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SUMMER SCHOOL ON PARTICLE PHYSICS

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PHENOMENOLOGY OF SUPERSYMMETRY

Lecture III & IV

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Please note: These are preliminary notes intended for internal distribution only.

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VI. Supersymmetry Breaking

Supersymmetry is unbroken if

$$Q_\alpha |0\rangle = \bar{Q}_{\dot\alpha} |0\rangle = 0 .$$

Recall that:

$$P^\mu = \frac{1}{4} \bar\sigma^{\mu\dot\beta\alpha} \{ Q_\alpha, \bar{Q}_{\dot\beta} \}$$

with for $\mu = 0$ reads:

$$H = P^0 = \frac{1}{4} (Q_1 \bar{Q}_{\dot 1} + \bar{Q}_{\dot 1} Q_1 + Q_2 \bar{Q}_{\dot 2} + \bar{Q}_{\dot 2} Q_2)$$

Unbroken SUSY then implies that at the potential minimum, $\langle 0 | H | 0 \rangle = 0$. Since:

$$V_{scalar} = \sum_i F_i^* F_i + \frac{1}{2} D^a D^a$$

$\langle 0 | H | 0 \rangle = 0$ implies that $\langle 0 | V_{scalar} | 0 \rangle = 0$, from which we conclude that

$$\langle 0 | F_i | 0 \rangle = 0$$
$$\langle 0 | D^a | 0 \rangle = 0 .$$

A second perspective:

For a chiral superfield,

$$\delta_\xi \psi_i(x) = i [\xi Q + \bar\xi \bar Q, \psi_i(x)] = -i\sqrt{2} \sigma^\mu \bar\xi \partial_\mu A_i - \sqrt{2} \xi F_i$$

By Lorentz invariance, $\langle 0 | \partial_\mu A | 0 \rangle = 0$. Thus,

$$\langle 0 | [\xi Q + \bar\xi \bar Q, \psi_i(x)] | 0 \rangle = -\sqrt{2} \xi \langle 0 | F_i | 0 \rangle$$

If $Q_\alpha |0\rangle = \bar Q_{\dot\alpha} |0\rangle = 0$, then $\langle 0 | F_i | 0 \rangle = 0$

Similarly for the super gauge multiplet:

$$\delta_\xi \lambda^a = i\xi D^a + \sigma^{\mu\nu}\xi F^a_{\mu\nu}$$

and since $\langle 0|F^a_{\mu\nu}|0\rangle = 0$, it follows that

$$\langle 0|[\xi Q + \bar{\xi}\bar{Q}, \lambda^a(x)]|0\rangle = i\xi\langle 0|D^a|0\rangle$$

If $Q_\alpha|0\rangle = \bar{Q}_{\dot{\alpha}}|0\rangle = 0$, then $\langle 0|D^a|0\rangle = 0$

Conversely, if either $\langle 0|F_i|0\rangle \neq 0$ for some i or $\langle 0|D^a|0\rangle \neq 0$ for some a, then supersymmetry is spontaneously broken.

<u>example</u>: O'Raifeartaigh mechanism ("F-type" breaking)

Consider the set of equations:

$$F_i^* = \frac{dW}{dA_i} = 0$$

and search for a solution, i.e. a choice of the A_i such that all equations $F_i^* = 0$ are fullfilled. Suppose a solution $A_i = v_i$ solves these equations. Choosing

$$\langle 0|A_i|0\rangle = v_i$$

automatically guarantees that $\langle 0|F_i|0\rangle = 0$.

If no solution exists, then it must be true that $\langle 0|F_i|0\rangle \neq 0$ for at least one i. Supersymmetry is spontaneously broken.

<u>Theorem</u>: If SUSY is spontaneously broken, then there exists a massless spin-1/2 fermion in the spectrum called the Goldstino.

<u>A Tree-level proof:</u>

$$V_{scalar} = \sum_i F_i^* F_i + \frac{1}{2} D^a D^a$$

$$F_i = \left(\frac{dW}{dA_i}\right)^*, \qquad D^a = -g A_i^* T_{ij}^a A_j$$

At the potential minimum where $\frac{\partial V}{\partial A_i} = 0$, $\quad A_i = \langle A_i \rangle$

notation: $\langle A_i \rangle \equiv \langle 0 | A_i | 0 \rangle$.

Then,

$$0 = \left(\frac{\partial V}{\partial A_i}\right)_{\langle A \rangle} = \sum_i \frac{\partial^2 W}{\partial A_i \partial A_j} F_i \Big|_{\langle A \rangle} - g A_i^* T_{ij}^a D^a \Big|_{\langle A \rangle}$$

Thus,

$$\sum_i \left\langle \frac{\partial^2 W}{\partial A_i \partial A_j} \right\rangle \langle F_i \rangle = g \langle A_i^* \rangle T_{ij}^a \langle D^a \rangle$$

The fermion masses of the theory arise from

$$-\mathcal{L}_m = \frac{1}{2} \left\langle \frac{d^2 W}{dA_i dA_j} \right\rangle \psi_i \psi_j - i\sqrt{2} g \langle A_i^* \rangle T_{ij}^a \psi_j \lambda^a + h.c$$

$$= \frac{1}{2} \begin{pmatrix} \psi_i & -i\lambda^b \end{pmatrix} \begin{pmatrix} \left\langle \frac{d^2 W}{dA_i \partial A_j} \right\rangle & \sqrt{2} g \langle A_j^* \rangle T_{ji}^a \\ \sqrt{2} g \langle A_i^* \rangle T_{ij}^b & 0 \end{pmatrix} \begin{pmatrix} \psi_j \\ -i\lambda^a \end{pmatrix}$$

I claim that

$$\begin{pmatrix} \left\langle \dfrac{d^2 W}{dA_i\, dA_j} \right\rangle & \sqrt{2}\, g \langle A_j^* \rangle T^a_{ji} \\ \sqrt{2}\, g \langle A_i^* \rangle T^b_{ij} & 0 \end{pmatrix} \begin{pmatrix} \langle F_j \rangle \\ \frac{1}{\sqrt{2}} \langle D^a \rangle \end{pmatrix} = 0$$

The requirement that the superpotential W is gauge invariant is as follows:

$$\frac{dW}{d\phi_i} (T^b)_{ij}\, \phi_j = 0$$

which can be re-written as:

$$\langle F_i \rangle T^a_{ji} \langle A_j^* \rangle = 0$$

after taking the vacuum expectation value.

Thus, the equation at the top of the page is a consequence of the potential minimum condition and the condition for a gauge invariant W.

That is, the fermion mass matrix has a zero eigenvalue, corresponding to the massless Goldstino. Moreover, the Goldstino is a linear combination of ψ_i and $-i\lambda^a$ which is given by the eigenvector exhibited above:

$$\tilde{G} = \begin{pmatrix} \langle F_j \rangle \\ \frac{1}{\sqrt{2}} \langle D^a \rangle \end{pmatrix} \quad \text{OR} \quad \tilde{G} = \langle F_j \rangle \psi_j - \frac{i}{\sqrt{2}} \langle D^a \rangle \lambda_a$$

up to an overall renormalization.

Sum rules of spontaneously broken SUSY

Examine the masses arising in a spontaneously broken SUSY theory, which contains a super-Yang-Mills theory coupled to matter.

Spin 1 masses

These arise from $\mathcal{L} = (\mathcal{D}_\mu A_i)(\mathcal{D}^\mu A_i)^*$, where

$$\mathcal{D}_\mu = \partial_\mu + igT^a V_\mu^a$$

which yields:

$$\mathcal{L}_{mass} = g^2 \langle A_i^* \rangle T^a_{ij} T^b_{jk} \langle A_k \rangle V_\mu^a V^{\mu b}$$

$$\equiv \frac{1}{2} M^2_{1\,ab} V_\mu^a V^{\mu b}$$

The spin-1 squared-mass matrix is then:

$$M^2_{1\,ab} = 2g^2 \langle A_i^* \rangle T^a_{ij} T^b_{jk} \langle A_k \rangle$$

Recall that $D^a = -g A_i^* T^a_{ij} A_j$ so

$$\frac{\partial D^a}{\partial A_i^*} = -g T^a_{ij} A_j$$

we can then write:

$$M^2_{1ab} = 2 \left\langle \frac{\partial D^a}{\partial A_j^*} \frac{\partial D^b}{\partial A_j} \right\rangle$$

so that

$$\mathrm{Tr}\, M_1^2 = 2 \left\langle \frac{\partial D^a}{\partial A_j^*} \frac{\partial D^a}{\partial A_j} \right\rangle$$

Spin 1/2 masses

We previously wrote down

$$M_{1/2} = \begin{pmatrix} \left\langle \frac{d^2W}{dA_i dA_j} \right\rangle & \sqrt{2} g \langle A_j^* \rangle T^a_{ji} \\ \sqrt{2} g \langle A_i^* \rangle T^b_{ij} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \left\langle \frac{dF_i^*}{dA_j} \right\rangle & -\sqrt{2} \left\langle \frac{\partial D^a}{\partial A_i} \right\rangle \\ -\sqrt{2} \left(\frac{\partial D^b}{\partial A_j} \right) & 0 \end{pmatrix}$$

It follows that

$$\mathrm{Tr}\, M^\dagger_{1/2} M_{1/2} = \left\langle \frac{\partial F_i}{\partial A_j^*} \frac{\partial F_i^*}{\partial A_j} \right\rangle + 4 \left\langle \frac{\partial D^a}{\partial A_i^*} \frac{\partial D^a}{\partial A_i} \right\rangle$$

Spin 0 masses

These arise from the scalar potential

$$V = \sum_i F_i^* F_i + \frac{1}{2} D^a D^a$$

We can write:

$$\mathcal{L}_{mass} = \frac{1}{2} (A_i \;\; A_j^*) M_0^2 \begin{pmatrix} A_k^* \\ A_\ell \end{pmatrix}$$

$$M_0^2 = \begin{pmatrix} \left\langle \frac{\partial^2 V}{\partial A_i \partial A_k^*} \right\rangle & \left\langle \frac{\partial^2 V}{\partial A_i \partial A_\ell} \right\rangle \\ \left\langle \frac{\partial^2 V}{\partial A_j^* \partial A_k^*} \right\rangle & \left\langle \frac{\partial^2 V}{\partial A_j^* \partial A_\ell} \right\rangle \end{pmatrix}$$

Computing the second derivatives using the scalar potential and noting that $\frac{\partial^2 D^a}{\partial A_k^* \partial A_i} = -g T^a_{ik}$, we end up with:

$$\mathrm{Tr}\, M_0^2 = 2\left\langle \frac{\partial F_m^*}{\partial A_k} \frac{\partial F_m}{\partial A_k^*} \right\rangle + 2\left\langle \frac{\partial D^a}{\partial A_k^*} \frac{\partial D^a}{\partial A_k} \right\rangle - 2g \langle D^a \rangle \mathrm{Tr}\, T^a$$

<u>The supertrace</u>

$$\mathrm{Str}\, M^2 \equiv \sum_i (-1)^J (2J+1) M_i^2 C_i$$

$$C_i^{\text{fermions}} = \begin{cases} 1 & \text{Majorana} \\ 2 & \text{Dirac} \end{cases} \qquad C_i^{\text{bosons}} = \begin{cases} 1 & \text{real} \\ 2 & \text{complex} \end{cases}$$

We have computed mass matrices using real vector fields and Majorana fermions. Complex scalars have already been counted properly. Thus,

$$\mathrm{Str}\, M^2 = 3\,\mathrm{Tr}\, M_1^2 - 2\,\mathrm{Tr}\, M_{1/2}^{\dagger} M_{1/2} + \mathrm{Tr}\, M_0^2$$

Inserting our expressions,

$$\boxed{\mathrm{Str}\, M^2 = -2g \langle D^a \rangle \mathrm{Tr}\, T^a}$$

Check the SUSY-conserving limit:

$$\langle D^a \rangle = 0, \quad \text{and} \quad \mathrm{Str}\, M^2 = M^2\, \mathrm{Str}\, 1$$

since the masses of all particles in a supermultiplet are degenerate. But $\mathrm{Str}\, 1 = n_B - n_F = 0$. Thus,

$$\mathrm{Str}\, M^2 = 0 \qquad \text{SUSY-conserved.}$$

Models of spontaneous SUSY-breaking

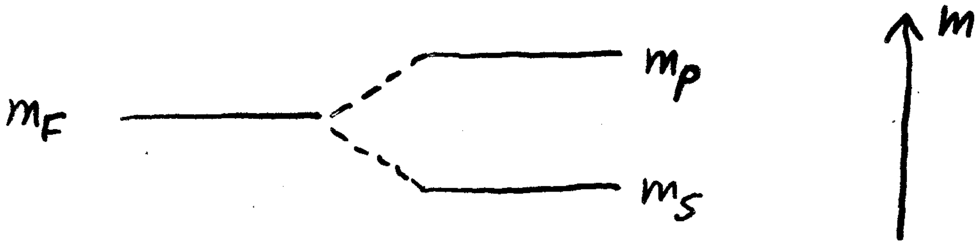
1. O'Raifeartaigh (F-type) $\langle F_i \rangle \neq 0$, $\langle D^a \rangle = 0$.

$$Str\, M^2 = 0$$

The masses of particles inside a supermultiplet can be split, but in such a way to preserve $Str\, M^2 = 0$.

<u>example</u>: a chiral superfield [complex scalar, Majorana fermion]

Write $A = \frac{1}{\sqrt{2}}(S + iP)$



This is very bad for realistic phenomenology, which requires all superpartner masses to be heavier then the observed fermion masses.

2. D-type SUSY breaking in non-abelian gauge theory.

For a non-abelian gauge group, $Tr\, T^a = 0$ and we again see that $Str\, M^2 = 0$.

3. D-type SUSY breaking in gauge theory with an abelian group.

<u>example 1</u>: the Standard Model: $SU(3) \times SU(2) \times U(1)$.

Here, $U(1)$ is hypercharge Y. But in the Standard Model, $Tr\, Y = 0$ when summed over Standard Model particles. Again, $Str\, M^2 = 0$.

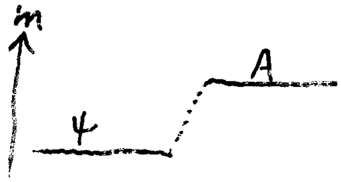
Example 2: D-type breaking à la Fayet-Iliopoulos

Consider a supersymmetric gauge theory based on $U(1)$ coupled to one chiral superfield with $g\xi > 0$ and $g = 1$*
(ξ = Fayet-Iliopoulos term and g = $U(1)$ gauge coupling).
Then, there is no superpotential and

$$V_{scalar} = \frac{1}{2}(\xi + gA^*A)^2$$

Minimize the potential. Clearly, $\langle A \rangle = 0$, so

$$\langle V_{scalar} \rangle = \frac{1}{2}\xi^2$$



$m_A^2 = g\xi$ (complex scalar)

$m_\psi = 0$ (superpartner of the scalar remains massless)

and the gauge multiplet (photon, photino) remain massless.
Since $D = -\xi - gA^*A$, we have $\langle D \rangle = -\xi$. Indeed,

$$Str\, M^2 = 2g\xi$$

[The 2 corresponds to the fact that A is a <u>complex</u> scalar.]

*<u>Note</u>: this theory has a $U(1)$-gauge anomaly. However, the same phenomeno[n] can be exhibite[d] in an anomaly free theory such as SUSY-QE[D]

<u>Remark</u>: Actually, in non-abelian SUSY Yang-Mills theory, only F-type SUSY breaking is allowed. When we minimize the scalar potential to determine $\langle A_i \rangle$, one can show that a choice of $\langle A_i \rangle$ which solves $F_i = \left(\frac{dW}{dA_i}\right)^*$ can always be chosen to satisfy $D^a = -gA_i^* T_{ij}^a A_j = 0$. The proof uses the holomorphicity of W.

Realistic models of spontaneous SUSY breaking, in which all superpartners of known fermions are very heavy, are extremely difficult to construct. They all require the existence of new physics beyond the Standard Model – either new gauge groups, new matter fields, or both.

One interesting loophole

The result $\mathrm{Str}\, M^2 = -2g \langle D^a \rangle \mathrm{Tr}\, T^a$ is a result that holds at tree level. Radiative corrections need not respect this result. So, one can try to build models in which $\mathrm{Str}\, M^2 = 0$ at tree level but $\mathrm{Str}\, M^2 > 0$ when radiative corrections are included. This is the strategy of gauge-mediated SUSY breaking which will be discussed later in these lectures.

However, one still needs to add new sectors of physics to accomplish the desired result.

A second interesting loophole – supergravity

In these lectures, supersymmetry is a global symmetry. The supersymmetry transformations $\delta_\xi \phi$ involve a space-time independent anti-commuting parameter, ξ. Suppose $\xi = \xi(x)$. What would I expect? Since

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

we see that local supertranslations necessarily require that ordinary space-time translations are a local symmetry. This is a theory of gravity plus supersymmetry, i.e. supergravity.

One of the massless supermultiplets of supersymmetry contains a helicity $\frac{3}{2}$ and a helicity 2 particle:

$$(\underbrace{\tilde{g}_{3/2}}_{\text{gravitino}},\ \underbrace{g_2}_{\text{graviton}})$$

Suppose we couple this multiplet to ordinary matter (chiral supermultiplets). In addition, suppose we spontaneously break the local supersymmetry.

Then, the mass of the graviton and gravitino must be split. The graviton stays massless (after all, we wish to keep general relativity and the infinite range gravitational force), so the gravitino must become massive. But how?

The massless gravitino has helicity $\pm\frac{3}{2}$.

The massive gravitino has four spin states $(m_s = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2})$

Spontaneous SUSY-breaking also generates a massless Goldstino (some linear combination of spin-½ states from the chiral multiplet).

The super-Higgs mechanism

The massless gravitino absorbs the Goldstino. The Goldstino (which is spin-½) provides the missing $m_s = \pm\frac{1}{2}$ spin states, so that the resulting gravitino is now massive (and the Goldstino is removed from the spectrum).

In spontaneously broken supergravity, the tree-level mass sum rule is modified. If N chiral supermultiplets are minimally coupled to supergravity,

$$\mathrm{Str}\, M^2 = (N-1)(2m_{3/2}^2 - \kappa^2 D^a D^a) - 2g D^a T^a$$

where $\kappa \equiv (8\pi G_N)^{1/2} = (8\pi)^{1/2}/M_{PL}$, $m_{3/2}$ = gravitino mass and D^a is evaluated for $A_i = \langle A_i \rangle$.

Typical models of interest have $\langle D^a \rangle = 0$, in which case

$$\mathrm{Str}\, M^2 = 2(N-1)m_{3/2}^2 .$$

If $m_{3/2} \gtrsim O(100\ \mathrm{GeV})$, then we would have a reason for why superpartners of the observed fermions have not yet been observed.

A phenomenological approach to SUSY-breaking

Two possibly viable mechanisms for generating phenomenologically acceptable masssplittings in supermultiplets are:

(i) generate mass-splitting due to SUSY-breaking by radiative effects. This requires new sectors of physics (where the SUSY-breaking resides) associated with a mass scale significantly higher than $O(1\ TeV)$.

(ii) generate SUSY-breaking by supergravity. Here, the relevant scale associated with SUSY-breaking is M_{PL}.

At energy scales of $O(1\ TeV)$, the low-energy effective theory is a broken globally supersymmetric theory. But the supersymmetry-breaking terms of this effective theory must be such that $\mathrm{Str}\, M^2 > 0$.

We therefore take a phenomenological approach and ask what are the possible supersymmetry-breaking terms that appear in the effective low-energy theory?

Soft SUSY-breaking — protection from quadratic divergences

In spontaneously-broken SUSY, we might worry that quadratic divergences reappear.

Consider the one-loop effective potential for a gauge theory coupled to matter:

$$V_{eff}(\phi) = \underbrace{V_{scalar}(\phi)}_{\text{tree-level scalar potential}} + V^{(1)}(\phi)$$

A one-loop computation, where the divergence is regulated by a momentum cut-off Λ yields:

$$V^{(1)}(\phi) = \frac{\Lambda^2}{32\pi^2} \,\mathrm{Str}\, M_i^2(\phi) + \frac{1}{64\pi^2}\,\mathrm{Str}\left\{ M_i^4(\phi)\left[\ln \frac{M_i^2(\phi)}{\Lambda^2} - \frac{1}{2}\right]\right\},$$

where $M_i^2(\phi)$ are the relevant mass-squared matrices in which the scalar vacuum expectation values are replaced by the corresponding scalar fields.

Note: a field-independent term proportional to $\Lambda^4\,\mathrm{Str}\,1$ has been omitted. In SUSY (unbroken or spontaneously broken), $\mathrm{Str}\,1 = n_B - n_F = 0$, so this term never arises.

We see that $\mathrm{Str}\,M^2 = 0$ guarantees that no quadratic divergences appear. But we saw that $\mathrm{Str}\,M^2 = 0$ was respected as long as $\mathrm{Tr}\,T^a = 0$, which is true in the Standard Model.

We define soft-SUSY-breaking to be supersymmetry breaking in which quadratic divergences remain absent.

One possible procedure due to Girardello and Grisaru is to examine the effect of a candidate term for SUSY-breaking, and compute its effect on $\mathrm{Str}\, M_i^2(\phi)$. If the contribution vanishes or is independent of ϕ, then no new quadratic divergence is generated.

note: constant contributions to $V(\phi)$ contribute to the vacuum energy. In models without gravity, such effects are unobservables. In models with gravity, they contribute to the cosmological constant. Why the cosmological constant is nearly zero is one of the great mysteries of fundamental theoretical physics.

Candidates for soft-SUSY-breaking terms

① $$-\delta\mathcal{L}_{soft} = m_{ij}^2 A_i A_j^* + [w(A) + h.c.]$$

where $w(A)$ is an arbitrary gauge-invariant cubic polynomial in the fields A (holomorphic).

$$\delta \operatorname{Tr} M_0^2(\phi) = 2 \operatorname{Tr} m^2$$

which does not depend on scalar fields.

② $-\delta\mathcal{L}_{soft} = \frac{1}{2}(m_{ab}\lambda^a\lambda^b + m^*_{ab}\bar{\lambda}^a\bar{\lambda}^b)$

where λ^a is the gaugino.

$$\delta\,\mathrm{Tr}\, M^\dagger_{1/2} M_{1/2} = \mathrm{tr}\, m^2$$

which is again field-independent.

③ $-``\delta\mathcal{L}_{soft}" = \frac{1}{2}(m_{ij}\psi^i\psi^j + m^*_{ij}\bar{\psi}^i\bar{\psi}^j)$

where ψ^i is the fermionic component of a chiral superfield.

$$\delta\,\mathrm{Tr}\, M^\dagger_{1/2} M_{1/2} = \frac{dF^*_i}{dA_j} m^*_{ij} + \frac{dF_i}{dA^*_j} m_{ij} + m_{ij} m^*_{ij}$$

which does contain field-dependent terms (in principle). Thus, this candidate may not yield soft-SUSY-breaking.

remark: the field-dependent terms can arise only in models with gauge singlets.

④ All SUSY-breaking dimension-4 terms are "hard".

Conclusion

$$-\delta\mathcal{L}_{soft} = m^2_{ij} A_i A^*_j + \frac{1}{2}[m_{ab}\lambda^a\lambda^b + h.c.] + [W(A) + h.c.]$$

where $W(A)$ is a cubic polynomial constitutes the most general soft-SUSY-breaking.*

* In models with no gauge singlets, one can in principle add in the omitted dimension-3 terms, although such terms typically do not arise in actual models of SUSY-breaking.

some SUSY jargon

$$-\delta\mathcal{L}_{soft} = m_{ij}^2 A_i A_j^* + \frac{1}{2}\left[m_{ab}\lambda^a\lambda^b + h.c.\right] + \left[w(A) + h.c.\right]$$

$$w(A) = c_i A_i + b_{ij} A_i A_j + a_{ijk} A_i A_j A_k$$

not permitted if there are no gauge singlets

the "B-terms"

the "A-terms"

Comments:

1. $-\delta\mathcal{L}_{soft} = m_{ij}^2 A_i A_j^* + \frac{1}{2}[m_{ab}\lambda^a\lambda^b + h.c.] + [w(A) + h.c.]$

arises precisely in a theory of broken supergravity coupled to matter and gauge fields, after integrating out the Planck scale physics to obtain a low-energy theory of broken global supersymmetry.

2. In some theories, one finds that $w(A)$ is proportional to the superpotential $W(\phi)\big|_{\phi=A}$, although this is not true in all cases.

Soft-SUSY-breaking: an effective theory perspective

Suppose I have a set of light chiral superfields ϕ, and a heavy chiral superfield Φ which I wish to integrate out of my theory. Assume that SUSY-breaking is generated because

$$\langle F_\Phi \rangle = f \neq 0$$

A possible term in the effective Lagrangian is:

$$\frac{1}{M}\int d^2\theta\, \Phi\, w(\phi)$$

$w(\phi)$ is holomorphic since $\Phi w(\phi)$ is a contribution to the superpotential. I need the $\frac{1}{M}$ (where M is a heavy scale associated with Φ) for dimensional reasons:

$[\int d^2\theta] = 1$, $[\Phi] = 1$, $[w] = 3$ for a cubic polynomial in

and of course $[\mathcal{L}] = 4$, since the action $\int d^4x\,\mathcal{L}$ is dimensionless

Since $\langle F_\Phi \rangle = f$, I can write $\langle \Phi \rangle = \theta\theta f$, so

$$\frac{1}{M}\int d^2\theta \; \theta\theta f \, w(\phi) = \frac{f}{M} w(A)$$

which is precisely of the form allowed by $\delta\mathcal{L}_{soft}$.

For phenomenology, I require $\frac{f}{M} \lesssim O(1 \text{ TeV})$.

For example, if $M = M_{PL}$ (as in broken supergravity), I would need $f \sim (10^{11} \text{ GeV})^2$, which indicates the scale of SUSY-breaking required in the fundamental theory to generate effective SUSY-breaking at the TeV-scale.

Another example:

$$\frac{1}{M^2}\int d^4\theta \; \phi_i^\dagger (e^{2gV})_{ij} \phi_j \; \Phi^\dagger \Phi$$

Again, take $\langle \Phi \rangle = \theta\theta f$ and evaluate in the Wess-Zumino gauge.

$$\frac{1}{M^2}\int d^4\theta \; \theta\theta\bar\theta\bar\theta f^2 \phi^\dagger e^{2gV} \phi = \frac{f^2}{M^2} A^* A$$

Gaugino masses:

From $\displaystyle \frac{1}{M}\int d^2\theta \, \Phi \, \mathrm{Tr}(W^\alpha W_\alpha) \longrightarrow \frac{-f}{M} \mathrm{Tr}\, \lambda^\alpha \lambda_\alpha$

VII. Supersymmetric extension of the Standard Model (finally!)

Steps to construct a supersymmetric extension of the Standard Model:

1. Add a superpartner to all Standard Model particles such that:

$$\mathrm{Str}\, 1 = n_B - n_F = 0$$

2. In step 1, new fermions are added which can generate a gauge anomaly. This will require a slight extension of the original Standard Model.

The standard model contains a Higgs boson with quantum numbers $(1, 2, 1)$ under $SU(3) \times SU(2) \times U(1)$.

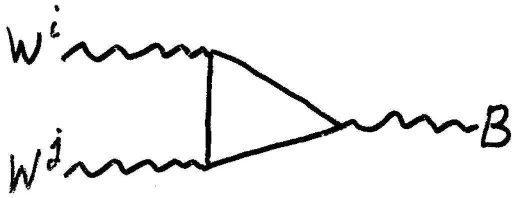
color singlet — weak doublet — hypercharge $Y = 1$.

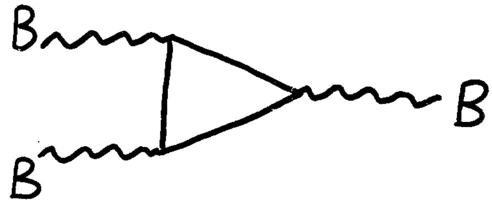
Its higgsino partner will generate an anomaly.
To avoid this problem, add a second Higgs doublet with opposite hypercharge: $(1, 2, -1)$. Now, the supersymmetric extended model contains two pairs of higgsinos

$$(1, 2, 1) \oplus (1, 2, -1)$$

which is now vector-like rather than chiral; the total contribution to the gauge anomalies cancel.

Cancellation of anomalies:





$$\mathrm{Tr}\, T_3^2 Y = 0 \qquad\qquad \mathrm{Tr}\, Y^3 = 0$$

(Remember the color factor when performing the trace!)

For example, for the Standard Model fermions,

$$(\mathrm{Tr}\, Y^3)_{SM} = 3\left(\frac{1}{27} + \frac{1}{27} - \frac{64}{27} + \frac{8}{27}\right) - 1 - 1 + 8 = 0$$

If we just had one Higgs doublet, then the (left-handed) Higgsino superpartners would be $(\tilde{H}^+, \tilde{H}^0)$ with $T_3 = \pm\frac{1}{2}$ and $Y = 1$.
Then, $\mathrm{Tr}\, Y^3 = (\mathrm{Tr}\, Y^3)_{SM} + 2$.

(Note: the only other supersymmetric fermions – the gauginos – have $Y=0$ and so do not contribute to the anomaly or its cancellation.)

By including Higgs multiplets in pairs, with opposite hypercharge, the Higgsinos' contributions to the anomaly will cancel.

Finally, it is worth pointing out that if we require the cancellation of pure gravitation and mixed gravitational/gauge anomalies (by substituting some or all gauge bosons above with gravitons), we deduce one more new requirement: $\mathrm{Tr}\, Y = 0$. Again this implies that we must include Higgs doublets in pairs with opposite hypercharge.

3. Include supersymmetric interactions

As we shall see, the most general set of allowed interactions contain some terms that violate baryon number B or lepton number L. These can be removed if a particular discrete symmetry is imposed.

4. Break the supersymmetry by adding the most general set of soft-supersymmetry breaking terms.

The result of steps 1–4:

The minimal supersymmetric extension of the Standard Model (MSSM). The MSSM is defined such that all B and L violating interactions are forbidden by a discrete symmetry.

To go beyond the MSSM, either:

(i) allow for some B or L violation at step 3

(ii) extend the matter sector of the MSSM and repeat steps 1–4

(iii) extend the gauge sector of the MSSM and repeat steps 1–4.

A summary of supersymmetric interactions

① Self-interaction of the gauge supermultiplet

origin of terms: $\frac{1}{2}\int d^2\theta \, \mathrm{tr}\, W^\alpha W_\alpha$ + h.c.

- self-coupling of gauge fields [dictated by non-abelian gauge theory]
- coupling of the gauge field to the gauginos
  (also fixed by the non-abelian gauge theory)

② Interaction of the gauge and matter supermultiplet

origin of terms: $\int d^4\theta \, \bar{\Phi} e^{2gV} \Phi$

- coupling of the gauge field to spin-0 matter
- coupling of the gauge field to spin-½ matter
  (these two are fixed by gauge invariance)
- coupling of the gaugino to spin-½ matter and its superpartner

$$\mathcal{L} = -i\sqrt{2}\, g \left(\bar{\lambda}^a \bar{\psi}_i T^a_{ij} A_j - A_i^* T^a_{ij} \psi_j \lambda^a \right)$$

this is a consequence of the supersymmetry

③ Self-interaction of the matter supermultiplet

- scalar potential

$$V_{scalar} = \sum_i F_i^* F_i + \frac{1}{2}\left[D^a D^a + (D')^2 \right]$$

if the gauge group contains a U(1) factor

- Yukawa interactions

$$\mathcal{L} = -\frac{1}{2}\left[\frac{d^2W}{dA_i dA_j} \psi_i \psi_j + \left(\frac{d^2W}{dA_i dA_j} \right)^* \bar{\psi}_i \bar{\psi}_j \right]$$

origin of these two terms: $\int d^2\theta \, W(\Phi)$ + h.c.

Steps 1 and 2: The spectrum of the MSSM

MSSM = **M**inimal **S**upersymmetric **S**tandard **M**odel

or

Minimal **S**uper**S**ymmetric Extension of the Standard Mo[del]

	Boson Fields	Fermionic Partners	$SU(3)_c$	$SU(2)_L$	$U($
Gauge Multiplets	g	[illegible]	[illegible]	[illegible]	[illegible]
	W	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Matter Multiplets					
leptons	[illegible]	[illegible]	1	2	-1
	[illegible]	[illegible]	1	1	2
quarks	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Higgs	H [illegible]	[illegible]	1	[illegible]	[illegible]
	H [illegible]	[illegible]	1	[illegible]	[illegible]

The matter multiplets of the MSSM originate from chiral superfields:

$$\hat{L}, \hat{E}, \hat{Q}, \hat{U}, \hat{D}, \hat{H}_1 \text{ and } \hat{H}_2$$

where hats will be used to indicate superfields.

For example,

$$\hat{E} = A_E + \sqrt{2}\,\theta\Psi_E - \theta\theta F_E$$

with $A_E = \tilde{e}_R^+$ and $\begin{pmatrix}\Psi_E\\0\end{pmatrix} = P_L e^+ = e_L^c$.

Supersymmetric interactions

The SUSY interactions are known once we specify the superpotential. The most general gauge invariant superpotential has the following form:

$$W = W_R + W_{NR}$$

WARNING: two sign conventions exist in the literature

where

$$W_R = \varepsilon_{ij}\left[h_\tau \hat{H}_1^i \hat{L}^j \hat{E} + h_b \hat{H}_1^i \hat{Q}^j \hat{D} + h_t \hat{H}_2^j \hat{Q}^i \hat{U} - \mu \hat{H}_1^i \hat{H}_2^j\right]$$

i and j are weak $SU(2)$ indices. Here $\varepsilon_{12} = -\varepsilon_{21} = +1$.

μ = supersymmetric Higgs mass parameter.

Generation labels have been suppressed; h_τ, h_b and h_t are actually 3×3 matrices. In a one-generation model,

$$h_\tau = \frac{\sqrt{2}m_\tau}{v_1}, \quad h_b = \frac{\sqrt{2}m_b}{v_1}, \quad h_t = \frac{\sqrt{2}m_t}{v_2}$$

where $\langle H_1^0\rangle = \frac{v_1}{\sqrt{2}}$, $\langle H_2^0\rangle = \frac{v_2}{\sqrt{2}}$

$$W_{NR} = \varepsilon_{ij}\left[\lambda_L \hat{L}^i \hat{L}^j \hat{E} + \lambda_L' \hat{L}^i \hat{Q}^j \hat{D} - \mu' \hat{L}^i \hat{H}_2^j\right] + \lambda_B \hat{U}\hat{D}\hat{D}$$

where generation labels are again suppressed. Note that λ_L must be antisymmetric upon interchange of $\hat{L}^i$ and $\hat{L}^j$.

One quickly observes that the terms in W_{NR} violate either baryon number (B) or lepton number (L):

$\boxed{\text{In the MSSM, set } W_{NR} = 0.}$

In this regard, the MSSM is not as elegant as the SM. Recall that if one imposes $SU(3)\times SU(2)\times U(1)$ on all possible SM interactions, one finds that all terms of dimension ≤ 4 preserve B and L. Not so in supersymmetry!

How does one impose $W_{NR} = 0$?

1. Matter parity

The MSSM does not distinguish Higgs and Quark/Lepton Superfields. Define matter parity such that all Quark/Lepton superfields change sign but the Higgs superfields do not.

In superspace, a chiral superfield is

$$\hat{\Phi} = A(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta F(x)$$

Under a continuous $U(1)_R$ symmetry, $\theta \to e^{i\alpha}\theta$ and $\hat{\Phi} \to e^{in\alpha}\hat{\Phi}$. The superfield has $R = n$. This means that the component fields have $R(A) = n$, $R(\psi) = n-1$, $R(F) = n-2$. The superpotential W must have $R(W) = 2$ in order that the theory conserve $U(1)_R$, since $[W]_F + h.c.$ appears in the supersymmetric Lagrangian.

Thus, to set $W_{NR} = 0$, choose

$$R = 1 \quad \text{for } \hat{H}_1, \hat{H}_2$$
$$R = 1/2 \quad \text{for } \hat{L}, \hat{E}, \hat{Q}, \hat{U}, \hat{D}$$

In fact, $U(1)_R$ is too restrictive. Consider the gauge super-multiplet, $\hat{V}$. Since $\hat{V}$ is real, we must have $R(\hat{V}) = 0$ which means that $R(V_\mu) = 0$, $R(\lambda) = 1$. That is, $U(1)_R$ forbids Majorana masses for the gaugino.

In [illegible]

called R-[illegible]

$$R = (-1)^{3(B-L)+2S}$$

for part[illegible] invariance $W_{NR} = 0$.

At this point, our model consists of a supersymmetric gauge field theory based on $SU(3)\times SU(2)\times U(1)$.

But, it is not yet realistic for two reasons:

- supersymmetry is an exact symmetry
- $SU(2)\times U(1)$ is unbroken

To illustrate the second point, examine the scalar potential:

$$V_{scalar} = \sum_i F_i^* F_i + \tfrac{1}{2}\left[D^a D^a + (D')^2\right]$$

The result:

$$V_{Higgs} = |\mu|^2\left[|H_1|^2 + |H_2|^2\right] \longleftarrow \text{from the F-terms}$$

$$+\tfrac{1}{8}(g^2+g'^2)\left[|H_1|^2 - |H_2|^2\right]^2 + \tfrac{1}{2}g^2|H_1^* H_2|^2$$

(exercise: derive this!)

from the D-terms

Clearly, $V_{Higgs} \geq 0$ and $H_1 = H_2 = 0$ minimizes the Higgs potential (giving $\langle V_{Higgs}\rangle = 0$ as expected for a supersymmetric vacuum). Hence, no $SU(2)\times U(1)$ breaking.

We shall see that the addition of soft-SUSY-breaking terms will also permit the breaking of the electroweak symmetry.

Soft-SUSY-breaking terms

Assume R-parity invariance

notation: $\tan\beta \equiv \frac{v_2}{v_1}$ $\qquad v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2$

$m_W = \frac{1}{2} g v$

$$
\begin{aligned}
V_{soft} = \; & m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_{12}^2 (\varepsilon_{ij} H_1^i H_2^j + h.c.) \\
& + M_Q^2 (\tilde{t}_L^* \tilde{t}_L + \tilde{b}_L^* \tilde{b}_L) + M_U^2 \tilde{t}_R^* \tilde{t}_R + M_D^2 \tilde{b}_R^* \tilde{b}_R \\
& + M_L^2 (\tilde{\nu}^* \tilde{\nu} + \tilde{\tau}_L^* \tilde{\tau}_L) + M_E^2 \tilde{\tau}_R^* \tilde{\tau}_R \\
& + \frac{g}{\sqrt{2}\, m_W} \varepsilon_{ij} \left[\frac{m_\tau A_\tau}{\cos\beta} H_1^i \tilde{\ell}_L^j \tilde{\tau}_R^* + \frac{m_b A_b}{\cos\beta} H_1^i \tilde{q}_L^j \tilde{b}_R^* \right. \\
& \qquad \left. + \frac{m_t A_t}{\sin\beta} H_2^i \tilde{q}^i \tilde{t}_R^* \right] \\
& + \frac{1}{2} \left[M_3 \tilde{g} \tilde{g} + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B} \widetilde{B} + h.c. \right]
\end{aligned}
$$

where $\tilde{\ell}_L \equiv \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$ and $\tilde{q}_L \equiv \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}$. I have written this for the case of one generation. For three generations, $M_Q^2, M_U^2, M_D^2, M_L^2, M_E^2$ are 3×3 matrices.

color code

—— scalar soft squared-mass terms

—— B-term $\quad m_{12}^2 \equiv b = B\mu$

—— A-terms

—— gaugino Majorana mass terms

Mass Eigenstates of the MSSM

(i) scalar-quark sector

In principle, I must diagonalize 6×6 matrices corresponding to the basis $(\tilde{q}_{iL}, \tilde{q}_{iR})$ $i=1,2,3$ ← generation labels.

$$M^2_{\tilde{t}} = \begin{bmatrix} M_Q^2 + m_t^2 + m_Z^2\cos 2\beta\left(\frac{1}{2} - e_u s_W^2\right) & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & M_U^2 + m_t^2 + m_Z^2\cos 2\beta\, e_u s_W^2 \end{bmatrix}$$

$$M^2_{\tilde{b}} = \begin{bmatrix} M_Q^2 + m_b^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} + e_d s_W^2\right) & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & M_D^2 + m_b^2 + m_Z^2\cos 2\beta\, e_d s_W^2 \end{bmatrix}$$

where $e_u = 2/3$, $e_d = -1/3$, $s_W^2 \equiv \sin^2\theta_W$ and $\tan\beta \equiv \langle H_2^0\rangle / \langle H_1^0\rangle$.

(ii) scalar-lepton sector

$$M_{\tilde{\nu}}^2 = M_L^2 + \tfrac{1}{2} m_Z^2 \cos 2\beta$$

$$M_{\tilde{\tau}}^2 = \begin{bmatrix} M_L^2 + m_\tau^2 - m_Z^2 \cos 2\beta \left(\tfrac{1}{2} - s_W^2\right) & m_\tau (A_\tau - \mu \tan\beta) \\ m_\tau (A_\tau - \mu \tan\beta) & M_E^2 + m_\tau^2 - m_Z^2 \cos 2\beta \, s_W^2 \end{bmatrix}$$

These results suggest that $\tilde{f}_L - \tilde{f}_R$ mixing is unimportant in the first two generations. In the third generation, top-squark ("stop") mixing is likely to be the most significant, while bottom-squark ("sbottom") mixing and tau-slepton ("stau") mixing may also be relevant if $\tan\beta \gg 1$.

<u>Remark</u>: expectations for $\tan\beta$

The Higgs-fermion Yukawa coupling are:

$$h_b = \frac{\sqrt{2}\, m_b}{v_1} = \frac{\sqrt{2}\, m_b}{v \cos\beta}, \qquad h_t = \frac{\sqrt{2}\, m_t}{v_2} = \frac{\sqrt{2}\, m_t}{v \sin\beta}$$

Perturbativity of couplings suggest that h_b and h_t should not be too large. This would imply (crudely) that:

$$1 \lesssim \tan\beta \lesssim \frac{m_t}{m_b}$$

(More convincing arguments exist, but will not be given here.)

(iii) charged SUSY fermions ($\tilde{\chi}_i^\pm$, $i=1,2$) <u>CHARGINOS</u>

Charginos are mixtures of gauginos and higgsinos. They arise from the SUSY-interaction

$$-\mathcal{L} = i\sqrt{2}\, g^a \left(\bar{\lambda}^a \bar{\psi}_i T^a_{ij} A_j - A_i^* T^a_{ij} \psi_j \lambda^a \right)$$

when the Higgs bosons acquire their vacuum expectation values.
Two other sources for mass terms are:

(i) $-\mathcal{L} = \frac{1}{2} \left\langle \frac{d^2 W}{dA_i\, dA_j} \right\rangle \psi_i \psi_j$ due to the μ-term

(ii) soft-SUSY-breaking Majorana mass term for the gaugino.

Writing

$$-\mathcal{L}_{mass} = \frac{1}{2} (\psi^+ \ \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c.$$

with:

$$\psi^+ = (-i\lambda^+ \ \ \psi_{H_2}^+)$$
$$\psi^- = (-i\lambda^- \ \ \psi_{H_1}^-)$$

we find:

$$X = \begin{pmatrix} M_2 & g v_2 \\ g v_1 & \mu \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

Mass diagonalization works as follows. Let:

$$\chi_i^+ = V_{ij} \psi_j^+$$
$$\chi_i^- = U_{ij} \psi_j^-$$

V, U unitary

Then,

$$-\mathcal{L}_{mass} = \chi_i^-(M_D)_{ij}\,\chi_j^+ + h.c.$$

where M_D is a diagonal matrix with positive entries and:

$$U^* X V^{-1} = M_D$$

To determine U and V, note that:

$$M_D^\dagger M_D = V X^\dagger X V^{-1}$$
$$M_D M_D^\dagger = U^* X X^\dagger U^{*-1}$$

so all we have to do is to diagonalize $X^\dagger X$ and $XX^\dagger$ and adjust the relative phases of U^* and V such that the entries of M_D are positive.

The chargino masses are the positive square roots of the eigenvalues of $X^\dagger X$ (or $XX^\dagger$):

$$m^2_{\chi_i^\pm} = \frac{1}{2}\Big\{|\mu|^2 + |M_2|^2 + 2m_W^2$$
$$\mp\Big[(|\mu|^2 + |M_2|^2 + 2m_W^2)^2 - 4|\mu|^2|M_2|^2$$
$$-4m_W^4\sin 2\beta + 8m_W^2\sin 2\beta\,\mathrm{Re}(\mu M_2)\Big]^{1/2}$$

<u>eigenstates</u>: $\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$ with $m_{\tilde{\chi}_1} < m_{\tilde{\chi}_2}$.

(iv) neutral SUSY fermions ($\tilde{\chi}_i^0$, $i=1,\dots,4$) <u>NEUTRALINOS</u>

Neutralinos are mixtures of neutral gauginos and higgsinos. Following the previous analysis, we obtain:

$$-\mathcal{L}_{mass} = \frac{1}{2}\psi_i^0 Y_{ij} \psi_j^0 + h.c.$$

with

$$\psi^0 = (-i\lambda' \;\; -i\lambda^3 \;\; \psi_{H_1}^0 \;\; \psi_{H_2}^0)$$

λ' is the hypercharge gangino ("bino")
λ^3 is the W^3-gaugino ("wino")

and

$$Y = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}.$$

Mass diagonalization:

$$\chi_i^0 = N_{ij}\,\psi_j^0$$

$s_W = \sin\theta_W$
$c_W = \cos\theta_W$
$s_\beta = \sin\beta$
$c_\beta = \cos\beta$

$$-\mathcal{L}_{mass} = \chi_i^0 (M_D)_{ij}\, \chi_j^0 + h.c.$$

Then:

$$N^* Y N^{-1} = M_D$$

The phases of N can be adjusted such that all entries of the diagonal matrix M_D are positive.

Limiting cases for the neutralino mass matrix:

(i) $M_1 = M_2 = \mu = 0$

$$\tilde{\chi}_1^0 = \tilde{\gamma} \qquad\qquad m = 0$$
$$\tilde{\chi}_2^0 = \tilde{H}_1^0 \sin\beta + H_2^0 \cos\beta \qquad\qquad m = 0$$
$$\tilde{\chi}_3^0 = \sqrt{\tfrac{1}{2}}\left[\tilde{Z} + \tilde{H}_1^0 \cos\beta - \tilde{H}_2^0 \sin\beta\right] \qquad\qquad m = m_Z$$
$$\tilde{\chi}_4^0 = \sqrt{\tfrac{1}{2}}\left[-\tilde{Z} + \tilde{H}_1^0 \cos\beta - \tilde{H}_2^0 \sin\beta\right] \qquad\qquad m = m_Z$$

where
$$\tilde{\gamma} = c_W \tilde{B} + s_W \tilde{W}^3 \qquad \text{"photino"}$$
$$\tilde{Z} = -s_W \tilde{B} + c_W \tilde{W}^3 \qquad \text{"zino"}$$

Clearly, nature is not very close to this limit.

(ii) $M_1, M_2, |\mu| >> m_Z$

$$\tilde{\chi}_i^0 = \left\{ \tilde{B}, \tilde{W}_3, \tfrac{1}{\sqrt{2}}(\tilde{H}_1^0 - \tilde{H}_2^0), \tfrac{1}{\sqrt{2}}(\tilde{H}_1^0 + \tilde{H}_2^0) \right\}$$

with masses $|M_1|$, $|M_2|$, $|\mu|$ and $|\mu|$ respectively

Standard notation: $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$.

(v) The MSSM Higgs sector

Including the soft-SUSY-breaking terms, the Higgs potential now reads:

SUSY-breaking (B-term)

$$V_{Higgs} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (m_{12}^2 \varepsilon_{ij} H_1^i H_2^j + h.c.)$$
$$+ \frac{1}{8}(g^2+g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g^2 |H_1^* H_2|^2$$

D-term

where $m_1^2 \equiv |\mu|^2 + m_{H_1}^2$

$m_2^2 \equiv |\mu|^2 + m_{H_2}^2$

F-term SUSY-breaking

Let us search for a minimum where

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

where the vacuum expectation values ("vev's") appear in the charge-neutral components.

$$\langle V_{Higgs} \rangle = \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2 - m_{12}^2 v_1 v_2 + \frac{1}{32}(g^2+g'^2)(v_1^2 - v_2^2)^2$$

Minimize this:

$$\frac{\partial V}{\partial v_1} = 0 \qquad m_1^2 = m_{12}^2 \frac{v_2}{v_1} + \frac{1}{8}(v_2^2 - v_1^2)(g^2+g'^2)$$

$$\frac{\partial V}{\partial v_2} = 0 \qquad m_2^2 = m_{12}^2 \frac{v_1}{v_2} - \frac{1}{8}(v_2^2 - v_1^2)(g^2+g'^2)$$

If we write:

$$\langle V_{Higgs}\rangle = \frac{1}{32}(g^2+g'^2)(v_1^2-v_2^2)^2 + \frac{1}{2}(v_1 \;\; v_2)\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix},$$

then, for the vev's to correspond to a potential <u>minimum</u>,

$$\det\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix} < 0$$

i.e. $\boxed{m_1^2 m_2^2 < m_{12}^4}$

A second condition arises, since for $v_1 = v_2$, $\langle V_{Higgs}\rangle = (m_1^2+m_2^2-2m_{12}^2)\frac{1}{2}$ which would be unbounded from below unless:

$$\boxed{m_1^2+m_2^2 \geq 2m_{12}^2}$$

<u>Technicality</u>: in deriving these two conditions, I assumed that m_{12}^2, v_1 and v_2 are real. If m_{12}^2 is complex, I can absorb its phase into a re-definition of the Higgs fields. Similarly, this phase freedom can be used to take $v_1, v_2 > 0$. For the more general case, replace m_{12}^2 by $|m_{12}^2|$ in the above two conditions.

Thus, we have demonstrated the existence of the desired $SU(2)\times U(1)$ breaking minimum.

To compute the Higgs mass spectrum, we write:

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 + h_1^0 + i a_1^0 \\ h_1^- \end{pmatrix} \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} h_2^+ \\ v_2 + h_2^0 + i a_2^0 \end{pmatrix}$$

From the potential, we compute the quadratic terms in the scalars to derive squared-mass matrices. Diagonalizing these matrices yield the physical states:

$(h_1^0, h_2^0) \longrightarrow h^0, H^0$ CP-even Higgs bosons, with $m_h < m_H$.

$(a_1^0, a_2^0) \longrightarrow A^0, G^0$

CP-odd Higgs boson

Goldstone boson that gives mass to the Z

$(h_1^\pm, h_2^\pm) \longrightarrow H^\pm, G^\pm$

charged Higgs bosons

Goldstone boson that gives mass to the $W^\pm$

Physical Higgs degrees of freedom = 5

$$h^0, H^0, A^0, H^\pm$$

It is easy to show that the CP-even Higgs boson arise simply by considering

$$\langle V_{Higgs}\rangle = \frac{1}{2}(v_1\ v_2)\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2\end{pmatrix}\begin{pmatrix} v_1 \\ v_2\end{pmatrix} + \frac{1}{32}(g^2+g'^2)(v_1^2-v_2^2)$$

and computing

$$M_{ij}^2 = \frac{\partial^2\langle V_{Higgs}\rangle}{\partial v_i \partial v_j}$$

In deriving this result, we use the minimum conditions to eliminate m_1^2 and m_2^2.

Then,

$$M^2 = \frac{1}{v_1^2+v_2^2}\begin{pmatrix} m_A^2 v_2^2 + m_Z^2 v_1^2 & -(m_A^2+m_Z^2)v_1 v_2 \\ -(m_A^2+m_Z^2)v_1 v_2 & m_A^2 v_1^2 + m_Z^2 v_2^2\end{pmatrix}$$

where:

$$m_A^2 \equiv \frac{m_{12}^2}{v_1 v_2}(v_1^2+v_2^2)$$

$$m_Z^2 = \frac{1}{4}(g^2+g'^2)(v_1^2+v_2^2)$$

We shall demonstrate shortly that m_A is the mass of the CP-odd Higgs boson.

Squared-mass eigenvalues

$$m_{H,h}^2 = \frac{1}{2}\left[m_A^2+m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2 2\beta}\right]$$

In diagonalizing the CP-even squared-Higgs mass matrix, the corresponding physical eigenstates are:

$$h = -h_1^0 \sin\alpha + h_2^0 \cos\alpha$$
$$H = h_1^0 \cos\alpha + h_2^0 \sin\alpha$$

where the CP-even mixing angle α is given by:

$$\cos 2\alpha = -\cos 2\beta \left(\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}\right)$$

$$\sin 2\alpha = -\sin 2\beta \left(\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}\right)$$

In our conventions, $v_1, v_2 > 0 \Rightarrow \tan\beta > 0$ so we can take $0 < \tan\beta < \frac{\pi}{2}$. It follows that $-\frac{\pi}{2} < \alpha < 0$ [since $\sin 2\beta > 0$].

We finish up by displaying the CP-odd and charged Higgs squared mass matrices:

$$M^2_{odd} = m_{12}^2 \begin{pmatrix} v_2/v_1 & 1 \\ 1 & v_2/v_1 \end{pmatrix}, \qquad M^2_{charged} = \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g^2\right)\begin{pmatrix} v_2^2 \\ v_1 v_2 \end{pmatrix}$$

Note that both these have zero eigenvalues (the Goldstone boson since the determinants are zero. Thus, the squared-masses of A and $H^\pm$ are obtained from the trace of each squared-mass mat

$$m_A^2 = m_{12}^2\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right)$$
$$m_{H^\pm}^2 = \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g^2\right)(v_1^2 + v_2^2) = m_A^2 + m_W^2$$

Summary:

$$m^2_{H,h} = \frac{1}{2}\left[m_A^2 + m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right]$$

$$m_A^2 = \frac{m_{12}^2}{v_1 v_2}(v_1^2+v_2^2)$$

$$m^2_{H^\pm} = m_A^2 + m_W^2$$

Observations

(i) $m_h^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2$

(ii) $m_H \geq m_Z$

(iii) $m_{H^\pm} \geq m_W$

(iv) m_A is not restricted

In fact, given $\tan\beta$ and m_A, all tree-level Higgs properties are determined.

Phenomenological disaster?

LEP has searched for the Higgs bosons of the MSSM. They conclude that $m_h > m_Z$. Does this mean that the MSSM is ruled out?

The radiatively corrected Higgs mass

At one-loop, corrections to the Higgs masses arise. The most important of these are due to:

In the SUSY-limit, these contributions would cancel. But, due to soft-susy breaking, there is now an incomplete cancellation. We can do a quick and dirty computation to expose the largest effect:

Recall the formula for the effective potential. Setting $\mathrm{Str}\, M^2(\phi) = 0$,

$$V_{eff}(\phi) = V_{tree}(\phi) + \frac{1}{64\pi^2} \mathrm{Str}\left\{ M_i^4(\phi) \left[\ln \frac{M_i^2(\phi)}{\Lambda^2} - \frac{1}{2} \right] \right\}.$$

Examine the contribution of t and $\tilde{t}$ to Str. For simplicity, we ignore stop mixing and keep only the dominant terms. Moreover, we set $M_Q = M_U \equiv M_S$. Then,

$$m_{\tilde{t}}^2 \equiv m_{\tilde{t}_1}^2 = m_{\tilde{t}_2}^2 = M_S^2 + \frac{1}{2} h_t^2 v_2^2$$

$$m_t = \frac{1}{\sqrt{2}} h_t v_2$$

$$V^{(1)}(v_1, v_2) = \frac{\overset{\text{color}}{3}}{64\pi^2} \cdot \underset{\text{complex}}{2} \cdot \underset{\tilde{t}_1, \tilde{t}_2}{2} \left[\left(M_S^2 + \frac{1}{2} h_t^2 v_2^2\right)^2 \left[\ln\left(\frac{M_S^2 + \frac{1}{2} h_t^2 v_2^2}{\Lambda^2} \right) - \frac{1}{2} \right] - \frac{1}{4} h_t^4 v_2^4 \left[\ln\left(\frac{\frac{1}{2} h_t^2 v_2^2}{\Lambda^2} \right) - \frac{1}{2} \right] \right]$$

Repeat our previous calculation of the CP-even Higgs squared-mass matrix

$$V = \frac{1}{32}(g^2+g'^2)(v_1^2-v_2^2)^2 + \frac{1}{2}(v_1 \;\; v_2)\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix} + V^{(1)}$$

$$\frac{\partial V}{\partial v_2} = 0 \quad \Rightarrow \quad m_2^2 = m_{12}^2 \frac{v_1}{v_2} - \frac{1}{8}(v_2^2 - v_1^2)(g^2+g'^2) - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2}$$

Using this condition,

$$\frac{\partial^2 V}{\partial v_2^2} = m_{12}^2 \frac{v_1}{v_2} + \frac{1}{4}(g^2+g'^2)v_2^2 - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2} + \frac{\partial^2 V^{(1)}}{\partial v_2^2}$$

From our previous expression for $V^{(1)}$,

$$\frac{\partial^2 V^{(1)}}{\partial v_2^2} - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2} = \frac{3}{8\pi^2} h_t^4 v_2^2 \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Remarkably, Λ drops out!

The effect of the t and $\tilde{t}$ is to modify M_{22}^2 of the CP-even squared-mass matrix:

$$\delta M_{22}^2 = \frac{3g^2 m_t^4}{8\pi^2 m_W^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Diagonalizing the matrix in the limit of $m_A \gg m_Z$ yields the new upper limit:

$$\boxed{m_h^2 \leq m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)}$$

We can repeat the previous computation to include the effects of stop mixing. Recall that:

$$M^2_{stop} = \begin{pmatrix} M_Q^2 + m_t^2 + D_L & m_t X_t \\ m_t X_t & M_U^2 + m_t^2 + D_R \end{pmatrix}.$$

D_L and D_R arise from the D-terms, these are $O(m_Z^2)$.

$$X_t \equiv A_t - \mu \cot\beta$$

The new upper bound, which is saturated when $\tan\beta \gg 1$, is:

$$m_h^2 \leq m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12 M_S^2}\right)\right]$$

where $M_S^2 \equiv \frac{1}{2}\left(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2\right)$.

"minimal" mixing occurs at $X_t = 0$

"maximal" mixing occurs at $X_t = \sqrt{6}\, M_S$

where "minimal" and "maximal" refer to the minimal and maximal Higgs mass bound as a function of X_t.

<u>note</u>: actually, the radiative correction would go negative for $X_t \gg M_S$. But the calculation is not trustworthy in this regime.

Beyond one-loop at fixed order.

- renormalization group improvement

  sums $h_t^2 \ln\left(\frac{M_S^2}{m_t^2}\right)$ terms to all orders

- identify the leading two-loop effects

  leading double logs

  subleading single logs

  $O(m_t^2 h_t^2 \alpha_s)$

  $O(m_t^2 h_t^4)$

  leading squark-mixing effects

- understand relation between on-shell and $\overline{MS}$-parameters.

CONCLUSION

$m_h \lesssim 125 - 135$ GeV

for maximal mixing

$m_h \lesssim 113 - 123$ GeV

for minimal mixing

taking $m_t = 175 \pm 5$ GeV

and $M_S \lesssim 2$ TeV

Carena, Espinosa, Quiros, Wagner

Carena, Quiros, Wagner

Haber, Hempfling and Hoang

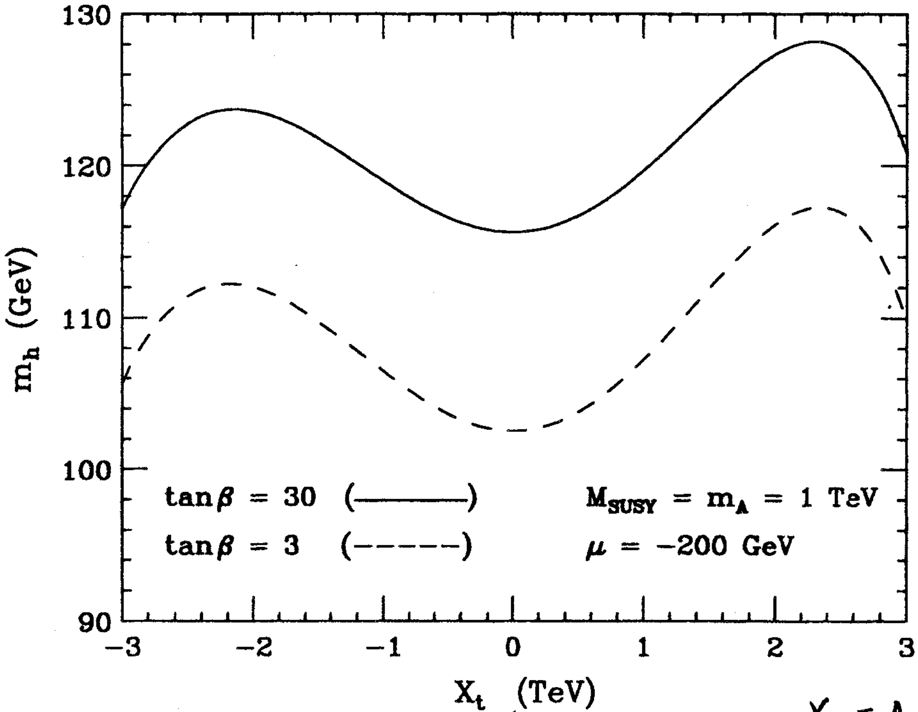
- -

Heinemeyer, Hollik and Weiglein

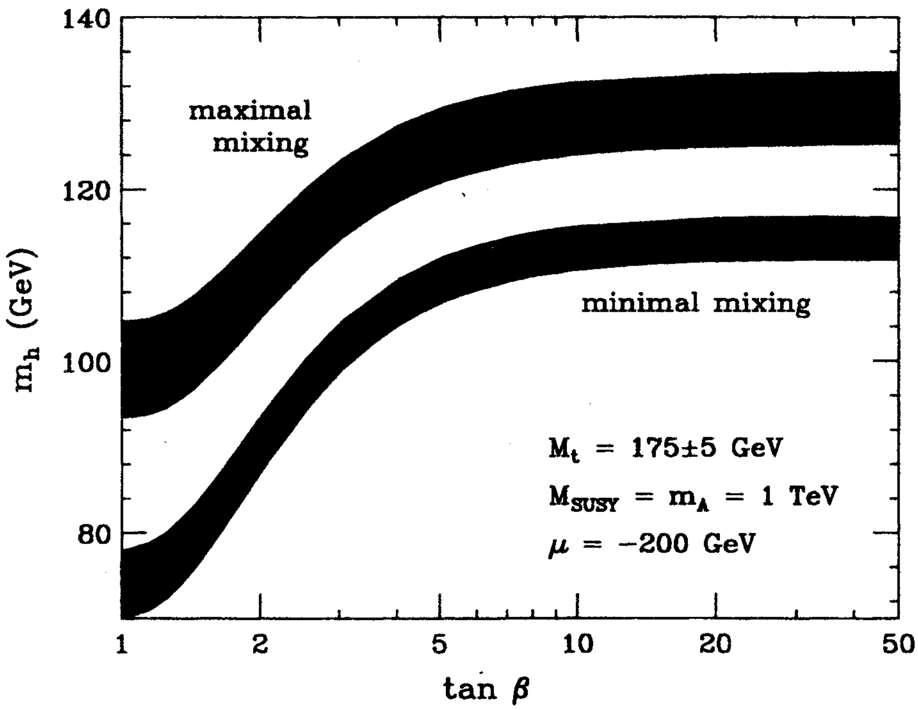
Zhang

Espinosa and Zhang

Carena, Haber, Heinemeyer, Hollik, Wagner and Weiglein



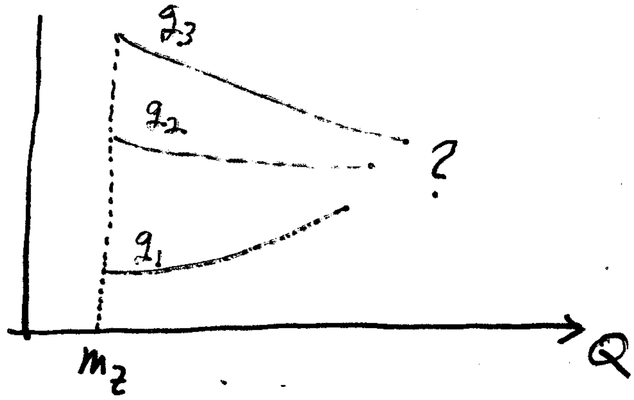
$X_t = A_t - \mu \cot\beta$



Carena, Espinosa, Quiros, Wagner
Haber, Hempfling, Hoang
Heinemeyer, Hollik, Weiglein
Espinosa, Zhang

Unification of Couplings

Grand unification predicts the unification of gauge couplings. Since the running of the coupling constants below the grand unified scale is dictated by the Standard Model particle spectrum, one can test the hypothesis of coupling constant unification.



A subtlety: normalization of the $U(1)_Y$ coupling

In $SU(2)_L \times U(1)_Y$ theory, the overall normalization of the $U(1)_Y$ coupling was a matter of convention. If g' is unified in a non-abelian group, then the relative normalization of g and g' is fixed. To work out the proper normalization, consider the covariant derivative:

$$D_\mu = \partial_\mu + i g_a T^a A_\mu^a$$

At scales above unification, we have complete unification, i.e. $g_a = g_U$ and $\mathrm{Tr}(T^a T^b) = T_R \delta^{ab}$.

Below unification,

$$g_a T^a A_\mu^a = g T^i W_\mu^i + g' \frac{Y}{2} B + \ldots$$

Thus, at the unification point,

$$g_U (W^3 T^3 + B T^0) = g W^3 T^3 + g' B \frac{Y}{2}$$

where T^0 is the properly normalized hypercharge generator when embedded in the grand unified group.

Thus, $g_U = g_3 = g_2 = g_1 =: g$ at the unification point

$$T^0 = \frac{g'}{g_1} \frac{Y}{2}$$

Using $\quad \mathrm{Tr}(T^3)^2 = \mathrm{Tr}(T^0)^2$

$$= \frac{1}{4} \frac{g'^2}{g_1^2} \mathrm{Tr}\, Y^2$$

we conclude that

$$\boxed{g_1^2 = g'^2 \frac{\mathrm{Tr}\, Y^2}{4\,\mathrm{Tr}(T^3)^2}} \implies \boxed{g_1^2 = \frac{5}{3} g'^2}$$

two component fields	T_3	Y	$\mathrm{Tr}\, T_3^2$	$\mathrm{Tr}\, Y^2$
ψ_{Q_1}	$1/2$	$1/3$	$3(1/4)$	$3(1/9)$
ψ_{Q_2}	$-1/2$	$1/3$	$3(1/4)$	$3(1/9)$
ψ_U	0	$-4/3$	$3(0)$	$3(16/9)$
ψ_D	0	$+2/3$	$3(0)$	$3(4/9)$
ψ_{L_1}	$1/2$	-1	$1/4$	1
ψ_{L_2}	$-1/2$	-1	$1/4$	1
ψ_E	0	$+2$	0	4
			2	$40/3$

don't forget the color factor of 3 for the quark fields!

assuming one generation of the Standard Model fills up complete representations of the grand unified gr

Coupling constant evolution

$$\frac{dg_i^2}{dt} = \frac{b_i g_i^4}{16\pi^2}$$

Solution:

$$\frac{1}{g_3^2(m_z)} = \frac{1}{g_U^2} - \frac{b_3}{16\pi} \ln\left(\frac{m_z^2}{M_X^2}\right)$$

$$\frac{1}{g_2^2(m_z)} = \frac{1}{g_U^2} - \frac{b_2}{16\pi} \ln\left(\frac{m_z^2}{M_X^2}\right)$$

$$\frac{1}{g_1^2(m_z)} = \frac{1}{g_U^2} - \frac{b_1}{16\pi} \ln\left(\frac{m_z^2}{M_X^2}\right)$$

Define:

$$\sin^2\theta_W(m_z) = \frac{g'^2(m_z)}{g^2(m_z)+g'^2(m_z)} = \frac{\frac{3}{5}g_1^2(m_z)}{\frac{3}{5}g_1^2(m_z)+g_2^2(m_z)}$$

$$= \frac{3}{8} - \frac{5}{32\pi}\alpha(m_z)(b_1-b_2)\ln\left(\frac{M_X^2}{m_z^2}\right)$$

$$\ln\frac{M_X^2}{m_z^2} = \frac{32\pi}{5b_1+3b_2-8b_3}\left(\frac{3}{8\alpha(m_z)} - \frac{1}{\alpha_s(m_z)}\right)$$

Introduce $\boxed{x \equiv \frac{1}{5}\left(\frac{b_2 - b_3}{b_1 - b_2}\right)}$

Then, we find:

$$\boxed{\sin^2\theta_W(m_Z) = \frac{1}{1+8x}\left[3x + \frac{\alpha(m_Z)}{\alpha_s(m_Z)}\right]}$$

To compute the b_i, we employ:

$$\boxed{b_i = \frac{2}{3} T_f(R_k) \prod_{k \neq j} d_f(R_j) + \frac{1}{6} T_s(R_k) \prod_{k \neq j} d_s(R_j) - \frac{11}{3} C_2(G_i)}$$

f = fermion
s = scalar

$\prod_{k \neq j} d(R_j)$ = multiplicity factors

$d(R) = \dim(R)$

$$\mathrm{Tr}\, T^a T^b = T(R)\, \delta^{ab}$$

$$(T^a T^a)_{ij} = C_2(G)\, \delta_{ij}$$

(**Note**: for the adjoint representation ($R = A$), we have: $T(A) = C_2(G)$.)

For $G = SU(N)$, $C_2(G) = N$
For $G = U(1)$, $C_2(G) = 0$

Note: using the properly normalized hypercharge generator, $\sqrt{\frac{3}{5}}\,\frac{Y}{2}$, it follows that:

$$T(R_1) = \left[\sqrt{\frac{3}{5}}\,\frac{1}{2} Y\right]^2 = \frac{3}{20} Y^2$$

fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$T(R_3)$	$\prod_{i\neq 3} d(R_i)$	$T(R_2)$	$\prod_{i\neq 2} d(R_i)$	$T(R_1)$	$\prod_{i\neq 1} d(R_i)$
$\begin{pmatrix}\psi_{Q_1}\\ \psi_{Q_2}\end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{1}{2}$	2	$\frac{1}{2}$	3	$\frac{1}{60}$	6
ψ_U	3^*	1	$-\frac{4}{3}$	$\frac{1}{2}$	1	0	3	$\frac{4}{15}$	3
ψ_D	3^*	1	$\frac{2}{3}$	$\frac{1}{2}$	1	0	3	$\frac{1}{15}$	3
$\begin{pmatrix}\psi_{L_1}\\ \psi_{L_2}\end{pmatrix}$	1	2	-1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2
ψ_E	1	1	2	0	1	0	1	$\frac{3}{5}$	1
$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$	1	2	1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2
$\begin{pmatrix}\tilde{\phi}^0\\ \tilde{\phi}^-\end{pmatrix}$	1	2	-1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2

$\frac{3}{20}y^2$

<u>example:</u> $$b_3 = \frac{2}{3}\left[\left(\tfrac{1}{2}\right)(2) + \left(\tfrac{1}{2}\right)(1) + \left(\tfrac{1}{2}\right)(1)\right]N_G - \frac{11}{3}(3)$$

Final result:

$$b_3 = \frac{4}{3} N_G - 11$$

$$b_2 = \frac{1}{6} N_H + \frac{4}{3} N_G - \frac{22}{3}$$

$$b_1 = \frac{1}{10} N_H + \frac{4}{3} N_G$$

where I have allowed for N_H copies of the Standard Model Higgs boson.

For the Standard Model, $N_G = 3$ and $N_H = 1$.

$$b_3 = -7$$

$$b_2 = \frac{-19}{6}$$

$$b_1 = \frac{41}{10}$$

$$x = \frac{1}{5}\left(\frac{b_2 - b_3}{b_1 - b_2}\right) = \frac{23}{218} = 0.1055$$

Remark: Notice that N_G drops out completely in the expression for x. (this is a special feature of the one-loop calculation). Thus, in this approximation, the success (or failure) of unification does not depend on the number of fermion generations.

Check the prediction of $\alpha_s(m_Z)$.

$$\alpha_s(m_Z) = \frac{\alpha(m_Z)}{(1+8x)\sin^2\theta_w(m_Z) - 3x}$$

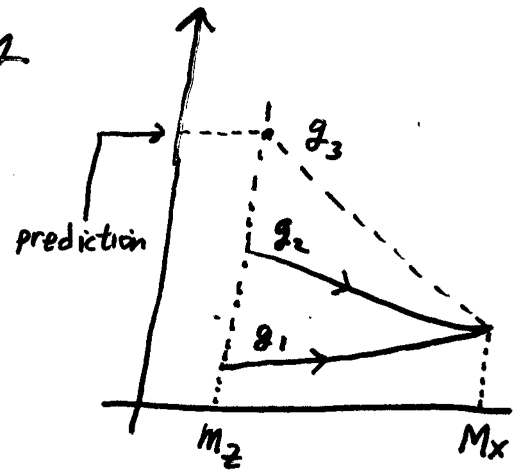
These are $\overline{MS}$-couplings.

$\sin^2\theta_w(m_Z)_{\overline{MS}} = 0.2315 \pm 0.0004$

$\alpha^{-1}(m_Z)_{\overline{MS}} = 127.90 \pm 0.09$

$x = 0.1055$

$\Longrightarrow \quad \alpha_s(m_Z) = 0.071$



to be compared with the world average:

$\alpha_s(m_Z) = 0.118 \pm 0.003$

Is this a hint for the minimal supersymmetric standard model (MSSM)?

exercise: Show that $x = \frac{1}{7}$ in the MSSM

$\Longrightarrow \alpha_s(m_Z) = 0.116$

Note: two-loop corrections to unification are not negligible. The prediction for $\alpha_s(m_Z)$ increases by roughly 0.01, not nearly enough to save Standard Model unification.

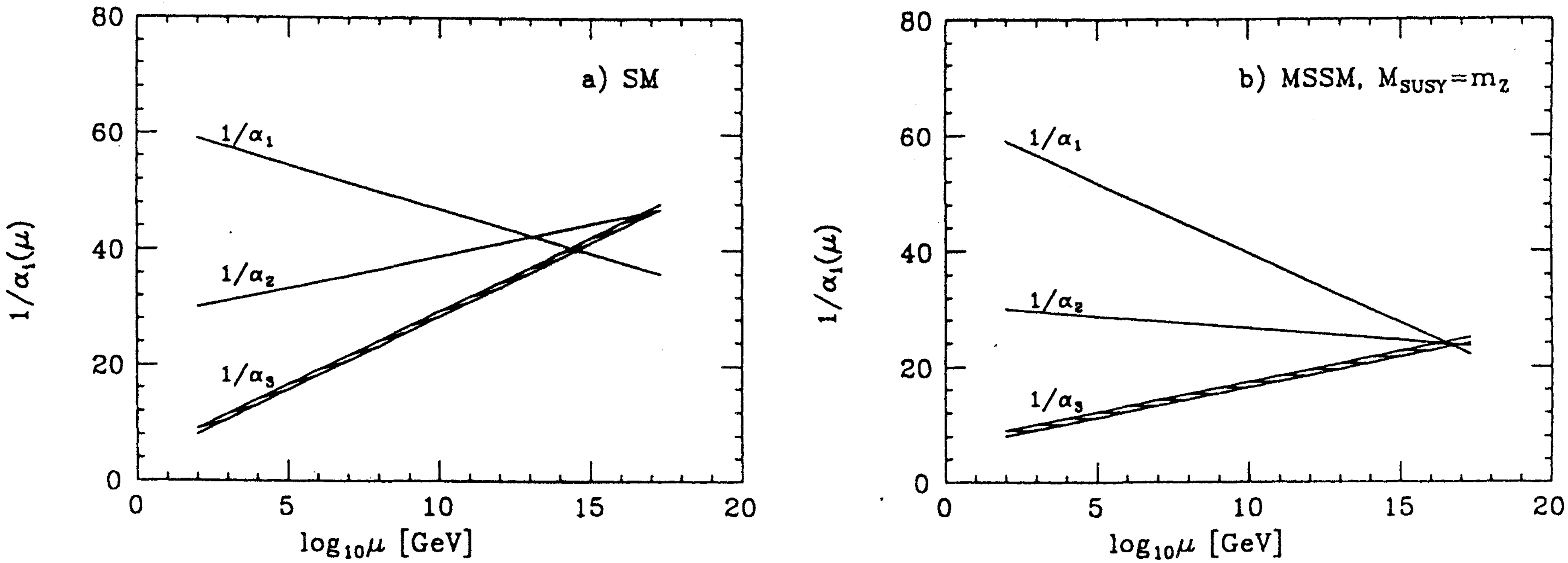


Figure 2: The running of the gauge couplings in the Standard Model (top) and its supersymmetric extension (bottom). Both figures assume $\alpha_S(m_Z) = 0.120 \pm 0.01$. In the lower frame an effective SUSY particle threshold at m_Z has been assumed; adapted from

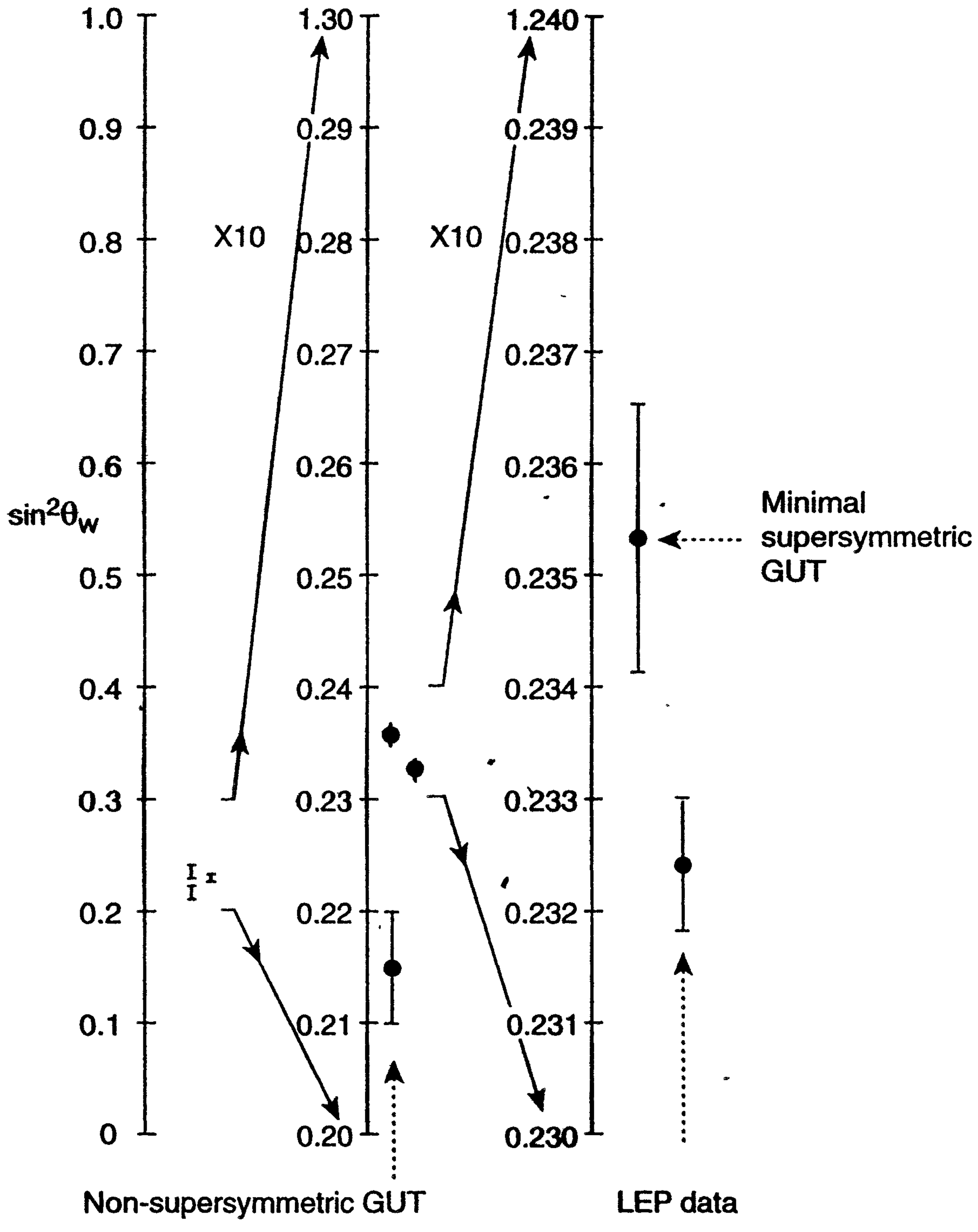


Fig. 8. Gee-whizz plot showing how well GUT predictions of $\sin^2\theta_W$ agree with the experimental data.

from J. Ellis (Lepton-Photon Conference Beijing 1995)

NMSSM

Remove the $\mu \hat{H}_1 \hat{H}_2$ term from the superpotential. Replace it with:

$$\lambda_1 \hat{H}_1 \hat{H}_2 \hat{N} + \lambda_2 \hat{N}^3$$

where λ_i is dimensionless. One obtains an effective μ, namely $\mu = \lambda_1 \langle \hat{N} \rangle$.

In this model, the number of neutralinos and neutral Higgs fields are increased.

(ii) The see-saw model of neutrino masses introduces a heavy right-handed neutrino. One can construct a supersymmetric extension of the seesaw model by introducing $\hat{N}$ as above, but with a different superpotential.

<u>remark</u>: in the MSSM, R-parity conservation is imposed to guarantee L conservation in the low-energy theory. The supersymmetric see-saw model is R-parity conserving since L is violated by two units in the neutrino mass term: $-\mathcal{L}_m = m_\nu \nu\nu + h.c.$

R-Parity Conserving Models

In the R-Parity-Conserving (RPC) MSSM:

- $R = (-1)^{3(B-L)+2S}$
- The LSP is stable
- Neutrinos are massless

To obtain neutrino masses consistent with RPC, one must violate L by two units. The simplest model is the supersymmetric extension of the seesaw.

One-generation model

$$W \ni \epsilon_{ij}\left[\lambda \hat{H}_U^i \hat{L}^j \hat{N} - \mu \hat{H}_D^i \hat{H}_U^j\right] + \tfrac{1}{2} M \hat{N}\hat{N}$$

$$V_{\rm soft} \ni m_L^2 \tilde{\nu}^* \tilde{\nu} + m_{\widetilde{N}}^2 \widetilde{N}^* \widetilde{N} + \left[\lambda A_\nu H_U^0 \tilde{\nu} \widetilde{N}^* + M B_N \widetilde{N}\widetilde{N} + \text{h.c}\right.$$

After EWSB, $\langle H_i^0 \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_u/v_d$, one obtains the usual seesaw result:

$$\mathcal{M}_N = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

where $m_D \equiv \lambda v_u$. Thus, taking $m_D \ll M$, $m_\nu \simeq m_D^2/M$.

The sneutrino masses are obtained by diagonalizing a 4×4 squared-mass matrix. Here, it is convenient to define:
$\tilde{\nu} = (\tilde{\nu}_1 + i\tilde{\nu}_2)/\sqrt{2}$ and $\tilde{N} = (\tilde{N}_1 + i\tilde{N}_2)/\sqrt{2}$.
Then, the squared-sneutrino mass matrix ($\mathcal{M}^2$) separates into CP-even and CP-odd blocks:

$$\mathcal{M}^2 = \tfrac{1}{2} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix} \begin{pmatrix} \mathcal{M}_+^2 & 0 \\ 0 & \mathcal{M}_-^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

where $\phi_i \equiv \begin{pmatrix} \tilde{\nu}_i & \tilde{N}_i \end{pmatrix}$ and $\mathcal{M}_\pm^2$ consist of the following 2×2 blocks:

$$\begin{pmatrix} m_{\tilde{L}}^2 + \frac{1}{2} m_Z^2 \cos 2\beta + m_D^2 & m_D[A_\nu - \mu \cot\beta \pm M] \\ m_D[A_\nu - \mu \cot\beta \pm M] & M^2 + m_D^2 + m_{\tilde{N}}^2 \pm 2 B_N M \end{pmatrix}.$$

To first order in $1/M$, the two light sneutrino eigenstates are $\tilde{\nu}_1$ and $\tilde{\nu}_2$, with corresponding squared masses:

$$m_{\tilde{\nu}_{1,2}}^2 = m_{\tilde{L}}^2 + \tfrac{1}{2} m_Z^2 \cos 2\beta \mp \tfrac{1}{2} \Delta m_{\tilde{\nu}}^2,$$

where $\Delta m_{\tilde{\nu}}^2 \equiv m_{\tilde{\nu}_2}^2 - m_{\tilde{\nu}_1}^2$. Writing $\Delta m_{\tilde{\nu}}^2 = 2 m_{\tilde{\nu}} \Delta m_{\tilde{\nu}}$,

$$r_\nu \equiv \frac{\Delta m_{\tilde{\nu}}}{m_\nu} \simeq \frac{2(A_\nu - \mu \cot\beta - B_N)}{m_{\tilde{\nu}}}$$

Three-generation model

In the three-generation model, one can choose various alternatives depending on the number of singlet superfields $\hat{N}$. Suppose that there are n_g SM generations.

- If there is only one $\hat{N}$ superfield, then

$$\mathcal{M}_N = \begin{pmatrix} 0 & (m_D)_j \\ (m_D)_i & M \end{pmatrix},$$

  where $(m_D)_i \equiv \lambda_i v_u$ [i labels generation], which yields (at tree-level) $n_g - 1$ massless neutrinos and one light neutrino with mass $m_\nu = \sum_i [(m_D)_i]^2$.

- If there are n_g singlets $\hat{N}_i$, then

$$\mathcal{M}_N = \begin{pmatrix} 0 & (m_D)_{\ell k} \\ (m_D)_{ij} & M_{ik} \end{pmatrix},$$

  where $(m_D)_{ij} \equiv \lambda_{ij} v_u$, which yields (at tree-level) n_g light neutrinos with nonzero mass. In this case, *all* light neutrinos have mass, since $\det \mathcal{M}_N = (\det m_D)^2 \neq 0$.

In all cases, if the unperturbed ($M \to \infty$) sneutrino masses are non-degenerate, then $(\Delta m_{\tilde{\nu}})_k \neq 0$ for *all* $k = 1, \ldots, n_g$.

R Parity Violating Models

- In a general R-Parity-Violating (RPV) model, both L and B are violated. The corresponding superpotential is

$$W = \epsilon_{ij}\Big[-\mu_\alpha \widehat{L}^i_\alpha \widehat{H}^j_u + \tfrac{1}{2}\lambda_{\alpha\beta m}\widehat{L}^i_\alpha \widehat{L}^j_\beta \widehat{E}_m + \lambda'_{\alpha n m}\widehat{L}^i_\alpha \widehat{Q}^j_n \widehat{D}_m - h_{nm}\widehat{H}^i_u \widehat{Q}^j_n \widehat{U}_m\Big] + (\lambda_B)_{pnm}\widehat{U}_p\widehat{D}_n\widehat{D}_m\,,$$

where α, $\beta = 0,\ldots,3$; m, n, $p = 1,2,3$ and $\hat{L}_0 \equiv \hat{H}_D$.

The RPC model is equivalent to introducing a $\mathbf{Z_2}$ matter parity. To avoid fast proton decay in the RPV model, one may introduce a $\mathbf{Z_3}$ triality, which conserves B. This is the unique choice for a (generation independent) discrete symmetry with no discrete gauge anomalies in a model consisting only of the MSSM superfields. [Ibanez, Ross]

Matter discrete symmetries

symmetry	$\widehat{Q}_n$	$\widehat{U}_n$	$\widehat{D}_n$	$\widehat{L}_n$	$\widehat{E}_n$	$\widehat{H}_U$	$\widehat{H}_D$
$\mathbf{Z_2}$	-1	-1	-1	-1	-1	$+1$	$+1$
$\mathbf{Z_3}$	ω	ω^{-1}	ω^{-1}	$+1$	$+1$	$+1$	$+1$

Note: $\omega \equiv e^{i\pi/3}$

The B-conserving RPV model

$\hat{H}_D$ and $\hat{L}_i$ are indistinguishable $Y=-1$ weak doublets

- Neutrinos mix with neutralinos $\Longrightarrow \quad m_\nu \neq 0$
- Sneutrinos mix with Higgs bosons $\Longrightarrow \quad \Delta m_{\tilde{\nu}} \neq 0$

  $\Delta m_{\tilde{\nu}}$: sneutrino–antisneutrino mass-splitting

Denote $\hat{H}_D$ by $\hat{L}_0$ ($\hat{L}_i \to \hat{L}_\alpha \quad \alpha = 0,1,2,3$)

(MSSM)$_{\rm R}$	(MSSM)$_{\rm B}$
$\mu \hat{H}_D \hat{H}_U$	$\mu_\alpha \hat{L}_\alpha \hat{H}_U$
$h^\ell_{jk} \hat{H}_D \hat{L}_j \hat{E}_k$	$\lambda_{\alpha\beta k} \hat{L}_\alpha \hat{L}_\beta \hat{E}_k$
$h^D_{jk} \hat{H}_D \hat{Q}_j \hat{D}_k$	$\lambda'_{\alpha jk} \hat{L}_\alpha \hat{Q}_j \hat{D}_k$
$b H_D H_U$	$b_\alpha \tilde{L}_\alpha H_U$
$a^\ell_{jk} H_D \tilde{L}_j \tilde{E}_k$	$a_{\alpha\beta k} \tilde{L}_\alpha \tilde{L}_\beta \tilde{E}_k$
$a^D_{jk} H_D \tilde{Q}_j \tilde{D}_k$	$a'_{\alpha jk} \tilde{L}_\alpha \tilde{Q}_j \tilde{D}_k$
$M^2_D H^\dagger_D H_D + (M^2_{\tilde{L}})_{ij} \tilde{L}^\dagger_i \tilde{L}_j$	$(M^2_{\tilde{L}})_{\alpha\beta} \tilde{L}^\dagger_\alpha \tilde{L}_\beta$
v_d	v_α

We define: $\quad v_d^2 = \sum v_\alpha^2\,, \quad \mu^2 = \sum \mu_\alpha^2\,, \quad b^2 = \sum b_\alpha^2$

and $v^2 \equiv v_u^2 + v_d^2 = (246~\text{GeV})^2$, $\quad \tan\beta \equiv v_u/v_d$

$$W = \epsilon_{ij} \; \left[-\mu_\alpha \hat{L}^i_\alpha \hat{H}^j_U + \tfrac{1}{2}\lambda_{\alpha\beta m} \hat{L}^i_\alpha \hat{L}^j_\beta \hat{E}_m + \lambda'_{\alpha n m} \hat{L}^i_\alpha \hat{Q}^j_n \hat{D}_m \right.$$
$$\left. - h_{nm} \hat{H}^i_U \hat{Q}^j_n \hat{U}_m \right]$$

$$V_{\text{soft}} = (M^2_{\widetilde{Q}})_{mn} \, \widetilde{Q}^{i*}_m \widetilde{Q}^i_n + (M^2_{\widetilde{U}})_{mn} \, \widetilde{U}^*_m \widetilde{U}_n + (M^2_{\widetilde{D}})_{mn} \, \widetilde{D}^*_m \widetilde{D}_n$$
$$+ (M^2_{\widetilde{L}})_{\alpha\beta} \, \widetilde{L}^{i*}_\alpha \widetilde{L}^i_\beta + (M^2_{\widetilde{E}})_{mn} \, \widetilde{E}^*_m \widetilde{E}_n + m^2_U |H_U|^2$$
$$- (\epsilon_{ij} b_\alpha \widetilde{L}^i_\alpha H^j_U + \text{h.c.}) + \epsilon_{ij} [\tfrac{1}{2} a_{\alpha\beta m} \widetilde{L}^i_\alpha \widetilde{L}^j_\beta \widetilde{E}_m$$
$$+ a'_{\alpha n m} \widetilde{L}^i_\alpha \widetilde{Q}^j_n \widetilde{D}_m - (a_U)_{nm} H^i_U \widetilde{Q}^j_n \widetilde{U}_m + \text{h.c.}]$$
$$+ \tfrac{1}{2} \left[M_3 \, \widetilde{g} \, \widetilde{g} + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B} \widetilde{B} + \text{h.c.} \right]$$

$$V_D = \tfrac{1}{8} g^2 \Big\{ \Big(|H_U|^2 - \sum_\alpha |\widetilde{L}_\alpha|^2 - \sum_m |\widetilde{Q}_m|^2 \Big)^2 - 2 \sum_{\alpha \neq \beta} |\epsilon_{ij} \widetilde{L}^i_\alpha \widetilde{L}^j_\beta|^2$$
$$+ 4 \sum_\alpha |H^{i*}_U \widetilde{L}^i_\alpha|^2 - 2 \sum_{m \neq n} |\epsilon_{ij} \widetilde{Q}^i_m \widetilde{Q}^j_n|^2$$
$$+ 4 \sum_m |H^{i*}_U \widetilde{Q}^i_m|^2 - 4 \sum_{\alpha m} |\epsilon_{ij} \widetilde{L}^i_\alpha \widetilde{Q}^i_m|^2 \Big\}$$
$$+ \tfrac{1}{8} g'^2 \Big[|H_U|^2 - \sum_\alpha |\widetilde{L}_\alpha|^2 + 2 \sum_m |\widetilde{E}_m|^2 + \tfrac{1}{3} \sum_m |\widetilde{Q}_m|^2$$
$$- \tfrac{4}{3} \sum_m |\widetilde{U}_m|^2 + \tfrac{2}{3} \sum_m |\widetilde{D}_m|^2 \Big]^2 .$$

Neutrino masses: Tree level

In the $\{\widetilde{B}, \widetilde{W}^3, \widetilde{h}_U, \nu_\alpha\}$ basis the 7×7 mass matrix, $M^{(\mathrm{n})}$ is

$$\begin{pmatrix} M_1 & 0 & m_Z s_W v_u/v & -m_Z s_W v_\beta/v \\ 0 & M_2 & -m_Z c_W v_u/v & m_Z c_W v_\beta/v \\ m_Z s_W v_u/v & -m_Z c_W v_u/v & 0 & \mu_\beta \\ -m_Z s_W v_\alpha/v & m_Z c_W v_\alpha/v & \mu_\alpha & 0_{\alpha\beta} \end{pmatrix}$$

Two zero eigenvalues: two massless neutrinos

Five non-zero eigenvalues: four $\tilde{\chi}^0$ and one ν

$$-\det{}' M^{(\mathrm{n})} = m_Z^2 \mu^2 M_{\tilde{\gamma}} \cos^2\beta \, |\hat{v} \times \hat{\mu}|^2$$

$\sin^2\xi \equiv |\hat{v} \times \hat{\mu}|^2 \equiv 1 - (\hat{v} \cdot \hat{\mu})^2$ measures the alignment of v_α and μ_α

$$m_\nu = \frac{\det{}' M^{(\mathrm{n})}}{\det M_0^{(\mathrm{n})}} = \frac{m_Z^2 \mu M_{\tilde{\gamma}} \cos^2\beta \sin^2\xi}{m_Z^2 M_{\tilde{\gamma}} \sin 2\beta + M_1 M_2 \mu} \sim m_Z \sin^2\xi$$

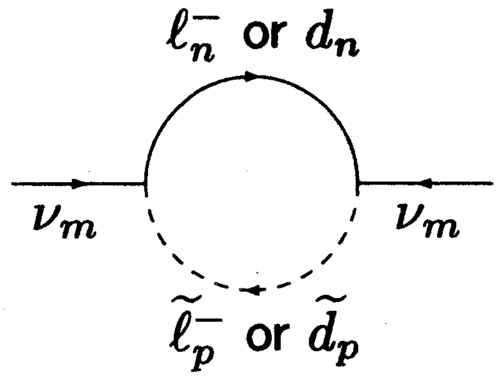
At tree level, $m_\nu \neq 0 \iff \sin\xi \neq 0$

$M_{\tilde{\gamma}} \equiv M_1 c_W^2 + M_2 s_W^2$

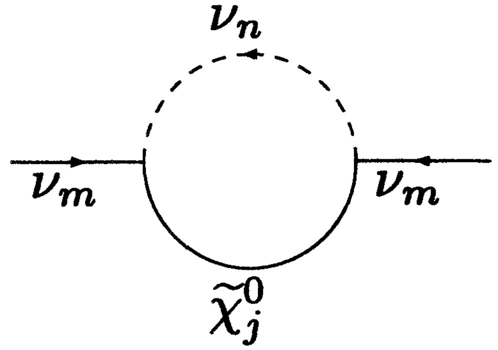
Neutrino masses: Loop effects

Contributions at one loop:

- Lepton–slepton loops and down type quark–squark loops. Proportional to trilinear lepton number violating interactions



- Sneutrino and neutralinos loops. Proportional to sneutrino–antisneutrino mass splitting. Exist in any model with lepton number violation



Sneutrino–neutralinos loops

$$m_\nu^{(1)} = \frac{g^2 \Delta m_{\tilde{\nu}}}{32\pi^2 \cos^2\theta_W} \sum_j f(y_j)|Z_{jZ}|^2 \sim 10^{-3}\Delta m_{\tilde{\nu}}$$

where $f(y_j) = \sqrt{y_j}\,[y_j - 1 - \ln(y_j)]\,/(1-y_j)^2$, Z_{jZ} projects out the $\widetilde{Z}$ eigenstate from $\tilde{\chi}_j^0$, and $y_j \equiv M_{\tilde{\nu}}^2/M_{\tilde{\chi}_j^0}^2$

This contribution exists in any model.

General structure of the one-loop mass:

$$(m_\nu)^{(\tilde{\nu})} \simeq (\text{loop factor}) \times (\text{RPV parameters})$$

If the sizes of the RPV parameters that enter here are roughly the same as the RPV Yukawas that contribute to $(m_\nu)^{(f)}$, then we would expect $(m_\nu)^{(\tilde{\nu})}$ to be the dominant one-loop contribution to the neutrino mass

$$\frac{(m_\nu)^{(\tilde{\nu})}}{(m_\nu)^{(f)}} \sim \frac{1}{\lambda_f^2} \gg 1$$

where λ_f is a down-type Higgs-fermion Yukawa coupling.

Sneutrino–antisneutrino mass splittings

In L-violating RPV models, $\Delta L = 1$ interactions (acting twice) yield $\Delta L = 2$ neutrino masses and sneutrino–antisneutrino mass splitting. The latter arises as a consequence of a squared-mass term: $m^2_{\Delta L=2}\tilde{\nu}\tilde{\nu}$ +h.c.

One expects

- Large ($\sim m_Z$) $\Delta L = 0$ SUSY breaking mass
- Small ($\sim m_\nu$) $\Delta L = 2$ "Majorana" mass

The sneutrino squared-mass matrix is schematically

$$\begin{pmatrix} m^2_{\tilde{\nu}} & m^2_{\Delta L=2} \\ m^2_{\Delta L=2} & m^2_{\tilde{\nu}} \end{pmatrix}$$

This results in sneutrino–antisneutrino mixing and small mass splitting of order $\Delta m_{\tilde{\nu}} \sim m_\nu$.