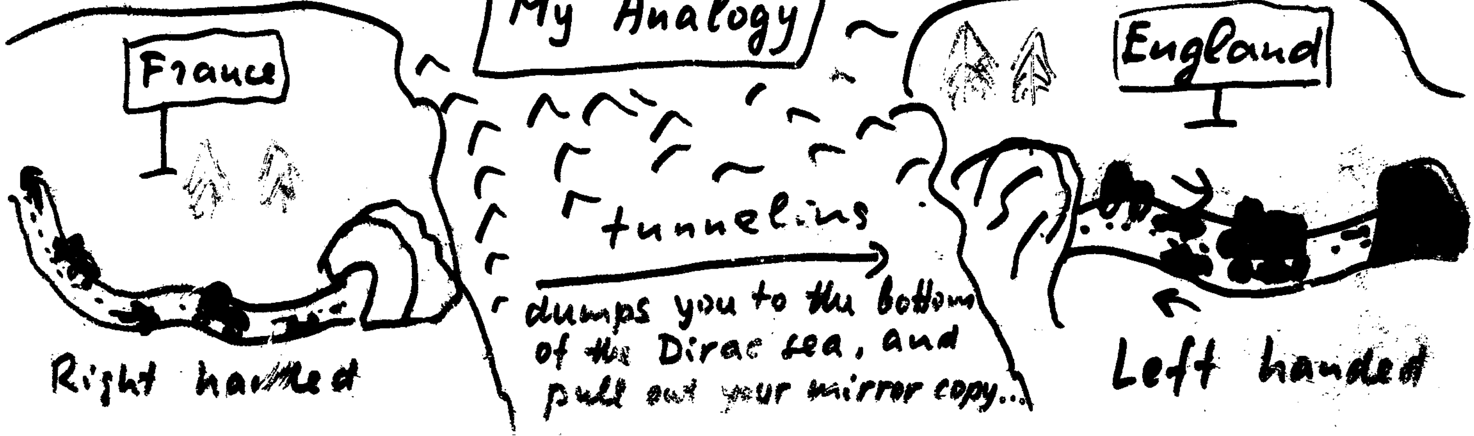
$+ \text{(v.v.)}$

$\boxed{\text{'t Hooft effective interaction}}$

[Strange quark is so

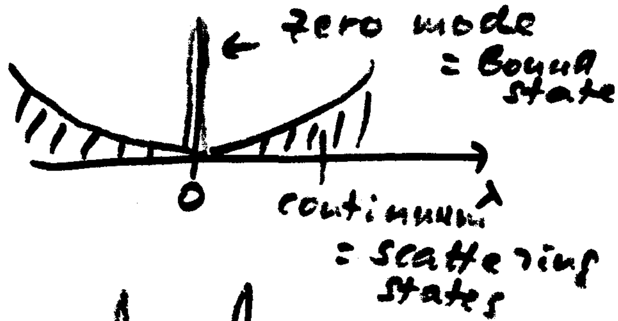
As any effective Lagrangian, it can be used in any channel

My Analogy



Spectrum of Dirac Eigenvalues

- Ex.1 Single instanton

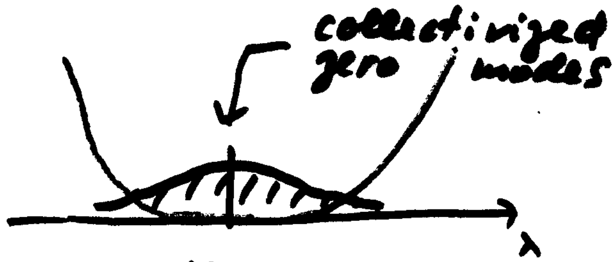


- Ex.2 $\bar{I}I$ "molecule"

$\begin{pmatrix} 0 & T_{IA} \\ T_{AI} & 0 \end{pmatrix}$ $\quad T = \langle \psi_I | \not{D} | \psi_A \rangle$

$\left.\frac{dN}{d\lambda}\right|_{\lambda=0} = 0$ no chiral sym. breaking

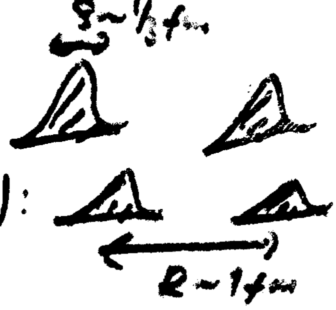
- Ex.3 "Instanton Liquid"



$\left.\frac{dN}{d\lambda}\right|_{\lambda=0} \neq 0$ chiral sym. is broken!

If so; two basic predictions:

→ ① $\sum_{(\text{small } \lambda)} |\psi_\lambda(x)|^2$ should look like sum of zero modes (bumps):



(If other mechanism → e.g. long-range gluons, monopoles etc → other pictures)

→ ② Each bump should be **locally chiral** $\gamma_5 \psi = \pm \psi$

Both seen on the lattice!

Low-lying Fermion Modes, Topology and Light Hadrons in Quenched QCD

Thomas DeGrand, Anna Hasenfratz
Department of Physics, University of Colorado, Boulder, CO 80309 USA
(February 21, 2001)

We explore the properties of low lying eigenmodes of fermions in the quenched approximation of lattice QCD. The fermion action is a recently proposed overlap action and has exact chiral symmetry. We find that chiral zero-eigenvalue modes are localized in space and their positions correlate strongly with the locations (as defined through the density of pure gauge observables) of instantons of the appropriate charge. Nonchiral modes are also localized with peaks which are strongly correlated with the positions of both charges of instantons. These correlations slowly die away as the fermion eigenvalue rises. Correlators made of quark propagators restricted to these modes closely reproduce ordinary hadron correlators at small quark mass in many channels. Our results are in qualitative agreement with the expectations of instanton liquid models.

I. INTRODUCTION

Is there a particular physical mechanism in QCD which is responsible for chiral symmetry breaking? If so, what other qualitative or quantitative features of QCD depend on this mechanism? The leading candidate for the source of chiral symmetry breaking is topological (instanton) excitation of the gauge field, which couples to the quarks through the associated fermion zero modes (or near-zero modes, after mixing) leading to chiral symmetry breaking via the Banks-Casher [1] relation. An elaborate phenomenology built on the interactions of fermions with instantons is said to account for many of the low energy properties of QCD (for a review, see Ref. [2,3]).

Lattice simulations can in principle address this issue, and indeed this is a large and active area of research. However, nearly all results, be they from pure gauge operators or from fermions, are contaminated by one kind of lattice artifact or another, which cloud the picture.

The problem is, that typically, pure gauge topological observables depend on the operator used. The dominant features of the QCD vacuum seen in any lattice simulation are just ultraviolet fluctuations, as they would be for any quantum field theory. To search for instantons (or other objects), one must invent operators which filter out long distance structure from this uninteresting noise. Some quantities (like the topological susceptibility in $SU(3)$ gauge theory) are less sensitive to filtering, but some (like the size distribution of topological objects) are more so, and most results are controversial (see Ref. [4] for a recent summary).

Perfect action topological operators [5,6] ...

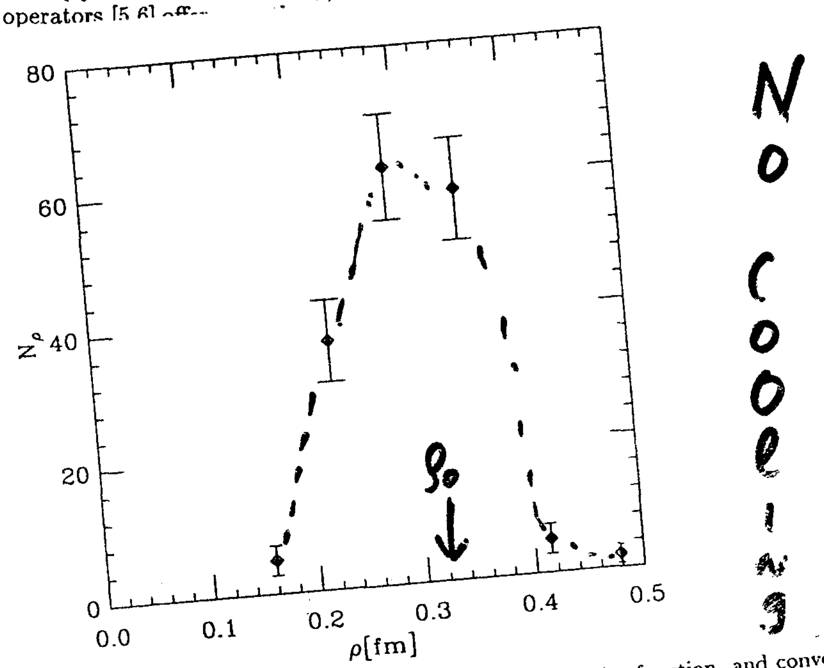


FIG. 13. Instanton number density vs size, extracted from the fermion chirality density function, and converted to physical units using a nominal lattice spacing of $a = 0.12$ fm.

How good is chiral sym. on the lattice these days?

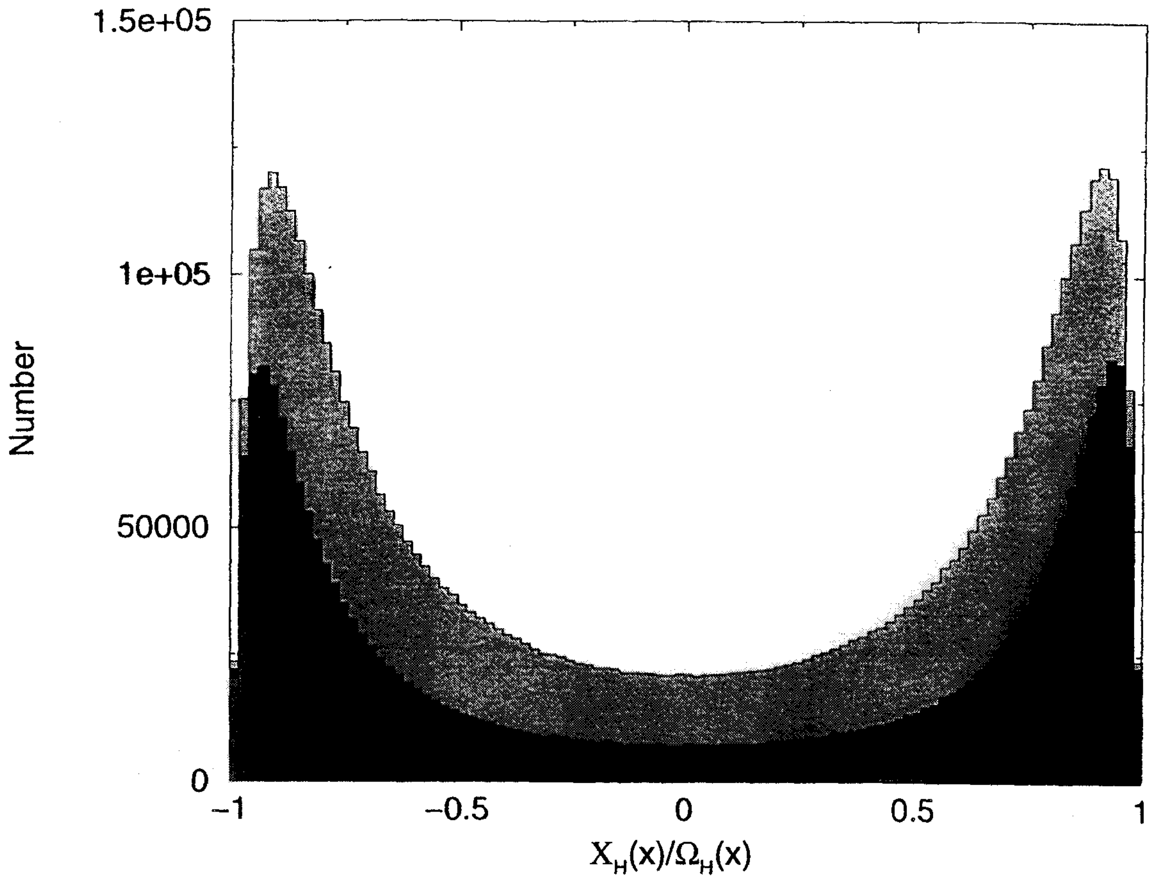


FIG. 5. The same quantities as in Fig. 4, but for non-zero mode eigenvectors and again for the Iwasaki action. The double-peak structure is a feature expected in instanton-dominated models of the QCD vacuum.

T. Blum, N. Christ... hep-lat/0105006
Domain wall fermions

One of 4 replies to
N. Isgur et al, 2001 who ~~have~~ not seen 2 maxima
and thus questioned instanton mechanism...

Chiral Random Matrix Theory (Jac Verbaarschot)

- Theory for fluctuations of QCD Dirac eigenvalues in the extreme infrared domain

- This part of the spectrum plays an essential role in the mechanism of chiral symmetry breaking and its restoration in the quark gluon phase

- Chiral Random Matrix Theory partition function

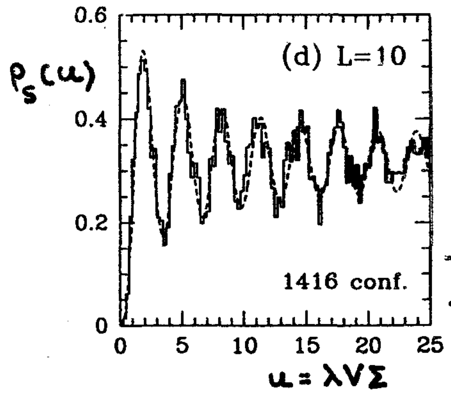
$$Z_{chRMT} = \int dW \det{}^{N_f}\begin{pmatrix} m & iW \\ iW^+ & m \end{pmatrix} e^{-N\Sigma^2 \operatorname{Tr} WW^+}$$

Σ: chiral condensate

W: $N \times (N+\nu)$ matrix (Shuryak-JV, NPA 93)

- Comparison with lattice QCD simulations

(Et al.-JV-Wettig, PRL 98)



--- chRMT (analytical)

— lattice (histogram)

$$\rho_s(u) = \lim_{V\to\infty} \frac{1}{V\Sigma}\, \rho\!\left(\frac{u}{V\Sigma}\right)$$

ρ: spectral density of Dirac operator

- Universal behavior in the ergodic domain ($\lambda \ll F_\pi^2/\Sigma\sqrt{V}$)

Osborn-JV, PRL 98

Sener-JV, PRL 98

Z_{QCD}

$\downarrow$

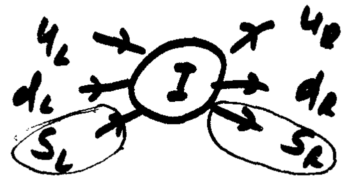
Z^{chiral} ← determined uniquely by chiral symmetry

$Z_{chRMT} \rightarrow Z^{chiral} \leftarrow Z_{Random\ Flux}$

How to describe the instanton-induced effects?

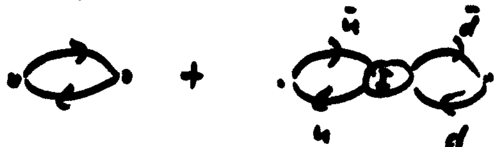
Way #1: First integrate away the color field A_μ^a

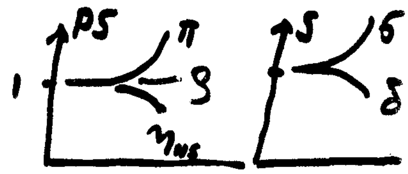
$\Rightarrow$ 't Hooft effective interaction (Like Nambu-Jona-Lasinio...)



if needed ...

Example: short distance behaviour of the correlators

$\Pi(x) =$ 

$\Rightarrow$ 

does not exist for V, A!

↑ 93 Paper, E.S. check numbers...

Example: Sum of  for π... is good!

(Diakonov, Petrov 86) $\to$ Mean field approxim.

... but, how to solve it to <u>all</u> orders?

Way #2 First do fermions, in 'atomic' approximation, then average over coll. variabl.

$$Z \sim \int d\Omega \, e^{-S_g} \left[\det(i\not{D} + im)\right]^{N_f}$$

↑ collective coordinates 12 / instanton

↑ gluonic interaction

↑ fermionic interaction

can be computed numerically, in "zero mode zone"

$\Rightarrow$ All orders in 't Hooft interaction included

Interacting Instanton Liquid Model

IILM → how we do statistical mechanics in 4d

The partition function of the instanton liquid

The main assumption underlying the instanton model is that the full partition function can be approximated by relevant gauge configurations, which are superpositions of instantons and anti-instantons.

Sum of all vacuum graphs

$$Z = \frac{1}{N_+! N_-!} \int \prod_i^{N_+ + N_-} [d\Omega_i \, d(\rho_i)] \exp(-S_{int}) \prod_f^{N_f} \det(\hat{D} + m_f) .$$

$= \sum$ 

loops

Here $d\Omega_i = dU_i \, d^4 z_i \, d\rho_i$ is the measure in the space of collective coordinates, color orientation, position and size. For the gauge group $SU(3)$ there is a total of 12 collective coordinates per instanton.

Fluctuations around the multi-instanton configuration are included in gaussian approximation for the individual instantons ('t Hooft 76). To two loop accuracy it reads

$$d(\rho) = C_{N_c} \rho^{-5} \beta_1(\rho)^{2N_c} \exp\left(-\beta_2(\rho) + (2N_c - \frac{b'}{2b}) \frac{b}{2b'} \frac{1}{\beta_1(\rho)} \log(\beta_1(\rho)) \right)$$

$$C_{N_c} = \frac{4.6 \exp(-1.86 N_c)}{\pi^2 (N_c - 1)!(N_c - 2)!}$$

where $\beta_1(\rho)$ and $\beta_2(\rho)$ are the one and two loop beta functions

$$\beta_1(\rho) = -b \log(\rho\Lambda), \qquad \beta_2(\rho) = \beta_1(\rho) + \frac{b'}{2b} \log(\frac{2}{b} \beta_1(\rho)),$$

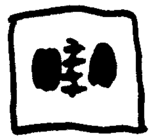
$$b = \frac{11}{3} N_c - \frac{2}{3} N_f \qquad b' = \frac{34}{3} N_c^2 - \frac{13}{3} N_c N_f + \frac{N_f}{N_c}.$$

In principle, there is wide selection of possibilities, e.g.

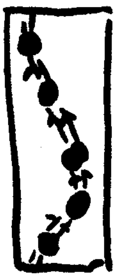
1. The ordered system, or "instantonic crystal". (never occured)

2. Disordered "liquid": provides chiral symmetry breaking as needed

3. Nearly ideal "gas" of instanton-anti-instanton pairs ("molecules"): this is what happens in QUark Gluon Plasma phase.



4. Long polymer chains of alternating I and $\bar{I}$ or diquark condensates" those are color superconductor phases at high density



But the main point now is that we cannot just select the phase we prefer. It is really impossible to say what phase is actually the case under before calculations are made. Chiral symmetry has been found to be broken for $N_f < N_F^{critical} = 5$. (T.Schafer,ES,J.Verl 1995.)

We can do calculations to all orders!

→ IILM

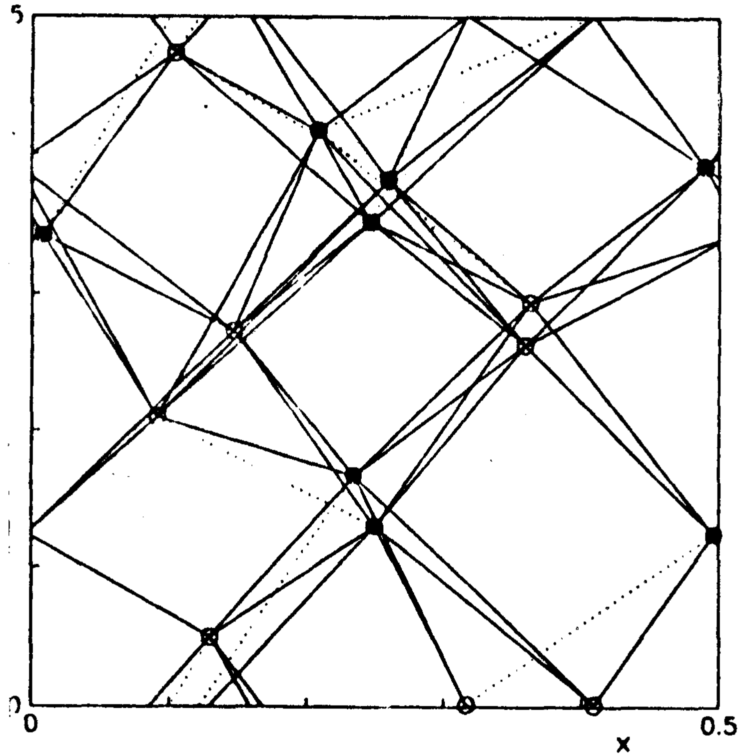
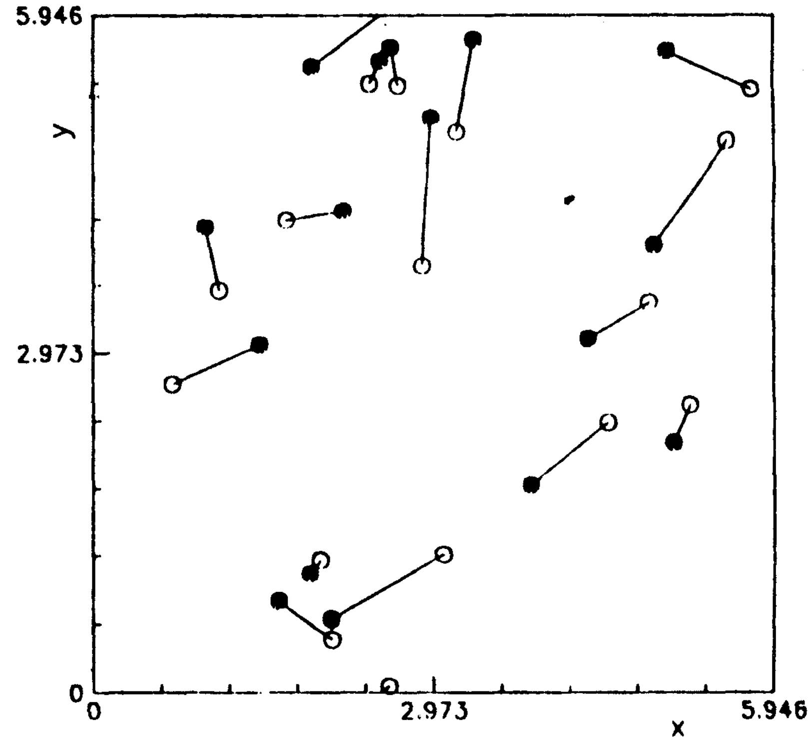


Fig. 1a

solid phase

$\frac{n}{\Lambda_{PV}^4} = 256$

too large...



$N_f = 3$

$N_c = 3$

$m_u = m_d = m_s = 0$

molecular phase

Fig. 1b

$\frac{n}{\Lambda_{PV}^4} = 0.0256$

too small,

(but happens at $T \gtrsim 200$ MeV ...)

INSTANTON LIQUID

PARTITION FUNCTION

$$Z = \frac{1}{N_+!\,N_-!} \int \prod_i^{N_+ + N_-} \left[dU_i\, d^4 z_i\, \mu(\rho_i)\, d\rho_i \right] \mathrm{Exp}(-S_{int}) \prod_{i=1}^{N_f} \mathrm{DET}(\not{D} + m_f)$$

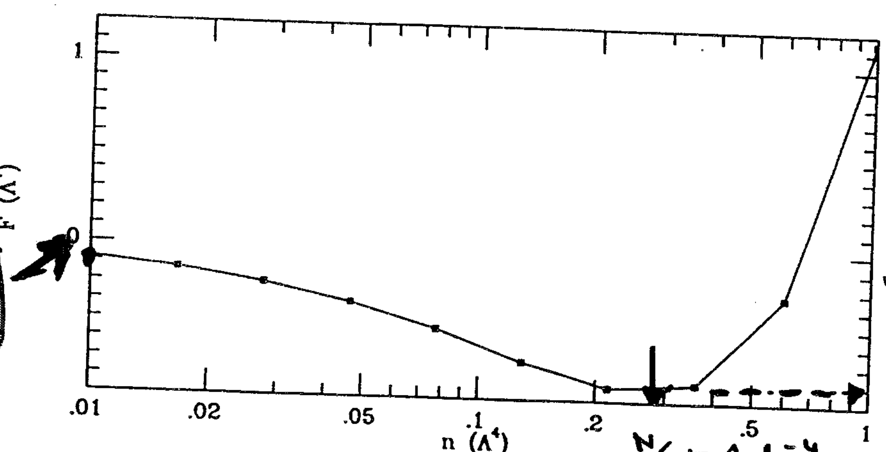
$$\mu(\rho) = C_{N_c} \beta^{2N_c} \rho^{-5} \exp(-\beta)$$

$$\beta(\rho) = -b \log(\rho \Lambda_{QCD})$$

REE ENERGY

$F = -\frac{1}{V_4} \log(Z)$

Perturb. vacuum



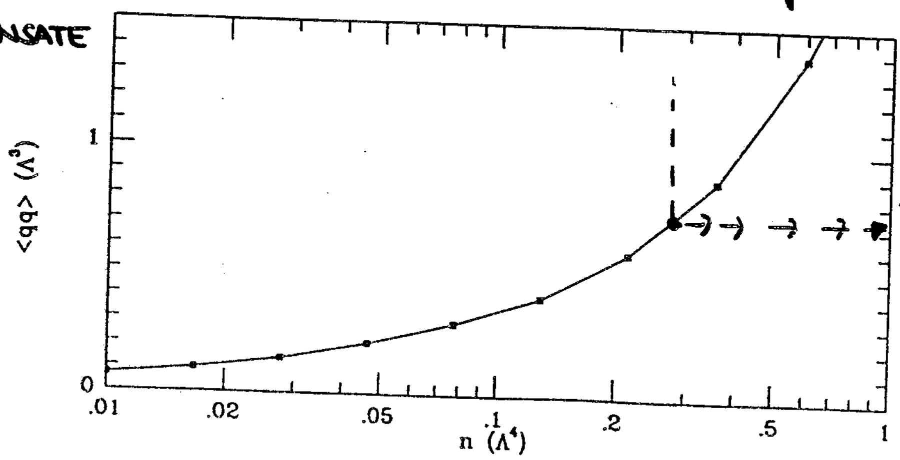
"BAG PRESSURE"

$\boxed{\varepsilon = -465\,\mathrm{MeV}/\mathrm{fm}^3}$

$N/V = 1\ \mathrm{fm}^{-4}$

tunneling shifts the ground state **down**, as usual

UARK CONDENSATE



QUARK CONDENSATE

$\boxed{\langle \bar{q}q \rangle = -(240\,\mathrm{MeV})^3}$

PURE GAUGE, RATIO ANSATZ

Lecture 1: Conclusions

- The so called "chiral scale" $\Lambda_\chi \sim 1\ GeV$ separates pQCD and effective theories: we will try to describe both
- Tunneling between topologically non-equivalent classical vacua is described by instantons
- Instantons form a relatively dilute ensemble: $(\rho/R)^4 \sim (1/3)^4$
- Fermionic zero modes and chiral anomaly are explained by "infinite hotel story": the level movement during tunneling
- New interaction - the 't Hooft Lagrangian, with $2N_f$ legs, provides explicit U(1) Chiral symmetry breaking
- However SU(N_f) Chiral symmetry breaking is more complicated: it is spontaneous one, which exists only in thermodynamic limit and only as multi-instanton effect

E. Shuryak

Lecture 2

Correlators and Hadronic Structure

Recommended Review

E. Shuryak RMP 65 (1993) 1

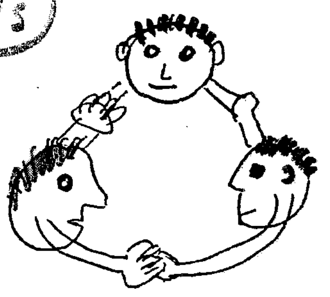
Hadronic Structure and the QCD correlation functions.

- Correlators as a bridge between hadronic and partonic worlds
- Example: vectors and axial correlators
- Other mesonic channels
- Baryonic correlators and Diquarks.
- Hadronic structure and the lowest Dirac eigenvectors

Few Old Models

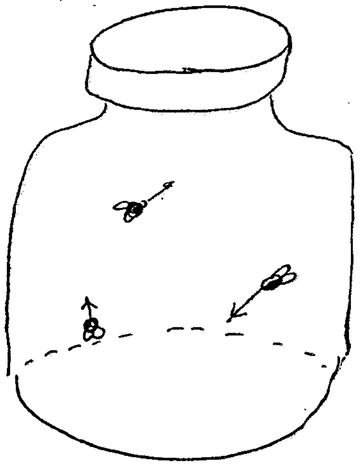
(a)

60's



Nonrelativistic Quarks

(B)



70's

MIT Bag

80's

(c)

Skyrmion

(d)

Chiral Bag

emphasize quite different physics ...

(Vax & Larkin) 1961

I have to mention also Nambu-Jona-Lasinio (NJL) model:

QCD vacuum $\Longrightarrow$ superconductor

Because of (BCS-type) interaction

$$\mathcal{L} = G(\bar{\psi}\Gamma\psi)^2$$

(Does it exist in QCD?)

Hadronic properties/models of light and heavy quark hadrons are quite different

- – Useful approximations are opposite: for u,d s it is the chiral limit $m \to 0$, for c,b,t it is the heavy quark symmetry limit, with $m \to \infty$ (ES 82, Isgur, Wise 86 ...)

Why heavy quark theory does not need chir. sym. breaking / insto ?

- – Heavy quark hadrons are remarkably insensitive to chiral symmetry breaking, pions and sigmas, instantons and all that.
  Example 1. Compare $\psi' \to J/\psi\overbrace{\pi\pi}^{\sigma}$ and $\rho' \to \rho\overbrace{\pi\pi}^{\sigma}$. Same quantum numbers ($\pi\pi$ in 0^{++} or $\underline{\sigma}$ state), about the same energy released. If SU(4) symmetry be true, should have the same width: the difference in fact is about a factor of 1000, 100 MeV vs 100 keV.
  Example 2.Instantons dominate light quark physics, but (their strong fields notwithstanding) they contribute to the static quark potential only few percents of their value, at large distances giving only $\delta M \approx 50MeV$

- – Light quark hadrons are remarkably insensitive to confinement.

Example 1. In spectroscopic calculations with quark model, string tension should be reduced to a fraction of $\sigma = 1$ GeV/fm. Why?
Fig

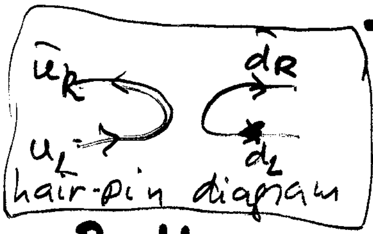
Example 2. Complete correlation functions/wave functions at all distances are calculated in the instanton model, without confinement.

Example 3. Cooling the lattice configuration one is killing confinement, but hadronic correlation functions/wave functions change very little.
→ fig

OZI rules are $\boxed{\text{badly}}$ violated in $0^{\pm}$ channels

OZI rule → ρ, ω are nearly degenerate ⇒ $\boxed{\text{not } \varphi}$

→ η, η' are <u>nearly</u> SU(3) 8 and 1 states



Very <u>small</u> if vector, axial...
Very <u>large</u> if scalar, pseudoscalar

<u>Both</u> point to instanton-induced vertex:

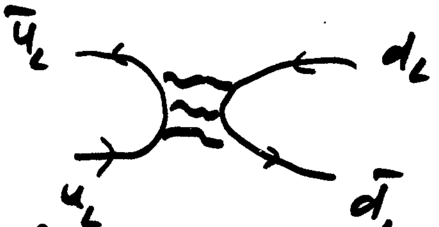
V, A	⟶	0	
π	⟶	−	same magnitude all at small distances $x \sim \rho \sim 1/3$ fm
σ,	⟶	−	
η'	⟶	+	
δ	⟶	+	

σ channel recently checked again on the lattice
Isgur, Thacker hep-lat/0005006 !

Whether PDG does or does not call σ a resonance ⟹ There exist strong attraction <u>there</u>

Not only for $\pi\pi$, but for $\bar{q}q$ at small distances)

- So, NJL were right 40 years ago, $m_\sigma \approx 2 m_{const}$ whatever many models said in 60, 70, 80...
- One cannot get it perturbatively, to <u>any</u> order



Quark chirality <u>is not flipped</u> in pQCD

General idea: why CORRELATORS?

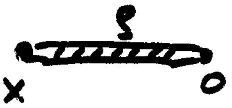
First we remind the general facts:

Let $j_i(x)$ be some operators

The corelation function $K_{ij}(x) \equiv \langle 0 | T[j_i(x)\, j_j(0)] | 0 \rangle$

We discuss only $x^2 < 0$ (space-like, virtual processes)

Two limits:

① $x^2 \to 0$	② $x^2 \to \infty$
quark-gluon language	hadronic language
e.g. $j_\mu = \bar{\psi} \gamma_\mu \psi$ electromagnetic current	
	$\langle j_\mu(x)\, j_\mu(0) \rangle \sim f_\rho^2 \exp(-m_\rho \lvert x \rvert)$ + (excited states)
$\langle j_\mu(x)\, j_\mu(0) \rangle = \dfrac{\text{const}}{x^6} +$ + corrections	

People say: ① is trivial (consequence of asympt. freedom)

② is interesting $\to f_h, m_h \ldots$

This is much studied on the lattice

But: A lot of interesting things are in between...

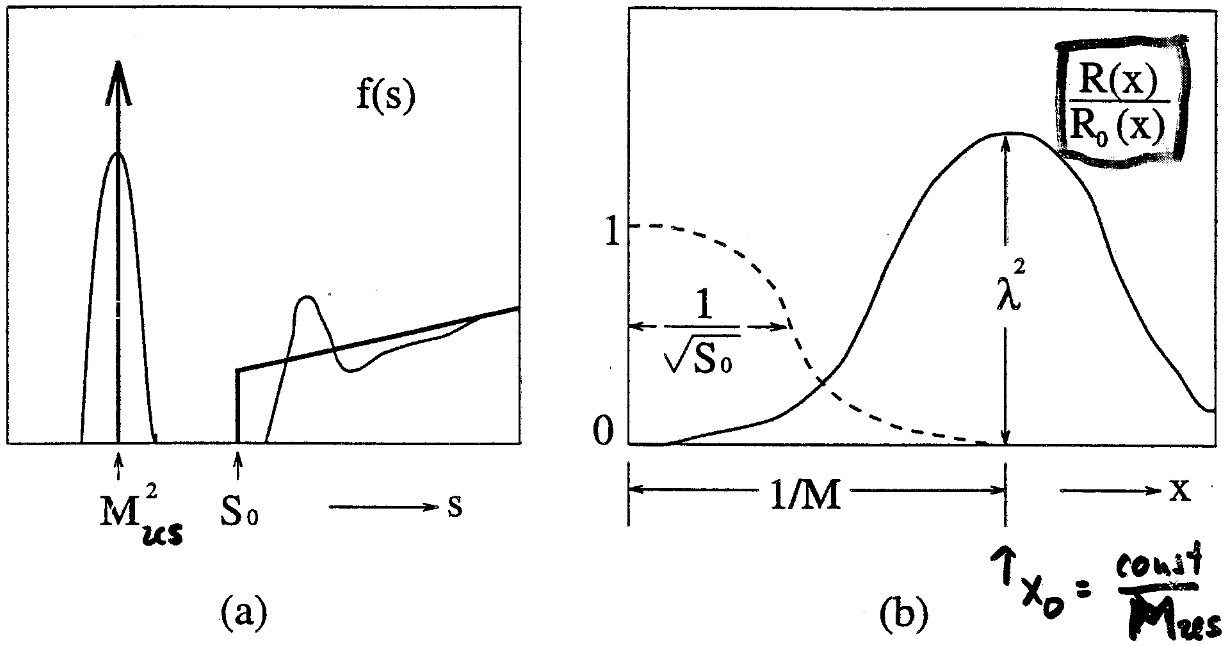
$x \simeq \left(\frac{1}{3} - 1\right) fm$

Important information about $\bar{q}q$, qqq interaction!

(analog of $\delta_\ell(k)$ for nuclear forces)

$\underline{R(x)} : \langle\ \bar\Psi\Gamma\Psi(x)\ \ \bar\Psi\Gamma\Psi(0)\ \rangle$

Energy spectral density $\Rightarrow$ Coordinate represent. of the correlator



(a) (b)

$\boxed{\text{3 param. fit...}}$

$\underline{R_0(x)} \equiv$  $: \quad \frac{const}{x^6}$

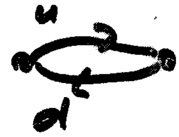
$\boxed{\text{2 currents by dim.}}$

Classification of correlation functions

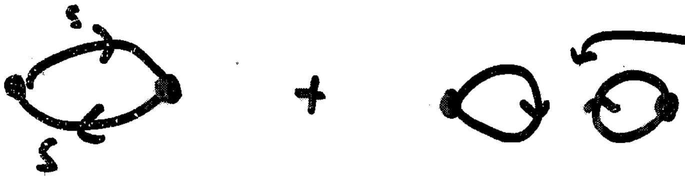
Flavored versus **non-flavored** ones

e.g. $j: \bar{u}\Gamma d$ is '**flavored**'

$(\bar{d}\Gamma u)^+ = \bar{u}\Gamma d \qquad \Gamma = 1, i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$

j  j^+ is the only diagram

but e.g. strange currents $\bar{s}\Gamma s$ are '**unflavored**



momentum can be tranfered by the glue ...

if $\Gamma = 1$ (scalar case) this goes to $\langle\bar{s}s\rangle^2$ at $|x-y| \to \infty$

Diagonal versus **non-diagonal** ones

e.g. $K_{scalar} = \langle T\, \bar{u}d(x)\, \bar{d}u(0)\rangle = \sum |\langle 0|j|n\rangle|^2 e^{-E_n|x|}$

diagonal

positive and monotonously decaying **in average**

$K^\mu_{PS-A} = \langle T\, \bar{u}\gamma_5 d(x)\, \bar{d}\gamma_\mu\gamma_5 u(0)\rangle = \sum_n \langle 0|j_5|n\rangle\langle n|j_{\mu 5}|0\rangle e^{-E_n x}$

non-diagonal $\to$

not conjugate, so any sign can be!

"Kinematics" of correlation functions

One can classify correlation functions considering quark paths, recognising two different types of diagrams:
(i)the one-loop ones,  e.g. $\langle \bar{u}\Gamma d \cdot \bar{d}\Gamma u \rangle$
and (ii) the two-loop diagrams  eg. $\langle \bar{u}\Gamma u \; \bar{d}\Gamma d$

The main portion of lattice work deal with one-loop diagrams, and therefore with the I=1 channels (the reason is technical, and can actually be overcomed). For those one can make some general statements.

First of all, following Weingarten,one may use the following relation for the propagator in backward direction

$$S(x,y) = -\gamma_5 S^+(y,x)\gamma_5$$

Second, one can decompose it into Dirac matrices

$$S = \Sigma a_i \Gamma_i$$

a_1 and a_μ are 'visible'
others are 'hidden'

where

$$\Gamma_i = 1, \gamma_5, \gamma_\mu, i\gamma_5\gamma_m u, i\gamma_\mu\gamma_n u(\mu \neq \nu)$$

Third step: one can consider all diagonal one-loop correlators of the type

$$\Pi = Tr(S(x,y)\Gamma_i S(y,x)\Gamma_i)$$

, and perform the traces.

For pseudoscalar (pion) correlator one has a sum of all coefficients squared:

$$\Pi_{PS}/\Pi_{PS}^{free} = (|a_1|^2 + |a_5|^2 + |a_\mu|^2 + |a_{\mu 5}|^2 + |a_{\mu\nu}|^2)/|a_0|^2$$

(all +)!
[grows strongly]

while e.g. the scalar one is

$$\Pi_S/\Pi_S^{free} = (-|a_1|^2 - |a_5|^2 + |a_\mu|^2 + |a_{\mu 5}|^2 - |a_{\mu\nu}|^2)/|a_0|^2$$

few minuses
→ and decay!

$$\boxed{\frac{\Pi^{PS}}{\Pi_{free}} > \frac{\Pi^{S} + \Pi^{T}_{\mu\nu\ldots}}{\Pi_{free}}} \Rightarrow m_\pi < m_\delta, m_\rho, m_A$$

As a result, Weingerten inequality follows: **the pseudoscalar correlator should exceed the scalar one at all distances.**

We did more → PS is lighter than S, V, A

The non-trivial thing is that physical pion is very light, while scalars are heavy, and therefore for $x > .5$ fm the scalar correlator is practically zero. It means **there is a very delicate cancellation** between different components of the propagator!

Similar relations for vector (ρ) and axial (A_1) channels:

$$\Pi_V/\Pi_V^{free} = (2|a_1|^2 - 2|a_5|^2 + |a_\mu|^2 - |a_{\mu 5}|^2)/|a_0|^2$$

$$\Pi_A/\Pi_A^{free} = (-2|a_1|^2 + 2|a_5|^2 + |a_\mu|^2 - |a_{\mu 5}|^2)/|a_0|^2$$

and Verbaarschot inequalities follow:

$$\Pi_{PS}/\Pi_{PS}^{free} > (1/2)(\Pi_V/\Pi_V^{free} + \Pi_A/\Pi_A^{free})$$

$$\Pi_{PS}/\Pi_{PS}^{free} > (1/4)(\Pi_V/\Pi_V^{free} - \Pi_A/\Pi_A^{free})$$

(Witten has found another interesting inequality between vector and axial correlators, but this holds in momentum representation, and therefore we do not discuss it here.)

As these inequalities are identities, they are satisfied for any configuration of the gauge field, and therefore theoretically are not very restrictive. However, they can be used to check consistency of experimental data, as discussed below.

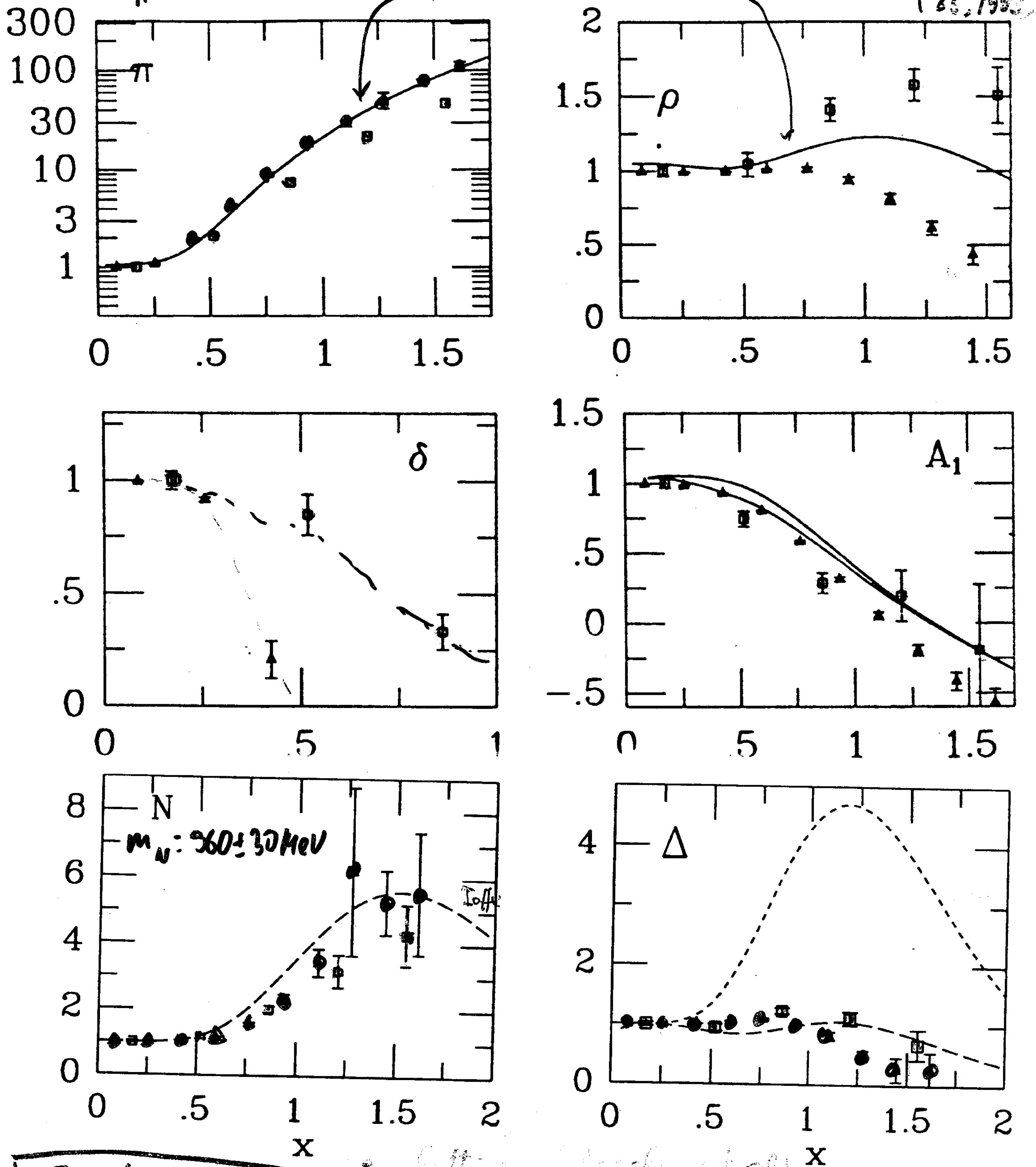
On the other hand, the (diagonal) correlators themselves are positive **monotonously decreasing functions**, as is clear from the spectral decomposition.

It is trivial experimentally, but produce the non-trivial limitations for the ensemble of vacuum fields. Some configurations do produce *negative* correlators, especially the scalar ones: as a result, their weight in the ensemble of vacuum fields should not be too large. !

Instanton vacuum cannot be too dilute!

$m_\pi = 142 \pm 12$ MeV

lines are from experimental data (Shuryak Rev. Mod. P. 65, 1993)



random instanton liquid model → ▲ - RILM (Shuryak, Verbaarschot)
Shaefer → Nucl. Phys. B. 93

$\downarrow \rho \quad \downarrow a_1 \quad \downarrow \pi$

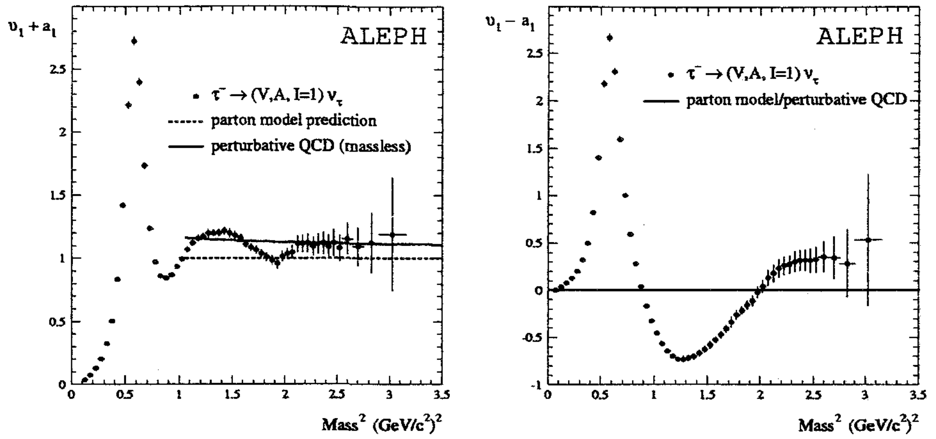


Figure 1: Spectral functions $v(s) \pm a(s) = 4\pi^2(\rho_V(s) + \rho_A(s))$ extracted by the ALEPH collaboration.

Recent example of V,A from τ decays

(Data - ALEPH, Theory - T.Schafer, ES, 2000)

The correlation functions are calculated from the spectral representation

$$\Pi_{V,A}(x) = \int ds\, \rho_{V,A}(s) D(\sqrt{s}, x)$$

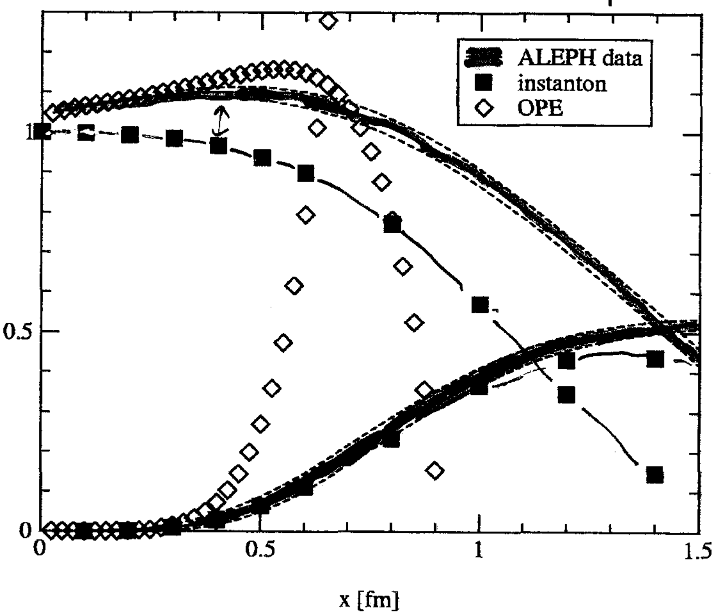
where $D(m,x) = m/(4\pi^2 x) K_1(mx)$ is the Euclidean coordinate space propagator of a scalar particle with mass m. The l.h.s. was calculated in the random ensemble of instantons with standard n, ρ. The agreement is stunning: it is there for ALL distances (including 10% pert.correction $1+\alpha_s/\pi$.). One can see how the charge runs...

↓ CORRELATORS Themselves change by ~4 orders ↓ of magnitude

$\bigcirc = \frac{\#}{x^6}$

V+A

$\frac{\Pi}{\bigcirc}$ Free propagation



← our m_π is somewhat too large → m_q was ~20 MeV (finite size)

V-A

Figure 2: Euclidean coordinate space correlation functions $\Pi_V(x) \pm \Pi_A(x)$ normalized to free field behavior. The solid lines show the correlation functions reconstructed from the ALEPH spectral functions and the dotted lines are the corresponding error band. The squares show the result of a random instanton liquid model and the diamonds the OPE fit described in the text.

→ The model (with $n = 1\,fm^{-4}$, $\rho = 1/3\,fm$ fixed in 1982) works with few percent accuracy

→ No sign of repulsive $\frac{\langle \bar{q} q \rangle^2}{r^2}$?

(Unlike π etc) where attraction at small x is strong

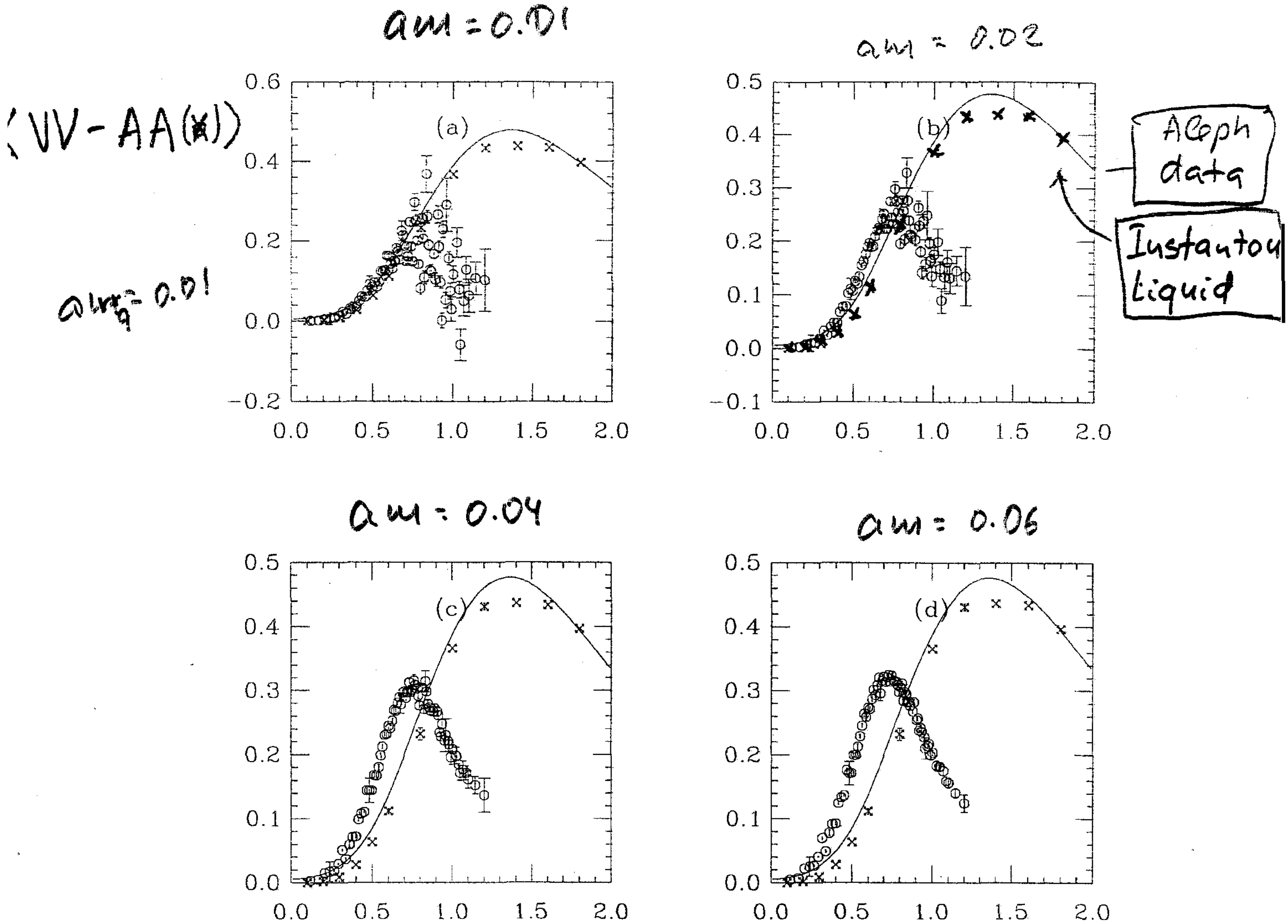


FIG. 10. Comparison of the difference point-to-point vector and axial vector correlators from the overlap action (octagons) and with the instanton model of Ref. [39] (fancy crosses) and ALEPH $\tau-$lepton decay, as extracted by Ref. [39] (lines). (a) $am_q = 0.01$ ($\pi/\rho \simeq 0.34$); (b) $am_q = 0.02$ ($\pi/\rho \simeq 0.50$); (c) $am_q = 0.04$ ($\pi/\rho \simeq 0.61$); (d) $am_q = 0.06$ ($\pi/\rho \simeq 0.64$).

T. DeGrand et al
Lattice results for different quark masses

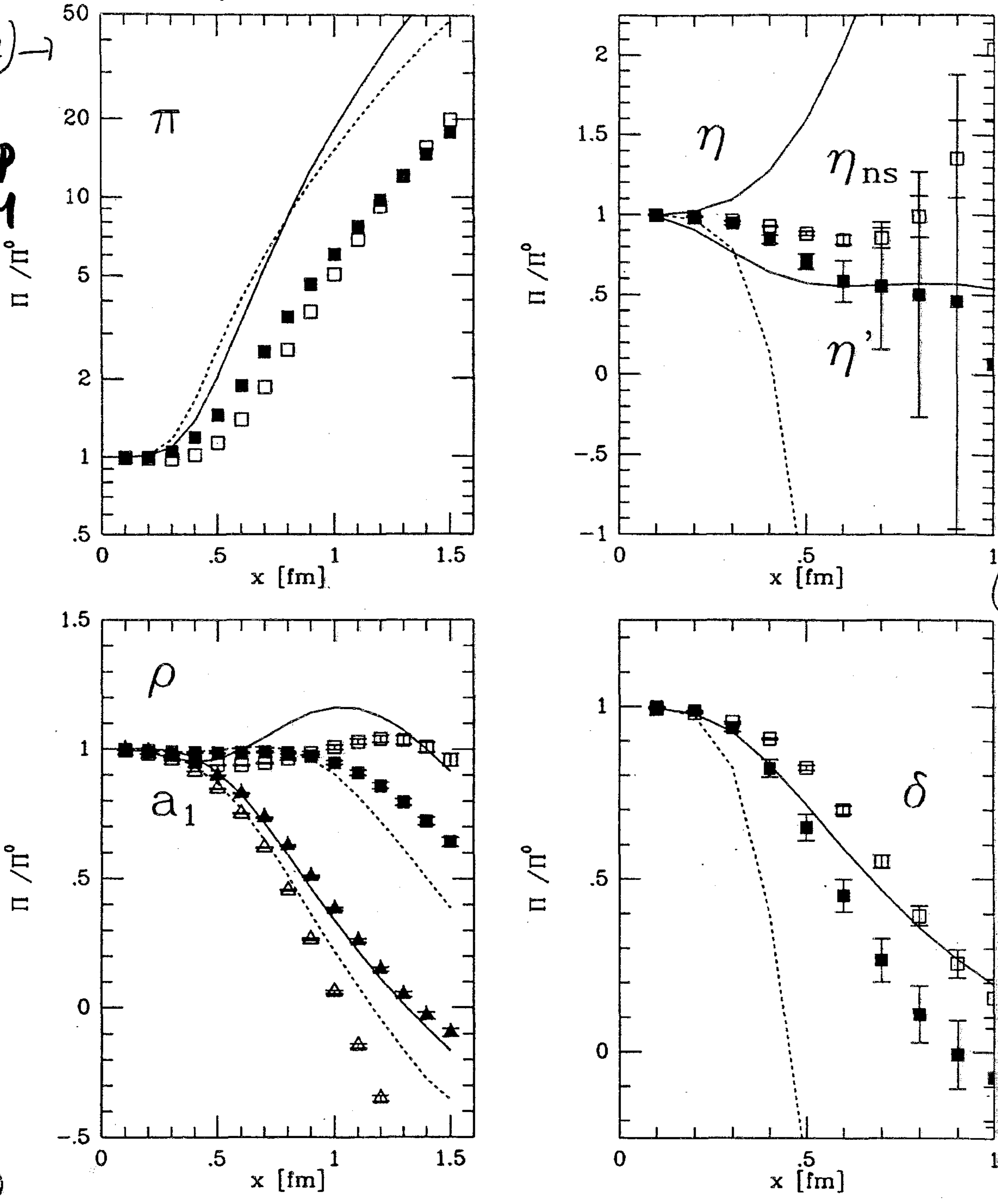
Interacting Instantons

attraction | **Repulsion**

attractive →

— = exp

... RILM



repulsive

$= f_0$ in modern notation.

$\boxed{\begin{matrix} I=1 \\ S=0 \end{matrix}}$

neutral →

Solid lines → "experiment" | Dashed lines - random liquid

FIG. 1.

Two variants of the gluonic interaction

□ open → "ratio ansatz"

■ closed → "streamline + core"

effects of $\bar{I}I$ correlations are seen, mostly δ, η_{ns}

		streamline	quenched = Random	ratio ansatz	RILM
m_π (measured)	[GeV]	0.265	0.268	0.128	0.284
m_π (extr.)	[GeV]	0.117	0.126	0.067	0.155
λ_π	[GeV^2]	0.214	0.268	0.156	0.369
f_π	[GeV]	0.071	0.091	0.183	0.091
m_ρ	[GeV]	0.795	0.951	0.654	1.000
g_ρ		6.491	6.006	5.827	6.130
m_{a_1}	[GeV]	1.265	1.479	1.624	1.353
g_{a_1}		7.582	6.908	6.668	7.816
m_σ	[GeV]	0.579	0.631	0.450	0.865
m_δ	[GeV]	2.049	3.353	1.110	4.032
$m_{\eta_{ns}}$	[GeV]	1.570	3.195	0.520	3.683

correct quark masses

TABLE III. Meson parameters in the different instanton ensembles. All quantities are given in units of GeV. The current quark mass is $m_u = m_d = 0.1\Lambda$. Except for the pion mass, no attempt has been made to extrapolate the parameters to physical values of the quark mass.

		streamline	quenched	ratio ansatz	RILM
m_N	[GeV]	1.019	1.013	0.983	1.040
λ_N^1	[GeV^3]	0.026	0.029	0.021	0.037
λ_N^2	[GeV^3]	0.061	0.074	0.048	0.093
m_Δ	[GeV]	1.428	1.628	1.372	1.584
λ_Δ	[GeV^3]	0.027	0.040	0.026	0.036

TABLE IV. Baryon parameters in the different instanton ensembles. All quantities are given in units of GeV. The current quark mass is $m_u = m_d = 0.1\Lambda$.

N-Δ splitting comes naturally!

TABLES

	streamline	quenched	ratio ansatz	RILM
n	$0.174\Lambda^4$	$0.303\Lambda^4$	$0.659\Lambda^4$	$1.0\ \mathrm{fm}^4$
$\bar\rho$	$0.64\Lambda^{-1}$	$0.58\Lambda^{-1}$	$0.66\Lambda^{-1}$	0.33 fm
	(0.42 fm)	(0.43 fm)	(0.59 fm)	
$\bar\rho^4 n$	0.029	0.034	0.125	0.012
$<\bar q q>$	$0.359\Lambda^3$	$0.825\Lambda^3$	$0.882\Lambda^3$	$(264\,\mathrm{MeV})^3$
	$(219\,\mathrm{MeV})^3$	$(253\,\mathrm{MeV})^3$	$(213\,\mathrm{MeV})^3$	
Λ	306 MeV	270 MeV	222 MeV	-

TABLE I. Bulk parameters of the different instanton ensembles.

channel	current	matrix element	experimental value
π	$j_\pi^a = \bar q\gamma_5\tau^a q$	$<0\vert j_\pi^a\vert\pi^b> = \delta^{ab}\lambda_\pi$	$\lambda_\pi \simeq (480\,\mathrm{MeV})^2$
	$j_{\mu 5}^a = \bar q\gamma_\mu\gamma_5\frac{\tau^a}{2}q$	$<0\vert j_{\mu 5}^a\vert\pi^b> = \delta^{ab}q_\mu f_\pi$	$f_\pi = 93$ MeV
δ	$j_\delta^a = \bar q\tau^a q$	$<0\vert j_\delta^a\vert\delta^b> = \delta^{ab}\lambda_\delta$	
σ	$j_\sigma = \bar q q$	$<0\vert j_\sigma\vert\sigma> = \lambda_\sigma$	
η_{ns}	$j_{\eta_{ns}} = \bar q\gamma_5 q$	$<0\vert j_{\eta_{ns}}\vert\eta_{ns}> = \lambda_{\eta_{ns}}$	
ρ	$j_\mu^a = \bar q\gamma_\mu\frac{\tau^a}{2}q$	$<0\vert j_\mu^a\vert\rho^b> = \delta^{ab}\epsilon_\mu\frac{m_\rho^2}{g_\rho}$	$g_\rho = 5.3$
a_1	$j_{\mu 5}^a = \bar q\gamma_\mu\gamma_5\frac{\tau^a}{2}q$	$<0\vert j_{\mu 5}^a\vert a_1^b> = \delta^{ab}\epsilon_\mu\frac{m_{a_1}^2}{g_{a_1}}$	$g_{a_1} = 9.1$
N	$\eta_1 = \epsilon^{abc}(u^a C\gamma_\mu u^b)\gamma_5\gamma_\mu d^c$	$<0\vert\eta_1\vert N(p,s)> = \lambda_1^N u(p,s)$	
N	$\eta_2 = \epsilon^{abc}(u^a C\sigma_{\mu\nu} u^b)\gamma_5\sigma_{\mu\nu} d^c$	$<0\vert\eta_2\vert N(p,s)> = \lambda_2^N u(p,s)$	
Δ	$\eta_\mu = \epsilon^{abc}(u^a C\gamma_\mu u^b)u^c$	$<0\vert\eta_\mu\vert N(p,s)> = \lambda^\Delta u_\mu(p,s)$	

TABLE II. Definition of various currents and matrix elements used in this work.

The diquark issue:

Are scalar-isoscalar (ud) pairs deeply bound, even without confining strings?

- The "small N_c" point of view

At $N_c=2$ diquarks are colorless hadrons

What do we know about their spectrum?

New symmetry exist in $N_c=2$ theory: $\psi \to \bar{\psi}$ uniting $\bar{q}q$ (mesons) and qq (diquark) (Pauli-Gürsey) into common multiplet

Goldstones (for u,d only) $\to$ 3π, $\underbrace{\text{scalar } ud \text{ (and its anti)}}_{\text{massless}}$ are very deeply bound

Vectors (ρ + vector diquarks) $m \approx 2 m_{const}$ not deeply bound

- Both perturb. and instanton forces have the same ratio

$$\frac{(qq)}{(\bar{q}q)} = \frac{1}{N_c-1} \to \begin{cases} -1 & N_c=0 \text{ QED} \\ 1 & N_c=2 \\ \boxed{1/2} & N_c=3 \\ 0 & N_c \to \infty \end{cases}$$

So, real world is exactly in between "small" and "large" N_c $\to$ $\boxed{1/2}$

Half Binding?

- Phenom. $\Sigma_c - \Lambda_c \simeq 170$ MeV | But we need detailed theory...

$\Delta - N \simeq 300$ MeV etc

Nucl. Phys. B412 (94) 143

attractive channel

(ud) scalar

vector

$I=1, S=1$

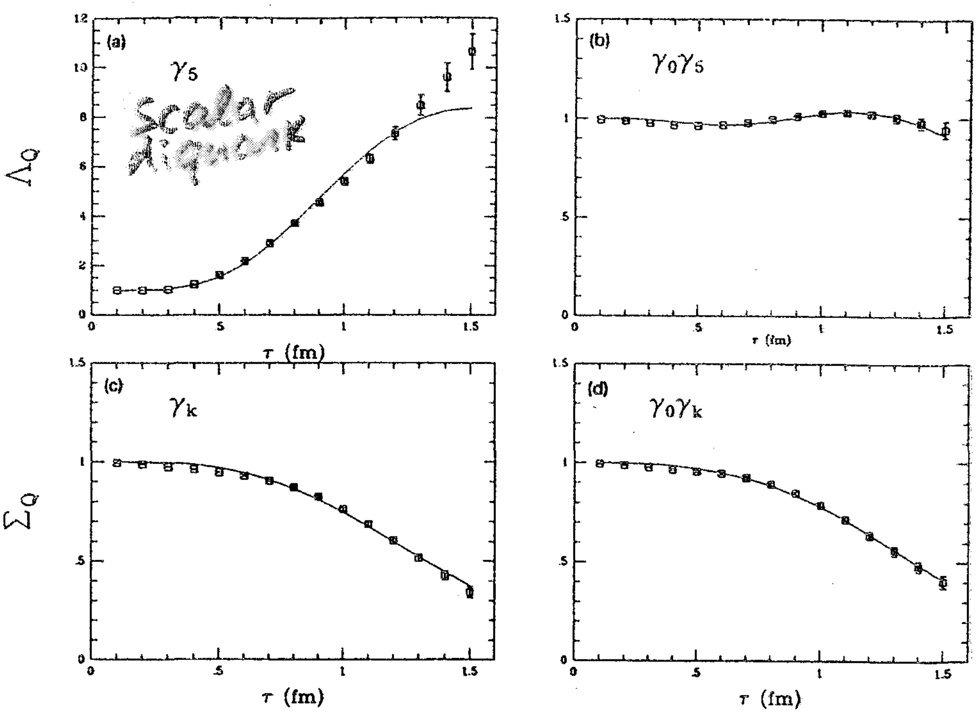


Fig. 2. Correlation functions for Λ-type diquarks (a-b) and Σ-type diquarks (c-d). The diquark channels are labeled by the Γ matrix defining the non relativistic current. The solid lines correspond to the heavy-light parametrization discussed in section 3.

Diquarks in the instanton model

TABLE 1

Numerical results from fitting the diquark correlation functions in the RILM with a "diquark resonance plus continuum" model. The parameters are defined in section 2 of the text.

	This work	Other information	Comment
m_S	420 ± 30 MeV	234 MeV	NJL model [27]
$m_{A,V}$	940 ± 20 MeV	824 MeV	NJL model [27]
m_T	570 ± 20 MeV		
g_S	0.225 ± 0.011 GeV^2	0.135 ± 0.025 GeV^2	QCD sum rules [28]
$g_{A,V}$	0.244 ± 0.010 GeV^2		
g_T	0.134 ± 0.004 GeV^2		

doing (diquark + heavy) somewhat reduce the effect

$m(\Sigma_c) - m(\Lambda_c) \simeq 200$ MeV only

Where are the Non-instanton effects?

●: uncooled, ○: cooled

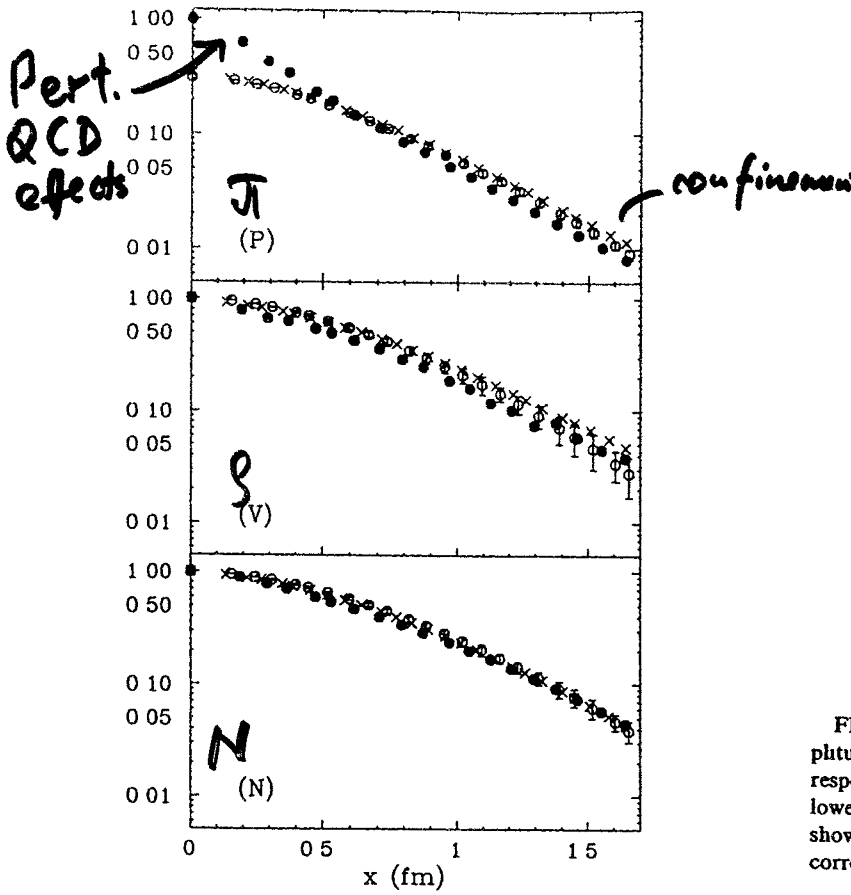


FIG 10 Comparison of uncooled and cooled density-density correlation functions for the pion, ρ, and nucleon. The solid circles denote the correlation functions calculated with uncooled QCD, the open circles with error bars show the results for 25 cooling steps, and the crosses denote the results for 50 cooling steps. The ρ and pion results are compared for $M_\pi^2 = 0.16$ GeV^2 and the nucleon results are compared for $M_\pi^2 = .36$ GeV^2. As in Figs 8 and 9, the separation is shown in physical units using values of a from Table I. All correlation functions are normalized to 1 at the origin, except for the cooled pion correlation functions, which are normalized to have the same volume integral as the uncooled pion result. Errors for the uncooled results and for 50 steps, which have been suppressed for clarity, are comparable to those shown for 25 steps.

Negele et al. 93

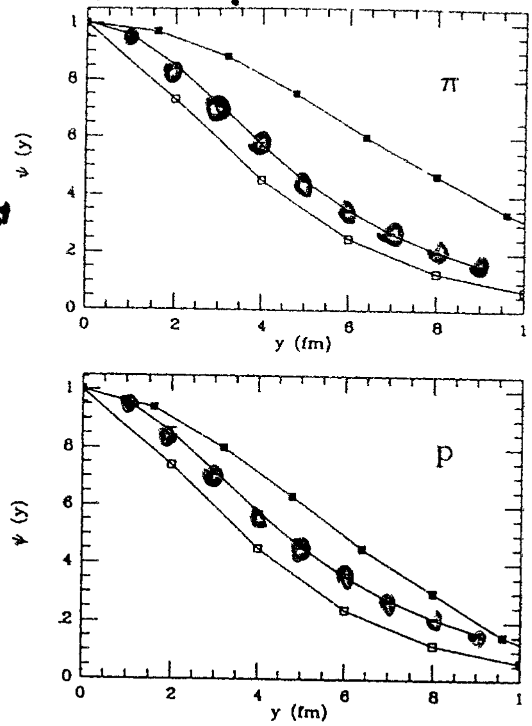


FIG 4 Comparison of pion and proton Bethe-Salpeter amplitudes calculated in the random instanton model with the corresponding results of lattice calculations reported in [3,4]. The lowest curve is the full lattice result, the curve in the middle shows the result in the instanton model, and the upper curve corresponds to the lattice result after cooling.

Schäfer + E.S.

○ = RILM

- Amazing prediction of the Instanton model

Tiny fraction of the Dirac spectrum is <u>sufficient to describe light hadrons</u>

$$\not{D}\psi_\lambda = \lambda\psi_\lambda$$

"Collectivized" zero modes have "20 MeV" scale mentioned

It looks like momentum, $(\not{\partial} e^{ipx} = \hat{p} e^{ipx})$ which is true for free fermions but very different for those in the QCD vacuum

$p \sim \frac{1}{\rho} \sim 600$ MeV: for partons OK

$|\psi_\lambda|^2$

$\frac{1}{(x^2+\rho^2)^3}$ well localized yet $\lambda \approx 0$

(Like wave function in liquid metal)

<u>Lattice studies</u>

(Ivanenko, Negele 97
A. Hasenfratz, T. DeGrand 2000)

$$S(x,y) = \sum_\lambda \frac{\psi_\lambda(x)\,\psi_\lambda^+(y)}{\lambda + im}$$

Let us split the sum to 2 parts $|\lambda| \lessgtr \boxed{\lambda_0}$ param.
Keep only the 1st (small λ) and see what happens

→ only ~ 20-40 modes are enough (out of 10^5-10^6 on the lattice)

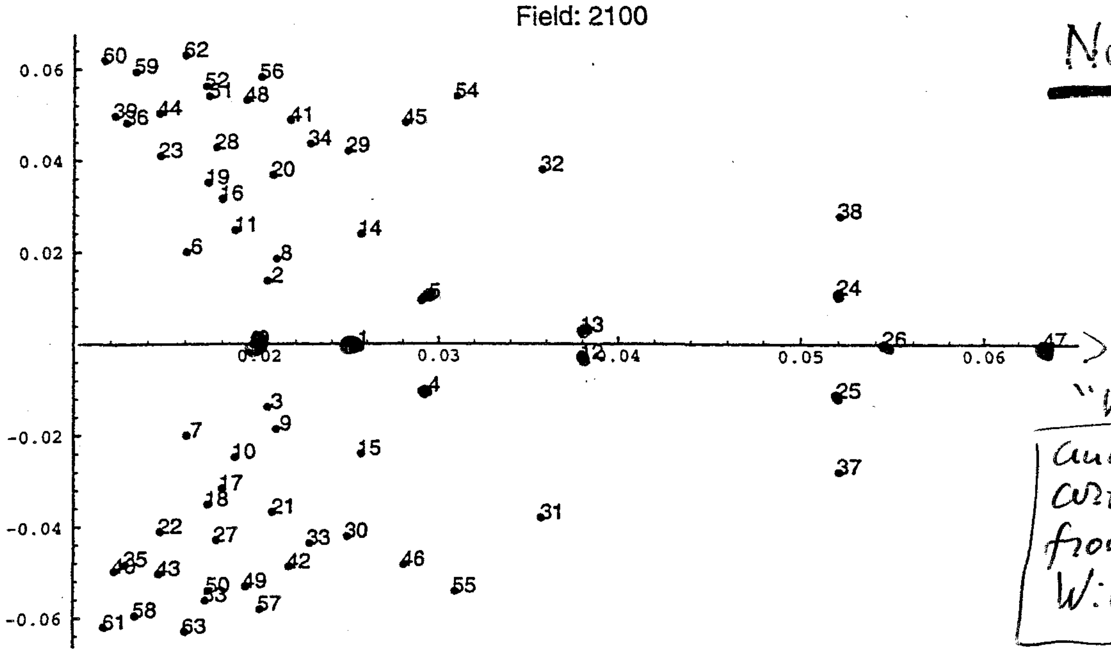
→ one can find instantons <u>without</u> cooling

$$\sum_{\lambda<\lambda_0} |\psi_\lambda(x)|^2 \Rightarrow$$ 

T. Ivanenko thesis
(MIT)

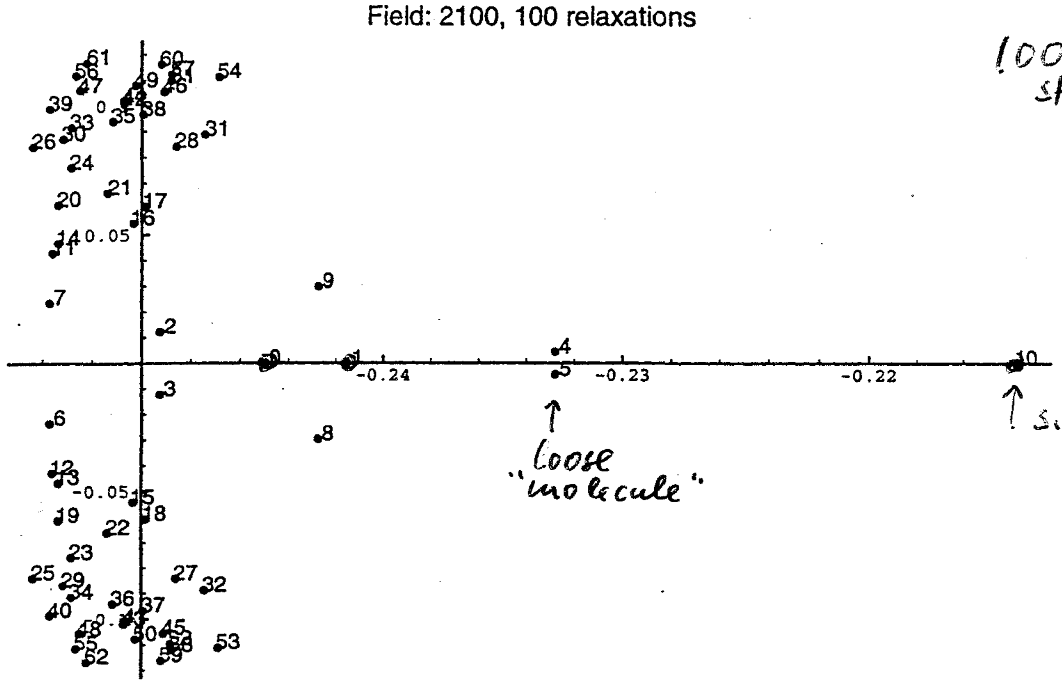
No cooling

λ ↑



"m"
and lattice artefacts from the Wilson term

100 cooling steps



↑ "loose molecule"

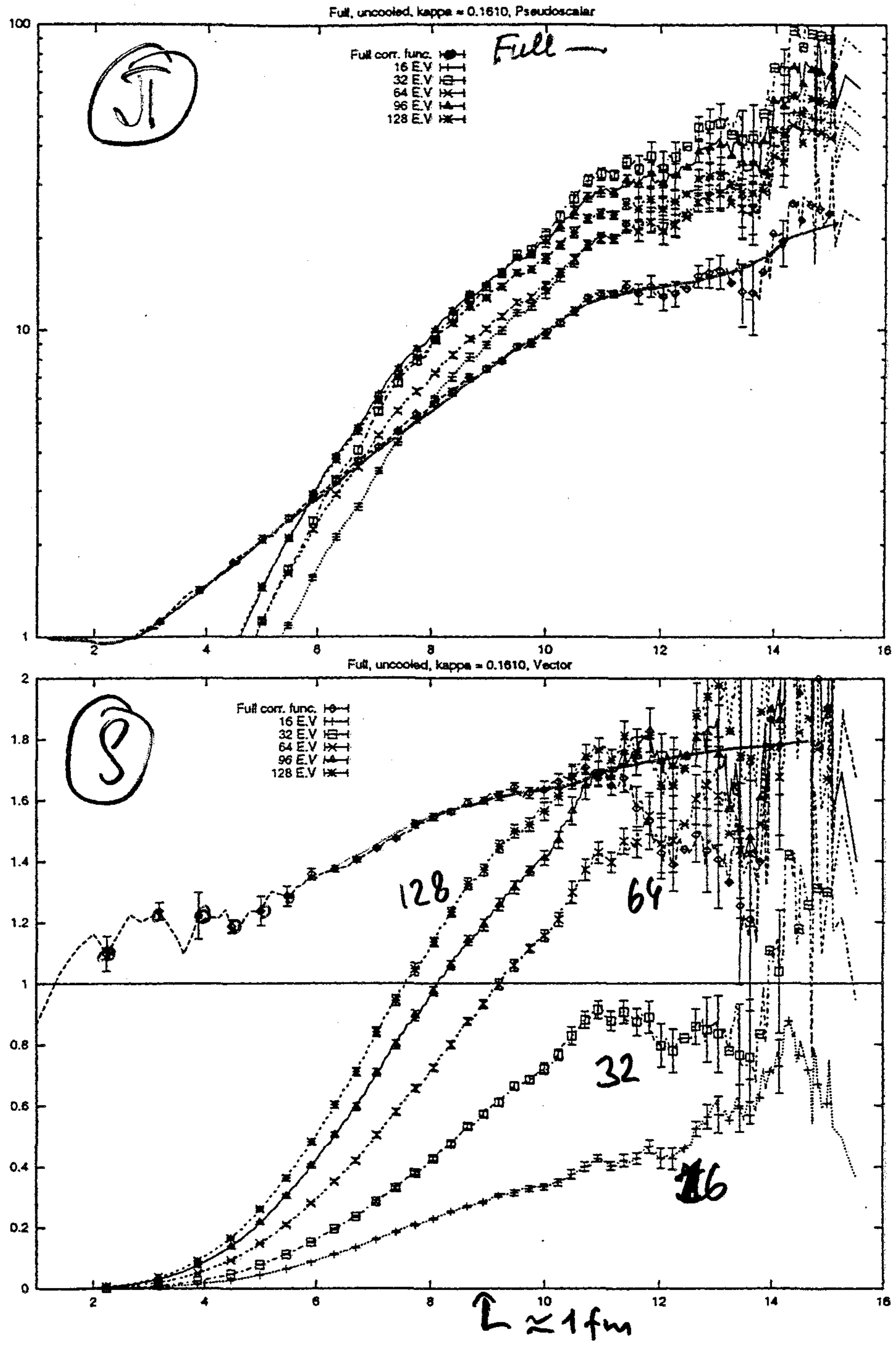
↑ single

Figure 5-8: The lowest 64 modes of the Dirac operator on one selected lattice after 0 and 100 relaxation steps. $\kappa = 0.1600$ on both graphs and large negative values on the lower graph indicated that κ_c for this configuration is much lower (≈ 0.125).

$$S = \sum_\lambda \frac{\psi_\lambda(x)\,\psi_\lambda^*(y)}{\lambda + i m}$$ using only <u>lowest</u> modes

Note: log plot → $\frac{\Pi(x)}{\Pi_{free}(x)}$

Total # of modes $\sim 10^6$ only $16 \div 128$ lowest are used



← correct magnitude

Figure 5-18: Normalized correlation functions for <u>full QCD configuration</u>, $\kappa_v = 0.1610$, $m_\pi a = 0.2823(4)$, $m_q a = 0.0351$

$\simeq 500$ MeV

$a \simeq 0.11$ fm

Ivanenko, Negele 97

53 cap 2

Conclusions

- – The non-perturbative QCD bounds the applicability of pQCD at rather large scale, ranging from about 1 GeV in many cases, to 3-4 GeV in the glueball world. Still, in some cases (V+A) all non-perturbative effects cancel to few percent level... Also Twist 3 in DIS.
- – This scale determines surprisingly small size of constituent quarks and flux tubes, the main element of our main tool, THE QUARK MODEL.
- – Understanding of what exactly happens at this scale is the top priority issue. JLAB+ and new lattice projects are going to address it.
- – The best model which works quantitatively for light quark problems is the instanton liquid model, with many elements verified on the lattice. It also quantitatively explains OZI rule violation, etc.

E. Shuryak

<u>Lecture 3</u>

<u>QCD at finite T / μ_B</u>

QCD at finite T and density

- The phases of QCD at the T-μ Phase Diagram
- Color Superconductivity, forces etc
- Color-Flavor locking phase, new pions etc
- Asimptotically large densities and pQCD superconductivity
- More exotic phases (crystalline ones, the pion or kaon condesates)

The QCD Phase Diagram

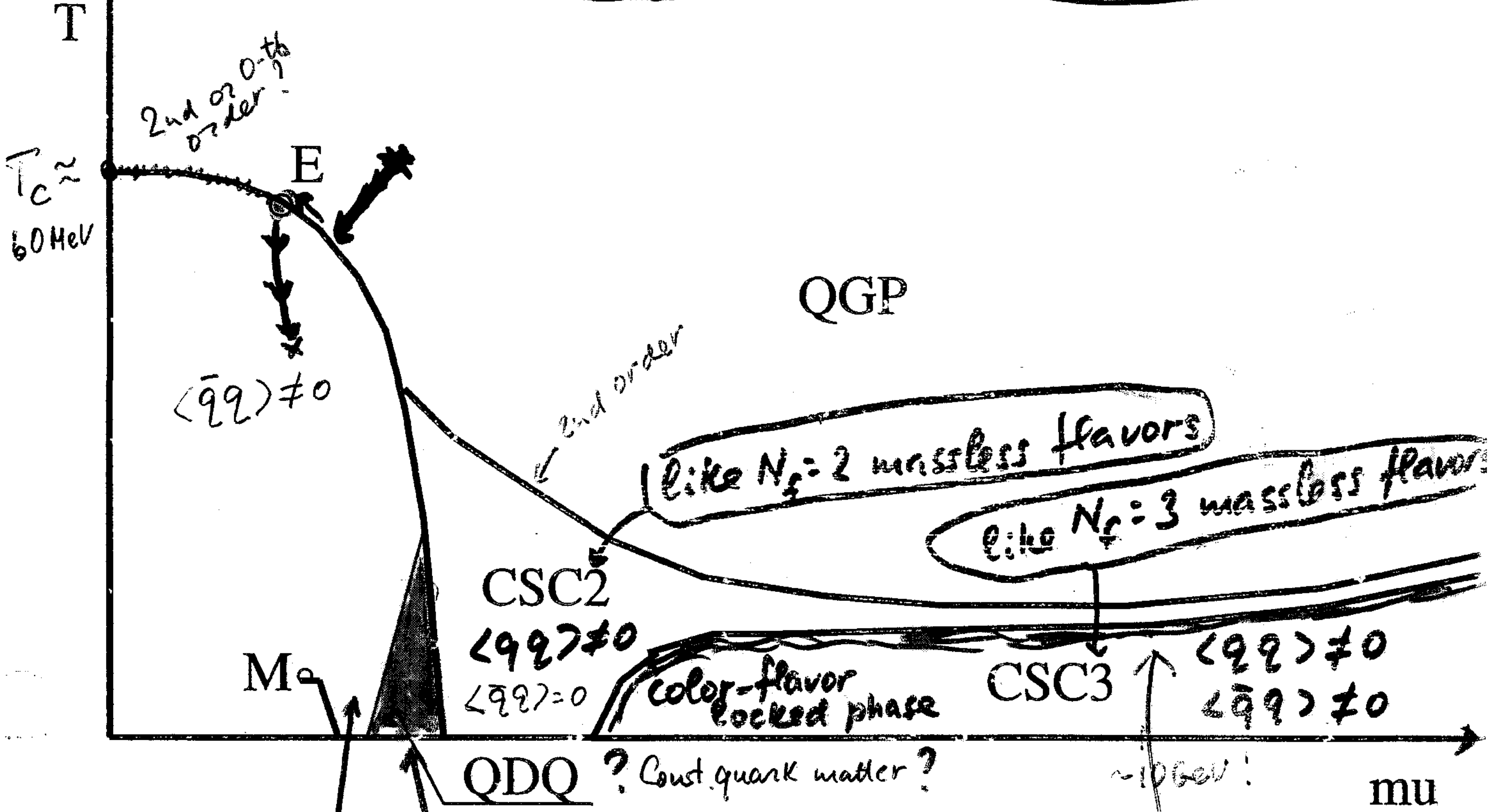


FIG. 1. Schematic QCD phase diagram, on the chemical potential μ - temperature T plane. Small T and mu region corresponds to ordinary hadronic matter, with broken chiral symmetry. The point M (from "multifragmentaion") is the endpoint of nuclear liquid-gas phase transition. The point E is another endpoint, in which the first order line goes second order (for $m_u, m_d = 0$) or disappears for finite light quark masses. CSC2 and CSC3 indicate the $N_f = 2$ and $N_f = 3$-type superconducting phases. The hypothetical intermediate quark-diquark phase is indicated by QDQ.

Nuclear matter

LOFF phase "shell" ?

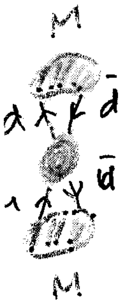
(*) Deconfinement ?

New competitor here: the chyral crystal

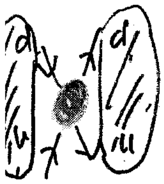
56 cap 3

Instantons and triality

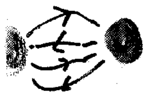
There are three attractive channels, which compete



- instanton-induced attraction in $\bar{q}q$ channel leads to χ-symmetry breaking, also η' mass – $U(1)$ problem



- instanton-induced attraction in qq leads to color superconductivity, It decreases with N_c as $1/(N_c-1)$] same at $N_c=2$, 1/2 in QCD



- light-quark-induced attraction of $\bar{I}I$ leads to pairing of instantons into "molecules". effect increases with N_f → no $\langle\bar{q}q\rangle$ for $N_f > N_f^{crit} \sim 5$!

The first two phases can be approximately described by Mean Field Approx.
→ not the last!

QCD at finite T

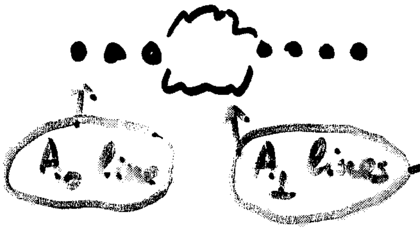
- Quark-Gluon Plasma: screening vs anti-screening
- Lattice results about Phase diagram and the OrhidEquation of State
- Chiral symmetry restoration and $\bar{I}I$ molecules

Quark-Gluon **Plasma** (QGP)

- As. freedom = $\boxed{\text{anti}}$screening of charge
  unlike QED

In the <u>Coulomb</u> gauge (I.B. Khriplovich 68,
it is seen better than in <u>covariant</u> ones

(Politzer, Gross-Wilczek 72)

$\Pi_{00} =$ 

 + (no logs)

()

$\# \left[+ \frac{1}{12} N_c \log \underbrace{q^2} \right]$

covariant $q^2 \to$ so it has imaginary part thus sign is <u>plus</u> (as sign of Im Π dictates)

$\# \left[(-1) N_c \log \vec{q}^{\,2} \right] \to -\frac{11}{3} N_c$

non-cov. $\vec{q}^{\,2}$

There is no physical state and no Im part

- At finite T / μ (and, or) $\Rightarrow$ **Still** <u>screening</u> (ES 1978)
  like QED

$\Rightarrow$ Thus the name

$\Pi_{00}\,(\underline{\omega = 0, q \to 0}) = \# g^2 T^2$ or $\# g^2 \mu_B^2 \to \frac{1}{q^2 + \Pi_{00}}$

→ in matter Lorentz covariance is lost, so result depends on ω/q, even if they $\to 0$.

→ E.g. $\Pi_{00}\,(\omega \ll q \to 0)$ is called "effective mass" of a gluon, it is also $\sim g^2 T^2$

- The "dangerous" diagram does not contribute, the last one does

Brief Summary of the Theory of Quark-Gluon Plasma

- Resummation leads to results not expandable in g^2

  e.g.  cactus-like diagrams give

  $$\delta\Omega \sim \underbrace{g^3}_{\text{"Plasmon" term}} T^4$$

  (Grand therm. potential, Ω) — E.S. 76, J. Kapusta 78

- Consistent resummation of HTL (Hard Thermal Loops with $p \sim T$) needs also vortèces modification

  E. Braaten, R. Pisarski; Wang, Taylor

- Terms in Ω up to g^5 are calculated — P. Arnold ..., E. Braaten

  $\to$ Convergence is bad, only when g is so small that corresponding scale $\mu \sim T \sim 10^6$ GeV!

  $\to$ while lattice results show rapid onset of a trend already at $T \simeq 2 \div 3 T_c \sim \frac{1}{2}$ GeV

- "Improved" resummations using quasiparticles explain these results, with accuracy ~ several % !

  F. Karsch; E. Braaten, Strickland; J.P. Blaizot, E. Iancu

- However, since $\Pi_\perp(\omega=0, q\to 0) \to 0$ ← $\boxed{\text{ES 76}}$

  there is no magnetic screening and long-range magnetic fields form $\boxed{\text{3d YM}}$ theory, with is known to be confining

  But remains to be controled by lattice only

  $\to$ $\delta\Omega \sim g^6 T^4$ is not calculable

F. Karsch 2001 hep-ph/0103314

EoS from Lattice QCD

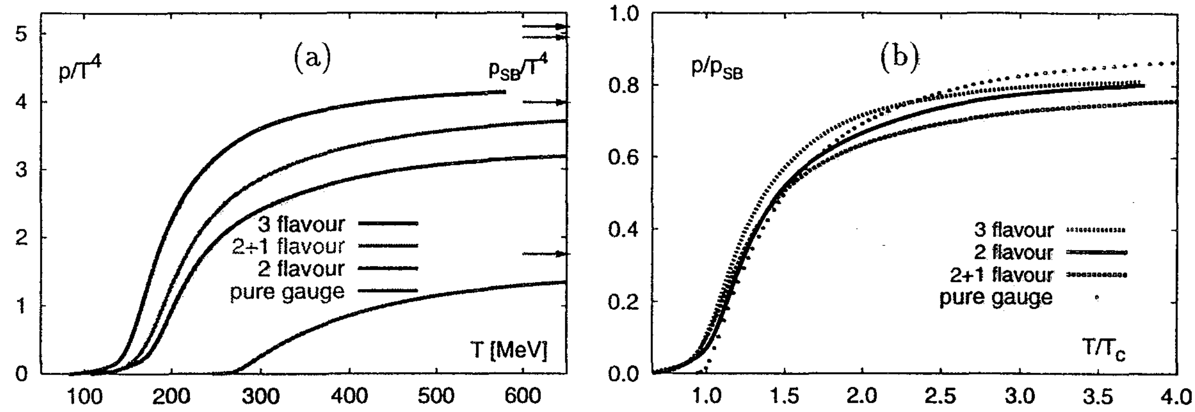


Figure 4. The pressure in QCD with $n_f = 0$, 2 and 3 light quarks as well as two light and a heavier (strange) quark. For $n_f \neq 0$ calculations have been performed on a $N_\tau = 4$ lattice using improved gauge and staggered fermion actions. In the case of the SU(3) pure gauge theory the continuum extrapolated result is shown. Arrows indicate the ideal gas pressure p_{SB} as given in Eq. 3.

$T_c^{QCD} \simeq 160$ MeV

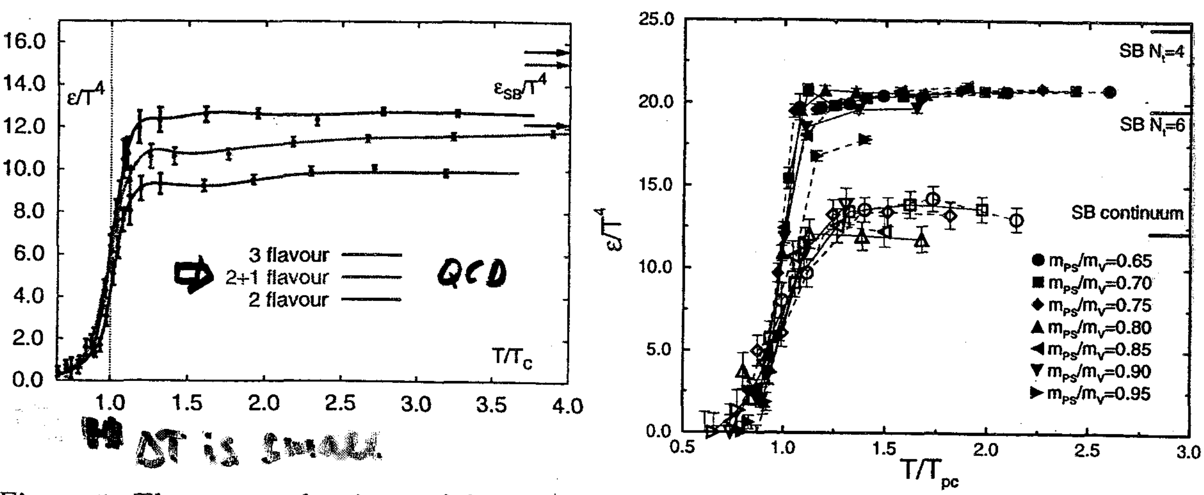
3. The Equation of State

LH8 $\Longleftarrow$ $LH \simeq 9 \cdot T_c^4 = 750 \frac{\text{MeV}}{\text{fm}^3}$

($\approx$ LH8 we use in H2H)

"Latent Heat"

LH



ΔT is small

Figure 5. The energy density in QCD. The left (right) figure shows results from a calculation with improved staggered (Wilson) fermions on lattices with temporal extent $N_\tau = 4$ ($N_\tau = 4$, 6). Arrows in the left figure show the ideal gas values ϵ_{SB} as given by Eq. 3.

Comment:

Pure glue $\Rightarrow$ "deconfinement" (agrees with string tension argument) $T_d \simeq 260$ MeV

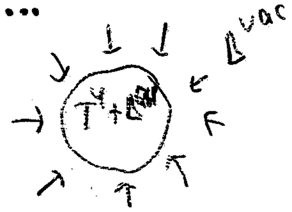
QCD with light quarks $\Rightarrow$ "chiral restoration" ph.t. $T_c \simeq 160$ MeV

(Karsch et al) Both are surprisingly small ...

1. $\boxed{\text{Why they are so different ?}}$

Stay tuned...

2. **Energetics** is very different



as can be seen from a QGP side

$$p^{QGP} = \left((\#)\, T^4 + \underbrace{\cancel{B^{QGP}}}_{\text{small?}} - B^{vac}\right) \text{ should stay } > 0 \,!$$

* For pure glue $(T = T_d)$ the first term is large and matches the vacuum energy $\boxed{B^{vac} \gtrsim 1\ \mathrm{GeV/fm^3}}$ so $p^{QGP}(T = T_d + \varepsilon)$ is small but positive

* For QCD $(T = T_c)$ the first term is too small

So, all non-perturbative fields cannot disappear at $T \gtrsim T_c$: $B^{QGP} \gtrsim \frac{1}{2} B^{vac}$!

(G. Brown, V. Koch - 92)

"Hard glue", or "epoxy" does not melt

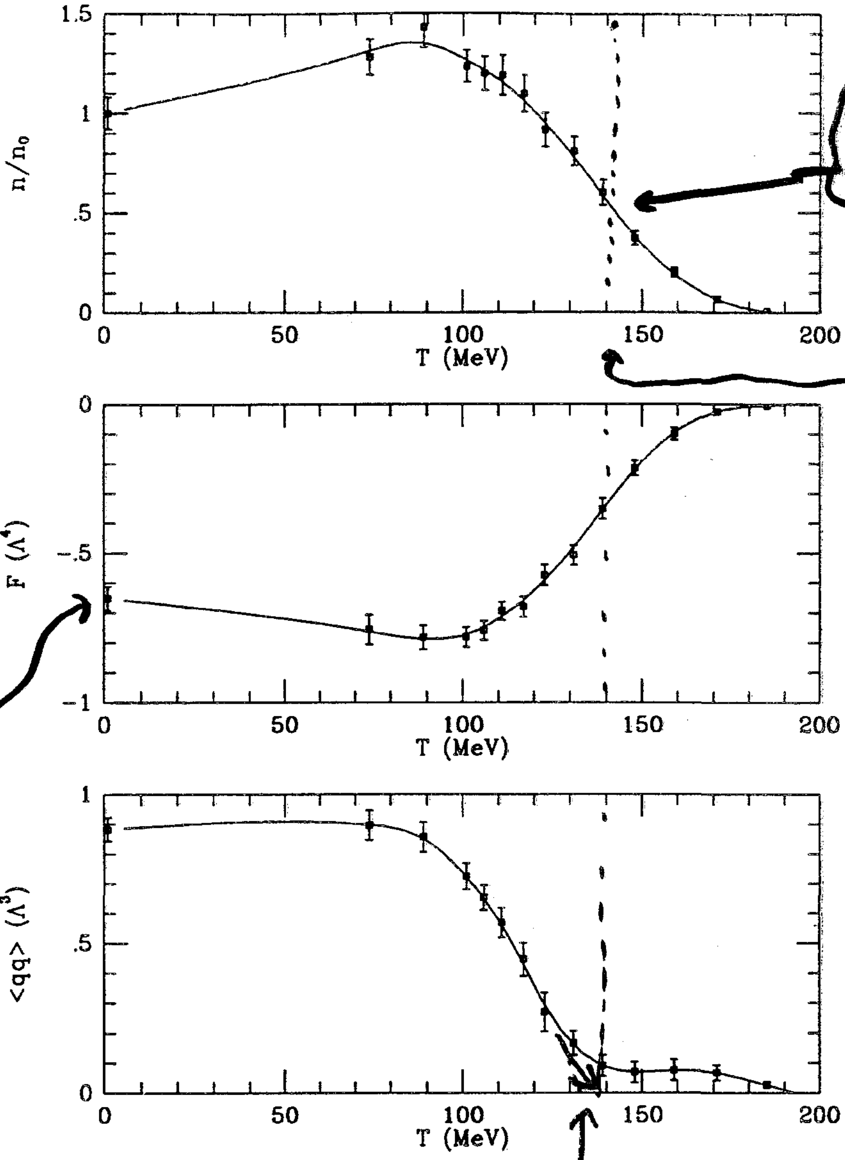
$\boxed{\text{What is it made of ?}}$

Instanton Liquid at Finite T

T. Schafer
ES.
J. Verbaarschot
PRD 94

$\boxed{\text{QCD}}$

Half glue is "epoxy"!



1/2!

Pisarski-Yaffe factor switched in for $T > T_c$

$\mathcal{E}_{vac}$

T_c

FIG. 9.

The quark condensate "melts" at $T = T_c$, but the gluon one does not! "Molecules" are the "<u>epoxy</u>"!

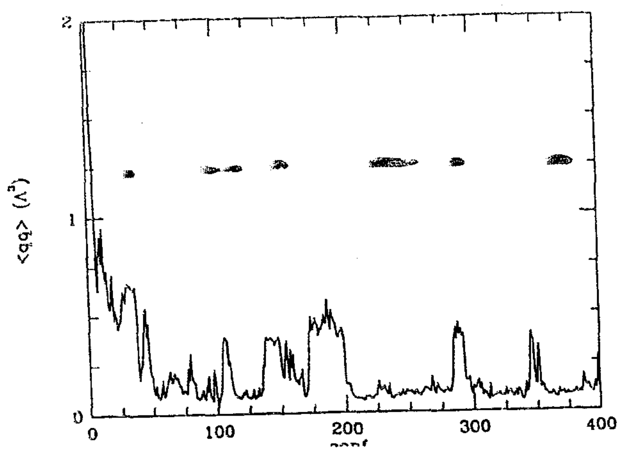
(glue which survives T_c!)

QCD ⇒ Thermal Histories

$T \downarrow$

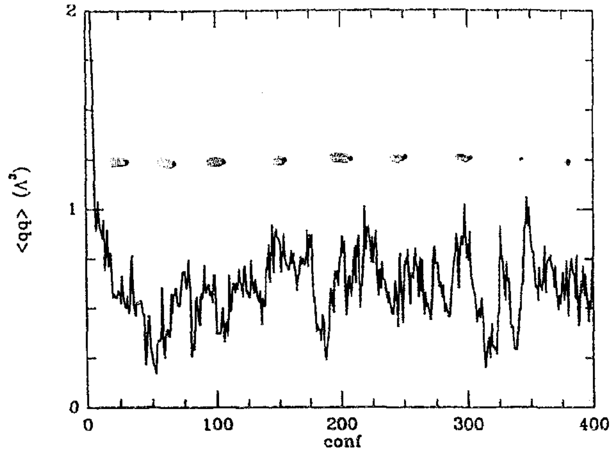
$\beta =$ [illegible]

153 MeV



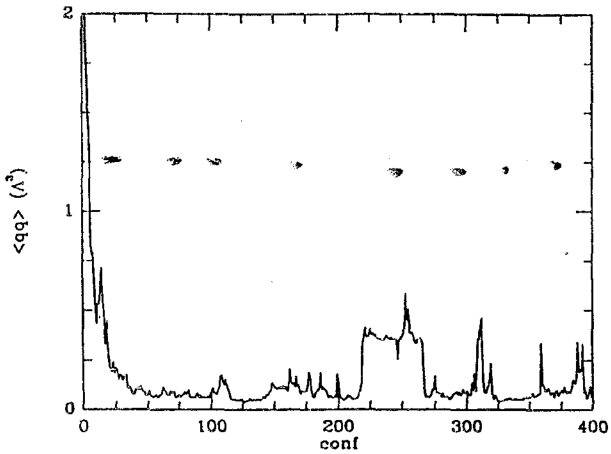
$\beta = 1.75$

T = 114 MeV



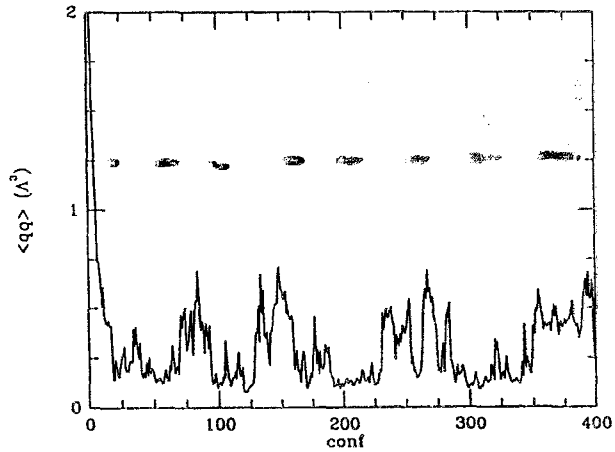
$\beta = 1.200$

166 MeV



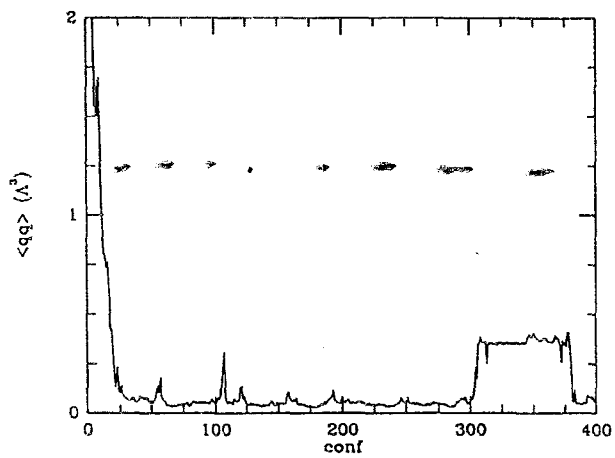
$\beta = 1.500$

133 MeV



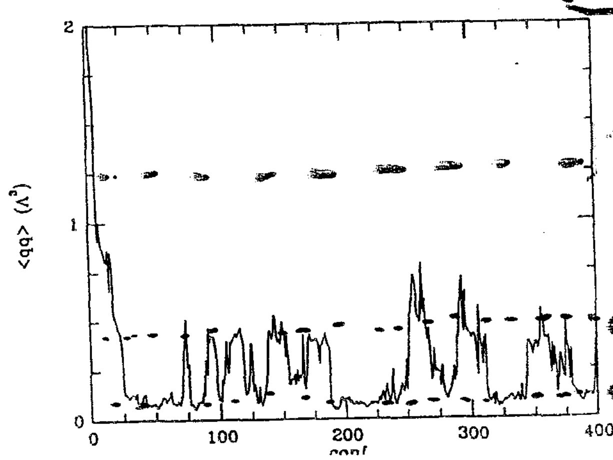
$\beta = 1.150$

180 MeV



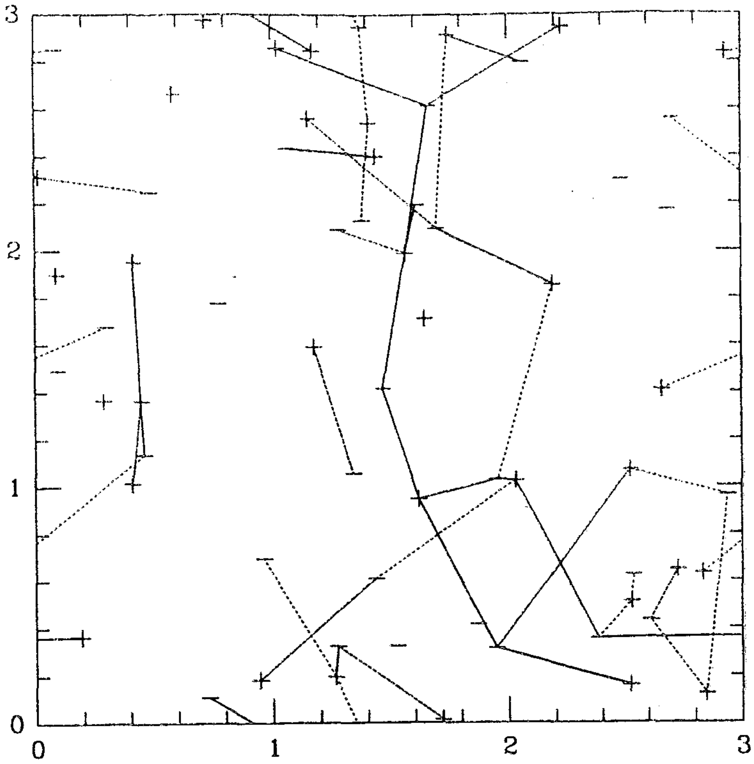
$\beta = 1.400$

142 MeV $\approx T_c$

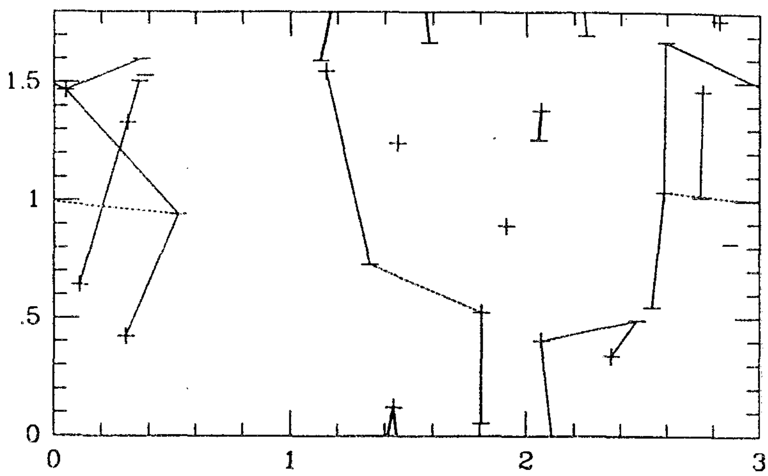


⇒ weak 1-st order

T = 75 MeV

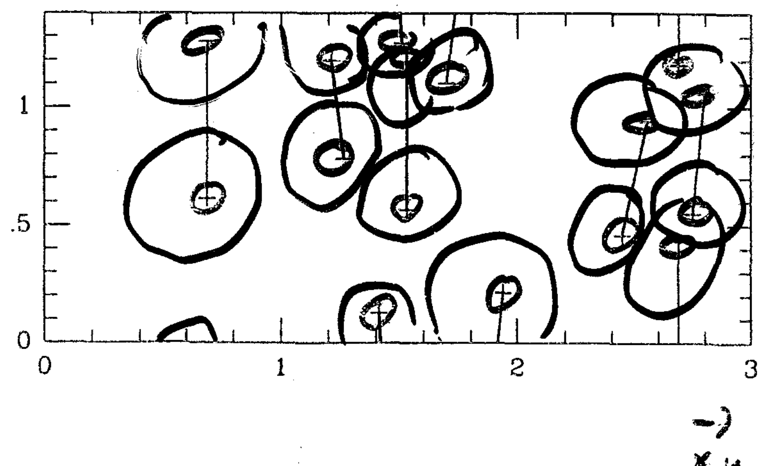


T = 123 MeV
(T < T$_c$)



τ ↑

T = 158 MeV
(> T$_c$)

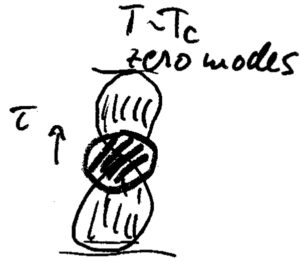


only molecules noticeably polarized in τ direction

→ x, y, z

CHIRAL SYMMETRY RESTORATION

(based on T.Schaefer, E. Shuryak and J. Verbaarschot, The chiral phase transitions and the Instanton Molecules; SUNY-NTG-94-24,Stony Brook.) PRD(95)



- – At growing T, quark motion becomes anisotropic
  Example: zero mode of the "caloron" ($T \neq 0$ instanton)

$$\psi(\tau, r) \sim |\sin(\pi T \tau)/\cosh(\pi T r)|$$

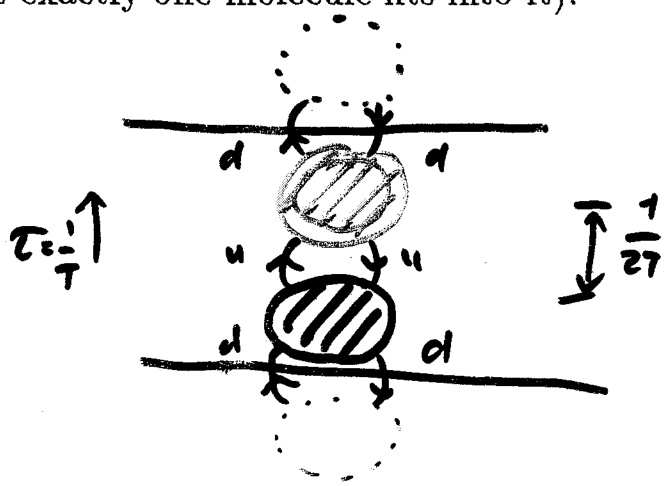
oscillatory in time, **exponentially decaying** in space.

- – A "pairing" of instantons (leading to formation of $\bar{I}I$ molecules even at T=0) becomes much stronger, if I and $\bar{I}$ are at the same point

$$detD \sim |\sin(\pi T \tau)/\cosh(\pi T r)|^{2N_f}$$

- – Rapid "polarization" of **even one** molecule was found at $T \approx T_c$, (which allows one to identify T_c as approximately the size of the "Matsubara box", such that exactly one molecule fits into it).

$\Delta\tau = \frac{1}{2T}$ half box
$\Delta r = 0$ same pouint



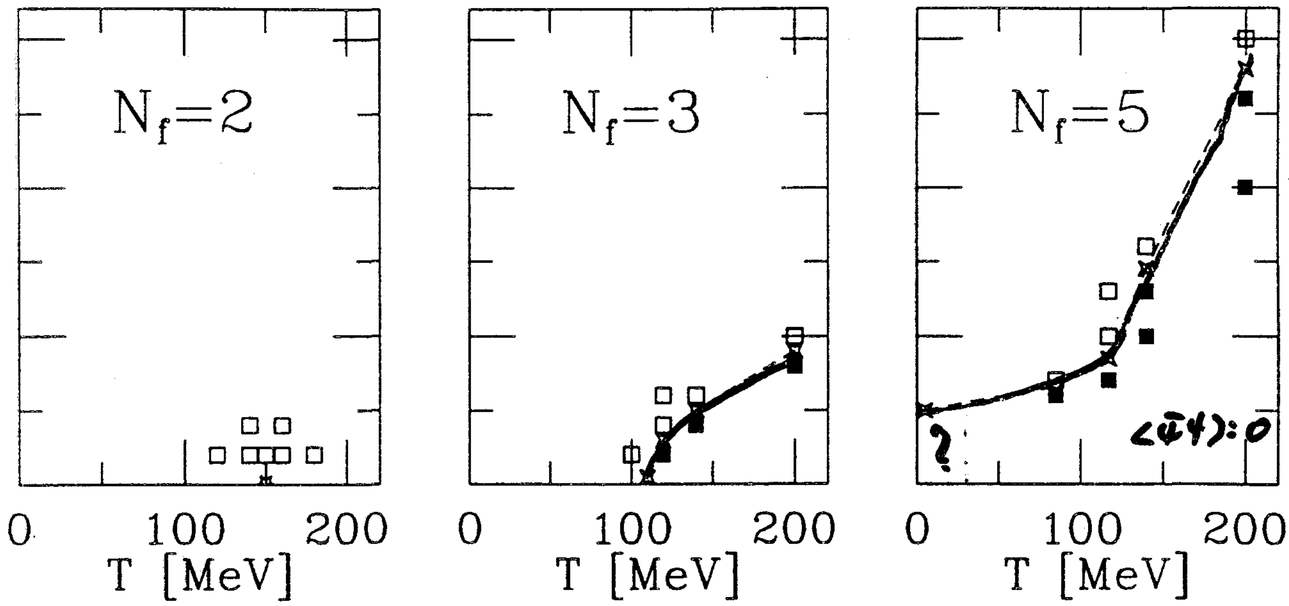
Appears as maximum in Z at T ≃ 110 MeV
By T=140MeV it dominates!

$\frac{1}{T_c} \simeq \boxed{\text{size of a molecule}}$!

⟸ For N_f = 2 to 3
(not true for smaller and larger N_f ...)

Increasing N_f (# of flavors) T. Shafer + E.S. 95

FIG. 7.



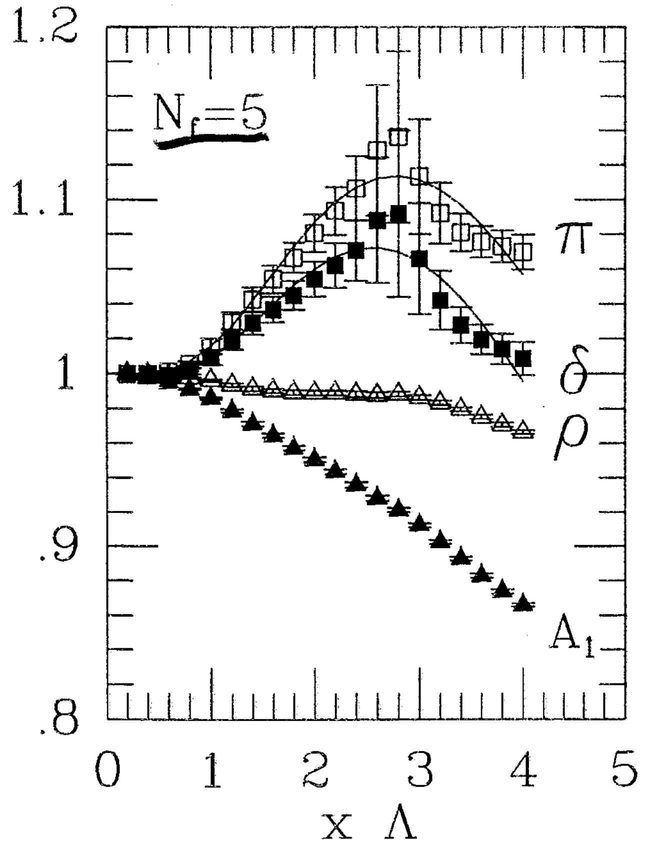
↑ 2-nd order

Phase diagram of IILM

at $N_f > 5$ and $m \to 0$
molecular vacuum
$\langle\bar\psi\psi\rangle = 0$

But $(\pi - \sigma - \delta - \eta')$ $U(N_f) \times U(N_f)$
multiplot
is <u>Bound</u>
($m \neq 0$, not Goldstone... of course)

Are there bound states in the chirally symmetric phase?



$m_{quark} = 0.1\,\Lambda$ used (and fitted from the correlators)

$\Rightarrow$ So chiral sym. is not exact

Yes: $m_\pi = m_\delta = m_\sigma = m_{\eta'} = \dots$

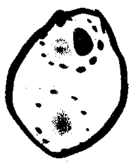
$= 1.4\,\Lambda \ (\pm 0.3\,\Lambda)$

All other channels ($N, \dots$) are consistent with free quarks

Color Supercond: Brief history

50's BCS...
60's Gorkov eqn
70's QGP, screening

- **- 70-80's: Quarks of different colors are attracted perturbatively:** S. C. Frautschi (Erice1978),F. Barrois, Nucl. Phys. B129, 390 (1977),D. Bailin and A. Love, Phys. Rep. 107, 325 (1984). $\Delta \sim 1\, MeV$ **only...**



N

Δ

- **-** T.Schaefer, E.S.,J.Verbaarschot Nucl. Phys. B412, 143 (1994): **ud scalar diquarks are very deeply bound in the instanton model, being a very robust element of Nucleons (octet) baryons, as opposed to Δ (decuplet) ones.** Sorry: too many phenomenological hints to mention here: weak decays, formfactors, fluctuations of the N cross section...

- **- First attempts to study instantons at finite density numerically** T. Schäfer, Phys. Rev D57 (1998) 3950.: **diquarks persist, even at high** μ : "polymers"

- Alford + Rajagopal → How magnetic field decomposes into "new photon" and "new gluo"
  - new photon: penetrates
  - new gluo: screened away (by monopoles)

- Zahed et al
  Son and Stephanov
  "K^-" $\bar{u}\gamma_5 s \to$ $\langle s\bar{u}\rangle + \langle \bar{u}\gamma_5 d\rangle$
  $\sim m_u$ not m_s

  If $m_q \neq 0$, then $m_\pi^2 \neq 0$ as in vacuum → GOR
  But $m_\pi^2 \sim m_q (\frac{\Delta}{\mu})$
  very small at large μ → very light π

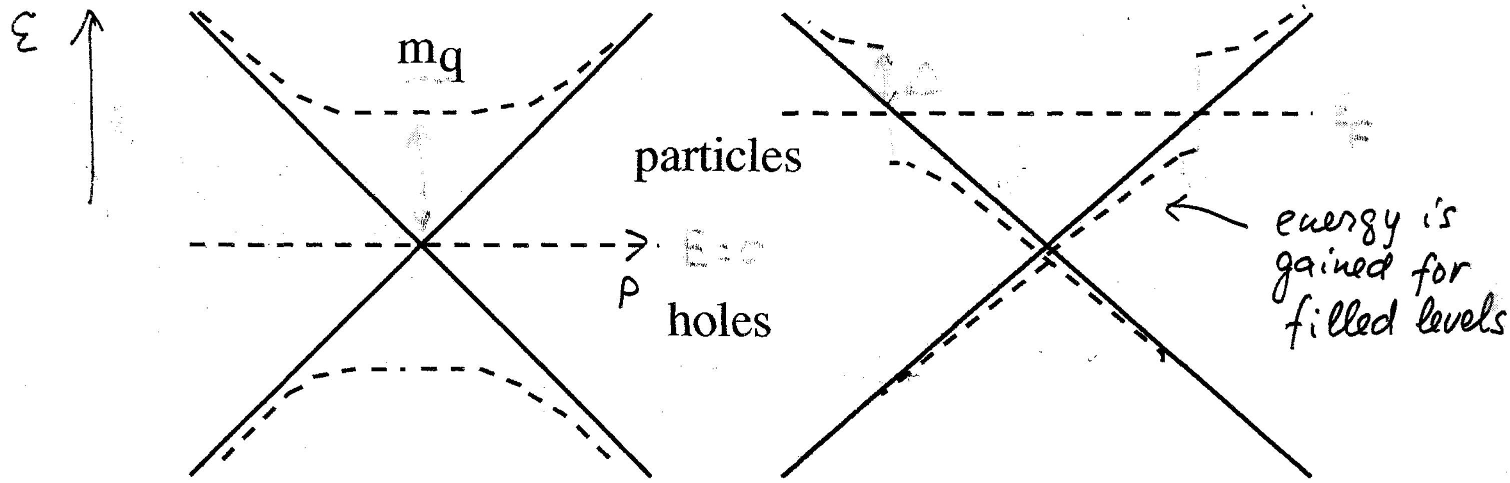
- P. Bedaque — In stars, if $\mu_u \neq \mu_d$ (more d)
  → "mixed phase", symmetric ($\mu_u = \mu_d$) + "remaining d"
  $\boxed{\text{another one}\uparrow}$
  (Can it bring even CFL ?)

- Son and Stephanov → very assymetric matter
  $\mu_u = -\mu_d$ → rotates toward pions, like 2 colors
  π condensate!

- Schafer — 1 flavor → spin-color locking
  gaps $\sim \frac{1}{100}$ of large ones

- Rapp, Shuryak, Zahed → Chiral crystal

Why transition from particle-hole to particle-particle pairing ?



vacuum

superconductor

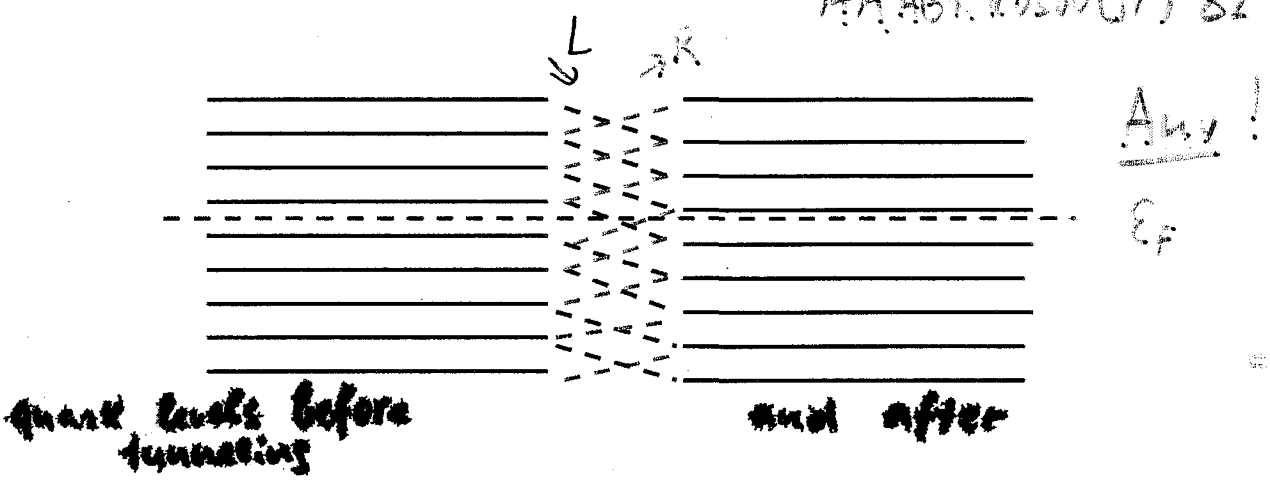
Energy versum momenta: the blue dashed line show the dispersion curves for vacuum and dense matter. It has discontinuity at two different places, the surface of the Dirac sea and Fermi sphere.

* Cooper instability
Δ is helped by a log $\Rightarrow$
any coupling works...

Why instantons?

- \- **It is the strongest non-perturbative effect generally** → (my talks at Lattice 93, 94)
- \- **Explanes quantitatively chiral symmetry breaking in vacuum. Gap is large:** $(m_{constituent} = 330 - 400 MeV)$
- \- **Anomaly is not eliminated by adding** $\mu\gamma_4$ **to the Dirac operator**[1]

AAAbrikosov(Jr) 82



- \- **But at very high density instanton effect are suppressed by the Debye screening (E.S.1982)**

$$dn(\rho, \mu) \approx dn(\rho, \mu = 0) exp(-N_f \rho^2 \mu^2)$$

[1] In random matrix models people have used a simple-minded approach: adding $\mu\gamma_4$ to the Dirac operator represented as a random matrix with some density of zero modes (leading to quark condensate). If μ is sufficiently large, eigenvalues move away from zero and chiral symmetry gets restored.

Two colors: a very special theory

- - The opposite to the large N_c limit: Baryons are degenerate with mesons: Pauli-Gursey symmetry (Unlike SUSY different number)

- - symmetry breaking is $SU(2N_f) \to Sp(2N_f)$ For $N_f = 2$ the coset $K = SU(4)/Sp(4) = SO(6)/SO(5) = S^5$ 5 massless modes: pions plus scalar diquark S and its anti-particle $\bar{S}$

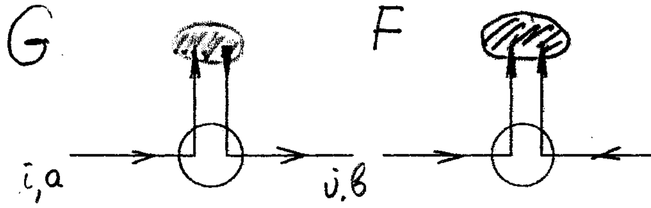
- - RSSV1: finite μ breakes rotates the 5-dim sphere. Scalar diquark (not sigma meson) becomes massive. more in: Kogut, Stephanov and Toublan hep-ph/9906346

$\langle\bar{q}q\rangle \to \langle qq\rangle$ the same magnitude $\Downarrow$ $\Delta \simeq 400$ MeV

- - Fermionic determinant is real: lattice simulations possible. Results by Karsh, Dagotto et al of mid-80's make sense! See recent work by S.Hands et al.

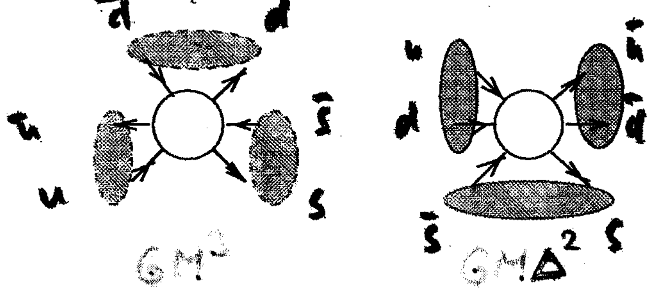
[redacted]

how the calculations are done: the mean field way



mass operator anomalous Gorkov operator

The mass operator include two types of diagrams: total energy of Fermi gas ("kinetic energy" is calcualated with it. Note: condensates are matrices in C/F



The ("potential energy") operator also two types of diagrams: For instantons and $N_f = 3$ it looks like this, where green area are $<\bar{q}q>$ and magenta $<qq>$ or its conjugate.

Then one minimizes the sum and get gap equations There are many because all masses/condensates are color-flavor matrices.

Approximations: (i) The coupling constant G is treated as constant. (ii) No clustering included. so far

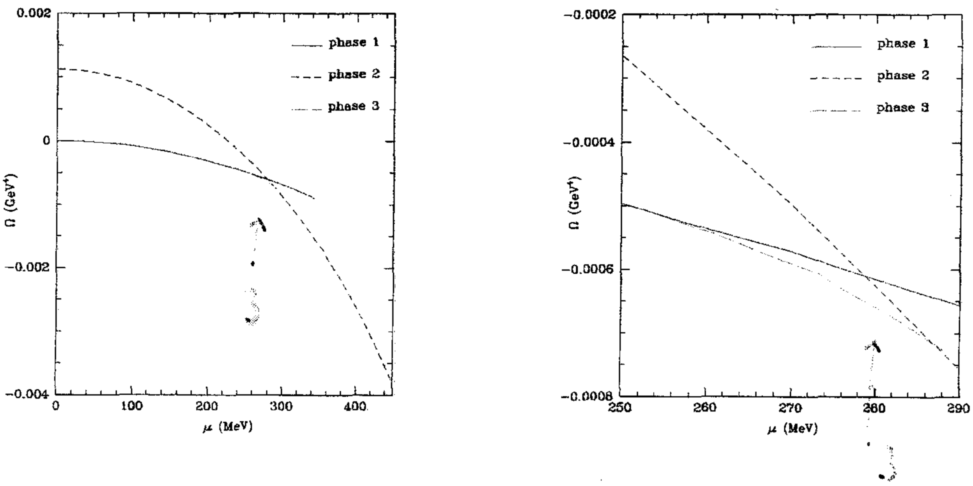
In early simplified studies (ARW1 and RSSV1) the possible intermediate phase (between vacuum and CSC2) - Fermi gas of constituent quarks, where both $M, \Delta \neq 0$ - was unstable.

$\Omega = -pV$

$\langle\bar{q}q\rangle \neq 0$ 1

$\langle qq\rangle \neq 0$ 2

both 3



In our latest study (RSSV hep-ph/9904353) , including formfactors coming from instanton zero modes we found that it survives...

Phase 3 (supercond. of "const. quarks") is as good an approx. to nuclear matter as mean field can provide

(nucleons exist outside MF, even without cofinement

The "continuity" issue ($N_f=3$)

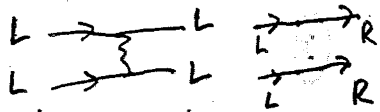
T.Schafer and F.Wilczek (98) pointed out that CSC3 phase has not only the same (?) symmetries as hadronic matter, but also very similar excitations:
The condensates conveniently mix color with flavor

quark language	hadronic language	comment
8 gluons →	8 massive vect. mes.	Meissner eff.
3*3 quarks →	8+1 "baryons"	$\Lambda(1405)$?
8 massless pions →	remain	if $< \bar{q}q > \neq 0$
γ is combined with "hypercharge" g →	massless γ_{inside}	like γZ in SM
Singlet scalar $U(1)_b$ →	H condensate	???

But: It is still not transparent for $\gamma_{outside}$ (something Weinberg/Salam should not have warried about) so it will levitate in an ordinary magnet, or reflect light from the surface...
Is it the correct condensation pattern of $N_f = 3$ nuclear matter?

The one-gluon exchange

The operator $(\gamma_\mu t^a)(\gamma_\mu t^a)$: no chirality flip



Strength depends on momentum transfer Q (it is after all the Rutherford-like scattering)

Electric exchanges are Debye screened at $Q \sim g\mu$ (like instantons)

Magnetic ones got no screening at T=0, only Landau damping

(One has also to take care of time delay effects, since we now speak of relativistic bound state bound by exchanges of propagating quanta... Eliashberg eqn.)

T.D.Son, hep-ph/9812287 therefore found a "double log" in the gap equation

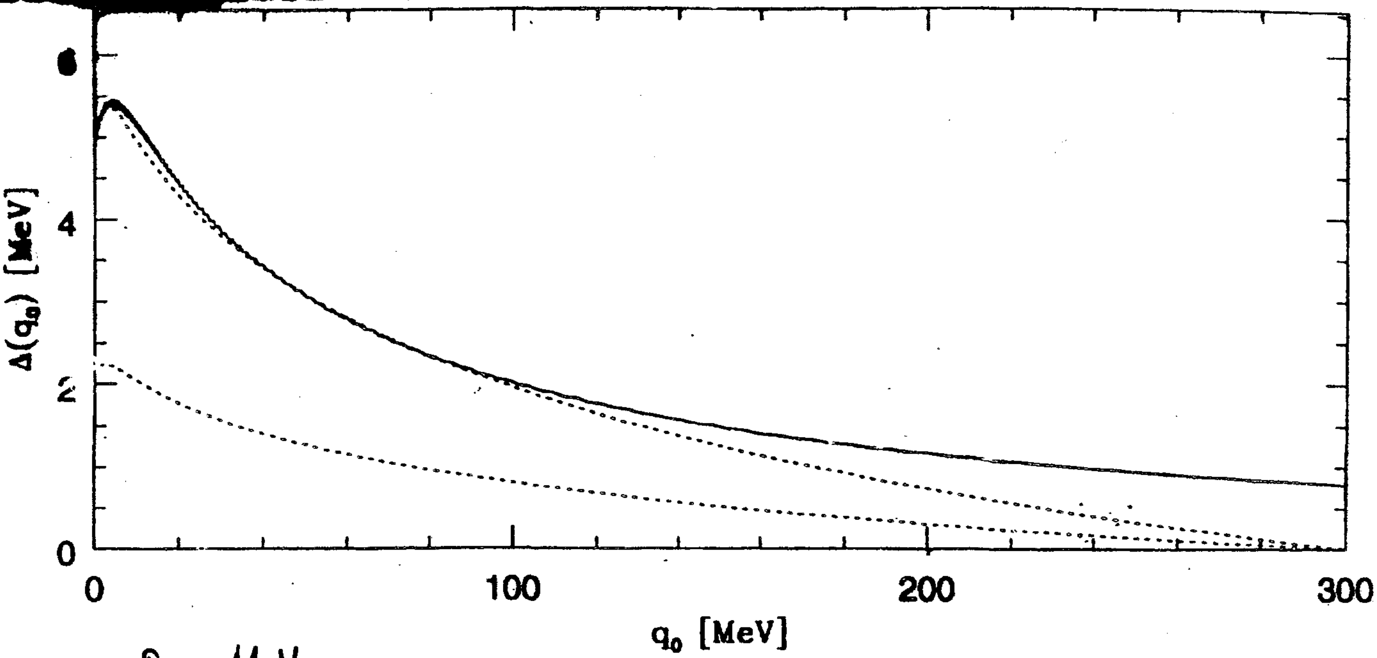
$$1 = const\, g^2\, log^2 \Delta$$

one log → BCS
the other $\int \frac{d\theta}{\theta}$ →

thus unusual answer: $\Delta \sim \mu exp(-\frac{3\pi^2}{\sqrt{2}g})$

ELIASHBERG EQUATION

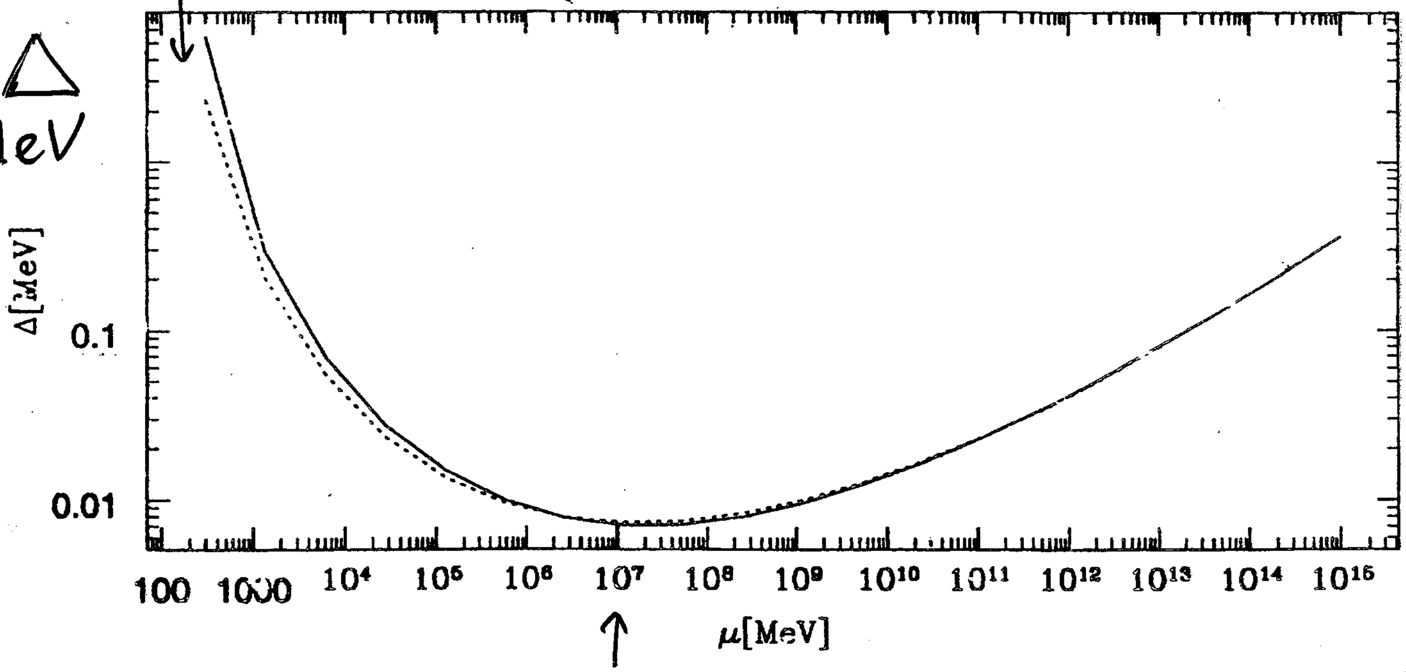
T. Schafer
following
T. D. Son



few MeV

Δ

MeV



10 GeV

SUMMARY

- \- **Finite density QCD is very rich, displaying a Higgs-like CSC2 phase ($< ud >_3 \neq 0$) or a combination of that with chirally asymmetric CSC3 phase ($< qq > \neq 0, < \bar{q}q > \neq 0$), with 1st order transition between them**
- \- **Instantons dominate at intermediate $\mu \sim 400 MeV$, but become Debye screened away at high μ. Triality of channels ($\bar{q}q, qq and \bar{I}I$ is the key to interplay of 3 major phases, hadronic, color superconduction and QGP.**
- \- **Magnetic gluons overtake electric ones at large μ, and the absolute value of the condensate grows with μ.** Phases are continuous!!

instantons → ~~molecule~~ electric gluons → magnetic gluons

——→ μ grows