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SECOND EUROPEAN SUMMER SCHOOL on MICROSCOPIC QUANTUM MANY-BODY THEORIES and their APPLICATIONS

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BOSE-EINSTEIN CONDENSATES WITHIN A PERIODIC POTENTIAL

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Experiments with a Rb Bose-Einstein condensate

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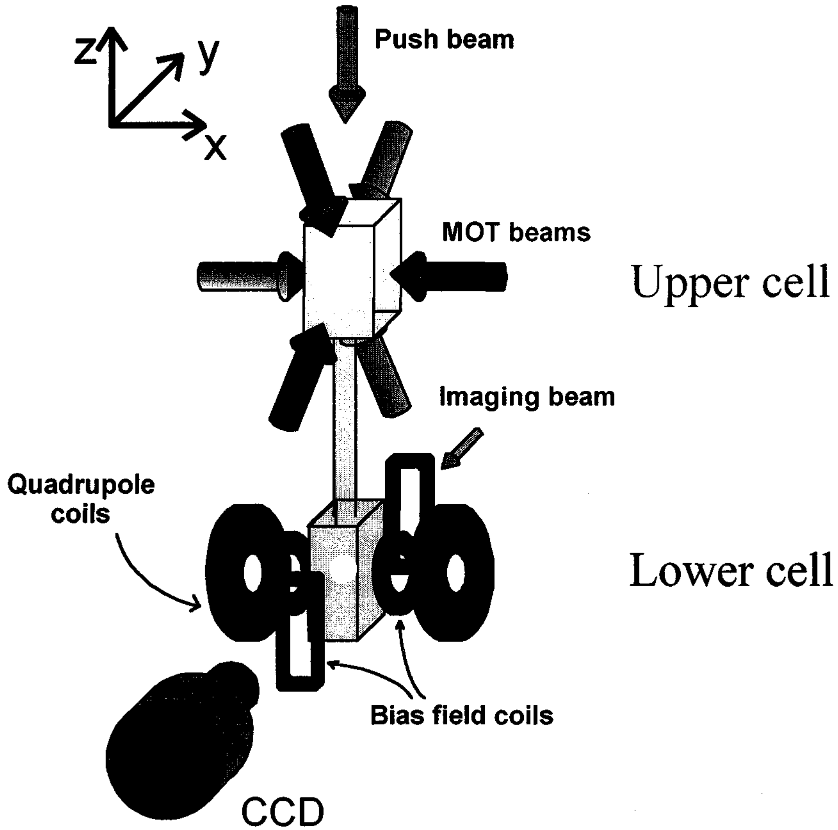
Outline

- Experimental set-up
  - BEC in a triaxial TOP trap
  - Trap characteristics
  - Atom-micromotion
- Condensate in Optical lattice
  - Bragg diffraction
  - Bloch-oscillations
  - Tunneling in optical lattices
  - Landau-Zener transitions
  - Atom-atom interactions
  - The effective potential and beyond
- Condensate ionization
- Conclusions

Experimental setup



- Double MOT (Magneto Optical Trap) apparatus

In the lower cell:

- High vacuum
- Strong magnetic confinement of cold atoms
- Optical detection (Absorption imaging)

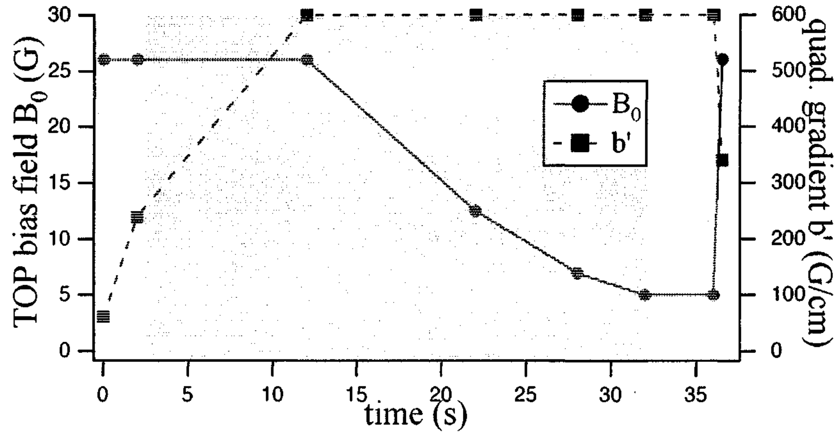
Experimental timing

• During a cycle atoms are collected in the upper MOT and then transferred and accumulated to the lower MOT

Once the lower MOT has been filled ($\approx 5 \cdot 10^7$ atoms):

• compressed-MOT phase and molasses phase
• optical pumping into the $|F=2, m_F=2\rangle$ ground state and transfer into the TOP trap
• compression and evaporative cooling

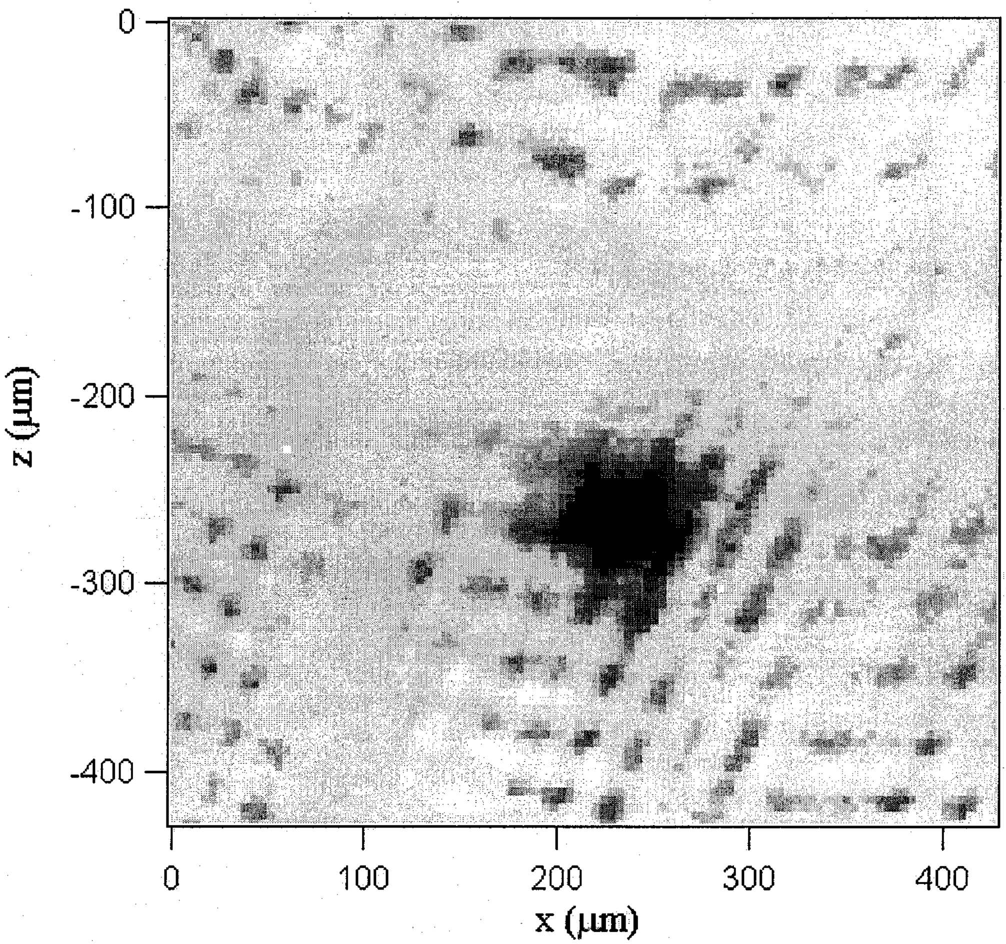
Compression and evaporative cooling sequence:



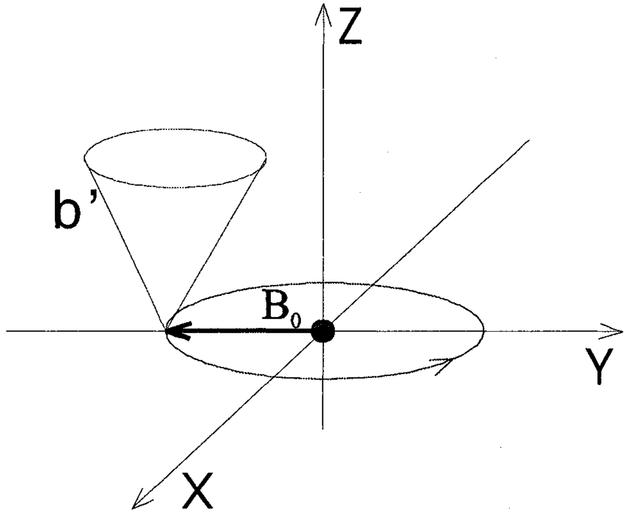
• Once the condensate is formed, release of the atoms from the trap and time of flight

•Monitoring is at a variable time before release

Experimental results for BEC



The TOP trap

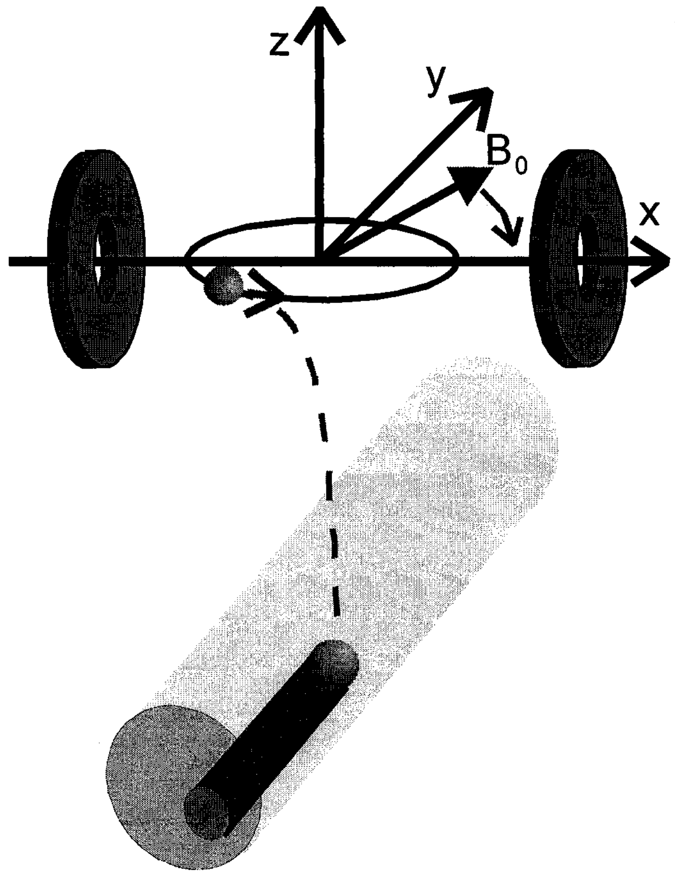


$$\vec{B} = \begin{pmatrix} 2b'x \\ -b'y \\ -b'z \end{pmatrix} + \begin{pmatrix} B_0\cos(\Omega t) \\ B_0\sin(\Omega t) \\ 0 \end{pmatrix}$$

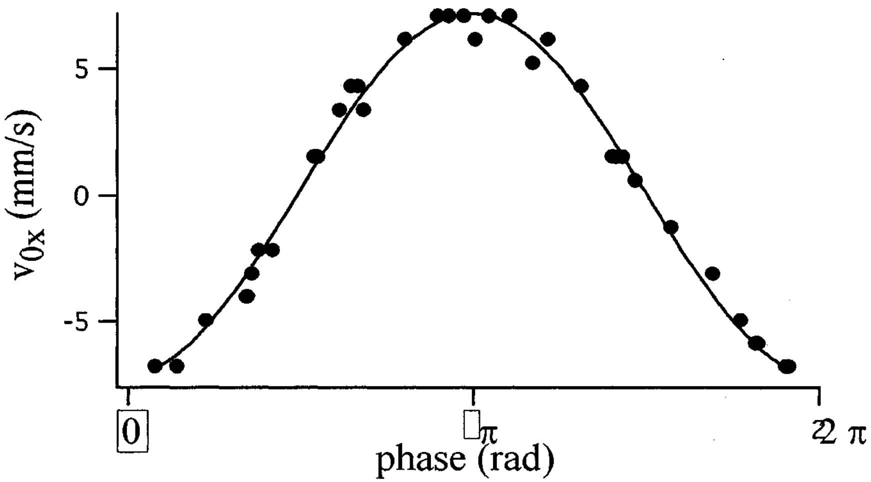
•in a TOP trap particles are subject to a
•*time-dependent inhomogeneous* magnetic field

• in our geometry the bias field rotates in a plane containing
• the symmetry axis of the quadrupole: triaxial TOP trap

Micromotion: Time of flight method



- atoms move in elliptical orbit
- trap switched off
- →atoms fly off with tangential velocity
- atoms are imaged after 8 ms of free fall



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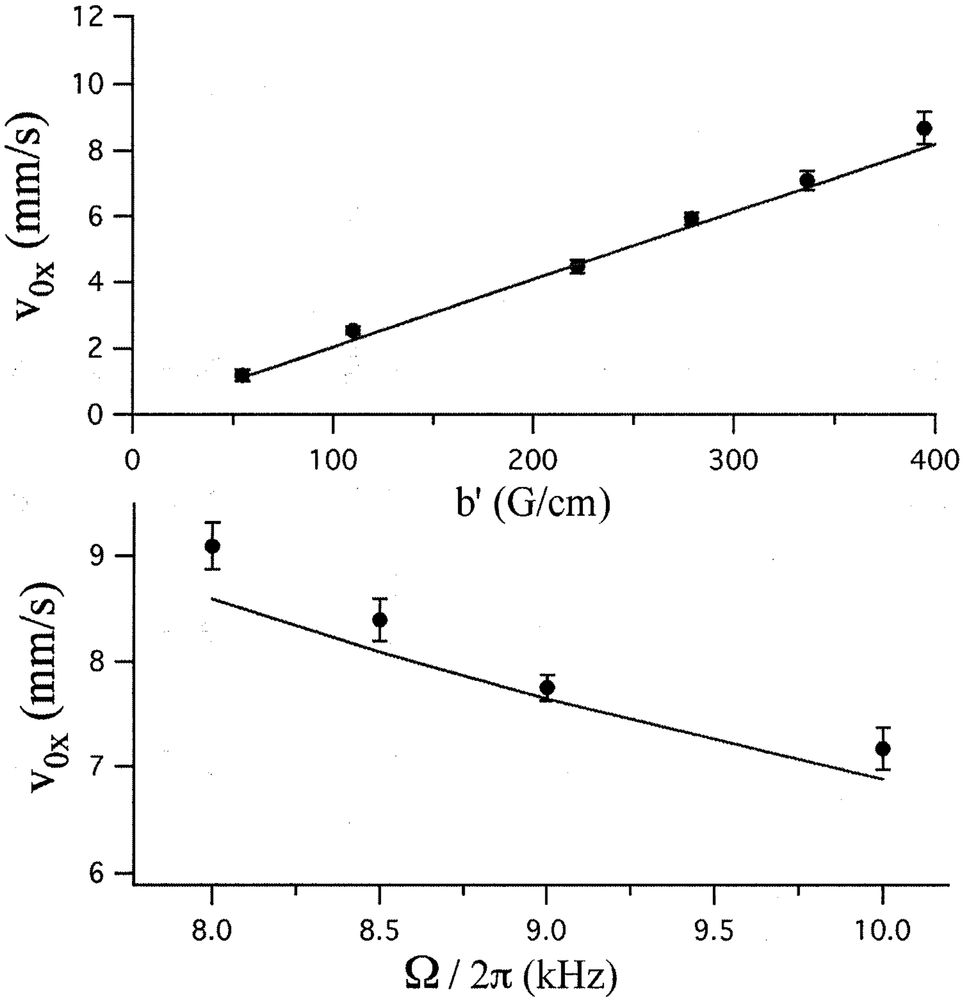
Micromotion results

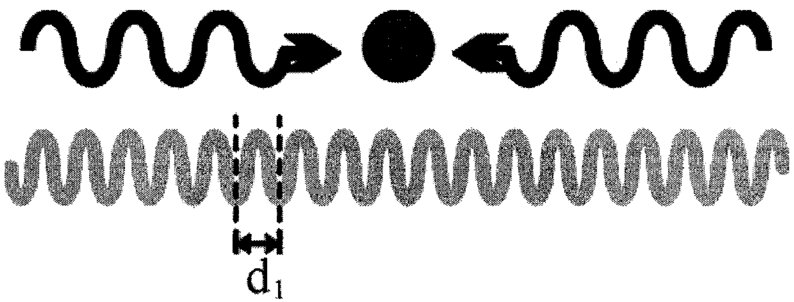
In the limit of small sag,
the velocity of the harmoni
atomic micromotion is

$$V_{0x} = 2\mu b' /(M\,\Omega)$$

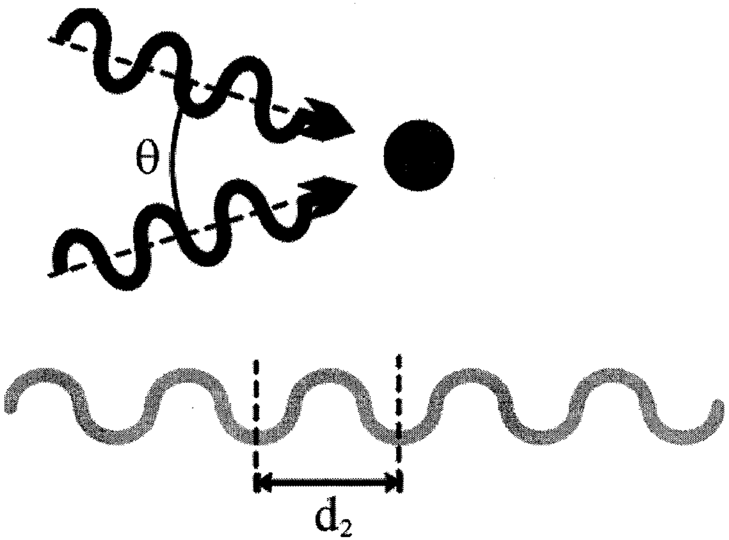


•*J.H. Müller, et al,*
•*Phys. Rev. Lett.,* 85, *4454 (2000)*

1-D optical lattices



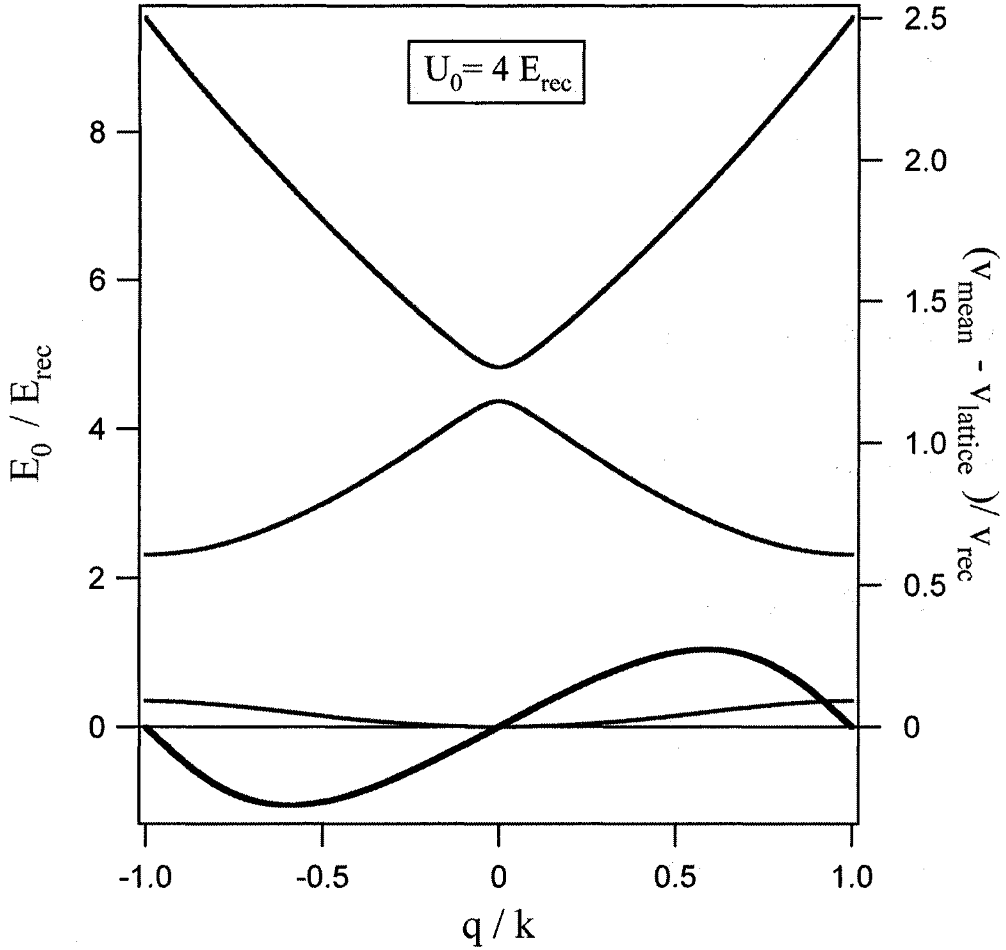
$$d_1 = \lambda/2 = 0.39\mu m$$



$\Theta = 28°$
Δ = 25-30 GHz
$I_{lattice}$ = 230 mW/cm^2

$$d_2 = \frac{\lambda}{2\sin(\vartheta/2)} = 1.56\mu m$$

BEC in periodic potentials



Band structure
for 1-D lattice

$$U_0 = \frac{1}{3}\frac{\Gamma^2 s_0}{\Delta}$$ lattice depth

$$s_0 = I/I_{sat}$$

$\Delta \approx$ blue detuning from the ^{87}Rb resonance line

$$E_{rec} = \frac{\hbar^2 k^2}{2m_{Rb}} \qquad k = 2\pi / \lambda_L$$

Atomic momentum within Lattice

Atoms with a given quasimomentum are delocalized over the lattice potential wells.
Bloch states of quasimomentum q within the n band, are coherent superposition of a number of plane waves, momentum states.

$$|n,q\rangle = \sum_{m=-\infty}^{m=\infty} a_{n,q}(m)\,|p = q + 2m\hbar k_L\rangle$$

Transitions between momentum states $|p = q + 2m\hbar k_L\rangle$ are produced by the periodic potential. Lattice transfers momentum to atoms in units of

$$2\hbar k_L = 2\hbar \frac{2\pi}{d}$$

Those transitions interpreted as Bragg scattering of the atoms by the periodic potential.

Loading the Condensate within Lattice

•Sudden loading of the BEC into the lattice, turning the optical lattice on abruptly:

several Bloch states in different lattice bands are populated

•Adiabatic loading of the BEC into the lattice:

time for turning on the lattice longer than the time scales of the system (*)

•In our experiment ramping of one lattice beam intensity (with duration 200μs) produces a condensate in the $|0,q>$ state. Control, releasing the condensate from the trap, that only one quasimomentum in the fundamental band is loaded.

(*) Berg-Sørenson and Møller, Phys. Rev. A 58, 1480 (1998), Choi and Niu, Phys. Rev. Lett. 82, 2022 (2000), Band, Malomed and Trippenbach, cond-mat/0108114

Collective condensate dynamics

Solve the time-dependent Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=\left\{\frac{p^2}{2M}+V_{ext}(x)+N\frac{4\pi\hbar^2 a}{M}|\psi|^2-Fx\right\}\psi(x,t)$$

where both the external potential and the nonlinear term are periodic in space.

This equation is equivalent to that of a single atom within a periodic structure.
However it contains a different physical meaning, determining the time-dependent state of the condensate and its excitations.

Theoretical Parameters(*)

Tunneling rate between neighbouring lattice wells:

$$\gamma = \frac{1}{\hbar}\mathrm{Re}\left\{\int d^3r\,\psi_n^*(\vec{r})\left[-\frac{\hbar^2}{2M}\nabla^2 + V_{ext}(\vec{r})\right]\psi_{n+1}(\vec{r})\right\}$$

Atom-atom interaction within each lattice well:

$$\kappa = \frac{2\pi\hbar a_s}{M}\int d^3r\,|\,\psi_n(\vec{r})\,|^4 = \frac{2\pi\hbar a_s}{M}\int d^3r\,|\,\psi_{n+1}(\vec{r})\,|^4$$

(*) Javanainen, Phys. Rev. A 60, 4902 (1999), Orzel et al, Science 291, 2386 (2001).

BECs in a periodic potential: theory

- Gross-Pitaevskii equation:

$$i\hbar\frac{\partial\phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial x^2} + V_0\cos(k_L x)\phi + \frac{4\pi n\hbar^2 a}{m}|\phi|^2\phi$$

- rescaled:

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + U_0\cos(x)\phi + C|\phi|^2\phi$$

- where

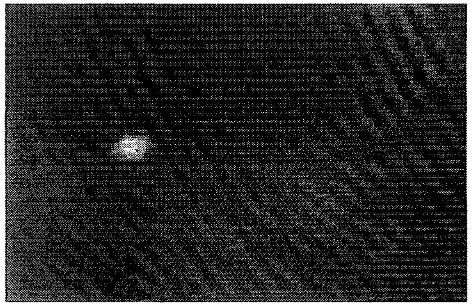
$$C = \frac{\pi n a_s}{k_L^2} = \frac{n a_s d^2}{4\pi}$$

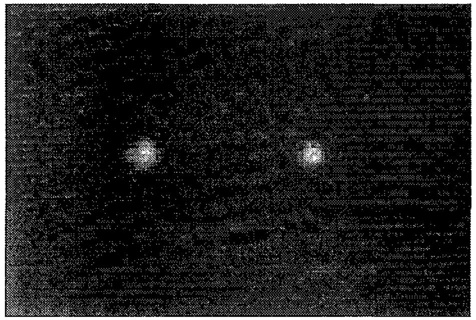
and

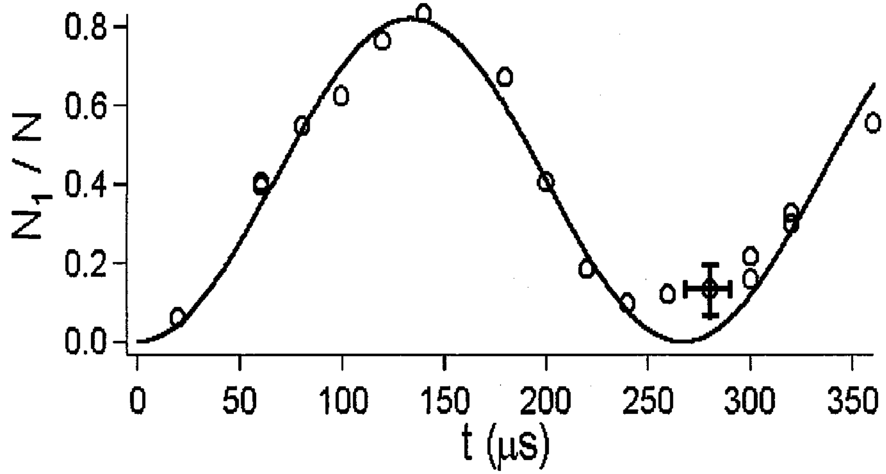
$$U_o = \frac{2mV_o}{\hbar^2 k_L^2}$$

Rabi oscillations

• drive transitions between momentum states by applying a pulsed moving lattice satisfying the Bragg condition
• as a function of time, the two momentum states exhibit a Rabi oscillation







In optical lattice with d = 0.39 μm when $\delta = 4E_{rec} / \hbar$ = 15.08 kHz

$\Rightarrow$ $\Omega_{Rabi} \approx 3.6$ kHz

$\Rightarrow$ $U_0 = 2\,\hbar\,\Omega_{Rabi} \approx 2.1\,E_{rec}$

N_1 = number of atoms in the momentum state $|p = 2\hbar k\rangle$

Coherent acceleration of BECs-I

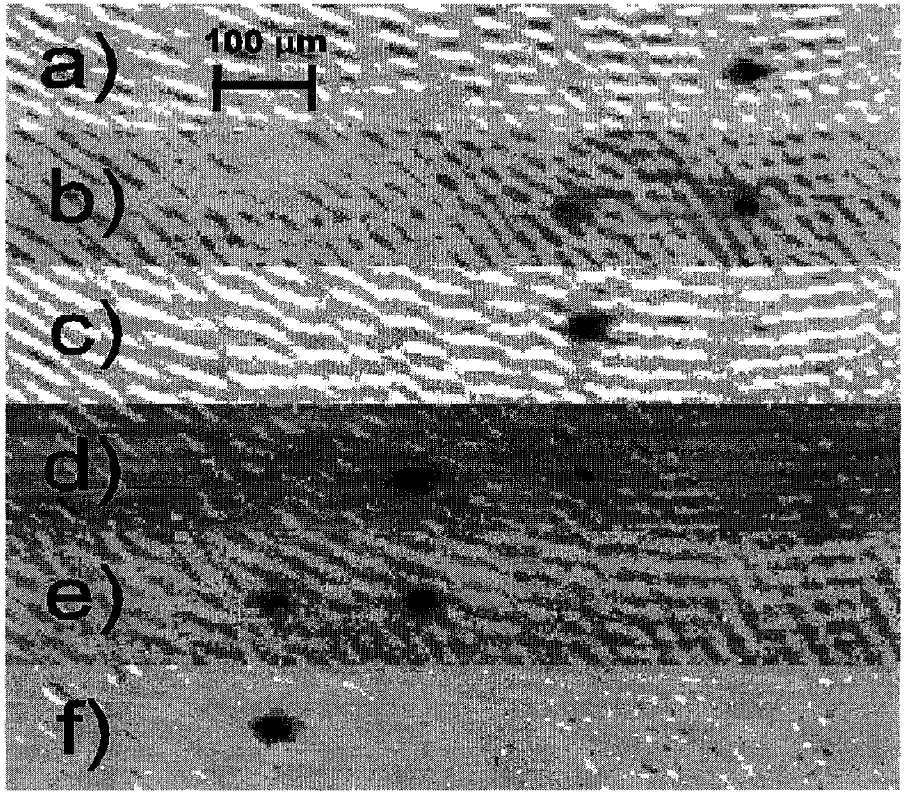
1. Coherent acceleration:

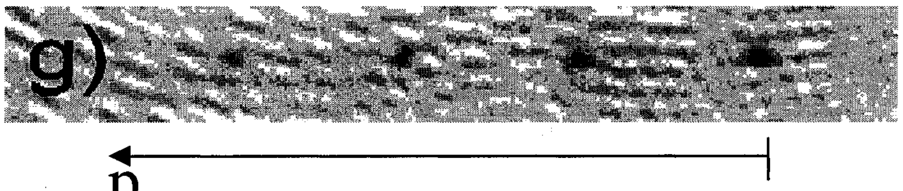
linear increase of the detuning δ between the beams;
duration 2-3 ms

$$a = \frac{\lambda}{2}\frac{d\delta}{dt}$$

2. Take snapshots after different interaction times

Coherent acceleration of BECs-II





$U_0 = 2.3\ E_{rec}$

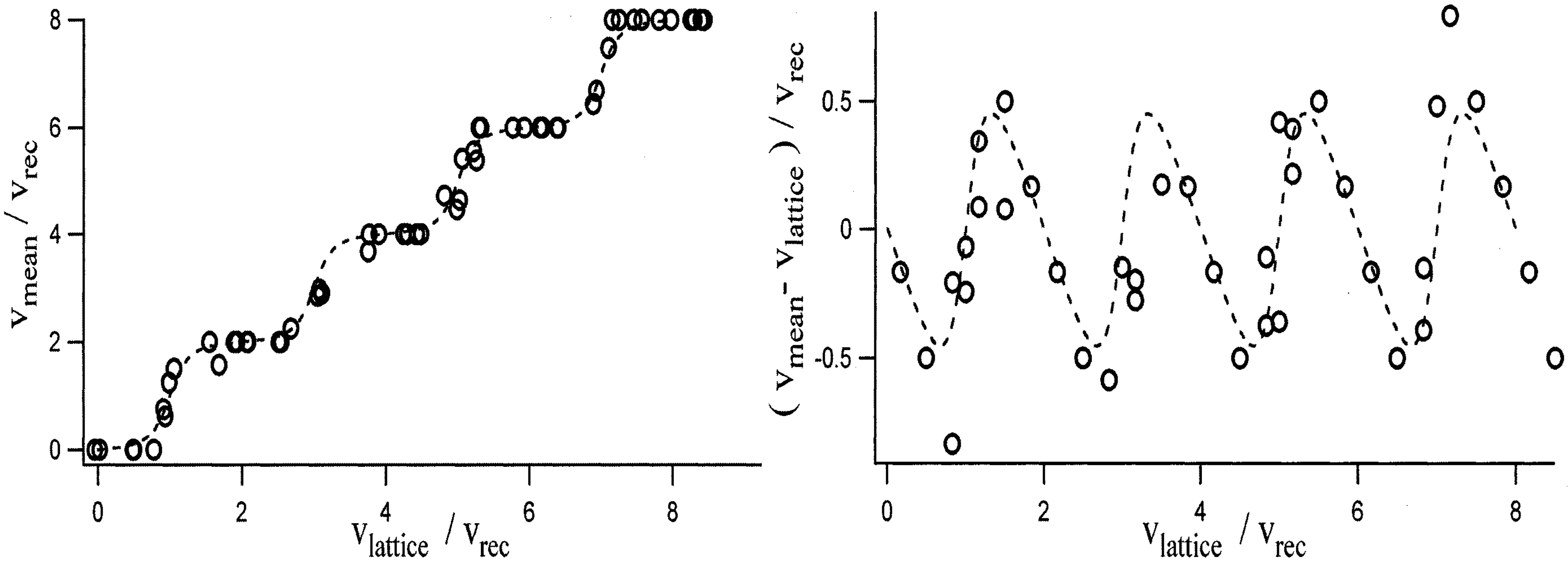
Adiabatic passage between momentum states $|\ p = 2m\hbar k >$ and $|\ p = 2(m+1)\hbar k >$

In a) -f): a = 9.81 m/s^2 acceleration applied for 0.1, 0.6, 1.1, 2.1, 3.0, 3.9 ms respectively

In g): a = 25 m/s^2 applied for 2.5 ms.

Coherent acceleration of BECs-III

$d = 0.39\ \mu m \quad U_0 = 2.3\ E_{rec} \quad a = 9.81\ ms^{-2}$



In the rest frame of the lattice:
Bloch oscillations

Lattice & micromotion

$d = 1.56\ \mu m$
$U_0 = 30\ E_{rec}$

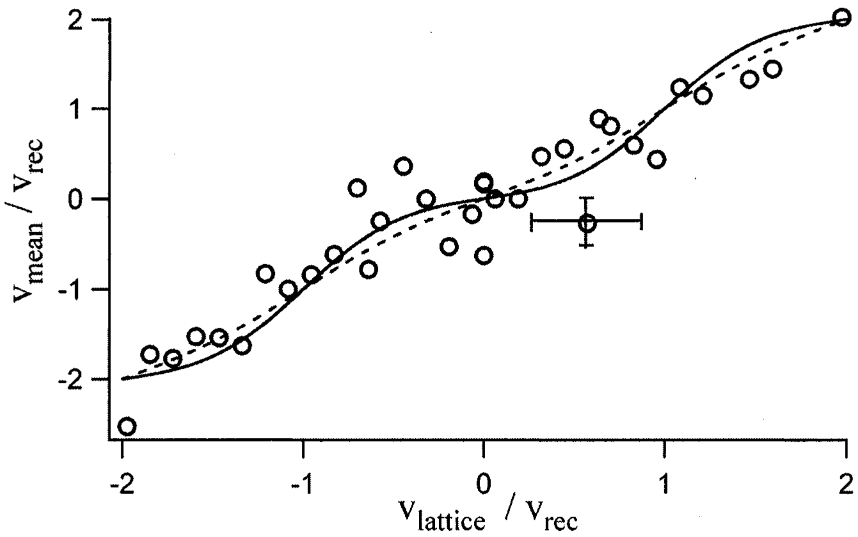
$\Rightarrow$ The lowest band is almost flat

Initial velocity of the condensate

$$v \approx v_{micromotion} \approx v_{rec}$$

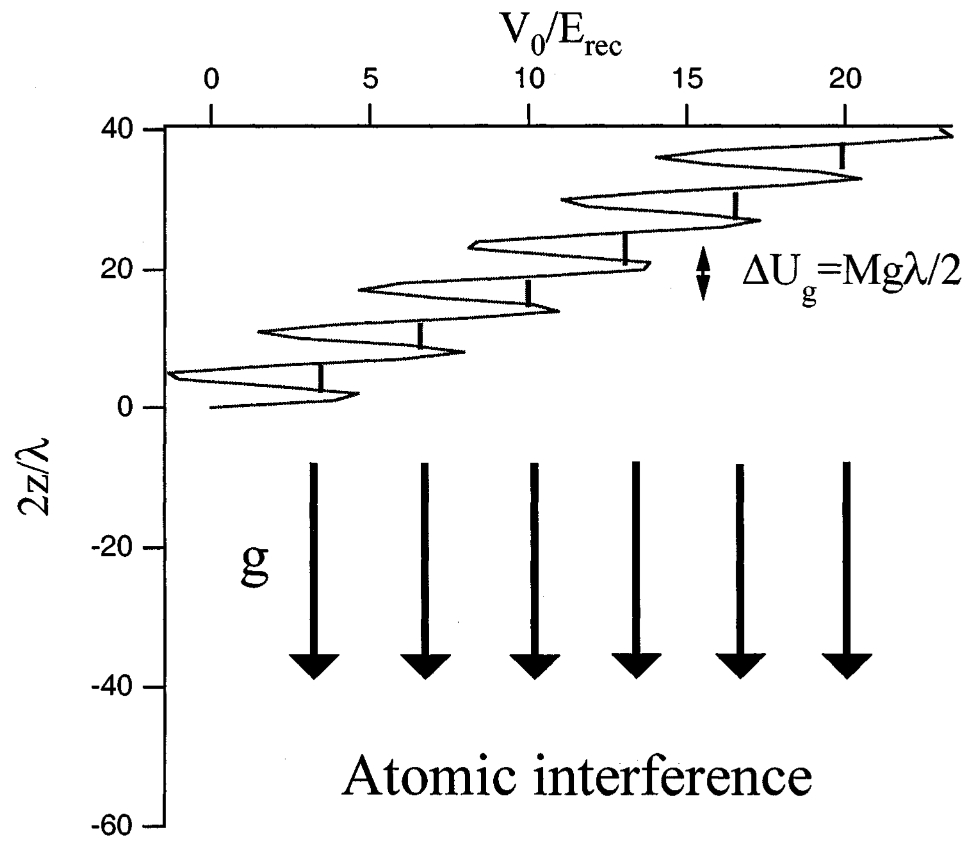
$\Downarrow$

- Intrap experiment
- Compensation of the micromotion in the rest frame of the lattice: Phase modulation of one lattice beam



Quantum interference of falling BEC

Following Anderson and Kasevich, Science 282, 1686 (1998)



Initial velocities:

$$v_i = n\frac{2\hbar k_L}{M}$$

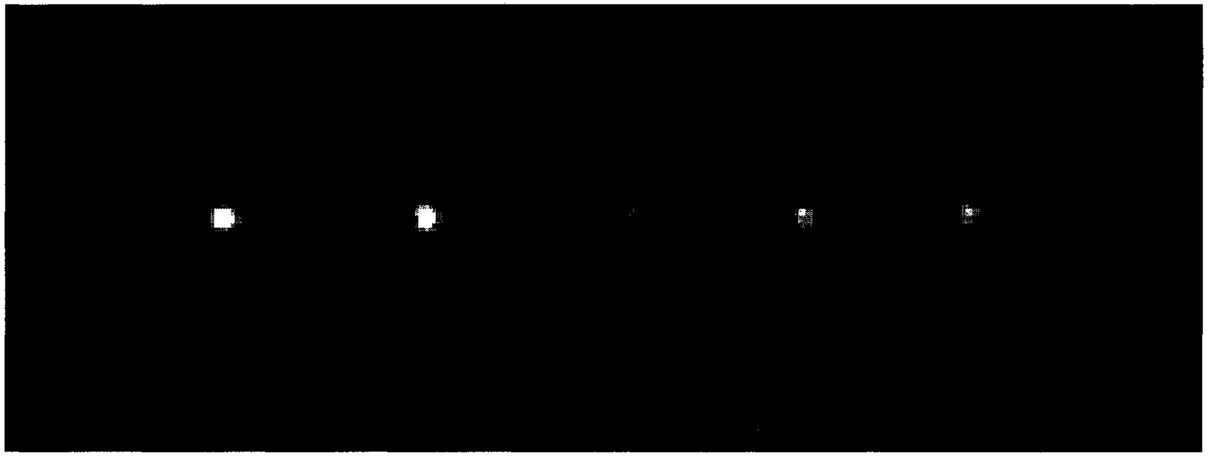
Arrival times for falling through a path L with

$$v_i^2 \ll Lg$$

$$t_f = \frac{v_i}{g} = n\frac{2\hbar k_L}{Mg} = n\frac{2}{Mg\lambda}h$$

Tunneling in optical lattices I

• when condensate is accelerated above a critical velocity, atoms can tunnel into higher-lying bands or the continuum



• this tunneling can be used in order to detect mean-field effects through a reduction of the effective potential seen by the atoms

see also Anderson and Kasevich, Science **282**, 1686(1998)

Landau-Zener tunneling

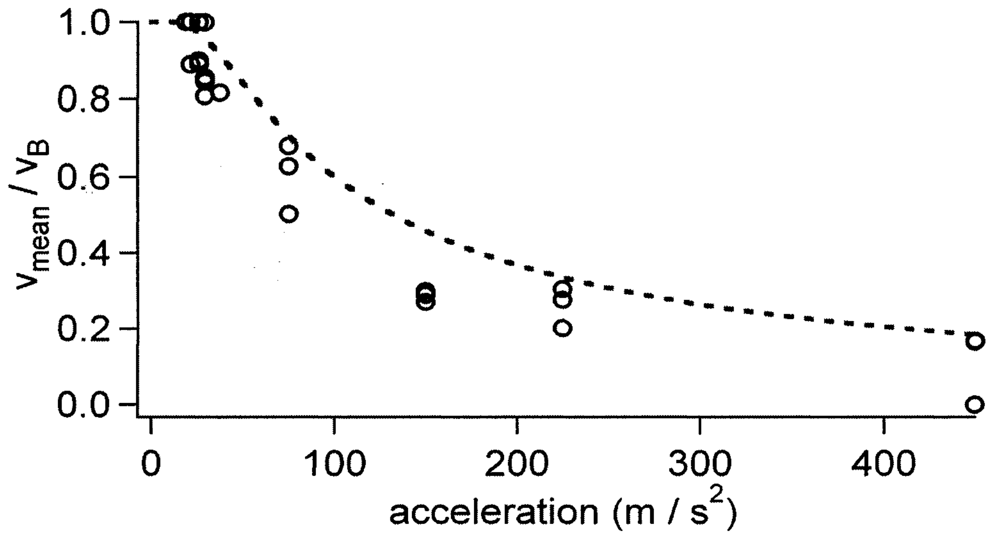
When $q/\mathrm{v}_{rec} = n$

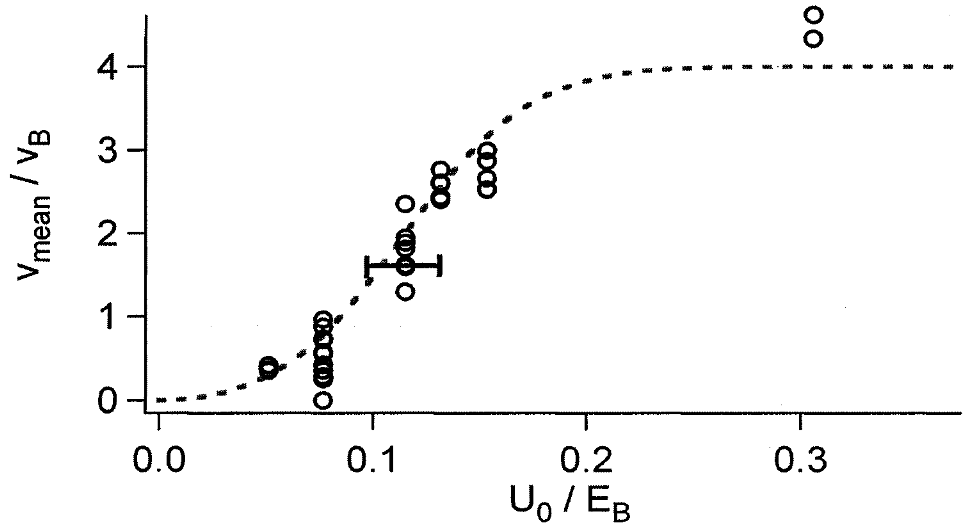
$n = \pm 1, \pm 2, \ldots$

a fraction r of atoms undergoes tunneling into the first excited band

$$r = \exp(-a_c/a)$$

$$a_c = \frac{\pi U_0}{16\hbar^2 k}$$





Effective periodic potential for BEC

- Rescaled Gross-Pitaevskii equation:

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + U_0\cos(x)\phi + C|\phi|^2\phi$$

- in the perturbative limit, find[1]

$$i\frac{\partial\phi}{\partial t} = -\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + U_{eff}\cos(x)\phi$$

- $U_{eff} = \frac{U_0}{1+4C}$ where $C = \frac{n a_s d^2}{4\pi}$

•effective potential in presence of interactions

→ Potential experiences by the atoms is reduced owing to the nonlinearity introduced by interactions

[1] Choi and Niu, PRL **82**, 2022 (1999)

Measurement of the effective potential-I

• Because the optical lattice potential is modified, the tunneling can be used in order to detect mean-field effects through a reduction of the effective potential seen by the atom:

$$r = \exp(-a_c/a)$$

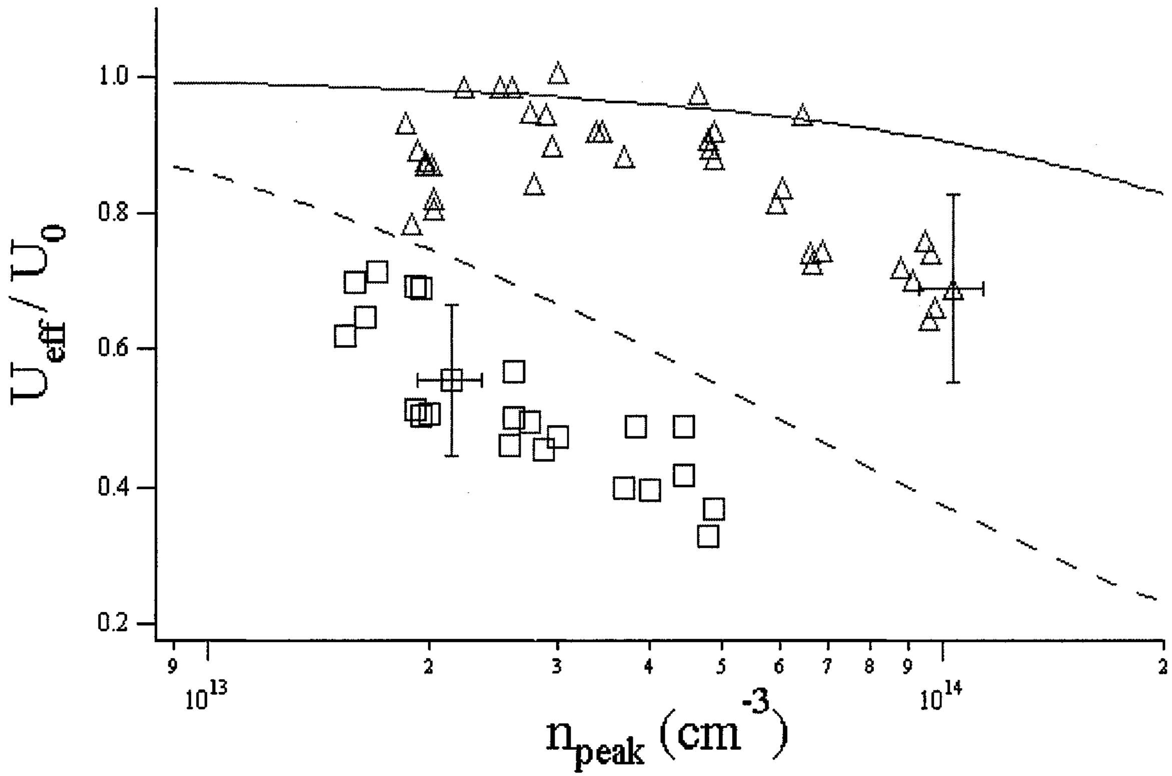
$$a_c = \frac{\pi U_{eff}}{16\hbar^2 k}$$

- measurement of tunneling probability
- derive U_{eff} from the tunneling probability
- in-trap measurement for transverse confinement
- change trap frequency to vary density

Measurement of the effective potential-II

Experimental results:

- Change trap frequency to vary density



Different lattice geometries to enhance C
Red line and data;
Counterpropagating beams
Blue line and data:
angle-tuned geometry

Measurement of the effective potential-III

Choi and Niu theory: $$C = \frac{1}{4\pi} n a_s d^2$$

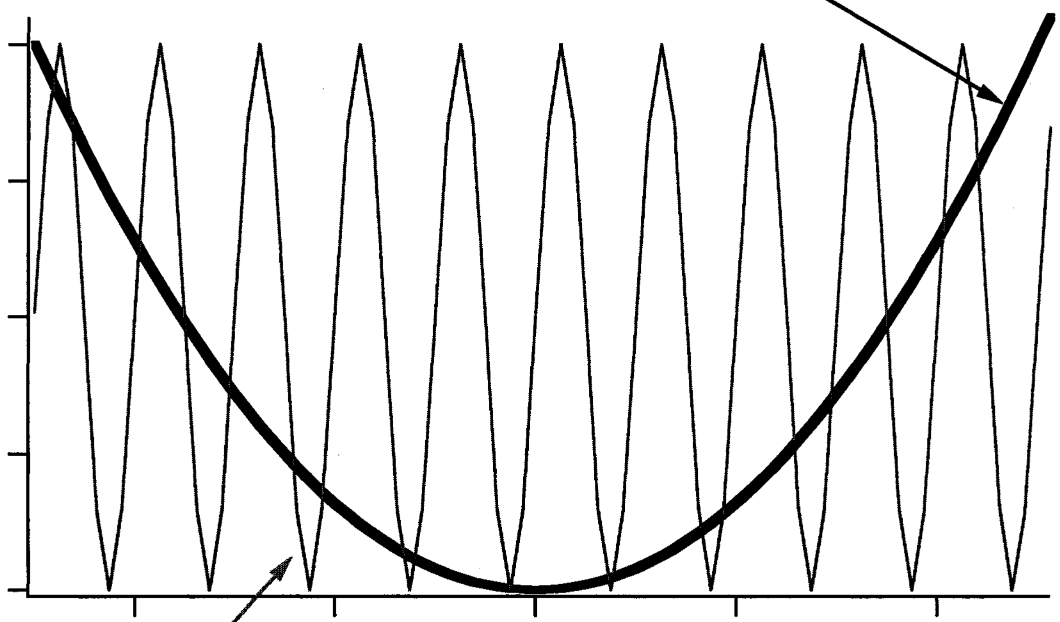
Our definition with $\bar{n}$ average: $$C = \frac{1}{4\pi} \bar{n} a_s d^2$$

Analyses by Jackson et al, Phys. Rev. A 58, 2417 (1998) and Steel and Zhang, cond-mat/9810284, with harmonic transverse confinement:

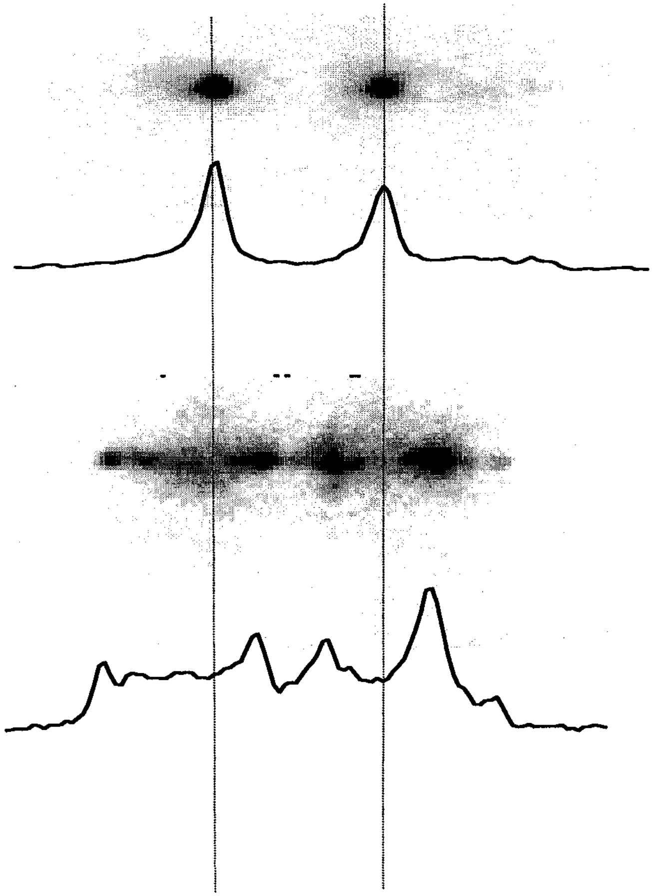
$$C = 2 \frac{N}{(a_{ho\perp})^2 (2k_w + 1) d} a_s d^2$$

Measurement of the effective potential-IV

Finite size role?



Nonlinear effects in the counterpropagating configuration



New momentum states appear, which are not only shifted by the lattice momentum !

Instabilities in optical lattices

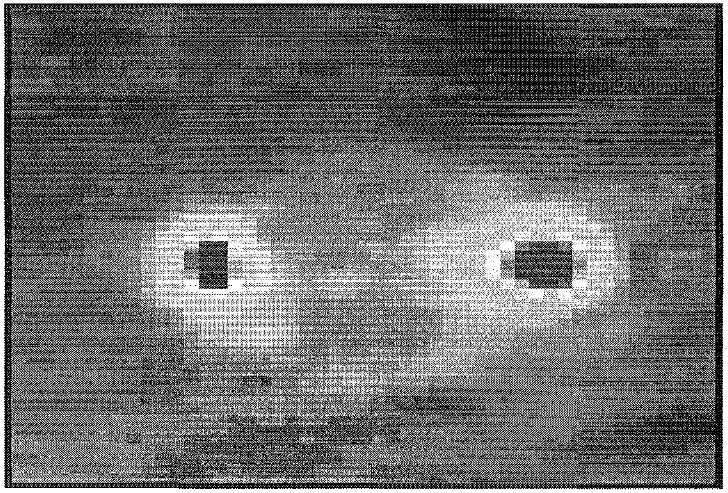
- Berg-Sørensen and Mølmer, Phys. Rev. A 58, 1480 (1998)
  Rayleigh-Bénard type instability in the transverse coordinates may take place.

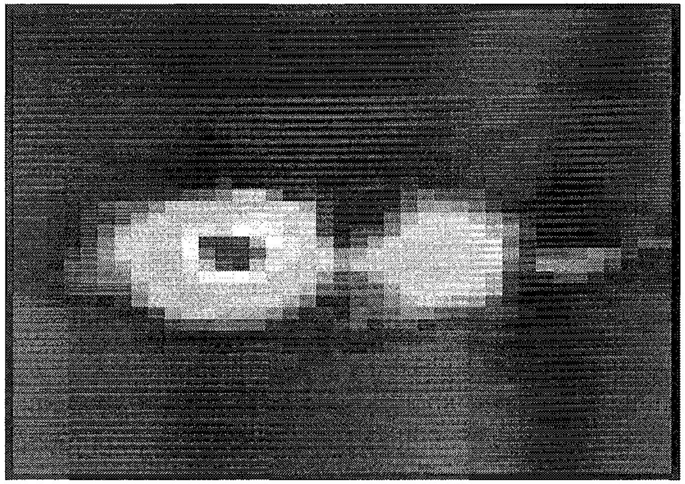
- Wu and Niu, cond-mat/0009455
  Dynamical instability, resulting in period doubling and other sort of symmetry breaking of the system, for small accelerations. Thus $a > \frac{\hbar^2 k^3}{10 m^2}$ it is requested.

- Konotop and Salerno, cond-mat/0106228
  Modulation instabilities for BEC in optical lattices

Bragg spectroscopy at high densities

•Observation of s-wave scattering collisions leads to a collisional halo corresponding to occupation of other momentum states:





angled lattice configuration

See also
Chikkatur et al: Phys. Rev. Lett. 85, 483 (2000)
Greiner et al: cond-mat/0105105

Other nonlinear effects in BECs

• Mott insulator transition through interplay between tunneling and on-site interactions (Jaksch *et al.*, PRL **81**, 3108)

• breakdown of Bloch oscillations due to dynamical instabilities at zone edges (Wu and Niu, cond-mat/0009455)

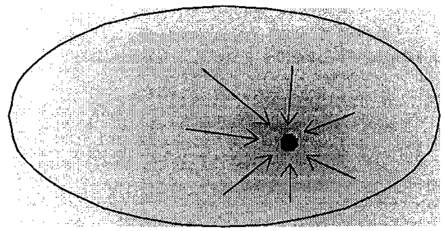
• inhibition of phase-space diffusion in a delta-kicked harmonic oscillator (Gardiner *et al.*, PRA **62**, 023612)

Ion-condensate Interactions

• Experiments using non dissipative traps permit to study energy transfer from charged particles to neutrals.

•Typical interaction energy between an ion and a polarizable Rb ground state atom at densities of $\sim 10^{14} cm^{-3}$:

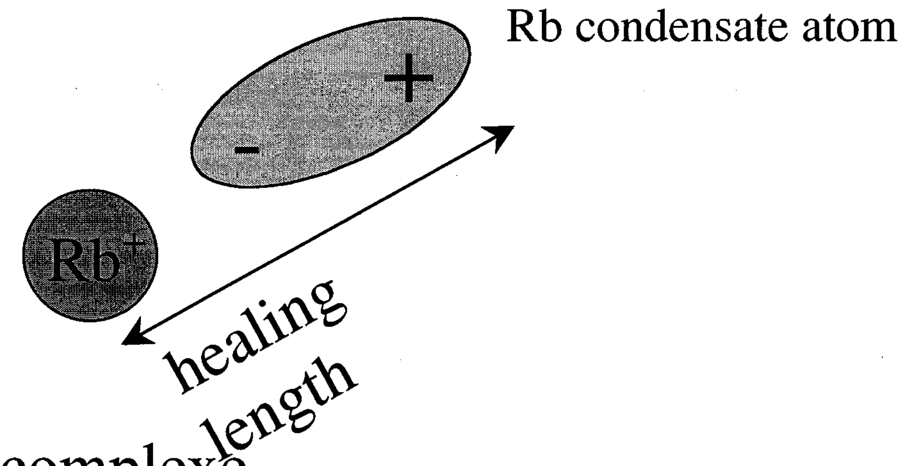
$$E/k_B \approx 100 \text{ nK}$$



•Experimentally the interaction time depends critically on stray electric fields.

Ions in superfluid helium[1] and in Rb condensate

- Ions as microscopic probe particles
- Formation of ion complexes:



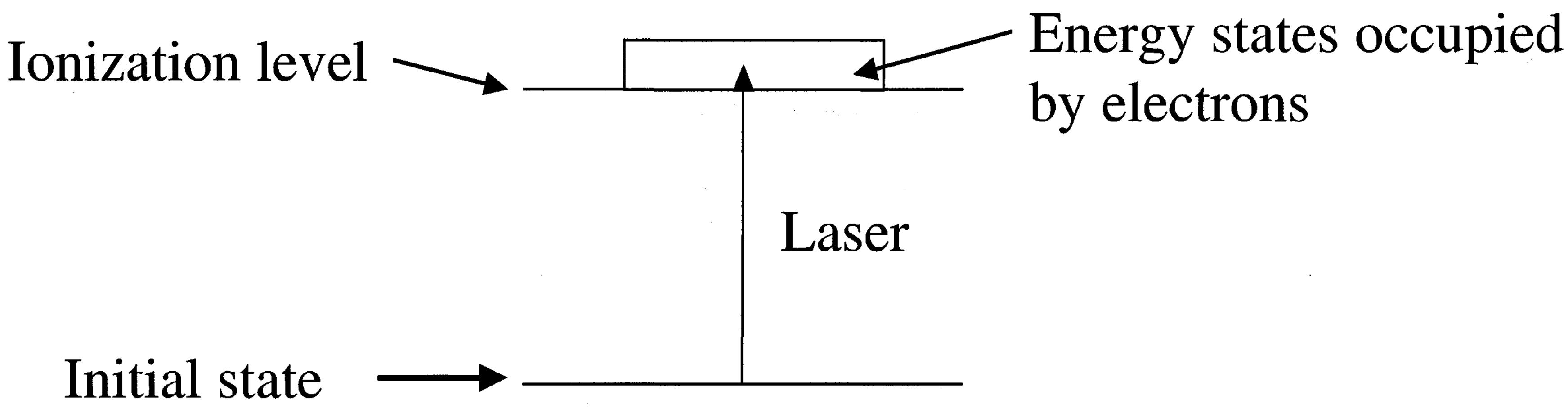
- Effective mass of the complexe
- Mobility of the ion complexe diverges at T=0
- Creation of vortices by ion motion

[1]Refs: G. Careri, in "*Progr. Low Temperature Physics*", vol.III, p. 58, 1961; and F. Reif, in "*Quantum Fluids*", eds. N. Wieser and D.J. Amit, (Gordon and Breach, 1970) p.165.

Condensate ionization

Fermi-Dirac statistics of produced electrons.

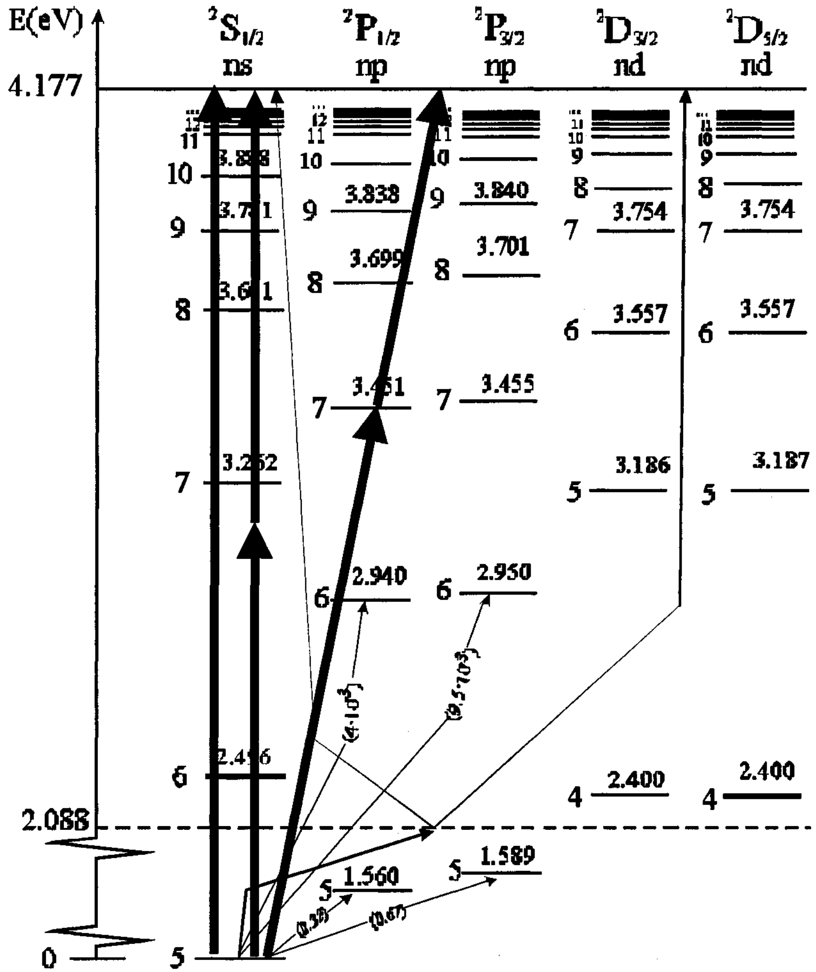
Ionization probability proportional to $1-n_e$, where n_e represents the cell occupation of the final states.



I. Mazets, *Quant. Sem. Opt.* 10, 675 (1998)

P. Zoller, *private communication*

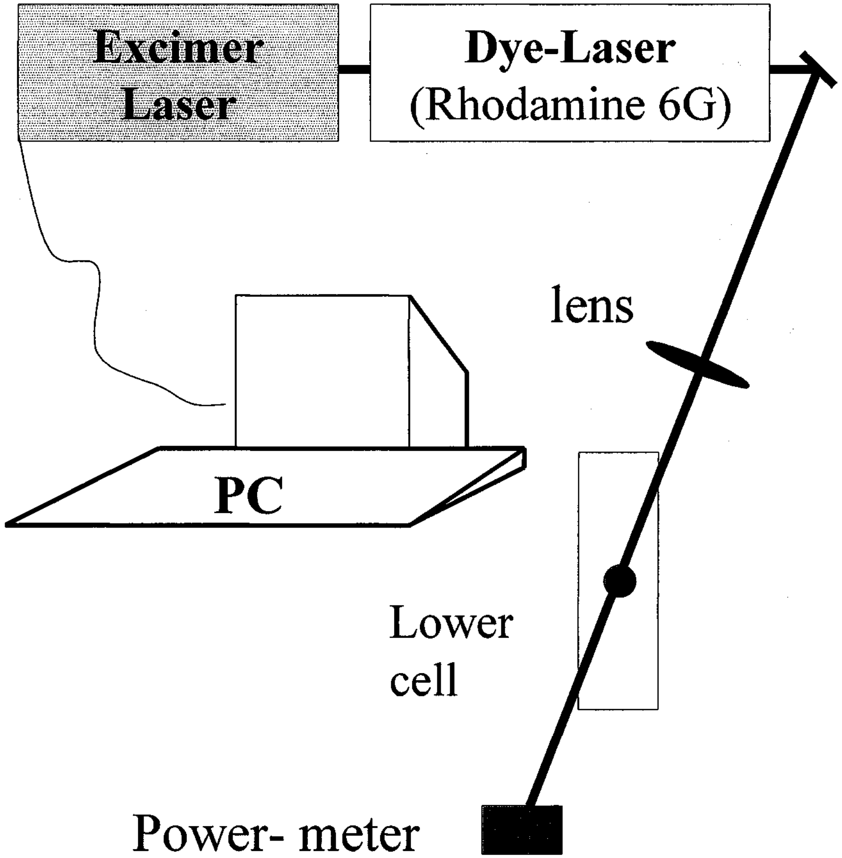
Diagram of Rb Energy Levels

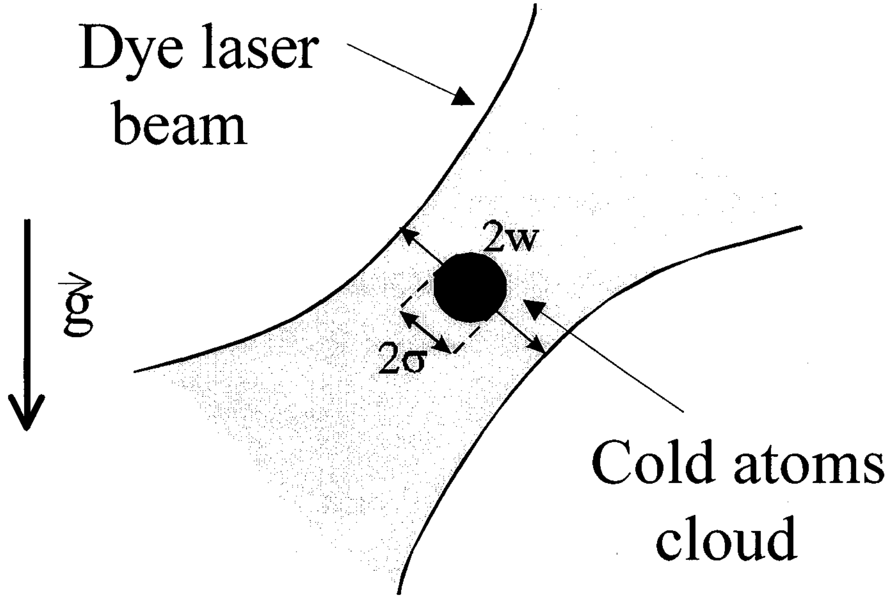


•1 or 2 photon ionization from the ground state.

•Monitor the condensate containing electrons and ions

Ionization geometry





- σ depends on experimental parameters: from 70 to 10 μm
- $w \cong 100\ \mu m$

Conclusions

•We have loaded a Bose-Einstein condensate into one-dimensional off resonant optical lattices.

•We accelerated the condensate by chirping the frequency difference between the two lattice beams.

•For small values of the lattice well-depth, Bloch oscillations were observed.

•Landau-Zener leading to a breakdown of the oscillations was studied.

•A regime of instabilities was reached at large atomic densities.

•The BEC ionization introduces a new area at the border between atomic physics and solid state physics.