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SUMMER SCHOOL
on
LOW-DIMENSIONAL QUANTUM SYSTEMS:
Theory and Experiment
(16 - 27 JULY 2001)

PLUS

PRE-TUTORIAL SESSIONS
(11 - 13 JULY 2001)

BILAYER QUANTUM HALL PSEUDO-FERROMAGNETS

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Bilayer Quantum Hall Pseudo-ferromagnets

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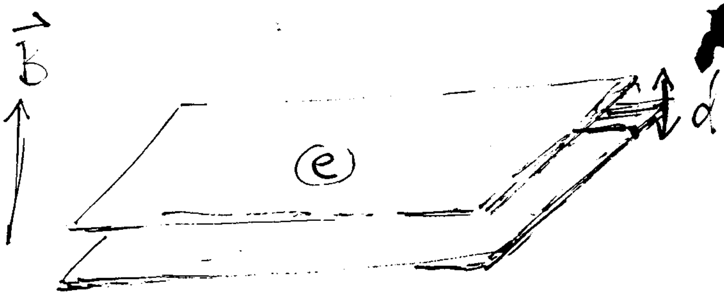

Korea Institute for Advanced Study
Seoul
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Bilayer Quantum Hall Pseudo-ferromagnets

Leo Radzihovsky
University of Colorado at Boulder

work with: Leon Balents

PRL **86**, 1825 (2001)

discussions with: S Girvin and A MacDonald

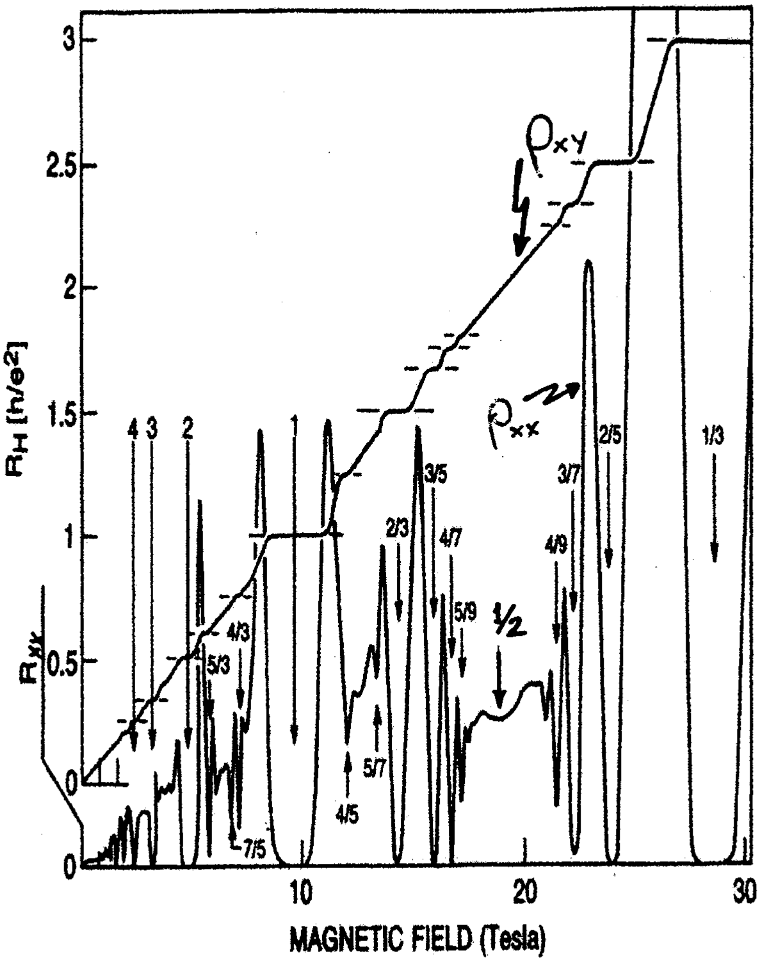
supported by: NSF, Packard, Sloan Foundations

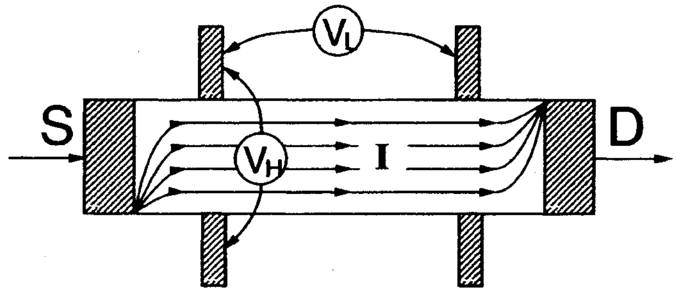
Outline

- **Introduction to double-layer quantum Hall effect**
- **Eisenstein experiments**
- **Bilayer QH as ideal Stoner ferromagnet**
- **Broken symmetries**
- **Theory of interlayer phase coherence and tunneling**
    - Model
    - Predictions
    - Calculational highlights
- **Open questions and conclusions**

Single-layer QHE

- **Experiment** D.Tsui, '90





$$\rho_{xy} = \frac{V_H}{I} \quad , \quad \rho_{xx} = \frac{V_L}{I}$$

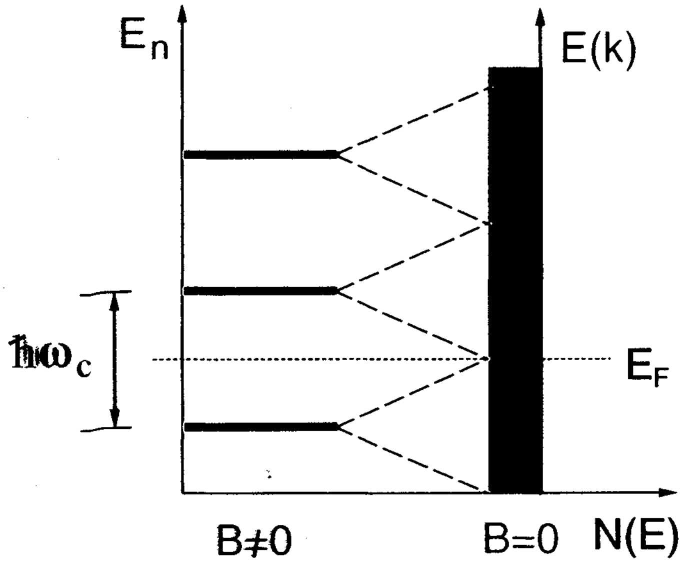
- **Phenomenology of the QHE**

  - $\sigma_{xx} = \rho_{xx} = 0$, dissipationless flow (cf. insulator and superconductor)
  - $\sigma_{xy} = \nu e^2/h$, "quantized", with $\nu \in$ integers (IQHE) and simple fractions (FQHE) (cf. classical Hall effect: $\sigma_{xy}^{(class.)} = \frac{nec}{B} = \frac{n}{n_B}\frac{e^2}{h}$)
  - gapped state

Hints at the QHE

- **IQHE**: 2d *noninteracting* electron under strong B-field

$$\frac{\hbar^2}{2m}\left(\vec{\nabla} - i\frac{2\pi}{\phi_0}\vec{A}\right)^2 \psi_{n,k} = E_n \psi_{n,k}$$



- Landau levels: $E_n = \hbar\omega_c(n + 1/2)$
- Macroscopic degeneracy: $N_B = BL^2/\phi_0$
- Spectrum suggests dissipationless flow when LL are full and E_F in a gap.

- **FQHE**: interactions are essential

  - Laughlin's state: $\psi = \prod_{i<j}^N (z_i - z_j)^{1/\nu} e^{-\frac{1}{4}\sum_i^N |z|^2}$
  - (Jain) Composite fermions: FQHE $\rightarrow$ IQHE 
  - (Girvin + MacDonald) Composite bosons: FQHE $\rightarrow$ superfluid 

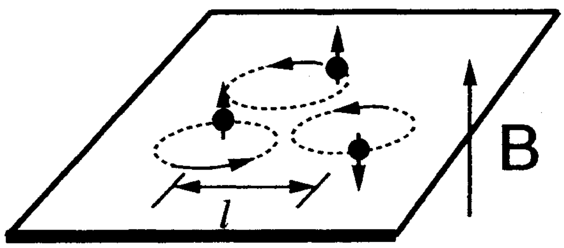
What about electron spin?

unimportant when $E_{Zeeman} = g\hbar\omega_c \gg \Delta_{QH}$

but can be very small! in semiconductors

Multi-component QH States

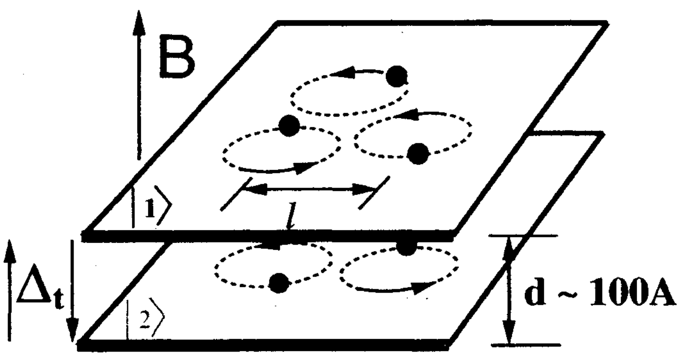
- **Spin** degeneracy in single-layer, when $g \to 0$ (Sondhi, et al. '93)



$$E_{Zeeman} = \mu B = g\hbar\omega_c \to 0$$

$\nu = 1$ QH
Ferromagnet
driven by
Coulomb interactions
"$\nu = 1$ is a fraction too"

- **Layer-index pseudo-spin** degeneracy in double-layer, when $\Delta_{tunneling} \to 0$ (Fertig '89, MacDonald, et al.'90, Wen and Zee '92, Ezawa and Iwazaki '92)



$$\ell = \sqrt{\frac{\phi_0}{2\pi B}}$$

$$\left.\begin{array}{l} |1\rangle \Rightarrow |\uparrow\rangle \\ |2\rangle \Rightarrow |\downarrow\rangle \end{array}\right\}\ \text{pseudo-spin}$$

$\nu = 1$ QH
pseudo-ferromagnet

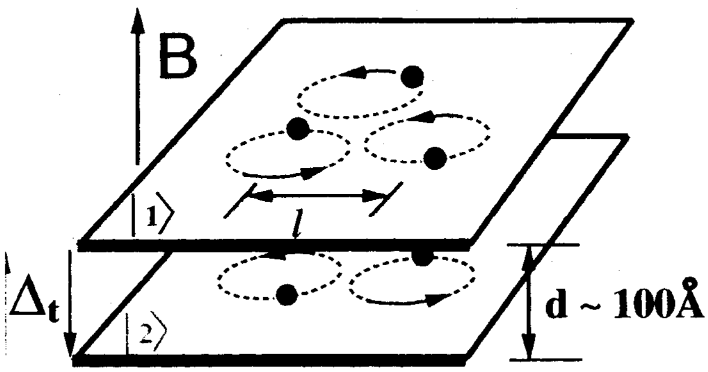
QH ground state as a special case of Halperin's lmn *state:*

$$\psi_{lmn} = \prod(z_i - z_j)^l \prod(w_i - w_j)^m \prod(z_i - w_j)^n$$

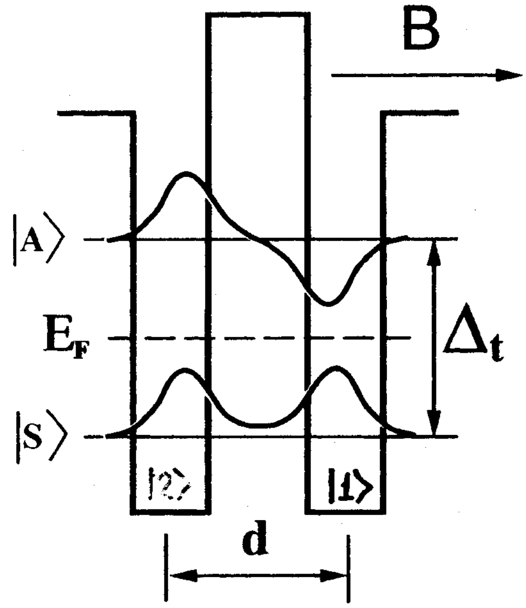
with $l = m = n$, i.e., the mmm QH state

Noninteracting $\nu = 1$ Bilayer

$\nu = \nu_1 + \nu_2 = 1, \qquad H_{tunnel} = -\Delta_t(c_1^\dagger c_2 + c_2^\dagger c_1)$



$\ell = \sqrt{\frac{\phi_0}{2\pi B}}$



- Single-particle quantum Hall gap $= \Delta_t$
- Valid when $\Delta_t \gg E_{Coulomb} \sim \frac{e^2}{\epsilon d}$

Can the QH state survive in the limit
$\Delta_t \to 0 \ll E_{Coulomb}$?

YES!

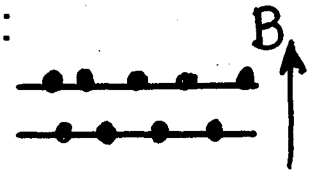
- $\psi_{111} = \prod(z_i - z_j)\prod(w_i - w_j)\prod(z_i - w_j)$ Halperin's 111 state
- exhibits a bulk *massless* $SU(2)$ mode related to $N_1 - N_2$ (Wen and Zee '92, Ezawa and Iwazaki '92)
- cf. single layer with spin (Sondhi, et al. '93) $|\downarrow\rangle \Rightarrow |A\rangle$
  $\nu = 1$, $E_{Zeeman} = g\hbar\omega_c \to 0$ $|\uparrow\rangle \Rightarrow |S\rangle$

Early Experimental Evidence

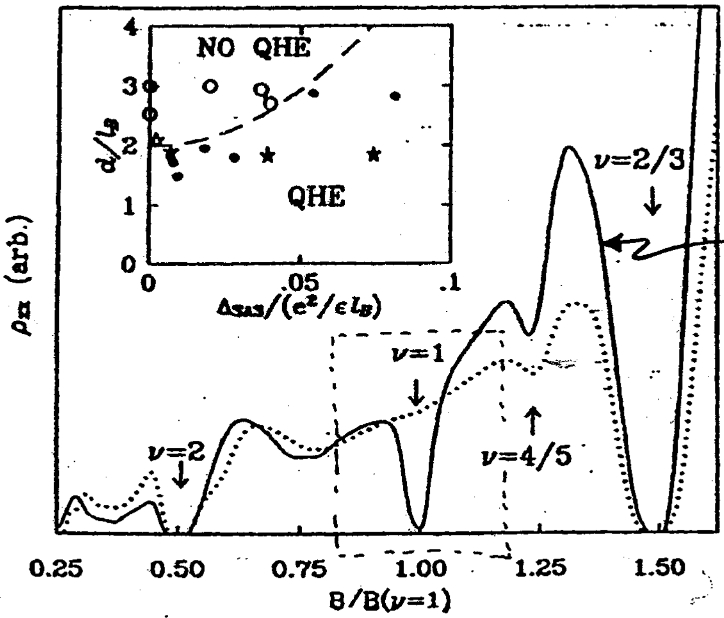
A. MacDonald, et al.
PRL 65, 775 (1990)

Murphy, et.al., PRL
72, 728, '94

- **Vanishing of ρ_{xx} at $\nu = 1$:**



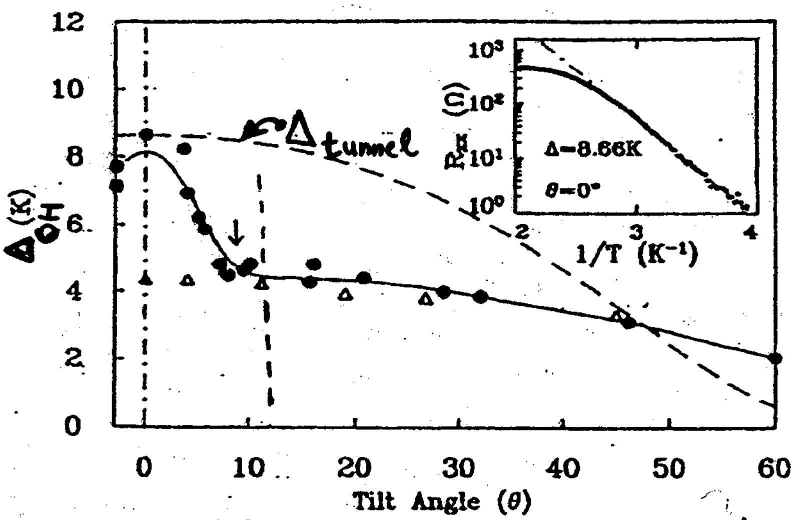
$\} \; \nu = \frac{n}{n_\phi} = 1$

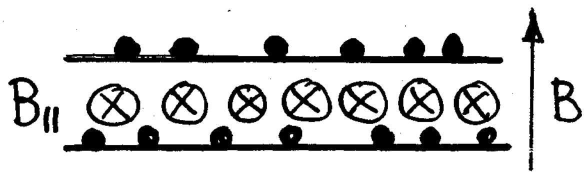


$\nu = 1$ bilayer QHE
for $d = 30\text{Å}$, $N_{tot} = 1.26 \times 10^{11}\,\text{cm}^{-2}$

(larger spacing, higher density)

- **$B_{||}$-induced QH gap collapse:**





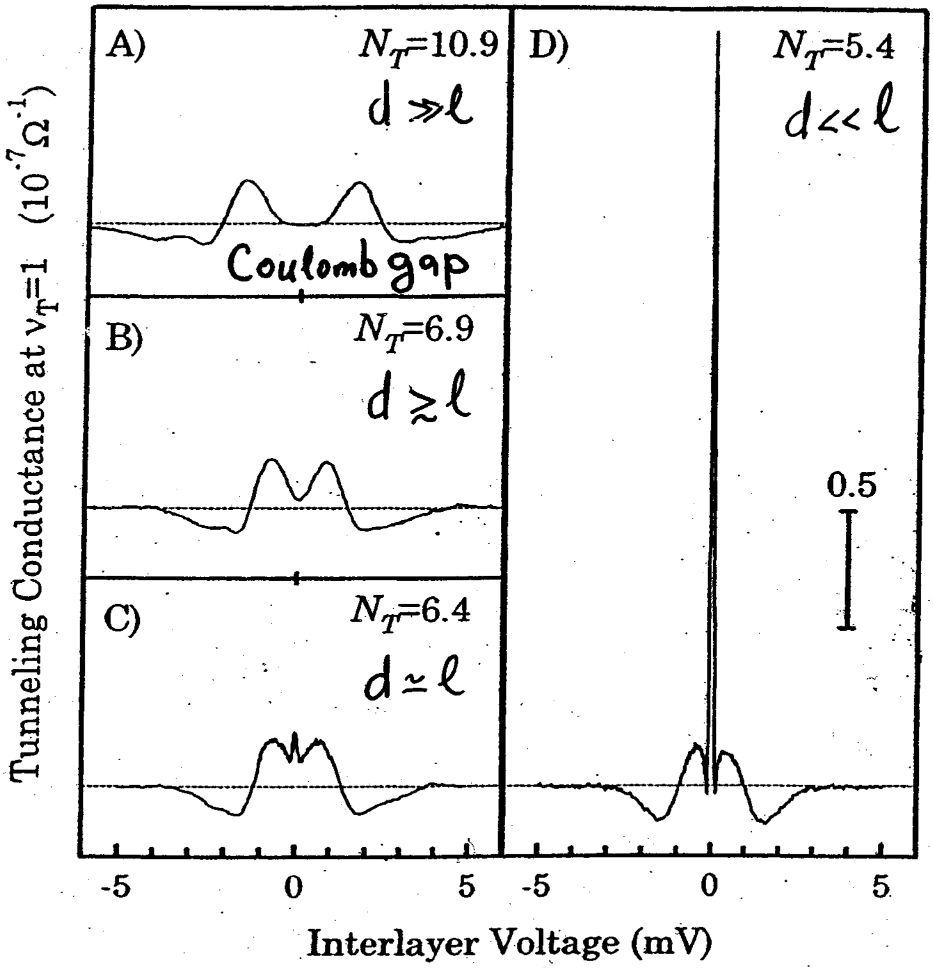
Commensurate   Incommensurate
transition

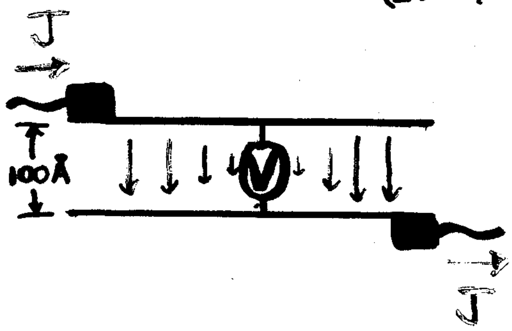
K. Yang, et al, PRB 54, 11644 (1996)

Recent Tunneling Experiments

- **Appearance of zero-bias peak for $\ell > d$:**

[Spielman, etal, PRL 84, 5808 (2000)]





- $T = 40\,mK$
- $\nu = 1$
- $\rho_s = \frac{1}{2} K$
- $\Delta_t = 0.1\,mK$
- $d = 100\,Å$
- $\Delta_J = 0.07\,K$

- Go through transition by adjusting ℓ relative to d keeping $\nu = 1$
- Quantum transition takes place when interlayer interaction e^2/d dominates over intralayer one e^2/ℓ spontaneous interlayer coherence for $d < \ell$

Finite T_{KT} Transition

(Spielman, etal. 2000)

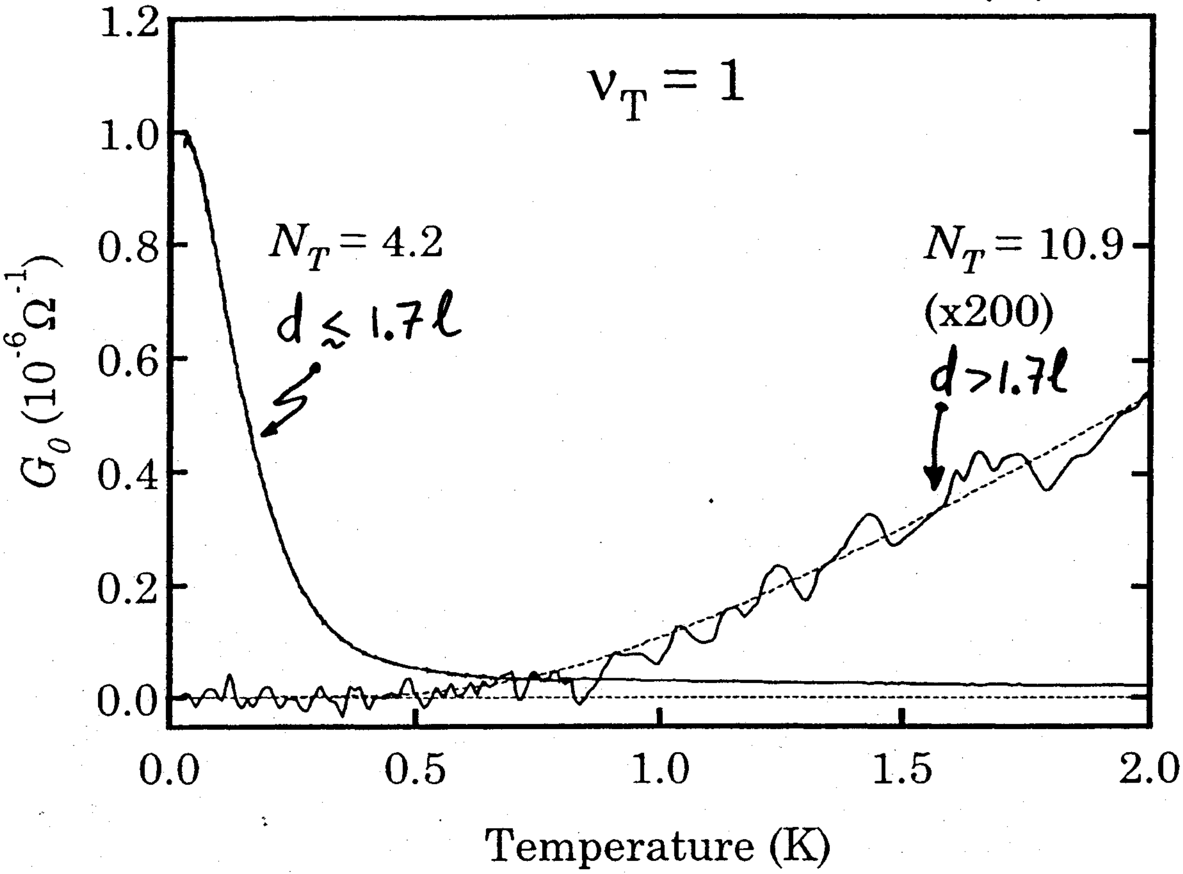
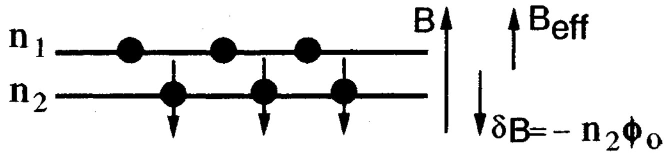


FIG. 3. Temperature dependence of the zero bias tunneling conductance at $\nu_T = 1$ at high and low densities. Note that the high density data has been magnified by a factor of 200.

Pictures of the Ground State

- **BEC of composite bosons** **(Wen and Zee '92)**

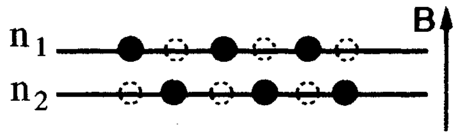
  e_1 sees e_2 carrying a unit of flux, leading to $B_1^{eff} = B - n_2\phi_0$



$$\psi_{111} = \prod(z_i - z_j)\prod(w_i - w_j)\prod(z_i - w_j)$$

- **BCS of interlayer excitons**

  electrons in layer 1 pair with holes in layer 2 via BCS

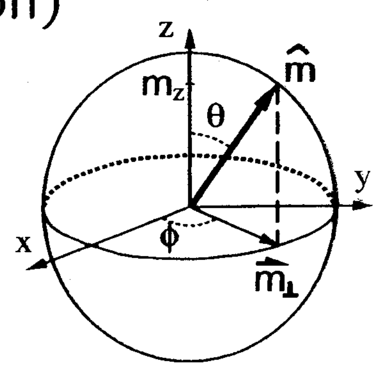


$$|\psi\rangle = \prod_X \left(\cos\frac{\theta_x}{2}\; e^{i\phi_{x1}} c^\dagger_{x_1} + \sin\frac{\theta_x}{2}\; e^{i\phi_{x2}} c^\dagger_{x_2}\right)|0\rangle$$

- **Pseudo-FM in layer index** **(K.Moon, et al.'95, K. Yang, et al.'96)**

$$\left.\begin{array}{l} |1\rangle \Rightarrow |\uparrow\rangle \\ |2\rangle \Rightarrow |\downarrow\rangle \end{array}\right\} \text{pseudo-spin}$$

  - ideal itinerate Stoner pseudo-FM (cf. Hund's rule spin polarization)
  - $E^{exch.}_{Coul.} > E_{kinetic} = 0$
  - $\vec{m} = c^\dagger_\alpha \vec{\sigma}_{\alpha\beta} c_\beta$
    $= \left(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\right)$



$$|\psi\rangle = \prod_{ij}(z_i - z_j)|\rightarrow\rightarrow\rightarrow\ldots\rightarrow\rangle$$

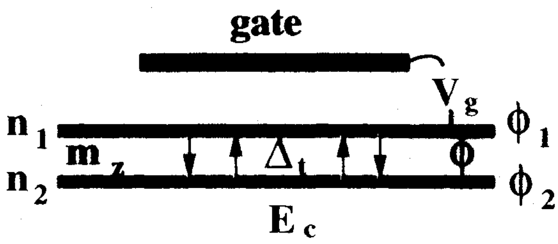
$$|\uparrow\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\phi} \end{pmatrix}$$

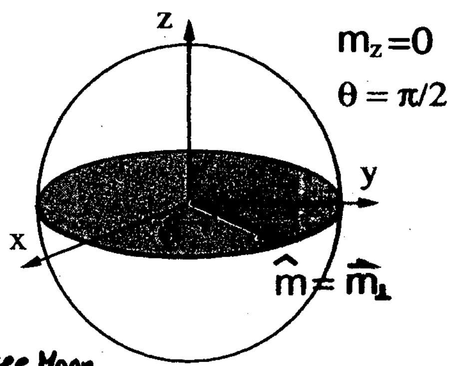
Goldstone Mode Hamiltonian

- $SU(2)$ **invariance for** $\Delta_t = E_c = 0$

  $\mathcal{H}_{SU(2)} = \frac{1}{2}J|\nabla\hat{\mathbf{m}}|^2, \qquad |\hat{m}| = 1$

- **Symmetry-breaking energies:**





  - Bilayer charging energy E_c

    $H_c = \frac{1}{2}E_c(n_1 - n_2)^2$
    $\quad = \frac{1}{2}E_c m_z^2$

    easy-plane anisotropy,
    breaking $SU(2) \to U(1)$

    also long-range dipolar interaction (see Moon et al)

  - Interlayer tunneling energy Δ_t

    $H_{tunnel} = -\Delta_t(c_1^\dagger c_2 + c_2^\dagger c_1)$
    $\quad = -\Delta_t m_x = -\Delta_t m_\perp \cos\phi$

    pseudo-B field along x,
    breaking $U(1)$ (pseudo-spin) symmetry

  - Gate V_g

    $H_{gate} = -V_g(n_1 - n_2)$
    $\quad = -V_g m_z$

    pseudo-B field along z,
    breaking $U(1)$ (pseudo-spin) symmetry

Model

- **Low energy modes**: (Moon, et al.'95)
  - relative charge density and phase
    $\widehat{m}_z = \widehat{n}_1 - \widehat{n}_2$, $\widehat{\phi} = \widehat{\phi}_1 - \widehat{\phi}_2$
  - canonical commutation relation
    $[\widehat{m}_z, \widehat{\phi}] = i$
- **Continuum field-theory**:
  - model Hamiltonian

$$\widehat{H} = \int d^2r \left[\underbrace{\frac{\rho_s}{2}|\vec{\nabla}\widehat{\phi}|^2}_{\text{exchange}} \underbrace{- \frac{\Delta_t}{2\pi\ell^2}\cos\widehat{\phi}}_{\text{tunneling}} \underbrace{+ \frac{E_c}{2\pi\ell^2}\widehat{m}_z^2}_{\text{charging}} \right]$$

  - coherent state Action

$$S = \int dtd^2r \left[\underbrace{\frac{\hbar}{\ell^2} m_z \partial_t \phi}_{} - \frac{\rho_s}{2}|\vec{\nabla}\phi|^2 + \frac{\Delta_t}{2\pi\ell^2}\cos\phi - \frac{E_c}{2\pi\ell^2} m_z^2 \right]$$

    Berry's "$p\dot{q}$" term $\vec{A}[\widehat{m}] \cdot \partial_t \widehat{m} = (1 - \cos\theta)\partial_t \phi$,
    encoding precessional FM dynamics

  - charging energy E_c gaps out m_z, which can therefore be integrated out exactly (cf. free particle with $p\dot{q} - p^2/2m$ and superconductor with n, ϕ)

$$S = \int dtd^2r \left[\frac{\rho_s}{2} \left(\frac{1}{v^2}(\partial_t\phi)^2 - |\vec{\nabla}\phi|^2 \right) + \frac{\Delta_t}{2\pi\ell^2}\cos\phi \right]$$

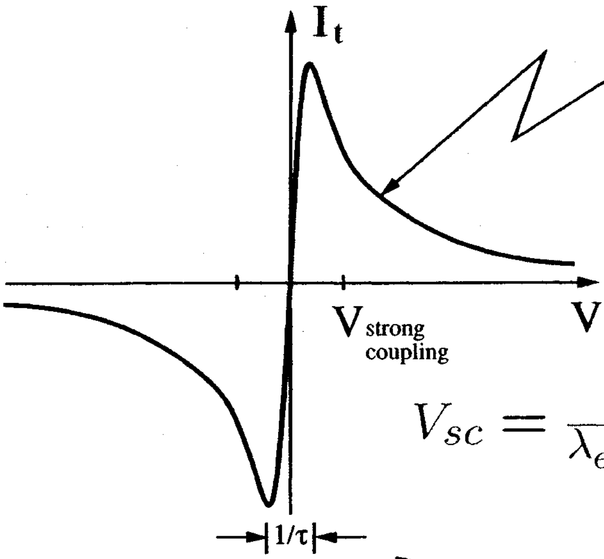
$v = 4\pi^{\frac{1}{2}} \frac{\ell}{\hbar} \sqrt{E_c \rho_s} \approx \Delta_J \lambda/\hbar \approx 10^4 \text{m/sec}$

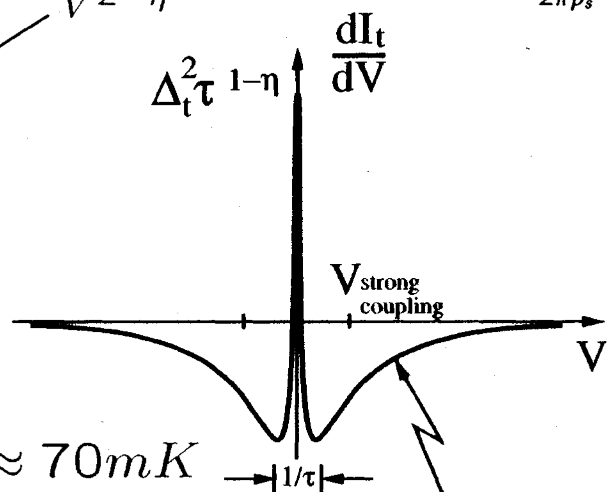
Perturbative IV Predictions

L. Balents, L. R., PRL 86, 1825 (2001)
(also see: A. Stern, et al.; M. Fogler, et al. PRL)

- **Clean bilayer**

$$J_t \sim \left(\frac{\Delta_t}{V}\right)^2 \left(\frac{V}{T}\right)^\eta \mathrm{sgn}(V) \sim \frac{1}{V^{2-\eta}} \qquad \eta(T) = \frac{k_B T}{2\pi\rho_s}$$





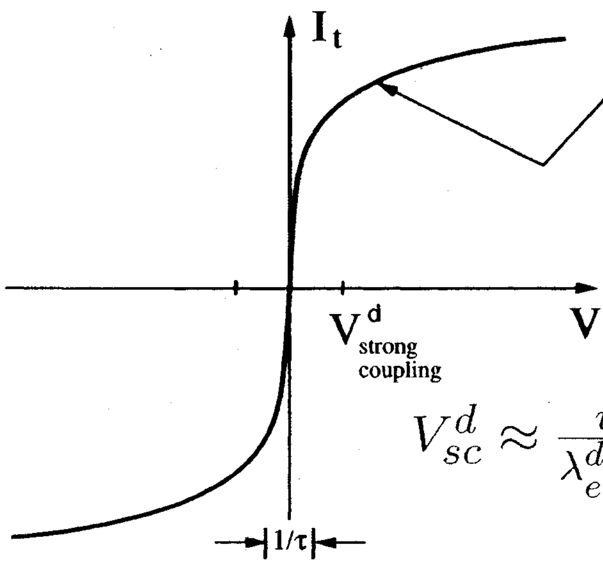
$$V_{sc} = \frac{v\hbar}{\lambda_{equil.}} \approx 70mK$$

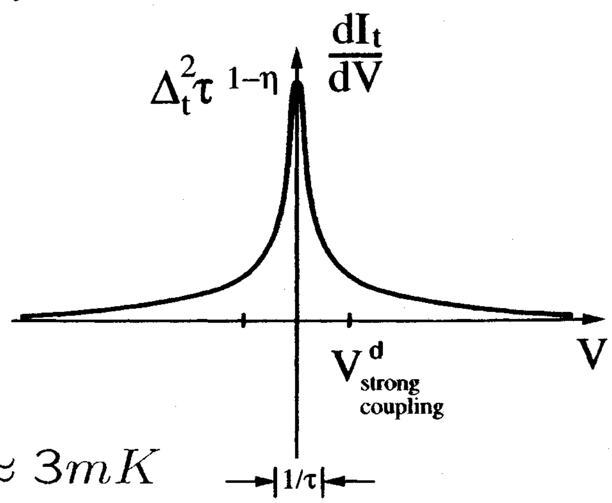
negative differential conductance

decoherence and strong coupling effects

- **Dirty bilayer**

$$J_t \sim \Delta_t^2 \left(\frac{\xi}{\ell}\right)^2 \left(\frac{V}{T}\right)^\eta \mathrm{sgn}(V) \sim V^\eta$$



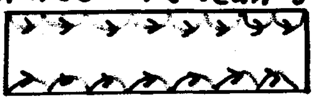


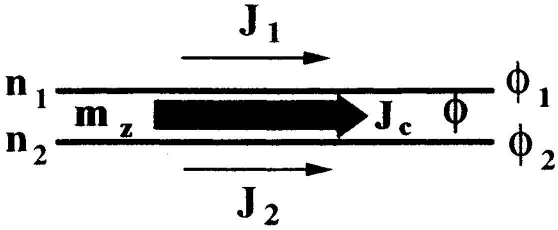
$$V^d_{sc} \approx \frac{v\hbar}{\lambda^d_{equil.}} \approx 3mK$$

Phenomenology

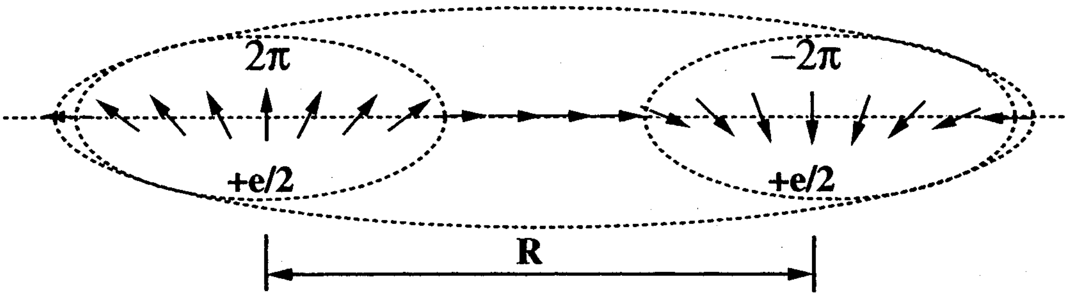
- **QHE in charge (symmetric) channel:**
  $\vec{J}_c = \vec{J}_1 + \vec{J}_2$

  carried by edge states in clean sample



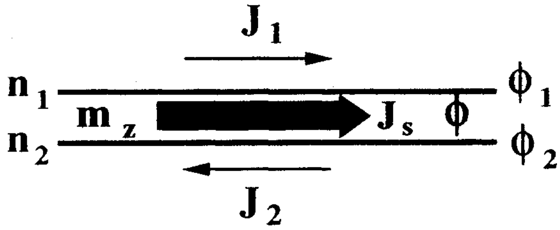


- **Quasi-particle excitation gapped by Δ_{QH}:**
  meron-antimeron pair with EM charge density
  $\delta n(\mathbf{r}) = -\frac{e}{8\pi}\epsilon_{\mu\nu}\hat{m}\cdot(\partial_\mu\hat{m}\times\partial_\nu\hat{m})$



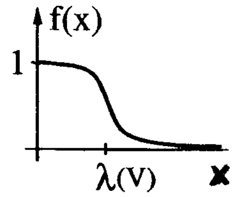
- **Bulk gapless Goldstone mode ($\Delta_t \to 0$):**
  $\phi = \phi_1 - \phi_2$
  symmetry breaking of pseudo-spin $U(1)$
  associated with $N_1 - N_2$ charge conservation

- **Persistent pseudo-spin (antisymmetric) currents:**
  $\vec{J}_s = \vec{J}_1 - \vec{J}_2 = \frac{e\rho_s}{\hbar}\vec{\nabla}\phi$

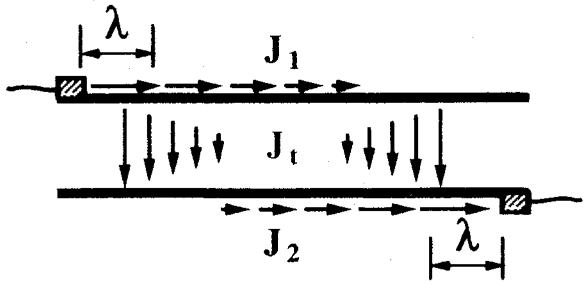


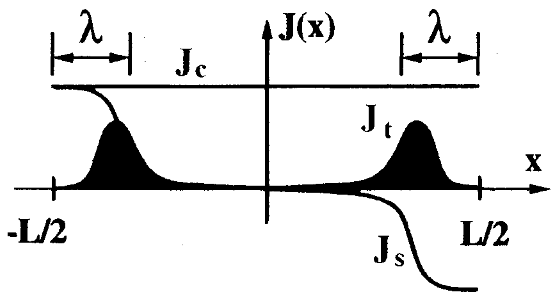
Strong Coupling and Screening

- **Current "Meissner" effect:** $J(x,V) = J(V) f(\frac{x}{\lambda})$



  - due to *explicit* breaking of staggered $U(1)$ by tunneling
  - nonextensive staggered and tunneling currents confined to sample edges





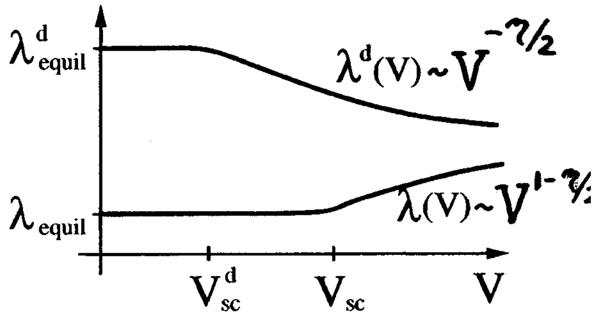
- **Screening lengths**
  - clean limit:

cut off by V $$\lambda(V) = \lambda_{equil.} \begin{cases} \left(\frac{\lambda_0}{\xi_T}\right)^{\frac{-\eta}{4-\eta}} \left(\frac{\lambda_0}{\xi_V}\right) \left(\frac{T}{V}\right)^{\eta/2}, & V > V_{sc} \\ 1, & V < V_{sc} \end{cases}$$

$\hbar, k_B T$ renorm. $\lambda_{equil.} = \lambda_0 (\frac{\lambda_0}{\xi_T})^{\frac{\eta}{4-\eta}} \approx 150\ell \approx 4\mu m$

strong coupling $\lambda_0 = \ell\sqrt{2\pi\rho_s/\Delta_t} \approx 4\mu m$

$$\left(\mathcal{H} = \tfrac{1}{2}\rho_s|\vec{\nabla}\phi|^2 - \tfrac{\Delta_t}{2\pi\ell^2}\cos\phi\right)$$

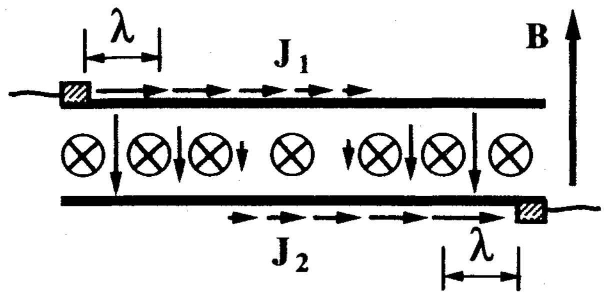


  - dirty limit:
    $\lambda_d(V)$ same as $\lambda(V)$ with $\xi_V \equiv \frac{\hbar v}{V} \to \xi_d$
    $\lambda^d_{equil.} = \lambda_0 (\frac{\lambda_0}{\xi_d})^{\frac{2}{2-\eta}} (\frac{\lambda_0}{\xi_T})^{\frac{\eta}{2-\eta}} \approx 25\lambda_{equil.} \approx 100\mu m$

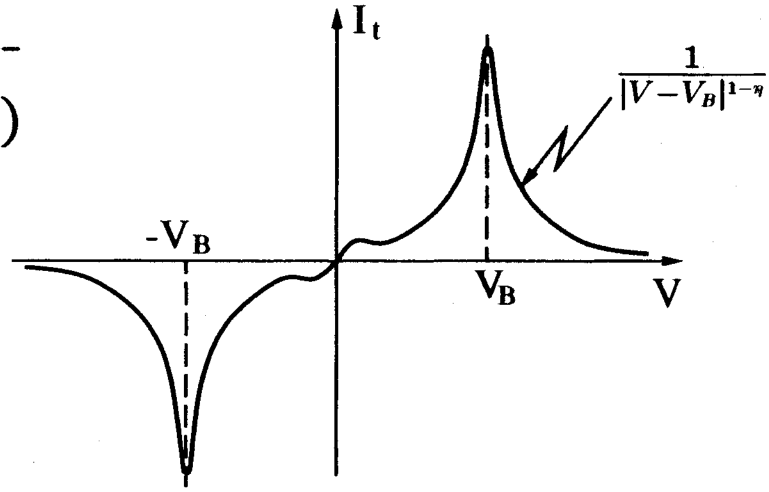
- **Other important scales**
  - disorder correlation length: $\xi_d \approx 12\ell \approx 0.2\mu m$
  - thermal length: $\xi_T = \hbar v/T = \lambda_0 \frac{\Delta_J}{T} \approx 10^3\ell \approx 20\mu m$
  - sample size: $L \approx 250\mu m$
  - tunneling energy: $\Delta_t \approx 0.1mK \approx 0.01\mu eV$
  - Josephson plasma energy: $\Delta_J \approx 70mK \approx 7\mu eV = \frac{\hbar v}{\lambda_0}$
  - exchange stiffness: $\rho_s \approx 0.5K \approx 0.05meV$

$$\Rightarrow I_t(V) \approx L_y \lambda(V) J_t(V) \sim \begin{Bmatrix} \frac{1}{V^{1-\eta/2}}, & \text{clean} \\ V^{\eta/2}, & \text{dirty} \end{Bmatrix}, \text{ for } L_x > \lambda(V) \text{ \& } V > V_{sc}$$

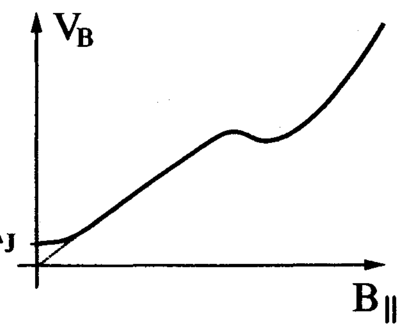
IV with In-plane $B_{||}$ Field



- $B_{||}$ splits the zero-bias peak to $\pm V_B(B_{||})$



- $eV_B(B_{||}) = \hbar\omega(2\pi d B_{||}/\phi_0)$ maps out the Goldstone mode dispersion $\hbar\omega(Q)$

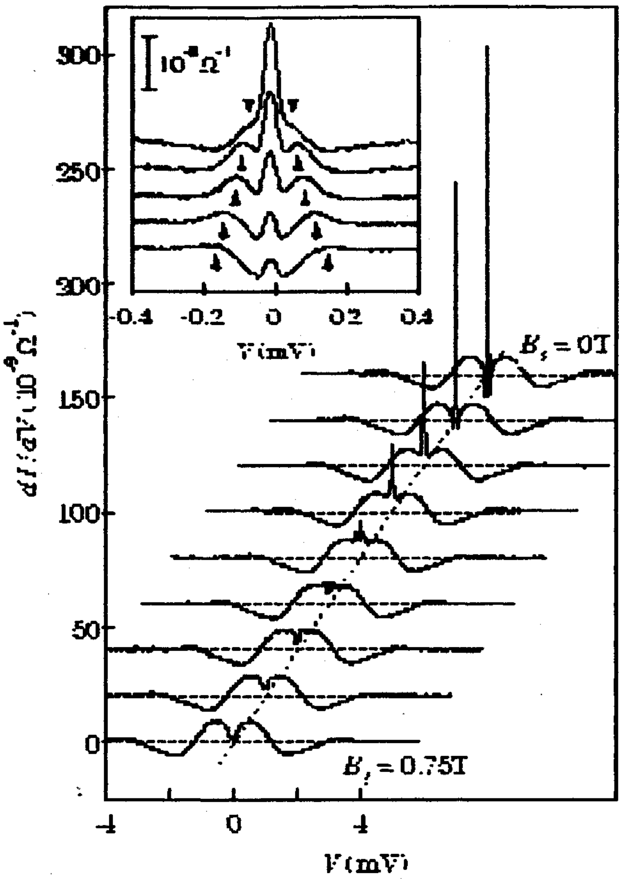


- **These are consequence of:**
  - single low energy mode ϕ dominating tunneling (cf. Fermi-liquid → Fermi-liquid tunneling: $I(V) = 2e|\Delta_t|^2 N_R N_L \int_{-eV}^{0} d\varepsilon \propto V$)
  - $B_{||}$ imposing an in-plane momentum $Q = 2\pi d B_{||}/\phi_0$
  - 2d energy and momentum conservation

Experimental IV's with $B_{||}$

I. Spielman, et al. condmat/0012094

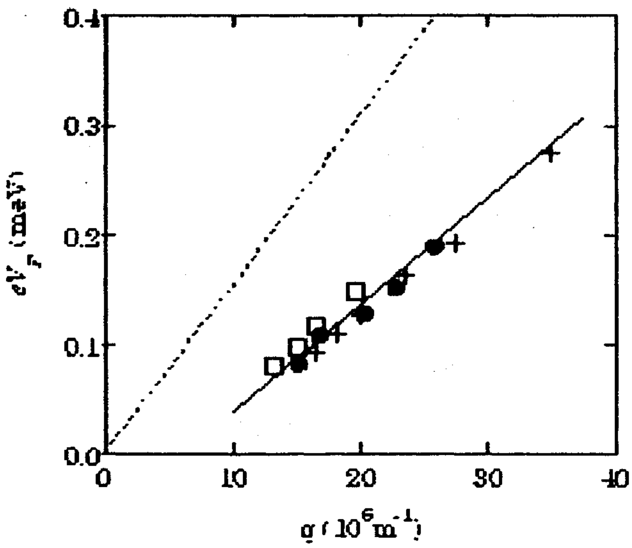
- **Split of IV peak with $B_{||}$: $\lesssim \frac{1}{2}$ Tesla**



Note: central peak persists

Direct measure of Goldstone mode (ϕ) spectral fnc.

- **Dispersion of the peak:**



$v \simeq 1.5 \times 10^{4}$ m/s

Calculational Highlights

- **Effective Action**:

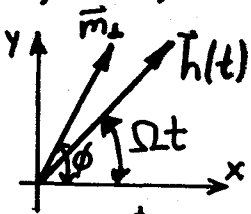
$$S_E = \int_{\tau,r} \left\{ \frac{\rho_s}{2} \left(\frac{1}{v^2} (\partial_t \phi)^2 + |\vec{\nabla}\phi|^2 \right) - \frac{\Delta_t}{2\pi\ell^2} \cos\left[\phi - Qx - \Omega t\right] \right\}$$

$$Q = B_{||} \frac{2\pi d}{\phi_0}, \quad \Omega = eV/\hbar$$

- **Calculational details**:

  - Tunneling current: $I_t = e\langle \dot{\hat{N}}_1 \rangle = -\frac{ie}{\hbar} \langle \left[\hat{N}_1, \hat{H} \right] \rangle$

    $\Longrightarrow J_t(V, B_{||}) = \frac{e\Delta_t}{2\pi\ell^2\hbar} \langle \sin\hat{\phi} \rangle = \langle \frac{e}{2\pi\ell^2\hbar} (\hat{z} \times \vec{m}_\perp(t)) \cdot \vec{h}(t) \rangle$



$\vec{m}_\perp(t) = \int_{-\infty}^{t} K(t-t') \vec{h}(t')$

  - Perturbation theory in Δ_t:

    $$J_t(V, B_{||}) = J_0 \frac{\Delta_t}{2\pi\ell^2\hbar} \mathrm{Re}\left[C(\mathbf{r}, t)\right]_{Q\hat{\mathbf{x}},\Omega}$$

    $$C(\mathbf{r}, t) = \theta(t) \langle \left[e^{i\hat{\phi}(\mathbf{r},t)}, e^{-i\hat{\phi}(\mathbf{0},0)} \right] \rangle_0$$

  - *Equilibrium* screening and crossover to strong coupling: $\lambda_{equil.} = \lambda_0 (\frac{\lambda_0}{\xi_T})^{\frac{\eta}{4-\eta}}$, for $\rho_s >> \Delta_t$ fluctuations at short time/length scales suppress effective tunneling, as can be studied via RG and matching onto strong coupling description

Additional Details

- Physics of *nonequilibrium* screening: $\lambda(V) > \lambda_{\text{equil.}}$
  - Steady state current conservation:
    $\frac{e}{\hbar}\rho_s \nabla^2\phi = -J_t(V) \Longrightarrow \lambda(V) = \sqrt{\frac{e\rho_s}{\hbar J_t(V)}}$

    .f. superconductor. $\vec{\nabla}\times\vec{B} = \vec{J}$
  - Perturbative IR divergences are softened but *not* eliminated by V even for $V > V_{sc}$
- Quenched disorder:
  - random phase shifts $\delta(\mathbf{r})$:
    $\Delta_t \sin\phi(\mathbf{r},t) \to \Delta_t \sin[\phi(\mathbf{r},t) - \delta(\mathbf{r})]$
  - random vector potential $\vec{a}(\mathbf{r})$: dominant
    $\vec{\nabla}\phi \to \vec{\nabla}\phi - \vec{a}(\mathbf{r})$
    with $\vec{a}(\mathbf{r}) = \delta V_-(\mathbf{r})\hat{z}\times\vec{\nabla}V_+(\mathbf{r})$ (Stern, et al.)
    same qualitative physics for weak disorder and quantum gauge glass for strong $\xi_{ss} \sim T^{-\nu}$
    $\xi_{ss}/\sigma \ll \tau_{ss} \sim e^{\frac{1}{T^\alpha}}$
- Crossover to classical hydrodynamics:
  $\partial_t m_z = -\sigma_t m_z + D\nabla^2 m_z$

$$
\begin{aligned}
J_t(V,\Delta_t) &= b^{-\eta/2} J_t(Vb, \Delta_t b^{2-\eta/2}) \\
&\sim \Delta_t^2 V^{-(2-\eta)}, \text{ for } eV \gg \Delta_t \\
&\sim \Delta_t^{\frac{2}{4-\eta}} V, \text{ for } eV \ll \Delta_t
\end{aligned}
$$

$$\Longrightarrow \sigma_t(\Delta_t) \sim \Delta_t^{\frac{2}{4-\eta}} \gg \Delta_t$$

Open Questions

- **width and height of the tunneling peak**

  $\hbar/\tau$ $\quad$ $\tau^{1-?}\Delta^{\beta}$

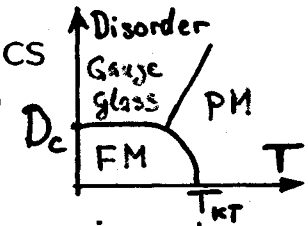
  dominant source of dissipation: vortices, leads, ... ?

  central ($V=0$) peak even for $B_\parallel \neq 0$?

- **quantum gauge glass physics** $\frac{\xi_{st}}{v} \ll t \ll \tau_{g\delta} \sim e^{\frac{1}{T}}$

  implications for tunneling and phason dynamics

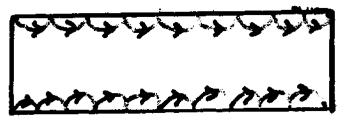
  (see lectures I & II on pinned elastic media)



- **crossover to hydrodynamics**

  generation of the effective dissipative $\gamma\partial_t\phi$ dynamics at long scales $\gamma(\Delta_t)$?

- **role and nature of edge states**



  essential for geometry of Eisenstein experiments?

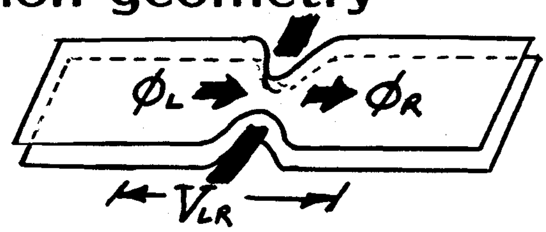
- **spontaneous interlayer phase coherence in the absence of Δ_{QH}**

  pseudo-spin superfluid QH insulator?

- **Josephson junction geometry**

  $I_s = I_{crit}\sin(\phi_L - \phi_R)$

  $\frac{d\phi_{LR}}{dt} = eV_{LR}/\hbar$

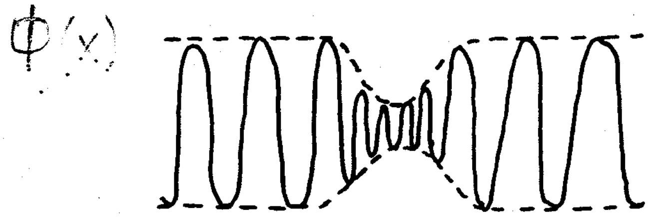


- **broader picture**

  phase transitions out of the pseudo-FM and relation to decoupled HLR and other QH states? see J. Schliemann, et al. condmat/0006309

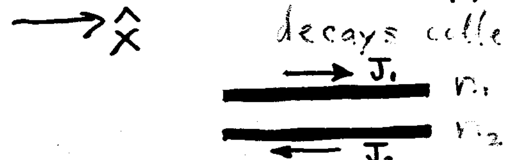
Work in Progress

- **decay of persistent staggered currents via phase slips** (with J. Kyriakidis, condmat/0010329)

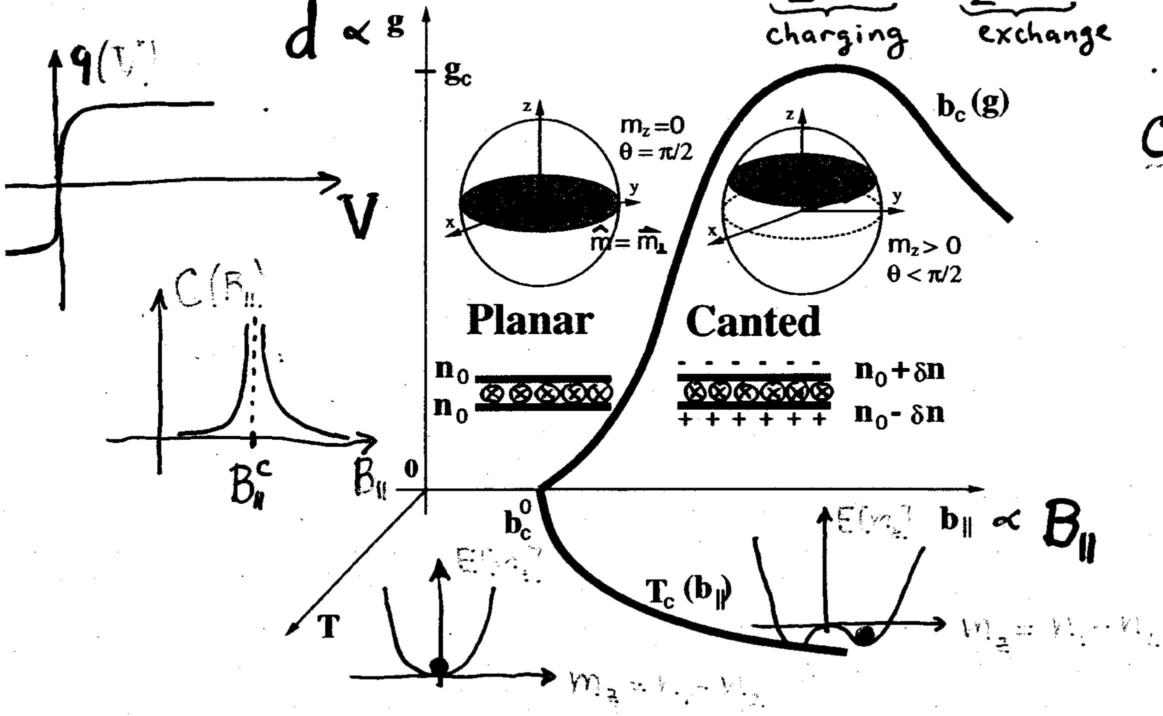


$(J_1 - J_2)\hat{x} = \vec{J}_{staggered} = \frac{e}{\hbar}\rho_s \vec{\nabla}\phi$

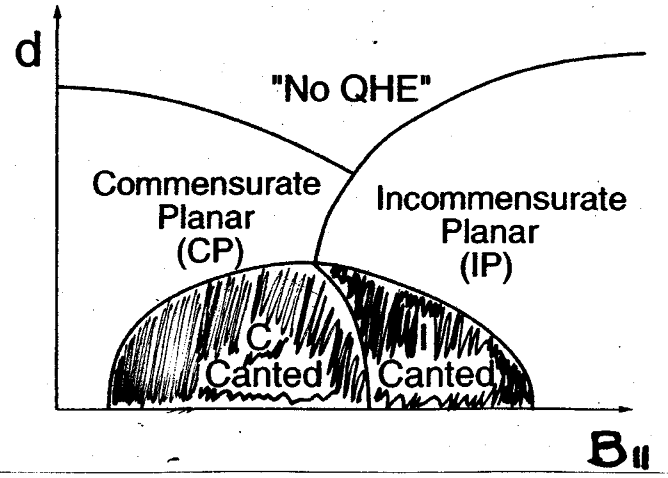
decays collectively



- **quantum interlayer charging transition** (cond-mat/0104128) $H = \underbrace{\frac{\beta}{2} m_z^2}_{\text{charging}} + \underbrace{\frac{\rho_s}{2}|\vec{\nabla}\phi|^2}_{\text{exchange}} - \underbrace{\tilde{\Delta}\cos(\phi - B_\parallel x)}_{\text{twists the phase } \phi}$



Competition of:
Exchange Energy
&
Charging Energy



Also see HFA:
Abolfath
L.R.
MacDonald