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SMR/1328/10

School on the Physics of Equatorial Atmosphere

(24 September - 5 October 2001)

Earth's Magnetic Field and Effects on Charged Particles

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EARTH'S MAGNETIC FIELD AND EFFECTS ON CHARGED PARTICLES

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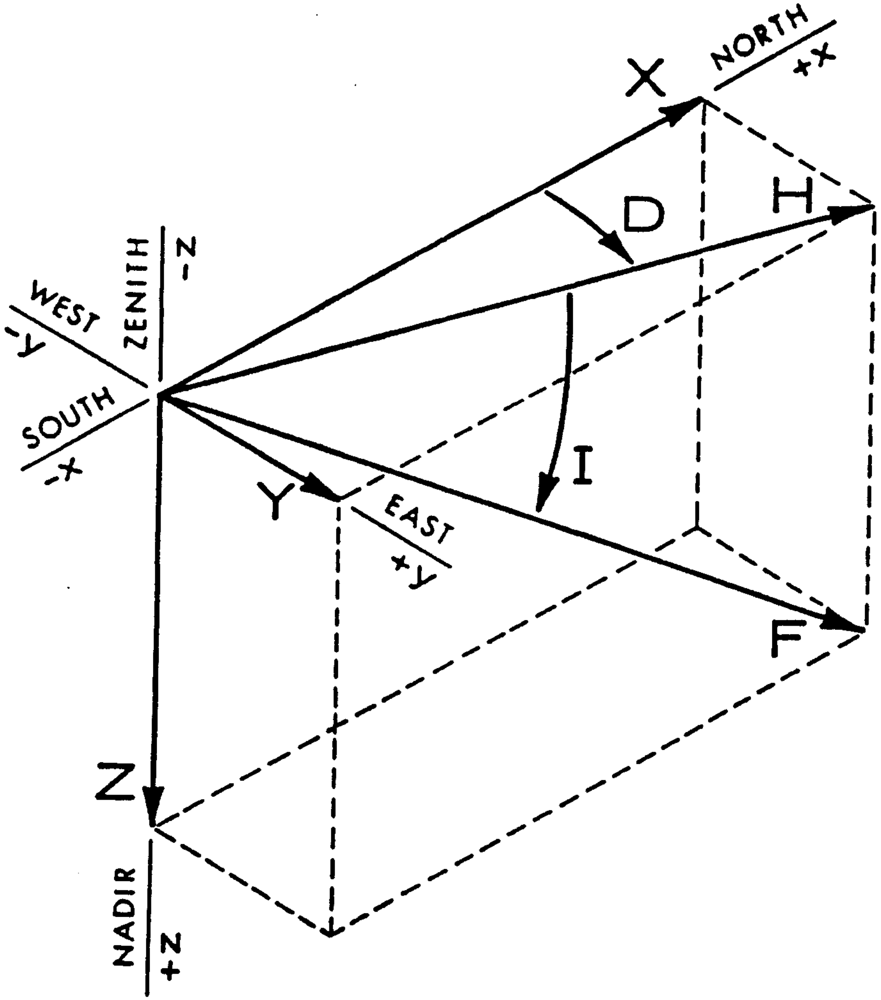
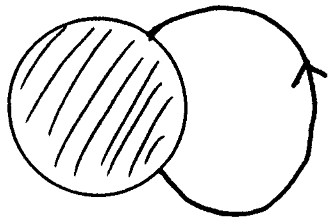
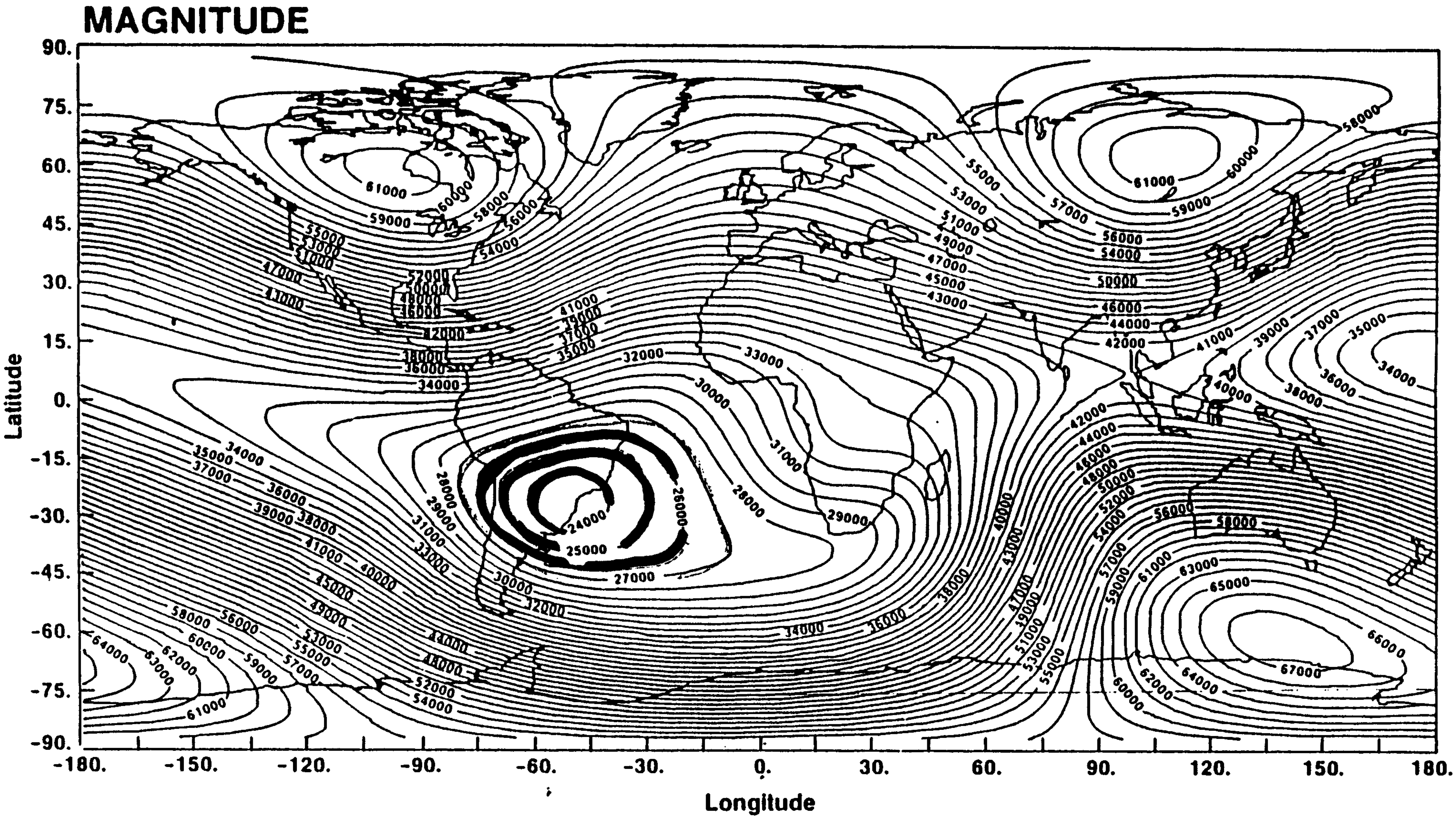
- **Magnetic Field Morphology and Nomenclature**
- **Spherical Harmonic Description**
  - **Simple dipole approximation**
  - **Tilted dipole approximation**
- **Dipole Coordinate System**
- **B-L Coordinate System "L-shells"**
- **Magnetosphere Morphology**
- **Magnetosphere/Solar Wind Interaction; Magnetosphere Circulation**
- **Plasmasphere**
- **Particles in Magnetic Fields: gravitational drift; gradient drift; mirroring**
- **Radiation Belts; South Atlantic Anomaly**

NOMENCLATURE FOR EARTH'S MAGNETIC FIELD

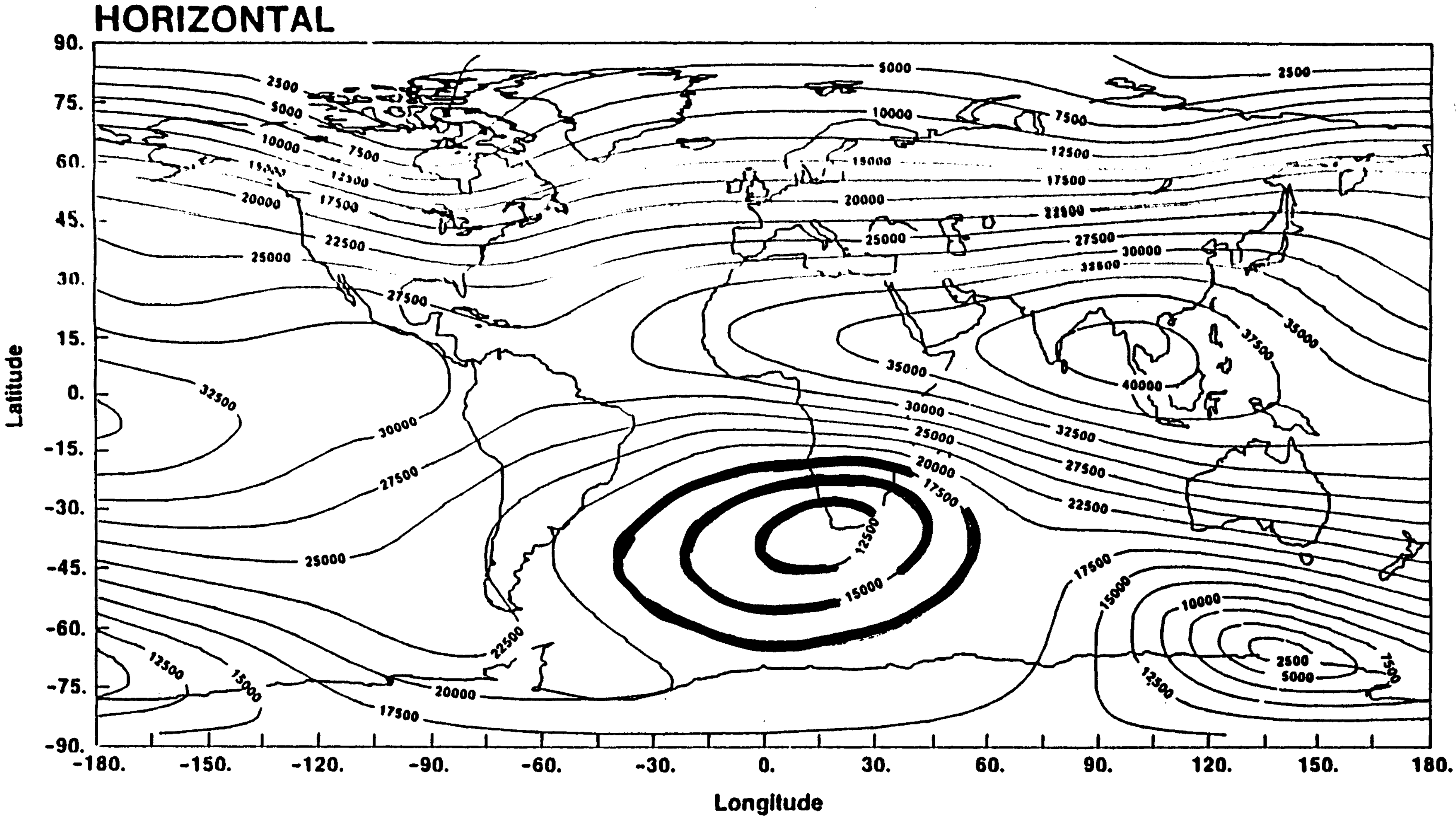


F: total field
H: horizontal component
X: northward component
Y: eastward component
Z: vertical component
D: declination
I: inclination

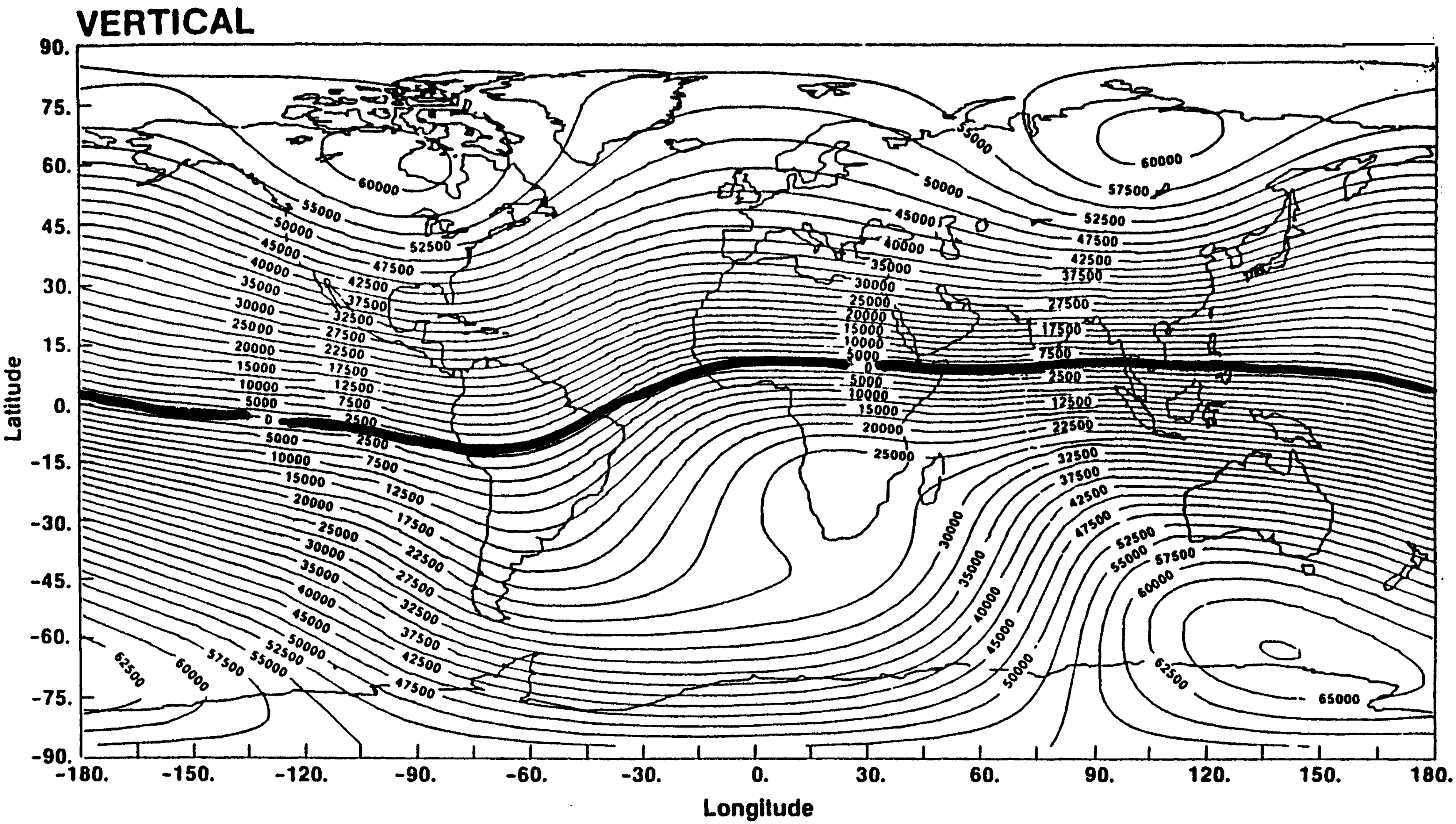




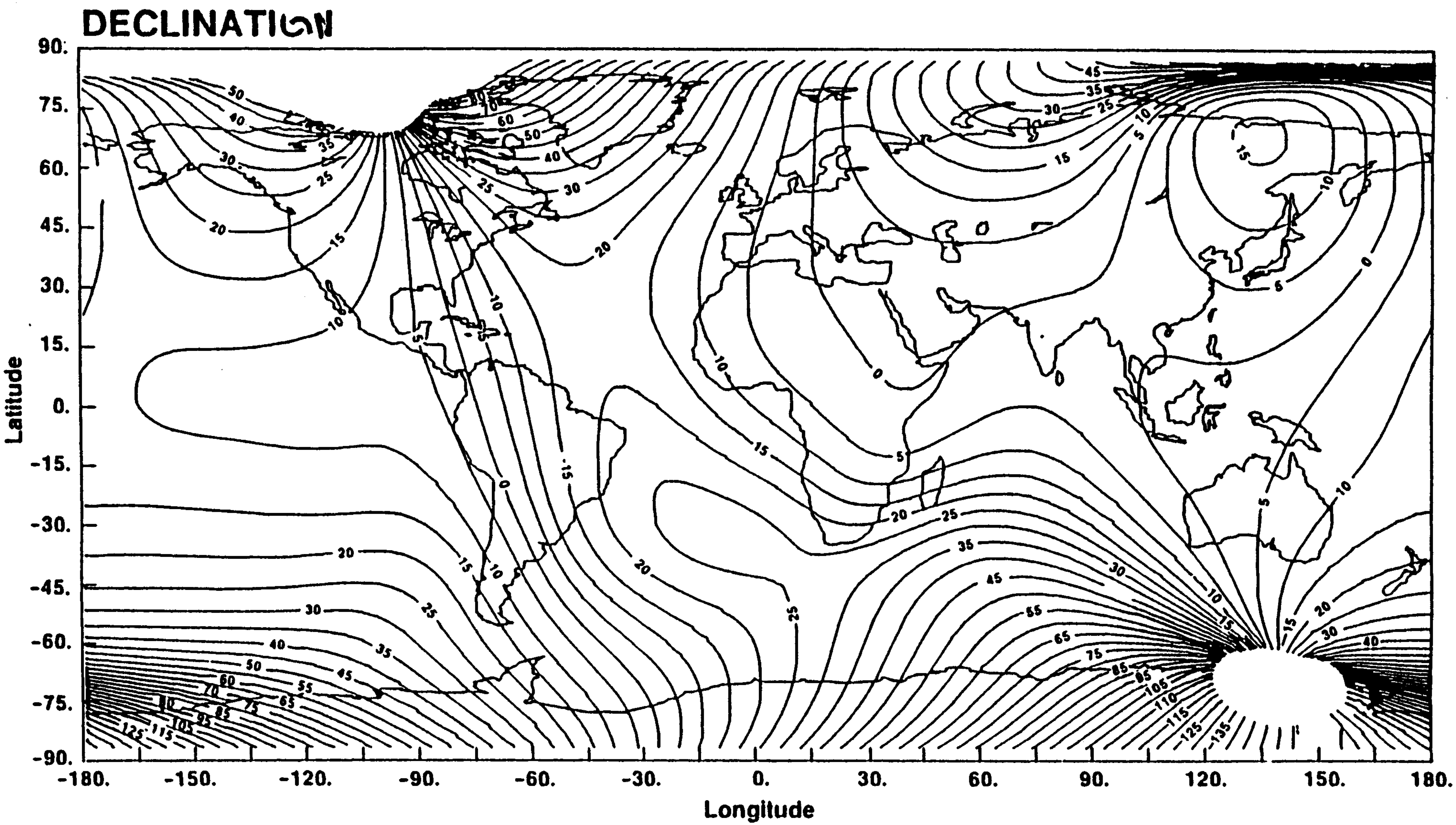
Contours of constant total field F at the surface of the earth from the model IGRF 1980.0



Contours of constant horizontal field H at the surface of the earth from the model IGRF 1980.0



Contours of constant vertical field Z at the surface of the earth from the model IGRF 1980.0



Contours of constant declination D at the surface of the earth from the model IGRF 1980.0

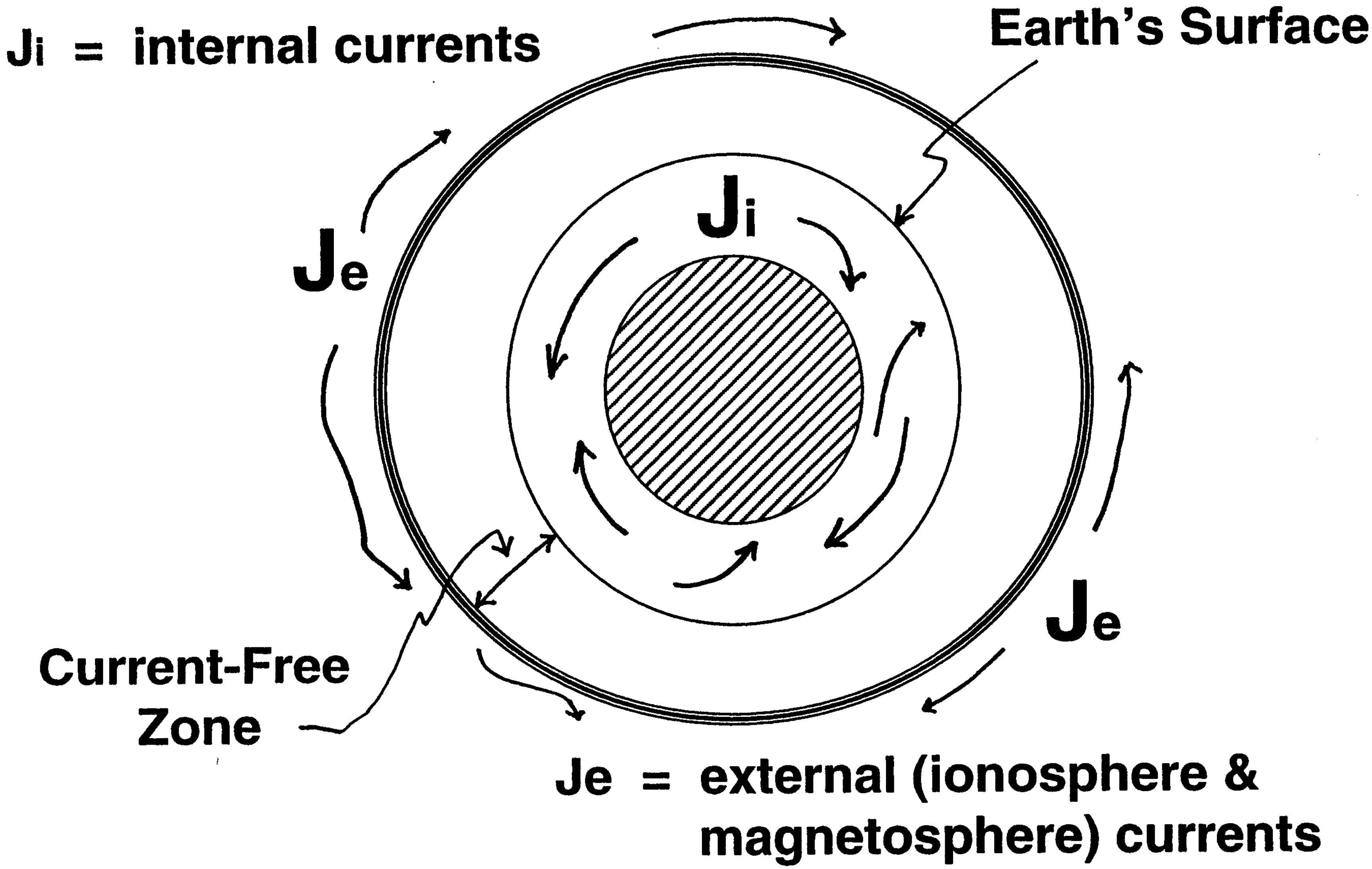
The magnetic field at the surface of the earth is determined mostly by internal currents with some smaller contribution due to external currents flowing in the ionosphere and magnetosphere (see following figure).

In the current-free zone

$$\nabla \times \vec{H} = \frac{4\pi}{c} J = 0$$

and since $\vec{B} = \mu \vec{H}$, **then**

$$\vec{\nabla} \times \vec{B} = 0 \quad \Rightarrow \quad \vec{B} = -\vec{\nabla} V$$


Ji = internal currents
Earth's Surface
Ji
Je
Je
Current-Free
Zone
Je = external (ionosphere &
magnetosphere) currents

Also, another of Maxwell's equations,

$$\vec{\nabla}\cdot\vec{B}=0$$

The combination $\vec{\nabla}\cdot\vec{B}=0$ **and** $\vec{B}=-\vec{\nabla}V$

$$\Rightarrow \qquad \nabla^2 V=0$$

(Laplace's equation)

Where V = the <u>magnetic scalar potential</u> .

- **The magnetic scalar potential can be written as a spherical harmonic expansion, in terms of the Schmidt function, a particular normalized form of Legendre Polynomial:**

$$V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} P_n^m(\cos\theta)$$ **degree = n order = m**

$$\times \left[\left(\frac{a}{r}\right)^{n+1} \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda \right) \right.$$ **internal sources**

$$\left. + \left(\frac{a}{r}\right)^{-n} \left(A_n^m \cos m\lambda + B_n^m \sin m\lambda \right) \right]$$ **external sources**

r = radial distance θ = colatitude λ = east longitude

a = radius of earth (geographic polar coordinates)

SIMPLE DIPOLE Approximation

For a single magnetic dipole at the earth's center, oriented along the geographic axis,

- **n = 1 m = 0 and** $P_1^0(\cos\theta) = \cos\theta$

- **Further we can neglect external sources in comparison to internal sources,**

$$\Rightarrow \qquad V = a g_1^0 \left(\frac{a}{r}\right)^2 \cos\theta$$

$$V = a g_1^0 \left(\frac{a}{r}\right)^2 \cos\theta$$

The field components are:

$$\mathbf{X} = \frac{1}{r}\frac{\partial V}{\partial \theta} = -g_1^0 \left(\frac{a}{r}\right)^3 \sin\theta$$ **(northward)**

$$\mathbf{Y} = 0$$ **(eastward)**

$$\mathbf{Z} = \frac{\partial V}{\partial r} = -2 g_1^0 \left(\frac{a}{r}\right)^3 \cos\theta$$ **(radially downward)**

Total field magnitude:

$$\sqrt{X^2+Y^2+Z^2} = g_1^0\left(\frac{a}{r}\right)^3\left(1+3\cos^2\theta\right)^{1/2}$$

A "dip angle" , **I, is also defined:**

$$\tan I = \frac{Z}{\sqrt{X^2+Y^2}} = 2\cot\theta$$

- **I is positive for downward pointing field**

$$I = \tan^{-1}[2\cot\theta]$$ **(simple dipole field)**

- **Note that**

g_1^0 = **southward component of the magnetic field at the equator at r = a**

≈ -.30G (Gauss)

1γ **= 10^{-5} G**

= 10^{-9} T (Tesla or Wbm^{-2})

TILTED DIPOLE Approximation

As the next best approximation, let

n = 1 , m = 1

We find that

$$P_1^1(\cos\theta) = \sin\theta$$ **and**

$$V = a\left(\frac{a}{r}\right)^2 \left\{ g_1^0 \cos\theta + \left(g_1^1 \cos\lambda + h_1^1 \sin\lambda \right) \sin\theta \right\}$$

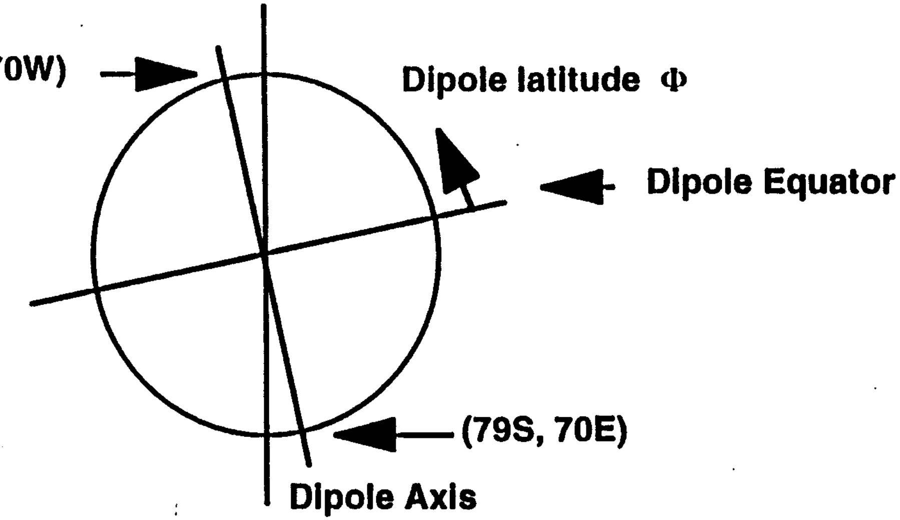
where λ = longitude (+ eastward).

$$\left[\left(g_1^0\right)^2+\left(g_1^1\right)^2+\left(h_1^1\right)^2\right]^{1/2}$$

- This "tilted dipole" is tipped 11.5° towards 70°W longitude and has an equatorial field strength of .312G.

It is often more convenient to order data, formulate models, etc., in a magnetic coordinate system. We will now re-write the tilted dipole in that coordinate system rather than the geographic coordinate system.

DIPOLE COORDINATE SYSTEM



$$\sin\Phi = \sin\varphi \sin\varphi_o + \cos\varphi \cos\varphi_o \cos(\lambda - \lambda_o)$$

$$\sin\Lambda = \frac{\cos\varphi \sin(\lambda - \lambda_o)}{\cos\Phi}$$

where

φ = **geographic latitude**

λ = **geographic longitude** φ_o, λ_o = **79°N, 290°E**

- The above formulation representing dipole coordinates (sometimes called geomagnetic coordinates) is now more or less the same as that on p. 10 & 11.

- We will now write several relations in terms of dipole latitude, Φ (instead of geographic colatitude, θ) and dipole longitude Λ.

- Dipole longitude is reckoned from the American half of the great circle which passes through (both) the geomagnetic and geographic poles; that is, the zero-degree magnetic meridian closely coincides with the 291°E geographic longitude meridian. Therefore,

$$V = -\frac{M \sin \Phi}{r^2}$$

$M = g_1^o a^3$ in our previous notation = "dipole moment"

$= 8.05 \pm .02 \; 10^{25}$ G-cm^3

From the above expression we can derive the following:

$$H = \frac{1}{r}\frac{\partial V}{\partial \Phi} = -\frac{M\cos\Phi}{r^3} \qquad Z = \frac{\partial V}{\partial r} = \frac{2M\sin\Phi}{r^3}$$

$$\tan I = \frac{Z}{H} = -2\tan\Phi$$

$$|B| = \sqrt{H^2 + Z^2} = \frac{M}{r^3}\left(1 + 3\sin^2\Phi\right)^{1/2}$$

These look very much like the simple dipole approximation in geographic coordinates.

locus of points following D, magnetic flux density

A field line is defined by $\frac{dr}{Z} = \frac{rd\Phi}{H}$ **, so**

$$r\frac{d\Phi}{dr} = \frac{H}{Z} = -\frac{1}{2\tan\Phi}$$ **and we may integrate to get**

$$r = r_o \cos^2\Phi$$

- **This is the equation for a field line.**

- **If $\Phi = 0$ (dipole equator), $r = r_o$; r_o is thus the radial distance to the equatorial field line over the equator, and its greatest distance from earth.**

• The point (magnetic or dipole latitude) where the line of force meets the surface is given by

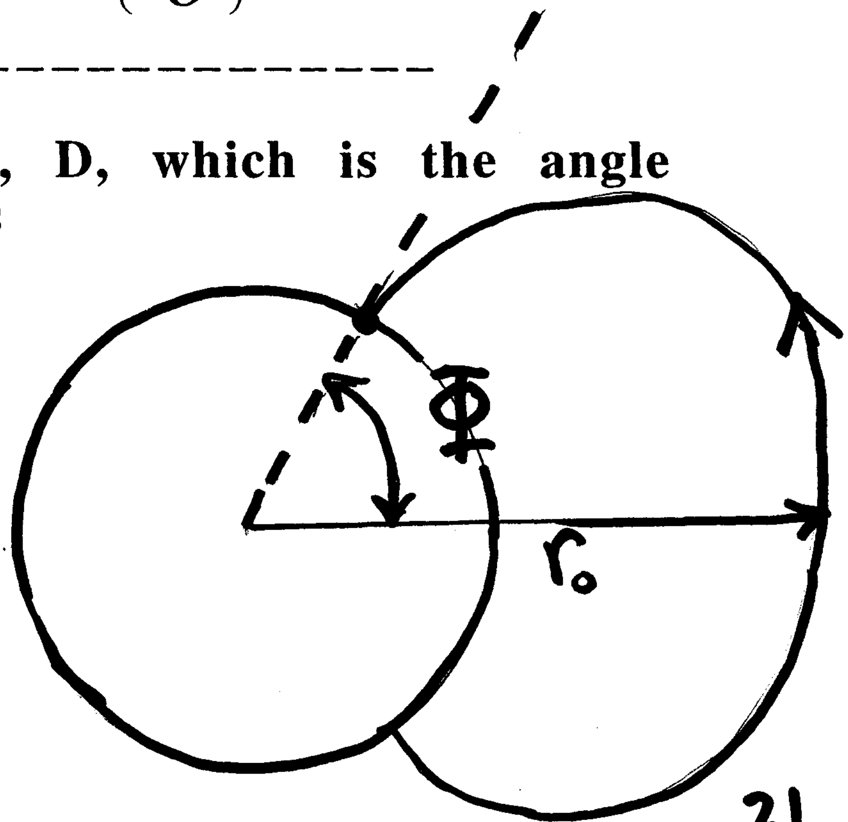
$$\frac{a}{r_o} = \cos^2 \Phi \Rightarrow \cos \Phi = \left(\frac{a}{r_o} \right)^{1/2}$$

• Note also that the declination, D, which is the angle between H and geographic north, is

tanD = Y/X

X = H cosD

Y = H sinD



The B-L Coordinate System "L-shells"

Let us take our previous equation for a dipolar field line

$$r = r_o \cos^2 \Phi$$

and re-cast it so the radius of the earth (a) is the unit of distance. Then R = r/a and

$$R = R_o \cos^2 \Phi$$

where R and R_o are now measured in earth radii. The latitude where the field line intersects the earth's surface (R = 1) is given by

$$\cos \Phi = R_o^{-1/2}$$

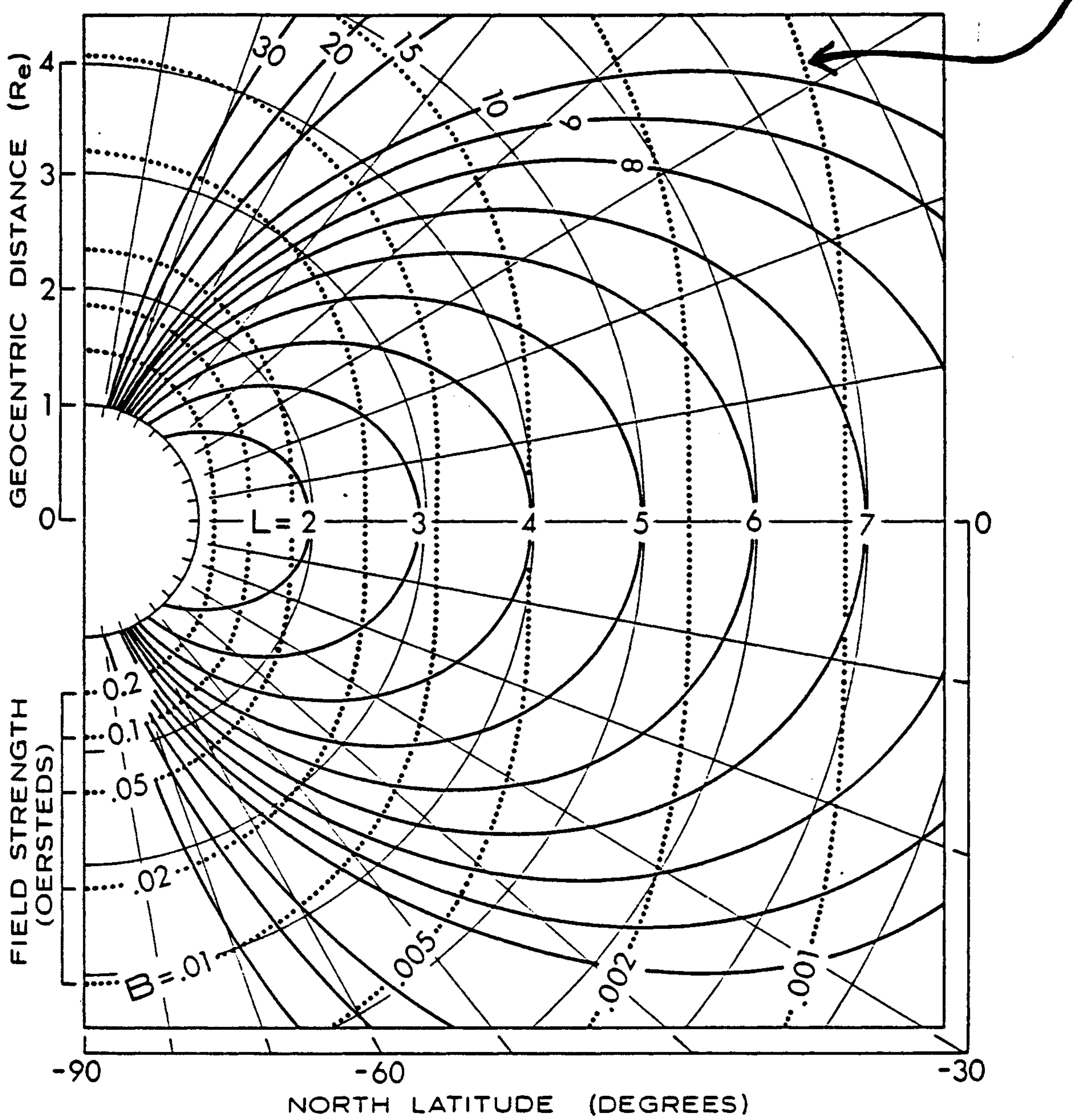
- Now we will discuss an analagous L-parameter (or L-shell) nomenclature for <u>non-dipole</u> field lines, often used for radiation belt and magnetospheric studies.

- We will understand its origin better when we study radiation belts; basically, the L-shell is the surface traced out by the guiding center of a trapped particle as it drifts in longitude about the earth while oscillating between mirror points.

- For a <u>dipole field</u> the L-value is the distance, in earth radii, of a particular field line from the center of the earth ($L = R_0$ on the previous page), and the L-shell is the "shell" traced out by rotating the corresponding field line around the earth.

- Curves of constant B and constant L are shown in the figure on the next page. Note that on this scale, the L-values correspond very nearly to dipole field lines.

Note that lines of constant B are ellipses, approximately



The B-L coordinate system. The curves shown here are the intersection of a magnetic meridian plane with surfaces of constant B and constant L (The difference between the actual field and a dipole field cannot be seen in a figure of this scale)

• By analogy with our previous formula for calculating the dipole latitude of intersection of a field line with the earth's surface, we can determine an invariant latitude in terms of L-value:

$$\cos \Lambda = L^{-1/2}$$

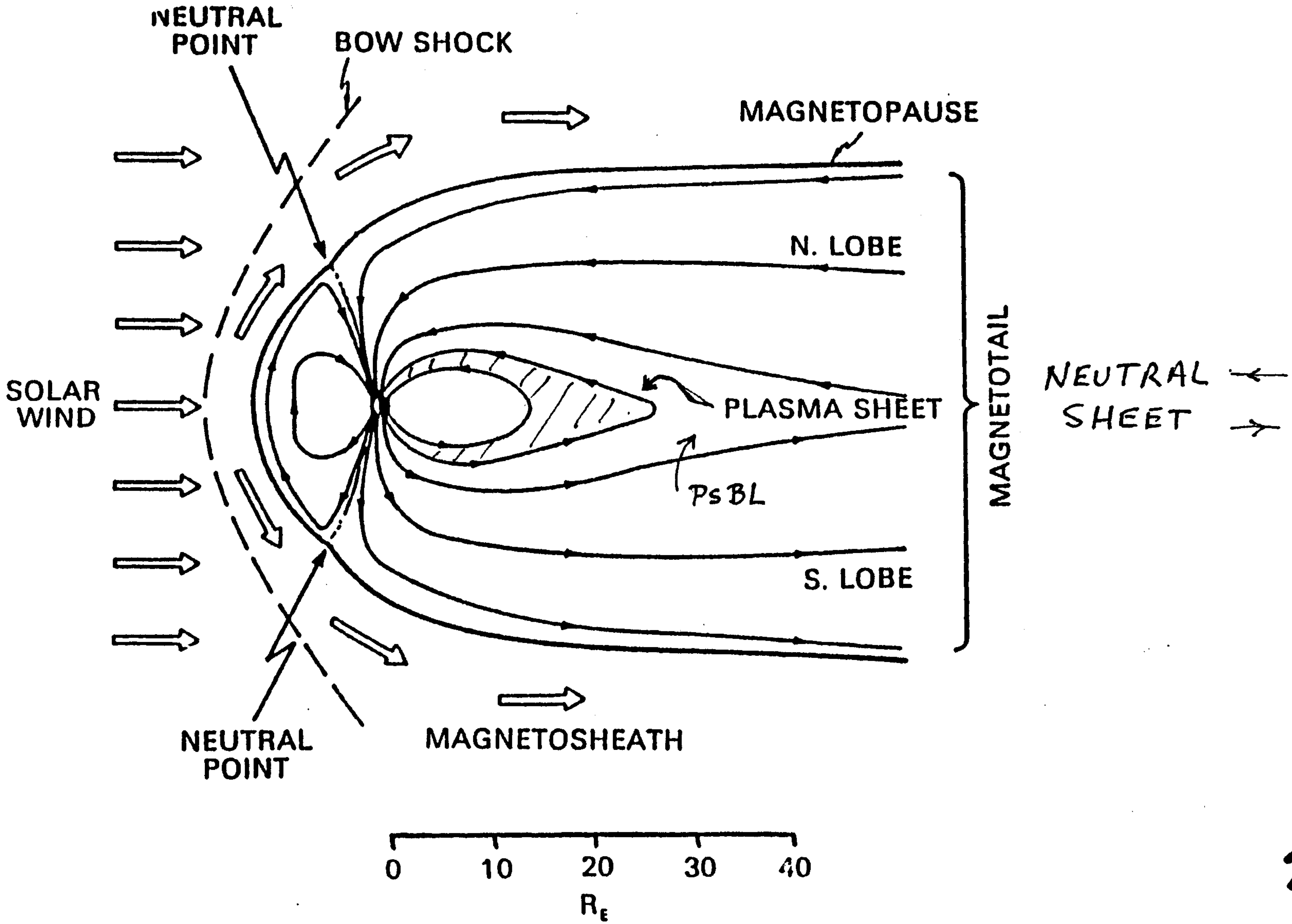
where Λ = invariant latitude

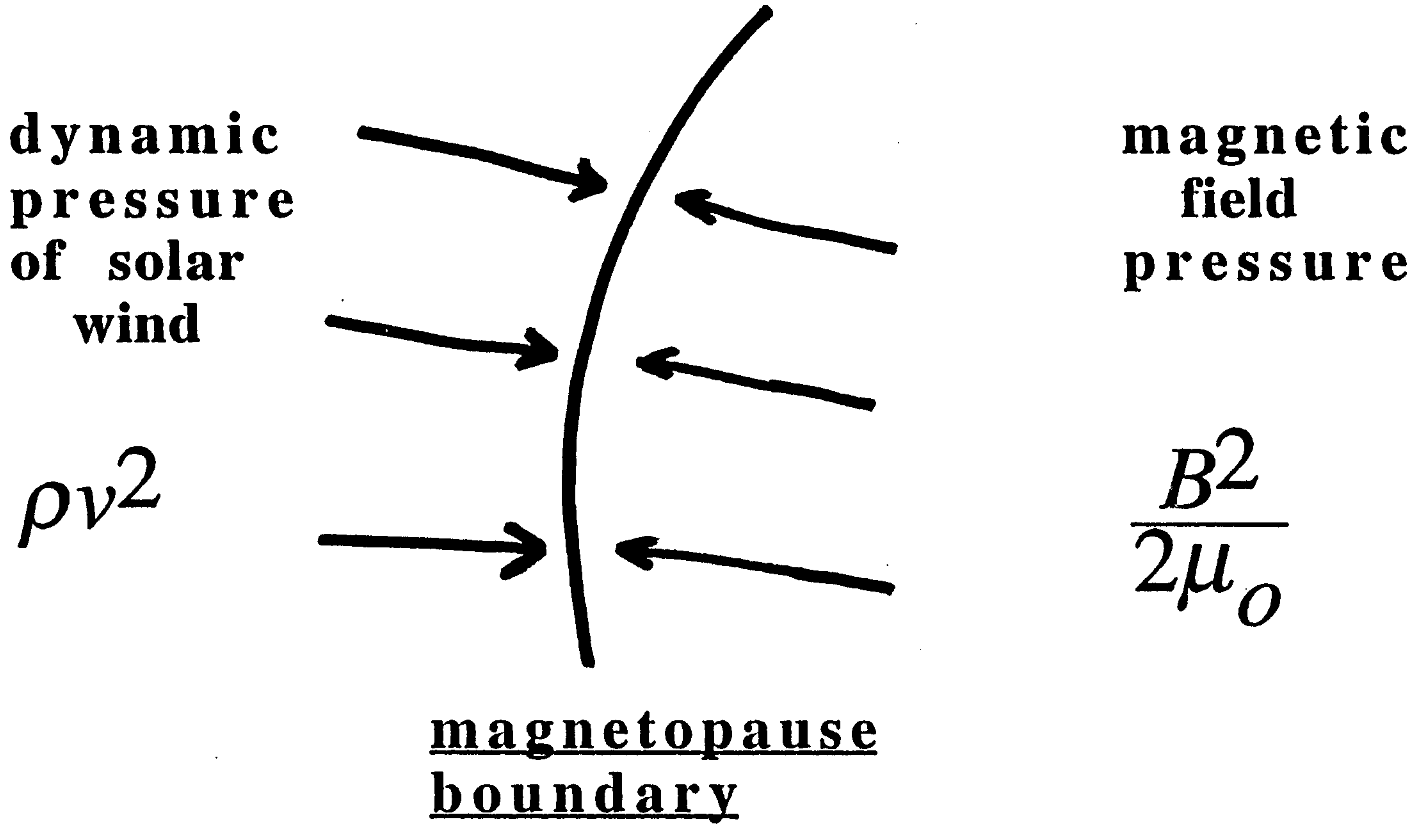
• Here L is the actual L-value (i.e., not that associated with a dipole field).

• Along most field lines L is constant to within about 1% and thus it usefully serves to identify field lines even though they may not be strictly dipolar.

THE MAGNETOSPHERE

- As the magnetized solar wind flows past the earth, the plasma interacts with the earth's magnetic field and confines the field to a cavity, the magnetosphere.



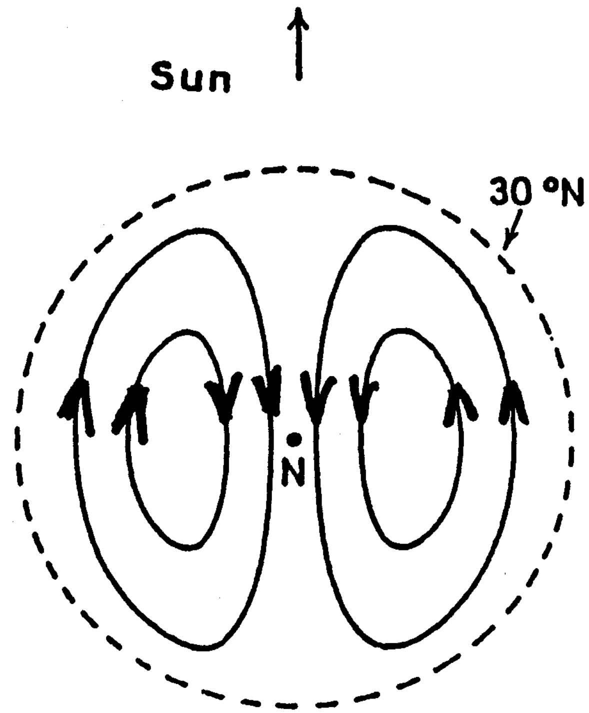

dynamic pressure of solar wind
ρv^2
magnetic field pressure
$\frac{B^2}{2\mu_o}$
magnetopause boundary

MAGNETOSPHERIC CIRCULATION

Historically, the S_q^P current system (as inferred from ground magnetometers) led to the first speculations about circulation patterns in the magnetosphere, and theories about solar wind - magnetosphere interactions.

Given that e- are bound to field lines in the E-region (where ionospheric current flows) and ions are stationary by comparison, the following motions of field lines (electrons, opposite to conventional current flow) are inferred from the S_q^P current patterns:

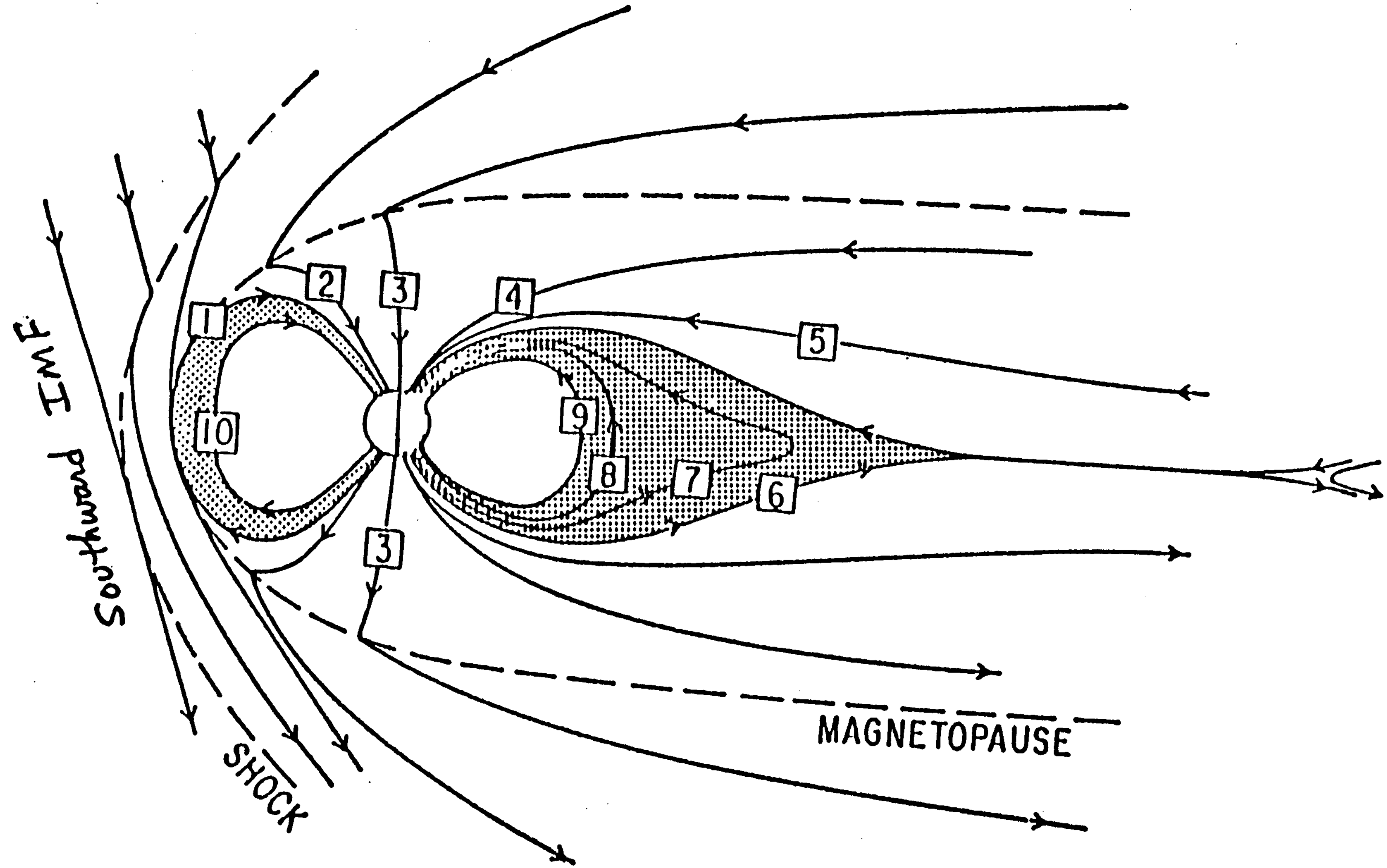
motion of "feet" of field lines



• This led Axford and Hines (1961) to propose the "viscous interaction" theory, whereby some unspecified "effective friction process" transfers momentum from the solar wind across the magnetopause, and establishes a circulation pattern within the magnetopause.

• The more accepted concept, based on the work of Dungey, involves magnetic connection or merging (between the terrestrial magnetic field and the IMF) on the dayside and reconnection on the night side. The following figure (polar plane view) shows the sequences of magnetic merging, reconnection, and convection, where the numbers represent the time in hours after a field line has merged on the dayside with a southward-directed IMF ($B_Z < 0$):

1 - 2	IMF merges with TMF
2 - 5	The connected field lines are swept back by the solar wind
6 - 7	Reconnection of the TMF occurs
8 - 10	Field lines convect back to the dayside (this must happen or an accumulation of magnetic energy would be implied on the nightside)


Southward IMF
SHOCK
MAGNETOPAUSE
1
2
3
4
5
6
7
8
9
10
3

PARTICLES IN THE MAGNETOSPHERE

- **The main particle populations are:**

  -- plasmasphere

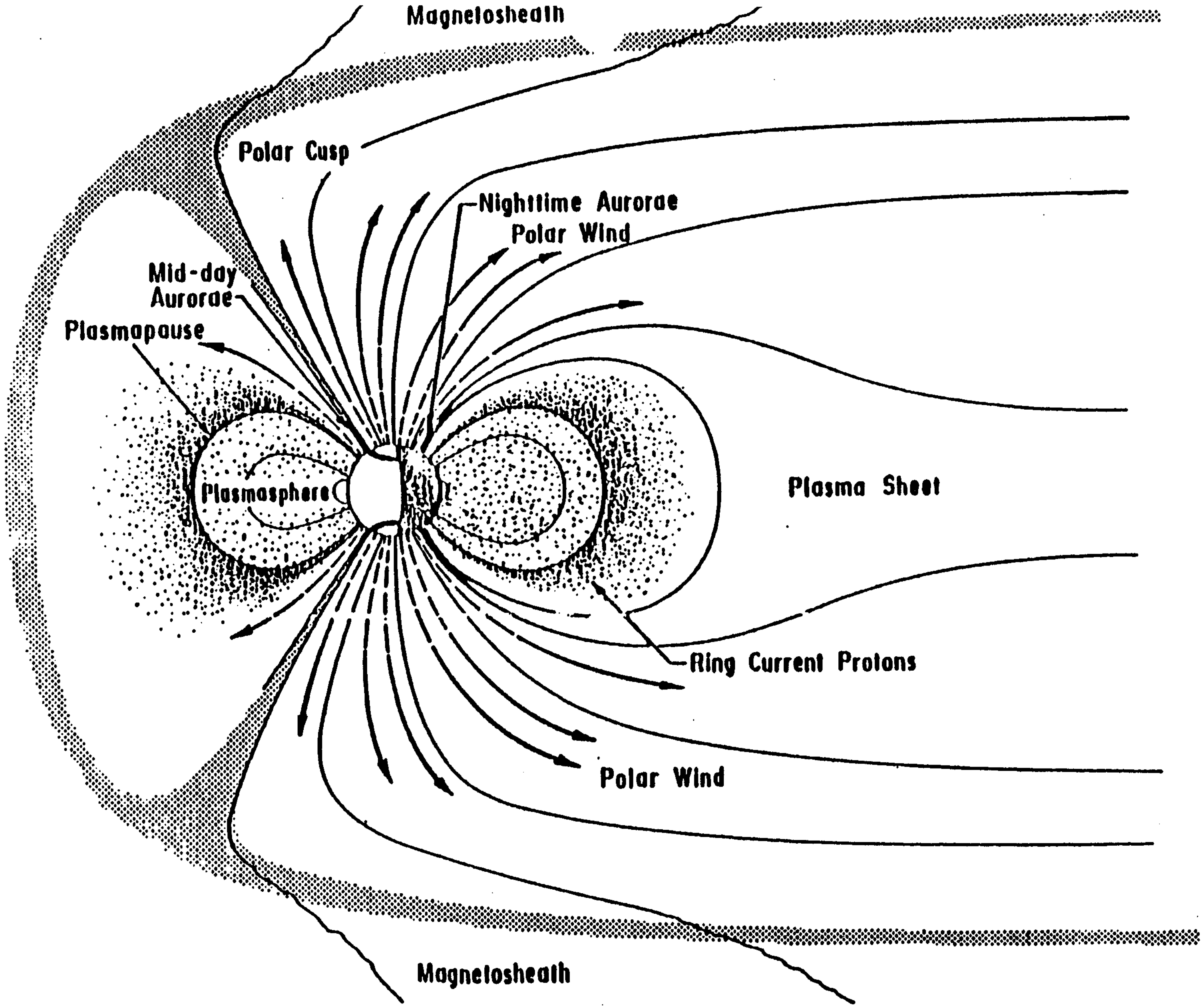
  -- radiation belts

  -- plasma sheet

  -- boundary layers

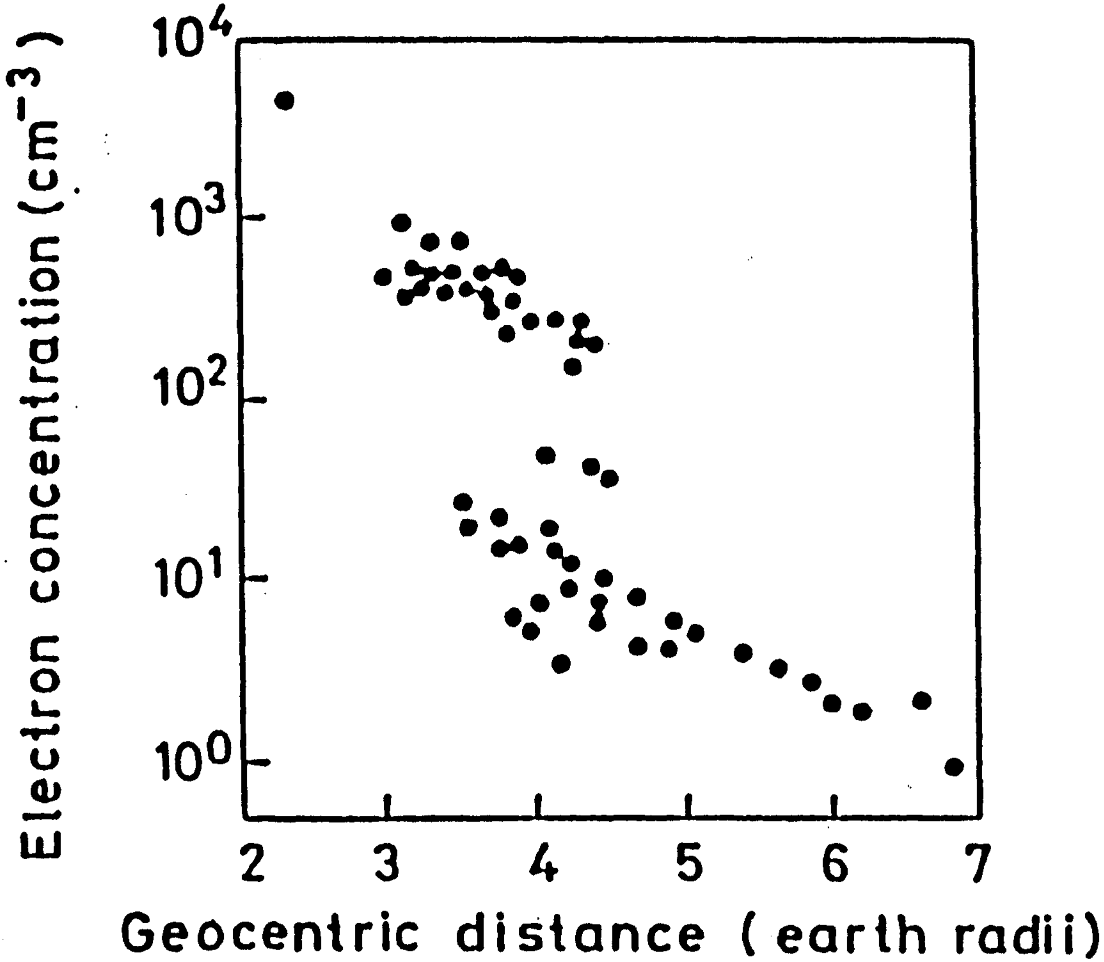
- **We have discussed the radiation belts extensively, and the plasma sheet to some extent. We will return to the plasma sheet when discussing magnetic storms.**

- **The plasmasphere represents the relatively cold ionospheric plasma (~ .3 eV or T ~ 2000 K) which is co-rotating with the earth (frictional coupling).**



Cross section through the earth's magnetosphere showing the particle regions.

The outer boundary of the plasmasphere, at about 4 R_E, is where the plasma density undergoes a sudden drop. This is the <u>plasmapause</u>.



THE RADIATION BELTS

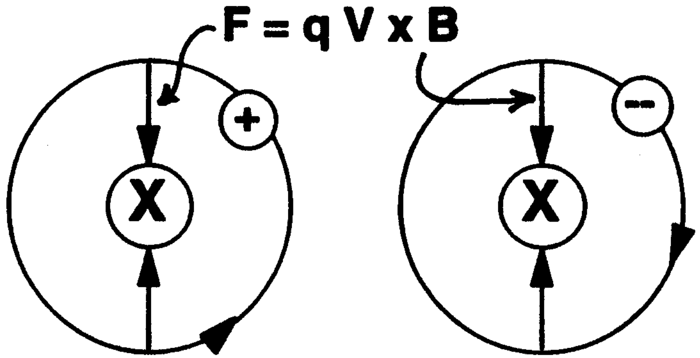
In a collisionless magnetoplasma, in the absence of any other forces, charged particle follow circular paths around the B-field determined by a balance between the Lorenz force and the centrifugal force:

q = charge

B = magnetic field strength

m = mass

v = velocity



B - field into paper

gyroradius $$r = \frac{mv}{qB}$$

gyrofrequency $$\Omega = \frac{qB}{m}$$

In the magnetosphere, additional forces act on the particles

due to:	producing:
gravity	a current
E-fields	an E x B plasma drift
non-uniform B-field	ring current
B-field curvature	curvature drift
converging/diverging field lines	mirroring

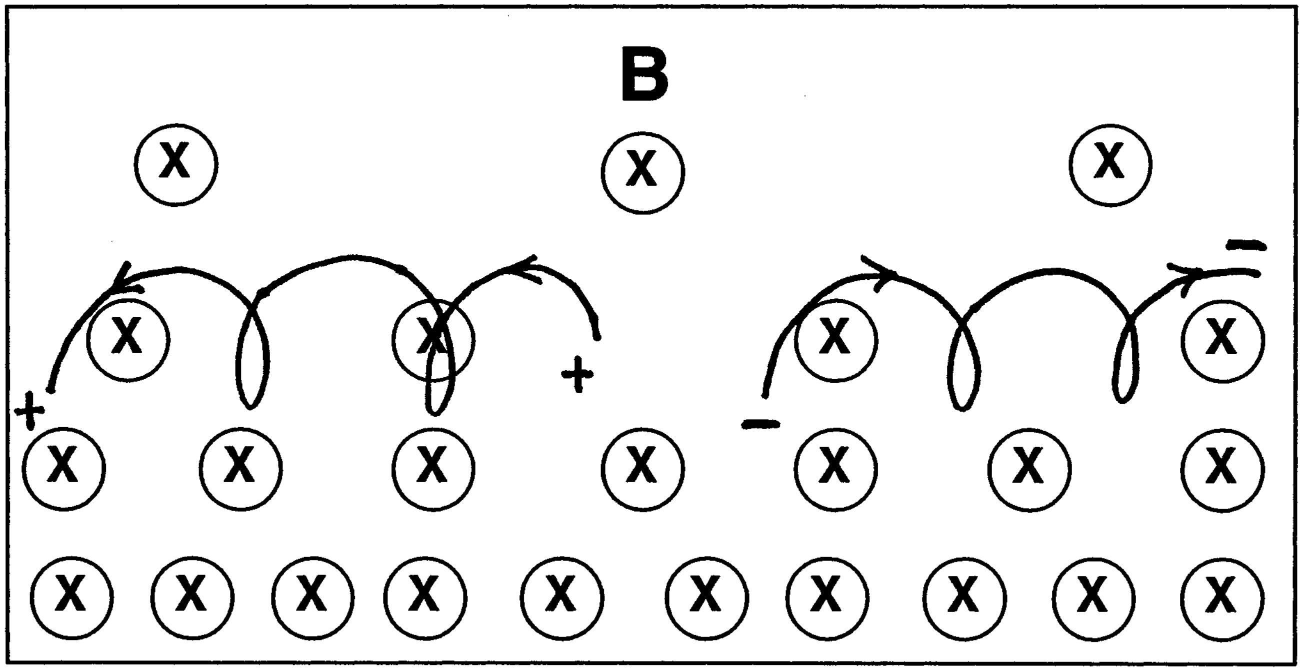
In general, a force F produces a drift $\vec{v}_D = \dfrac{\vec{F} \times \vec{B}}{qB^2}$

Where q represents the charge, and can be + or negative. In this abbreviated presentation, let us consider the conditions leading to the ring current and mirroring since these are of some relevance to the equatorial region.

NON-UNIFORM MAGNETIC FIELD

Now consider a non-uniform magnetic field (i.e., |B| varies spatially). The gyroradius is smaller when |B| is larger:

$$r_c = \frac{mv}{qB}$$



<--------- net current flow

In this case the force on the particle is proportional to the gradient of $\vec{B}$:

$$\vec{F} = -\frac{1}{2} m v_{\perp}^{2} \frac{\nabla \vec{B}}{B}$$

where $v_{\perp}$ is the particle's velocity around the magnetic field.

The so-called *gradient drift* is

$$\vec{v}_{grad} = \frac{1}{2} \frac{m v_{\perp}^{2}}{e} \frac{\vec{B} \times \nabla B}{B^{3}} = \mu \frac{\vec{B} \times \nabla B}{e B^{2}}$$

where μ = the particle's "magnetic dipole moment"

The above gradient drift is primarily responsible for the westward ring current, mainly responsible for the Dst variation on the ground.

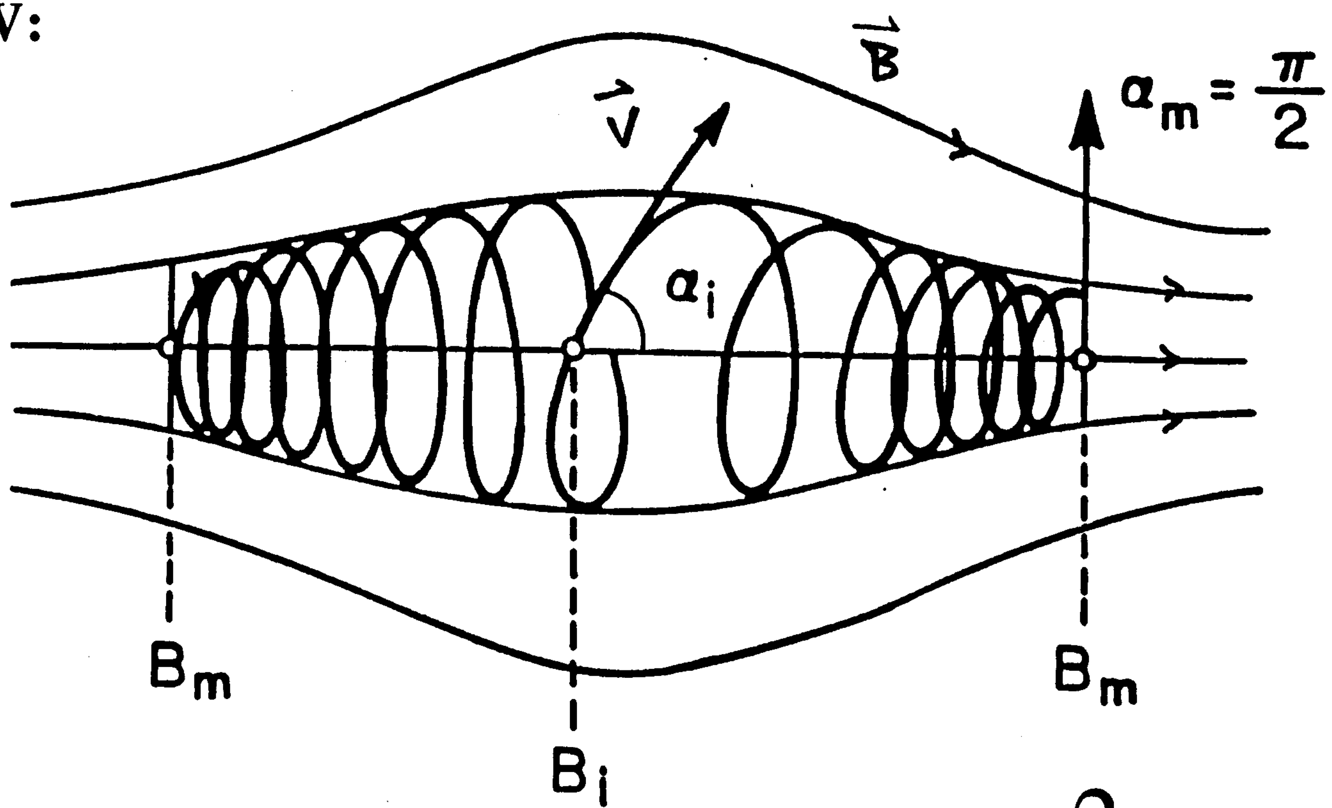
In a similar fashion, by qualitatively considering the forces on a particle encountering converging field lines, we would find that particles would encounter a force opposite in direction to their motion, which, if sufficiently strong, would cause them to reverse direction, or "mirror". Let us consider this more quantitatively.

If we require that no forces act on the particle *in the direction of its motion,* then the magnetic flux enclosed by the particle's circular path must remain constant (else, according to Faraday's law a change of magnetic flux will induce an electric field acting on the particle). This leads to the condition

magnetic moment $\mu = \frac{1}{2}\frac{mv^2{\perp}}{B}$ = constant,

which is called the "first adiabatic invariant".

If we define the pitch angle to be the angle between B and V:



then

$$v_{\perp} = v \sin\alpha \quad \text{and} \quad \mu = \left[\frac{1}{2} m v^2\right] \frac{\sin^2\alpha}{B} = const$$

A charged particle trajectory in a magnetic "bottle". Conservation of the first adiabatic invariant can cause the spiraling particle to be reflected where the magnetic field is stronger. This causes the particle to be trapped by the magnetic field.

Since $\frac{1}{2}mv^2 = const$ **, then <u>α must increase as B increases</u>, and correspondingly the distribution of K.E. between** $v_\perp$ **and** $v_\parallel$ **changes:**

$$v_\perp = v\sin\alpha$$
$$v_\parallel = v\cos\alpha$$

If α increases to 90° before the particle collides vigorously with the neutral atmosphere, the direction of $v_\parallel$ **will change sign (at the "mirror point") and the particle will follow the direction of decreasing B.**

For a given particle the position of the mirror point is determined by the pitch angle as the particle crosses the equator (i.e., where the field is weakest) since

$$\frac{\sin^2 \alpha}{B} = const \Rightarrow \frac{\sin^2 \alpha_{eq}}{B_{eq}} = \frac{1}{B_M}$$

$$\Rightarrow \quad B_M = \frac{B_{eq}}{\sin^2 \alpha_{eq}}$$

Therefore, the smaller α_{eq} the larger B_M , and the lower down in altitude is the altitude of B_M.

• Particles will be lost if they encounter the atmosphere before the mirror point.

• Obviously this will happen if α_{eq} is too small, because that then requires a relatively large B_M (|B| at the mirror point).

• The equatorial pitch angles that will be lost to the atmosphere at the next bounce define the <u>loss cone</u>, which will be seen as a depletion within the <u>pitch angle distribution</u>.

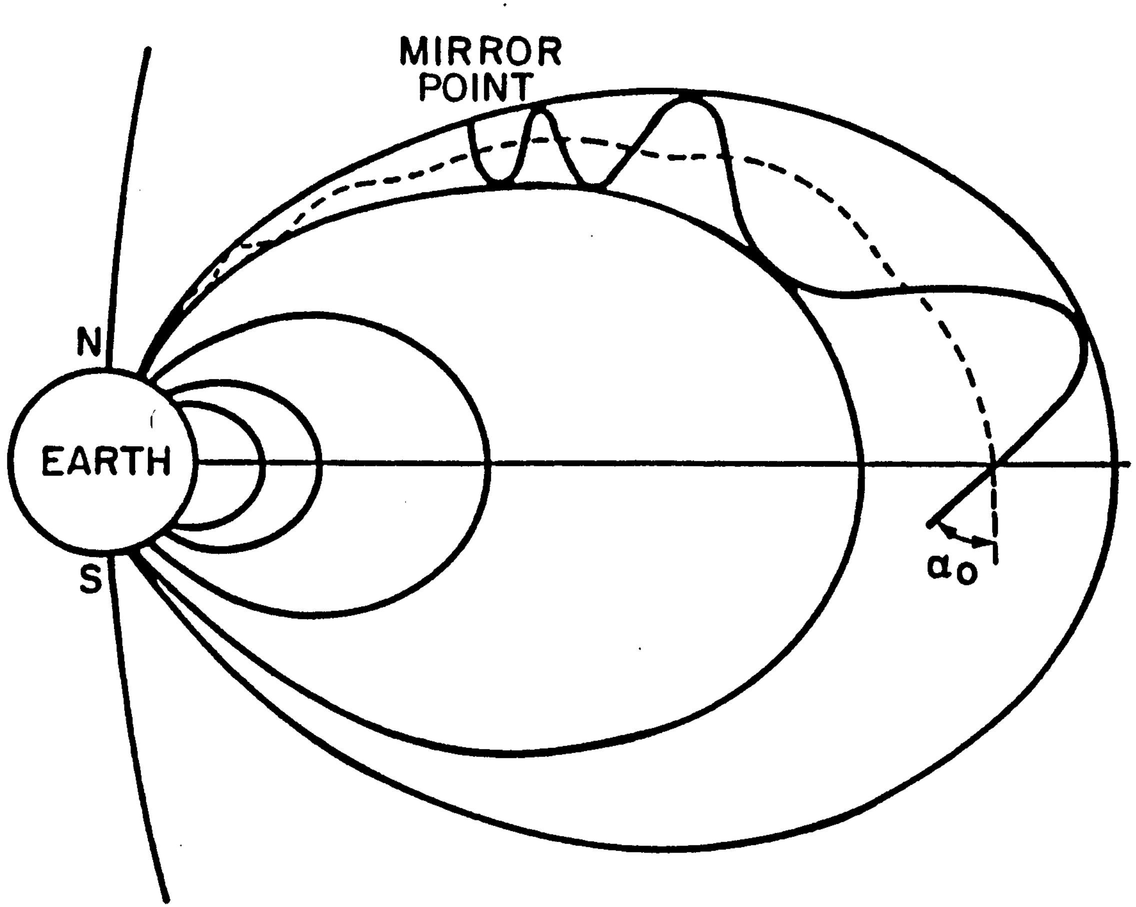
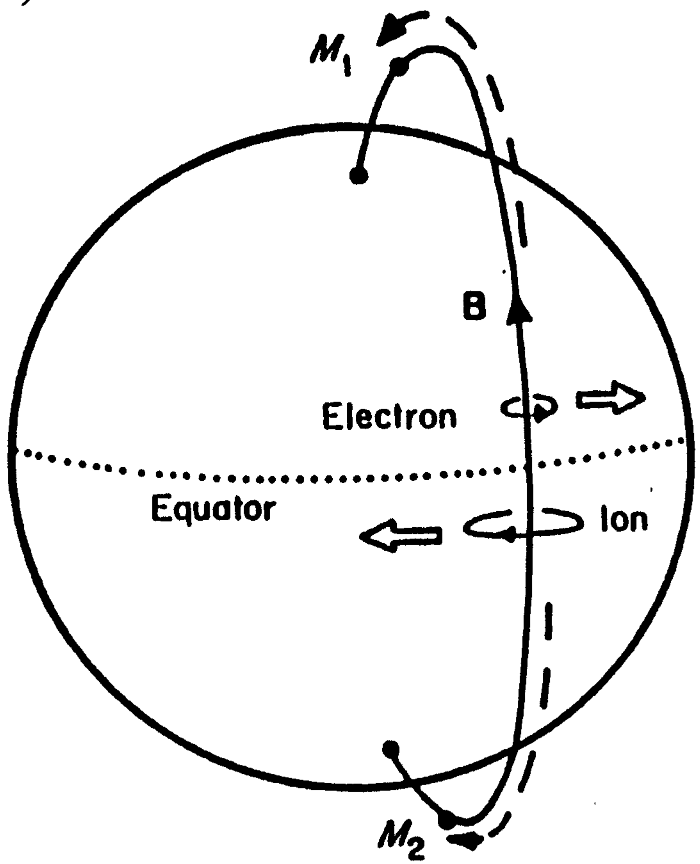
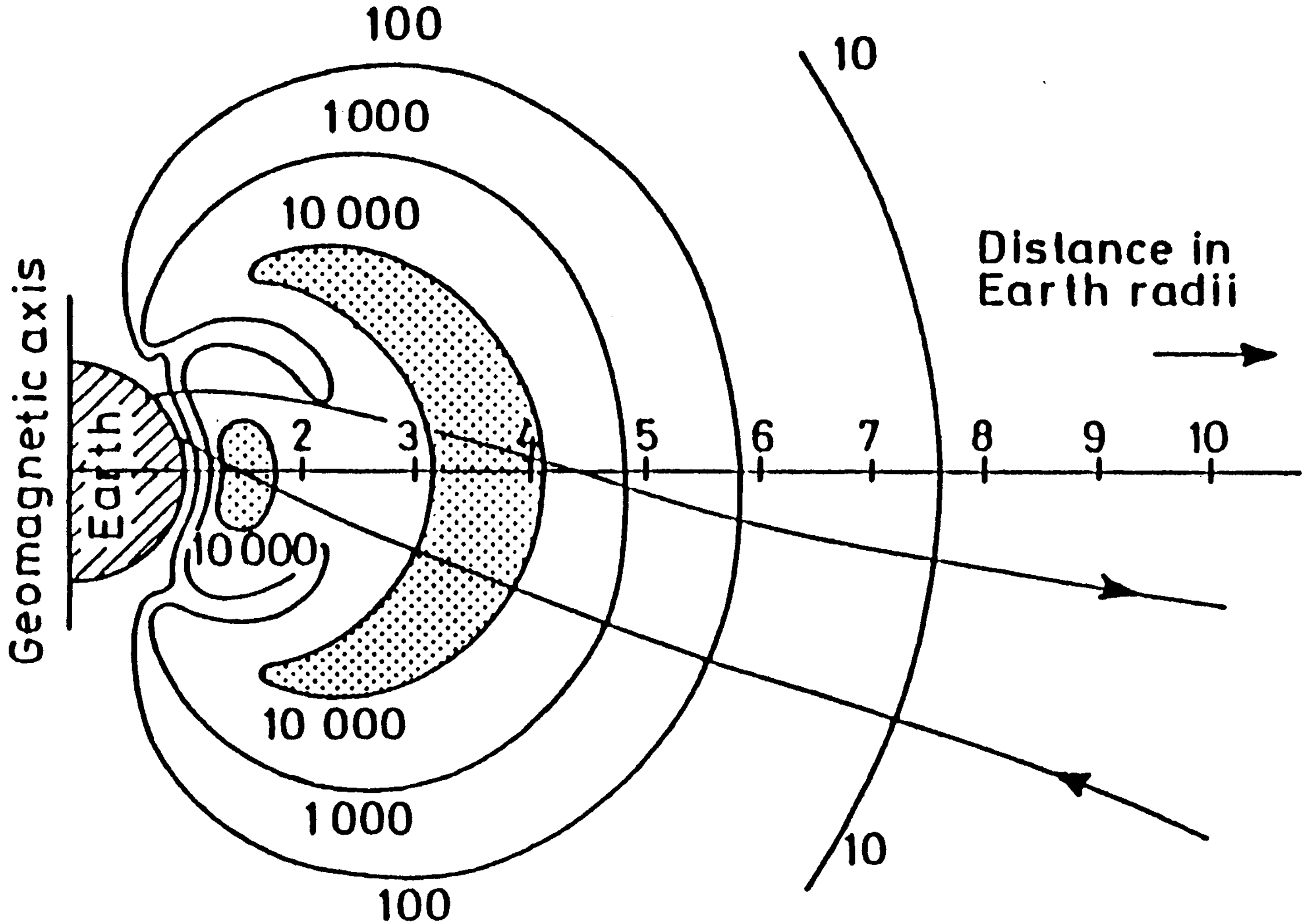


Illustration of magnetic mirroring in a dipolar magnetic field. The single particle trajectory shown in the solid line is for a particle outside the atmospheric bounce loss cone and the dashed line represents the trajectory of a particle inside the loss cone. The latter particle will encounter the denser parts of the earth's atmosphere (mirror point height nominally below 100 km) and will thus precipitate from the radiation belts.

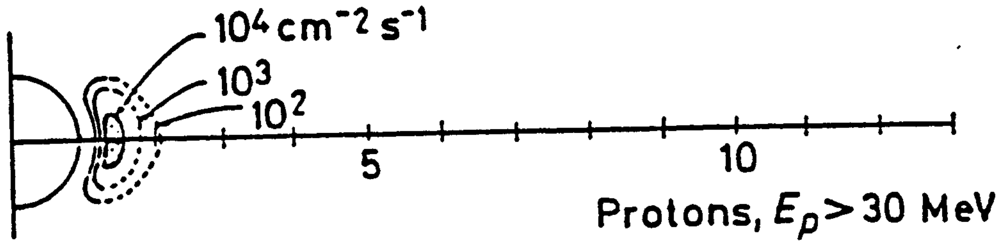
Some typical periods of particle motion for 1Mev particles at 2000 km altitude at the magnetic equator

	Tg gyroperiod	Tb bounce period	Td drift period
electrons	7µs (r=.3km)	.1s	50 min.
protons	4ms (r=10km)	2 s	30 min.

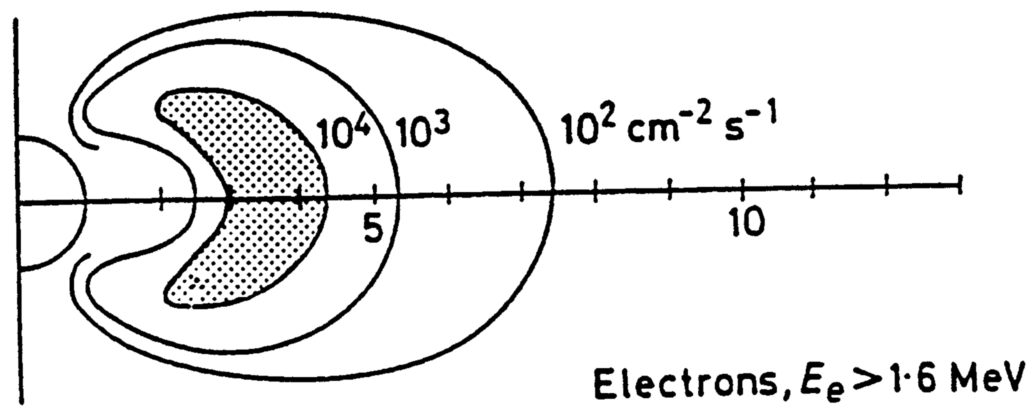




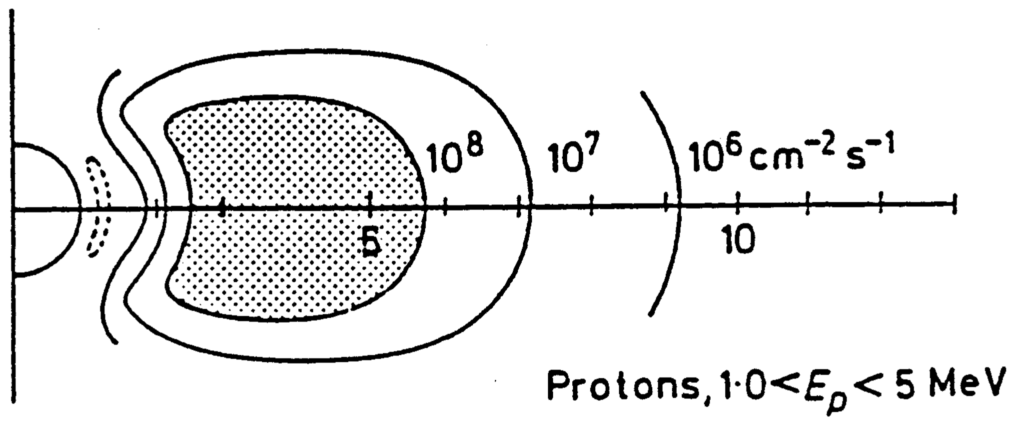
Van Allen's first map of the radiation belts, showing counting rates of the Pioneer-3 Geiger counter. (After J. A. Van Allen and L. A. Frank, *J. Geophys. Res.* 64, 1683, 1959, copyright by the American Geophysical Union)



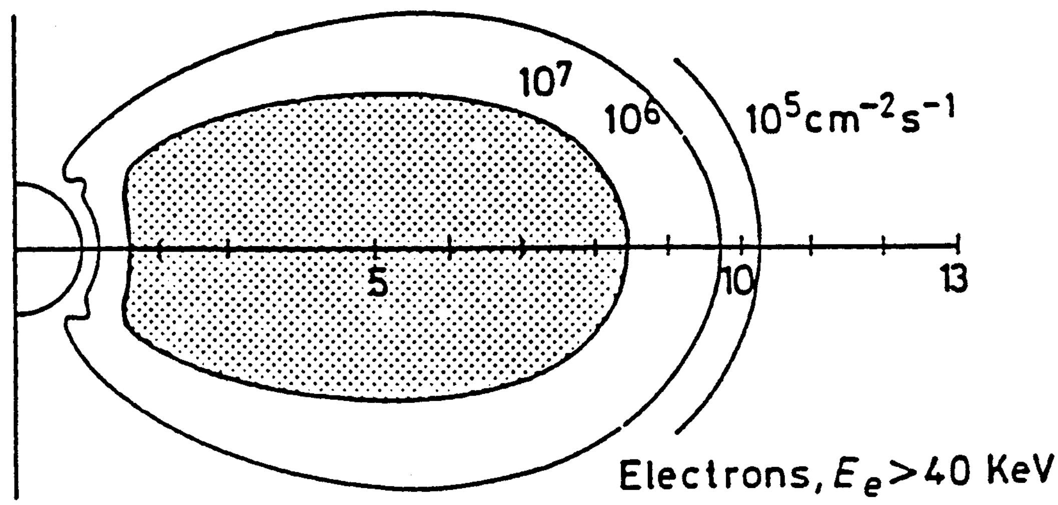
10^4 cm^{-2} s^{-1}
10^3
10^2
5
10
Protons, $E_p > 30$ MeV



10^4
10^3
10^2 cm^{-2} s^{-1}
5
10
Electrons, $E_e > 1{\cdot}6$ MeV



10^8
10^7
10^6 cm^{-2} s^{-1}
5
10
Protons, $1{\cdot}0 < E_p < 5$ MeV

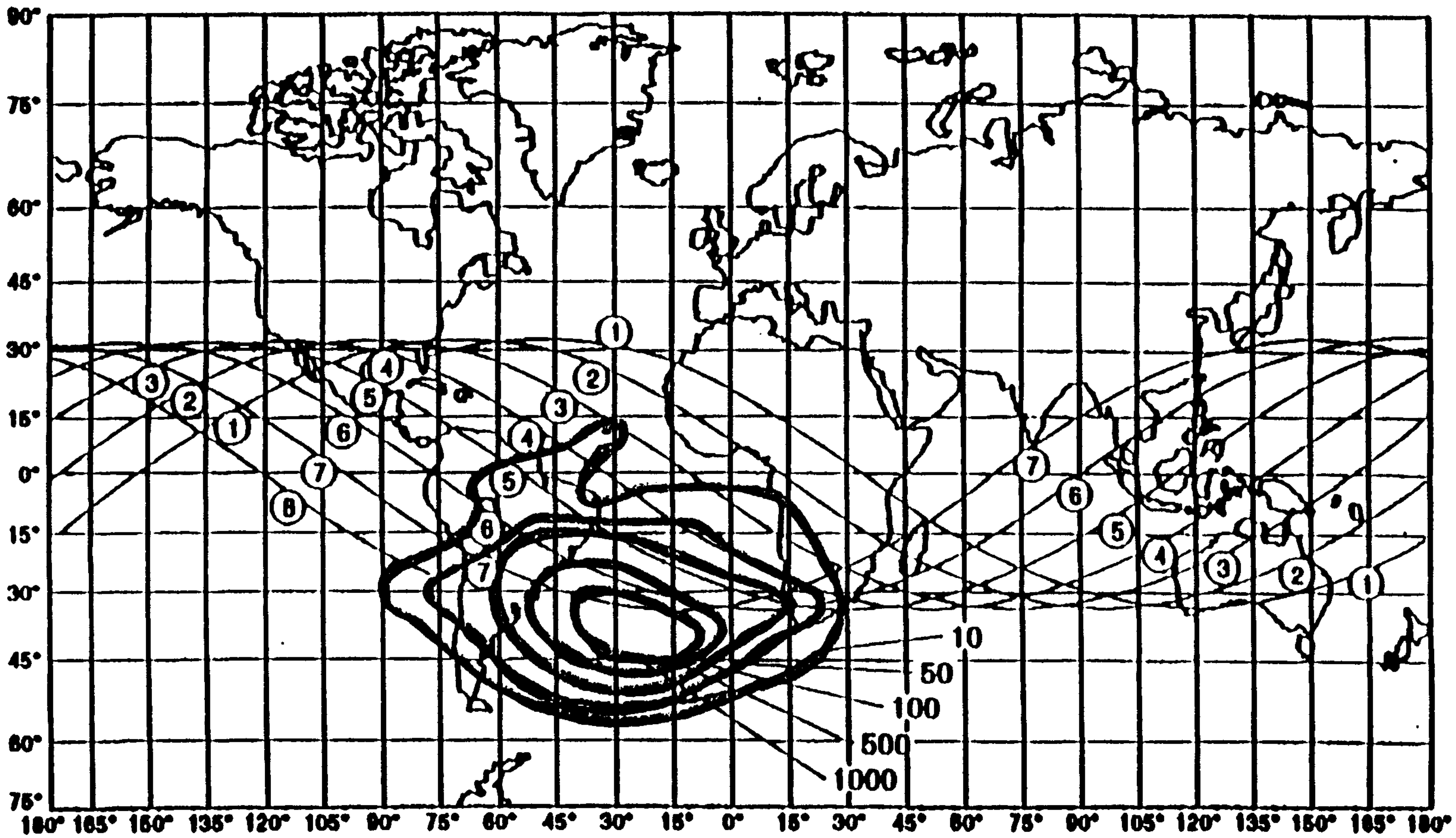


10^7
10^6
10^5 cm^{-2} s^{-1}
5
10
13
Electrons, $E_e > 40$ KeV

- Near the coast of Brazil, a decrease in the intensity of earth's magnetic field exists called the South Atlantic Magnetic Anomaly. This causes an increase in the energetic particle fluxes encountered, for instance, at LEO. Proton fluxes near 300 km associated with the anomaly are shown in the following figure, with the ground track of a 30° inclination satellite superimposed.

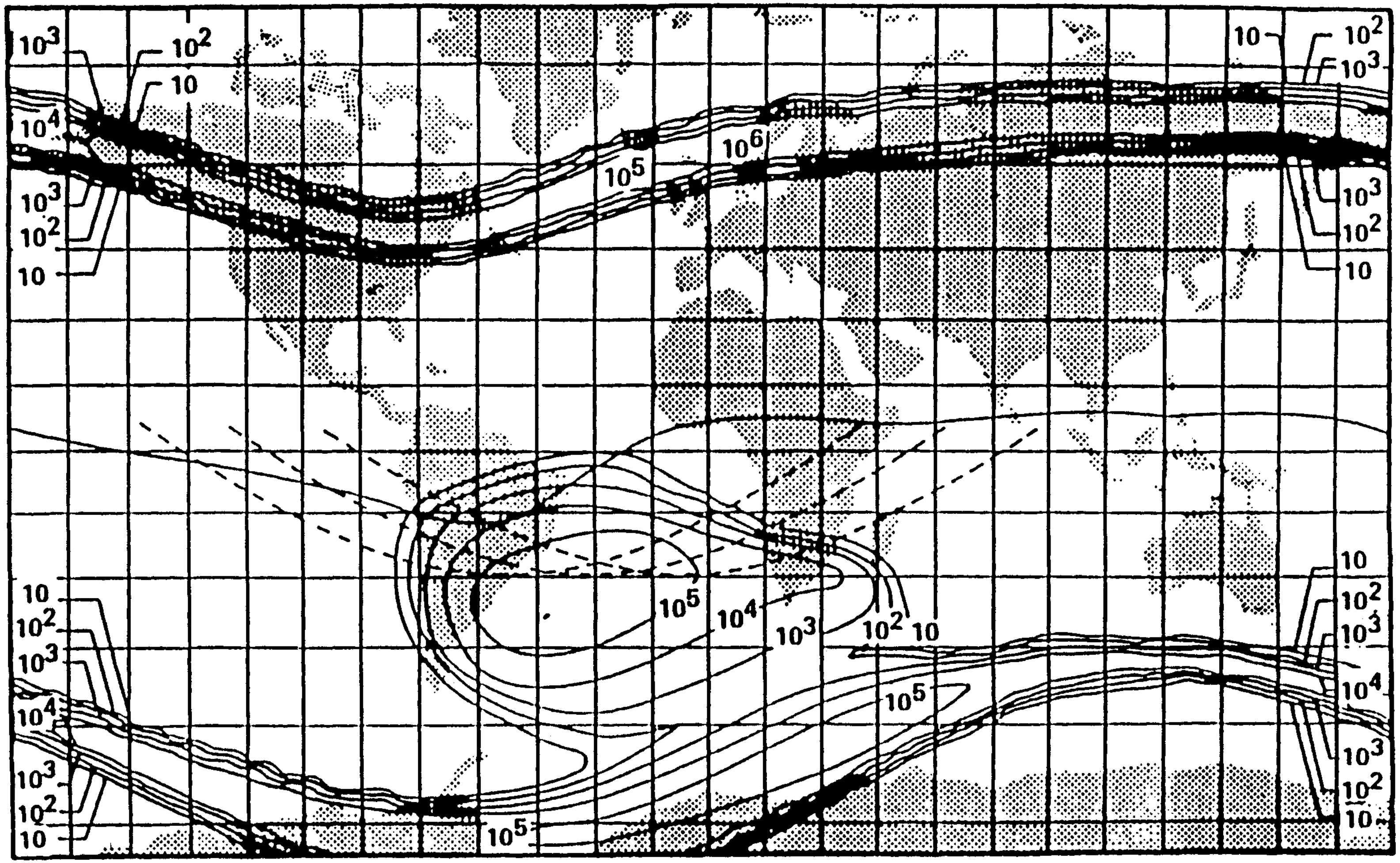
    --- Since the magnetic field is weaker, and particles mirror at a constant magnetic field strength, these particles find themselves mirroring at much lower altitudes in this geographical region.

- The subsequent figure illustrates the electron fluxes for energies > 0.5 MeV at 400 km. Note that in addition to the South Atlantic Anomaly, that at high altitudes particles from the low-altitude extension of the radiation belts (or "horns") are apparent.



OMNIDIRECTIONAL FLUX (PROTONS/CM² SEC) ENERGY > 30 MeV

PROTON FLUX DENSITIES AT AN ALTITUDE OF 296 KILOMETERS IN THE SOUTH ATLANTIC ANOMALY. THIS IS A REGION OF LOW MAGNETIC FIELD



Electron Constant Flux Contours at 400 km Alti-tude.(E>0.5 MeV)[11]