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SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

APPLICATIONS OF INTEGRABLE MODELS TO QUASI-1D QUANTUM MAGNETS AND MOTT INSULATORS

F.H.L. ESSLER
University of Warwick
Department of Physics
Gibbet Hill Road
Coventry
CV4 7AL
U.K.

These are preliminary lecture notes, intended only for distribution to participants

Applications of integrable models to quasi-1D quantum magnets and Mott insulators

F.H.L. Essler (Warwick)

<u>Colaborators:</u>

A.M. Tsvelik (BNL)
D. Controzzi (Princeton)
F. Gebhard and E. Jeckelmann (Marburg)

Mott Insulators

- Optical conductivity
- Formation of "Mott-Hubbard Excitons"
- weakly coupled Mott insulators
  crossover MI $\longrightarrow$ Fermi liquid

Quasi-1D Quantum Magnets

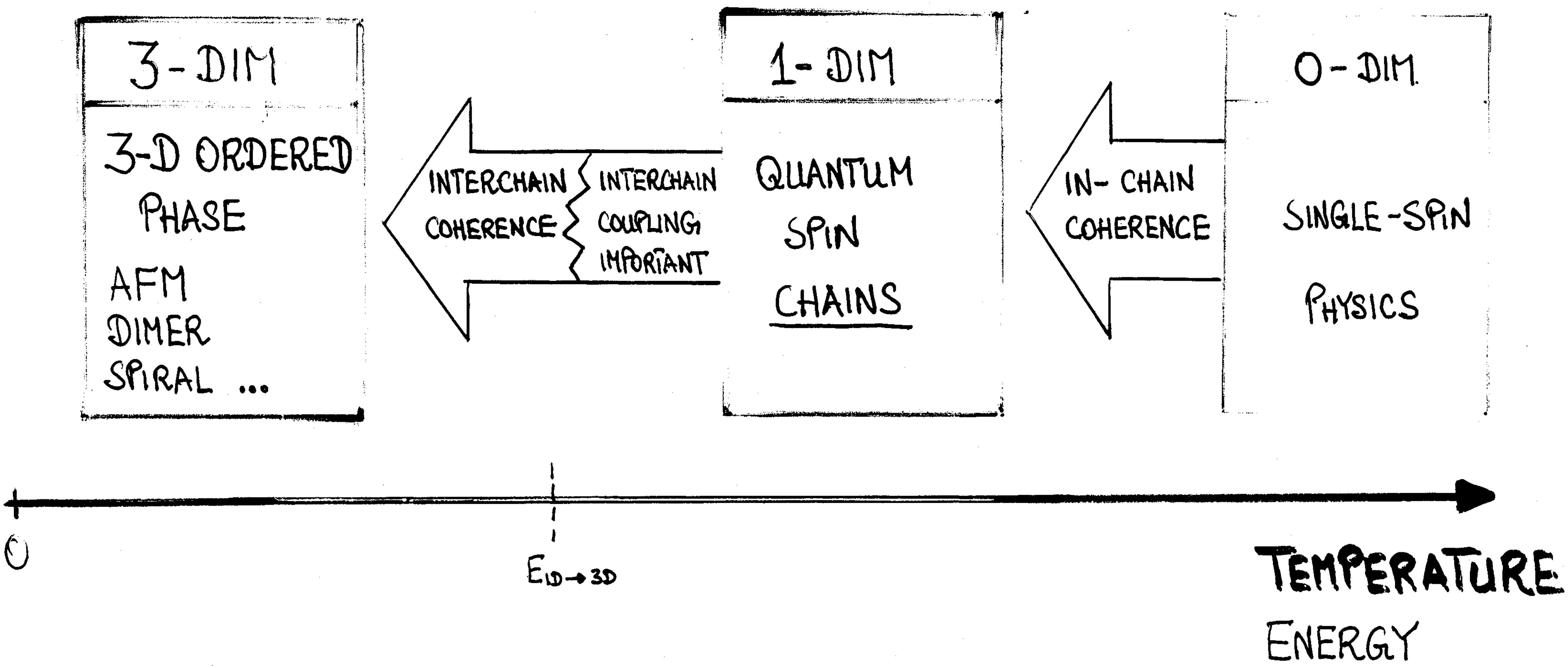
- "Dimensional Crossover"

  The "High-energy" physics in the ordered phase of spin-1/2 antiferromagnets
- "Field induced gap systems":

  quantum solitons and "breathers" in CuBenz

  [IAN AFFLECK'S LECTURES]

"SEPARATION OF SCALES"



Some Generalities

- Gapless quasi-1D systems can be treated by means of CFT techniques ($\longrightarrow$ reviews by e.g. Schulz, Voit, book by GNT)
- We are interested in quasi-1D systems with a **gap**. These are much more difficult to handle.
- We will do so using field-theory techniques. This imposes some limitations.

  Magnets:

$$H = J \sum_n \vec{S}_n \cdot \vec{S}_{n+1} + h \sum_n (-1)^n S_n^z \ .$$

  Field theory applies at energies/temperatures $\omega, T \ll J$ if the gap is **small** $M(h) \ll J$; J plays the role of a UV cutoff.

  Mott Insulators:

$$H = -t \sum_{n,\sigma} \left[c_{n,\sigma}^\dagger c_{n+1,\sigma} + \text{h.c.} \right] + U \sum_k n_{k,\uparrow} n_{k,\downarrow}$$

  Field theory applies for $\omega, T, M(U) \ll t$. **This does not imply that $U \ll t$.**

  We will further assume that $T \ll M(U)$ so that we can neglect temperature corrections. The results will apply in the regime

$$T_{1D \to 3D} \ll T \ll M(U) \ll t \ .$$

Dynamical Properties of 1D Mott insulators

Tough old problem: strongly correlated with a **gap**.

Candidates/Examples:

- quasi-1D antiferromagnets [$KCuF_3$] [HUBBARD $\xrightarrow[\frac{U}{t}\to\infty]{}$ HEISENBERG]
- carbon nanotubes
- organic conductors e.g. Bechgaard salts

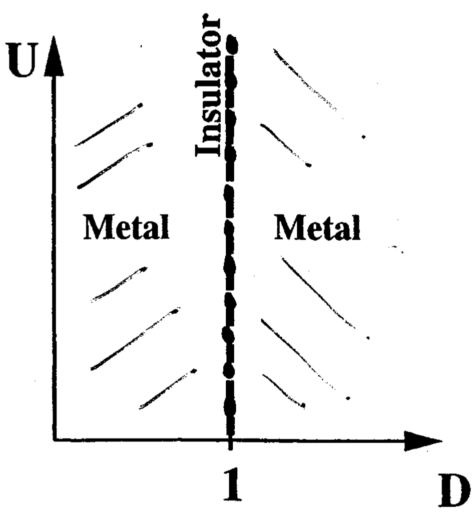
<u>1D Mott transition:</u>

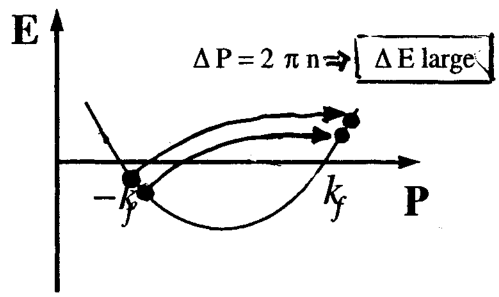
Simplest example: Hubbard model

$$\boxed{H = -t\sum_{n,\sigma}\left[c^{\dagger}_{n,\sigma}c_{n+1,\sigma} + \text{h.c.}\right] + U\sum_{k} n_{k,\uparrow}n_{k,\downarrow}}$$

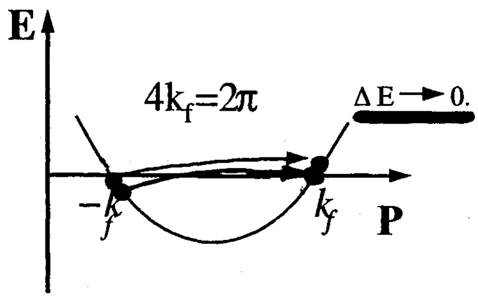
Phase diagram:

[LIEB/WU]





incommensurate filling



half filling

Field Theory of 1D Mott insulators

1D Metal $\equiv$ **Luttinger liquid**

$$H_{\rm LL} = H_{\rm charge} + H_{\rm spin}$$

$$\mathcal{H}_{\rm spin} = \frac{1}{2}\left[\frac{1}{K_s}\left(v_s\partial_x\phi_s\right)^2 + K_s\left(\partial_t\phi_s\right)^2\right]$$

$$\mathcal{H}_{\rm charge} = \frac{1}{2}\left[\frac{1}{K_c}\left(v_c\partial_x\phi_c\right)^2 + K_c\left(\partial_t\phi_c\right)^2\right]$$

$\longrightarrow$ spin-charge separation. $K_{s,c}$ contain information on interactions

Umklapp only affects the charge sector

$\longrightarrow H_{\rm MI} = \tilde{H}_{\rm charge} + H_{\rm spin}$

$$\tilde{\mathcal{H}}_{\rm charge} = \frac{1}{2}\left[\frac{1}{K_c}\left(v_c\partial_x\phi_c\right)^2 + K_c\left(\partial_t\phi_c\right)^2\right] + \underbrace{\lambda\cos\sqrt{8\pi}\phi_c}_{\text{"half-filled" Umklapp}}.$$

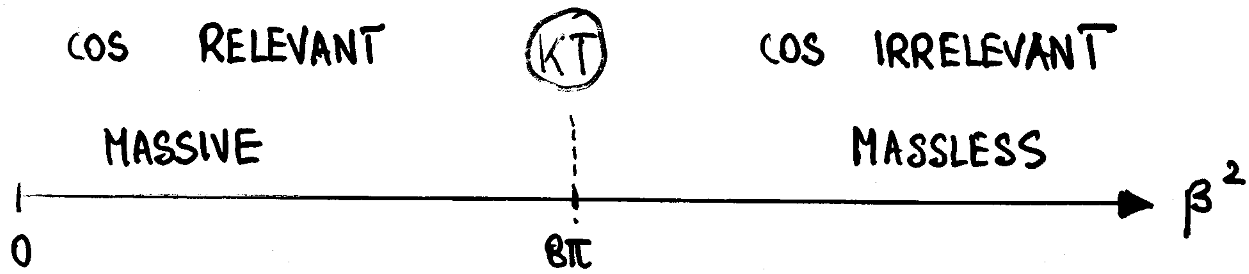
This is quite general! [$K_c = 1$ for Hubbard
$K_c < 1$ for "extended" Hubbard]

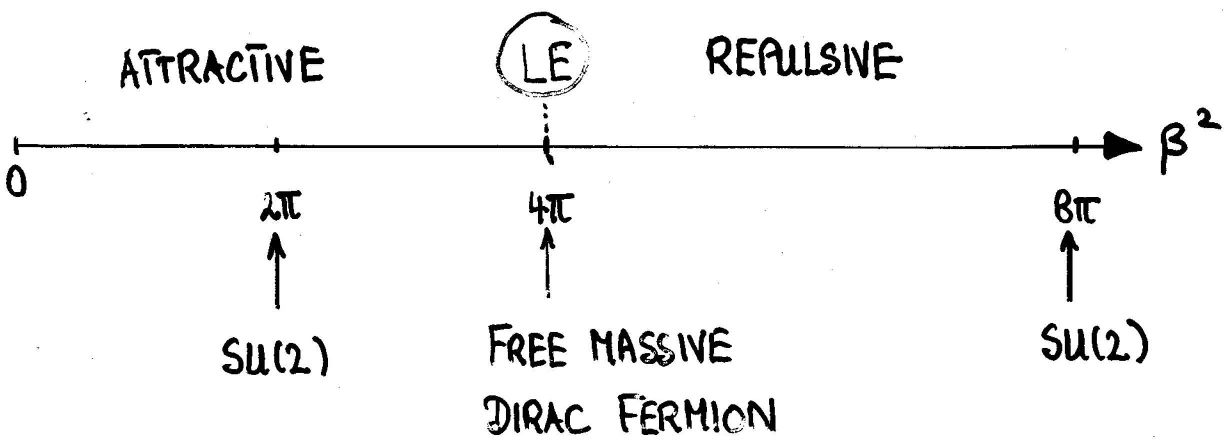
Spectrum in the charge sector: Gap M.

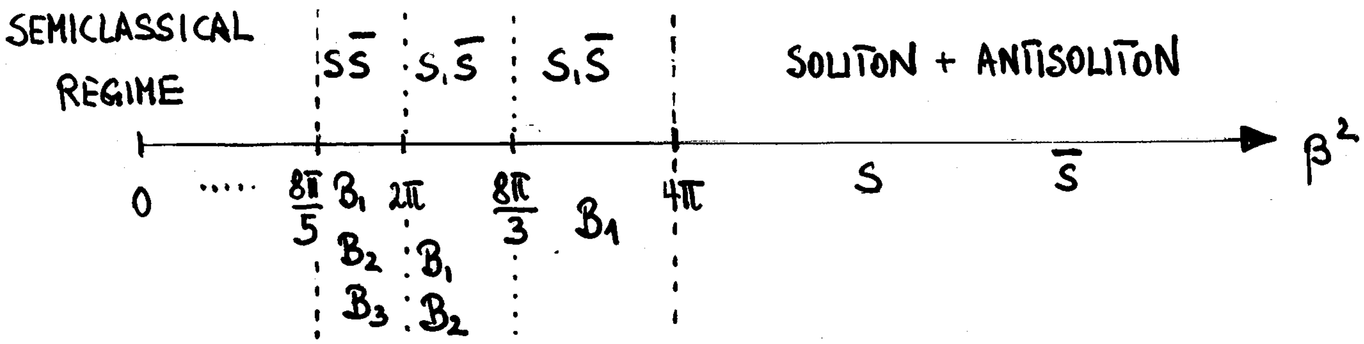
- $K_c > \frac{1}{2}$ "holon" and "antiholon" only. (spinless charge $\mp e$ carriers)
- $K_c = \frac{1}{2}$ "Luther-Emery point"
- $K_c < \frac{1}{2}$ holon, antiholon and several excitons ($h\bar{h}$ bound states)

SOME FACTS ON SINE-GORDON

$$\boxed{\mathcal{L} = \frac{1}{2}\int dx\,(\partial_\mu\phi)^2 - \lambda\int dx\,\cos\beta\phi}\qquad \beta = \sqrt{8\pi K_c}$$







"Luther-Emery" Point

At the special point $\beta = \sqrt{4\pi}$, the SGM

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \lambda \cos\sqrt{4\pi}\phi$$

is equivalent to a free massive Dirac fermion

$$\mathcal{L} = \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi\right]$$

This is a free theory $\longrightarrow$ easy to calculate correlation functions local in fermions.

In the literature it is often stated that

- Correlation functions at the LE point are representative for the whole SGM ("universal").

  This is incorrect; the LE point is quite special.

- the physics of the Hubbard model is described by its LE point.

  The Hubbard model does not have a LE point.

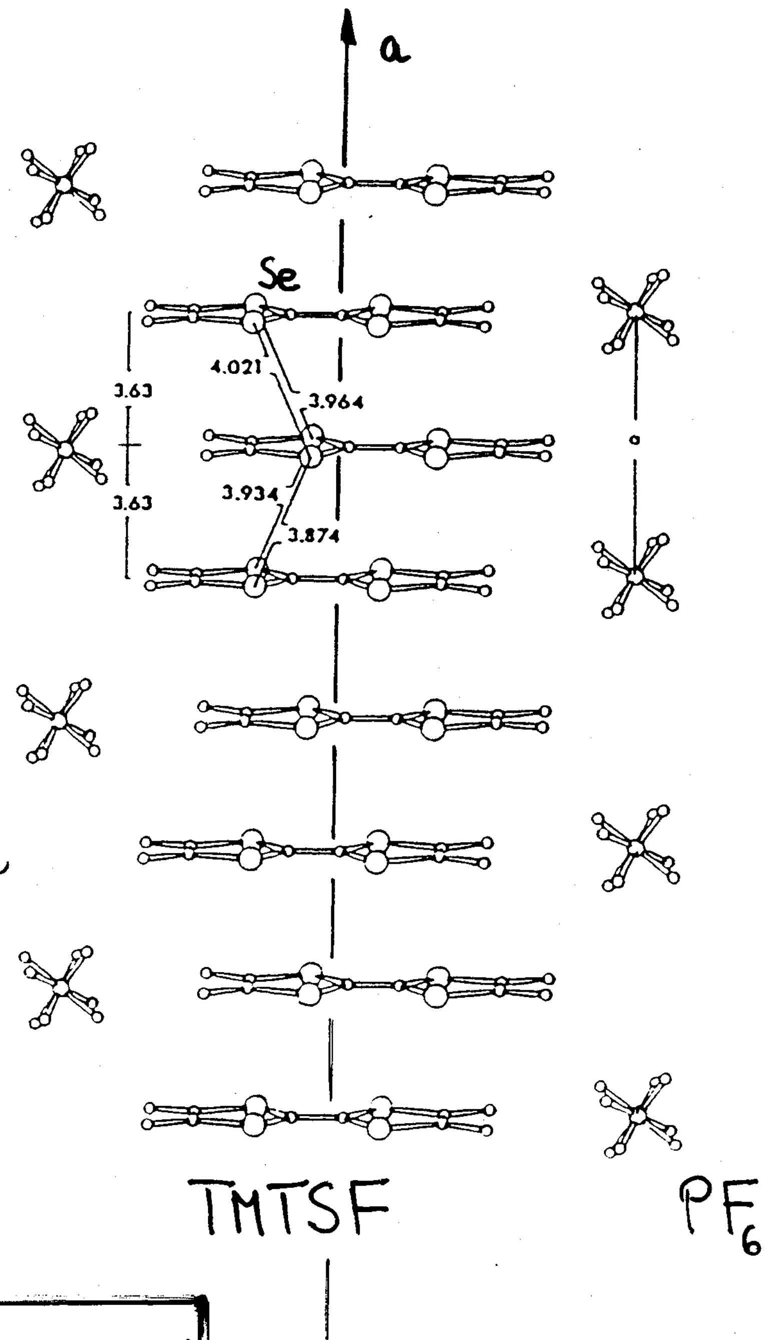
  The easiest way to see that is to note that free fermions have a 2-particle S-matrix $S = -1$, whereas the exact S-matrix of the Hubbard model [Essler/Korepin '94] is

$$\begin{aligned} S(k_1, k_2) &= -\frac{\Gamma(-i\frac{\lambda}{2})}{\Gamma(i\frac{\lambda}{2})}\frac{\Gamma(\frac{1}{2}+i\frac{\lambda}{2})}{\Gamma(\frac{1}{2}-i\frac{\lambda}{2})}\left(\frac{i}{\lambda-i}I - \frac{\lambda}{\lambda-i}P\right), \\ \lambda &= \frac{\sin(k_1)-\sin(k_2)}{2U/t}. \end{aligned}$$

IMPOSSIBLE TO HAVE $S \to -1$ AND LORENTZ INVCE

ORGANIC CONDUCTORS

EXAMPLE: $(TMTSF)_2 PF_6$ [TETRA METHYLSELENAFULVALEN]

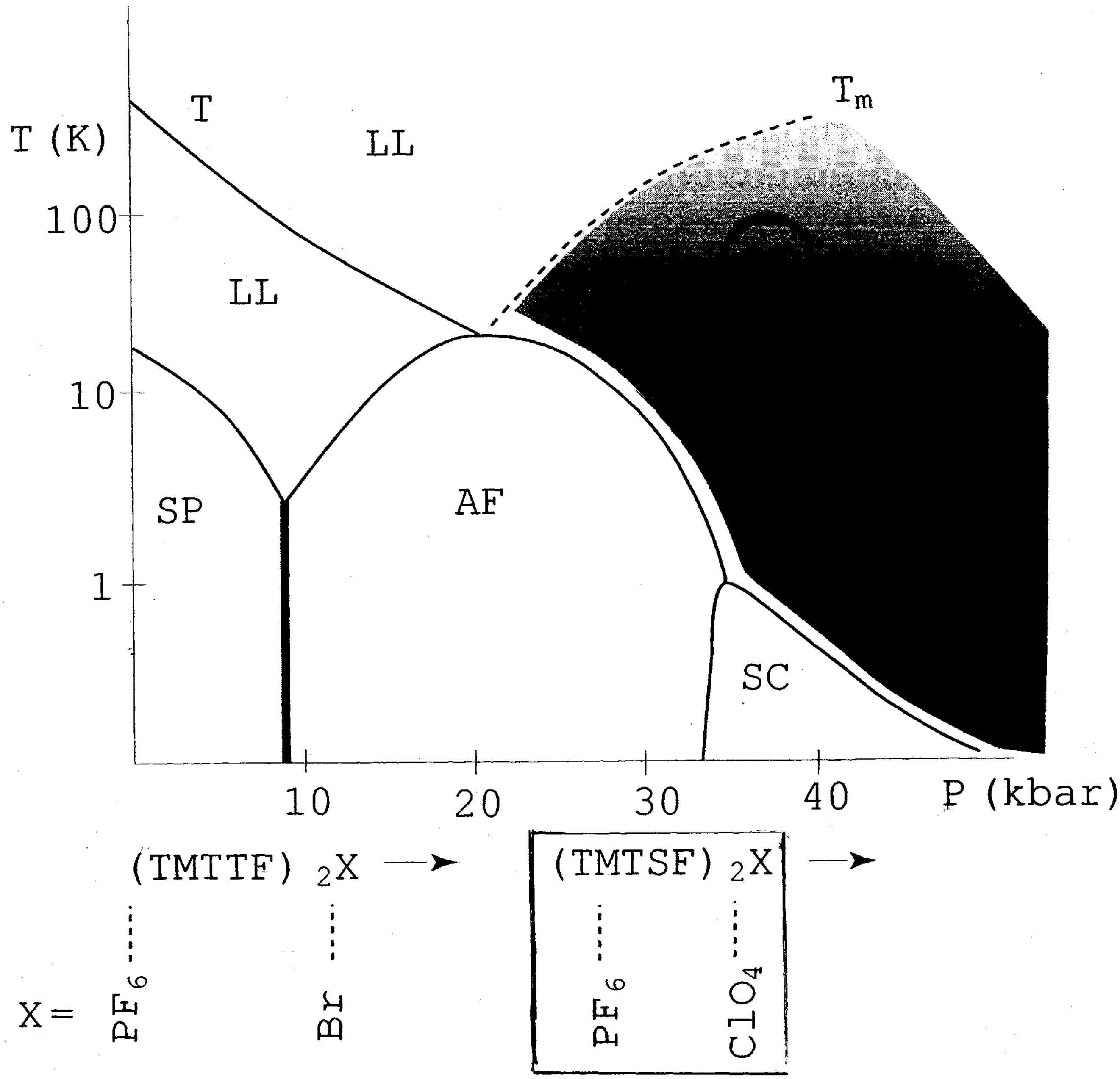


$\boxed{t_a : t_b : t_c \simeq 250\,meV : 25\,meV : 1\,meV}$

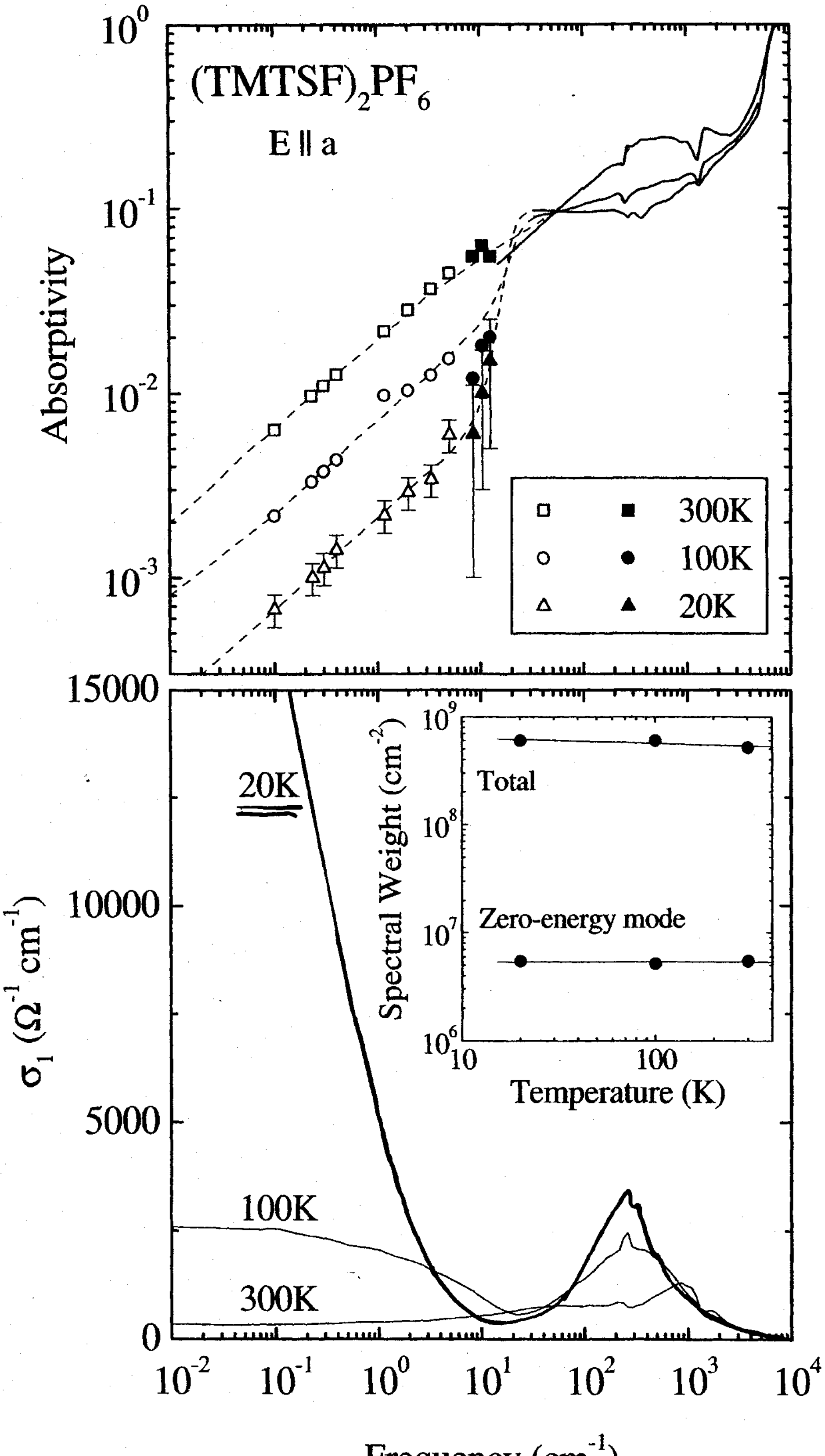
$\boxed{\frac{3}{4} - \text{FILLED}}$

C. BOURBONNAIS & D. JÉROME

COND-MAT/9903101

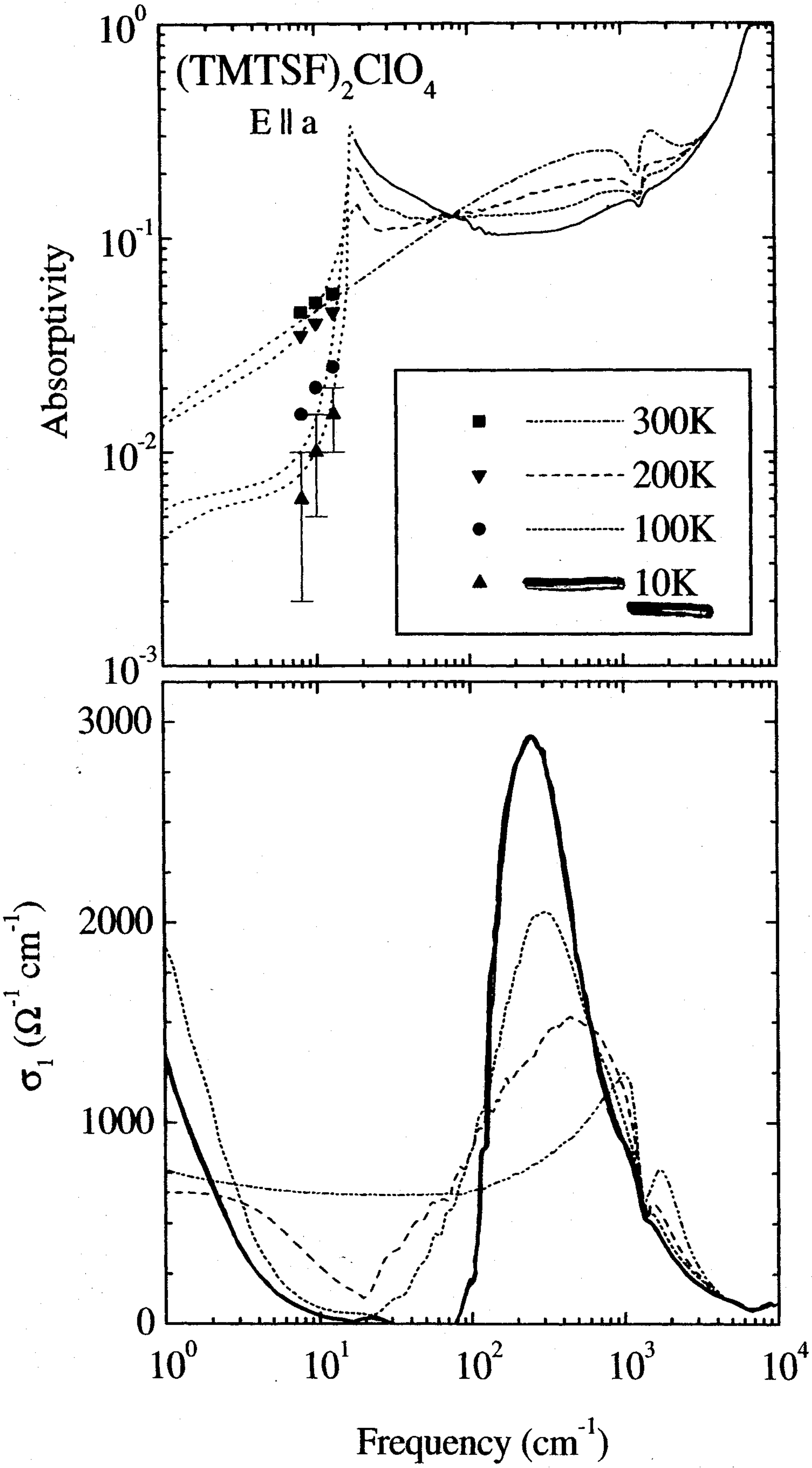


A. Schwartz et. al.
PRB 58, 1261 (1998)

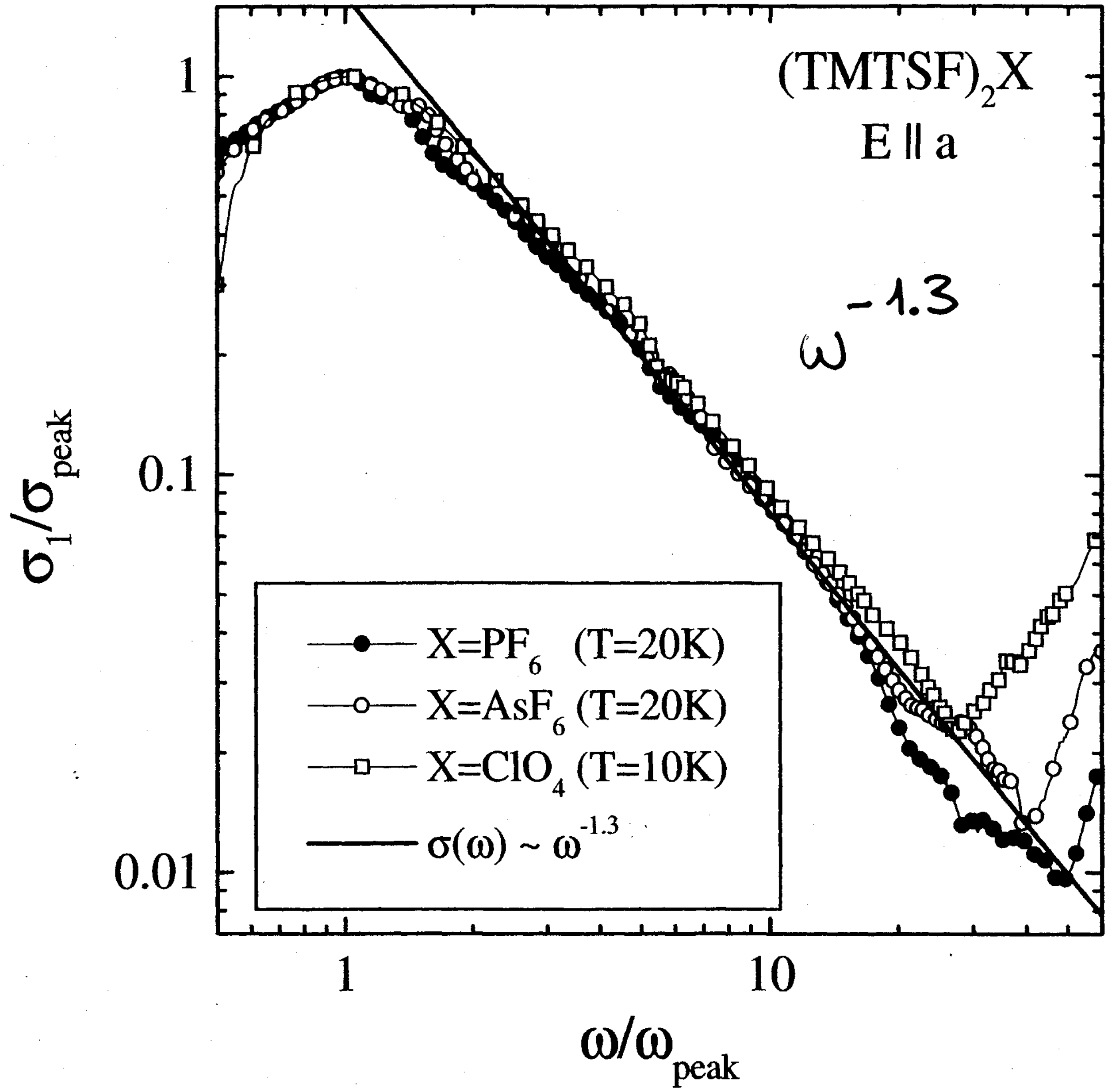


1D-3D CROSSOVER
SCALE

A. Schwartz et al
PRB 58, 1261 (1998)



A. Schwartz et al
PRB 58 1261 (1998)



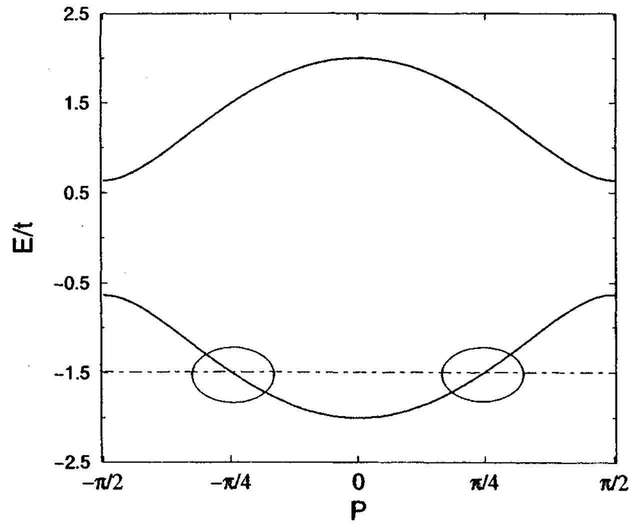
Situation in the Bechgaard salts

- quarter-filled
- small dimerization

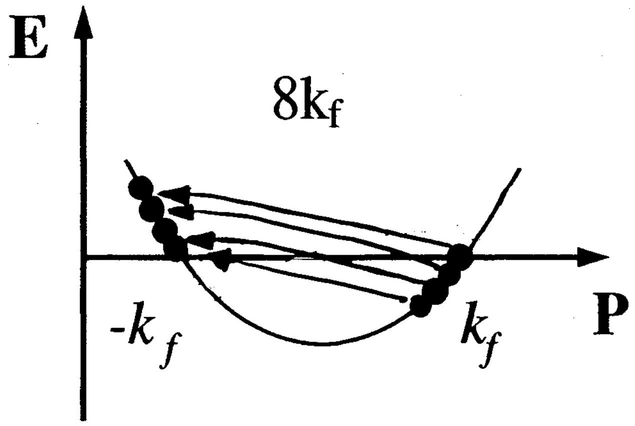
$$
\begin{aligned}
H &= -\sum_{n,\sigma}(t+(-1)^n\delta)\left[c^\dagger_{n,\sigma}c_{n+1,\sigma}+\text{h.c.}\right] \\
&\quad +U\sum_k n_{k,\uparrow}n_{k,\downarrow}+V_1\sum_k n_k n_{k+1}+V_2\sum_k n_k n_{k+2}\dots \\
&= H_0+H_{\text{int}}.
\end{aligned}
$$

Diagonalize H_0 :

$$
\begin{aligned}
H_0 &= \sum_k \epsilon(k)[l^\dagger_\sigma(k)l_\sigma(k)-u^\dagger_\sigma(k)u_\sigma(k)] \\
\epsilon(k) &= -2\sqrt{t^2\cos^2 k+\delta^2\sin^2 k}\ .
\end{aligned}
$$



- Interaction terms mix the 2 bands
- Integrate out upper band and "high-energy" regions of lower band $\longrightarrow$ effective Hamiltonian for degrees of freedom around $\pm k_F$.
- $\longrightarrow$ generates double Umklapp (T. Giamarchi)



"double Umklapp"

Bosonize the resulting Hamiltonian $\longrightarrow$

$$\begin{aligned}
\mathcal{H} &= \mathcal{H}_{\text{charge}} + \mathcal{H}_{\text{spin}} \\
\mathcal{H}_{\text{spin}} &= \frac{1}{2}\left[(v_s\partial_x\phi_s)^2 + (\partial_t\phi_s)^2\right] \\
\mathcal{H}_{\text{charge}} &= \frac{1}{2}\left[\frac{1}{K_c}(v_c\partial_x\phi_c)^2 + K_c(\partial_t\phi_c)^2\right] + \lambda\cos\sqrt{8\pi}\phi_c + \mu\cos 2\sqrt{8\pi}\phi_c.
\end{aligned}$$

$v_{s,c}, K_{s,c}, \lambda, \mu$ are functions of $U/t, V_1/t, V_2/t$

"Double Sine-Gordon Model" (Delfino/Mussardo '98, Fabrizio/Gogolin/Nersesyan '99) not integrable.

Study limiting cases:

- $\lambda \propto \delta$ and therefore $\lambda \ll \mu$ is possible $\longrightarrow$ double Umklapp should dominate at "high" energies $\rightarrow$ **"SGM$_1$"**
- If dimerization is dominant physical ingredient $\longrightarrow$ neglect double Umklapp $\rightarrow$ **"SGM$_2$"** (THIS HAPPENS AT $K_c > 1/4$)
- if both processes are important $\rightarrow$ trouble.

IN WHAT FOLLOWS WE WILL CONSIDER THE MODEL

$$\boxed{\begin{aligned} \mathcal{L} &= \mathcal{L}_s + \mathcal{L}_c \\ \mathcal{L}_s &= \frac{1}{2}\left(\partial_\mu \varphi_s\right)^2 \\ \mathcal{L}_c &= \frac{1}{2}\left(\partial_\mu \varphi_c\right)^2 - \lambda \cos\beta\varphi_c \end{aligned}}$$

IN 2 CASES :

(i) <u>HALF-FILLED MOTT INSULATOR</u>

$$\beta = \sqrt{8\pi K_c}$$

(ii) <u>QUARTER-FILLED MOTT INSULATOR</u>

$$\beta = \sqrt{32\pi K_c}$$

<u>N.B.</u> THE MAIN DIFFERENCE IS THE RELATION OF LATTICE OPERATORS AND OPERATORS IN THE SGM IN THE 2 CASES

EXAMPLES: ELECTRON OPERATORS

$$c_{j\sigma} \sim e^{ik_F x} R_\sigma(x) + e^{-ik_F x} L_\sigma(x)$$

$$L_\sigma = \eta_\sigma \, e^{\frac{i}{4}\left[\beta \varphi_c - \frac{8\pi}{\beta}\theta_c\right]} \, e^{\pm i\sqrt{\frac{\pi}{2}}\left[\varphi_s - \theta_s\right]} \qquad \text{HALF FILLED}$$

$$L_\sigma = \eta_\sigma \, e^{\frac{1}{4}\left[\frac{\beta}{2} \varphi_c - \frac{16\pi}{\beta}\theta_c\right]} \, e^{\pm i\sqrt{\frac{\pi}{2}}\left[\varphi_s - \theta_s\right]} \qquad \tfrac{1}{4}\text{-FILLED}$$

CURRENT OPERATOR

$$J = -\frac{iet}{\hbar} \sum_{j,\sigma} c^{+}_{j\sigma} c_{j+1\sigma} - c^{+}_{j+1\sigma} c_{j\sigma}$$

[N.B. DOES <u>NOT</u> COMMUTE WITH LATTICE HAMILTONIAN]

$$J \sim \partial_t \varphi_c \qquad \text{BOTH FOR } \tfrac{1}{2} \text{ AND } \tfrac{1}{4} \text{ FILLED}$$

CALCULATE OPTICAL CONDUCTIVITY :

$$\boxed{\begin{array}{l} \sigma(\omega) = \frac{1}{\omega} \; \mathrm{Re} \int dx \int_0^\infty dt \; \langle [j(t,x), j(0,0)] \rangle \, e^{i\omega t} \\[2em] j(t,x) = A \, \partial_t \varphi \qquad \underline{\text{CURRENT OPERATOR}} \end{array}}$$

CONSIDER $T = 0$ ONLY

<u>SPECTRAL REPRESENTATION :</u>

$$\langle \mathcal{O}(x,t) \, \mathcal{O}^+(0,0) \rangle = \sum_n e^{iE_n t - i P_n x} \, \underbrace{|\langle 0 | \mathcal{O}(0,0) | n \rangle|}^{2}$$

CAN CALCULATE THESE <u>EXACTLY</u>

USING INTEGRABILITY OF SGM

$$\sigma(\omega) = \frac{2\pi^2 A}{\omega} \sum_n |\langle 0 | j(0,0) | n \rangle|^2 \, \underbrace{\delta(P_n) \, \delta(\omega - E_n)}$$

NEED ONLY FINITE # OF

MATRIX ELEMENTS FOR GIVEN ω

WE OBTAIN AN EXPANSION OF $\sigma(\omega)$ OF THE FORM

$\beta^2 \geq 4\pi$

$$\sigma(\omega) = \underbrace{\underbrace{\underset{\substack{\uparrow \\ S\bar{S}}}{\sigma_2(\omega)}}_{\substack{\text{EXACT FOR} \\ \omega < 4M}} + \underset{\substack{\uparrow \\ S\bar{S}S\bar{S}}}{\sigma_4(\omega)}}_{\substack{\text{EXACT FOR} \\ \omega < 6M}} + \sigma_6(\omega) + \ldots.$$

$\beta^2 \leq 4\pi$

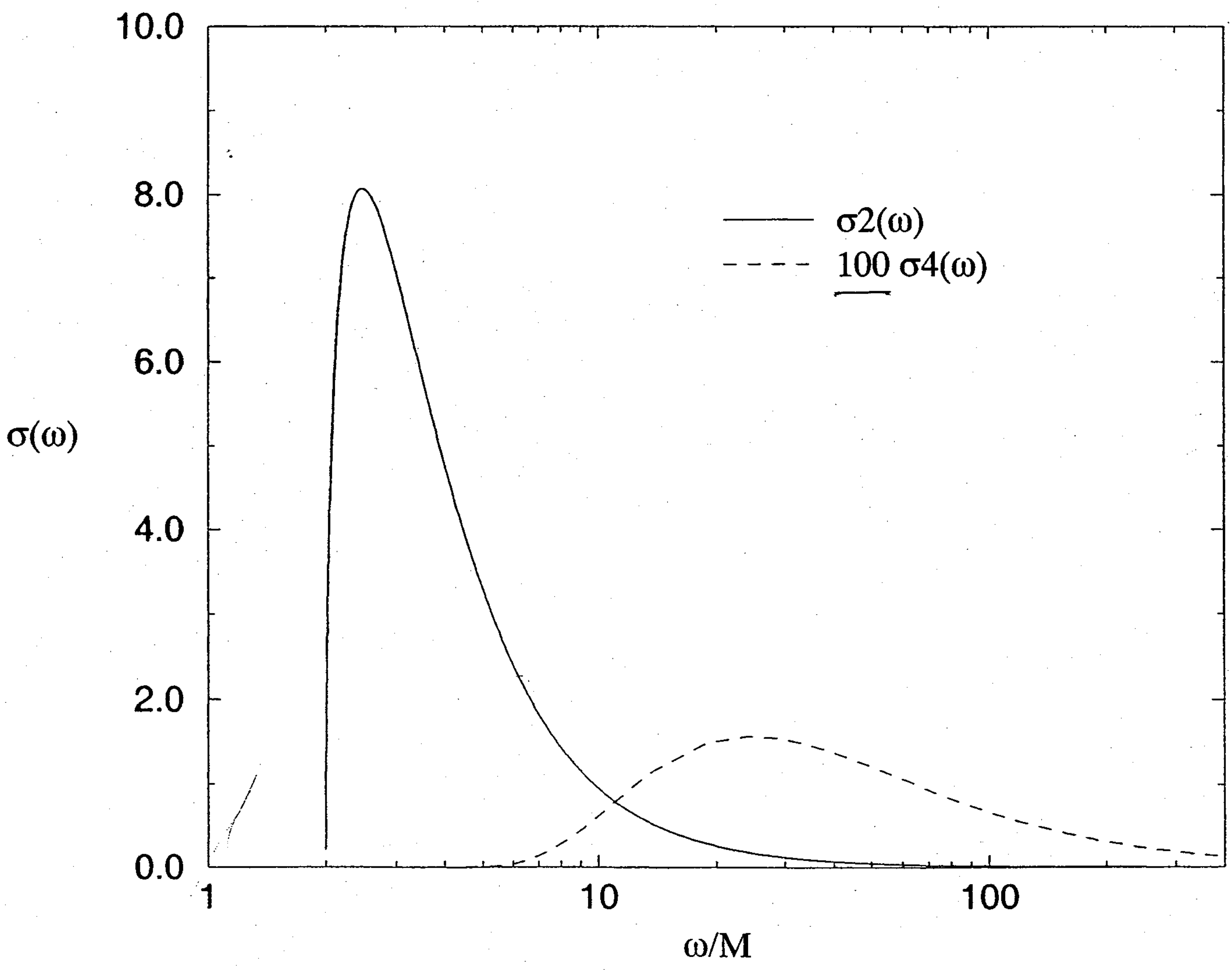
$$\sigma(\omega) = \underset{\substack{\uparrow \\ \text{BREATHER CONTRIB.}}}{\sigma_B(\omega)} + \sigma_{S\bar{S}}(\omega) + \sigma_{B_1 B_2}(\omega) + \ldots.$$

$\longrightarrow$ δ-fns

$K_c = 0.9$

$\longrightarrow$ 4 PART. CONTRIBUTION ALREADY <u>TINY</u> !

$(\omega \lesssim 100\,M)$



CONFORMAL PERTURBATION THEORY [ZAMOLODCHIKOV]

AROUND THE UV FIXED POINT

$$S_0 = \int d^2x \; \frac{1}{2}(\partial_\mu \varphi_c)^2$$

ONE CAN DEVELOP A PERTURBATIVE EXPANSION IN

$$S_1 = -2\mu \int d^2x \; \cos\beta\varphi_c$$

$$\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2) \rangle = \frac{\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, e^{-S_1} \rangle_0}{\langle e^{-S_1} \rangle_0}$$

$$= \frac{\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2) \left\{ 1 + 2\mu \int d^2x \cos\beta\varphi_c + \dots \right\} \rangle_0}{\langle 1 + 2\mu \int d^2x \cos\beta\varphi_c + \dots \rangle_0}$$

$$\Rightarrow \boxed{\sigma(\omega) = A\, 2^{9-4\beta^2} \left(\frac{\pi^2 \beta}{\Gamma(2\beta^2)} \right)^2 \mu^2\, \omega^{4\beta^2 - 5} + \dots}$$

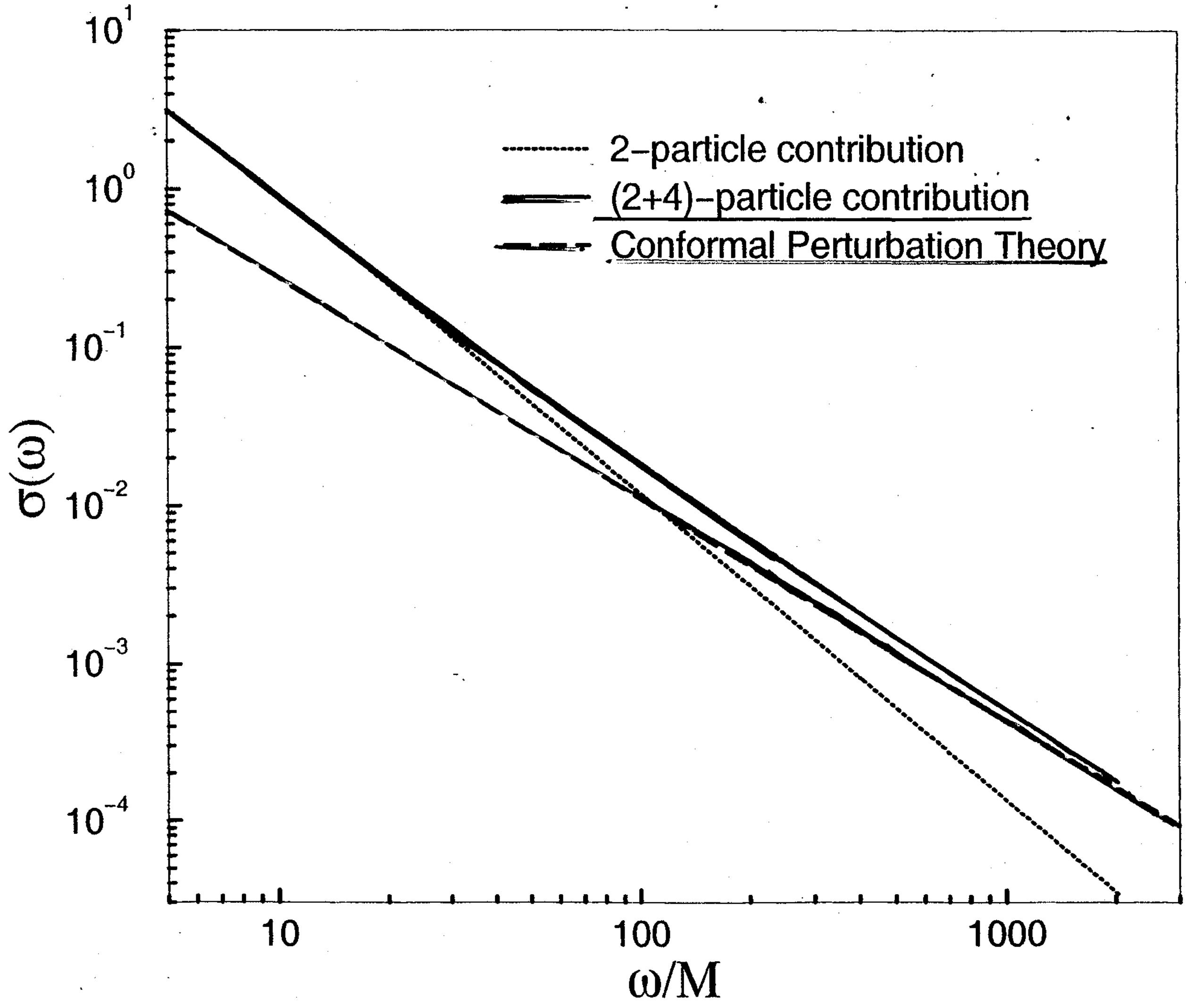
WHAT IS THE RELATION BETWEEN μ AND THE PHYSICAL MASS M ? TBA $\rightarrow$

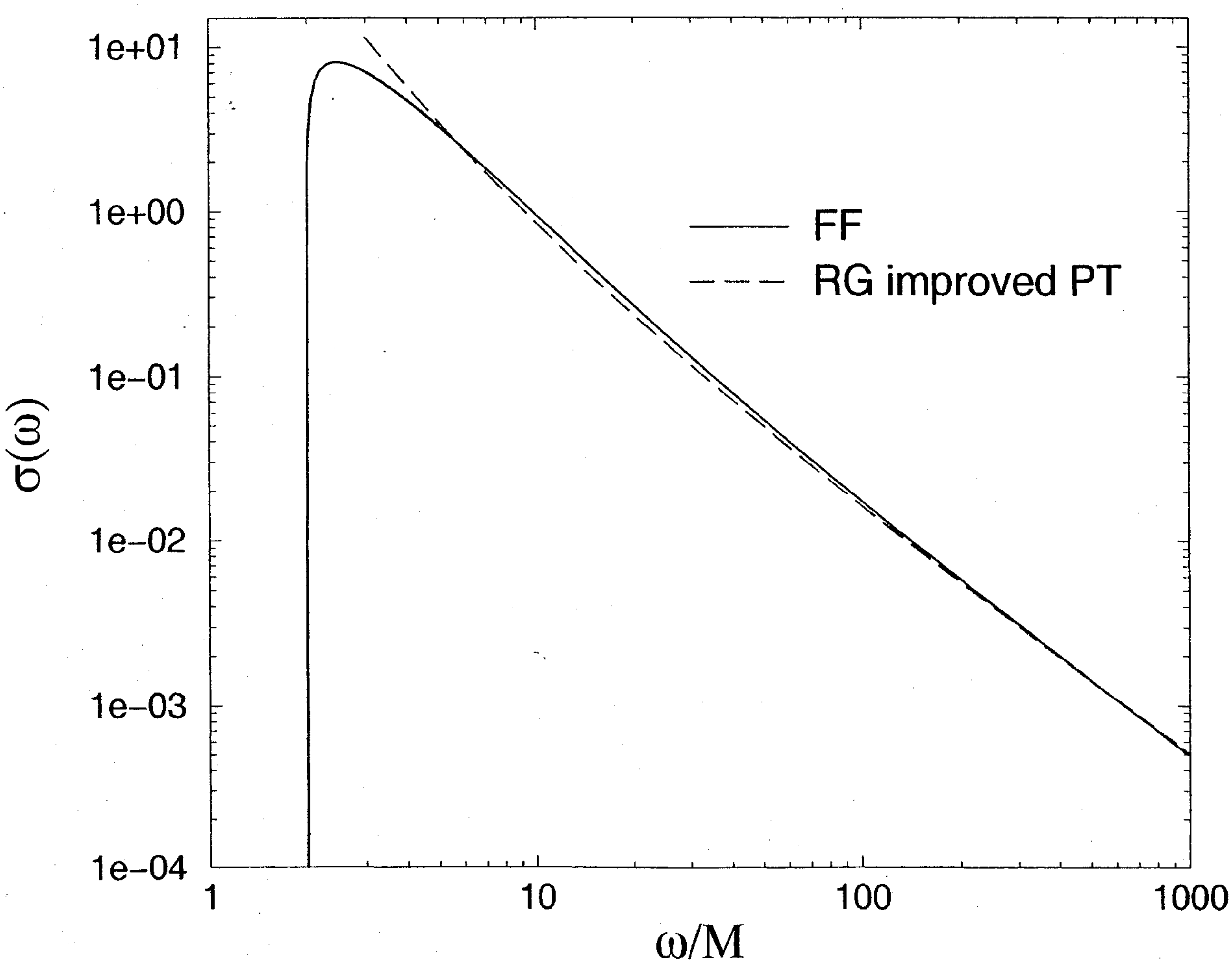
$$\mu = \frac{\Gamma(\beta^2)}{\pi\, \Gamma(1-\beta^2)} \left[\frac{M \sqrt{\pi}\, \Gamma(\frac{1}{2} + \xi/2)}{2\, \Gamma(\xi/2)} \right]^{2-2\beta^2}$$

[AL.B. ZAMOLODCHIKOV '95]

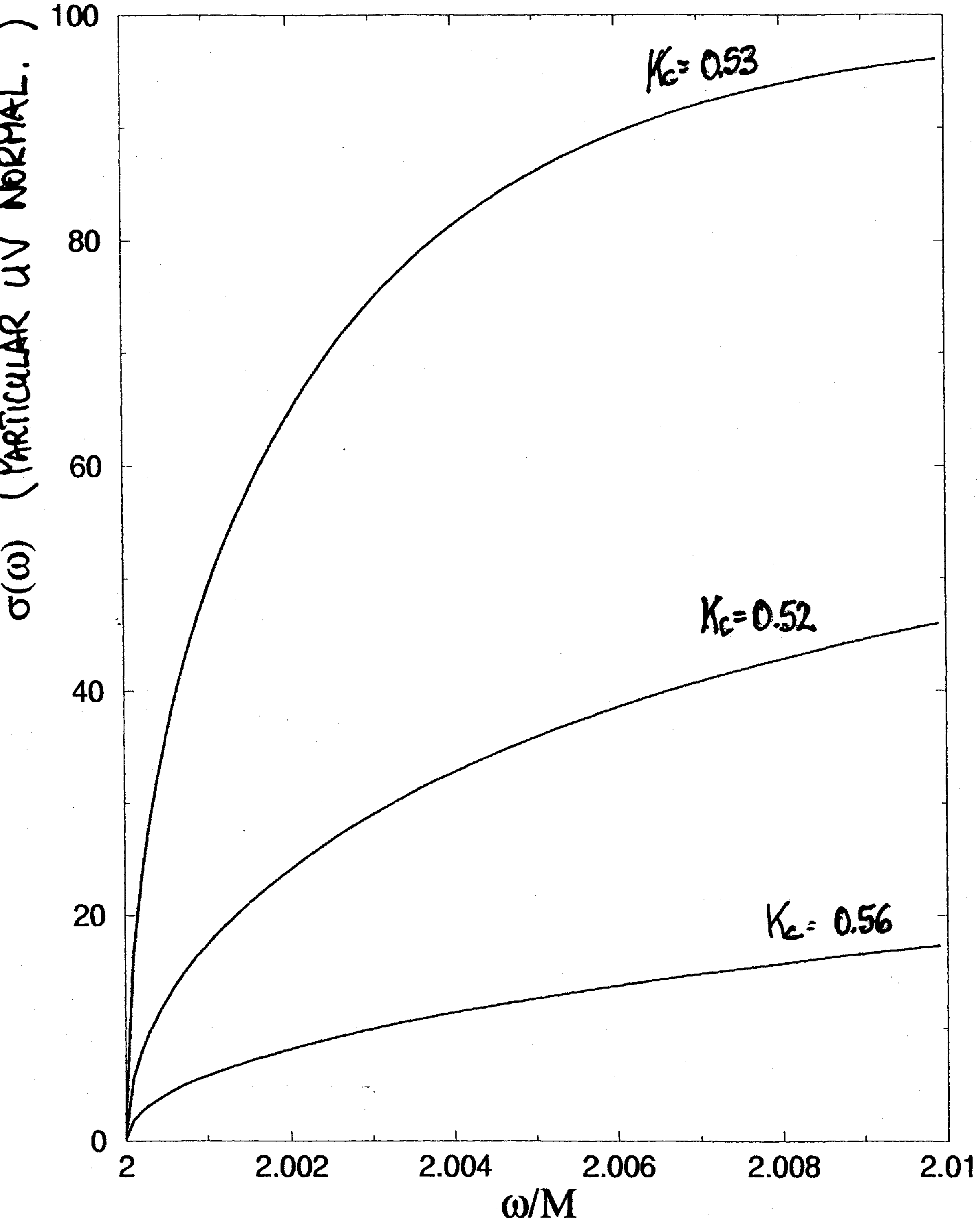
HIGH-ENERGY BEHAVIOUR :

COMPARISON TO (CONFORMAL) PERTURBATION THEORY

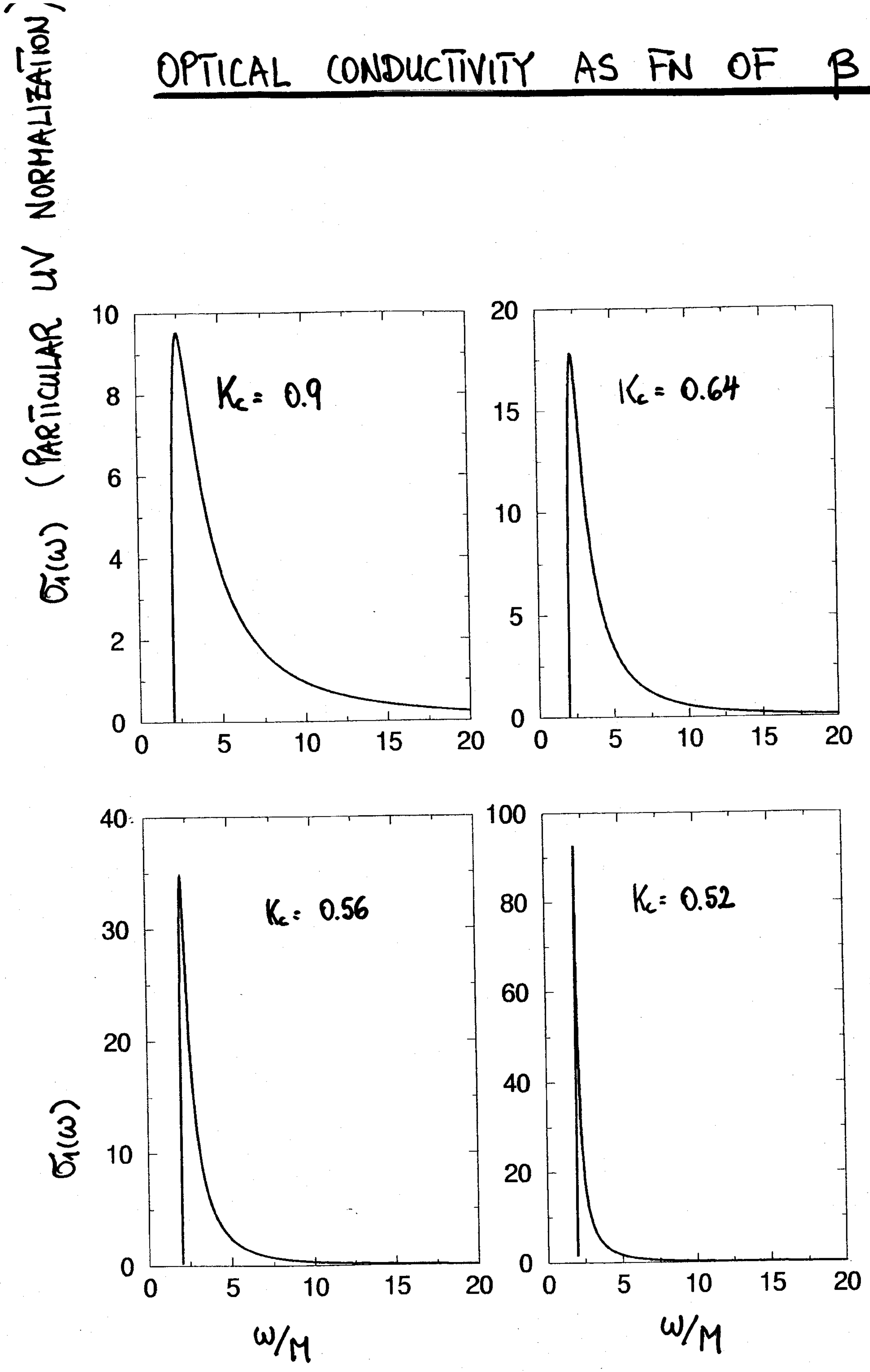


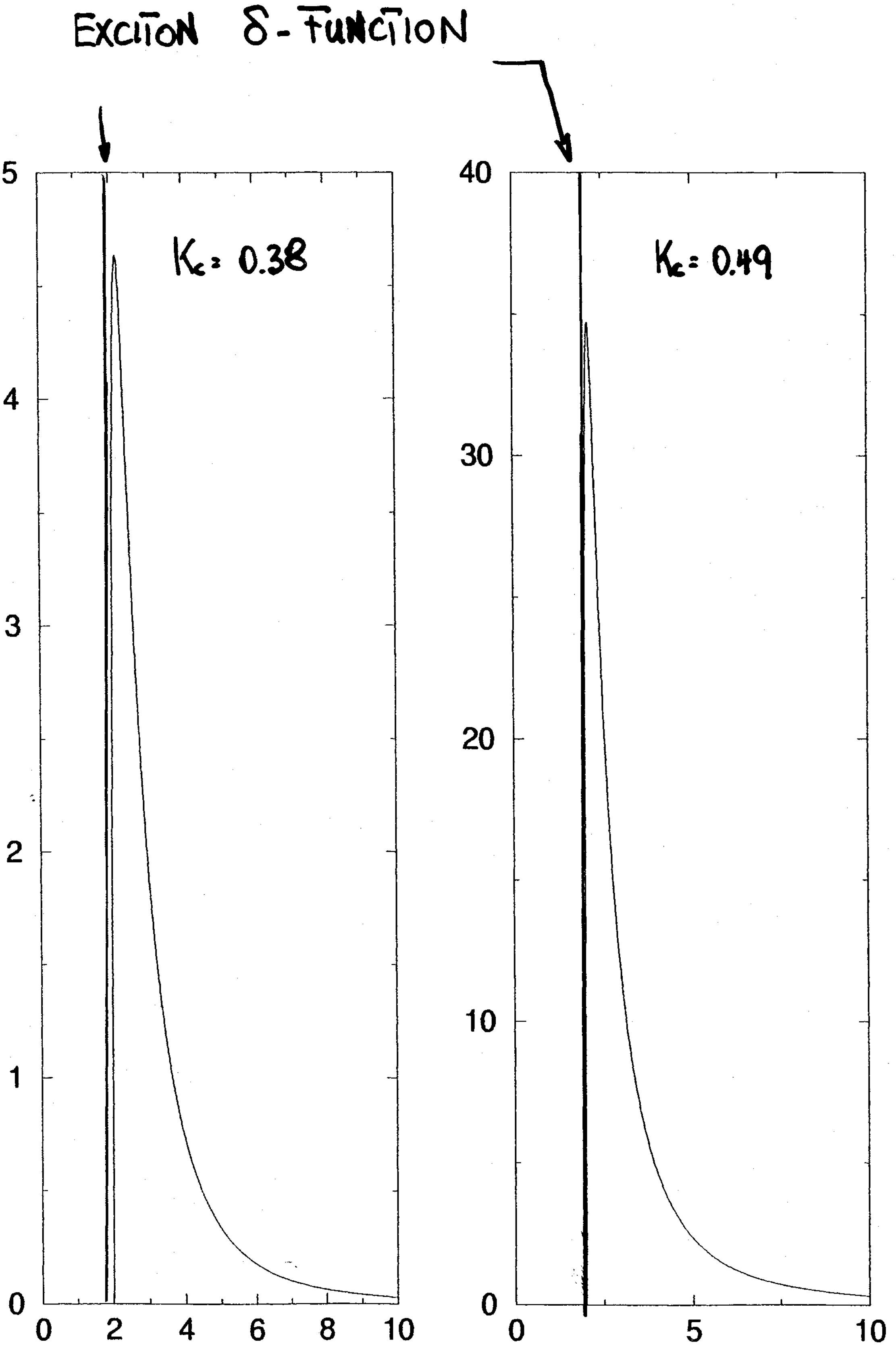

1e+01
1e+00
1e-01
1e-02
1e-03
1e-04
σ(ω)
1
10
100
1000
ω/M
FF
RG improved PT

THRESHOLD BEHAVIOUR



OPTICAL CONDUCTIVITY AS FN OF β





78 % of spectral weight

8 % of spectral weight

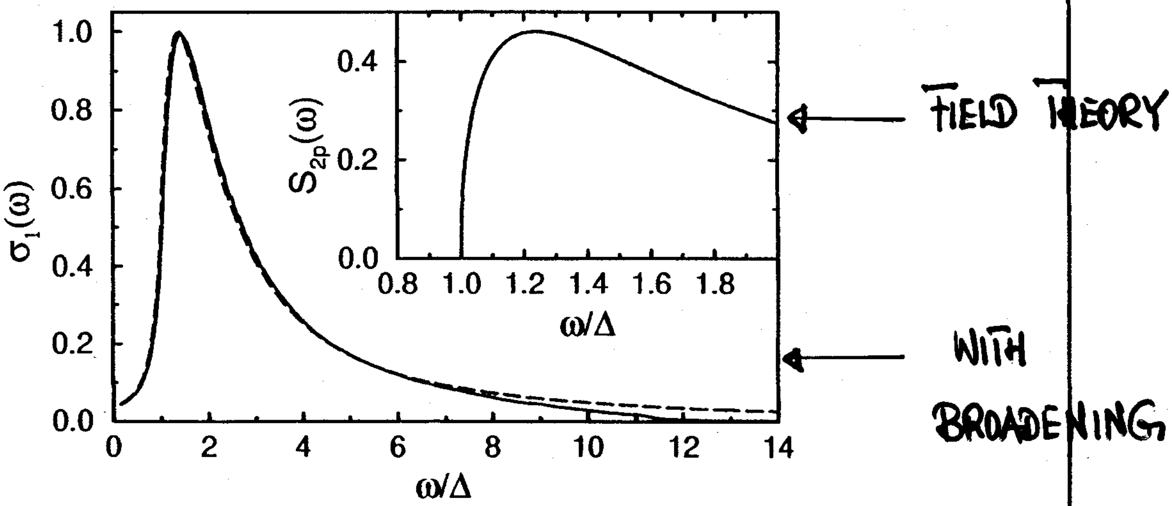
Comparison to Dynamical DMRG

DDMRG calculates

$$\sigma_\eta(\omega) = -\frac{1}{\omega}\mathrm{Im}\ \sum_n \frac{|\langle 0|\hat{J}|n\rangle|^2}{\omega-(E_n-E_0)+i\eta}$$

for specific lattice models on 128 sites.

<u>half-filled Hubbard model $U \simeq 3t$, $\eta = 0.1t$:</u>



Extended Hubbard models: $\longrightarrow$ exciton formation

EXTENDED HUBBARD MODELS @ $\frac{1}{2}$- FILLING

$$H = -t \sum_j c^{+}_{j\sigma} c_{j+1\sigma} + h.c. + U \sum_j n_{j\uparrow} n_{j\downarrow}$$

$$+ V_1 \sum_j n_j n_{j+1} + V_2 \sum_j n_j n_{j+2}$$

$$\Big\downarrow \quad \begin{array}{l} U, V_1, V_2 \ll t \\ V_2 < V_1 < U \end{array}$$

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_C$$

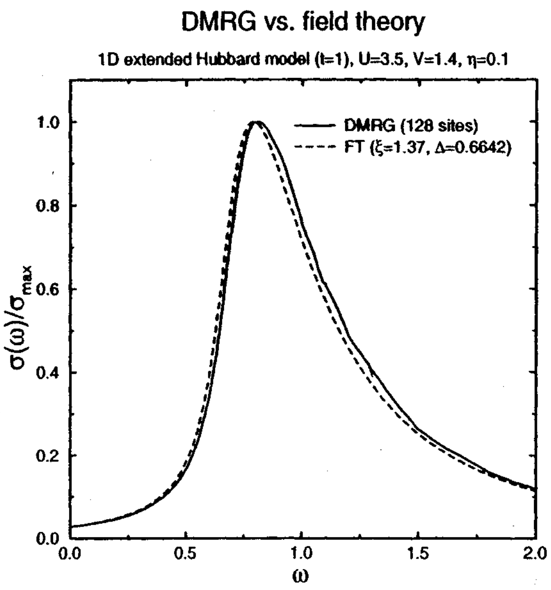
$$\mathcal{L}_S = \frac{1}{2} (\partial_\mu \varphi_s)^2 + \text{MARGINALLY IRRELEVANT}$$

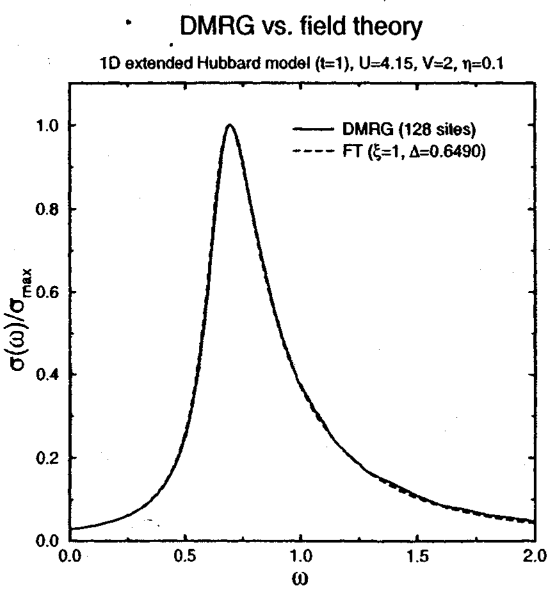
$$\mathcal{L}_C = \frac{1}{2} (\partial_\mu \varphi_c)^2 - \lambda \cos \beta \varphi_c$$

$$\beta(U, V_1, V_2) \le \sqrt{8\pi}$$

BY TUNING U, V_1, V_2 AND DROPPING THE RESTRICTION $U, V_j \ll t$ WE CAN TUNE β FROM $\sqrt{8\pi}$ $[V_j = 0]$ INTO THE ATTRACTIVE REGIME $\beta < \sqrt{4\pi}$

<u>half-filled extended Hubbard model:</u> [E. Jeckelmann (2001)]



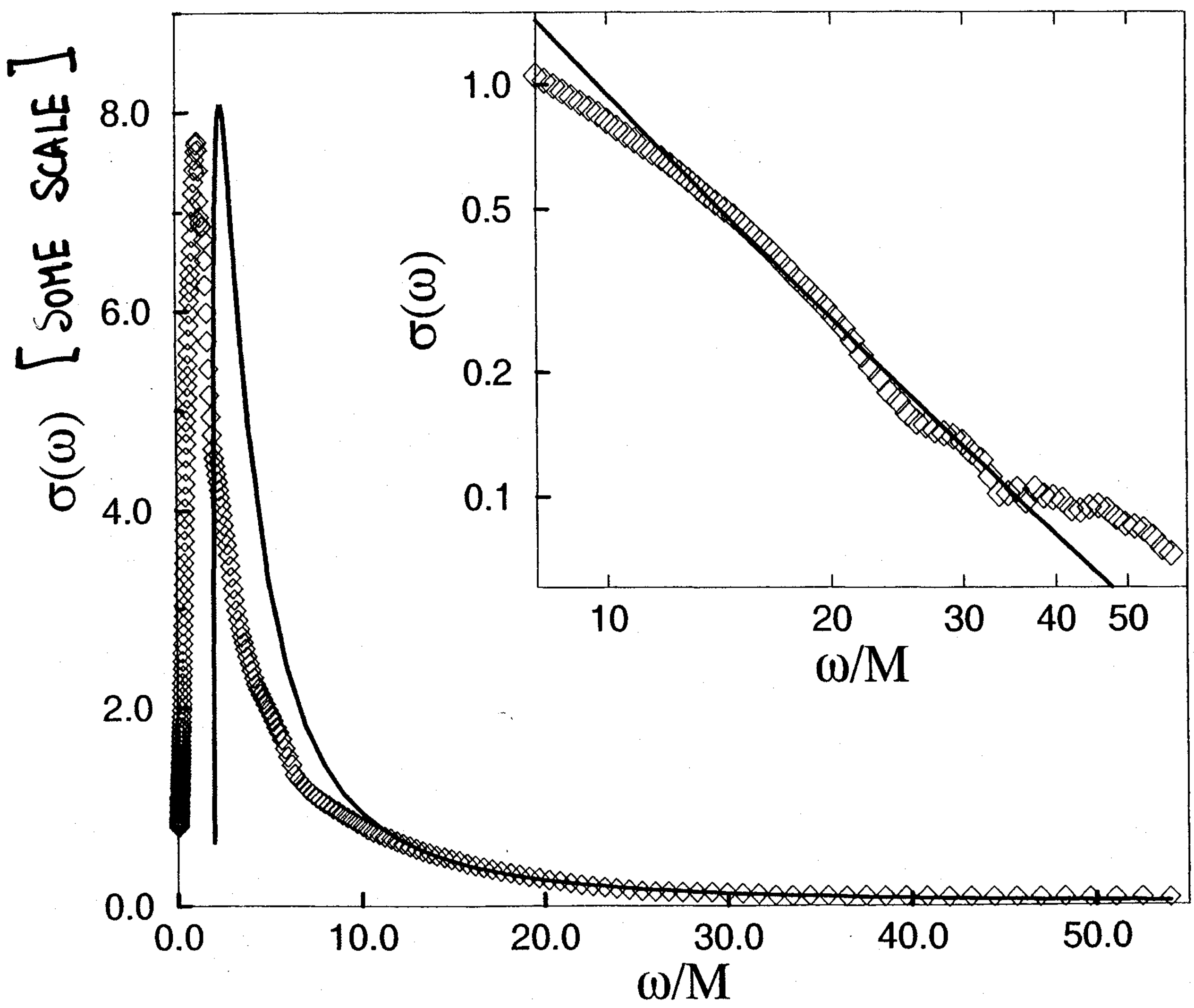


COMPARISON TO $(TMTSF)_2 PF_6$

$\frac{\beta^2}{8\pi} \simeq 0.9$ ($K_c \simeq 0.24$) ($K_c \simeq 0.95$)

[DOUBLE UMKLAPP] [DIMERIZATION]

c.f. T. GIAMARCHI
ET AL



Single-Particle Green's function

1. Half-Filled Case. $(\beta^2 = 8\pi K_c)$

$$c_{j,\sigma} \propto \sqrt{a_0}\,[\exp(ik_F x)\; R_\sigma(x) + \exp(-ik_F x)\; L_\sigma(x)]\ ,$$

$$\begin{aligned} L_\sigma &= \eta_\sigma\ \exp\left(\frac{i}{4}\left[\beta\Phi_c - \frac{8\pi}{\beta}\Theta_c\right]\right)\exp\left(\frac{i}{4}\tilde{\sigma}[\Phi_s - \Theta_s]\right), \\ R_\sigma &= \eta_\sigma\ \exp\left(-\frac{i}{4}\left[\beta\Phi_c + \frac{8\pi}{\beta}\Theta_c\right]\right)\ \exp\left(-\frac{i}{4}\tilde{\sigma}\,[\Phi_s + \Theta_s]\right). \end{aligned}$$

$\Theta_{c,s} = \frac{-i}{v_{c,s}} \int_{-\infty}^{x} dy\, \partial_\tau \phi_{c,s}(\ldots$

$\tilde{\sigma} = \pm\sqrt{8\pi}, \sigma = \uparrow, \downarrow$, η_σ Klein factors $\eta_\sigma^2 = 1$, $\eta_\uparrow\eta_\downarrow = -\eta_\downarrow\eta_\uparrow$.

- spin sector: free boson $\longrightarrow$ easy to get correlation functions
- charge sector: use form factor approach. Complicated operators but formfactors have recently been determined by Lukyanov/Zamolodchikov (2001). Form factor expansion looks like

$$\int \frac{d\theta}{2\pi}\langle 0|\mathcal{O}^\dagger(x,t)|\theta\rangle_{-}\,{}_{-}\langle\theta|\mathcal{O}(0,0)|0\rangle + \ldots$$

where next contribution is from intermediate states with 2 solitons and 1 antisoliton. Take only single-soliton intermediate state into account

$$\langle R_\sigma(x,\tau)R^\dagger_\sigma(0,0)\rangle \simeq \frac{Z_0(\beta)}{2\pi}\frac{\exp[-m\sqrt{\tau^2 + x^2 v_c^{-2}}]}{\sqrt{(v_s\tau - ix)(v_c\tau - ix)}}\ .$$

Fourier transform and analytically continue$\rightarrow$ retarded Green's function $(\alpha = v_s/v_c)$

$$\boxed{\begin{aligned} G^{(R)}_{RR}(\omega,q) &= -Z_0(\beta)\sqrt{\frac{2}{1+\alpha}}\frac{\omega + v_c q}{\sqrt{m^2 + v_c^2 q^2 - \omega^2}} \\ &\times\left[\left(m + \sqrt{m^2 + v_c^2q^2 - \omega^2}\right)^2 - \frac{1-\alpha}{1+\alpha}(\omega + v_c q)^2\right]^{-\frac{1}{2}} \end{aligned}}$$

HOW GOOD AN APPROXIMATION IS THIS ?

(1) **LOW ENERGIES / LARGE DISTANCES :**

→ GOOD BY DEFINITION [CORRECTIONS $\propto$ e^{-3mr}]

(2) **"HIGH" ENERGIES / SHORT DISTANCES :**

$$\langle R_\sigma(x,\tau)\, R_\sigma^+(0,0)\rangle \propto \frac{1}{\sqrt{v_s\tau - ix}}\ \frac{1}{(v_c\tau - ix)^{\gamma}} \qquad \text{(LL)}$$

$$\gamma = \frac{K_c}{4} + \frac{1}{4K_c}\,.$$

$K_c = 1$ [SU(2) SYMM. IN CHARGE SECTOR "η- PAIRING" IN HUBBARD]

⇒ FF RESULT GETS SHORT DISTANCE ASYMPTOTICS RIGHT !

RG: $$\langle R_\sigma(x,\tau)\, R_\sigma^+(0,0)\rangle = \frac{Z(r)}{2\pi(v\tau - ix)} \quad \text{for } K_c = 1,\ v_s = v_c = v$$

$$Z(r) \simeq 1 + \frac{3}{8\ln mr} + \dots \qquad (r \to 0)$$

FF : $$\langle R_\sigma(x,\tau)\, R_\sigma^+(0,0)\rangle = \frac{Z_0}{2\pi}\,\frac{e^{-mr}}{v\tau - ix}$$

$$Z_0 = 0.9218 \qquad (\text{LZ '01})$$

Spectral Function

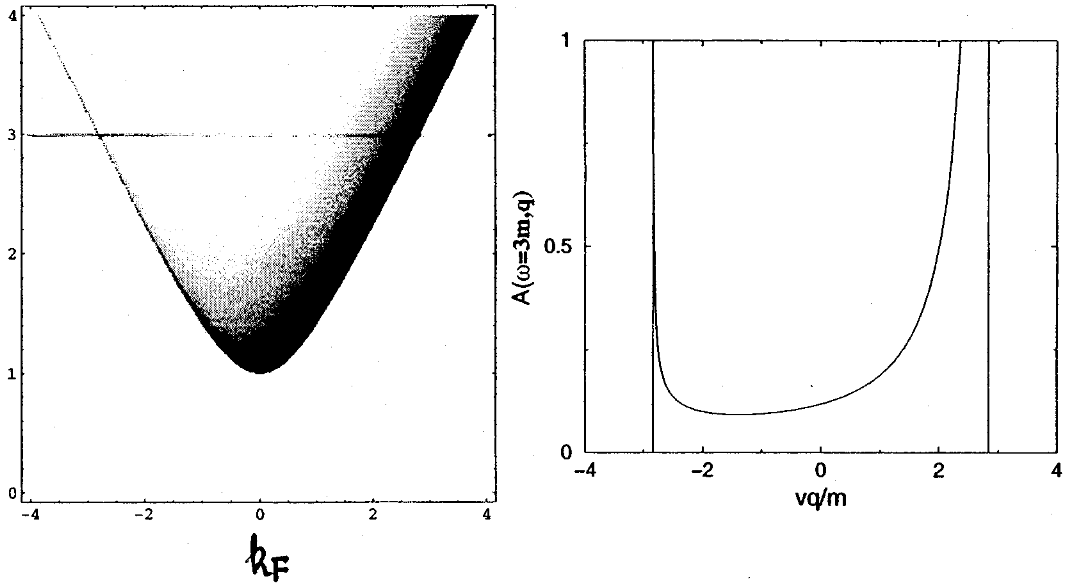


Figure 1: (a) Density plot of the spectral function $A_{RR}(\omega, q)$ as a function of ω and vq/m for $v_s = v_c = v$. (b) Constant energy ($\omega = 3m$) scan of the spectral function for $\alpha = 1$.

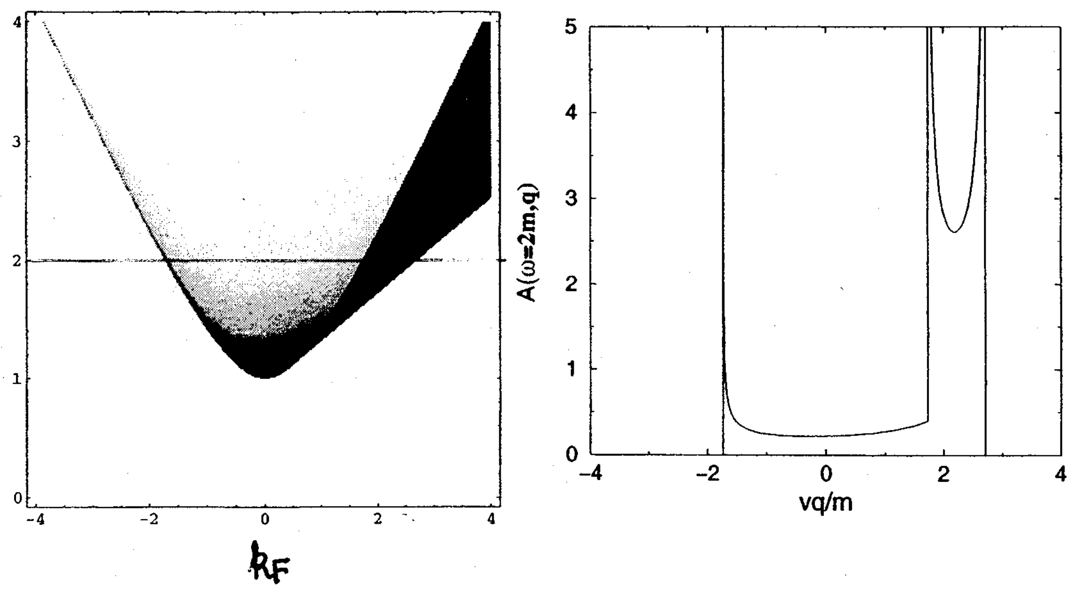
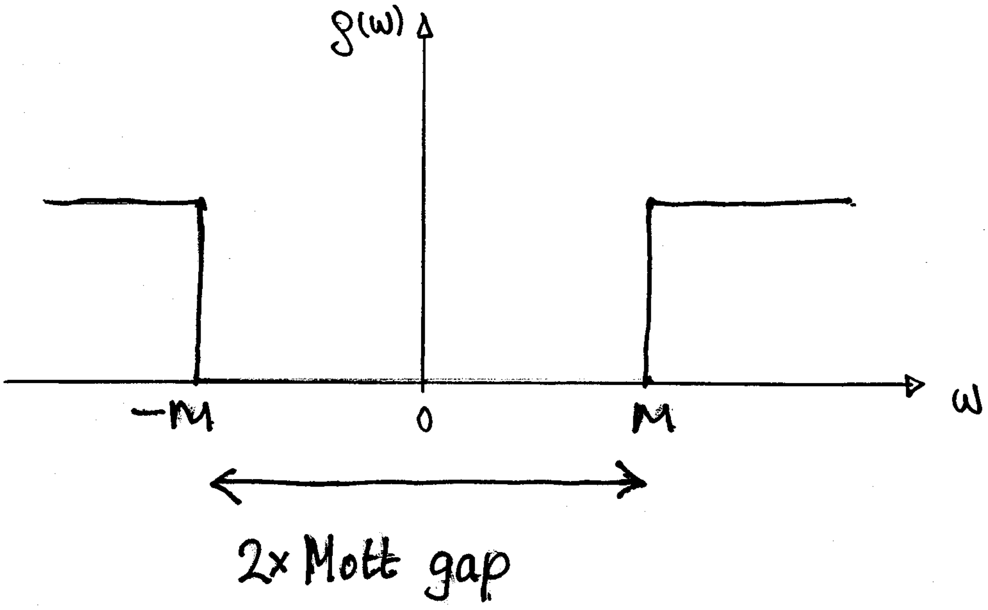


Figure 2: (a) Density plot of the spectral function $A_{RR}(\omega, q)$ as a function of ω and $v_c q/m$ for $v_s/v_c = 0.4$. (b) Constant energy ($\omega = 3m$) scan of the spectral function for $\alpha = 0.4$.

DENSITY OF STATES

$$\rho(\omega) = \int dq \quad A(\omega, q)$$

$$= \frac{Z_0}{\pi v_s v_c} \quad \Theta(|\omega| - m)$$



Single-Particle Green's function

2. Quarter-Filled Case.($\beta^2 = 32\pi K_c$)

$$c_{j,\sigma} \propto \sqrt{a_0}\,[\exp(ik_F x)\ R_\sigma(x) + \exp(-ik_F x)\ L_\sigma(x)]\ ,$$

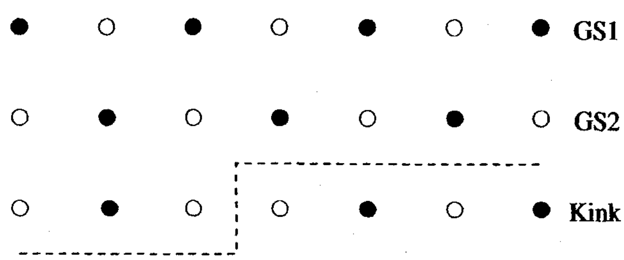
$$\begin{aligned} L_\sigma &= \eta_\sigma\ \exp\left(\frac{i}{4}\left[\frac{\beta}{2}\Phi_c - \frac{16\pi}{\beta}\Theta_c\right]\right)\exp\left(\frac{i}{4}\tilde{\sigma}[\Phi_s - \Theta_s]\right), \\ R_\sigma &= \eta_\sigma\ \exp\left(-\frac{i}{4}\left[\frac{\beta}{2}\Phi_c + \frac{16\pi}{\beta}\Theta_c\right]\right)\ \exp\left(-\frac{i}{4}\tilde{\sigma}\,[\Phi_s + \Theta_s]\right). \end{aligned}$$

$\tilde{\sigma} = \pm\sqrt{8\pi}\ \ \sigma = \uparrow, \downarrow$, η_σ Klein factors $\eta_\sigma^2 = 1$, $\eta_\uparrow\eta_\downarrow = -\eta_\downarrow\eta_\uparrow$.

- spin sector: free boson $\longrightarrow$ easy to get correlation functions
- charge sector: use form factor approach [Lukyanov/Zamolodchikov (2001)]. Form factor expansion looks like

$$\int \frac{d\theta_1}{2\pi}\frac{d\theta_2}{2\pi}\langle 0|\mathcal{O}^\dagger(x,t)|\theta_1\theta_2\rangle_{--}\ {}_{--}\langle\theta_1\theta_2|\mathcal{O}(0,0)|0\rangle + \ldots$$

i.e. lowest contribution comes from **two** solitons. Can understand this intuitively by considering $U - V$ extended Hubbard model with "large" V:



The kink has charge 1/2 $\longrightarrow$ electron operator must couple to **two** kinks.

use the explicit expressions for the form factors⟶

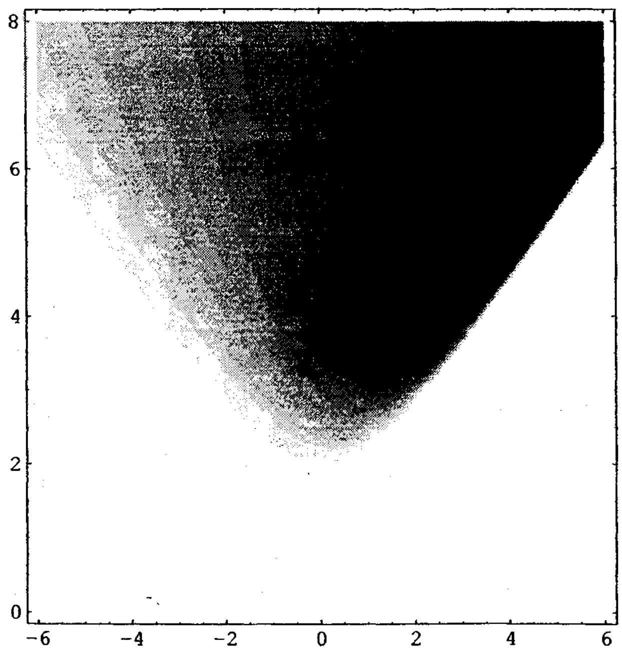
$$G^{(\sigma)}_{RR}(\omega,q) = Z_2(\beta/8)\sqrt{\frac{2v_c}{1+\alpha}}\int_{-\infty}^{\infty}\frac{d\theta}{\sqrt{2\pi}}\frac{|G(2\theta)|^2}{\sqrt{c(\theta)}}$$

$$\times\frac{\omega+v_c q}{\sqrt{c^2(\theta)-s^2}}\left[\left(c(\theta)+\sqrt{c^2(\theta)-s^2}\right)^2 - \frac{1-\alpha}{1+\alpha}(\omega+v_c q)^2\right]^{-\frac{1}{2}},$$

$s^2=\omega^2-v_c^2q^2$, $c(\theta)=2M\cosh\theta$, $\alpha=v_s/v_c$,

$$\begin{aligned} G(\theta) &= iC_1\sinh\theta/2 \\ &\quad\times\exp\left(\int_0^\infty\frac{dt}{t}\frac{\sinh^2(t[1-i\theta/\pi])\sinh(t[\xi-1])}{\sinh 2t\ \sinh\xi t\ \cosh t}\right), \\ C_1 &= \exp\left(-\int_0^\infty\frac{dt}{t}\frac{\sinh^2(t/2)\ \sinh(t[\xi-1])}{\sinh 2t\ \sinh\xi t\ \cosh t}\right), \end{aligned}$$

$\xi=\beta^2/(1-\beta^2)$

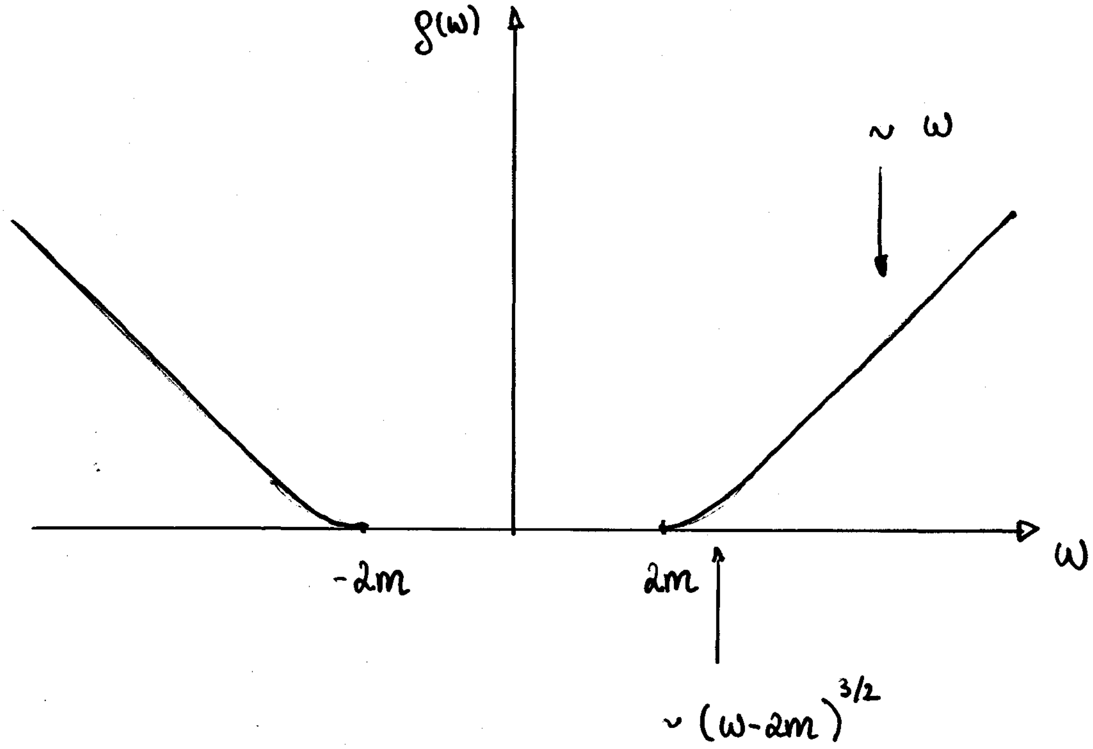


No singularities !!

DENSITY OF STATES [$\frac{1}{4}$ FILLING]

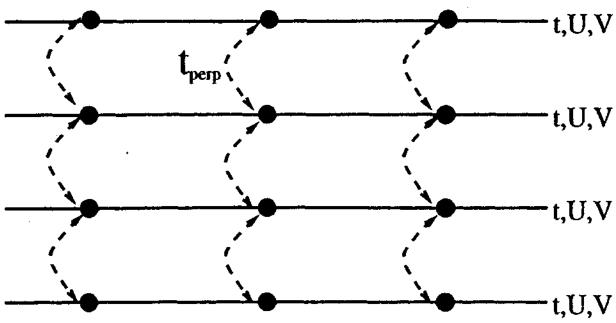
$$\rho(\omega) = \text{const} \int_0^{\operatorname{arccosh}\frac{\omega}{2m}} d\theta \quad \frac{|G(2\theta)|^2}{\sqrt{\cosh\theta}}$$

$$G(\theta) = \text{const} \times \sinh\frac{\theta}{2} \; e^{\int_0^\infty \frac{dt}{t} \frac{\sinh^2 t\left(1-\frac{i\theta}{\pi}\right)}{\sinh 2t} \frac{\sinh t(\xi-1)}{\sinh \xi t \, \cosh t}}$$



Weakly coupled Mott insulators

HALF-FILLED CASE

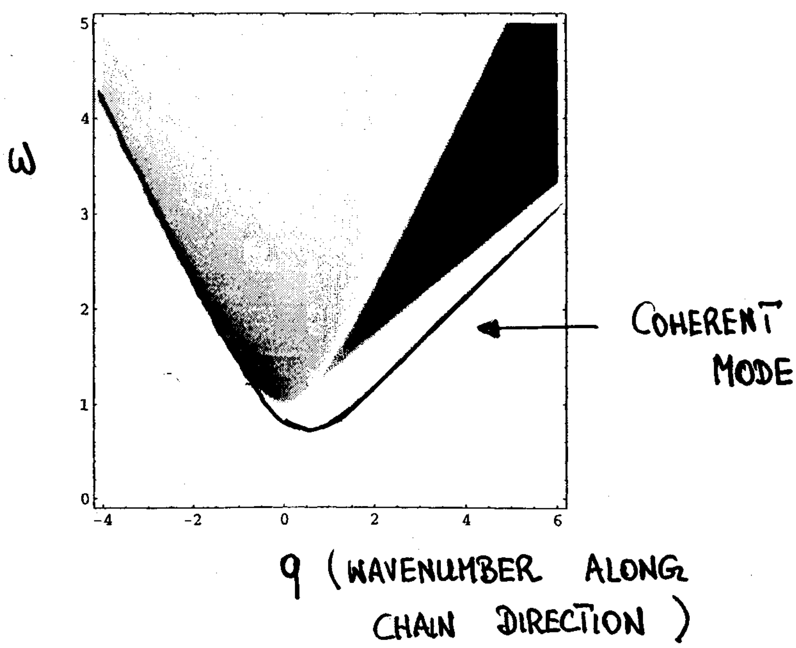


Take the interchain hopping into account in Random Phase Approximation

$$\begin{aligned} G_{3\mathrm{D}}(\omega, q, \vec{k}) &= \frac{G_{1D}(\omega, q)}{1 - t_\perp(\vec{k})\ G_{1D}(\omega, q)} \\ t_\perp(\vec{k}) &= t_\perp \sum_{\vec{a}} \exp(i\vec{k}\cdot\vec{a}) \end{aligned}$$

$\longrightarrow$ can have a pole $\equiv$ collective mode (with quantum numbers of an electron)

Spectral Function:

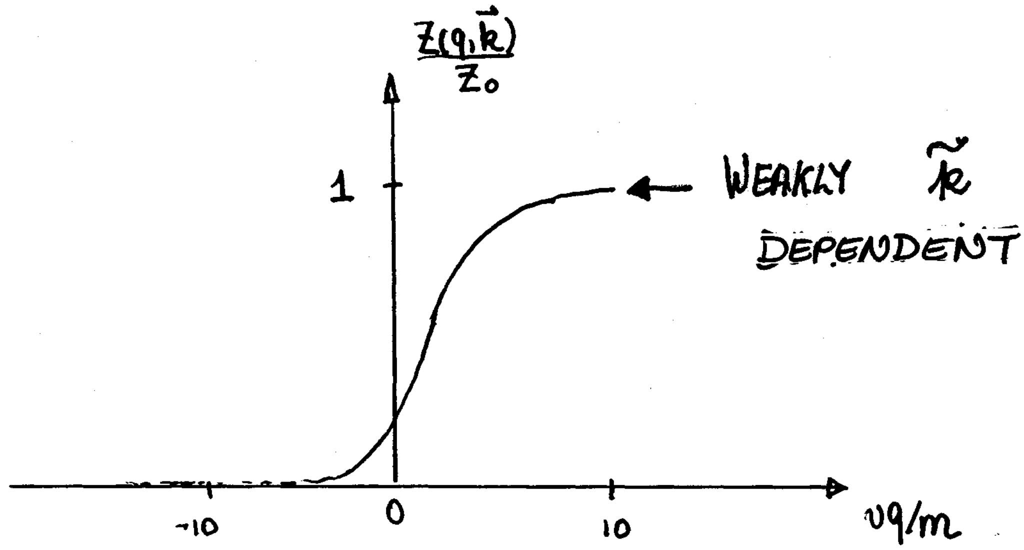


$\frac{v_s}{v_c} = 0.4$

IN THE VICINITY OF THE COHERENT MODE

WE HAVE

$$G^{(R)}_{RR}(\omega, q, \vec{k}) \simeq \frac{Z(q,\vec{k})}{\omega - \varepsilon(q,\vec{k})}$$



$Z(q,\vec{k})$ VERY SMALL WITHIN THE

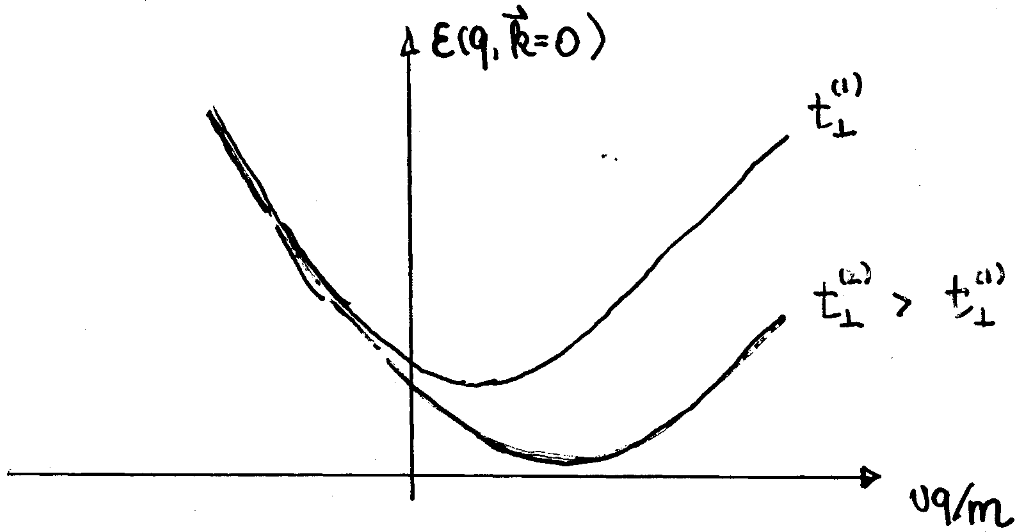
NONINTERACTING FERMI SURFACE

FORMATION OF A FERMI SURFACE

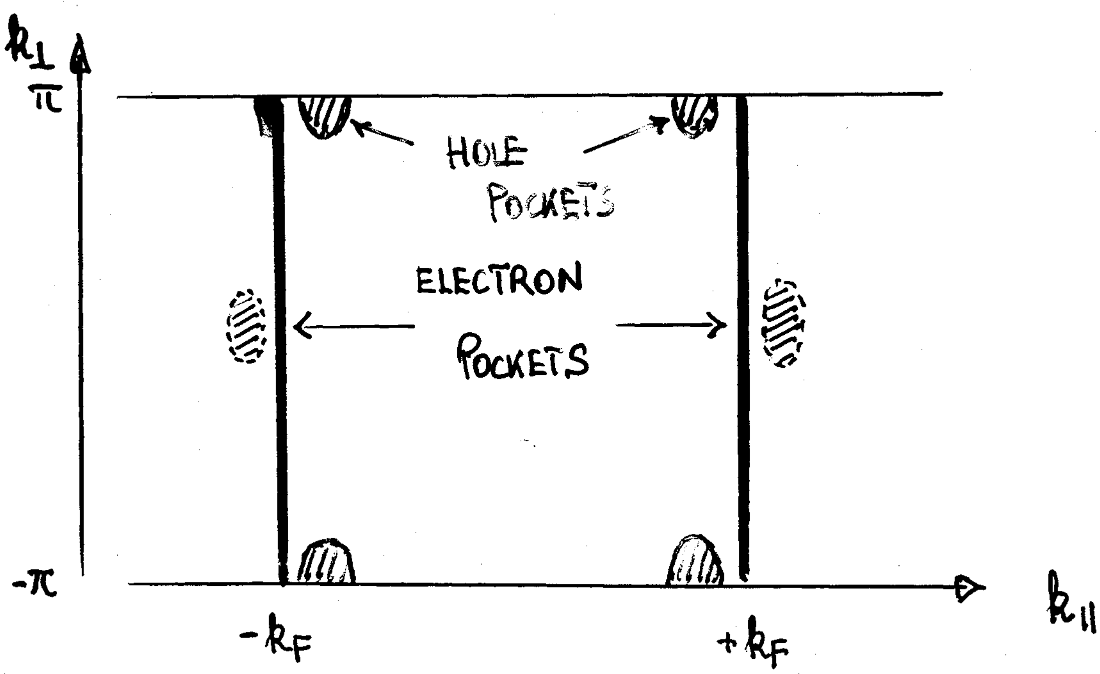
INCREASE TRANSVERSE HOPPING AMPLITUDE $t_\perp$

$\Longrightarrow$ COHERENT MODE EVENTUALLY BECOMES SOFT

$\longrightarrow$ SMALL ELECTRON AND HOLE POCKETS FORM !



EXAMPLE: D=2 SQUARE LATTICE



Luttinger's Theorem

\# of particles $\propto$ volume of mtm space included in the surface defined by the singularities of $\ln[G(\omega=0,\vec{p})]$.

$\rightarrow$ Poles or <u>zeroes</u> of $\ln G$.

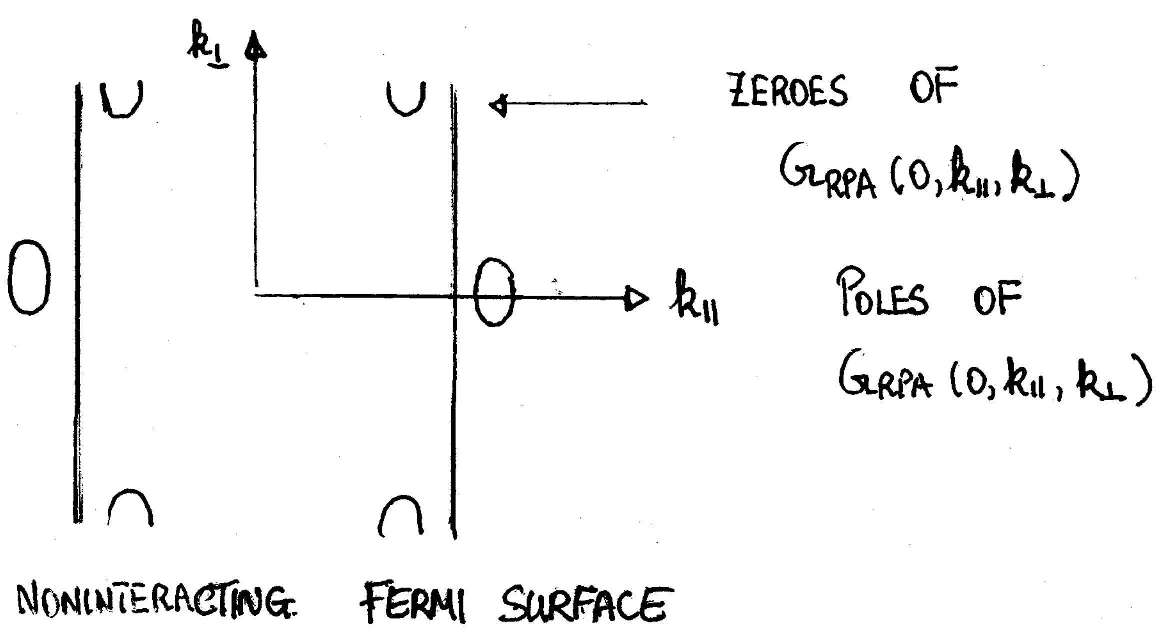
(i) **1D Mott insulator**

"$U=0$" $\Rightarrow$ Poles at $\pm k_F$

"$U>0$" $\Rightarrow$ $G_{\alpha\alpha}(0,q)$ has **zeroes** at $q=0$ (i.e. $\pm k_F$)

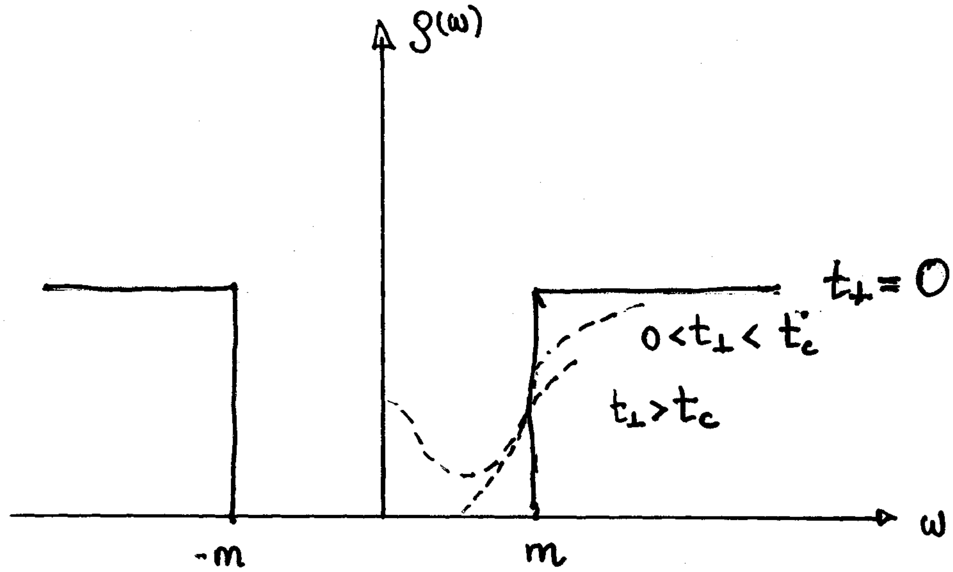
$\Rightarrow$ Luttinger's thm holds.

(ii) **Quasi-2D Mott insulator**



DENSITY OF STATES

(i) $D=2$, SQUARE LATTICE



→ GAP GETS FILLED IN

→ PEAK AT $\omega = 0$ DEVELOPS

(ii) $D=3$, CUBIC LATTICE

NO PEAK AT $\omega = 0$ DEVELOPS FOR $t_\perp > t_c$