LONGITUDINAL VORTICES IN GRANULAR FLOWS

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Abstract :

We analyse a new instability observed in rapid granular flows down rough inclined planes. For high inclinations and flow rates, the free surface of the flow experiences a regular deformation in the transverse direction. This instability is associated with the formation of longitudinal vortices in the granular flow. From the experimental observations, we propose a mechanism for the longitudinal vortex formation based on the concept of granular temperature. This mechanism is studied in the framework of the kinetic theory of rapid granular flows. The results of a linear stability analysis agree qualitatively with the experimental observations.

1 Introduction

In fluid mechanics, the development of instabilities can dramatically affects the dynamics of a flow, creating coherent structures and controlling the transition to turbulence. For granular flows, the relevance of these hydrodynamic concepts and the existence of a similar scenario are open questions. Whereas granular materials present some characteristics of fluids, they do not behave like classical fluids (Jaeger et al. 1996). The dissipative nature of the grain interactions and the lack of scale separation between the grain size and the flow scale represent difficulties for an hydrodynamic description of granular flow. One way to better understand the specificity of granular flows is to investigate the instabilities that can develop. In this study we present a new instability leading to the formation of longitudinal vortices in rapid granular flow down rough inclined plane. Although such structures are well know in fluid mechanics (gortler vortices – Saric 1994, streaks in boundary layers -Kachanov 1994) they have not been observed in granular flows.

2 Experiment

The experimental set-up consists of a rough inclined plane with a reservoir containing the granular material (Fig. 1a). The plane is a glass plate (130cm long, 30cm wide) made rough by gluing one layer of particles onto its surface. The gate of the tank can be suddenly opened in order to provide an opening h_g constant across the bed. The results presented here are carried out with sand 0.25mm in mean diameter but the same observations can be made with other kind of granular material (coarse sand 0.8mm in mean diameter and quasi-monodispersed glass beads of 0.5mm diameter). The two control parameters of this experiment are the angle of inclination of the plane θ and the opening of the gate h_g.

The instability is observed when the inclination and the opening are large enough. In this regime, the flow is accelerating down the slope. At a certain distance from the entrance, the free surface shows a very regular pattern of longitudinal streaks parallel to the flow direction (Fig. 1a). The bright and dark streaks result from a free surface deformation in the transverse direction. The wavelength λ depends on the control parameters but is typically two/three times the average depth h of the flowing granular layer.

To understand the origin of the instability, we have carefully studied the structure of the flow. Velocity measurements at the free surface have shown that the streaks correspond to the formation of longitudinal vortices. Two contra-rotative vortices form one wavelength for the surface deformation. (Fig. 1b). We have also measured the volume fraction (defined as the volume occupied by the grains divided by the total volume). The mean volume fraction is low (around 0.2-0.3) , indicating that the instability occurs in a dilute regime. Moreover, we have noticed that the density varies periodically together with surface deformation: crests are dilute and deeps are dense.

The Fig. 1b is a sketch of the instability structure deduced from the experimental observations. The heavy part of the flow is going down while the light part is going up. This last result suggests that the density plays an important role in the instability mechanism. When the instability appears, the flow is rapid and dilute. Its dynamic is thus mainly controlled by the collisions between particles and/or boundaries. Keeping this in mind, we propose the following instability mechanism. Because of collisions with the rough bottom, particles close to the plane are strongly agitated while particles near the free surface remain `frozen'. In a collisional regime, the agitation of the grains is analogous to a `granular temperature' (Campbell 1990). Since the flow is accelerating, the `granular temperature' at the bottom increases during the flow. Consequently, the density at the bottom decreases until it becomes smaller close to the plane than above. At this stage, the flow is mechanically unstable under gravity because the heavy material is above the light one. This leads to the formation of longitudinal vortices in the granular flow. There is a close analogy between this situation and the case of a liquid heated from below (Rayleigh-Benard instability). When a liquid is flowing down a hot plate, one can indeed observe longitudinal rolls convection (Sparrow 1969). However, the `granular temperature' in our case is created during the flow by the shearing at the rough boundary.

The proposed mechanism is based on the existence of an inverse density profile. However, in a dissipative granular gas, the density profile results form a complex equilibrium between gravity, collisions and dissipation and its prediction is not straightforward. In order to study the relevance of the proposed mechanism, we have theoretically investigated the instability of the flow down rough plane in the framework of the kinetic theory.

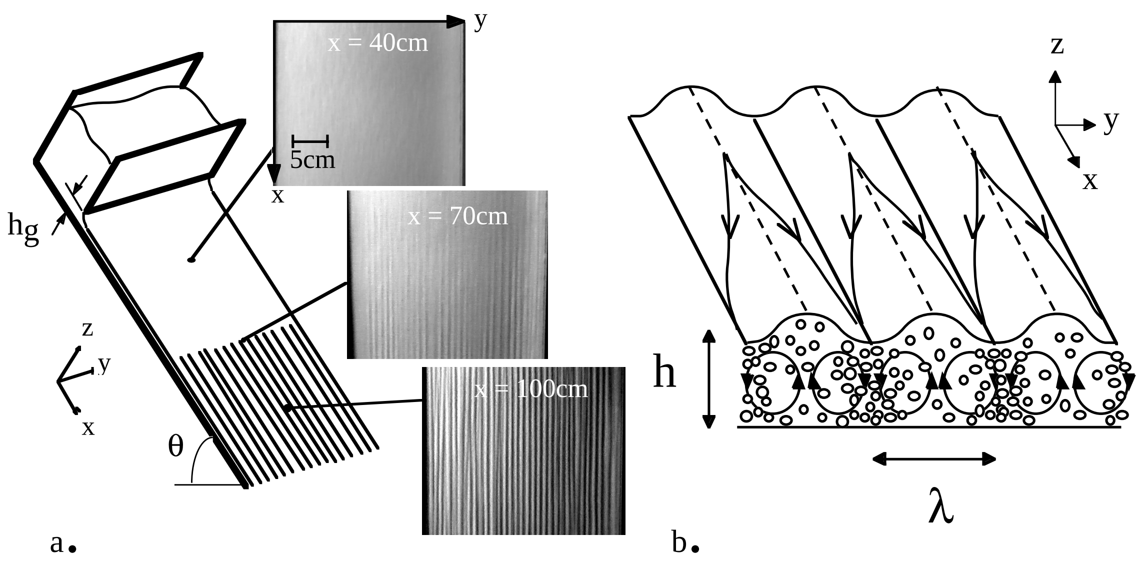

FIG. 1a : *Experimental set up. The three pictures are top view of the free surface of the flow lighted from the side (0,25mm sand, θ = 41°, h_g= 13mm).* b *:Sketch of the flow in a cross section showing the longitudinal vortices and the density variations*

3 Theory

The kinetic theory of granular flow is inspired by the kinetic theory of dense molecular gases (Lun et al 1984, Goldhirsch 1999). The difference lies in the inelasticity of the collisions which intrinsically induces a coupling between the flow and the temperature. The kinetic theory gives constitutive equations for the density ρ, the velocity $\mathbf{u}$ and the granular temperature T (defined as the average of the square of the velocity fluctuations) :

$$\frac{d\rho}{dt} = -\rho \nabla . \mathbf{u} \qquad (1)$$

$$\rho \frac{d\mathbf{u}}{dt} = \rho \mathbf{g} - \nabla . \mathbf{P} \qquad (2)$$

$$\frac{1}{2} \rho \frac{dT}{dt} = -\mathbf{P} : \nabla \mathbf{u} - \nabla . \mathbf{q} - \gamma \qquad (3)$$

In these equations, $\mathbf{P}$ is the stress tensor, $\mathbf{q}$ the heat flux and γ the loss of energy due to inelastic collisions. For instantaneous and inelastic collisions, the kinetic theory gives the expression for all the coefficient in the constitutive equations as a function of the temperature, velocity, density of the medium and their gradients. We have used the Lun et al (1984) equations with boundary conditions proposed by Jenkins and Richman (1986) for studying the linear stability of a stationary uniform flow down an inclined plane. The first step consists in computing the basic stationary uniform flow from eqs. 1, 2, 3 and the boundary conditions. Then the equations are linearized around the basic state ((ρ_0(z), u_0(z), T_0(z)) for harmonics perturbations f(z)exp(σt+iky) where Re(σ) gives the growth rate. We then obtain 5 ordinary differential equations (one for the mass, three for the momentum, one for energy) with boundary conditions at the rough bottom and at infinity. The problem is then an eigenvalue problem: for a given wavenumber k, nonzero solutions exist only for specific values of σ.

We have solved the problem numerically using a Chebyshev spectral collocation method (Malik 1990). We were then able to get the dispersion relation dispersion σ(k) and the corresponding density, velocity and temperature profiles for the perturbation.

The Fig. 2a shows two typical density profiles of the basic state. For a given inclination, one continuously evolves from a normal density profile (with a density decreasing with the distance from the plane) to an inverse profile by continuously increasing the flow rate. A stability analysis shows that non inverse profile are always stable to transverse perturbation (i.e Re(σ)<0 for all k). The inverse profile is can be unstable: Re(σ)>0 for some wavenumbers. The velocity perturbation eigen functions are in this case longitudinal vortices (Fig. 2b). The density perturbation is positive in the ascending part and negative in the descending parts, in qualitative agreement with the experimental observations. By varying k at a given inclination and flow rate, one can plot the dispersion relation σ(k) (Fig. 3). Above a threshold, a range of wavenumber is unstable. The most amplified mode is of the order of few times the thickness.

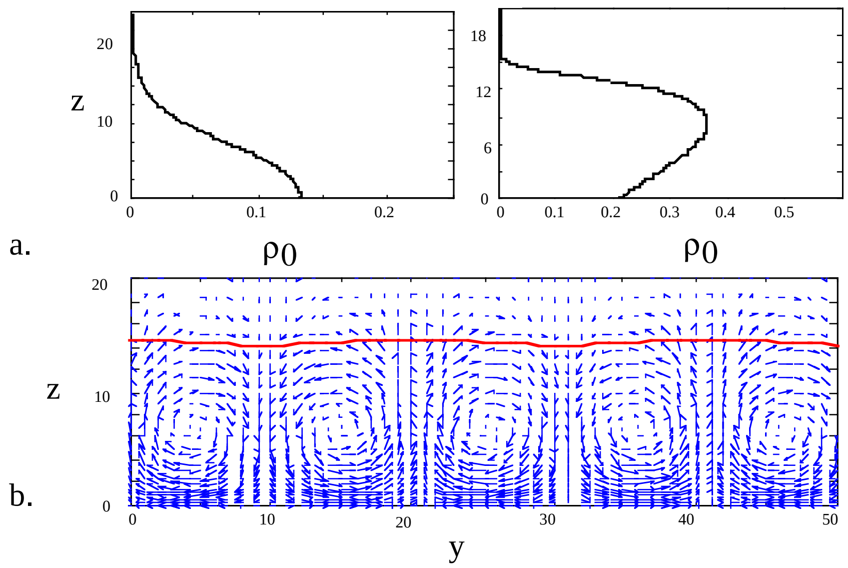

FIG. 2. a *: Density profiles (made dimensionless by the particle density) as a function of the distance to the rough plane (in particle diameter unit) for the basic flow for two flow rates at θ=21. Left: low flow rate. Right: high flow rate.*

b: Velocity perturbations in a section perpendicular to the flow in an unstable situation

These results are compatible with our experimental observations and show the relevance of the instability mechanism proposed to explain the formation of the granular vortices. However the agreement is only qualitative. The threshold and growth rates are not quantitatively predicted. This is not surprising because the validity of the kinetic theory is known to be limited (Goldhirsch 1999). However, this study shows that the coupling between the shear flow and the agitation of the particles is enough to capture the formation of longitudinal vortices.

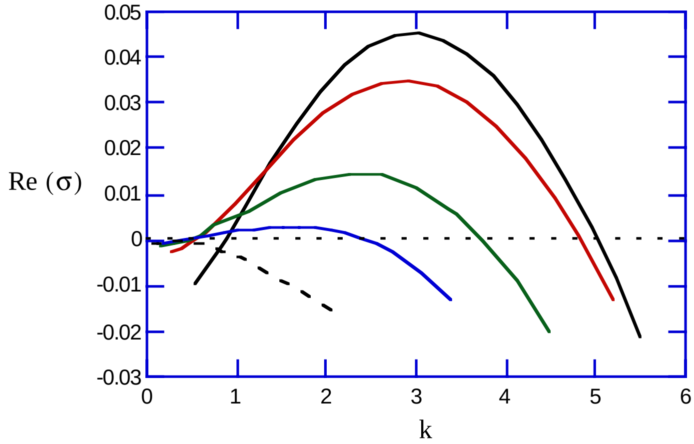

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FIG. 3. *Growth rate Re(σ) versus the k (in thickness units) for θ =21°, the dimensionless flow rate varying form 56 to 19.*

4 Conclusion

We have presented a new instability observed in rapid granular flows on rough inclined plane. The instability induces the formation of longitudinal vortices. From experimental measurement we have been able to proposed a mechanism to explain the formation of the vortices based on concept of granular temperature. The linear stability analyses in the frame work of the kinetic theory of granular dissipative gases have shown that the proposed mechanism is relevant.

Références

Campbell, C. S. 1990 Rapid granular flows, *Annu. Rev. Fluid Mech.* **22**, pp. 57 –92.
Forterre et Pouliquen. 2001 Longitudinal vortices in granular flows, *preprint.*
Goldhirsch, I. 1999 Scales and kinetics of granular flows, *Chaos* **9**, pp. 659-672.
Jaeger, H. J., Nagel, S. R. et Behringer, R. P. 1994 Granular solids, liquids, and gases, *Rev. Mod. Phys.* **68**, pp. 1259-1273.
Jenkins, J. T. et Richman, M. W. 1986 Boundary conditions for plane flows of smooth, nearly elastic, circular disks, *J. Fluid. Mech.* **171**, pp. 53-69.

Kachanov, Y. S. 1994 Physical mechanisms of laminar boundary-layer transition, *Annu. Rev. Fluid Mech.* **26**, pp. 411-482.

Lun, C. K. K., Savage, S. B., Jeffrey, D. J. et Chepurniy, N. 1984 Kinetic theories for granular flows : inelastic particles in Couette flow and slightly inelastic particles in a general flowfield, *J. Fluid. Mech.* **140**, pp. 223-256.

Malik, M. R. 1990 Numerical methods for hypersonic boundary layer stability, *J. Comp. Phys.* **86,** pp. 376-413.

Saric, W. S. 1994 Görtler vortices, *Annu. Rev. Fluid Mech.* **26**, pp. 379-409.

Sparrow, E. M. et Husar, R. B. 1969 Longitudinal vortices in natural convection flow on inclined plates, *J. Fluid. Mech.* **37**, pp. 251-255.