$Q_0 = c\,\Delta\zeta$. As a consequence, for a front propagating in the same direction as the grains i.e. downward the pile, c and Q_0 have the same sign, so the moving part is above the static one ($\Delta\zeta > 0$). On the contrary, a front propagating upward (with c and Q_0 of opposite signs) has a flowing side below the static side ($\Delta\zeta < 0$).

The second classification also corresponds to similar front properties : the two starting fronts are both close to shocks and keep steady shapes, while the two stopping fronts are continuously flattening. We will investigate this property using Saint-Venant approach. Introducing the free surface profile ζ and the local particle flux Q, the mass conservation and the momentum equation read [10] :

$$\partial_t \zeta + \partial_x Q = 0 \quad (1)$$

$$\partial_t Q + \partial_x E = F \quad (2)$$

where E is the energy and F the forces acting on the flowing layer.

Let us first discuss the effect of advection. If the grains velocity v is assumed to be uniform inside the flowing layer ($v = U$, see [11,12]), the advection does not distort the fronts shapes since it just translates them. This is not the case if this velocity profile presents a strong gradient Γ, as observed experimentally [13,14]. Then, higher grains in the front go faster than the lower ones and thus, advection tend to flatten the "start-up" and "stop-down" fronts and to sharpen the "start-down" and "stop-up" fronts. Assuming that the velocity profile is linear ($v = \Gamma z$) over the whole flow depth H (so that $Q = \Gamma H^2/2$ and $E = \Gamma^2 H^3/3$), the momentum conservation (2) becomes an equation governing the erosion/accretion of static grains :

$$\partial_t \zeta + \Gamma H \partial_x H = 0 \quad (3)$$

$$\partial_t H + \Gamma H \partial_x H = \frac{F}{\Gamma H} \quad (4)$$

As in solid friction, the flowing layer gains momentum when the free surface slope is larger than μ_{stop}. Thus, the force term should at least take into account gravity and a global friction acting on the flowing layer [12] : $F = gH(-\partial_x \zeta - \mu_{stop})$, and the momentum equation becomes :

$$\partial_t H + \Gamma H \partial_x H = \frac{g}{\Gamma}(-\partial_x \zeta - \mu_{stop}) \quad (5)$$

For a front propagating at a constant speed c (i.e. whose characteristics depend only on $x - ct$) on a static sandpile (of slope μ_{stop}), the free surface profile ζ verifies : $\partial_t \zeta = -c(\mu_{stop} + \partial_x \zeta)$. Using this relation, ζ can be eliminated between the conservation of matter and the momentum equation leading to :

$$\partial_t H + \left(1 + \frac{g}{\Gamma c}\right) \Gamma H \partial_x H = 0 \quad (6)$$

The gravity turns out to behave like advection but its action depends on the sign of c: it tends to sharpen the "start-down" and the "start-up" fronts and to flatten the "stop-down" and "stop-up" fronts.

Globally, advection and gravity have the same effect for the very sharp "start-down" front (Fig. 1 a) and for the very flat "stop-down" front (Fig. 1 d), since c is positive. For the two other fronts, advection and gravity compete each other. Experiments show that $-g/c$ is greater than Γ ($\Gamma \simeq 0.4\sqrt{g/d}$ and $c \simeq -\Gamma d$). Thus, the "start-up" front tends to make a shock (Fig. 3 c) and the "stop-up" front to flatten (Fig. 3 b).

To show the effect of advection and gravity, we have sought for fronts propagating with a steady shape. Seeking for flattening fronts does not change the result. Indeed the Saint-Venant equations (3,5) admit a solution

$$H = \frac{(x/t)^2}{\Gamma(x/t) + g}$$

flattening as $1/t$ which corresponds to the "stop-down" ($x > 0$) and "stop-up" ($x < 0$) fronts. However in this solution, the limit between flowing and static grains does not move. In order to recover the propagation of these fronts, a further effect has to be taken into account : the trapping of grains at small flowing heights [15]. Indeed, when the free surface angle φ is smaller than the starting angle φ_{start}, a single moving grain stops, trapped by the bumps of the static pile. The trapping of grains is responsible for the hysteretic transition between flowing and static states [15] i.e. of the difference between φ_{start} and φ_{stop} : for a flowing height H smaller than one grain, the flowing layer freezes under a larger friction $\mu \simeq \mu_{start}$. Including this effect, we explain both the flattening of the "stop-down" and "stop-up" fronts and their propagation (Fig. 4).

The trapping effect can be directly evidenced in the experiment. The periodic dynamics presented above, with fronts propagating between the two sides of the cell, are observed for a moderate grain flux. For a too high flow rate, $Q > 150 - 200\ g^{1/2}d^{3/2}$ for the avalanches, $Q > 40 - 80\ g^{1/2}d^{3/2}$ for the anti-avalanches, a continuous flow can be observed (as in the rotating drum experiment [16,13]). But for a too small flux, $Q < 3\ g^{1/2}d^{3/2}$ and $Q < 8\ g^{1/2}d^{3/2}$ respectively, we obtain a new regime (Fig. 5) [17]. When the sand accumulates in the filling cell, it forms a bump which starts flowing when the slope exceeds φ_{start}. In the intermediate flux range, the propagating front is constantly fed by the incoming flux so that it propagates down to the bottom. But for a too small flux, we can nearly consider that only the small quantity of grains in the bump moves. While going down it spreads, and the flowing height decreases below the trapping height. The small avalanche thus freezes quickly before reaching the cell's boundary, leaving a step at the place where the front was. The next avalanche starts and spreads the same way, but as it would stop at the same length, it find the step and thus propagates one step further, leaving a flat surface where it has flown,