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SPINON-HOLON INTERACTION IN THE SUPERSYMMETRIC t - J MODEL WITH $1/r^2$-INTERACTION

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Spinon-Holon interaction in the supersymmetric $t-J$-model with $1/r^2$-interaction.

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Motivations.

- **Angle REsolved Photoemision Spectra measrements in quasi-one dimensional insulators ($SrCuO_2$, Sr_2CuO_3).**

  (From Z. X. Shen et. al., Phys. Rev. B56, 15589 (1997)).

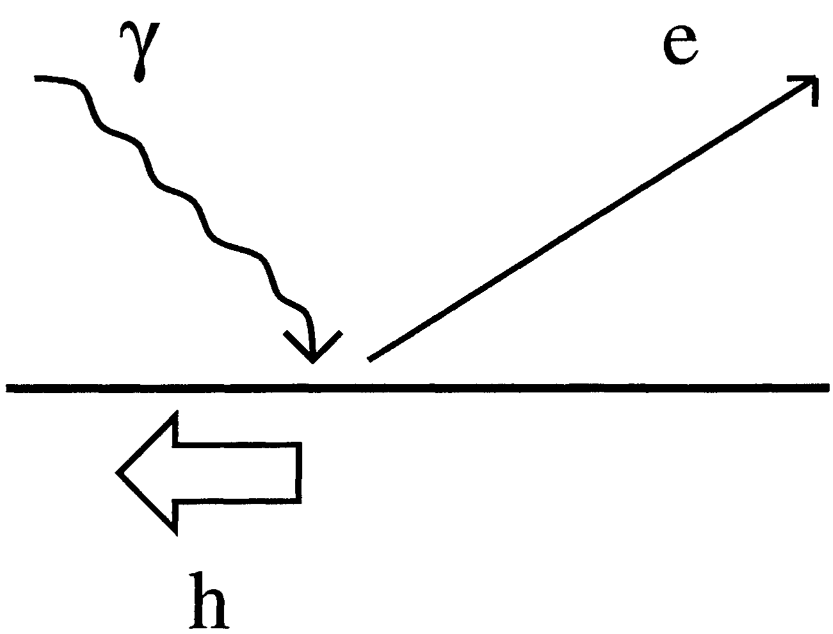


Figure 1 Photoemission with recoiling hole

- **Divergences at threshold followed by a branch cut should be seen in a clean experiment. We show that divergences at threshold followed by broad spectra are the main consequence of spinon-holon interaction.**

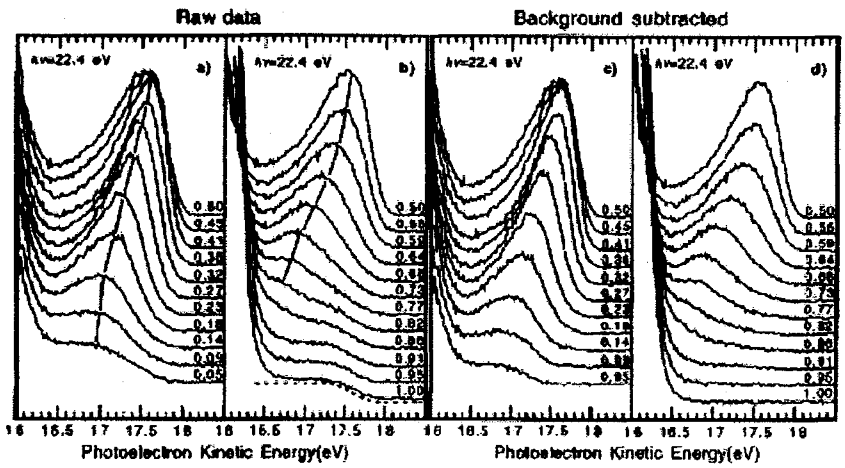
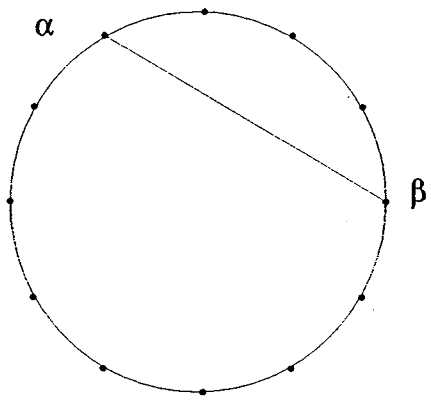


Figure 2: ARPES spectra.

Content of the talk.

- The model: Kuramoto-Yokohama Hamiltonian, Ground state and elementary excitations: spinons and holons;
- One-spinon one-holon eigenstates;
- Real-space representation for one-spinon one-holon function: Equation of motion and its solution. Spinon-holon attraction;
- Hole spectral density of states and spinon-holon interaction;
- Back to physics: Experimental consequences of spinon-holon interaction. Hole instability and broad spectra;
- Conclusions and further developments.

Supersymmetric $t-J$-model with $1/r^2$-interaction.



$z_\alpha = e^{\frac{2\pi i \alpha}{N}}$

Figure 3: Kuramoto-Yokohama interaction.

- **Strong on-site Coulomb repulsion $\Rightarrow$ Gutzwiller projector P**
- **Kuramoto-Yokohama Hamiltonian**

$$H_{KY} = \frac{J}{2}\left(\frac{2\pi}{N}\right)^2 \sum_{\alpha\neq\beta} \frac{1}{|z_\alpha - z_\beta|^2} P\Big\{\vec{S}_\alpha \cdot \vec{S}_\beta$$

$$-\frac{1}{2}\sum_\sigma c^\dagger_{\alpha\sigma} c_{\beta\sigma} + \frac{1}{2}(n_\alpha + n_\beta) - \frac{1}{4} n_\alpha n_\beta - \frac{3}{4}\Big\}P$$

-

$$n_{\alpha\sigma} = c^\dagger_{\alpha\sigma} c_{\alpha\sigma} \quad ; \quad \vec{S}_\alpha = \frac{1}{2}\sum_{\sigma\sigma'} c^\dagger_{\alpha\sigma} \vec{\tau}_{\sigma\sigma'} c_{\alpha\sigma'}$$

$$P = \prod_\alpha (1 - n_{\alpha\uparrow} n_{\alpha\downarrow})$$

Ground-State at 1/2-filling

- **Disordered spin-liquid state (N even):**

$$\Psi_{GS}(z_1, \ldots, z_{\frac{N}{2}}) = \prod_{i<j=1}^{\frac{N}{2}} (z_i - z_j)^2 \prod_{t=1}^{\frac{N}{2}} z_t \qquad z_j = \uparrow$$

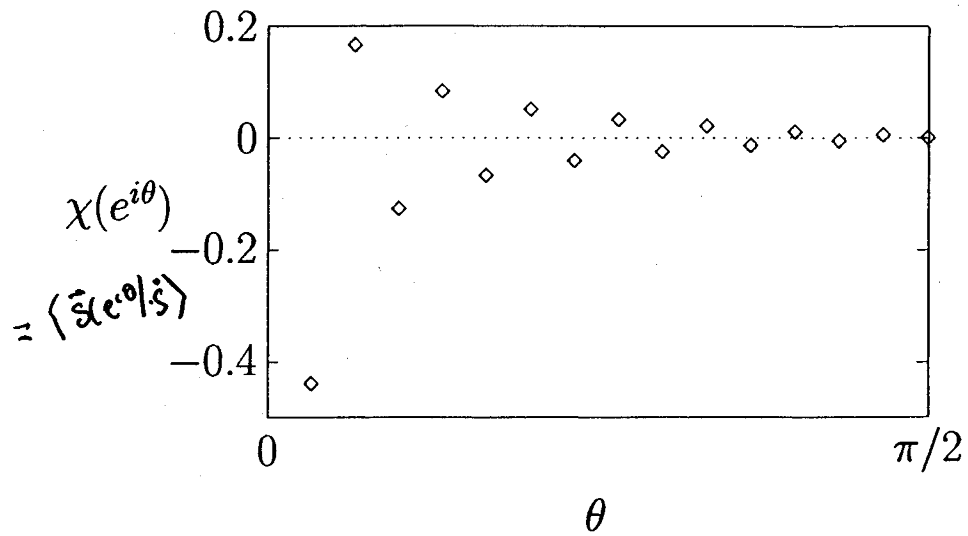


Figure 4: Spin-spin correlation function for the HS ground-state.

- **Energy:**

$$E_{GS} = -J(\frac{\pi^2}{24})(N + \frac{5}{N})$$

Elementary excitations: spinons

- **Spinon at s (N odd, $M = (N-1)/2$):**

$$\Psi_s^{\mathrm{sp}}(z_1, \ldots, z_M) = \prod_{j=1}^{M} (z_j - s) \prod_{i<j=1}^{M} (z_i - z_j)^2 \prod_{t=1}^{M} z_t$$

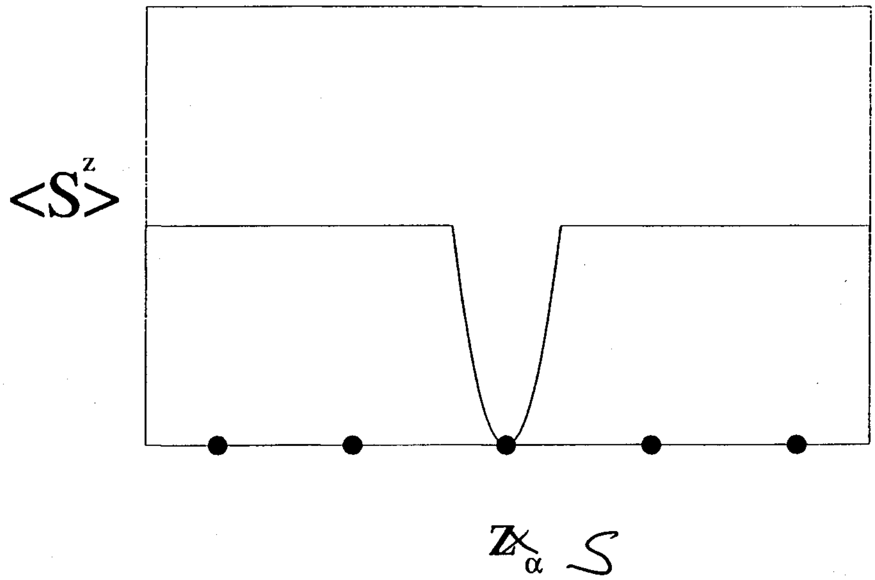


Figure 5: Localized Spinon at z_α.

Propagating Spinon:

$$\Psi_m^{\rm sp}(z_1,\ldots,z_M) = \frac{1}{N}\sum_s (s^*)^m \Psi_s(z_1,\ldots,z_M)$$

$$q_m = \frac{\pi}{2}N - \frac{2\pi}{N}(m+\frac{1}{2}) \pmod{2\pi}$$

$$E^{\rm sp}(q_m) = \frac{J}{2}\left[\left(\frac{\pi}{2}\right)^2 - q_m^2\right] \pmod{\pi}$$

$$-\frac{\pi}{2} \le q_m \le \frac{\pi}{2}$$

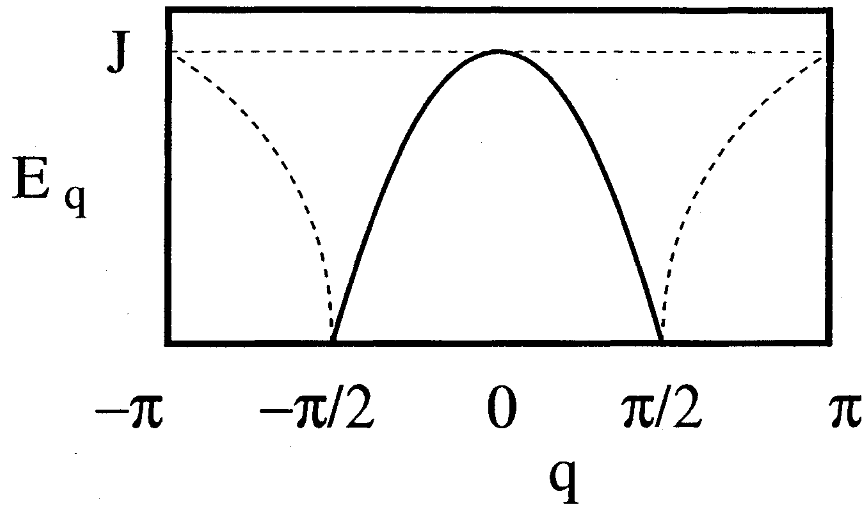


Figure 6: One-spinon dispersion relation.

Thermodynamic limit ⇒ no difference between even and odd-N chains

Spinons are alleged spin-1/2 elementary excitations of the HSM.

Elementary excitations: holons

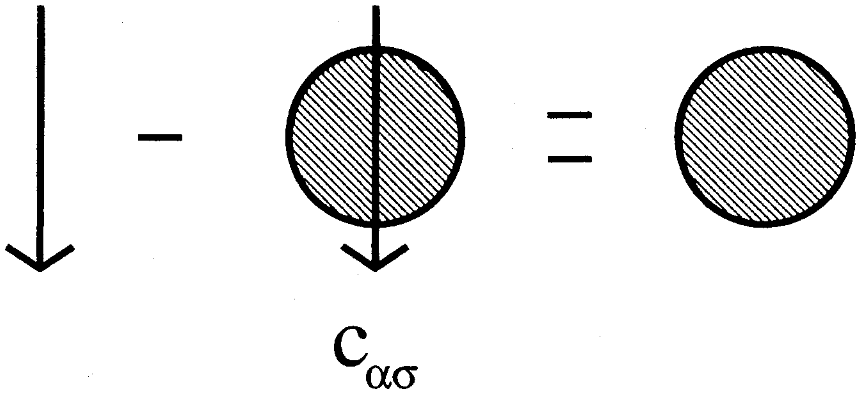


Figure 7: Birth of a holon .

- **Negative-energy propagating one-holon eigenstate (filling = $(N-1)/2N$)**

$$\Psi_n^{\text{ho}}(z_1,\ldots,z_M|h) = h^n \prod_j^M (z_j - h) \prod_{i<j}^M (z_i - z_j)^2 \prod_t^M z_t$$

$1 \leq n \leq M+1$, $h \equiv$ **location of the empty site**

$$q_n = \frac{\pi}{2}N + \frac{2\pi}{N}(n - \frac{1}{2}) \;\; (\text{mod} 2\pi)$$

$$E^{\text{ho}}(q_n) = -\frac{J}{2}\left[\left(\frac{\pi}{2}\right)^2 - q_n^2\right] \;\; (\text{mod } \pi)$$

- **(Part of) state for a localized holon at h_0**

$$\Psi_{h_0} = \sum_{n=1}^{M+1} h_0^{-n} \Psi_n^{\mathrm{ho}}(z_1, \ldots, z_M | h)$$

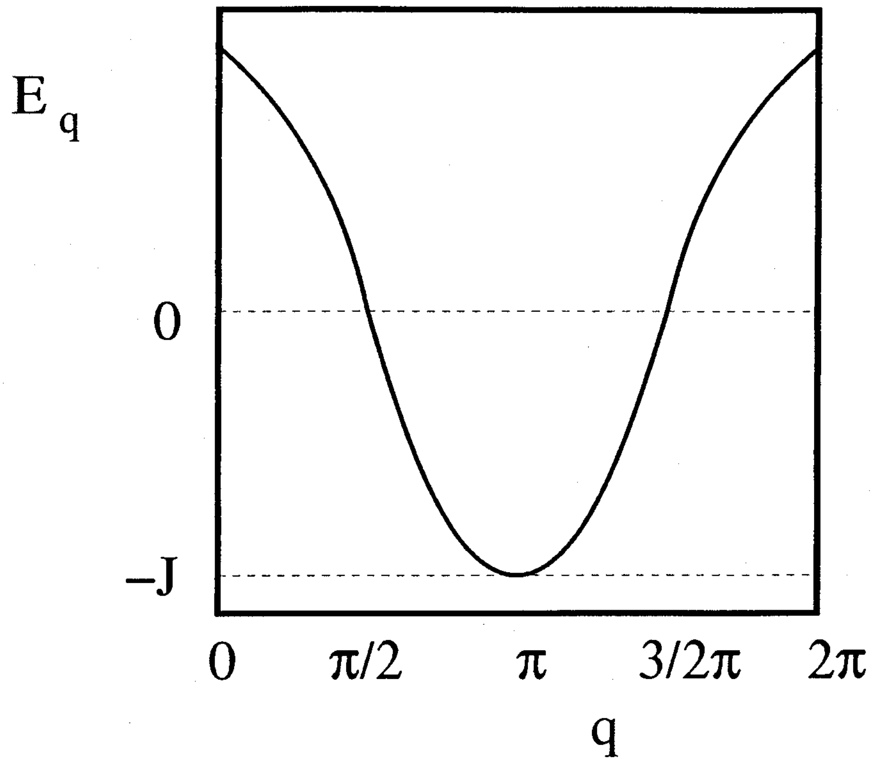


Figure 8: One-Holon dispersion relation.

One-spinon one-holon eigenstates of the KYM.

- **Spinons and holons keep their integrity when many of them are present in the same state**
- **Propagating holon, localized spinon**

$$\Psi_s^n = h^n \prod_j^M (z_j - s)(z_j - h) \prod_{i<j}^M (z_i - z_j)^2 \prod_t^M z_t$$

- **Propagating holon, propagating spinon**

$$\Psi_{mn}(z_1, \ldots, z_M | h) = \frac{1}{N} \sum_s (s^*)^m \Psi_s^n(z_1, \ldots, z_M | h)$$

- **Diagonalization of H_{KY} $\Rightarrow$ one-spinon one-holon energy eigenstate**

$$\Phi_{mn} = \sum_{\ell=0}^{m} a_\ell \Psi_{m-\ell, n-\ell} \;\; (m - n + 1 < 0)$$

$$\Phi_{mn} = \sum_{\ell=0}^{M-m} a_\ell \Psi_{m+\ell, n+\ell} \;\; (m - n + 1 \geq 0)$$

$$a_\ell = -\frac{1}{2\ell} \sum_{k=0}^{\ell-1} a_k \;\; ; \; a_0 = 1$$

- **Energy eigenvalues**

$$E^{\mathrm{sp,ho}}(q_m, q_n) = -J(\frac{\pi}{24})^2(N + \frac{5}{N})$$

$$+[E^{\mathrm{sp}}(q_m) + E^{\mathrm{ho}}(q_n)] - \frac{\pi J}{N} \frac{|q_m - q_n|}{2}$$

- **As for a many-spinon solution, as $N \Rightarrow \infty$ (thermodynamic limit)**

$$E^{\mathrm{sp,ho}}(q_m, q_n) \approx E_{GS} + [E^{\mathrm{sp}}(q_m) + E^{\mathrm{ho}}(q_n)]$$

  = Sum of the energies of single isolated spinon and holon

- **As for two-spinon states, this <u>does not imply</u> that spinons and holons do not interact**

Spinon-holon interaction

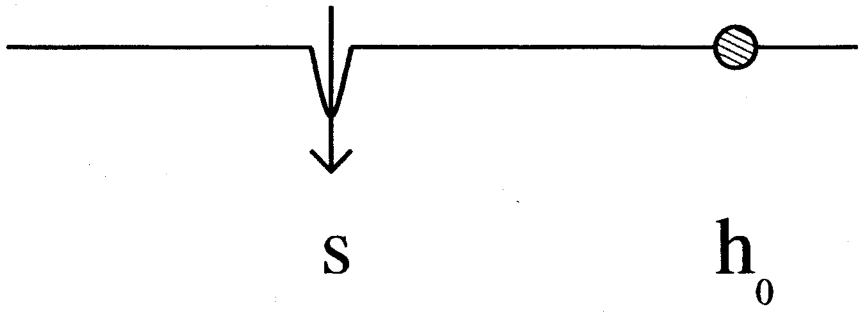


Figure 9: Localized spinon at s and localized holon at h_0

- **State for a localized spinon at s and a localized holon at h_0**

$$\Psi_{sh_0} = \sum_{n=1}^{M+2} \sum_{m=0}^{M} s^m h_0^{-n} \Psi_{mn}(z_1, \ldots, z_M | h)$$

$\Psi_{sh_0} \equiv$ "coordinate representation" of a one-spinon one-holon eigenstate: the representation is partially overcomplete

- **The set Φ_{mn} is a set of eigenstates of $\mathcal{H}_{KY}$: it provides a basis for the states Ψ_{sh_0}**

$$\Psi_{sh_0} = \sum_{n=1}^{M+2} \sum_{m=0}^{M} s^m h_0^{-n} p_{mn}(\frac{s}{h_0}) \Phi_{mn}$$

$p_{mn}(z) \equiv$ wavefunction for a spinon-holon pair in a state with energy $E^{\text{sp,ho}}(q_m, q_n)$

- **Equation of motion for the spinon-holon wave-function**

$$(E(q_m, q_n)-E_{GS})\langle\Phi_{mn}|\Psi_{sh_0}\rangle = \langle\Phi_{mn}|(\mathcal{H}_{\text{KY}}-E_{GS})|\Psi_{sh_0}\rangle$$

$$= J(\frac{2\pi}{N})^2\Big\{\Big[(M - s\frac{\partial}{\partial s})s\frac{\partial}{\partial s} + h_0\frac{\partial}{\partial h_0}(1 + \frac{N}{2} + \frac{\partial}{\partial h_0})$$

$$+\frac{1}{2}\left(\frac{h_0+s}{h_0-s}\right)\left(s\frac{\partial}{\partial s} + h_0\frac{\partial}{\partial h_0} + 1\right)\Big]\langle\Phi_{mn}|\Psi_{sh_0}\rangle$$

$$+\frac{h_0}{s-h_0}\left(\frac{s}{h_0}\right)^{\nu}\langle\Phi_{mn}|\Psi_{sh_0}\rangle\Big\}$$

$\nu = M$ **if** $m-n+1<0$, $\nu = 0$ **otherwise.**

- **"Dirac-like" differential equation for** $p_{mn}(z)$

1. $m-n+1<0$

$$\left[2\frac{d}{dz} - \frac{1}{1-z}\right]p_{mn}(z) + \frac{z^{M-m-1}}{1-z}p_{mn}(1) = 0$$

2. $m-n+1\geq 0$

$$\left[2\frac{d}{d\left(\frac{1}{z}\right)} - \frac{1}{1-\frac{1}{z}}\right]p'_{mn}(z) + \frac{\left(\frac{1}{z}\right)^m}{1-\frac{1}{z}}p'_{mn}(1) = 0$$

- **First-order differential equation ("relativistic" spectrum), interaction potential diverging at short distances** ($\sim 1/x$)

- **Solutions**

1.

$$p_{mn}(z) = \sum_{k=0}^{M-m-1} \frac{\Gamma[k+\frac{1}{2}]}{\Gamma[\frac{1}{2}]\Gamma[k+1]} z^k$$

2.

$$p'_{mn}(z) = \sum_{k=0}^{m} \frac{\Gamma[k+\frac{1}{2}]}{\Gamma[\frac{1}{2}]\Gamma[k+1]} \left(\frac{1}{z}\right)^k$$

- **The nature of spinon-holon interaction is figured out by looking at $|p_{mn}(e^{i\theta})|^2$, the probability of finding a spinon and a holon at a distance θ one from another.**

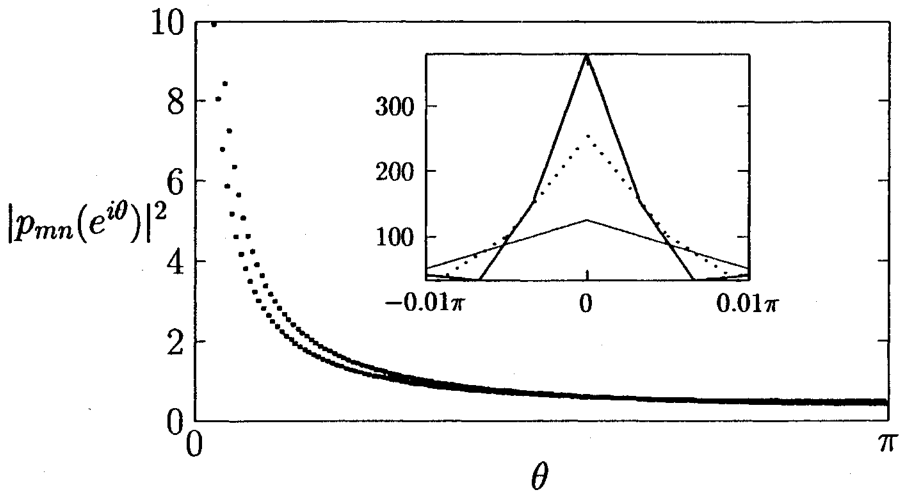


Figure 10: Square of the spinon-holon wavefunction, $|p_{mn}(z)|^2$, for $m = N/2-1$ and $N = 600$. The probability peaks up at short separations between spinon and holon, but it does not depend on the distance at large separations. The inset show the function around the origin for $N = 200, 400, 600$.

- **Large spinon-holon separation $\Rightarrow |p_{mn}(e^{i\theta}))|^2$ independent of $\theta \Rightarrow$ noninteracting particles.**
- **Short separation $\Rightarrow$ huge enhancement in $|p_{mn}(e^{i\theta})|^2 \Rightarrow$ short-range attraction between spinon and holon.**
- **The enhancement gets sharpened and peaks up as $N \to \infty$.**

Measurable effect: hole spectral density of states.

- **Strong enhancement in the probability for a spinon and a holon to be at the same site $\Rightarrow$ enhancement in the probability for a hole to decay into a spinon and a holon**

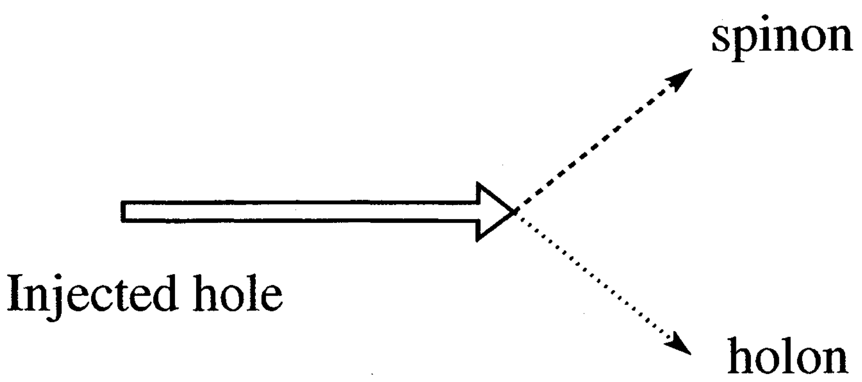


Figure 11: Hole decay mode

- **Hole spectral density**

$$A(\omega, q) = \Im m \Big\{ \sum_X \frac{\langle X | \Sigma_{h_0} (h_0)^{-q} c_{h_0 \uparrow} | \Psi_{GS} \rangle |^2}{\langle X | X \rangle \langle \Psi_{GS} | \Psi_{GS} \rangle}$$

$$\times \left[\frac{1}{\omega + (E_X - E_{GS}) - i0^+} + \frac{1}{\omega - (E_X - E_{GS}) + i0^+} \right] \Big\}$$

- **One-spinon one-holon contribution to $A(\omega, q)$**

$$A^{\text{sp,ho}}(\omega, q) = \Im m \frac{1}{\pi} \Big\{ \sum_{n=1}^{M+1} \sum_{m=0}^{M} \frac{\delta_{k-m+l} p_{mn}^2(1)}{\omega + i0^+ - (E_{mn} - E_{GS})}$$

$$+\sum_{n=1}^{M+1}\sum_{m=0}^{M}\frac{\delta_{k-m+l}(p'_{mn})^2(1)}{\omega+i0^+-(E_{mn}-E_{GS})}\Big\}\frac{\langle\Phi_{mn}|\Phi_{mn}\rangle}{\langle\Psi_{GS}|\Psi_{GS}\rangle}$$

- **Thermodynamic limit**

$$A^{\mathrm{sp,ho}}(\omega,q)=2\Im m\int_0^{\pi}\frac{dq_{\mathrm{ho}}}{\pi}\Big\{\int_0^{q_{\mathrm{ho}}}\frac{dq_{\mathrm{sp}}}{\pi}\sqrt{\frac{\pi-q_{\mathrm{sp}}}{q_{\mathrm{sp}}}}$$

$$+\int_{q_{\mathrm{ho}}}^{\pi}\frac{dq_{\mathrm{sp}}}{\pi}\sqrt{\frac{q_{\mathrm{sp}}}{\pi-q_{\mathrm{sp}}}}\Big\}\frac{\delta(q-q_{\mathrm{sp}}+q_{\mathrm{ho}})}{\omega-\mu+i0^+-E(q_{\mathrm{sp}},q_{\mathrm{ho}})}$$

$$=\frac{1}{\pi^2Jq}\sqrt{\frac{J[q+\frac{\pi}{2}]^2-\omega}{\omega-J[q-\frac{\pi}{2}]^2}}$$

$$\times\Theta\left[\omega-J[q-\frac{\pi}{2}]^2\right]\Theta\left[J[\frac{\pi^2}{4}+q(\pi-q)]-\omega\right]$$

Total instability of the hole excitation ⇒ no Landau's quasiparticle pole ⇒ singular threshold followed by a broad spectrum

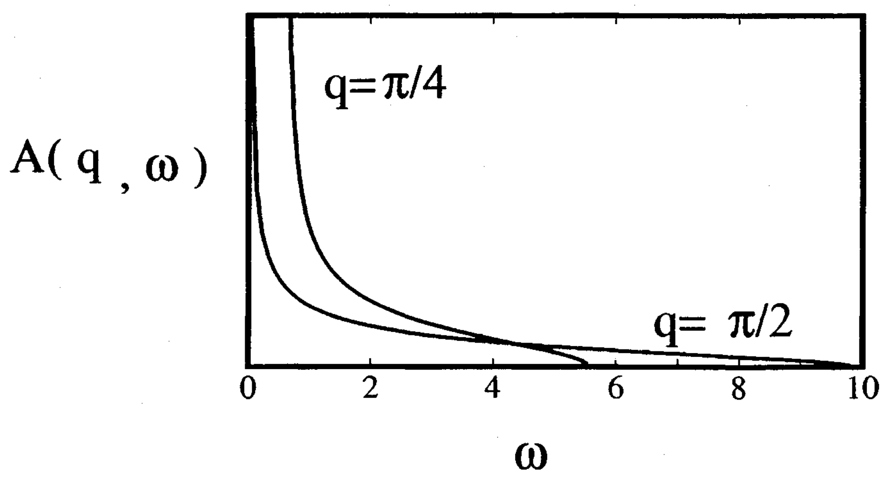


Figure 12: $A(q,\omega)$ vs. ω at fixed q.

Conclusions.

- **In the thermodynamic limit the resonant enhancement $p_{mn}(1)$ turns into the square-root singularity in the hole spectral density**
- **Branch cut ≡ hole instability. It is a consequence of the enhancement in the probability for spinon and holon to be at the same position, that is, of the SPINON-HOLON ATTRACTION**
- **Hence: Spinon-holon attraction ⇒ broad spectra and square-root singularity at threshold**

References.

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