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THE CHERN-SIMONS APPROACH TO THE t-J MODEL: BASIC IDEAS AND 1D RESULTS

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THE CHERN-SIMONS APPROACH TO THE t-J MODEL: BASIC IDEAS AND 1D RESULTS

P.A. Marchetti

based on

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P.M., Z.B. Su, L. Yu Phys Rev B 58 (1998) 5808

P.M., J.H. Dai, Z.B. Su, L. Yu J Phys Cond Matt 12 (2000) L329

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Main underlying physical motivation: attempt to understand the low-energy physics of high T_c cuprates

Model Hamiltonian : 2D $t-J$

$$H = P_G \left[\sum_{<ij>} -t\, c_i^{+} c_j + J\, \vec{S}_i \cdot \vec{S}_j \right] P_G$$

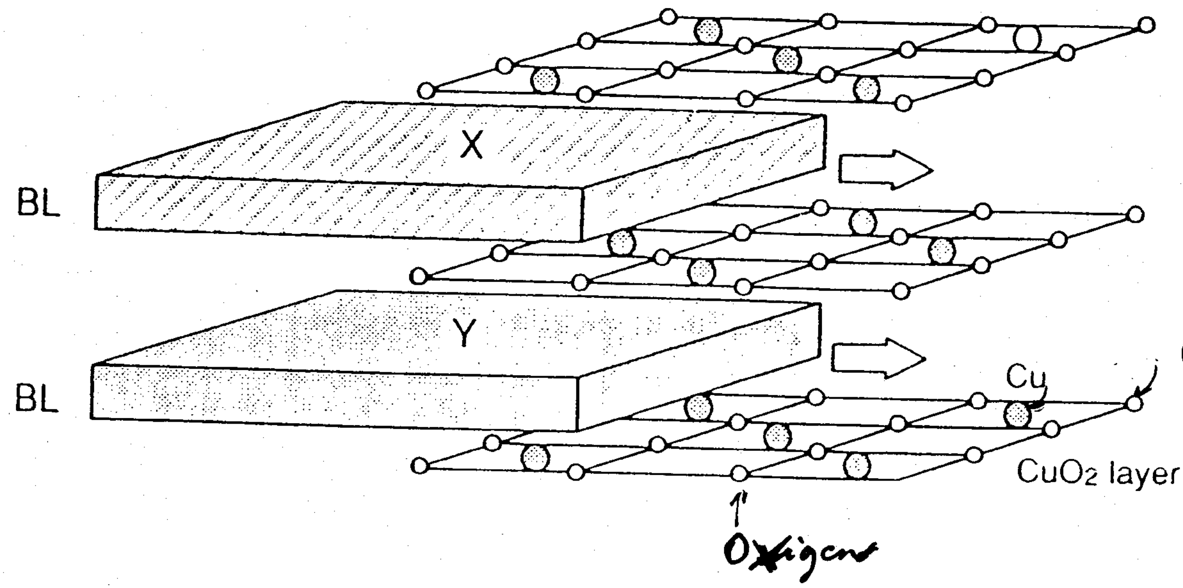
$\vec{S}_i = c^{+}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$, i are sites of 2D square lattice

P_G project out states with double occupation

"Justification": Undoped high T_c cuprates are AF insulators whose low-energy physics is dominated by CuO_2 planes well described by an AF Heisenberg hamiltonian on the lattice identified by Cu sites.

Doping these materials (converting them into metals or superconductors) effectively introduces holes (or electrons) in the CuO_2 planes, whose spin form a spin-singlet with the spin moment of the copper

Structure of undoped high T_c cuprates



Key active feature: CuO_2 planes
(lattice ~ square)

Doping effect: to introduce charges in the CuO_2 planes ($Cu^{2+} \to Cu^{2+\delta+}$)

Examples

Undoped material
(A F insulator)

Doped material

$$La_2 Cu O_4 \longrightarrow La_{2-\delta} Sr_\delta Cu O_4$$

$$Y Ba_2 Cu_3 O_7 \longrightarrow Y Ba_2 Cu_3 O_{7-\delta}$$

δ - doping concentration

$\Rightarrow$ Physics exhibits 2-dimensional features

Natural variables for phase diagram: T, δ

holes introduced by doping occupy hybridized O-orbitals around Cu sites forming a spin singlet

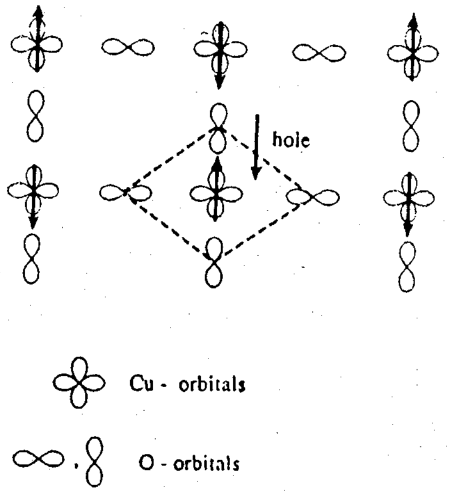


Figure 2. $3d_{x^2-y^2}$ Cu orbitals and the $2p_x, 2p_y$ O–orbitals in CuO_2 planes. The spin $\frac{1}{2}$ moments of the Cu sites and of a hole introduced by doping are indicated. The dashed lines indicate hybridization.

These spin singlets can hop between different Cu sites because each CuO_4 has an oxygen in common with the n.n. CuO_4.
Furthermore a strong on-site Coulomb repulsion inhibits the double occupation →

$$H_{t-J} = P_G \left[\sum_{\langle ij \rangle} -t c_i^+ c_j + h.c. + J \, \vec{S}_i \cdot \vec{S}_j \right] P_G$$

(in cuprates $J/t \sim 1/3$)

One can hope to gain some understanding of the unsolved 2D t–J model from its 1D counterpart which is exactly solvable in the limit $J/t \simeq 0$ and at the supersymmetric point $J = t$ (closer to physics in 2D)

1D t-J model at $J/t \sim 0$

Using Bethe Ansatz or CFT techniques one derives the following features

- spin-charge separation : the charge and the spin degrees of freedom (d.o.f.) are characterised by different physical behaviour, in particular their velocities are different $v_s \neq v_c$
- low energy physics of charged d.o.f. is described by a free spinless fermion field (holon)
- low energy physics of spin d.o.f. is described by an AF Heisenberg model in a squeezed chain, obtained omitting the unoccupied sites. The spin $\frac{1}{2}$ field f_α such that $\vec{S} = f^{+}_{\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} f_{\beta}$ in the squeezed chain is called spinon field

- the electron field is a product of a holon and a spinon fields together with a non-local dressing ("string") modifying the power law decays of its correlation functions at large scales

typical contributions are of the form

$$\frac{e^{i\frac{\pi}{2}\rho n}}{(x-iv_ct)^{\alpha_c^-}(x+ivt)^{\alpha_c^+}(x-iv_st)^{\alpha_s^-}(x+iv_st)^{\alpha_s^+}}$$

$n\in\mathbb{Z}$, $\alpha_c^{\pm},\alpha_s^{\pm}\in\mathbb{Q}$ $\qquad$ ρ density

e.g.

$$\langle\vec{S}(0,0)\cdot\vec{S}(x,t)\rangle \sim \frac{\cos\pi\rho x}{(x^2+v_s^2t^2)^{\frac{1}{2}}(x^2+v_c^2t^2)^{1/4}}$$

spin correlation

$$\langle\psi_\alpha(0,0)\,\psi_\alpha(x,t)\rangle\sim\frac{1}{(x+iv_st)^{\frac{1}{2}}(x^2+v_c^2t^2)^{1/16}}\left[\frac{e^{i\frac{\pi}{2}\rho x}}{(x+iv_ct)^{1/2}}+\frac{e^{i\frac{3\pi}{2}\rho x}}{(x+iv_ct)^{3/2}}\right]\text{etc.}$$

electron correlation

However all these results have been derived using techniques bound to 1D. Natural questions arise: in 2D the concept of spinon and holons is still useful? spin-charge separation is still active?

Aim of this talk is to introduce the Chern-Simons approach; naturally defined in 2D, it can be adapted to 1D using dimensional reduction and there it reproduces the above results

• Although no rigorous results are available in 2D, at the end I'll flash some results obtained with an extension of these ideas to 2D, comparing with experimental data for high T_c cuprates (resistivity, metal-insulator crossover)

Chern-Simons representations

- Given a group G and the corresponding gauge field W_μ (with values in the Lie algebra of G) one defines in $2+1$ dimension the Chern-Simons action by

$$S_{C.S.}(W) = \frac{1}{4\pi}\int d^3x \, \mathrm{Tr}\, \varepsilon^{\mu\nu\rho}\left[W_\mu \partial_\nu W_\rho + \frac{2}{3} W_\mu W_\nu W_\rho\right]$$

$$= \frac{1}{4\pi}\int d^3x \; \varepsilon_{ij}\left[W_i^a \partial_0 W_j^a + W_0^a \partial_i W_j^a + \frac{2}{3} f_{abc} W_i^a W_j^b W_0^c\right]$$

where $a, b \ldots$ are indices labelling the generators of Lie G and f_{abc} the structure constants ($=0$ if G is abelian)

- Given an action $S(\psi)$ in terms of a spin $\frac{1}{2}$ field ψ_α [α spin index], we define the G-gauge invariant action $S(\psi_\alpha, W_\mu)$ by inserting a minimal coupling of ψ_α with W_μ

i.e. replacing in the continuum the derivative ∂_μ by the covariant derivative $\partial_\mu - W_\mu$ and on the lattice

$$\psi^+_{i\alpha}\psi_{j\alpha} \text{ by } \psi^+_{i\alpha}\left(P e^{i\int_{\langle ij\rangle} W}\right)_{\alpha\beta}\psi_{j\beta}$$

where P is the path-ordering (necessary only for G non-abelian)

– Let χ_α be a new spin $\frac{1}{2}$ field, then <u>for a suitable choice of the gauge group G, the coefficient of the Chern-Simons action K_G, and the statistics of χ, the fermionic model described by the classical action $S(\psi)$ is equivalent to the model described by the classical action $S(\chi, W) + K_G S_{C.S}(W)$ in terms</u> of the fields χ_α, W_μ. For the lattice model non-double occupation constraint is necessary.

In particular correlation functions of $\Psi_\alpha(x)$ are represented in the new formulation by correlation functions of the fields $\left(P e^{i\int_x^\infty \mathcal{A}}\right)_{\alpha\beta} \chi_\beta(x)$

Examples

$G = U(1)$

$k_G = 1$ $\quad \chi$ bosonic $\xrightarrow{MFA}$ slave fermion

$k_G = -1$ $\quad \chi$ fermionic $\xrightarrow{MFA}$ slave boson

---

$G = U(1) \times SU(2)$

$k_{U(1)} = -2 \quad k_{SU(2)} = 1$ $\quad \chi$ fermionic

sucessful in reproducing 1D t-J

We call these new formulations of the original model Chern-Simons representations.

There is a strict equivalence between all these formulations if treated exactly, but each one suggests a different Mean Field Approx.

How Chern-Simons representation works

Intuitive idea:

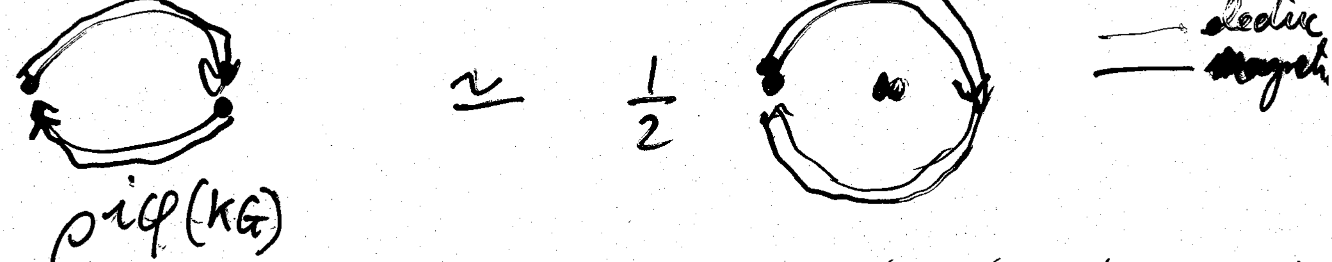
- the minimal coupling to W gives to X-particle a G-"electric" charge
- The action is linear in the time-component W_0, integrating it out we get the constraint

from $S(X,W) \rightarrow j_0 = \frac{k}{2\pi}\epsilon_{ij} W^{ij} \leftarrow$ from $k_G S_{c.s.}(W)$

where j_0 is the G-density of X and W^{ij} the field strength "magnetic" of W_i

$\Rightarrow$ each X-particle carries also a G-"magnetic" flux.

- exchange of X-particle produces an Aharonov-Bohm phase factor k_G-dependent $e^{i\varphi(k_G)}$. If $e^{i\varphi(k_G)} = \pm 1$ we choose X fermionic / bosonic

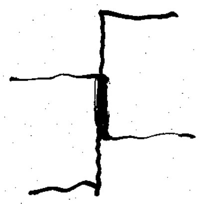


$e^{i\varphi(k_G)}$

so Chern-Simons coupling induce a statistics transmutation

- Does the Chern-Simons coupling have other effect? Where there are no χ-particles $j_0 = 0$, then $W_{ij} = 0$, but by gauge invariance this implies $W_\mu = 0$, i.e. no other effects

This explains also why we need the non-double occupation constraint for lattice theories, because for lattice trajectories of χ of the form



we cannot associate a well defined Aharonov-Bohm phase factor to their (finite) overlap, which should then be forbidden. (The probability of a finite overlap of trajectories in the continuum is 0)

Chern - Simons representation

Sketch in formulas for the partition function of a free fermion field ($\psi_\alpha(x)$ Grassmann variable)

action $S(\psi) = \int \psi^*_\alpha \partial_0 \psi_\alpha + \psi^*_\alpha \frac{\vec{\nabla}^2}{2m} \psi_\alpha$

$$Z = \int \mathcal{D}\psi \mathcal{D}\psi^* e^{-S(\psi)} = \sum_{N=0}^{\infty} \sum_{\alpha_1 \ldots \alpha_N} \int dx_1 \ldots dx_N$$

first quantized representation; spin indices

$$\sum_{\pi \text{ permutation}} (-1)^{\sigma(\pi)} \prod_{r=1}^{N} \int_{\substack{x_r(0) = x_r \\ x_r(\beta) = x_{\pi(r)}}} \mathcal{D}x_r(t) \; e^{-\frac{m}{2}\int_0^\beta \dot{\vec{x}}_r^2(t)\,dt}$$

fermion $(-1)^{\sigma(\pi)}$: $\sigma(\pi)$ = # exchanges in π

$$\boxed{\text{identity } (-1)^{\sigma(\pi)} = \int \mathcal{D}W \, e^{-k_G S_{CS}(W)} \operatorname{Tr} P e^{i\int_{\underline{x}} W}}$$

for suitable G and k_G (simple for $G = U(1)$, $k_{U(1)} = 1$)

$\underline{x} = \{ x_r(t),\ t \in [0,\beta],\ r = 1, \ldots N \}$

$$= \int \mathcal{D}W \, e^{-k_G S_{c.s}(W)} \sum_{N=0}^{\infty} \sum_{\alpha_1 \ldots \alpha_N} \int dx_1 \ldots dx_N$$

$$\sum_{\pi} \frac{1}{A} \prod_{r=1}^{N} \int_{\substack{x_r(0) = x_r \\ x_r(\beta) = x_{\pi(r)}}} \mathcal{D}x_r(t) \boxed{e^{-\frac{m}{2}\int_0^\beta \dot{\vec{x}}_r^2(t)\,dt} \operatorname{Tr} P e^{i\int_{\underline{x}} W}}$$

$\frac{1}{A}$: boson factor

$\Downarrow$

χ_α complex

$\boxed{\ldots} = \exp[-\text{action of } N \text{ particles minimally coupled to } W]$

$$= \int \mathcal{D}W \, e^{-k_G S_{c.s}(W)} \int \mathcal{D}\chi \mathcal{D}\chi^* \, e^{-\int \chi^*_\alpha (\partial_0 - W_0)_{\alpha\beta} \chi_\beta}$$

$$e^{-\int \chi^*_\alpha \frac{(\vec{\nabla} - \vec{W})^2_{\alpha\beta}}{2m} \chi_\beta}$$

Chern-Simons approach to 1D t-J

- Apply U(1) X SU(2) Chern-Simons representation in 2D, U(1) gauging the charge and SU(2) the spin global symmetries

Technically we use ~~the~~ for the quartic term the identity

$$\Psi^{*}_{i\alpha}\,\vec{\sigma}_{\alpha\beta}\,\Psi_{i\beta}\cdot\Psi^{*}_{j\gamma}\,\vec{\sigma}_{\gamma\delta}\,\Psi_{j\delta} =$$

$$= 2\,\Big|\underbrace{\Psi^{*}_{i\alpha}\,\Psi_{j\alpha}}\Big|^{2} - \Psi^{+}_{i\alpha}\Psi_{i\alpha}\,\Psi^{*}_{j\beta}\Psi_{j\beta}$$

for B_μ U(1) gauge field
V_μ SU(2) gauge field
replaced by $\;\big|\,\chi^{+}_{i\alpha}\big(P e^{i\int V}\big)_{\alpha\beta}\,e^{i\int B}\,\chi_{j\beta}\big|^{2}$

- formal spin-charge separation

$$\chi_\alpha = H\,\Sigma_\alpha$$

χ_α: spin $\frac{1}{2}$ fermion charged

H: spinless fermion charged

Σ_α: spin $\frac{1}{2}$ hard-core boson

P_G implemented by $\Sigma^{*}_\alpha\Sigma_\alpha = 1$

H is coupled to the $U(1)$ gauge field B_μ

Σ_α $\qquad\qquad\qquad\qquad SU(2) \qquad\qquad\qquad\qquad V_\mu$

- <u>dimensional reduction</u> $2D \to 1D$

restricting χ to a $1D$ sublattice (of coordinate $x^2=0$)

Action of $1D$ $t-J$ in $U(1)\times SU(2)$

Chern-Simons representation:

$$S = \int dt \sum_i H_i^* (\partial_0 - B_0 + \mu) H_i + H_i^* H_i \Sigma^*_{\alpha i} (\partial_0 \delta_{\alpha\beta} - (V_0)_{\alpha\beta}) \Sigma_{\beta i}$$

$$+ \sum_{\langle ij \rangle} -t\, H_i^* e^{i\int_{\langle ij\rangle} B} H_j\, \Sigma^*_{\alpha i} \left(P e^{i\int_{\langle ij\rangle} V}\right)_{\alpha\beta} \Sigma_{\beta j} + h.c.$$

$$+ \sum_{\langle ij \rangle} H_i^* H_i H_j^* H_j \left(\left[\Sigma^*_{\alpha i} \left(P e^{i\int_{\langle ij\rangle} V}\right)_{\alpha\beta} \Sigma_{\beta j} \right]^2 - \frac{1}{2} \right)$$

$$- 2 S_{c.s.}(B) + S_{c.s.}(V)$$

with constraint

$$\Sigma^*_{\alpha i} \Sigma_{\alpha i} = 1$$

3 gauge invariances

$U(1)$ $\qquad H_j \to H_j e^{i\Lambda(j)} \qquad B_\mu(x) \to B_\mu(x) + \partial_\mu \Lambda(x)$

$SU(2)$ $\qquad \Sigma_j \to R^+(j) \Sigma_j \qquad V_\mu(x) \to R^+(x) V_\mu(x) R(x) + R^+(x) \partial_\mu R(x)$

h/S $\qquad H_j \to H_j e^{i\xi_j} \qquad \Sigma_{\alpha j} \longrightarrow e^{-i\xi_j} \Sigma_{\alpha j}$

We gauge-fix $U(1)$ by Coulomb:

$$\partial^i B_i(x) = 0$$

We gauge-fix $SU(2)$ by choosing a "ferromagnetic" gauge:

$$\Sigma_j = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\Rightarrow$ spin d.o.f. are carried by V
charge H

- Since no-matter field live in the region $x_2 \neq 0$, but only B and V on S.c.s., we can integrate out B_2 and V_2 obtaining

$$\varepsilon^{ij} \partial_i B_j(x) = 0$$

$$\varepsilon^{ij} \left(\partial_i V_j^a + \varepsilon^{abc} V_i^b V_j^c \right)(x) = 0$$

$$\Rightarrow \begin{cases} B_i = 0 \\ V_i = g^{-1} \partial_i g \qquad g \in SU(2) \end{cases}$$

Action becomes

$$\Rightarrow S(H, g) = \int dt \sum_i \; H_i^\dagger (\partial_0 - \mu) H_i + H_i^\dagger H_i \left(g^{-1} \partial_0 g \right)_{11} +$$

$$\sum_{\langle ij \rangle} -t \; H_i^\dagger H_j \left(g_i^\dagger g_j \right)_{11} + \text{h.c.}$$

$$+ \sum_{\langle ij \rangle} J \; H_i^\dagger H_i \, H_j^\dagger H_j \left(|(g_i^\dagger g_j)_{11}|^2 - \frac{1}{2} \right)$$

How the squeezed chain appears?

1- optimisation

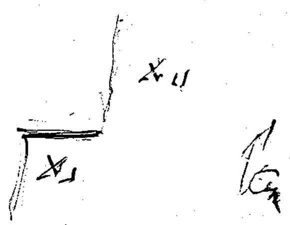
Consider the partition function of H in a g back-ground and express it in terms of a "first-quantisation" path-integral:

$$Z(H|g) \equiv \int \mathcal{D}H\,\mathcal{D}H^{\dagger}\, e^{-S(H,g)} =$$

$$\sum_{N=0}^{\infty} \frac{e^{\beta\mu N}}{N!} \sum_{j_1 \dots j_N} \int_{\substack{X_i(0)=j_i \\ X_i(\beta)=j_i}} \mathcal{D}X_1(t) \dots \mathcal{D}X_N(t) \prod_{i=1}^{N} e^{\int_{\underline{X}^{\parallel}} (g^{\dagger}\partial_0 g)_{11}\, dt}$$

$\beta = T^{-1}$

$$\prod_{\langle ij \rangle \in \underline{X}^{\perp}} t\,(g_i^{\dagger} g_j)_{11} \prod_{\langle ij \rangle \cap \underline{X} = \emptyset} e^{-J\int \left[|(g_i^{\dagger} g_j)_{11}|^2 - \frac{1}{2}\right] dt}$$



where $\underline{X} = \{X_1(t) \dots X_N(t)\}$

$\underline{X}^{\parallel}$ and $\underline{X}^{\perp}$ denote the components parallel and perpendicular to the time axis.

Furthermore we omitted the sum over permutations of $j_1 \dots j_N$ at β because, since the H are spinless fermion their trajectories by Pauli principle can not have intersections.

- We find an a-priori upper bound on $|Z(H)|$, an H-dependent g configuration saturating the bound ("optimal spin configuration") and consider it as a starting point for adding spin fluctuations.

The bound (reminiscent of "diamagnetic inequality") is simply a consequence of

$$|(g_i^+ g_j)_{11}| \le 1 \qquad \left| e^{\int (g^+ \partial_0 g)_{11}\, dt} \right| \le 1$$

and it is saturated by a configuration g satisfying

$$(g^+ \partial_0 g)_{11} = 0 \quad \text{on } \underline{X}^{\parallel}$$

$$|(g_i^+ g_j)_{11}| = 1 \quad \text{on } \underline{X}^{\perp}$$

$$(g_i^+ g_j)_{11} = 0 \quad \text{on } \langle ij \rangle \cap \underline{X} = \emptyset$$

[Technical remark: using paths one can optimise separately the Heisenberg and the hopping terms.]

The solution (g^m) is

$$g_j^m(t) = e^{i\frac{\pi}{2}\sigma_x \sum_{l<j} \sum_r \delta_{x_r(t),l}}$$

rewritten in terms of fields:

$$g_j^m(t) = e^{i\frac{\pi}{2}\sigma_x \sum_{l<j} H_l^+ H_l(t)}$$

Picture of the effect of g^m

$\uparrow \; \uparrow \; \circ \; \uparrow \; \uparrow \; \circ \; \uparrow$ original Σ configuration

empty site

$g^m \Sigma$ configuration

$\uparrow \; \downarrow \; \circ \; \uparrow \; \downarrow \; \circ \; \uparrow$

i.e. it flips the "spin" of Σ every two sites skipping those site which are empty.

Given a site of coordinate j we define $\tilde{j}(H)$ as the coordinate obtained subtracting from j the number of sites empty to the left of j and $[\tilde{j}] = \tilde{j} \bmod 2$. Expanding in spin fluctuations around g^m parametrised by $U \in SU(2)$ i.e. $g = U g^m \rightarrow$ action

$$S(H, U) = \int dt \sum_j H_j^* \left(\partial_0 + \mu + (U^\dagger \partial_0 U)_{[\tilde{j}][\tilde{j}]} \right) H_j$$

$$+ \sum_{\langle ij \rangle} -t \, H_i^* H_j \, (U_i^\dagger U_j)_{[\tilde{i}\tilde{j}]} + h.c.$$

$$+ \frac{J}{2} \sum_{\langle ij \rangle} H_j^* H_j H_i^* H_i \left\{ \left| (U_j^\dagger U_i)_{[\tilde{j}][\tilde{i}]} \right|^2 - \frac{1}{2} \right\}$$

Up to now no approximation made!

How ~~the~~ squeezed chain appear

2- MFA

- Mean Field for spin fluctuations on the hopping term for holons:

$$t \langle (U_i^{*} U_{j,[i][j]}) \rangle \equiv t_R$$

- since the motion of ~~the~~ charged d.o.f. is much faster ($t \gg J$) ~~than~~ the motion of the spin d.o.f., we replace $x_j(t)$ in the Heisenberg term by its time average, a straight line. By hard core exclusion ~~these~~ straight lines are at a distance μ^{-1} in lattice spacing units $\longrightarrow$ we obtain the squeezed chain omitting the empty sites and renormalising accordingly the lattice spacing.

MF action $\quad S_{MFA}(H, U) = S(\cancel{H}) + S(U)$

$$S(E) = \int dt \sum_i H_i^{*} (\partial_0 - \mu) H + \sum_{\langle ij \rangle} -t H_i^{*} H_j + h.c$$

free spinless fermion action

$$S(U) = \int\limits_{\text{squeezed chain}} dt \sum_j \left(U_i^\dagger \partial_0 U_j \right)_{[j][j]} +$$

$$+ \sum_{\langle ij \rangle} \frac{J}{2} \left[\left| \left(U_i^\dagger U_j \right)_{[i][j]} \right|^2 - 1 \right]$$

Setting $\quad U e^{i\frac{\pi}{2}\sigma_x j} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad$ hard-core spin $\frac{1}{2}$

with constraint $\quad b_\alpha^* b_\alpha = 1$

$$S(U) = S(b) = \int\limits_{\text{squeezed chain}} dt \sum_j b_{j\alpha}^* (\partial_0 - \mu) b_{j\alpha} +$$

$$+ \frac{J}{2} \sum_{\langle ij \rangle} b_i^* \frac{\vec{\sigma}}{2} b_i \cdot b_j^* \frac{\vec{\sigma}}{2} b_j$$

AF Heisenberg in the squeezed chain

Fermionizing b_α, calling f_α the fermion we obtain for the electron field the formula

$$\psi_{\frac{1}{2}} = \underset{\text{holon}}{H} \; \underbrace{e^{-i\frac{\pi}{2}\sum_{\ell<j} H_\ell^* H_\ell}}_{\text{string}} \underbrace{\left[e^{\mp i\frac{\pi}{2}\sum_{\ell<j} f_{\alpha\ell}^\dagger f_{\alpha\ell}} f_{\frac{1}{2}} \right]_{\text{squ. chain}}}_{\substack{f_\pm \\ \text{spinon}}}$$

$f_{z_j} = e^{i\pi j} f$

At large scale for Left and Right components

$$\langle H^*_{R/L}(0,0) H_{R/L}(x,t) \rangle \sim \frac{1}{x \pm i v t}$$

$$H_j = e^{i\pi\mu j} H_j^L + e^{-i\pi\mu j} H_j^R$$

$$\langle f_\pm^*(0,0)\, f_\pm(x,t)\rangle \sim \frac{1}{\sqrt{x \pm i v_s t}}$$

applying 1D bosonisation

So that e.g. for the spin correlation

$$\langle \vec{S}(0,0)\cdot\vec{S}(x,t)\rangle \sim \langle H H^+(0,0)\, e^{-i\pi\int_{-\infty}^{0} H^+H}$$

$$f_-(0,0)\, f_+(0,0)\; H^+H(x,t)\, e^{i\pi\int_{-\infty}^{x} H^+H}\, f_-^+(x,t)\, f_+^*(x,t)\rangle$$

$$+ h.c. \qquad \sim \quad H^+H = :H^+H: + \mu$$

neglecting

$$\mu^2 \quad \cos\pi\mu x \quad \langle e^{-i\pi\int_{-\infty}^{0} :H^+H:}\, e^{i\pi\int_{-\infty}^{x} :H^+H:}\rangle$$

$$\langle f_-(0,0)\, f_-^+(x,t)\rangle\, \langle f_+(0,0)\, f_+^+(x,t)\rangle$$

$$\sim \mu^2 \frac{\cos\pi\mu x}{(v_c^2t^2+x^2)^{1/4}\sqrt{x-iv_st}\;\sqrt{x+iv_st}}$$

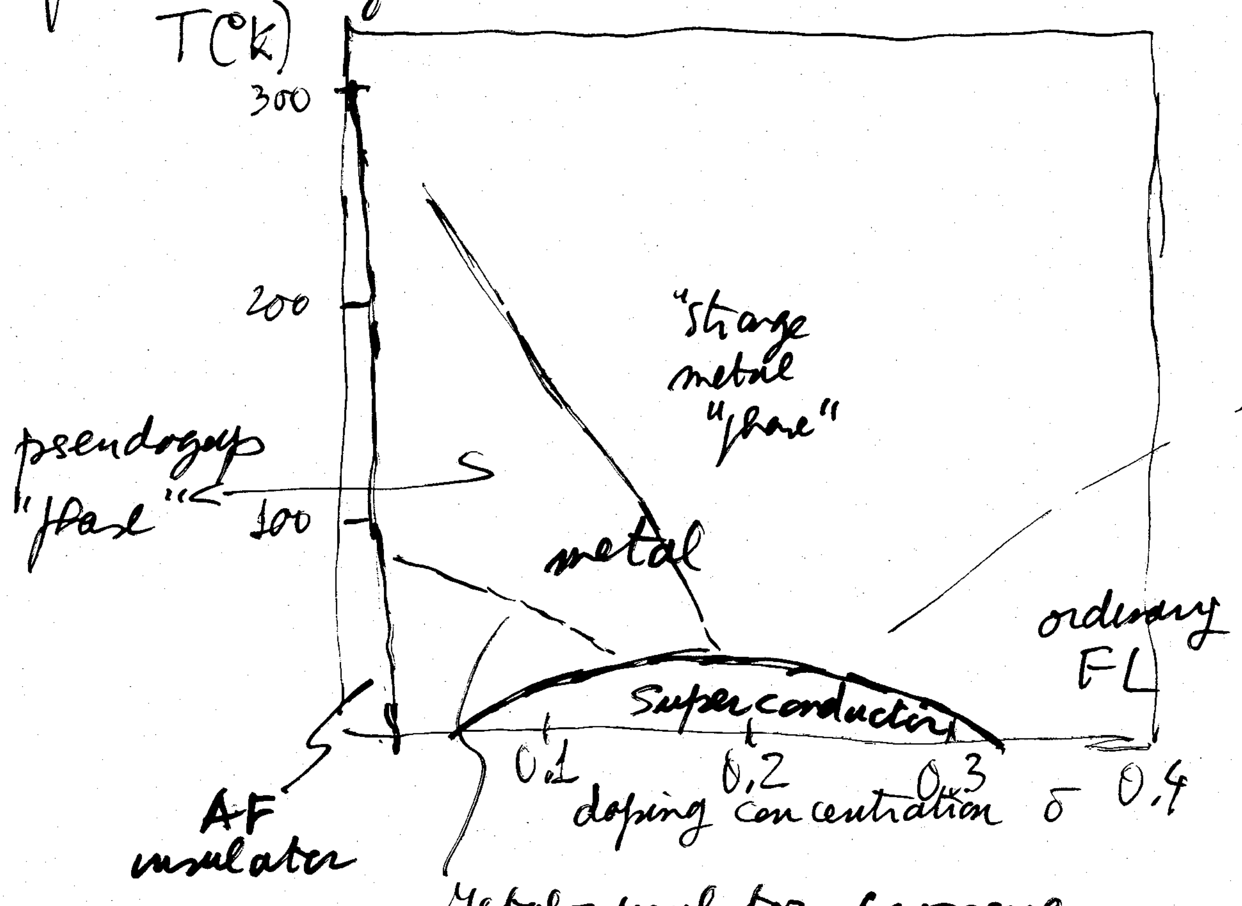
A more sophisticated treatment reproduces also the electron Green function in similar way:

$$\langle \psi_\alpha^+(0,0)\,\psi_\alpha(x,t)\rangle \sim \frac{1}{\sqrt{x+iv_st}}\;\frac{1}{(x^2+v_c^2t^2)^{1/16}}$$

$$\left\{\frac{e^{i\frac{\mu}{2}x}}{(x+iv_ct)^{1/2}} + \frac{e^{i\frac{3\pi}{2}\mu x}}{(x+iv_ct)^{3/2}}\right\} + h.c.$$

Back to 2D
High T_c cuprates

phase diagram



underdoped | overdoped

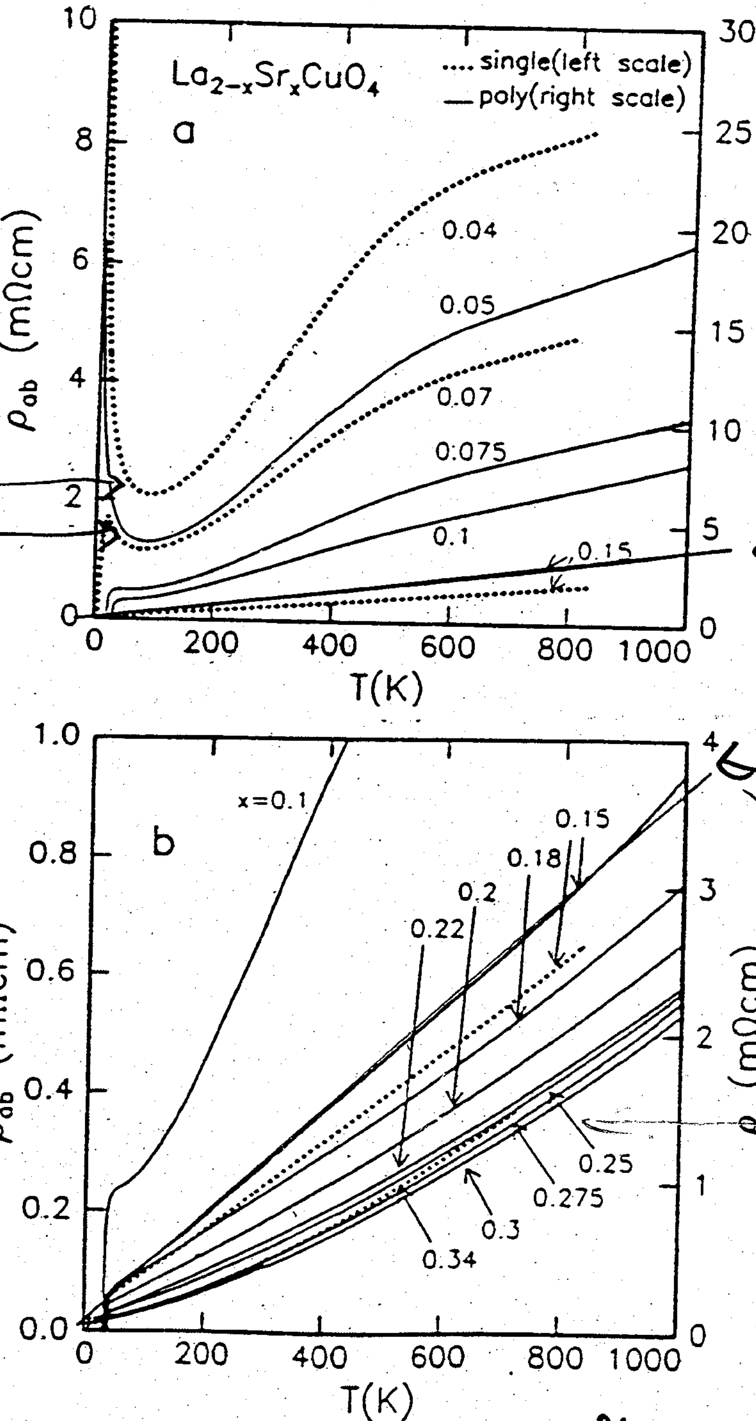
highest T_c = optimally doped

Difference between "phases" visible e.g. in the resistivity

Metal - insulator crossover
(pseudogap "phase")

linear in T resistivity - optimally doped
(strange metal "phase")

Takagi
Batlogg '92 ...
AT&T



under-
doped

over-doped

We apply $U(1)\times SU(2)$ C.S. representation to low T, δ 2D t-J (→ pseudogap phase)
$\underset{0}{\overset{\downarrow}{T}}$, $\underset{0}{\overset{\downarrow}{\delta}}$

What changes from 1D to 2D (approximate!)

$$\chi_\alpha = H \cdot \Sigma_\alpha$$

- gauge invariances

$U(1)$ field B_μ, $SU(2)$ field V_μ, h/s field A_μ

in 1D gauge fields has 0 physical d.o.f.
in 2D 1

1D	2D
modulo gauge transf $B\sim 0$	$e^{i\int_{\partial p} B} = -1$ flux π per plaquette
charge d.o.f H spinless fermion	via Hofstadter convert H into a Dirac fermion with pseudospin
$V\sim 0$	$\langle V_\mu V^\mu\rangle \neq 0$
spin d.o.f. (gauge fluct of V) $b_j \to$ continuum	$b_{j\alpha} \to z_\alpha(x)$ spin $\frac{1}{2}$ hardcore; term $\langle V_\mu V^\mu\rangle z_\alpha^* z_\alpha \to$ mass
$A\sim 0$ spinon - holons non interacting	$A_\mu \neq 0$ couples spinon z_α to holons

Low energy effective action

$S(\underset{\text{holon}}{\psi}, \underset{\text{spinon}}{z}, \underset{\text{h/s gauge}}{A}) =$

$$= \int d^3x \; z_\alpha^* \left[(\partial_0 - A_0)^2 + v_s^2 (\partial_i - A_i)^2 + m^2 \right] z_\alpha$$

$$+ \int d^3x \; \bar{\psi} \left(\gamma^0 (\partial_0 - A_0 - \delta) + (\partial_i - A_i) \gamma^i \right) \psi$$

i.e. massive spinons + gapless holons (finite Fermi Surface $k_F \sim \delta$) interacting via gauge fluctuations A_μ

In terms of the above action one can reproduce the striking phenomenon of metal-insulator crossover of resistivity in pseudogap phase

Sketch: in the regime where resistivity can be written as a sum of holon and spinon contributions (Ioffe-Larkin rule, T not too low)

$\Rightarrow$ spinon dispersion without gauge fluct. is "relativistic massive" $\omega \sim \sqrt{q^2 + m^2}$

Low energy effective action

$$S(\Psi, Z, A) =$$

holon spinon RS gauge

$$= \int d^3x \; Z^*_\alpha \left[(\partial_0 - A_0)^2 + v_s^2 (\partial_i - A_i)^2 + m^2 \right] Z_\alpha$$

$$+ \int d^3x \; \bar{\Psi} \left(\gamma^0 (\partial_0 - A_0 - \cancel{A_0}\delta) + (\partial_i - A_i)\gamma^i \right) \Psi$$

i.e. massive spinons + gapless holons (but Fermi surface $k_F \sim \delta$) interacting via gauge fluctuations A_μ

In terms of the above action one can reproduce the striking phenomenon of metal-insulator crossover of resistivity in pseudogap phase

Sketch: in the regime where resistivity can be written as a sum of holon and spinon contributions [illegible]

→ spinon dispersion without gauge fluct. is "relativistic massive" $\omega \sim \sqrt{q^2 + m^2}$

An approximate evaluation of the effect of gauge fluctuations for $q=0$ small ω produces a damping linear in T

$$\omega \sim \sqrt{m^2 + iT}$$

The spinon-current bubble $\Pi_s(\omega)$ behaves as

$$\operatorname{Im} \Pi_s(\omega) \underset{\omega \to 0}{\sim} \operatorname{Im} \frac{\omega\,(m^2+iT)^{1/4}}{\omega - \sqrt{m^2+iT}}$$

Resistivity turns out to be dominated by spinons and

$$\rho \sim \lim_{\omega \to 0} \left(\frac{\operatorname{Im} \Pi_s(\omega)}{\omega} \right)^{-1}$$

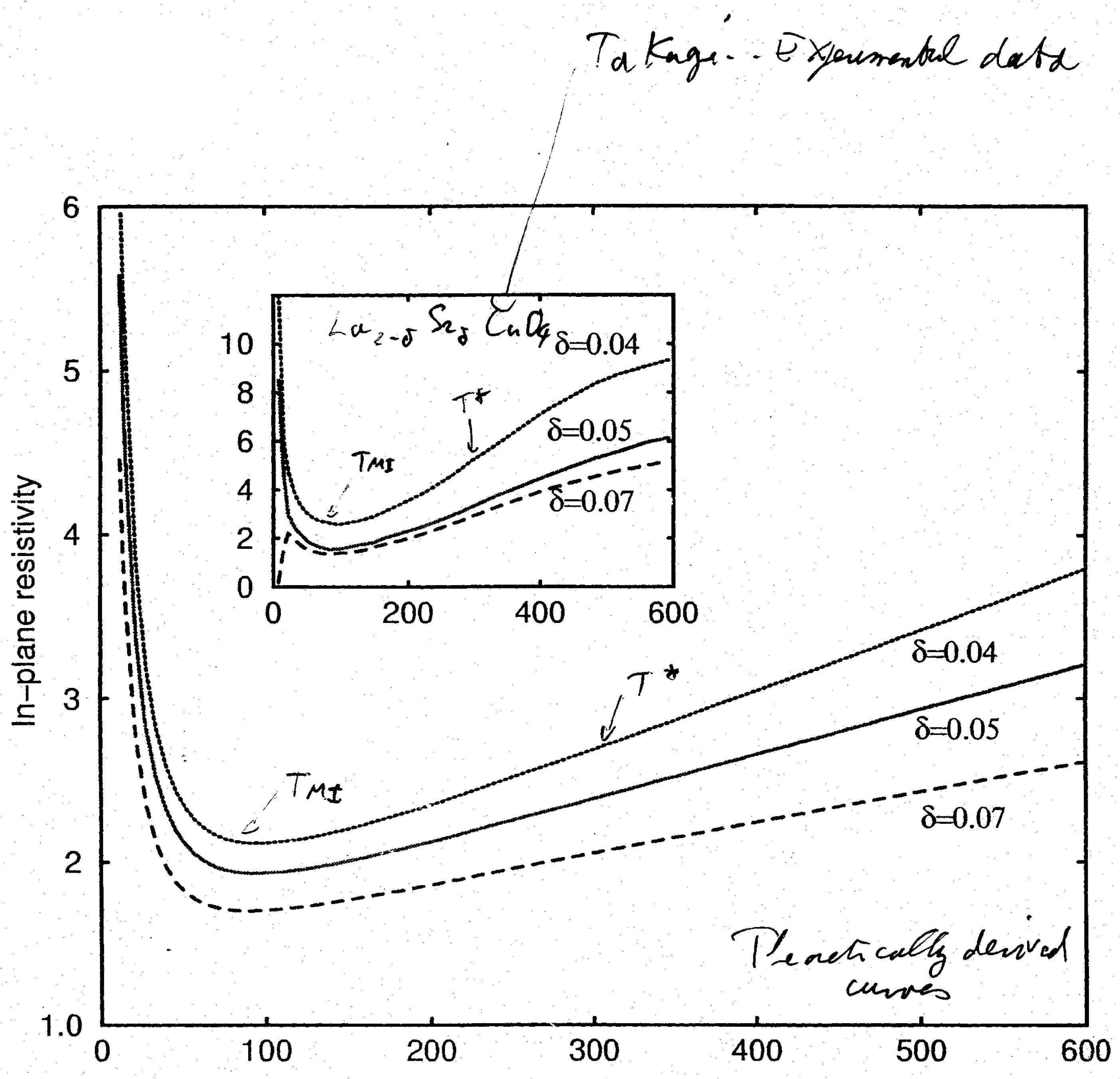
$$\sim \left(\operatorname{Im} \, (m^2 + iT)^{-1/4} \right)^{-1} =$$

$$= \frac{(m^4 + T^2)^{1/8}}{\sin \frac{1}{4} \operatorname{arctg} \frac{T}{m^2}}$$

$T \downarrow 0$ $\quad \frac{1}{T}$ insulating

$T \gg m^2$ $\quad T^{1/4}$ metallic

$\Rightarrow$ combined effect of the mass for the spinons induced by the SU(2) gauge field and the linear in T damping due to the h/s (A_μ) gauge field reproduce the metal-insulator crossover!


Takagi.. Experimental data
La2-δ Srδ CuO4
δ=0.04
δ=0.05
δ=0.07
T*
TMI
In−plane resistivity
Theoretically derived curves

Keimer. Experimental data