

SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

FULL FERROMAGNETIC SATURATION OF A
2D QUANTUM ANTIFERROMAGNET

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These are preliminary lecture notes, intended only for distribution to participants

Full Ferromagnetic Saturation of a 2D Quantum Antiferromagnet

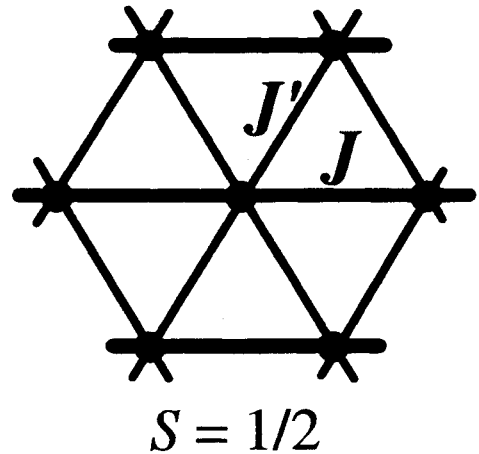
Radu Coldea

University of Oxford

D.A. Tennant (Oxford)

K. Habicht & P. Smeibidl (HMI, Berlin)

Z.Tylczynski, (Adam Mickiewicz U., Poland)



Outline of talk

1. Introduction
2. Fully-polarized phase of a quantum magnet
3. Experiments on frustrated magnet Cs_2CuCl_4
4. Transition out of full alignment (Bose condensation of magnons)
5. Conclusions

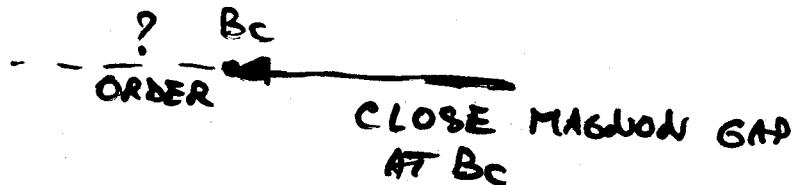
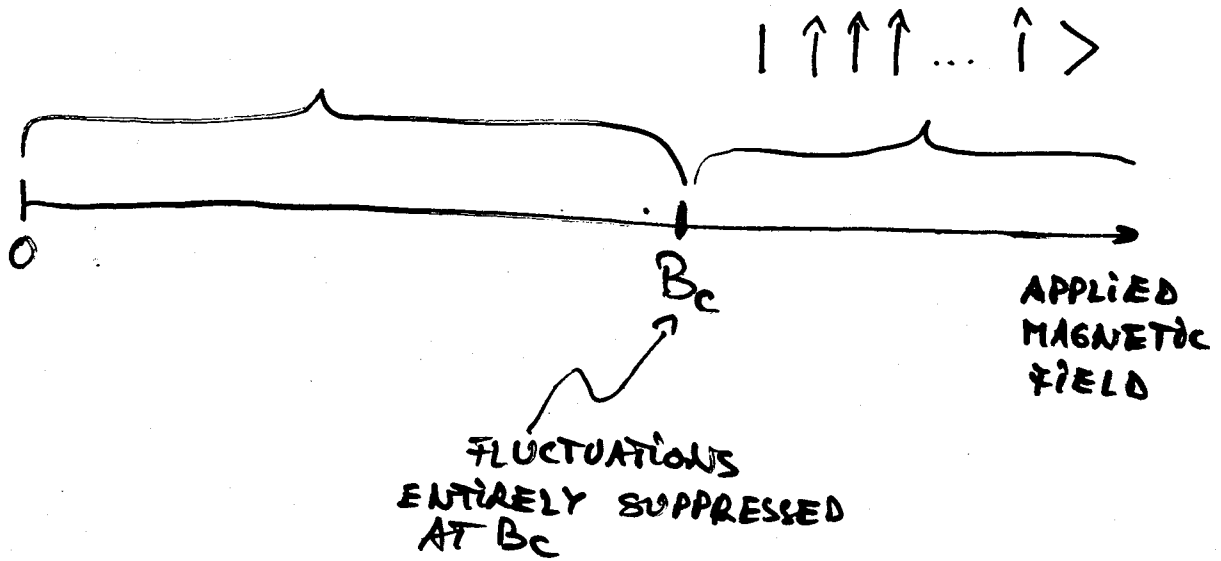
INTRODUCTION

- USE APPLIED MAGNETIC FIELDS TO MANIPULATE GROUND-STATE FLUCTUATIONS IN A QUANTUM MAGNET

PHASES SHOWING STRONG FLUCTUATIONS

E.g. QUANTUM RENORMALIZATIONS
DECONFINED SPINONS
.....

FULLY-POLARIZED PHASE
(NO FLUCTUATIONS)
SPIN-1 MAGNONS

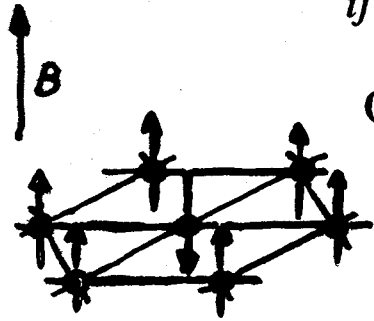


Excitations in the fully-polarized phase

- neutrons flip over one spin $S^- | \uparrow \uparrow \uparrow \uparrow \dots \rangle = | \uparrow \uparrow \downarrow \uparrow \uparrow \dots \rangle$

$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - h \sum_i S_i^z$$

$$= \sum_{ij} \frac{J_{ij}}{2} (S_i^- S_j^+ + S_i^+ S_j^-) + \sum_{ij} J_{ij} S_i^z S_j^z - h \sum_i S_i^z$$



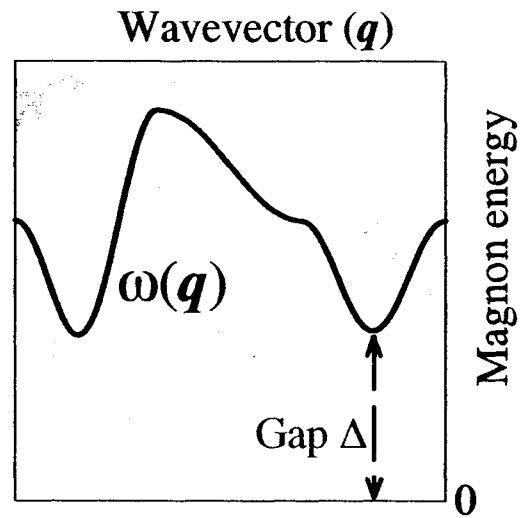
One spin flip $\Delta S^z = -1$
manifold of states

$$\left\{ | \uparrow \uparrow \downarrow \uparrow \uparrow \dots \rangle \right.$$

fully-aligned
eigenstate

$$| \uparrow \uparrow \uparrow \uparrow \dots \rangle$$

S^-



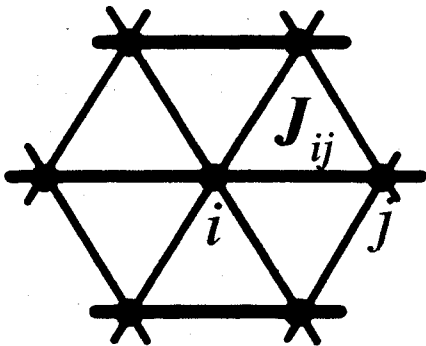
magnon
wavefunction

$$| \varphi_q \rangle = \frac{1}{\sqrt{N}} \sum_i e^{iqr_i} | \downarrow_i \rangle$$

energy

$$\omega(\mathbf{q}) = J(\mathbf{q}) - J(0) + h$$

exact
result



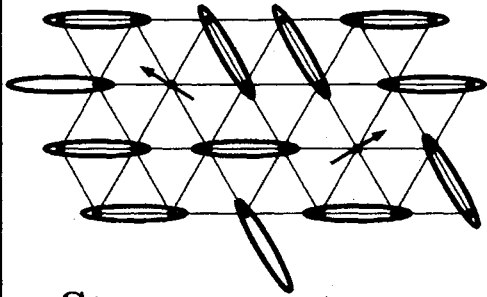
Fourier transform
of magnetic couplings

$$J(\mathbf{q}) = \frac{1}{2} \sum_{ij} J_{ij} e^{iqr_{ij}}$$

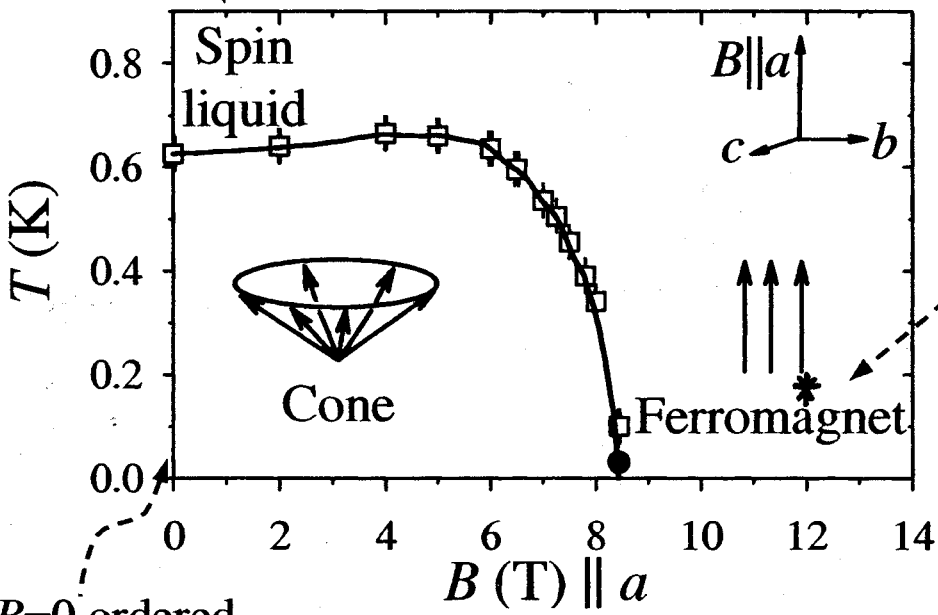
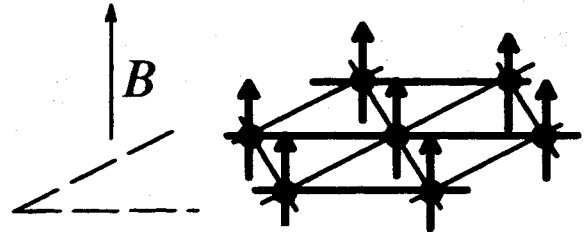
Zeeman
energy

Excitations wavefunctions are known exactly,
dispersion images exchange Hamiltonian

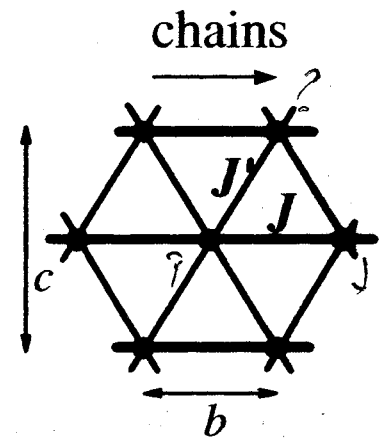
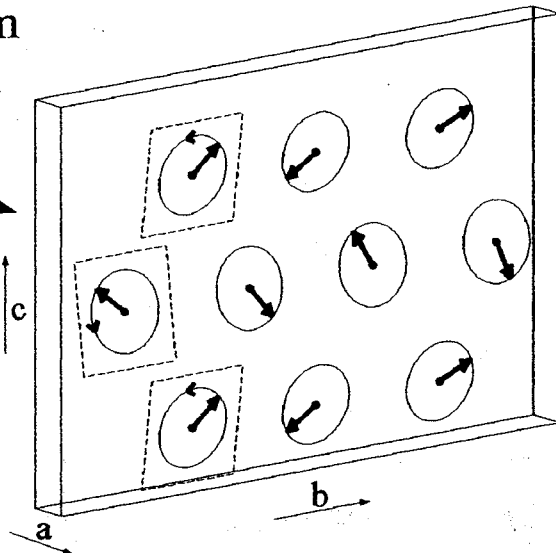
2D Frustrated quantum magnet Cs_2CuCl_4



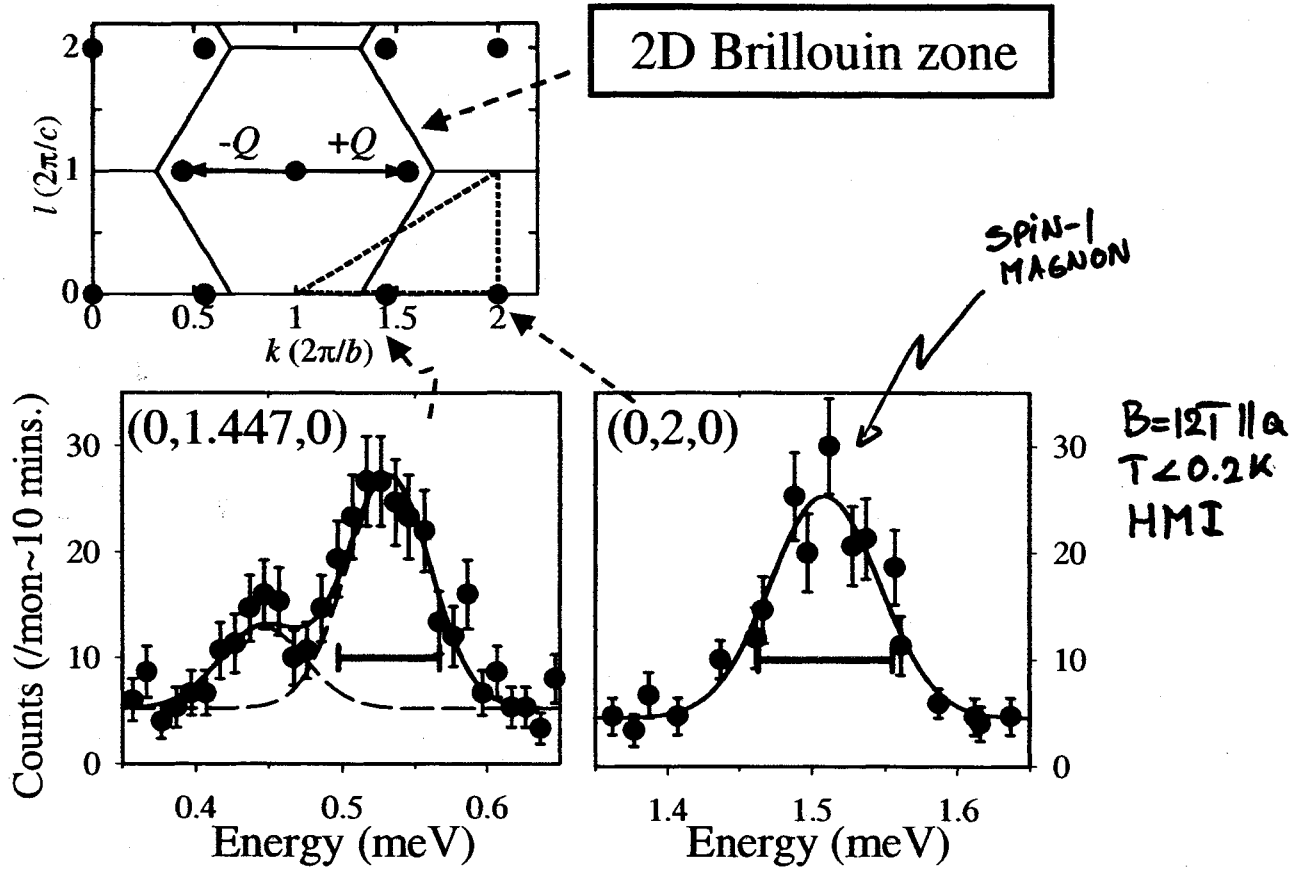
Fluctuations entirely suppressed, spin-1 magnons



At $B=0$ ordered spins form cycloids.

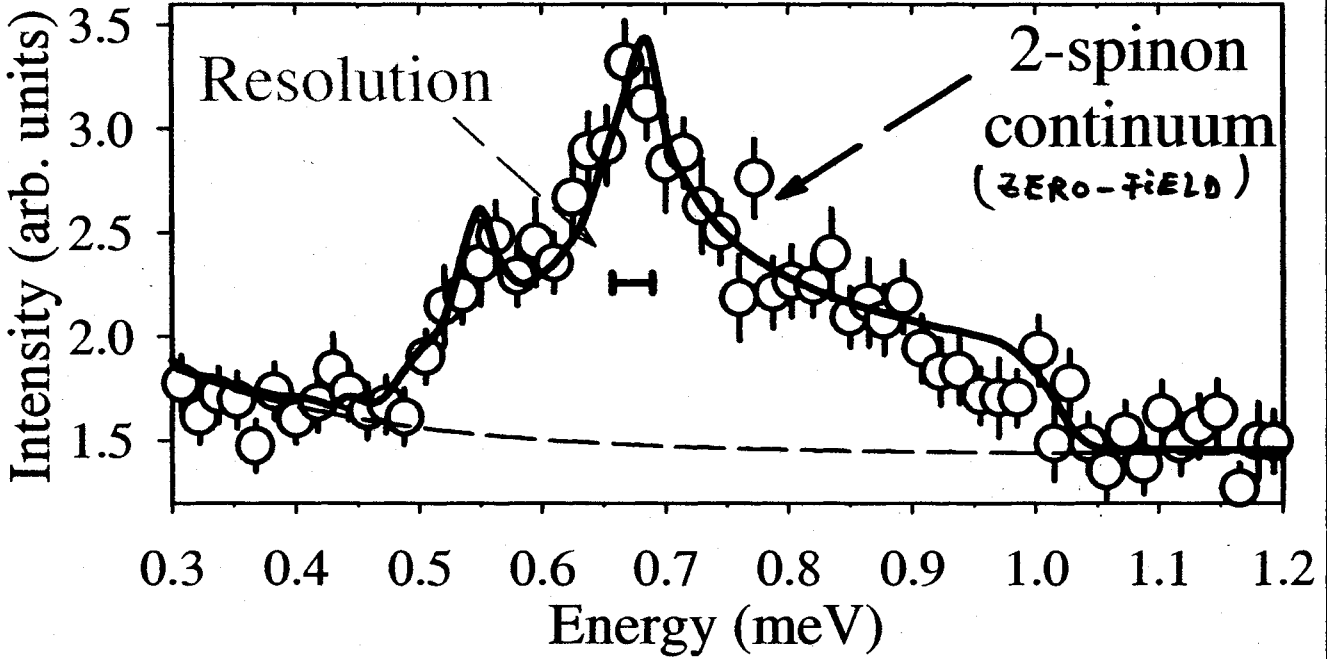


Excitations lineshapes in the fully saturated phase



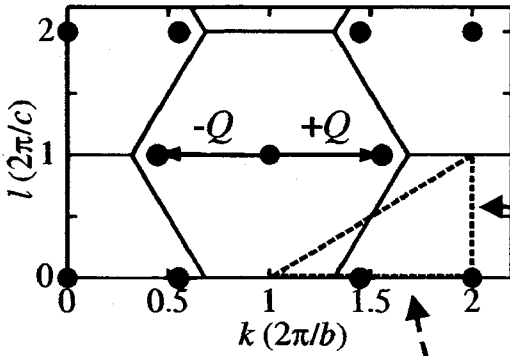
Lineshapes show well-defined, resolution-limited peaks as expected for spin-1 magnons

In contrast, in the spin-liquid phase ($B=0$) the lineshapes show a continuum arising from pairs of spin-1/2 spinons

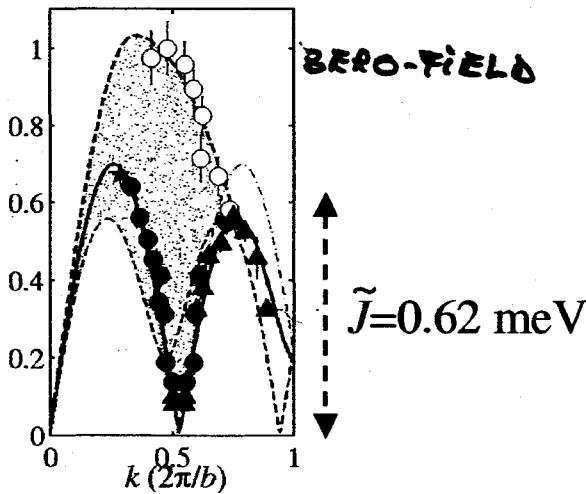
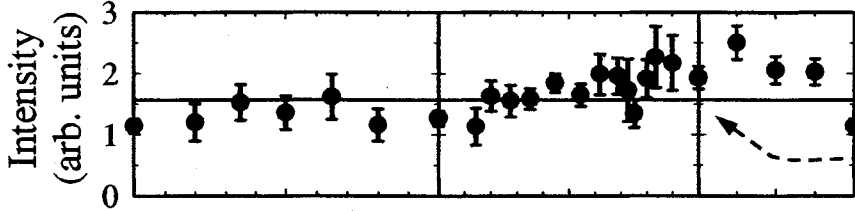
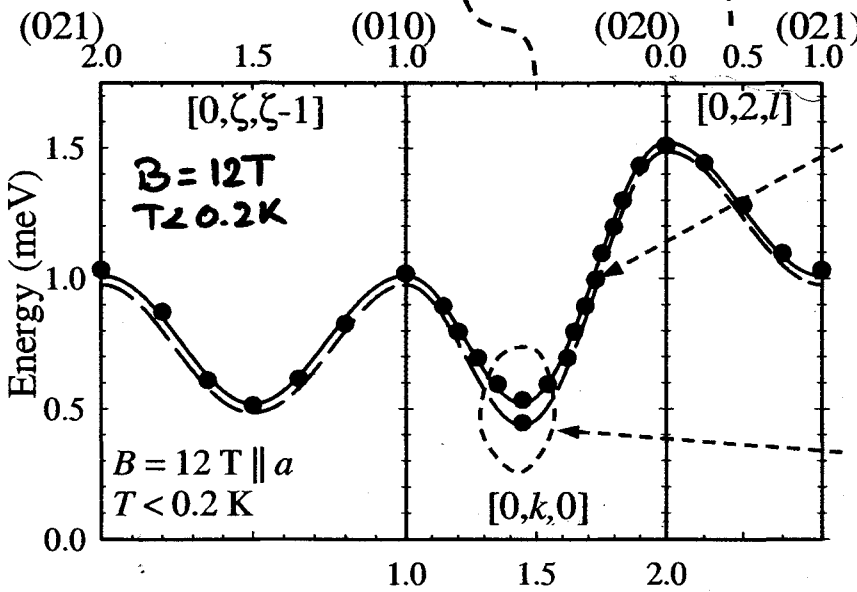
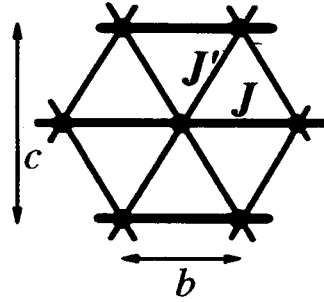


Dispersion relation and exchange constants

2D Brillouin zone



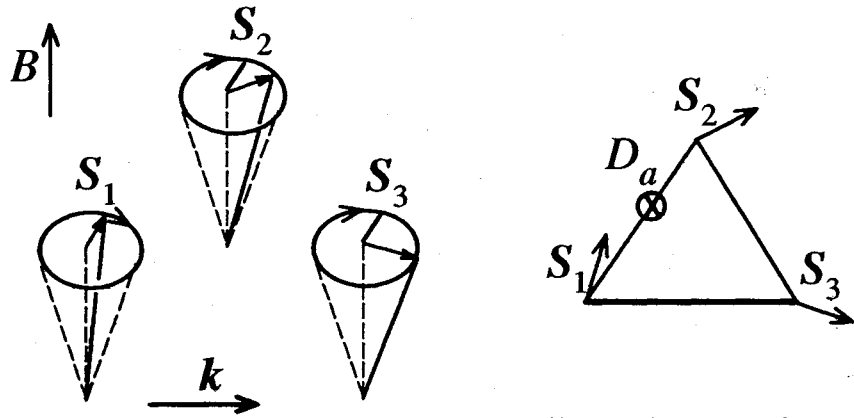
Triangular lattice



In contrast, in the $B=0$ spin liquid phase quantum fluctuations renormalize dispersion $R=\tilde{J}/J=1.65$ ($\pi/2$ in 1D)

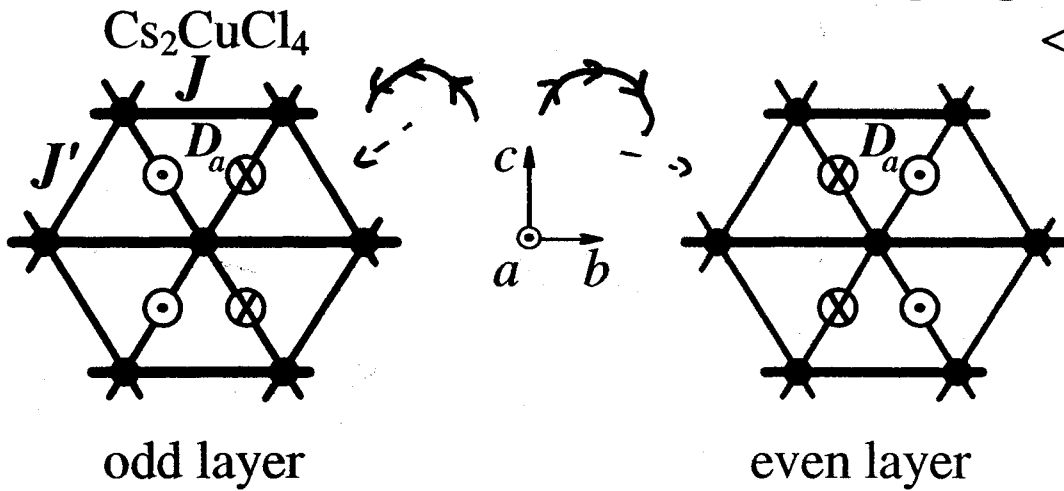
Splitting and Dzyaloshinskii-Moriya couplings

A $\Delta S^z = -1$ magnon is a spin-wave precession with a well-defined sense at given k

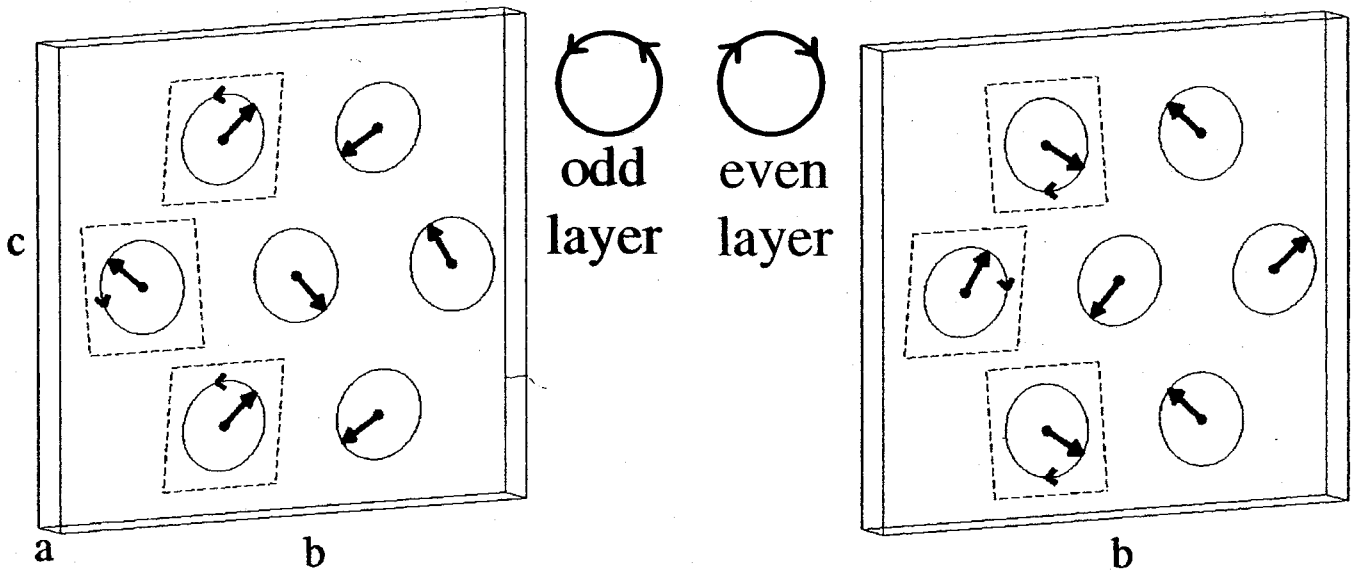


$$D_a (S_1 \times S_2) > 0 \text{ for } D_a > 0$$

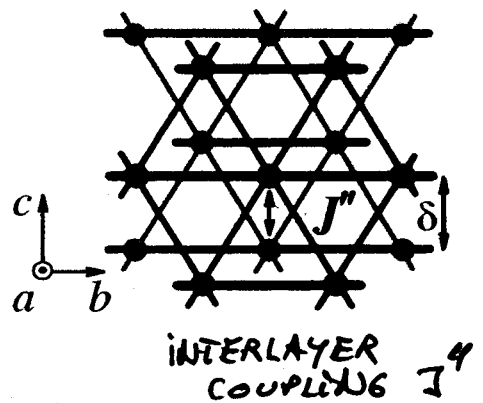
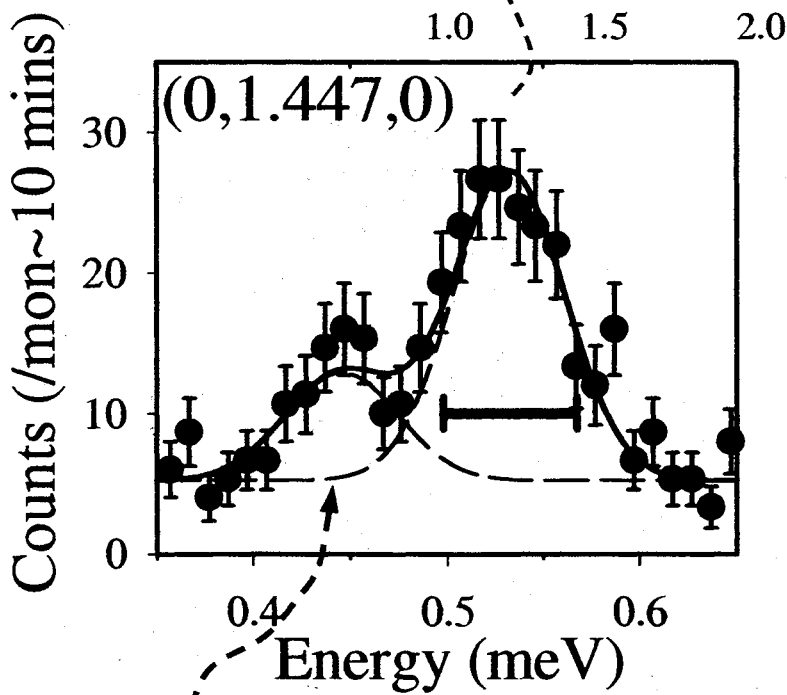
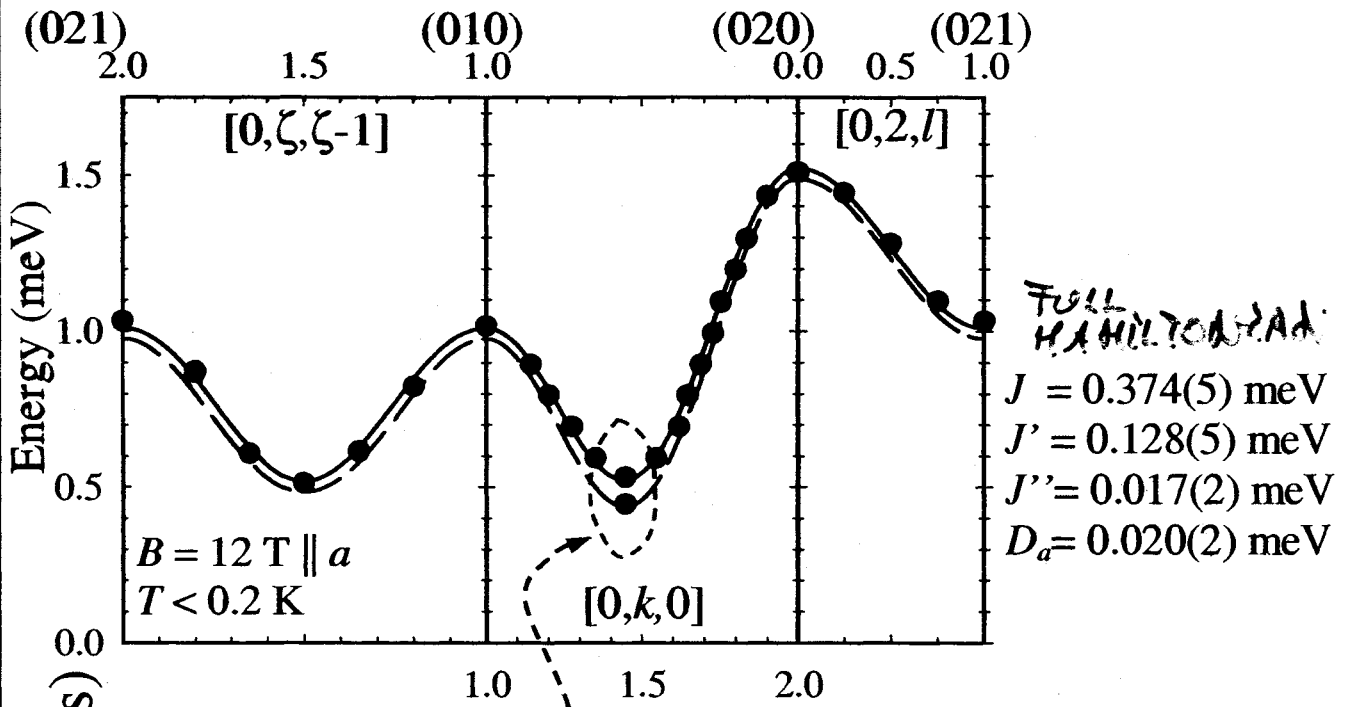
$$< 0 \text{ for } D_a < 0$$



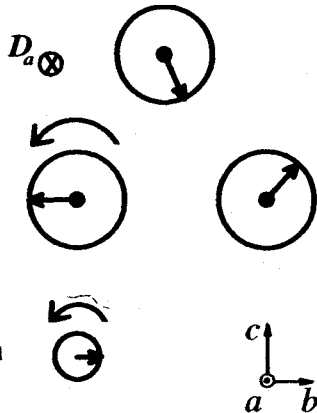
Local DM couplings favour same sense of rotation in one layer and opposite sense in adjacent layers
 \Rightarrow splitting between odd/even layers $\sim D(q)$
 \Rightarrow counter-rotating layers in the $B=0$ cycloids



Dispersion relation and magnon wavefunctions



pictorial representation



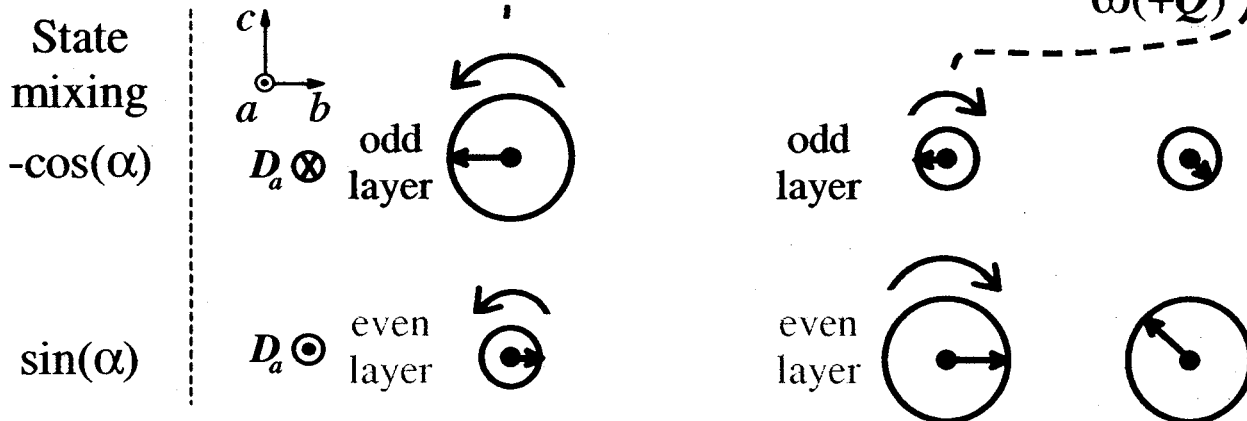
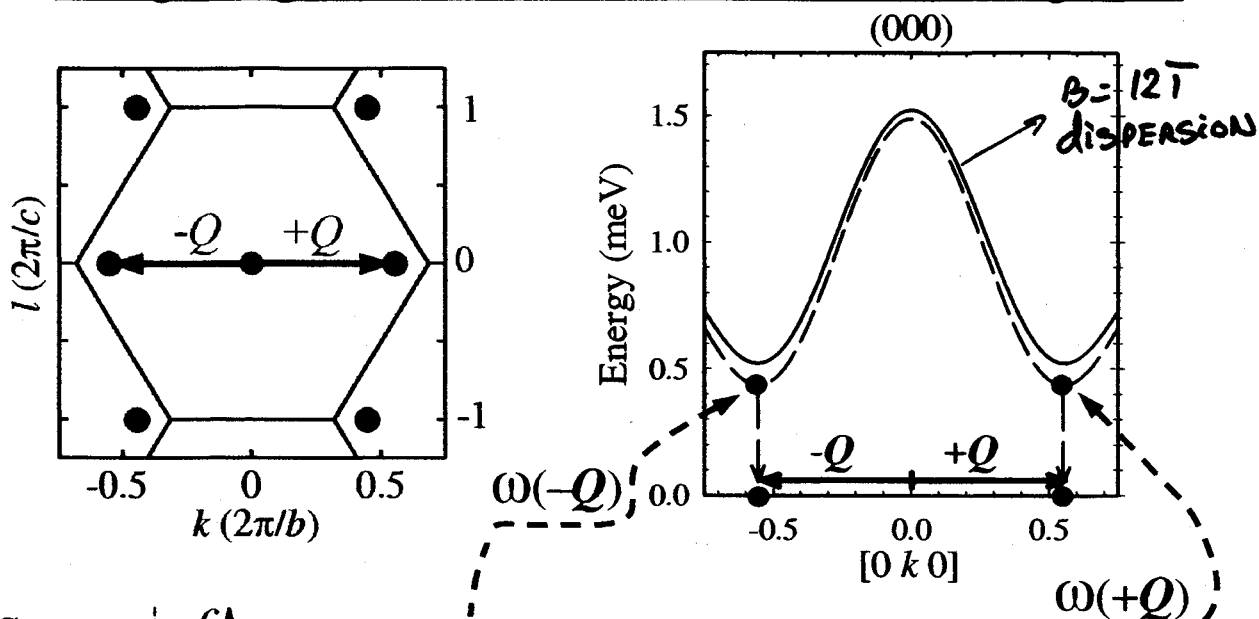
Mixing of eigenstates due to interlayer coupling J''

Magnon wavefunction

$$|\varphi\rangle = -\cos \alpha \frac{1}{\sqrt{N}} \sum_i^{\text{odd}} e^{iQr_i} |\downarrow_i\rangle + \sin \alpha \frac{1}{\sqrt{N}} \sum_j^{\text{even}} e^{iQr_j} |\downarrow_j\rangle$$

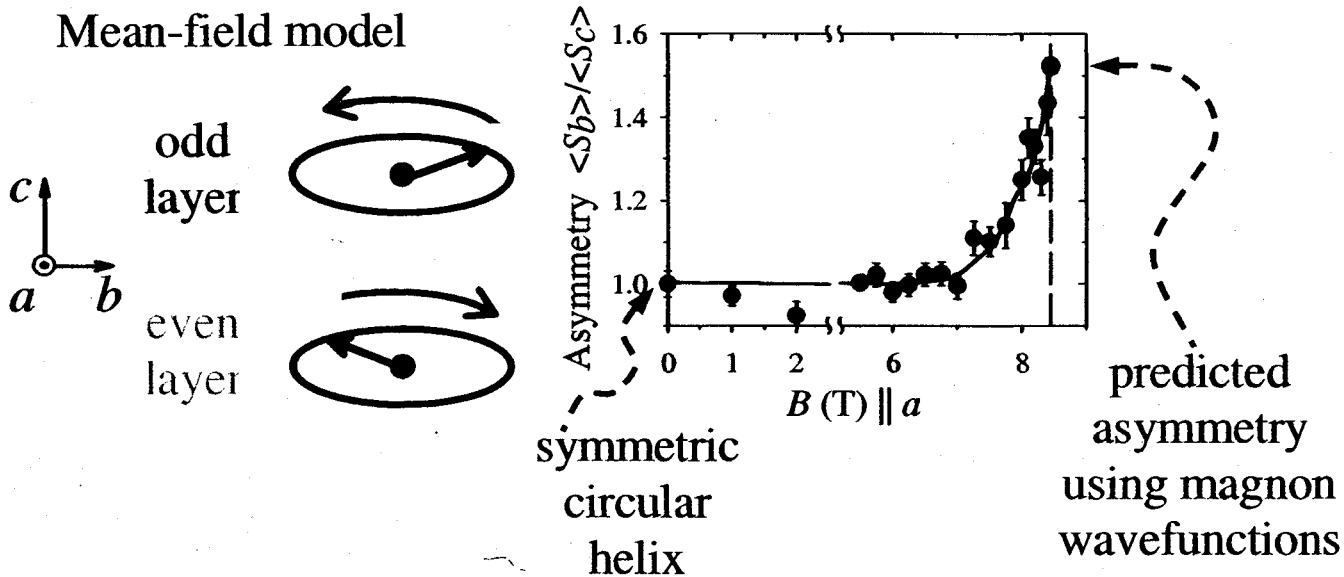
small $\alpha \sim 0.065 \pi$

Long-range order as a condensation of magnons



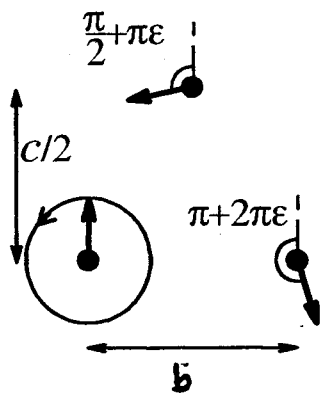
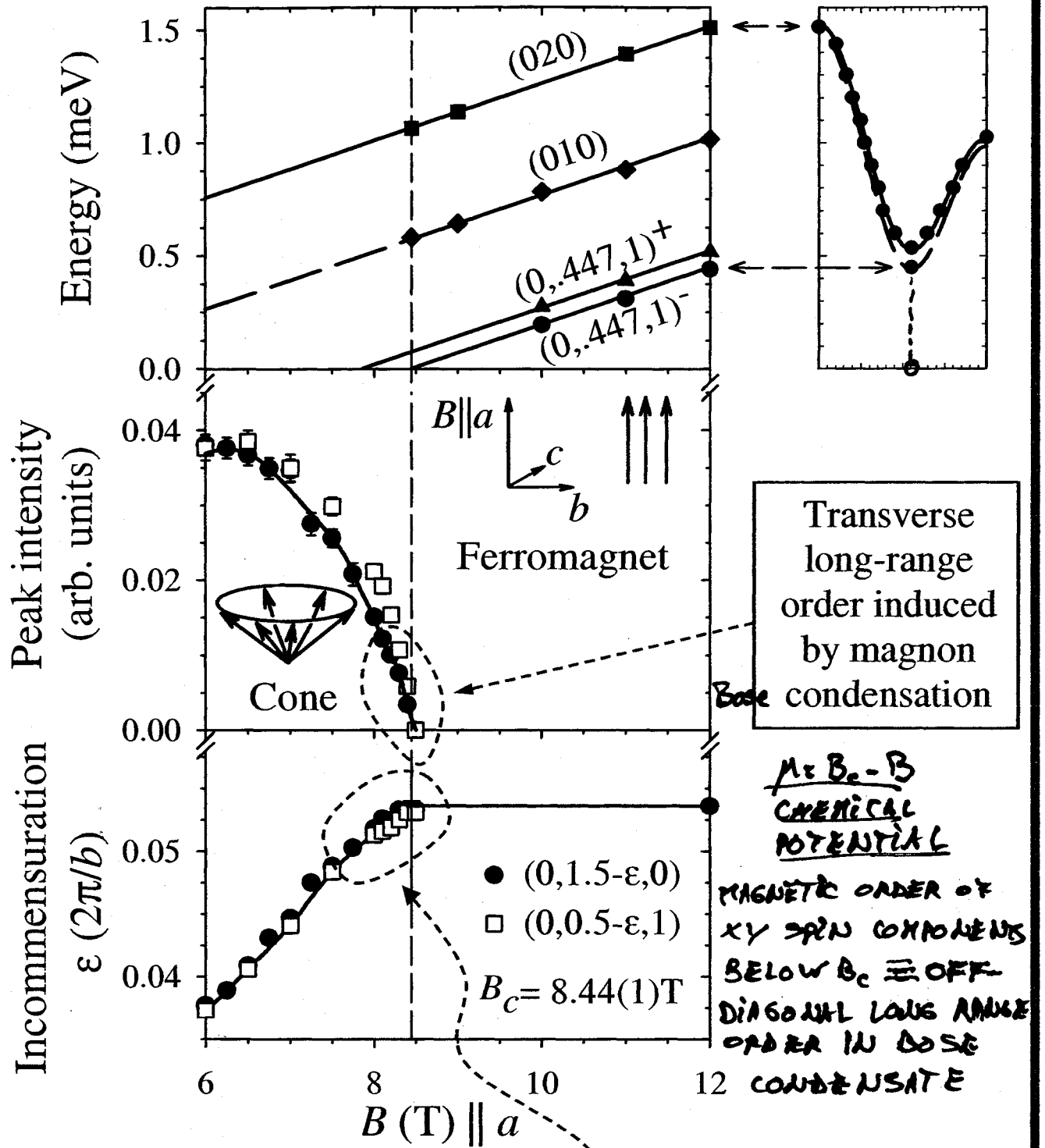
Simultaneous condensation at $-Q$ and $+Q \Rightarrow$ elliptical order

Mean-field model



The wavefunction of the magnetic order can be constructed from the wavefunctions of the condensed magnons

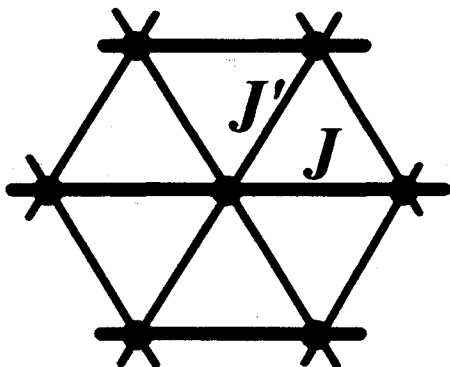
Closing the magnon gap and onset of long-range order



Change in incommensuration due to magnon interactions

Conclusions

1. Full ferromagnetic saturation prepares a magnet in a phase where the ground state and all excited states have exactly known wavefunctions. Experimental measurement of the excitations in this phase gives the bare Hamiltonian with absolute (unrenormalized) values for all couplings.
2. For strongly fluctuating magnets this is a uniquely powerful method to experimentally determine ground-state energies and quantum renormalizations.



2D frustrated quantum
magnet Cs_2CuCl_4
($J'/J=0.33$)

3. Closing the magnon gap at the saturation field allows studies of the conditions in which order arises from condensation of particles in the ground state ($B_c - B = \mu$ due to potential for Bose condensation)