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SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

CHIRALLY STABILIZED CRITICAL STATE IN 1D

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Chirally Stabilized Critical State In 1 D

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Azaria, Lecheminant, Nersesyan, PRB 58, R8881 (1998)
Azaria, Lecheminant, Nucl. Phys. B 575 [FS], 439 (2000)
Azaria, Lecheminant, PRB 62, 61 (2000)
Lecheminant, Gogolin, preprint (2001)

$$\boxed{\text{Introduction}}$$

→ 1D Interacting **Left-Right** moving Fermions :

$$\mathcal{H} = -i v_F \left(\sum_{r=1}^{k_R} R^+_{r\alpha} \partial_x R_{r\alpha} - \sum_{\ell=1}^{k_L} L^+_{\ell\alpha} \partial_x L_{\ell\alpha} \right) + g \vec{J}_R \cdot \vec{J}_L$$

- $R_{r\alpha}$, $L_{\ell\alpha}$: Right, Left moving Fermions

  $r, \ell = 1, \dots, k_{R}, k_{L}$ Flavor index

  $\alpha = \uparrow, \downarrow$ Spin index

- Currents in the Spin Sector:

$$\begin{cases} \vec{J}_R = \displaystyle\sum_{r=1}^{k_R} R^+_{r\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} R_{r\beta} \rightsquigarrow su(2)_{k_R} \text{ KM} \\ \vec{J}_L = \displaystyle\sum_{\ell=1}^{k_L} L^+_{\ell\alpha} \frac{\vec{\sigma}_{\alpha\beta}}{2} L_{\ell\beta} \rightsquigarrow su(2)_{k_L} \text{ KM} \end{cases}$$

→ 1-Loop RG : $\beta_g = \frac{g^2}{2\pi}$

↳ $g > 0$ $\boxed{\text{Marginal Relevant}}$

→ <u>Questions</u>:

- Nature of the IR phase
- Characterization of the Low-Energy Spectrum
- Correlation functions
- ⋮

$$\boxed{k_L = k_R = k}$$

→ $SU(2)_k$ WZNW models with current-current interaction

$$\mathcal{H} = \frac{2\pi v}{k+2}\left[\vec{J}_R^{\,2} + \vec{J}_L^{\,2}\right] + g\,\vec{J}_R \cdot \vec{J}_L\,, \qquad \vec{J}_{R,L} \ SU(2)_k$$

→ $g > 0$:

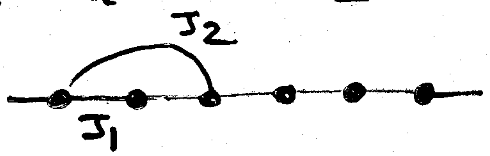
- Spectral Gap
- Strong-Coupling massive Phase
- Excitations: Massive Kinks

LeClair, Bernard, CMP (1999)

→ <u>Physical Realizations For $k_L = k_R = k = 1$:</u>

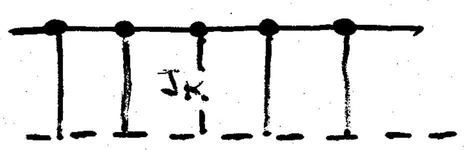
- $J_1 - J_2$ $S = 1/2$ Heisenberg chain

Haldane, PRB (1982)



- Kondo-Heisenberg chain, Incommensurate Filling

Sikkema et al., PRL (1997)

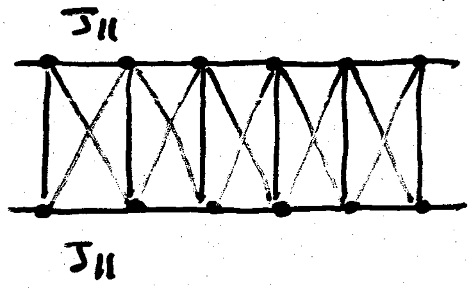


$S = 1/2$ chain

1DEG $\qquad J_K$: Kondo Coupling

- 2-leg Ladder with crossings:

Allen, Essler, Nersesyan, PRB (1999)



$J_{\perp}$, J_X

Special line: $J_{\perp} = 2\,J_X$

$\boxed{\text{Chiral Liquids: } k_L \neq k_R}$

→ 1-Loop β function does not depend on $k_{R,L}$

$$\Downarrow$$

$g > 0$ Marginal Relevant

$$\Downarrow$$

Same Physics as $k_L = k_R$: Massive Phase ?

→ Answer: $\boxed{\text{Criticality}}$

$k_R \neq k_L$:

Critical point

→ Origin of the Criticality: Polyakov, Wiegmann PLB (1984)

$k_R \to \infty$, $k_L \to \infty$ at Fixed $k_R - k_L > 0$

Chiral excess of Particles ($k_R > k_L$)

$$\Downarrow$$

No mass gap can be formed for <u>ALL</u> Particles

$$\Downarrow$$

Some degrees of freedom remain critical

Symmetry of the IR Fixed point:

$SU(2)_{k_R - k_L}$ WZNW

$\boxed{\text{Chirally Stabilized Critical Liquids}}$

$k_R < \infty$, $k_L < \infty$, $k_R > k_L$ Andrei, Douglas, Jerez PRB (1998)

→ <u>Symmetry of the IR FP:</u>

$$SU(2)_{k_R-k_L}\Big|_R \otimes \frac{SU(2)_{k_L} \times SU(2)_{k_R-k_L}}{SU(2)_{k_R}}\Bigg|_L$$

Consistent with:

- Global SU(2) symmetry
- Invariant Flow: 't Hooft anomaly matching
  - * $c_R - c_L = cte$
  - * # of level $= cte$
- $c_{IR} < c_{UV}$: c- theorem
- Value of c_{IR} checked by TBA

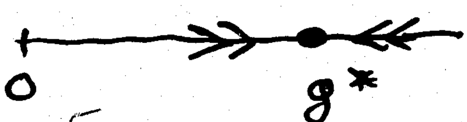
→ <u>New 1D NF liquid</u>: Non-Fermi liquid with <u>Universal</u> exponents

$\neq$ Luttinger Liquid

$\boxed{\text{Chirally Stabilized liquids}}$

→ Chiral FP checked by:

- Toulouse Limit approach: $k_R = 2$, $k_L = 1$ Azaria, L, Nersesyan PRB (1998)
- All-orders β- function: $k_R > k_L$ Leclair, hep-th (2001) Existence of a Fixed point



Possible Realizations of Chiral Spin Liquids (CSL)

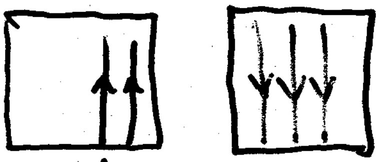
→ T- Breaking states: $k_R \neq k_L$

$\Downarrow$

Breakdown of the $R \leftrightarrow L$ symmetry

Exemple: Andrei, Douglas, Jerez PRB (1998)

Edge states in a paired sample of IQHE with



k_L k_R

$v_L = k_L \neq v_R = k_R$

Virtual Hopping

↳ Chiral FP

→ Time- Reversal Invariant Realizations of CSL:

Azaria, L, Nersesyan PRB (1998)

Leading part of the Continuum Limit of a Lattice System:

$$\mathcal{H} \simeq \mathcal{H}_1 + \mathcal{H}_2 \,, \qquad [\mathcal{H}_1, \mathcal{H}_2] = C$$

- $\mathcal{H}_{1,2}$ chirally asymmetric with current - current interaction
- $\begin{cases} t \to -t \\ \mathcal{H}_{1,2} \to \mathcal{H}_{2,1} \end{cases}$

$\Rightarrow$ Possible realizations of CSL physics in 1D systems: Spin Ladders, ...

→ **Problem:** **Backscattering Terms**

CSL Physics in Spin Ladders

↳ Only Current - Current Interactions

⇒ Suppression of backscattering terms:

$$\vec{n}_a \cdot \vec{n}_b$$

Strongly relevant perturbation $(\Delta = 1)$

→ **Two ≠ routes to CSL:**

- $\boxed{\text{Frustration}}$: Backscattering perturbation is suppressed Geometrically

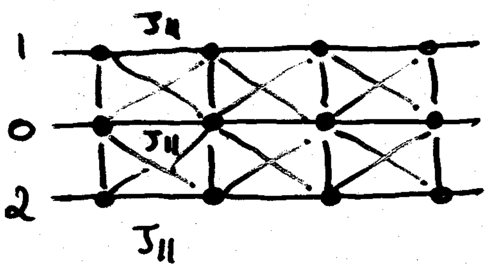
- $\boxed{\text{Asymmetric Dopings}}$:

Backscattering term becomes an oscillating piece

Spin Ladder with Crossings

→ <u>The model</u>: 3-Leg Ladder with Crossings

Azaria, L, Nersesyan, PRB (1998)

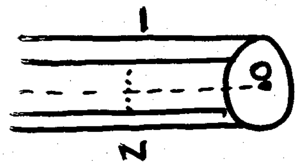


J_X

$J_\perp$

$J_{\parallel}$

Generalization:



→ <u>Continuum Limit</u>: $J_\perp, J_X \ll J_{\parallel}$

$$\mathcal{H} \simeq \frac{2\pi v}{3} \sum_{a=0}^{N} \left(\vec{J}_{aR}^{\,2} + \vec{J}_{aL}^{\,2} \right) + \left(J_\perp - 2J_X \right) \vec{n}_0 \cdot \sum_{a=1}^{N} \vec{n}_a$$

$$+ \left(J_\perp + 2J_X \right) \left(\vec{J}_{0L} + \vec{J}_{0R} \right) \cdot \left(\vec{I}_L + \vec{I}_R \right) + \gamma \sum_{a=0}^{N} \vec{J}_{aR} \cdot \vec{J}_{aL}$$

$\gamma < 0$, $\vec{J}_{0L,R}$ $su(2)_1$, $\vec{I}_{L,R} = \sum_{a=1}^{N} \vec{J}_{aL,R}$ $su(2)_N$

→ <u>Line</u>: $J_\perp = 2J_X$ No backscattering terms

<u>Neglecting</u>:
- Marginal irrelevant cc Interactio[ns]
- cc interactions of same chirality

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2, \qquad [\mathcal{H}_1, \mathcal{H}_2] = 0$$

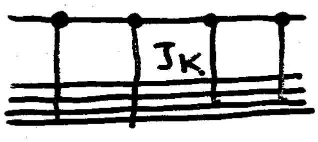
$\mathcal{H}_{1,2}$ chiral asymmetric $su(2)_1$, $su(2)_N$ WZNW ($N \geq 2$)

$\Rightarrow$ CSL properties in the IR limit

Asymmetric Dopings

→ **Kondo – Heisenberg chains:** Sikkema et al. PRL (1997)
Lehur PRL (1999)

- Overscreened KH:



N-channel

$S = 1/2$ Heisenberg
Multi-channel 1DEG

- Underscreened KH

Critical spin-S chain
1DEG

→ **Continuum Limit:** $J_K \ll t, J_H$ Incommensurate doping

Spin sector:

$$\mathcal{H} \sim \frac{2\pi v_s}{N+2}\left(\vec{I}_R^{\,2} + \vec{I}_L^{\,2}\right) + \frac{2\pi v_0}{3}\left(\vec{J}_{0R}^{\,2} + \vec{J}_{0L}^{\,2}\right) + g\left(\vec{J}_{0L}\cdot\vec{I}_R + \vec{J}_{0R}\cdot\right.$$

- $\vec{I}_{R,L}$ $su(2)_N$: uniform part of the { spin conduction density (overscreened case); Local moments $N=2S$ (underscreened case) }

- $\vec{J}_{0R,L}$ $su(2)_1$: uniform part of the { Local moments (overscreened case); spin conduction density (underscreened case) }

$\Downarrow$ $N \geq 2$ or $S \geq 1$

IR properties characteristic of CSL

Azaria, L, Nersoyan PRB (1998)
Andrei, Orignac, PRB (2000)
Azaria, L, NPB (2000)

Questions

→ IR properties of the Ladder with crossings, KH chains:

- Thermodynamic properties
- Leading asymptotic of Correlation functions
- Low T behavior of Physical Quantities
- Leading instabilities

→ Stability of the Chiral Fixed Point ?

$\Downarrow$

Need the UV- IR Transmutation of the Field

Our Approach: Toulouse Limit Solution :

Existence of a special point (Decoupling point)

- The model can be solved
- Correlation functions can be computed

Toulouse Limit Solution

Azaria, L, Nersesyan, PRB (1998)
L, Gogolin, preprint (2001)

→ Anisotropic Version:

$$\mathcal{H}_1 = \frac{2\pi v}{N+2}\vec{I}_R^{\,2} + \frac{2\pi v}{k+2}\vec{J}_{0L}^{\,2} + g_{\parallel} I_R^z J_{0L}^z + \frac{g_\perp}{2}\left(I_R^+ J_{0L}^- + \text{H.c.}\right)$$

$\vec{I}_R$ $su(2)_N$, $\vec{J}_{0L}$ $su(2)_k$ $N > k$

$g_\parallel, g_\perp > 0$

→ Effect of the Anisotropy: Leclair, hep-th (2001)

All-orders β function has an IR FP:

$$g_\parallel^* = g_\perp^* = \frac{2}{N\pi}$$: Restoration of the SU(2) symmetry at the IR FP

→ "Bosonization" of $su(2)_N$:

$\mathbb{Z}_N$ Parafermions : $\mathbb{Z}_N \sim \frac{su(2)_N}{u(1)_N}$ Zamolodchikov-Fateev, Sov JETP (1985)

↓

$u(1)$ rational CFT:

Compactified bosonic Field ϕ_s

Radius: $R_s = \sqrt{2N}$

↳ Representation:

$$\begin{cases} I_R^+ = \sqrt{N}\,\Psi_{1R} :e^{-i\sqrt{\frac{2}{N}}\phi_{sR}}: \\ I_R^z = \sqrt{\frac{N}{2}}\,\partial_x \phi_{sR} \end{cases}$$

Ψ_{1R} First parafermion current $(0, 1-\frac{1}{N})$

→ Boso- Parafermionization:

$$\mathcal{H}_1 = \frac{v}{4\pi}\left[(\partial_x \phi_{0L})^2 + (\partial_x \phi_{sR})^2\right] + \frac{g_{\parallel}\sqrt{Nk}}{2}\,\partial_x \phi_{0L}\,\partial_x \phi_{sR}$$

$$+ \mathcal{H}^0_L(Z_k) + \mathcal{H}^0_R(Z_N) + \frac{g_{\perp}\sqrt{Nk}}{2}\left(\Psi_{1R} :e^{-i\sqrt{\frac{2}{N}}\phi_{sR} - i\sqrt{\frac{2}{k}}\phi_{0L}}: \Psi^{+}_{1L} + H.c.\right)$$

→ Canonical Transformation: $\boxed{\text{Toulouse Basis}}$

$$\begin{pmatrix} \phi_{0L} \\ \phi_{sR} \end{pmatrix} = \begin{pmatrix} \operatorname{ch}\alpha & \operatorname{sh}\alpha \\ \operatorname{sh}\alpha & \operatorname{ch}\alpha \end{pmatrix} \begin{pmatrix} \overline{\Phi}_{2L} \\ \overline{\Phi}_{1R} \end{pmatrix}$$

- Diagonalisation: $\operatorname{th}2\alpha = -\dfrac{g_{\parallel}\pi\sqrt{Nk}}{v}$
- Decoupling of a degrees of freedom: α such that $\operatorname{th}\alpha = -\sqrt{\dfrac{k}{N}}$

→ Decoupling of $\overline{\Phi}_{1R}$

Toulouse point: $g^*_{\parallel} = \dfrac{2v}{\pi(N+k)}$

→ Toulouse Hamiltonian:

$$\mathcal{H}_1 = \frac{u}{4\pi}(\partial_x \overline{\Phi}_{1R})^2 + \frac{u}{4\pi}(\partial_x \overline{\Phi}_{2L})^2 + \mathcal{H}^0_L(Z_k) + \mathcal{H}^0_R(Z_N)$$

$$+ \frac{g_{\perp}\sqrt{Nk}}{2}\left[\Psi_{1R} :e^{i\sqrt{\frac{2(N-k)}{Nk}}\overline{\Phi}_{2L}}: \tilde{\Psi}^{+}_{1L} + H.c.\right]$$

$\overline{\Phi}_1$ massless compactified bosonic Field

Radius: $\overline{R}_1 = \sqrt{2(N-k)}$

$\boxed{\text{Two-Channel Case}}$

→ Simplest case: $N=2,\ k=1$

- $\overline{R}_1 = \sqrt{2} \rightsquigarrow su(2)_1$
- $\overline{R}_2 = 1 \rightsquigarrow$ Free-Fermion point
- $\mathcal{H}^0_R(\mathbb{Z}_2) = -\frac{iv}{2}\,\xi^3_R\,\partial_x\,\xi^3_R$, ξ^3_R Majorana Fermion

→ Bosonization:

$$:e^{-i\Phi_{2L}}: \sim \eta_L + i\,\zeta_L \qquad \mathbb{Z}_2 \text{ Majorana Fermi}$$

$$\Downarrow$$

$$\mathcal{H}_1 = \frac{u}{4\pi}(\partial_x\Phi_{1R})^2 + \frac{iu}{2}\zeta_L\,\partial_x\,\zeta_L$$

$$\underbrace{-\frac{iv}{2}\xi^3_R\,\partial_x\,\xi^3_R + \frac{iu}{2}\eta_L\,\partial_x\,\eta_L + i\,m\,\xi^3_R\,\eta_L}_{\text{Massive degrees of freedom}}$$

→ IR Fixed point:

$$\boxed{su(2)_1\big|_R \otimes \mathbb{Z}_2\big|_L}$$

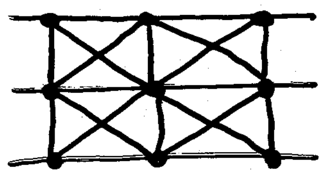
Full agreement with Andrei, Douglas, Jerez result:

$$su(2)_1\big|_R \otimes \underbrace{\frac{su(2)_1 \times su(2)_1}{su(2)_2}}_{\mathbb{Z}_2}\Bigg|_L$$

→ Toulouse basis:

$\rightsquigarrow$ UV-IR Transmutation of the Fields

$\boxed{\text{3- Leg Spin Ladder with Crossings}}$



→ Symmetry of the IR FP:

$$\boxed{SU(2)_1 \times \mathbb{Z}_2 \times \mathbb{Z}_2}$$

Φ_1 — Effective Heisenberg $S=1/2$ chain

$\xi_{R,L}$ — Chiral Stabilization

$\Xi^0_{R,L}$ — channel sector

$$c_{IR} = 1 + \frac{1}{2} + \frac{1}{2} = 2$$

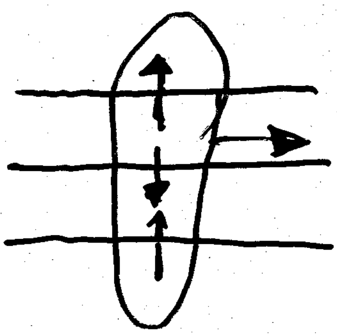
→ Elementary excitations:

- Spinons of the effective $S=1/2$ chain

Exact Relation: $\mathcal{J}^z_R = J^z_{1R} + J^z_{2R} + J^z_{0L}$

$\Downarrow$

IR spinons = Chirally asymmetric Correlated State of 2 spinons and 1 anti-spinon



- $\mathbb{Z}_2$ Massless Singlet Excitations:

$$\begin{cases} \Xi^0_{R,L} \leftrightarrow \text{Exchange symmetry of the surface chains} \\ \xi_{R,L} \leftrightarrow \text{chiral Stabilisation} \end{cases}$$

→ <u>Correlation functions</u>:

Slowest : between the spins of the surface chains

$$\langle \vec{S}_1(x)\cdot \vec{S}_1(0)\rangle \sim \frac{(-1)^{x/a_0}}{x^{3/2}}$$

$\Downarrow$

Low-T NMR relaxation rate : $\frac{1}{T_1} \sim \sqrt{T}$

$\neq$ cte $S=1/2$ Heisenberg

→ <u>Stability of the Chiral Fixed Point</u>:

- Neglected current-current interactions :

$$\left.\begin{array}{l} \vec{J}_{0R}\cdot\vec{I}_R + R\to L \\ -\gamma \sum_{a=0}^{2} \vec{J}_{aR}\cdot\vec{J}_{aL} \end{array}\right\} \xrightarrow{IR} \left\{\begin{array}{l} \text{Velocities Renormalized} \\ + \\ \text{Log - Corrections} \end{array}\right.$$

- Effect of the interchain backscattering :

$$\mathcal{H}_b = \tilde{g}\, \vec{n}_0 \cdot (\vec{n}_1 + \vec{n}_2) \qquad \tilde{g} = J_\perp - 2J_X$$

↳ Relevant perturbation ($d = 1/4$) at the Chiral FP : Mass gap in the $\boxed{\text{Ising Sectors}}$

$$\Delta \sim |\tilde{g}|^{4/7}$$

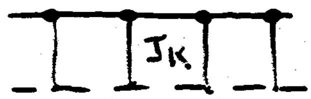
↳ Cross-over to the $c=1$ ($SU(2)_1$) FP of the 3-Leg Ladder

$\boxed{\text{But}}$ Possibility of an Intermediate Energy Regime : $\Delta \ll E \ll m$

Governed by the Chiral FP

Kondo – Heisenberg Chains

→ **Underscreened Case:**



Critical spin S

1DEG

- FP in the spin sector:

$$su(2)_{2S-1} \otimes \mathcal{M}_{2S+1}$$

← Minimal model Series

- e^- Green's Function:

$$S=1 \qquad \langle R_\sigma(x,\tau)\, R^+_\sigma(0,0)\rangle \sim \frac{1}{(v_F\tau - ix)^{1/2}\,(u_1\tau+ix)^{1/2}\,(u_2\tau - ix)}$$

Non-Fermi liquid state

- Dominant instabilities: $\langle O(x)\, O(0)\rangle \sim \dfrac{1}{x^\alpha}$

Spin-S case Decomposition: Andrei, Orignac, PRB (2000)

$$su(2)_{2S} \times su(2)_1 \longrightarrow su(2)_{2S-1} \times \mathcal{M}_{2S+1}$$

* Conventional Order parameters:

$$\alpha_{CDW} = \alpha_{SDW} = \alpha_{SS} = \alpha_{TS} = 2 + \frac{6}{2S+1} > 2 \qquad \boxed{\text{Suppressed}}$$

* Composite Pairing:

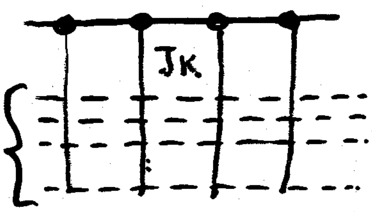
$$\begin{cases} O_{C\text{-}CDW} = \vec{n}\cdot\vec{O}_{SDW} \\ O_{C\text{-}S} = \vec{n}\cdot\vec{O}_{TS} \end{cases}$$

$$\alpha_{C\text{-}CDW} = \alpha_{C\text{-}S} = 2 - \frac{3}{2S+2} < 2 \qquad \boxed{\text{Enhanced}}$$

For $S=1$, Full agreement with the Toulouse approach

Azaria, L, PRB (2000)

→ <u>Overscreened Case</u>:

N channels 

spin 1/2 chain

Multichannel 1DEG

- IR Fixed point in the spin sector:

$$SU(2)_{N-1} \otimes \mathcal{M}_{N+1}$$

- Non Fermi liquid properties

- Leading instabilities: Andrei, Orignac, PRB (200

$$\begin{cases} \alpha_{CDW} = \alpha_{SDW} = \alpha_{SS} = \alpha_{TS} = 2 + \dfrac{6}{(N+1)(N+2)} \\ \alpha_{C-S} = \alpha_{C-CDW} = 3 - \dfrac{6}{N+2} \end{cases}$$

$N = 2$, OK with the Toulouse solution

For $N \leq 5$, $\boxed{\text{Composite Pairings}}$

are the dominant instabilities

Massless Flow

→ Toulouse limit in the general case $N>k$:

$$\mathcal{H}_1 = \frac{u}{4\pi}(\partial_x \bar\Phi_{1R})^2 + \frac{u}{4\pi}(\partial_x \bar\Phi_{2L})^2 + \mathcal{H}_L^0(\mathbb{Z}_k)$$
$$+ \mathcal{H}_R^0(\mathbb{Z}_N) + \frac{g_\perp\sqrt{Nk}}{2}\left(\Psi_{1R} :e^{-i\sqrt{\frac{2(N-k)}{Nk}}\bar\Phi_{2L}}: \tilde\Psi_{1L}^{+} + \text{H.c.}\right)$$

$\bar\Phi_1$ Massless bosonic Field with Radius $\bar R_1 = \sqrt{2(N-k)}$

$\rightsquigarrow$ $U(1)_{N-k}$ CFT

→ From the results:

- Existence of a stable IR FP in the anisotropic case
  $g_{\parallel}^* = g_\perp^*$ LeClair, hep-Th (2001)
- Symmetry of the IR FP:
  $$su(2)_{N-k}\Big|_R \otimes \frac{su(2)_{N-k}\times su(2)_k}{su(2)_N}\Bigg|_L$$

→ Massless Flow:

$$\mathcal{H} = \frac{u}{4\pi}(\partial_x \bar\Phi_{2L})^2 + \mathcal{H}_L^0(\mathbb{Z}_k) + \mathcal{H}_R^0(\mathbb{Z}_N)$$
$$+ g_\perp'\left(\Psi_{1R} :e^{-i\sqrt{\frac{2(N-k)}{Nk}}\bar\Phi_{2L}}: \tilde\Psi_{1L}^{+} + \text{H.c.}\right)$$

Flow To $\mathbb{Z}_{N-k}\Big|_R \otimes \frac{su(2)_k\times su(2)_{N-k}}{su(2)_N}\Bigg|_L$

$$\mathcal{H} = \mathcal{H}^0_L(U(1)) + \mathcal{H}^0_L(\mathbb{Z}_k) + \mathcal{H}^0_R(\mathbb{Z}_N) + \text{Interactions}$$

Flow to $\mathbb{Z}_{N-k}\big|_R \otimes \dfrac{SU(2)_k \times SU(2)_{N-k}}{SU(2)_N}\Big|_L$

→ $N=2,\ k=1$: OK

Bosonization ⇝ $\mathcal{H}^0_L(U(1)) = \mathcal{H}^0_L(\mathbb{Z}_2) + \mathcal{H}^0_L(\mathbb{Z}_2)$

→ $N = k+1,\ k \geq 1$ Radius of the Bosonic Field $= \sqrt{2k(k+1)}$

$$\mathcal{H}^0_L(U(1)) + \mathcal{H}^0_L(\mathbb{Z}_k) = \mathcal{H}^0_L(\mathbb{Z}_{k+1}) + \mathcal{H}^0_L(\mathcal{M}_{k+2})$$

$\mathbb{Z}_{k+1}$ degrees of fredom acquiere a gap

IR FP: $\mathcal{M}_{k+2}\big|_L$

→ Four-channel case: $N=4,\ k=1$

$\mathbb{Z}_4$ $c=1$ CFT ⇝ Bosonic Field φ_R

$$\Phi = \varphi_R + \Phi_{2L}, \quad \Theta = \varphi_R - \Phi_{2L}$$

$$\mathcal{H} \simeq \frac{1}{2}\left[(\partial_x \Phi)^2 + (\partial_x \Theta)^2\right] + g\left[\cos(\sqrt{6\pi}\,\Phi) + \cos(\sqrt{6\pi}\,\Theta)\right]$$

Self-dual Sine-Gordon model

≡ Continuum limit of 2D XY model + Symmetry Breaking Field

José et al, Wiegmann .... (80)

O(2) symmetry → $\mathbb{Z}_3$ Clock model

At the self dual point ⇝ 3-state Potts criticality

OK with the IR FP: $\mathbb{Z}_3\big|_R \otimes \mathcal{M}_5\big|_L$

≡ 3-state Potts Universality class