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SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

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# CONDUCTIVITY OF A LONG CLEAN WIRE

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These are preliminary lecture notes, intended only for distribution to participants

# Conductivity of a clean 1-d wire

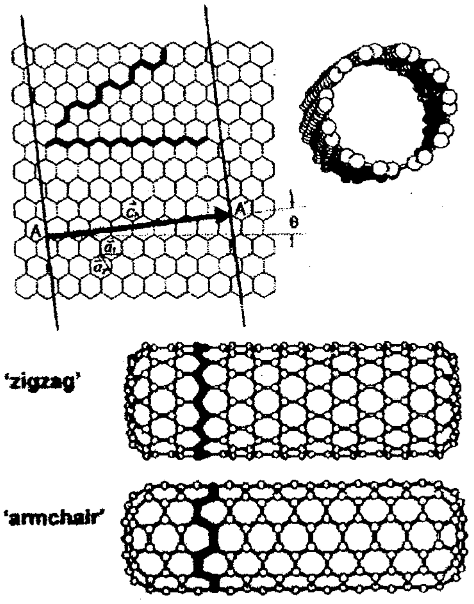
- Transport and weakly violated conservation laws
- Interacting electrons (RG, Luttinger liquids, bosonization and all that)
- Interacting electrons in the presence of a periodic potentail
- Transport and the Memory Functional Formalism
- Computation of the conductivity
- Complex dependence on Filling and Temperature

Achim Rosch - Karlsruhe University
Natan Andrei - Rutgers University

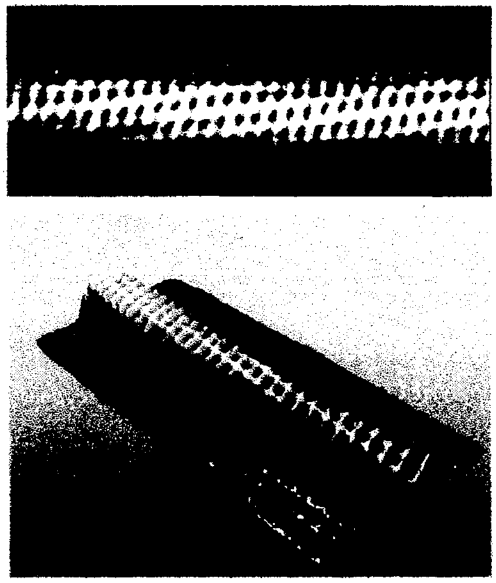
# An ideal 1d wire: carbon nanotube

## nanotubes: rolled up graphite sheets

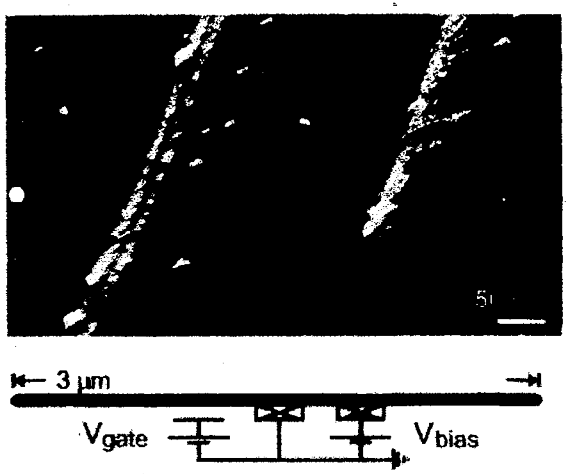
Various types of nanotubes




**Atomic resolution on the nanotubes**



**mK experiments on an *individual* nanotube**



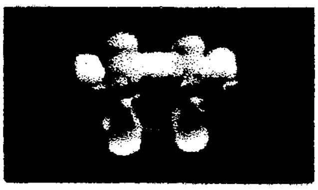

**contact resistance ~ 500 kΩ**

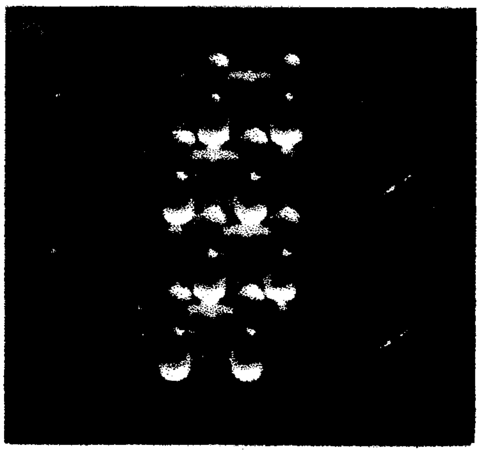
Tans, Devoret, Thess, Smalley, Geerligs, Dekker, Nature 386, 474 (1997)

# Weakly coupled 1d wires: Bechgaard salts

stacked molecules

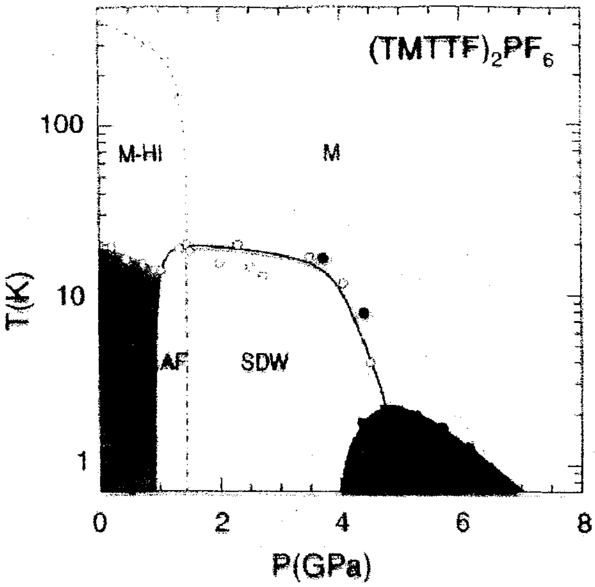
TMTSF



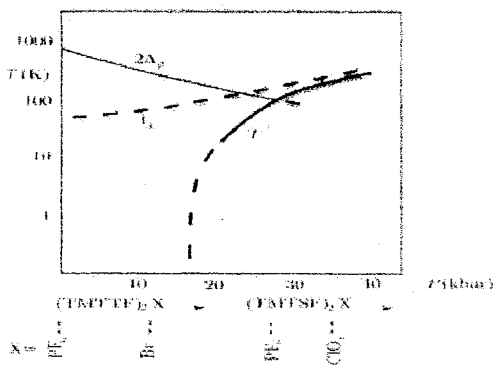


$(TMTSF)_2X$ $X=(PF_6)^-$ ,,,,

*by D. Jerome*

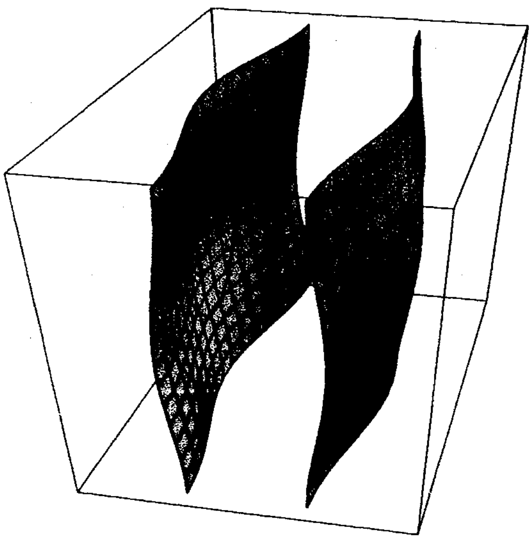



Correlation gap and dimensional cross-over




open Fermi surface:

two Fermi-sheets



# Historical remarks

- **most papers:**
  **neglect Umklapp away from 1/2 filling - (irrelevant operator)**
  $\rightarrow \quad \sigma(T > 0) = \sigma_{\mathbf{bulk}} = \infty$
  $\rightarrow \quad G = G_{\text{contact}} = \frac{2e^2}{h}$ (conductance)

- Giamarchi (91), $(4k_F - G)$ -Umklapp
  - perturbation theory (memory functional)
  $\rightarrow \sigma(T > 0) < \infty$
  - Luther-Emery transformation
  $\rightarrow \sigma = \infty$

- many papers: using PT result of Giamarchi or same result with different PT

- Giamarchi, Millis (92) (band structure effects)
  $\infty > \sigma(T > 0) > T^{-n}$

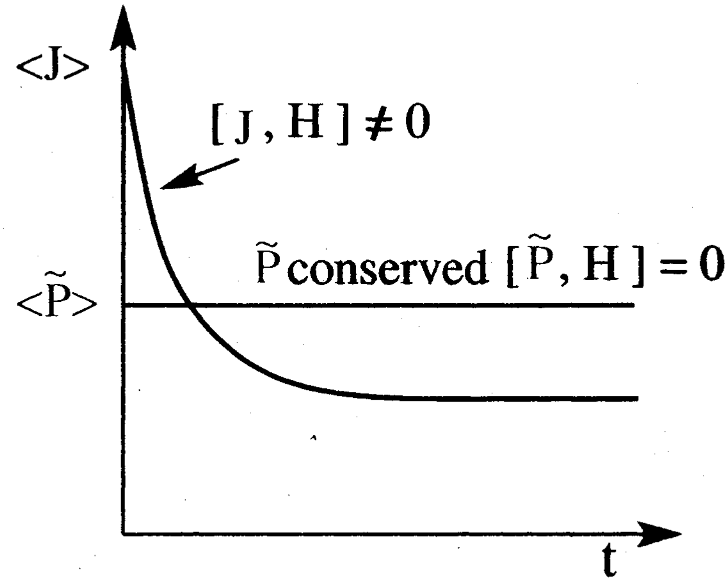
- Castella, Zotos *et al.* (95-97): *integrable* systems
  - infinite number of conserved quantities.
  Select $Q_1 ... Q_N$, $(\langle Q_n Q_m \rangle = \delta_{nm} \langle Q_n^2 \rangle)$
  then:

$$\begin{aligned} \sigma(T > 0, \omega) &= 2\pi D(T) \delta(\omega) + ... \\ D(T) &\geq \frac{1}{2} \sum_{n=1}^{N} \frac{(\chi_{JQ_n})^2}{\chi_{Q_n Q_n}} \end{aligned}$$

## Gedankenexperiment I

- $\tilde{P}$ *exactly conserved*:

prepare state with current $J$




How large is $\langle \tilde{P} \rangle$?

$$\frac{\langle \tilde{P} \rangle}{\langle J \rangle} = \frac{\chi_{\tilde{P}J}}{\chi_{JJ}}$$

How much $\lim_{t\to\infty}\langle J \rangle$ is induced by $\langle \tilde{P} \rangle$?

$$\frac{\langle J(t \to \infty)\rangle}{\langle \tilde{P} \rangle} = \frac{\chi_{J\tilde{P}}}{\chi_{\tilde{P}\tilde{P}}} \quad \Rightarrow \quad \frac{\langle J(t \to \infty)\rangle}{\langle J(t=0)\rangle} = \frac{\chi^2_{J\tilde{P}}}{\chi_{\tilde{P}\tilde{P}}\chi_{JJ}}$$

infinite conductivity at $T > 0$

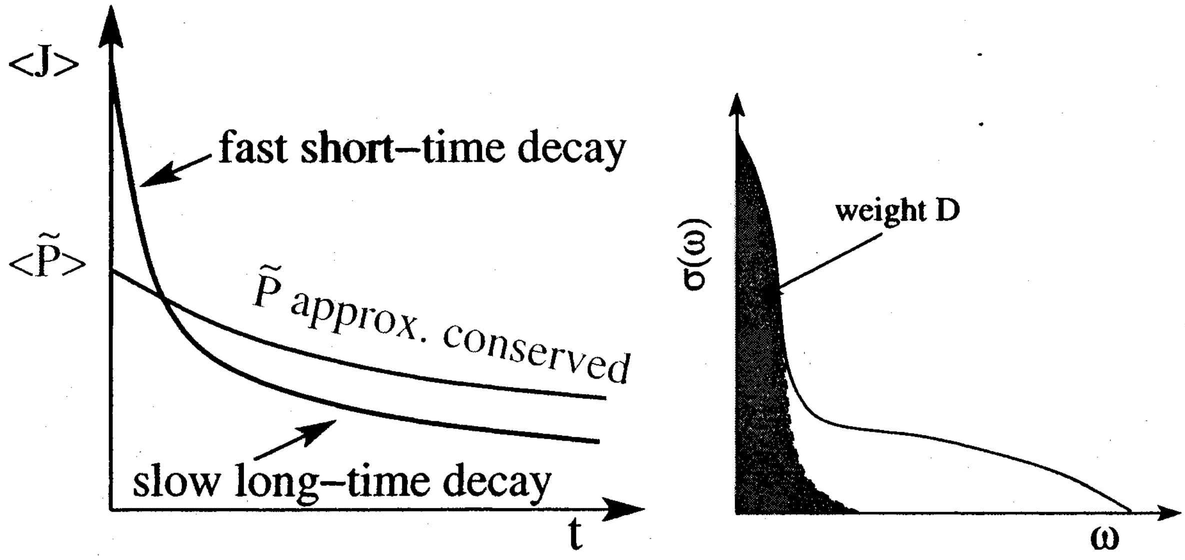
$$\mathrm{Re}\,\sigma(\omega) = 2\pi D \delta(\omega) + \sigma_{\mathrm{reg}}(\omega)$$

**Drude weight** $D = \frac{1}{2}\frac{\chi^2_{J\tilde{P}}}{\chi_{\tilde{P}\tilde{P}}}$

(exact if only $\tilde{P}$ conserved (Suzuki 71))

## Gedankenexperiment II

- $\tilde{P}$ *slowly decaying*:




peak in $\sigma(\omega)$, decay-rate of $\tilde{P}$ determines $\sigma(0)$

$$D = \frac{1}{2}\frac{\chi^2_{J\tilde{P}}}{\chi_{\tilde{P}\tilde{P}}}$$

## From the lattice to the continuum

- *A general Hamiltonian on a lattice*

$$H = H_0 + H_{e-e} + H_{lat}$$

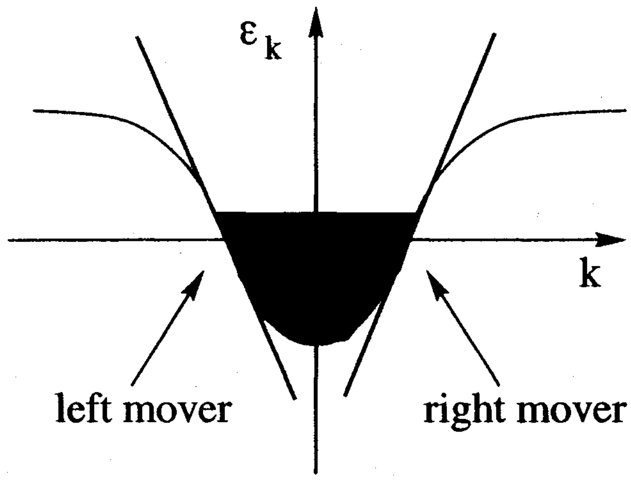
$$\begin{aligned} H_0 &= \sum \epsilon_k c_k^\dagger c_k \\ H_{e-e} &= \sum_{k \in BZ} V_{k_1,k_2} c_{k_1}^\dagger c_{k_2} c_{k_3}^\dagger c_{k_4} \delta_G(k_1 - k_2 + k_3 - k_4 \\ H_{lat} &= \text{some periodic lattice potential} \end{aligned}$$

with $\delta_G(k) = \sum_G \delta(k - G)$

- *How do we study low-energy, long-distance behaviour?*

- Do RG to obtain low-energy effective hamiltonian
- Alternatively, build effective hamiltonian "by hand"

# From the lattice to the continuum



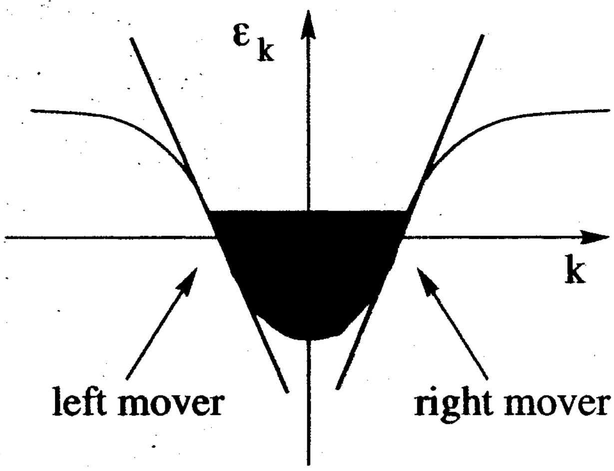

- keep modes: $k = \pm k_F + q,\ q \le \Lambda$
  $c_{n,\alpha} = \sum_k c_{k,a} e^{ikna} \approx e^{ik_F x}\psi_{R,\alpha}(x) + e^{-ik_F x}\psi_{L,\alpha}(x)$
  with:
  - $\psi_R^\dagger(x) = \sum e^{-iqx} c_{k_F+q}^\dagger$
  - $\psi_L^\dagger(x) = \sum e^{-iqx} c_{-k_F+q}^\dagger$

- low - energy effective Hamiltonian:
  Fixed point Hamiltonian + correction terms

  - Fixed point Hamiltonian:
  $H^* = H_{LL}$ (*Luttinger liquid*)
  - Correction Terms:
  $H^{corrections} = H_{\text{irr}} + H_{\text{umklapp}}$

# From the lattice to the continuum

## The Luttinger liquid




separate in slowly varying left- and right moving electrons:

$$\begin{aligned}\Psi_{\uparrow/\downarrow}(x) &= \Psi_{L,\uparrow/\downarrow}(x)e^{-ik_F x} \\ &\quad +\Psi_{R,\uparrow/\downarrow}(x)e^{ik_F x}\end{aligned}$$

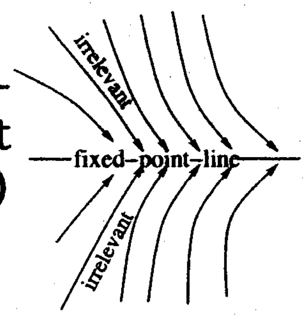
- lowest energies: linearize around Fermi energy fixed point: Luttinger liquid

$$\begin{aligned}H_{LL} &= v_F \int \left(\Psi^\dagger_{R\sigma} i\partial_x \Psi_{R\sigma} - \Psi^\dagger_{L\sigma} i\partial_x \Psi_{L\sigma}\right) + g\int \rho(x)^2 \\ &= \frac{1}{2}\int \frac{dx}{2\pi} \sum_{\nu=\sigma,\rho} v_\nu \left(K_\nu (\partial_x\theta_\nu)^2 + \frac{1}{K_\nu}(\partial_x \phi_\nu)^2\right)\end{aligned}$$

- bosonization: $\Psi_{L/R,\uparrow/\downarrow}(x) \propto e^{-i\Phi_{L/R,\uparrow/\downarrow}(x)}$

- spin-charge separation, non-Fermi liquid: power-laws

- deviation from $H_{LL}$ irrelevant: perturbation theory convergent, effects small at low temperature (exception: half-filling)




- dangerously irrelevant for conductivity

## From the lattice to the continuum

Classify deviations from LL-Hamiltonian

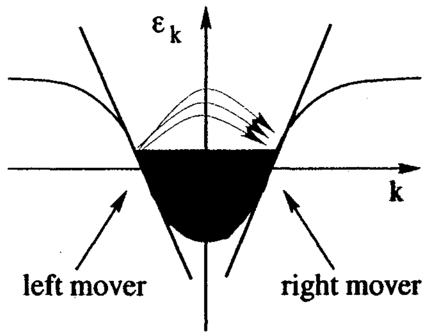
$$H = H_{LL} + H_{\text{irr}} + \sum_{n,m}^{\infty} H_{n,m}^{U}$$

Luttinger liquid + irrelevant terms + Umklapp

with:

$$\begin{aligned} P_T &= \sum_\sigma \int dx \left( \Psi_{R\sigma}^\dagger i\partial_x \Psi_{R\sigma} + \Psi_{L\sigma}^\dagger i\partial_x \Psi_{L\sigma} \right) \\ J_0 &= N_R - N_L = \sum_\sigma \int dx \left( \Psi_{R\sigma}^\dagger \Psi_{R\sigma} - \Psi_{L\sigma}^\dagger \Psi_{L\sigma} \right) \end{aligned}$$

- $H_{LL} = v_F \int \left( \Psi_{R\sigma}^\dagger i\partial_x \Psi_{R\sigma} - \Psi_{L\sigma}^\dagger i\partial_x \Psi_{L\sigma} \right) + g \int \rho(x)^2$
- $[H_{\text{irr}}, P_T] = [H_{\text{irr}}, J_0] = 0$
- Umklapp: $H_{n,m}^{U}$




$n$ fermions from L to R
+ lattice momentum
$\Delta k_{n,m} = n 2k_F - mG$

$$\begin{aligned} H_{1,m}^{U} &\approx g_{1,m}^{U} \sum_\sigma \int e^{i\Delta k_{1,m} x} \Psi_{R\sigma}^\dagger \Psi_{L\sigma} \rho_{-\sigma} + h.c. \\ H_{2,m}^{U} &\approx g_{2,m}^{U} \int e^{i\Delta k_{2,m} x} \Psi_{R\uparrow}^\dagger \Psi_{R\downarrow}^\dagger \Psi_{L\downarrow} \Psi_{L\uparrow} + h.c. \\ H_{3,m}^{U} &\approx g_{3,m}^{U} \int e^{i\Delta k_{3,m} x} \Psi_{R\uparrow}^\dagger \Psi_{R\uparrow}^\dagger \Psi_{R\downarrow}^\dagger \Psi_{L\downarrow} \Psi_{L\uparrow} \Psi_{L\uparrow} + h.c. \end{aligned}$$

## From the lattice to the continuum

The continuum Hamiltonian

$$\boldsymbol{H} = H_{LL} + H_{\mathrm{irr}} + \sum_{n,m}^{\infty} H^U_{n,m}$$

- The fixed point Hamiltonian

$$\begin{aligned} H_{LL} &= v_F \int \left( \Psi^\dagger_{R\alpha} i\partial_x \Psi_{R\alpha} - \Psi^\dagger_{L\alpha} i\partial_x \Psi_{L\alpha} \right) + g \int \rho^2 \\ &= \frac{1}{2} \int \frac{dx}{2\pi} \sum_{\nu=\sigma,\rho} v_\nu \left( K_\nu (\partial_x \theta_\nu)^2 + \frac{1}{K_\nu} (\partial_x \phi_\nu)^2 \right) \end{aligned}$$

- $H_{\mathrm{irr}}$ - *band strcture terms etc.* (need not be specified.)
- Umklapp terms $H^U_{n,m}$
  - transfer $n$ fermions from L to R (and vice versa) and lattice momentum $mG = m\frac{2\pi}{a}$
  - dangerously irrelevant

# From the lattice to the continuum

**Umklapp terms are of the form:**

$$
\begin{aligned}
H^U_{0,m} &\approx g^U_{0,m} \int e^{i\Delta k_{0,m} x} (\rho_L + \rho_R)^2 + h.c. \\
H^U_{1,m} &\approx g^U_{1,m} \sum_\sigma \int e^{i\Delta k_{1,m} x} \Psi^\dagger_{R\sigma} \Psi_{L\sigma} \rho_{-\sigma} + h.c. \\
H^U_{2,m} &\approx g^U_{2,m} \int e^{i\Delta k_{2,m} x} \Psi^\dagger_{R\uparrow} \Psi^\dagger_{R\downarrow} \Psi_{L\downarrow} \Psi_{L\uparrow} + h.c. \\
H^U_{2n,m} &\approx g^U_{2n,m} \int e^{i\Delta k_{2n,m} x} \\
&\quad \times \prod_{j=0}^{n-1} \frac{\partial^j_x \Psi^\dagger_{R\uparrow} \partial^j_x \Psi^\dagger_{R\downarrow} \partial^j_x \Psi_{L\downarrow} \partial^j_x \Psi_{L\uparrow}}{(j!/\alpha^j)^4} + h.c.
\end{aligned}
$$

**Momentum transfer:**

$$
\Delta k_{n,m} = n 2k_F - mG
$$

**Bosonized Umklapp term**

$$
H^U_{n,m,n_s} = \frac{g^U_{n,m,n_s}}{(2\pi\alpha)^n} \int e^{i\Delta k_{n,m} x} e^{i\sqrt{2}(n\phi_\rho + n_s \phi_\sigma)}
$$

Transfers:

- $n$ electrons with $n_S$ total spin
- $mG$ momentum absorbed by lattice

## Weakly violated conservation laws

Operators:

$$P_T = -i\sum_\sigma \int dx \left(\Psi^\dagger_{R\sigma}\partial_x\Psi_{R\sigma} + \Psi^\dagger_{L\sigma}\partial_x\Psi_{L\sigma}\right)$$

$$J_0 = N_R - N_L = \sum_\sigma \int dx \left(\Psi^\dagger_{R\sigma}\Psi_{R\sigma} - \Psi^\dagger_{L\sigma}\Psi_{L\sigma}\right)$$

- conserved on the Fermi-surface. Note $P \approx k_F J_0 + P_T$
- weakly violated away from it:
- **violation leads to degrading of electric current**
  - terms in $H_{\text{irr}}$ commute with both,
    $[H_{\text{irr}}, P_T] = [H_{\text{irr}}, J_0] = 0$
  - terms in $H^U$
    - do not commute with either $P_T$ or $J_0$
    - dangerously irrelevant

Observation: $[H^U_{2n,m}, \Delta k_{n,m} J_o + 2nP_T]$

- single Umklapp does not degrade the current completely
- need at least two Umklapps to have finite conductivity

## From the lattice to the continuum

Define pseudo-momentum $\tilde{P}_{n,m}$
with $\Delta k_{n,m} = n \cdot 2k_F - mG$

$$\tilde{P}_{n,m} = \frac{\Delta k_{n,m}}{2n}(N_R - N_L) + P_T$$

(without Umklapp: $\tilde{P}_{n,0}$ =usual momentum)

- Hamiltonian with single type of Umklapp conserves pseudo-momentum $\Rightarrow$ $\infty$ conductivity

$$\left[H_{LL} + H_{irr} + H^U_{n,m} \ , \ \tilde{P}_{n,m}\right] = 0$$

- interplay of two independent Umklapps $H^U_{n,m}, H^U_{n',m'}$ renders $\sigma$ finite

- second strongest Umklapp determines $\sigma(\omega = 0)$

How to calculate $\sigma(\omega)$ perturbatively?

- Problem: $\sigma$ and $1/\sigma$ singular function of perturbations for $\omega \to 0$

- full quantum-transport equations?
$\Rightarrow$ difficult (highly non-linear interaction of LL bosons)

- approximate conservation laws known
$\Rightarrow$ "hydrodynamic" description possible

- use Memory Matrix Formalism in space of slow modes (Mori (65), Zwanzig (61))
  - combined short-time and perturbative expansion for slow decay rates
  - short-time dynamics of slowest modes = long time behavior
  - weights of low-frequency peak exactly reproduced if time-scales well separated

## Transport and the Memory Function Formalism I

- Memory Functional Formalism: **study transport in the presence of approximate conserved quantities.**

**Mori (65), Zwanzig (61), Götze Wölfle (72), Giamarchi (91)**

- Scalar product in *operator* space

$$(A(t)|B) \equiv \frac{1}{\beta}\int_0^\beta d\lambda \left\langle A(t)^\dagger B(i\lambda)\right\rangle$$

- Static susceptibility

$$\chi_{AB} = \beta(A|B) \qquad t = 0$$

- Dynamic Correlation function

$$\begin{aligned} C_{AB}(z) &\equiv \int_0^\infty e^{izt}\,(A(t)|B)\,dt \\ &= \left(A\left|\frac{i}{z-L}\right|B\right), \quad LA = [H, A] \\ &= \frac{i}{\beta z}\int_0^\infty e^{izt}\,\langle[A(t), B]\rangle - \frac{(A|B)}{iz} \end{aligned}$$

- Conductivity

$$\sigma(\omega, T) = \beta C_{JJ}(\omega) = \beta(J|\frac{i}{\omega - L}|J)$$

# Transport and the Memory Function Formalism II

**Transport in the presence of several "slow" variables:**
$j_1 = J, j_2, \ldots, j_N$

- The conductivity

$$\sigma(\omega, T) = [(\hat{M}(\omega, T) - i\omega)^{-1}\hat{\chi}(T)]_{1,1}$$

- The susceptibility matrix

$$\hat{\chi}_{pq} = \beta(j_p|j_q)$$

- The memory matrix

$$\hat{M}_{pq}(\omega) = \beta \sum_r \left( \partial_t j_q \left| Q \frac{i}{\omega - QLQ} Q \right| \partial_t j_r \right) (\hat{\chi}^{-1})_{rp}.$$

- **The projection away from slow modes**

$$Q = 1 - \sum_{pq} |j_q)\beta(\hat{\chi}^{-1})_{qp}(j_p|.$$

Philosophy:

$\hat{M}$ non-singular in P.T.

- P.T. valid for short-time behavior

- P.T. also valid for long-time behavior of slowest modes (provided slow modes dynamics projected out - $Q$.)

## Intermezzo - conserved quantities

- If there are linear combination of $\{j_p\}$ that are conserved: $\tilde{J}_1, \ldots, \tilde{J}_S$

$\rightarrow$ expect $\infty$ dc - conductivity.

Indeed, carry out matrix inversion, project out zero - modes etc.

$$\sigma(\omega \rightarrow 0, T > 0) = i\frac{(\hat{\chi}\hat{\chi}_c^{-1}\hat{\chi})_{11}}{\omega + i0} + \sigma_{\mathrm{reg}}(\omega, T)$$

where:

- $\hat{\chi}_c^{-1} = \mathcal{P}_c(\mathcal{P}_c\hat{\chi}\mathcal{P}_c)^{-1}\mathcal{P}_c$

  $\mathcal{P}_c$ - projection on space of conserved variables

- $\sigma_{reg}(\omega, T)$ regular as long as all conserved currents are included.

  Thus

- $Re\ \sigma(\omega \rightarrow 0) = 2\pi D(T)\delta(\omega) = \pi(\hat{\chi}\hat{\chi}_c^{-1}\hat{\chi})_{1,1}\delta(\omega)$

  Determined by the overlaps of the current $J$ with conserved quantities, $\tilde{\chi}_{1,s}$

## The generic case

- All variables $j_1, \ldots, j_N$ decay slowly
- Restrict to two dimensional space

$$\begin{aligned} j_1 &= J \approx v_F J_0 \\ j_2 &= P_T \end{aligned}$$

- commute with all scattering processes on Fermi-surface

- longest decay rate, exponential in $T$, dominate transport

- can neglect other slow quantities at low - $T$,

decay as powers of $T$

(unless model is integrable e.g. $H_{LL} + H_{21}^U$ *relevant at 1/2 filling*)

# The calculation I

We can approximate:

- $L_{LL} = [H_{LL}, .]$. $\partial_t v_F J_0$ and $\partial_t P_T$ linear in $g^U_{n,m}$
- $L_{LL} P_T = L_{LL} J_0 = 0$, so no contribution from $Q$

Thus

$$\hat{M} \approx \sum_{nm} M_{nm}(\omega, T) \begin{pmatrix} v_F^2 (2n)^2 & -2n v_F \Delta k_{nm} \\ -2n v_F \Delta k_{nm} & (\Delta k_{nm})^2 \end{pmatrix} \hat{\chi}^{-1}$$

where

$$\hat{\chi} \approx \begin{pmatrix} 2v_F/\pi & 0 \\ 0 & \frac{\pi T^2}{3}\left(\frac{1}{v_\rho^3} + \frac{1}{v_\sigma^3}\right) \end{pmatrix}$$

$$M_{nm} \equiv (g^U_{nm})^2 M_n(\Delta k_{n,m}, \omega) \equiv \frac{\langle F; F\rangle^0_\omega - \langle F; F\rangle^0_{\omega=0}}{i\omega}.$$

with

- $F = [J_0, H^U_{nm}]/(2n)$
- $\langle F; F\rangle^0_\omega$ - retarded correlation function of $F$ with respect to $H_{LL}$.

# The calculation II

For $n$ arbitrary and $n_S = 0$, ($M_2$, Giamarchi 91)

$$M_n(\Delta k, \omega) = \frac{2\sin 2\pi K_\rho^n}{\pi^4 \alpha^{2n-2} v_\rho} \left[\frac{2\pi\alpha T}{v_\rho}\right]^{4K_\rho^n - 2} \frac{1}{i\omega} \times$$
$$\times [B(K_\rho^n - iS_+, 1 - 2K_\rho^n) B(K_\rho^n - iS_+, 1 - 2K_\rho^n)$$
$$- B(K_\rho^n - iS_+^0, 1 - 2K_\rho^n) B(K_\rho^n - iS_+^0, 1 - 2K_\rho^n)]$$

where

- $K_\rho^n = (n/2)^2 K_\rho$
- $S_\pm = (\omega \pm v_\rho \Delta k)/(4\pi T)$

Approximate forms:

$$\begin{aligned} & \approx \quad \frac{\alpha^{2-2n}}{\pi^2 \Gamma^2(2K_\rho^n) v_\rho T} \left(\frac{\alpha \Delta k}{2}\right)^{4K_\rho^n - 2} e^{-v_\rho \Delta k/(2T)} \\ M_n & \approx \quad \frac{(\alpha T/v_\rho)^{n^2 K_\rho - 1} (\alpha \Delta k)^{n_s^2 K_\sigma - 2}}{\Gamma^2(n_s^2 K_\sigma/2) v_\sigma^2 \alpha^{2n-3}} e^{-v_\sigma \Delta k/(2T)} \\ & \approx \quad T^{n^2 K_\rho + n_s^2 K_\sigma - 3} \end{aligned}$$

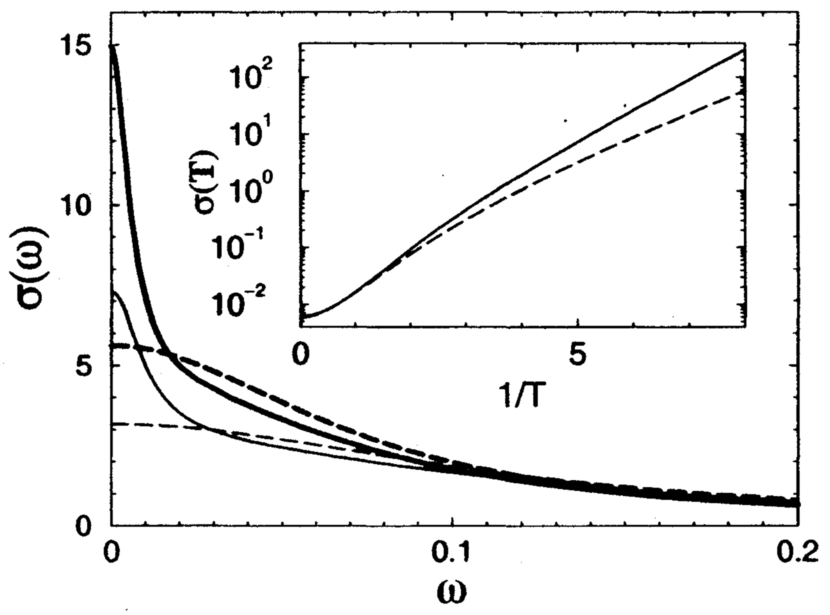
# One Umklapp, two Umklapps..

- One Umklapp term (insufficient to degrade current)
  → finite Drude peak, infinite dc - conductivity

$$D(T) \approx \frac{v_\rho K_\rho}{\pi} \frac{1}{1 + T^2 \frac{2\pi^2 n^2 K_\rho}{3(v_\rho \Delta k_{nm})^2}\left(1 + \frac{v_\rho^3}{v_\sigma^3}\right)}. \quad (1)$$

- Two Umklapp terms ($H^U_{n,m}, H^U_{n',m'}$)
  → finite dc - conductivity

$$\sigma(T, \omega = 0) = \frac{(\Delta k_{nm})^2 / M_{n'm'} + (\Delta k_{n'm'})^2 / M_{nm}}{\pi^2 (n \Delta k_{n'm'} - n' \Delta k_{nm})^2} \quad (2)$$




Conductivity for two Umklapp terms $H^U_{21}$ and $H^U_{20}$

$\Delta k_{21} = -1.5 \Delta k_{20}, K_\rho = 0.7, K_\sigma = 1.3, g_{20} = g_{21} = 1,$
$T = 0.18, 0.20.$

## Commensurate filling

- **Commensurate filling:**

$filling = \frac{m}{n} \rightarrow \Delta k_{nm} = 0$. Recall $k_F = (filling)\frac{\pi}{a}$

- Does dominant scattering process $H^U_{nm}$ relax the current?

- Depends on the overlap $\chi_{JP_T}$

- **Identity** $\chi_{JP_T} = \Delta\rho + o(e^{-\beta E_F})$

  $\Delta\rho = 2\Delta n/a$ - electron density deviation from commensurate filling.

- **3d array of wires** -

  $\Delta\rho$ is $T$ - independent, determined by charge neutrality

- **single wire** -

  $\Delta\rho(T) \sim T^2/(mv^3)$

  PH sym breaking $\sim k2/(2m)$

- **Replace**

  $\Delta k$ by $(\pi\Delta\rho)$.

# The conductivity

**Which of the scattering processes will dominate?**

- <u>intermediate T</u> : small $n$ (low order) - Pauli
- <u>lower T</u> : exponential factor prevails, smallest $\Delta k_{nm}$

- Close to commensurate filling $k_F \approx G\frac{m_0}{n_0}$

  dominant processes $H^U_{n_0 m_0}, H^U_{n_1 m_1}$ where

  $\Delta k_{n_0, m_0} \approx 0, \Delta k_{n_1, m_1} = \pm G/n_0$

  $\rightarrow (n_1 m_0 = \pm 1 \; mod \; n_0)$

  $\rightarrow n_1 = \gamma n_0, \; \gamma \sim 1$

  We find: The conductivity close to commensurability:

$$\begin{aligned} \sigma(k_F \approx Gm_0/(2n_0)) &\sim (\Delta n(T))^2 \exp[\beta v G/(2n_0)] \\ \sigma(k_F = Gm_0/(2n_0)) &\sim T^{-n_0^2 K_\rho - (n_0 \bmod 2)^2 K_\sigma + 3} \end{aligned}$$

- At typical incommensurate filling

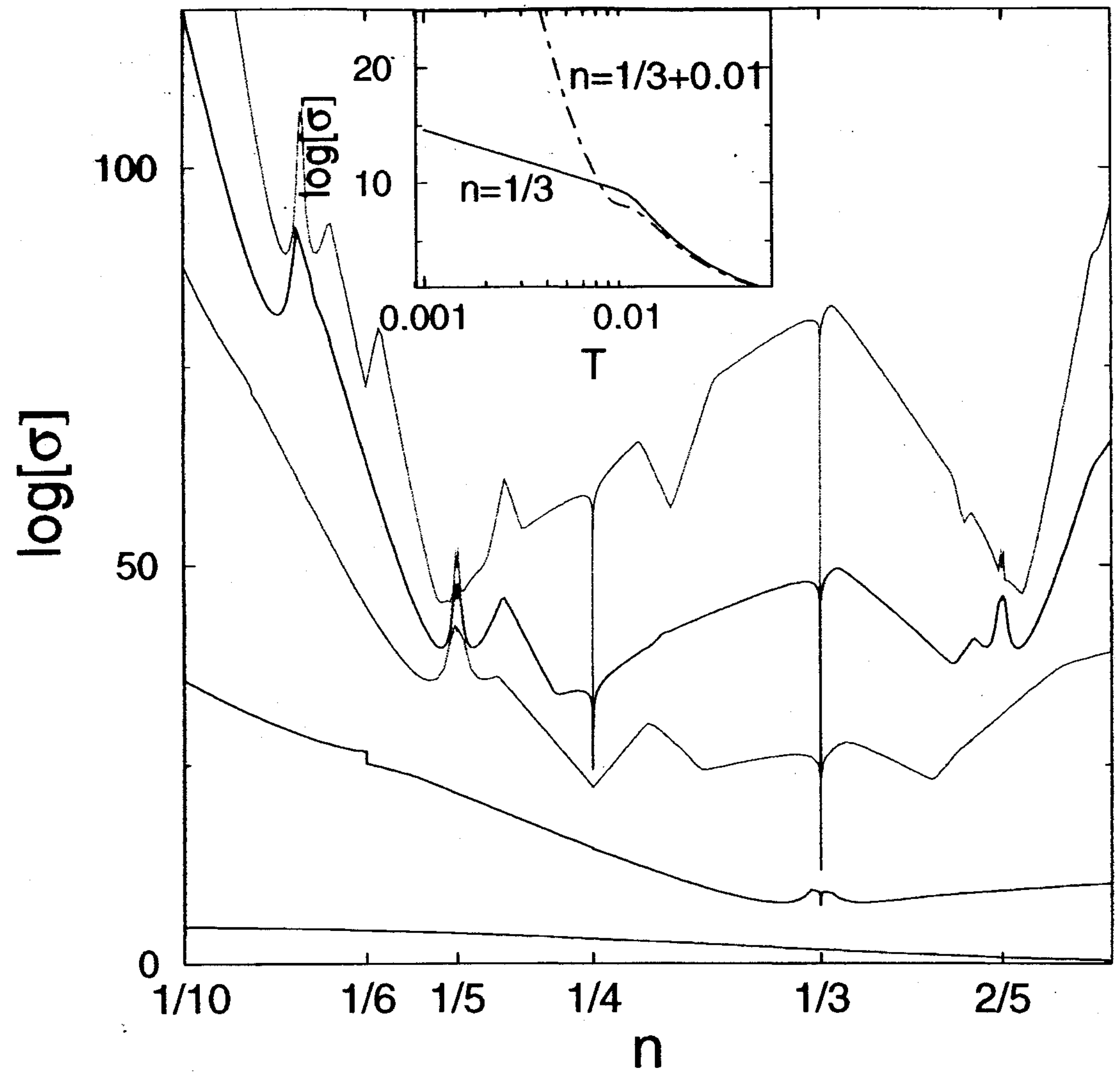
  Do saddle-point approximation with respect to $n$ of:

  $-\beta v G/(2N) + (\gamma N)^2 K \log[T]$

  We find: Typical conductivity:

$$\sigma_{\text{typical}} \sim \exp[c(\beta v G)^{2/3}]$$

Filling dependence of the conductivity




- Enhancement at commensurate filling $n$

- Dip at commensurate point: overlap of current $J$ and approx. conserved current $J_c = \Delta k_{nm} J_0 + P_T$ given by $\chi_{JJ_c} = n - n_{\text{commensurate}}$

$$\sigma(n \sim M/N) \approx \max\left[(\Delta n)^2 e^{\beta v G/N}, T^{-N^2 K_\rho}\right]$$