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SMR.1320 - 20

SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

INTERACTING RANDOM DIRAC FERMIONS IN SUPERCONDUCTING CUPRATES

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Interacting random Dirac fermions in superconducting cuprates

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in collaboration with:

J. Paaske and I.V. Gornyi (Karlsruhe),

W.A. Atkinson and P.J. Hirschfeld (Gainesville),

and A.G. Yashenkin (Chapel Hill)

Phys.Rev.Lett. **86**, *4668, 4672, 5982* (2001)

Outline of the talk

- Fermi surface vs Fermi points: Dirac-like excitations in strongly correlated fermion systems;
- Interacting quasiparticles and subdominant order parameters in layered **d-wave** superconductors;
- (De)localization theory for disordered Dirac fermions;
- Experimental signatures of quasiparticle localization in high-Tc cuprates .

Dirac fermions in condensed matter physics

- Effective description of statistical (**d+0**-dimensional) systems:

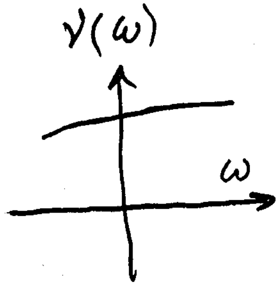
Ising model, random magnetic field, network models of Quantum Hall plateau transitions;

- Low-energy excitations in dynamical (**d+1**- dimensional) systems:

- layered **d-wave** superconductors (high-Tc cuprates);
- **p-wave** superconductors/superfluids (He3-A);
- semimetals (graphite);
- dichalcogenides (2H-TaSe2,..).

Fermi liquids vs. Dirac fermions

Fermi surface:



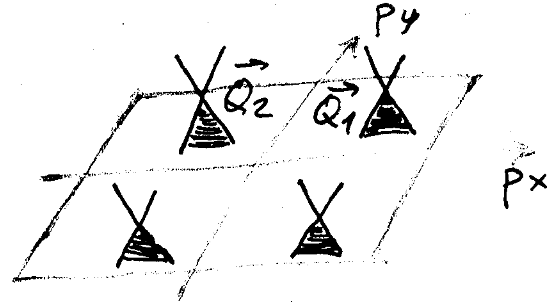
$$\varepsilon = v_F (p - p_F)$$

$$\nu(\omega) \simeq \mathrm{Const}$$

$$C(T) \sim T$$

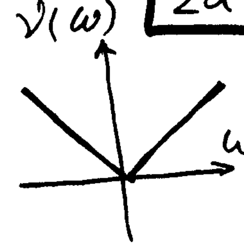
$$\chi_s(T) \sim \mathrm{Const}$$

Isolated Fermi points:



$\pm \vec{Q}_1 \, ; \, \pm \vec{Q}_2$

$\boxed{2d}$



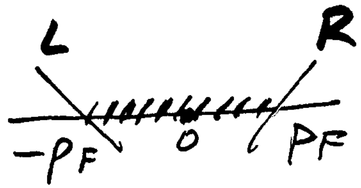
$$\varepsilon = v \, | \vec{p} - \vec{Q}_i |$$

$$\nu(\omega) \sim |\omega|^{d-1}$$

$$C(T) \sim T^{d}$$

$$\chi_s(T) \sim T^{d-1}$$

In 1d both are equivalent:
(almost)



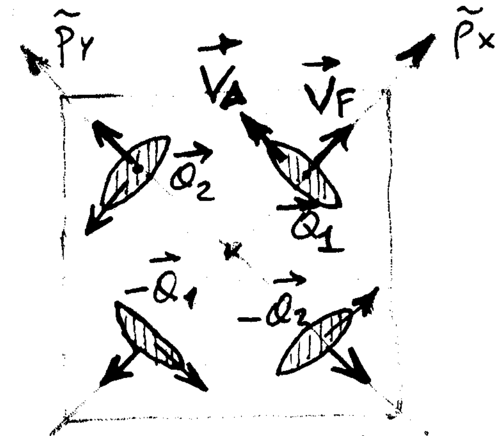
$\cong$

$$\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

Quasiparticles in planar d-wave superconductors

- Gor'kov-Nambu spinors and BdG Hamiltonian:

$$\psi^i_{\vec p\alpha} = \begin{pmatrix} c^i_{\vec p\alpha} \\ \epsilon_{\alpha\beta}\, c^{+}_{-\vec p\beta} \end{pmatrix} \qquad H = \sum_{\substack{i=1,2\\ \alpha=\uparrow\downarrow}} \psi^{+}_{i\,\vec p\alpha} \begin{pmatrix} \xi_{\vec p} & \Delta_{\vec p} \\ \Delta^*_{\vec p} & -\xi_{\vec p} \end{pmatrix} \psi^i_{\vec p\alpha}$$



- Lattice dispersion and gap function:

$$\underline{\xi_{\vec p}} = -2t(\cos p_x + \cos p_y) + \dots \qquad \Delta_{\vec p} = \underline{\Delta'_{\vec p}} + i\,\underline{\underline{\Delta''_{\vec p}}}$$

$$\underline{\Delta'_{\vec p}} = \Delta_0 (\cos p_x - \cos p_y)$$

- Low-energy qps:

$$E_p = \sqrt{\xi_p^2 + \Delta_p^2}$$

$$\underline{\underline{V_F}} = \frac{\partial \xi_{\vec p}}{\partial p_\perp} \qquad \underline{\underline{V_\Delta}} = \frac{\partial \Delta'_{\vec p}}{\partial p_\parallel}$$

$$\frac{V_\Delta}{V_F} \simeq 10 \div 20$$

$$\mathcal{H} = \psi_1^+ \left(v_F \tilde p_x \hat\sigma_3 + v_\Delta \tilde p_y \hat\sigma_1 + \Delta''_{Q_1} \hat\sigma_2 \right) \psi_1 + \psi_2^+ \left(v_F \tilde p_y \hat\sigma_3 + v_\Delta \tilde p_x \hat\sigma_1 + \Delta''_{Q_2} \hat\sigma_2 \right) \psi_2$$

- QP interactions (screened Coulomb, AFM fluctuations):

$$\boxed{\mathrm{Im}\Sigma = \tau^{-1}(\omega \sim T) \sim T^3}$$

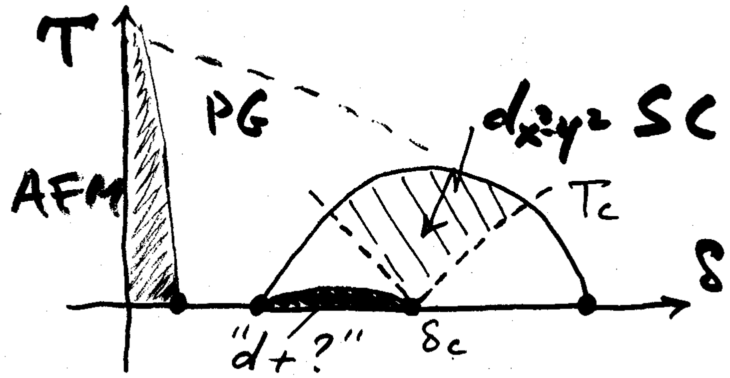
(D.Scalapino et al, '94)

$$\mathrm{Im}\,\Pi(\omega, q) \sim \theta(\omega - vq)$$

$$[\ \mathrm{FS}:\ \mathrm{Im}\,\Pi \sim \theta(vq - \omega)$$

Possible quantum-critical behavior in cuprates

- QP lifetime from experiment (ARPES, optical conductivity): $\boxed{\tau^{-1}(T) \sim T}$



Quantum-critical regime:

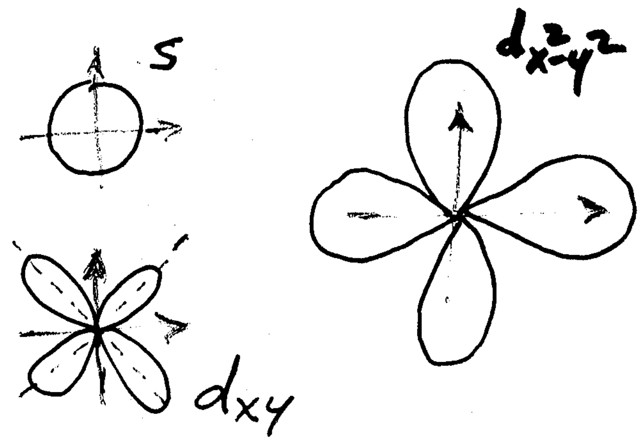
$$|\delta-\delta_c|^{\nu} \lesssim \frac{T}{T_c} < 1$$

- Competing ground states and qp scattering off the fluctuations of the corresponding order parameters (*S.Sachdev et al, '00*):

secondary pairing - fully gapped qp spectrum (**is**, **idxy**);
shifted nodes (**s**, **dxy**, **ig**); excitonic order (**p**, **dxy**).

Plausible options: $d_{x^2-y^2} \rightarrow d_{x^2-y^2} + i\,s$ (*T*-odd, **P**-even)

$d_{x^2-y^2} \rightarrow d_{x^2-y^2} + i\,d_{xy}$ (*T*-and **P**-odd)



NOT DUE TO: H; magn. imp.; surface

Nodal qps near second pairing transition

- Incipient order parameter fluctuations:

$$\mathcal{L}_\phi = \frac{1}{2c^2}(\partial_t \Phi)^2 - \frac{1}{2}(\nabla_i \Phi)^2 - \frac{m^2}{2}\Phi^2 - u\Phi^4 \quad \to -u:\Phi^4: + m_c^2 \Phi^2$$

$\boxed{\Delta'' \equiv \Phi \to -\Phi}$ Ising (Z_2) symmetry breaking at $\boxed{m^2 \leq m_c^2}$

- Couling to the nodal fermions:

$$\mathcal{L}_\psi = \sum_{\alpha=1}^{N} \bar{\Psi}_\alpha \left(i v_\mu \gamma_\mu \partial_\mu + g \cdot \Phi \hat{\Gamma} \right) \Psi_\alpha \qquad \Psi_\alpha = \begin{pmatrix} \psi_\alpha^1 \\ \psi_\alpha^2 \end{pmatrix}$$

$$\alpha = 1, 2 \to N$$

- Effective Lorentz-invariance (RG): $\tilde{v}_F = \tilde{v}_\Delta = \tilde{c} \to 1$

- Z_2--symmetry breaking in the presence of fermions:

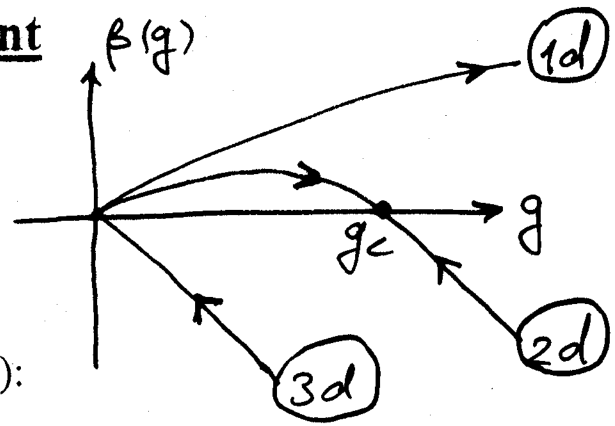
is: $\hat{\Gamma} = \hat{1} \otimes \hat{1}$, $\Phi_s \to -\Phi_s$ -- Chiral symmetry

$\Psi_\alpha \to (\hat{1} \times \hat{\sigma}_2)\Psi_\alpha$

id_{xy}: $\hat{\Gamma} = \hat{1} \otimes \hat{\sigma}_3$, $\Phi_d \to -\Phi_d$ -- Parity $x \to -x$

$\Psi_\alpha \to (\hat{\sigma}_1 \times \hat{\sigma}_1)\Psi_\alpha$ $\quad y \to y$

Strong-coupling infrared fixed-point



- $\mathcal{E}$ -- expansion in **d=3-$\mathcal{E}$** (*S.Sachdev et al, '00*):

$$\bar{\Psi}\Gamma\Psi\cdot\Phi : \; g_c^2 \sim \varepsilon \; ; \; u_c \sim \varepsilon \qquad |\Phi|^4$$

- Spontaneous chiral symmetry breaking (*DVK and J.Paaske, '00*):

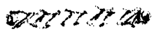
2d: $\underline{\underline{M}} \sim \langle \phi \rangle \neq 0$

$\boldsymbol{E} = v\sqrt{p^2+M^2}$

$$\boxed{1 = \frac{N g^2}{m^2}\int_0^{\Lambda} \frac{d^3p}{p^2+M^2}} \Rightarrow \underline{\underline{g_c}}^2 \sim \frac{m^2}{N\Lambda}$$

- 1/(4N)--expansion around the mean field solution:

$$G_q^{-1} = G_q^{(0)\,-1} + g^2 \cdot \int G_{q+k} \langle \phi\phi \rangle_k \, d^3k$$



$$\langle \phi\phi \rangle_k = (k^2 + m^2 - m_c^2 + \Pi_k)^{-1} \approx \Pi_k^{-1}$$

$$\Pi_k = g^2 \cdot \int G_{q+k} G_q \, d^3q \sim g^2 \cdot \sqrt{k^2}$$

$$\boxed{\begin{array}{l} [\Phi] = 1/2 \;\longrightarrow\; 1 + O(1/N) \\ u \to 0 \qquad g \to g_c \end{array}}$$

- Critical exponents: $\nu = 1 + \frac{8}{3\pi^2 N} + \ldots$ (MC: $\nu = 1.00$); $\eta = 1 - \frac{16}{3\pi^2 N} + \ldots$ (MC: $\eta = 0.7$

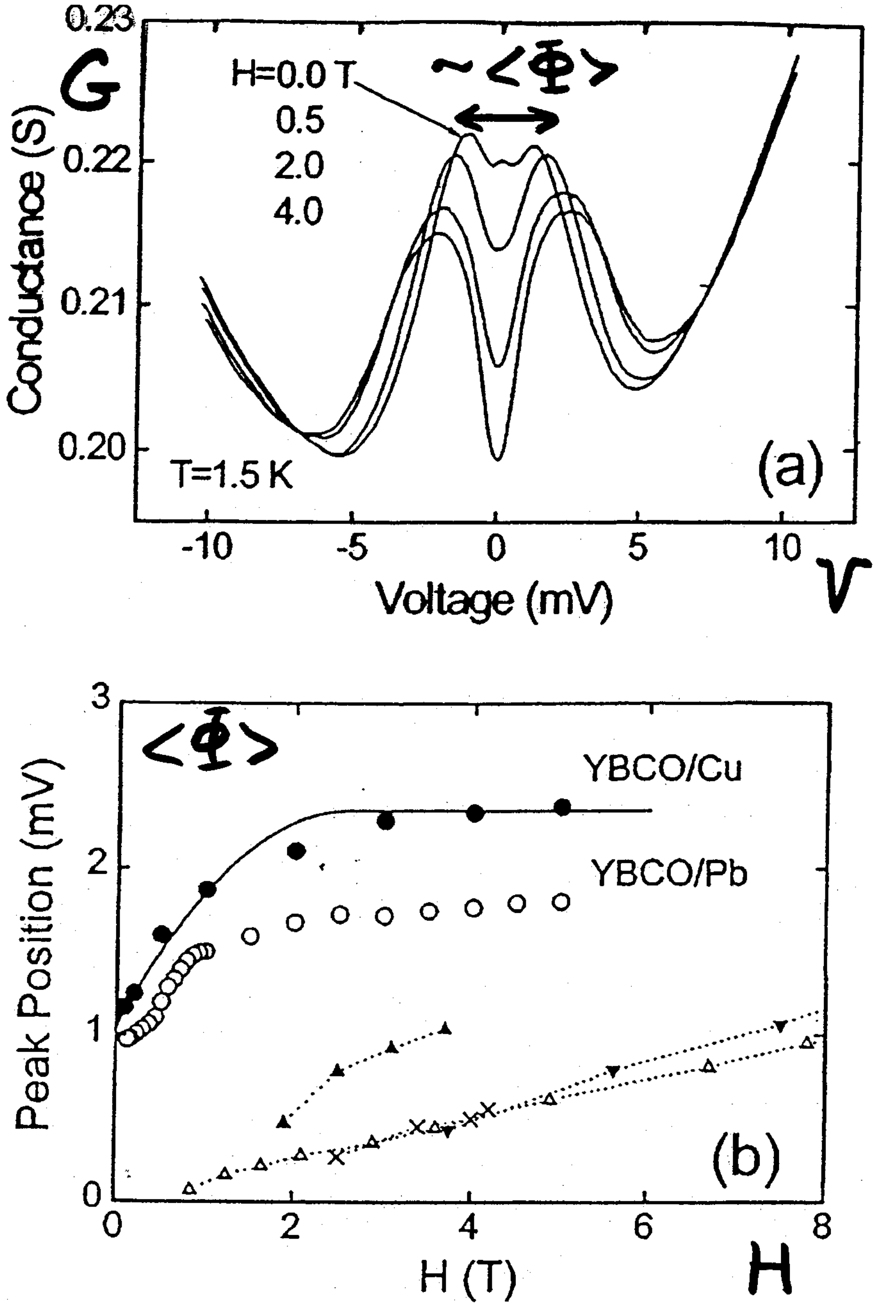
- Different universality class for different N, no Goldstone mode, ***F*** $[\langle \phi \rangle]$ - NON GL

S

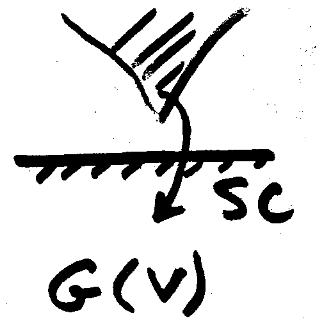
d_{xy}

NON BCS: $\frac{\Delta''(T=0)}{T_c^{*}} = 2\ln 2 \approx 1.39$

M. Covington et al '97



Evidence of a surface-induced "d+is" id' ?

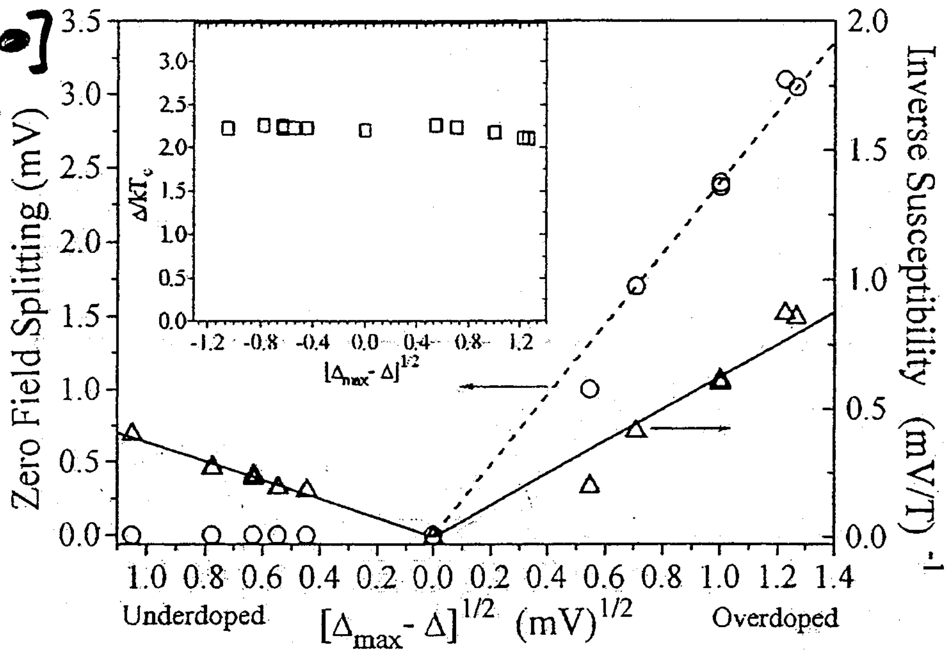


:G. 2. (a) Magnetic field dependence of the ZBCP from YBCO/Cu tunnel junction. A magnetic field induces rther splitting of the ZBCP. (b) A compendium of data on e magnetic field-induced splitting of ZBCP's. Data from BCO/Cu and YBCO/Pb [3] junctions are indicated by closed d open circles, respectively. The theoretical curve for the bdominant order parameter being A_{1g} (s wave) is shown as a ll line [14]. As a comparison, data from other junctions with ıgnetic scattering centers are included. These are represented (Δ) for $Ta/Ta_2O_5/Al$ [8], (▲) for $Sn/Sn_xO_y/Sn$ [23],) for Al/Ti-doped Al_2O_3/Al [24], and (▼) for a Au/Si:P hottky barrier tunnel junction [25].

G. Deutscher et al '01

$\langle \Phi \rangle$ [o] $(\delta-\delta_c)^\beta$

χ^{-1} [Δ] $\sim |\delta-\delta_c|^\gamma$



$\delta-\delta_c$

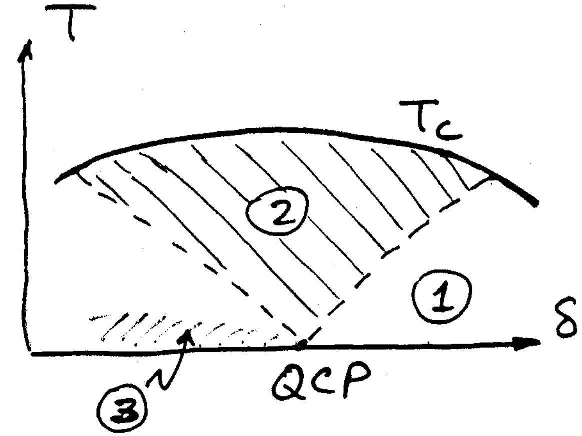
Figure 5 The spontaneous (at zero field) splitting of the zero bias conductance peak versus $[\Delta_{max}-\Delta]^{1/2}$ (circles) for doping ranging from slightly underdoped (T_c=83.6K down set) to slightly overdoped (T_c=85.6K down set) . $[\Delta_{max}-\Delta]^{1/2}$ is a quantity proportional to the doping level (see text).Triangles: the inverse susceptibility χ^{-1} for the same samples. The upper bound of χ^{-1} for the sample with $(\Delta_{max}-\Delta)=0$ is 0.08[mV/T]$^{-1}$. Solid lines: linear fits for both the underdoped and overdoped ranges. The lines extrapolate to zero at the same doping level where the spontaneous splitting appears. Dashed line linear fit for $\delta(0)$ on the overdoped side. Inset: $2\Delta/kT_{cW}$ for the samples measured.

Exp. : $\beta \simeq 1$, $\gamma \simeq 1$

Th. : $\beta = 0.87$, $\gamma = 1.25$

2+1 Ising : $\beta = 0.32$, $\gamma = 1.26$

Dirac fermion spectral function



- Solution of the Dyson eqs:

(1) $\omega, q \gg T$ $\quad G \propto \dfrac{\omega\gamma_0 - \vec{q}\vec{\gamma}}{(\omega^2 - q^2)^{1-\eta_\psi}}$

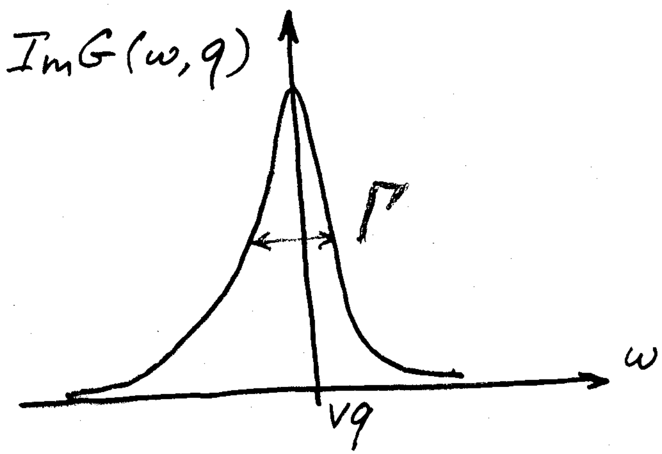
(2) $\omega, q \lesssim T$ $\quad G \propto T^{\eta_\psi} \cdot \dfrac{(\omega + i\Gamma)\gamma_0 - \vec{q}\vec{\gamma}}{(\omega + i\Gamma)^2 - q^2}$

(3) $M \neq 0$ $\quad G \propto \dfrac{1}{\omega^2 - q^2 - M^2}$

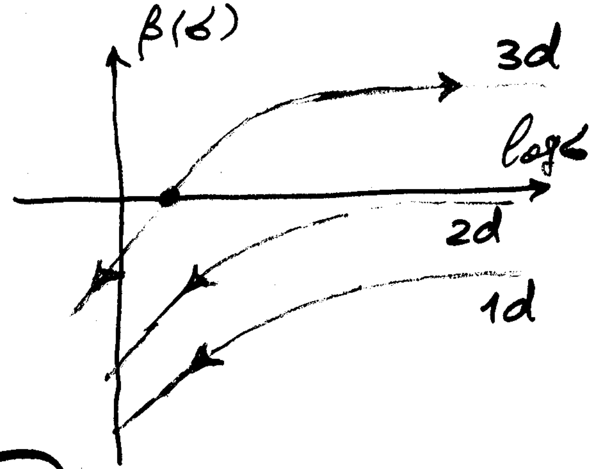
- Quasiparticle damping:

$$\Gamma(\omega, T, \delta - \delta_c) \sim \begin{cases} \max(\omega, \underline{\underline{T}}), & |\delta - \delta_c|^{\nu} \ll T/T_c \\ \max(\omega^3, T^3), & |\delta - \delta_c|^{\nu} \gtrsim T/T_c \end{cases}$$

- ARPES lineshape:



$$\boxed{\eta_\psi = \frac{2}{3\pi^2 N} + \ldots}$$



Disordered Fermi liquids

- Symmetries of the single-particle Hamiltonian: **SU(2)** and ***T***

- Three Gaussian ensembles:

Orthogonal (**SU(2)** & ***T***) → WL in 2d (potential impurity scattering);

$$\frac{d\ln\sigma}{d\ln L} = d-2 - \frac{\alpha}{\sigma}$$

$$\sigma_0 = p_F l \gg 1$$

Unitary (no ***T***) → "even weaker" WL in 2d (magnetic field or spin-flip);

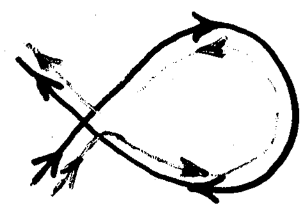
$$\frac{d\ln\sigma}{d\ln L} = d-2 - \frac{b}{\sigma^2}$$

$$L_\xi \gg l = v_F\tau$$

Simplectic (no **SU(2)**) → "anti"localization (spin-orbit scattering);

$$\frac{d\ln\sigma}{d\ln L} = d-2 + \frac{c}{\sigma}$$

- Single-particle DOS remains largely intact;



- Dephasing due to Coulomb e-e interactions:

$$\boxed{\frac{\delta\sigma(T)}{\sigma_0} \propto -\frac{1}{\sigma_0}\log\frac{\tau_\varphi(T)}{\tau}}$$

$$\boxed{\tau_\varphi^{-1}(T) \sim \frac{T}{\sigma}\log\sigma}$$

$$C(\omega,q) = \frac{1}{i\omega + Dq^2 + \tau_\varphi^{-1}}$$

Disordered Dirac fermions in *d-wave* superconductors

- Extra Hamiltonian symmetry: **p-h** transformation,

Novel coherence phenomena: impurity scattering + Andreev reflection;

- Energy and spin, but no charge, diffusion:

$$\partial_t \rho + \nabla_i J_i \neq 0 \qquad J_i = -\sigma^{ij} \nabla_j V$$

$$\partial_t h + \nabla_i Q_i = 0 \qquad Q_i = -\kappa^{ij} \nabla_j T$$

$$\partial_t \vec{S} + \nabla_i \vec{J}_i = 0 \qquad \vec{J}_i = -\sigma_s^{ij} \nabla_j \vec{H}$$

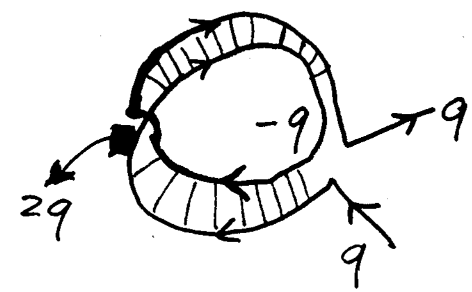
$$\boxed{\sigma_s = \nu \cdot D}$$

$$\boxed{\kappa / T = \sigma_s \cdot \frac{4\pi^2}{3} \frac{k_B^2}{\hbar^2}}$$

- Seven new universality classes (*Altland, Zirnbauer, '97*)

(different patterns of **SU(2)** and ***T*** – breaking);

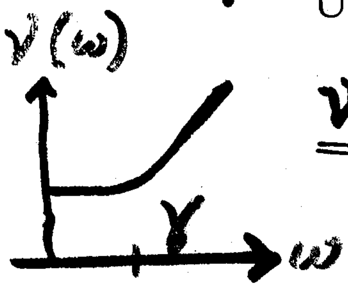
- Strong dependence on the type of disorder ;
- Single-particle DOS **is** affected by disorder;
- Self-consistent Born approximation (*P.A.Lee, '93*)

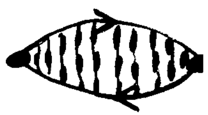


$$\underline{\underline{\mathrm{Im}\, \Sigma(0) = \gamma}} \sim \Delta e^{-1/\lambda}$$

$$\underline{\underline{\lambda}} \sim \frac{n_i |u_{imp}|^2}{v_F v_\Delta} \ll 1$$

- "Universal" limit:





$$\underline{\underline{\nu(0)}} \sim \frac{\gamma}{v_F v_\Delta} \log \frac{\Delta}{\gamma}$$

$$D \sim l_{tr} \sim \nu^{-1}(0)$$

$$\boxed{\frac{\kappa}{T}\Big|_{T \to 0} = \frac{\pi^2}{3} k_B^2 \left(\frac{1}{\pi^2} \cdot \frac{v_F^2 + v_\Delta^2}{v_F v_\Delta} \right)}$$

L. Taillefer et al, '97

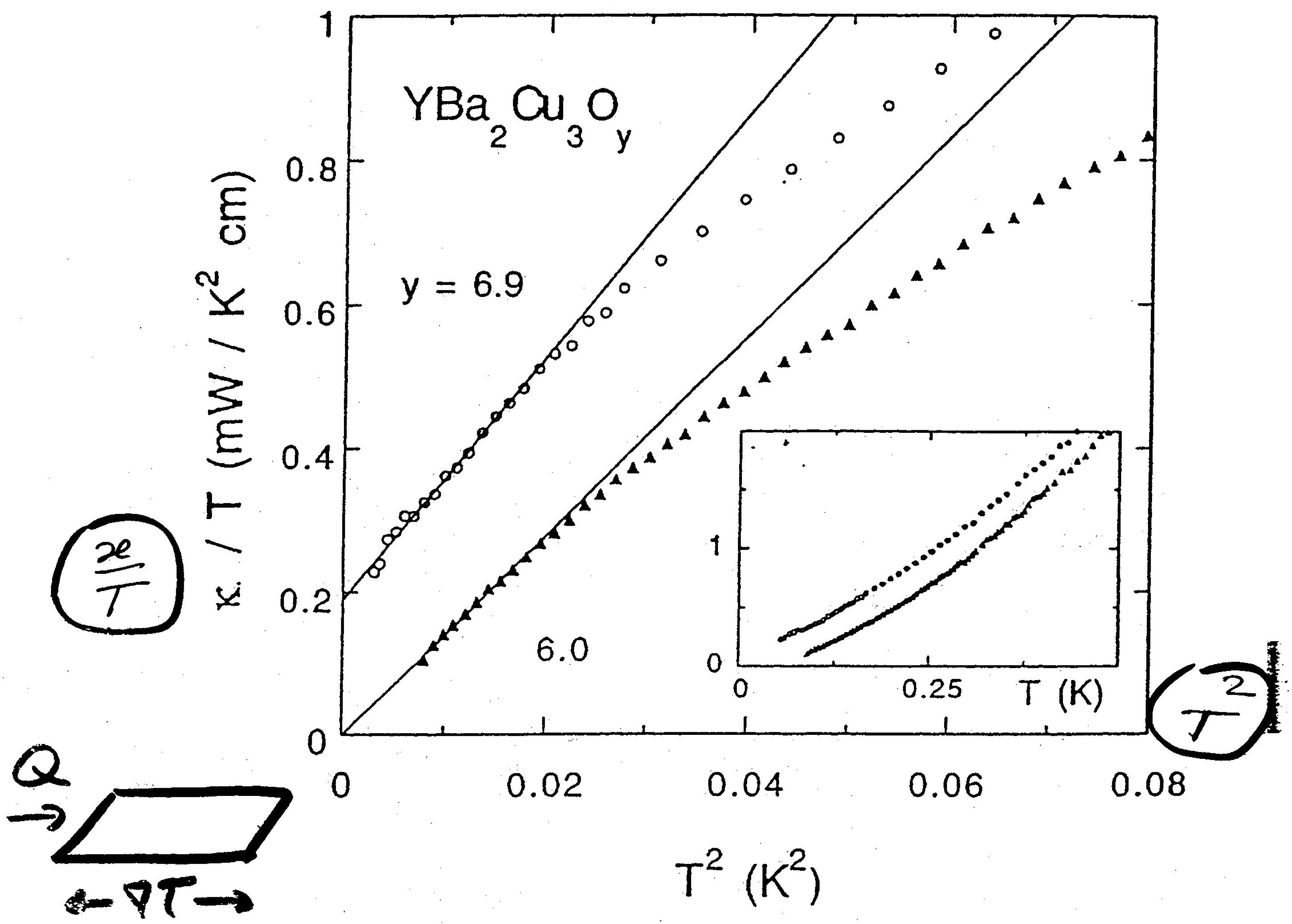


FIG. 1. *a*-axis thermal conductivity of the two $YBa_2Cu_3O_y$ crystals, one superconducting ($y = 6.9$; circles) and one insulating ($y = 6.0$; triangles). Main panel: κ/T vs T^2; lines are fits to $a + bT^2$ for $T < 0.15$ K. Inset: κ/T vs T.

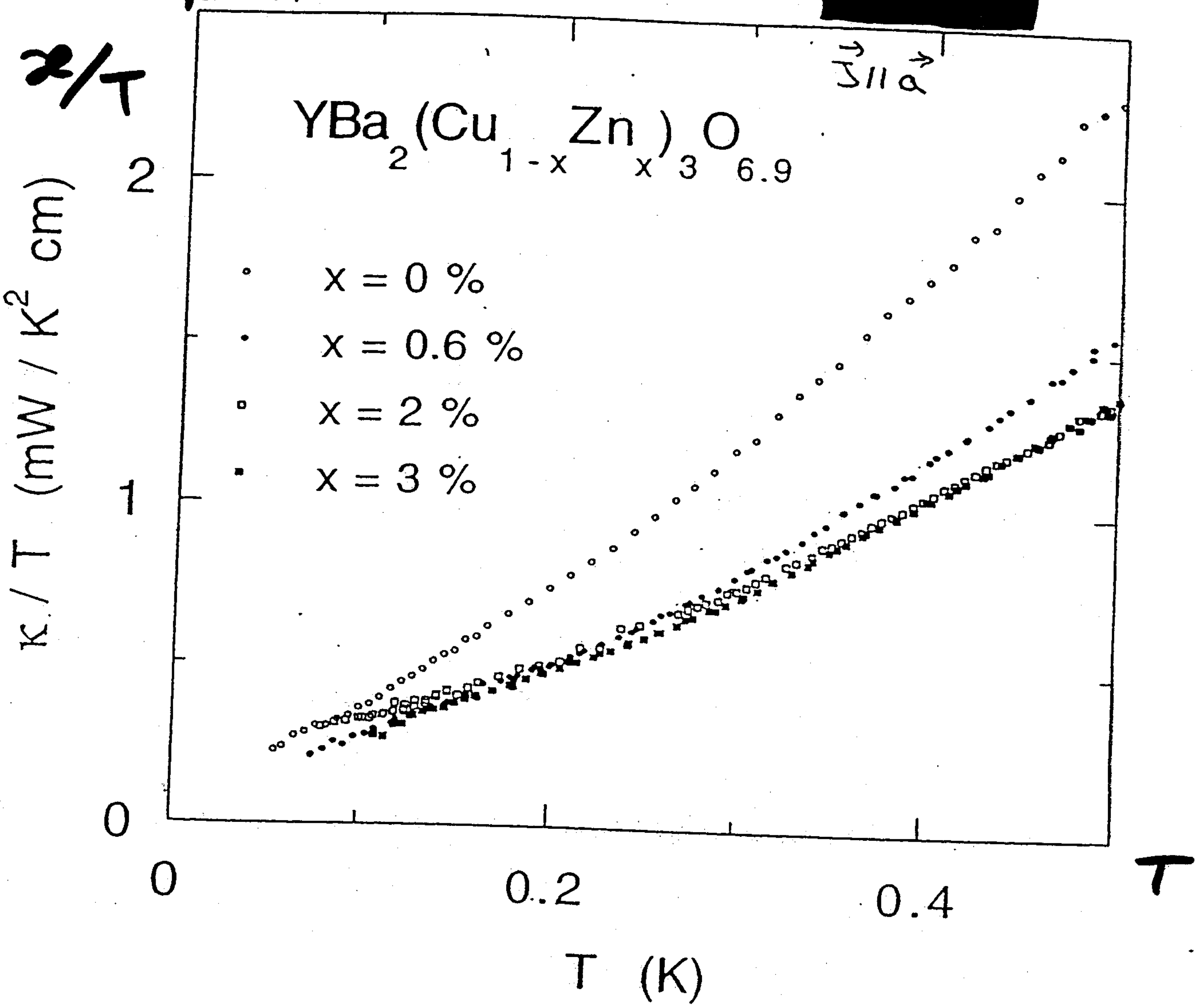
Universal limit $\varkappa/T\big|_{T\to 0}$

in (super) conducting

$YBa_2Cu_3O_{7-x}$

L. Taillefer et al '97

Taillefer '97 [redacted]



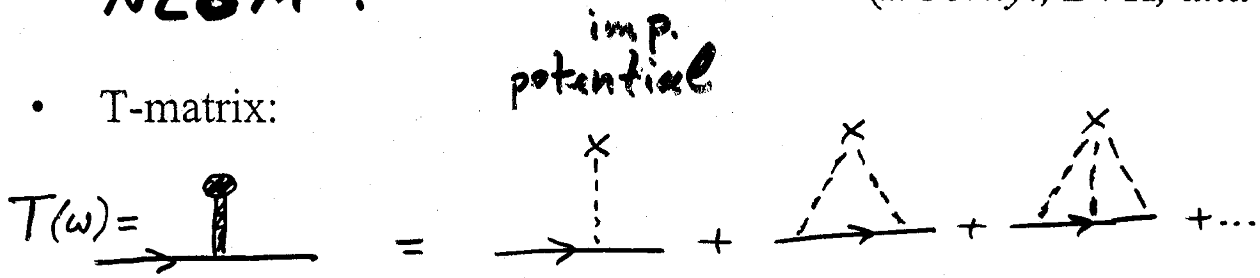
"Universal" thermal conductivity

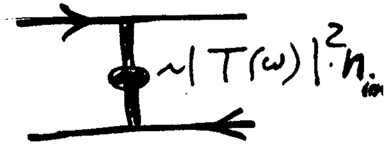
Beyond SCBA: diffusive regime

NLσM ?

(*I.Gornyi, DVK, and A.Yashenkin'00*)

- T-matrix:





$\hat S(\omega) = S_0\, \hat 1\times\hat 1$ (UL

$+ S_3\, \hat\sigma_3\times\hat\sigma_3$ (GL)

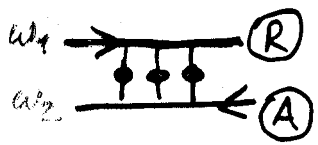
UL : $\varphi \to \pi/2$

GL : $\varphi \ll 1$

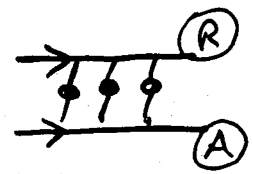
- Standard and novel diffusion modes:

$$\hat D(\hat C) = \sum_{\mu=0,1,2,3} D_\mu(C_\mu)\,\hat\sigma_\mu\times\hat\sigma_\mu \quad ; \quad \hat\sigma_0\equiv\hat 1$$

$$\hat D = \hat S + \hat S * \int GG\, d\vec p * \hat D$$



$D_{RA}(\omega_1-\omega_2)$;

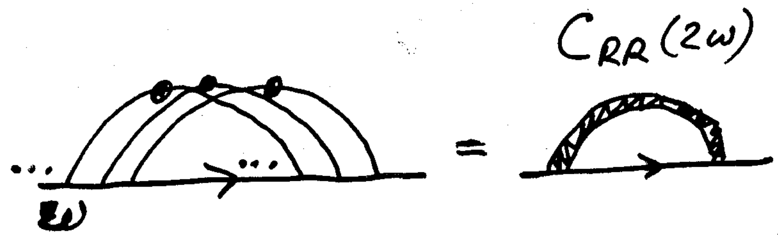


$C_{RA}(\omega_1-\omega_2)$;

$D_{RR\,(AA)}(\omega_1+\omega_2)$

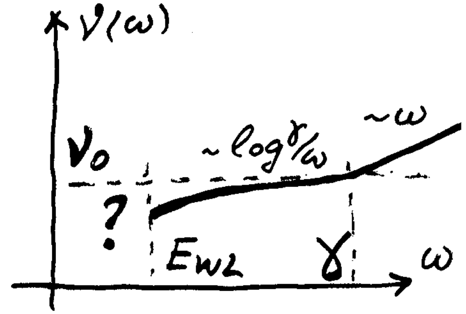
$C_{RR\,(AA)}(\omega_1+\omega_2)$

- DOS correction from the **C** – cooperon:



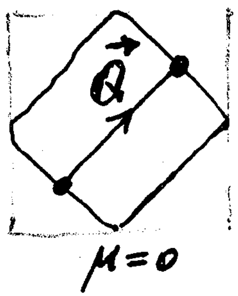
$$\boxed{\frac{\Delta\nu(\omega)}{\nu_0} \sim -\frac{1}{\sigma_s}\log\frac{\gamma}{\omega}}$$

$$\boxed{E_{WL}\sim \gamma\, e^{-\sigma_s}}$$



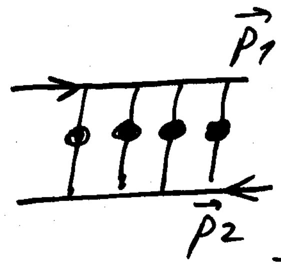
Lattice *d-wave* superconductor near half-filling : yet another discrete symmetry

- Additional doubling of the number of Goldstone diffusion modes



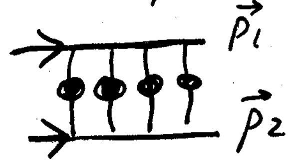
$$\xi_{\vec{p}} = -2t(\cos p_x + \cos p_y) - \mu$$

$$\boxed{\begin{array}{l}\xi_{\vec{p}} = -\xi_{\vec{p}+\vec{Q}} \\ \mu = 0\end{array}}$$



$$D^{\pi}_{\substack{RA\\RR}}(\varepsilon_1 \mp \varepsilon_2;\ \vec{p}_1 - \vec{p}_2 \approx \vec{Q})$$

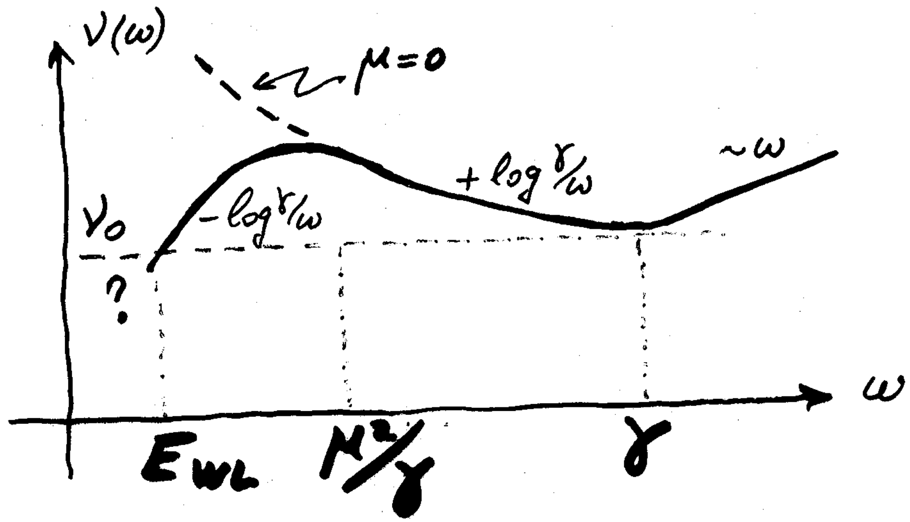
- Sign change of the DOS correction:



$$C^{\pi}_{\substack{RA\\RR}}(\varepsilon_1 \mp \varepsilon_2;\ \vec{p}_1 + \vec{p}_2 \approx \vec{Q})$$

$\underline{UL}$:

$\mu \ll t$



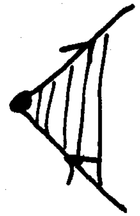
$$\boxed{\begin{array}{l}E_{WL} \sim \gamma\, e^{-\delta_s} \\ \delta_s \sim N \cdot \frac{v_F}{v_\Delta} \gg 1\end{array}}$$

Interaction effects in dirty d-wave superconductors

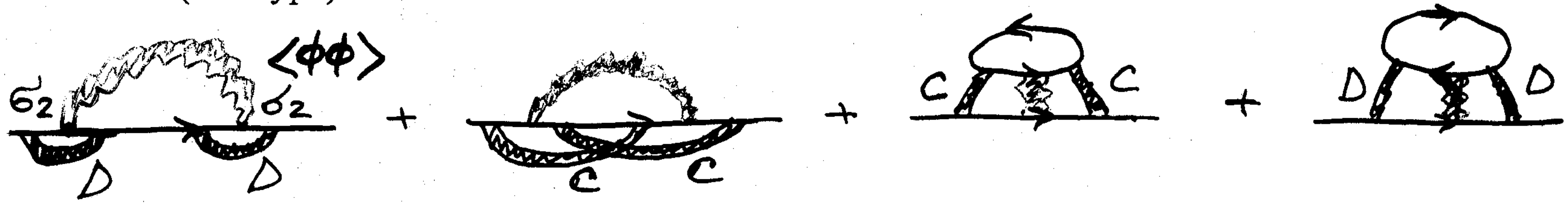
(DVK and A.Yashenkin, '00)

- Vertex corrections:

$\hat{1}, \hat{\sigma}_2$  $\sim \frac{1}{i\omega + Dq^2}$ diffusion-pole

$\hat{\sigma}_{1,3}$  no pole

- Interference (**AA**-type) corrections to **DOS:**

$\Delta_{int}\Sigma =$ 

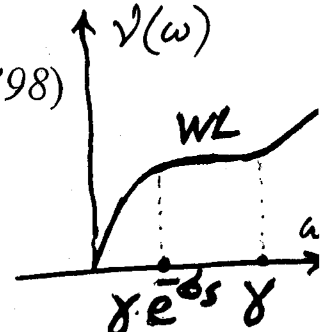
$$\frac{\Delta_{int}\nu(\omega)}{\nu_0} \sim \boxed{\oplus \frac{1}{\sigma_s} \log|\log \gamma/\omega|} \ll \frac{\Delta_{WL}\nu(\omega)}{\nu_0} \sim \ominus \frac{1}{\sigma_s} \log \frac{\gamma}{\omega}$$

- Thermodynamics: $\Delta C(T)/T \sim \Delta\chi_s \sim \boxed{+\log|\log \frac{\gamma}{T}|}$

- Dephasing rate:

$$\frac{\Delta_{WL}\sigma_s}{\sigma_s} \sim -\frac{1}{\sigma_s} \log[\gamma \underline{\underline{\tau_\phi}}(T)] \; ; \quad \boxed{\tau_\phi^{-1} \sim \frac{T}{\sigma_s} \cdot \frac{\log \sigma_s}{\log^2 \frac{\gamma}{T}}}$$

(De)localization properties of random Dirac fermions

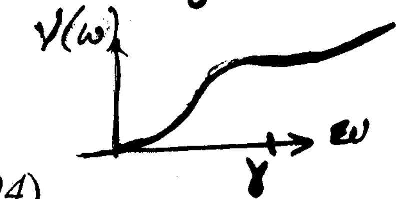


- Isotropic impurity scattering (short-range disorder) (*M.P.A.Fisher et al, '98*)

$$\nu(\omega) \sim |\omega| \qquad \sigma_s(T)\Big|_{T\to 0} = 0$$

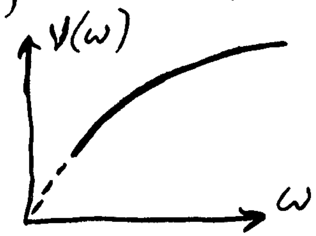
- Orbital magnetic field alone doesn't kill WL (but Zeeman field does)

[cf.: metallic Q-dot boundered by a superconductor in magnetic field]



$$\nu(\omega) \sim |\omega|^2 \qquad \sigma_s(T)\Big|_{T\to 0} = 0$$

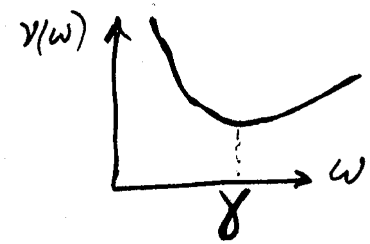
- Predominantly forward scattering (smooth disorder) (*A.Tsvelik et al, '94*)



$$\nu(\omega) \sim |\omega|^{\alpha} \qquad \alpha = \alpha(\lambda \sim n_i) < 1$$

- Imputities in unitarity limit (binary alloy) (*K.Pepin and P.A.Lee, '98*)

$$\nu(\omega) \sim \frac{n_i}{|\omega| \log^2 \frac{\Delta}{\omega}} \qquad \boxed{\mu = 0}$$



- Spin-orbit scattering (*T.Senthil et al'99*)

$$\nu(\omega) \sim \left|\log \frac{\Delta}{\omega}\right|^{1(1/2)} \qquad \sigma(T) \sim |\log T|$$

Experimental manifestations of different phases

- Pure **d** -- "Thermal insulator" phase:

Vanishing $\varkappa/T\big|_{T\to 0}$; $\sigma_s(T\to 0)$

Positive $\frac{d}{dH}\varkappa(H) \longleftrightarrow \frac{d}{dH}\Delta_{WL}\varkappa$

Linear DOS $\nu(\omega) \sim |\omega|$

- **d+is** -- "Even weaker" localization: $\nu(\omega) \sim |\omega|^2$, $\Delta_{xy} < \omega < \gamma$

Gapping of the nodal excitation spectrum $\nu(\omega) \sim e^{-\frac{Const}{|\omega|}}$, $\omega < \Delta_{xy}$

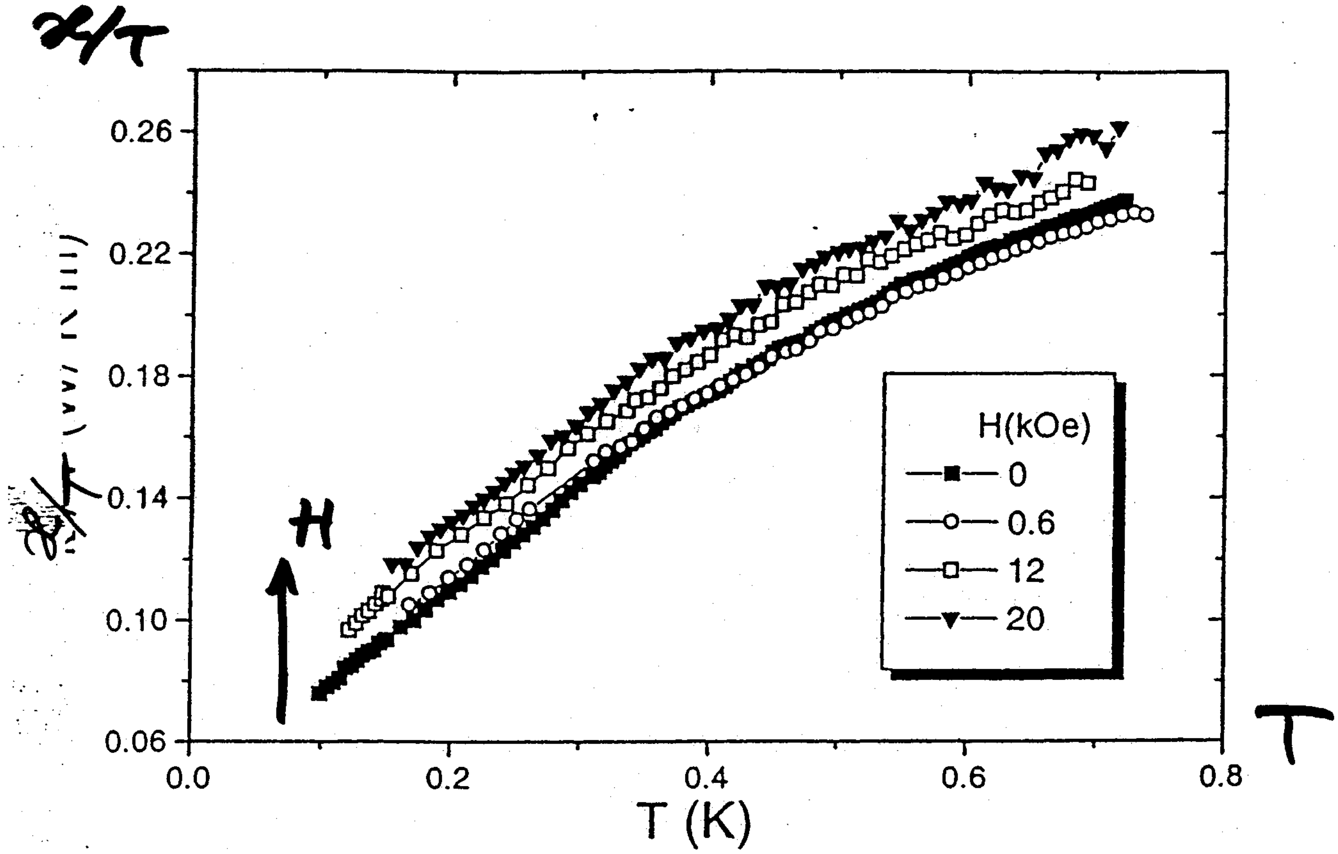
DOS tails

- **d+id'** -- "Thermal Quantum Hall" phase:

Quantized $\varkappa_H/T\big|_{T\to 0} = \frac{2\pi^2 k_B^2}{h^2}$ $\qquad Q_x = -\underline{\underline{\varkappa_H}}\nabla_y T$

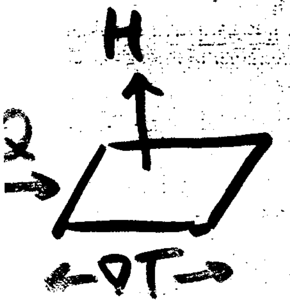
- Other possible experimental probes: spin injection/detection, tunneling, $C(T)$

H. Aubin et al '98



[FI]G. 2. Temperature dependence of the thermal conduc[tivit]y $\kappa(T)/T$ with a magnetic field applied above T_c. Note [the c]rossing of the 0.6 kOe and 0.0 kOe curves.

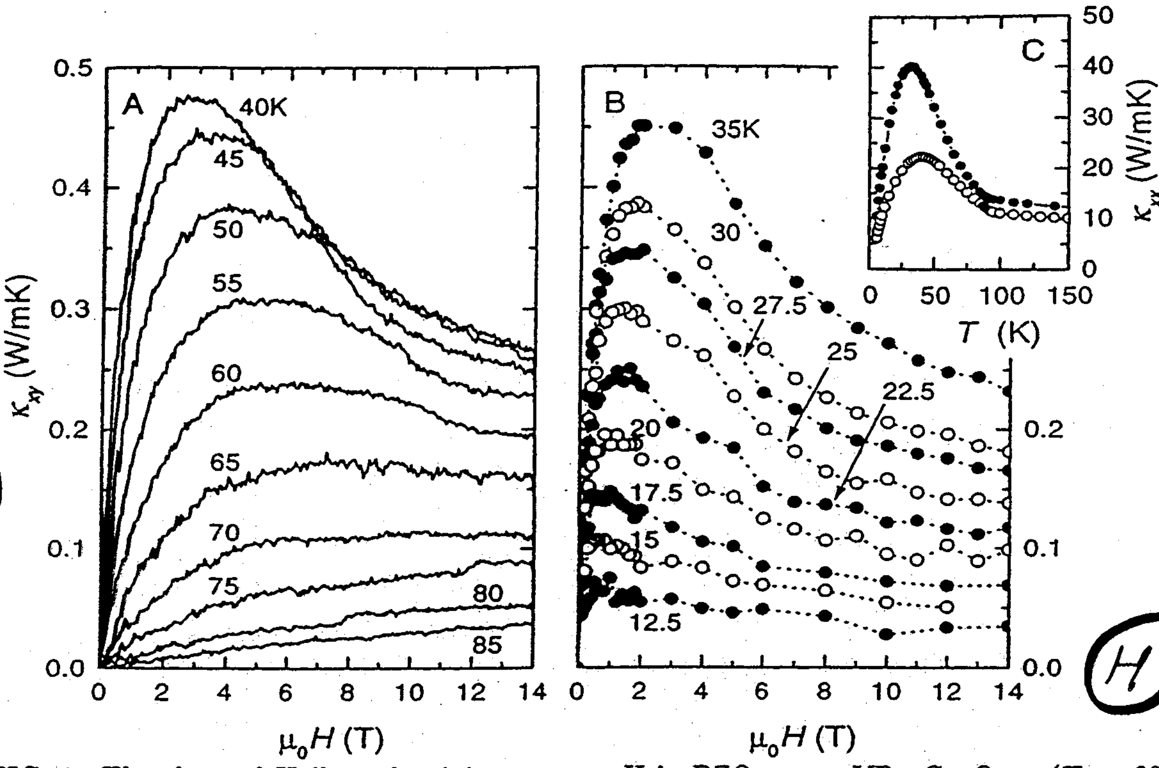
Evidence of a positive magneto (thermal) conductivity (?)



$$\frac{d\kappa}{dH} > 0$$

N. P. Ong et al '00

$\varkappa_H$



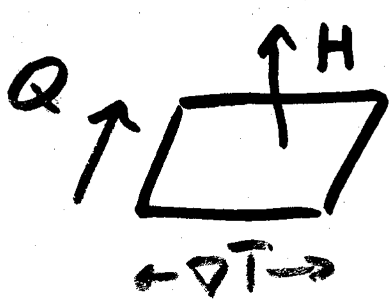
H

FIG. 1. The thermal Hall conductivity κ_{xy} vs. H in BZO-grown $YBa_2Cu_3O_{6.99}$ ($T_c = 89$ K) at high temperatures (85 to 40 K in Panel A), and low temperatures (35 to 12.5 K in Panel B). As T decreases below T_c, the initial slope κ^0_{xy}/B increases sharply. The prominent peak in κ_{xy} below 55 K is a new feature in BZO-grown YBCO. Panel C compares the zero-field $\kappa_{xx} \equiv \kappa_a$ in the BZO-grown crystal (solid circles) with a detwinned non-BZO grown crystal (open).

P. A. Lee + Simon '97

$$\varkappa_H = T^2 F\left(\frac{\sqrt{H}}{T}\right) \sim T \cdot \sqrt{H}$$

No quantization, as of today...



Conclusions

- The observed linear temperature dependence of the inverse qp lifetime in the superconducting state of the high-Tc cuprates suggests a possible quantum-critical behavior;

- Insights from relativistic theories allow one to identify the nature of the relevant QCP and its properties;

- The qp interactions specific for [redacted] this QCP do not necessarily alter the (de)localization scenarios proposed for the non-interacting random Dirac fermions;

- Experimental signatures of the conjectured QCP and associated effects of disorder can, in principle, be found in ARPES, tunneling, specific heat, and thermal/spin transport.