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School on the Physics of Equatorial Atmosphere

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Tides and their Impact on the Equatorial Atmosphere

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International School on the

Physics of the Equatorial Atmosphere and Ionosphere

ICTP Trieste

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Introduction

- **Atmospheric tides play an important role in atmosphere, especially in MLT and higher**
- **Propagating diurnal tide is especially important at latitudes less than ~35°**
- **Introduction to tidal theory**
- **Survey of impact of tides on atmosphere**

Source material:

Chapman and Lindzen, *Atmospheric Tides*, **1970**

M. E. Hagan, Lecture notes, Wave Workshop, Adelaide, September 1997

H. Volland, *Atmospheric Tidal and Planetary Waves,* **1988**

Tidal Forcing

- Atmospheric solar tides are driven by the absorption of solar radiation in the middle and lower atmosphere
- Two primary sources
  - Solar IR absorption by water vapour in the troposphere
  - Solar UV absorption by O_3 in the stratosphere
- Latent heat release in deep convective activity in the tropics is an additional tidal source

Governing Equations

$$\frac{d\bar{u}}{dt} + \bar{\Omega} \times \bar{v} = -\nabla \frac{p}{\rho}$$

$$\frac{\partial p}{\partial t} = -\rho \nabla \cdot \bar{u}$$

$$\rho c_v \frac{dT}{dt} = RT \frac{d\rho}{dt} + \rho J$$

$$p = \rho RT$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{u} \cdot \nabla$$

$$\bar{u} \cdot \nabla = \frac{u}{a \sin \theta} - \frac{v}{a} \frac{\partial}{\partial \lambda} + w \frac{\partial}{\partial z}$$

Coordinates :

λ **– longitude**

θ **– colatitude (from North Pole)**

z **– altitude**

velocity $\bar{u} = (u, v, w)$

pressure $= p(z, \theta)$

density $= \rho(z, \theta)$

Rotation $= \bar{\Omega}$

Temperature $= T(z, \theta)$

Heat capacity $= c_v$

Heating rate per unit vol $= J$

Assumptions

- **Linear steady-state theory**
- **Wave amplitudes small compared with background atmospheric fields**
- **No dissipation**
- **No latitudinal gradients**
- **Harmonic steady state solutions**

$$u = u', v = v', w = w'$$

$$p = p_o + p'$$

$$T = T_o + T'$$

$$\rho = \rho_o + \rho'$$

$$u', v', w', p', T', \rho' = \hat{u}, \hat{v}, \hat{w}, \hat{p}, \hat{T}, \hat{\rho} e^{i(s\lambda - \sigma t)}$$

s = **zonal wavenumber (+ve eastward)**

σ = **wave frequency**

The Classical Approximation

- A single differential equation in z and θ can be written in terms of perturbation geopotential → $\Phi' = \frac{p'}{\rho_o}$
- Equation is separable in terms of z and θ

$$\Phi' = \sum_n \Theta_n(\theta) G_n(z)$$

With the linearised horizontal momentum equations this allows for the expansion of the velocity perturbations

$$u' = \frac{i\sigma}{4\Omega^2 a} \sum_n U_n(\theta) G_n(z) \text{ and } v' = \frac{-\sigma}{4\Omega^2 a} \sum_n V_n(\theta) G_n(z)$$

These are the wind expansion functions.

Vertical Structure Equation

$$i\sigma H\left(\frac{1}{\rho_o}\frac{\partial}{\partial z}\rho_o\frac{\partial G_n}{\partial z}\right)+\frac{1}{\rho_o}\frac{\partial}{\partial z}(\rho_o\kappa J_n)=-\frac{i\sigma\kappa}{h_n}G_n$$

Where h_n is the separation constant (eigenvalue), $\kappa = R/c_p$, H is the scale height and J_n is the expansion of the thermal excitation

$$J=\sum\Theta_n J_n(z)$$

Assume an isothermal atmosphere and define reduced height

$x = z/H$ **and** $G' = G_n\sqrt{\rho_o}/N$

$$\frac{d^2G_n'}{dx^2}+\left(\frac{\kappa H}{h_n}-\frac{1}{4}\right)G_n'=\frac{1}{i\sigma\sqrt{\rho_o}N}\frac{d(\rho_o J)}{dx}$$

or

$$\frac{d^2G_n'}{dx^2}+\alpha^2G_n'=F(x)$$

Solutions: Forced Oscillations

(A) F(x) ≠ 0, "forced" oscillations with solution

$$G'_n \sim Ae^{i\alpha x} + Be^{-i\alpha x}$$

where $\alpha = \sqrt{\frac{\kappa H}{h_n} - \frac{1}{4}}$

Note, by removing H dependence can rewrite as

$$m = \sqrt{\frac{N^2}{gh_n} - \frac{1}{4H^2}}, \text{ where } N^2 = \frac{\kappa g}{H}$$

cf **dispersion relation for gravity wave**

$$m = \sqrt{\frac{N^2}{\omega^2} k_h^2 - \frac{1}{4H^2}}$$

h_n often referred to as "equivalent depth"

Propagating Solutions

$\alpha^2 > 0$ then $0 < h_n < 4\,\kappa H$ combined with "radiation condition" (no source at x = ∞) implies

$$G_n' \sim e^{i\alpha x}$$

Sign of α determines whether wave is upward (+) or downward (-) propagating

Trapped or Evanescent Solutions

$\alpha^2 < 0$ then $h_n < 0$ or $h_n > 4\,\kappa H$. Outside the source region the solution is of form

$$G_n' \sim e^{-|\alpha| x}$$

Solutions: Free or Lamb Waves

(B) $F(x) = 0$: Single solution that satisfies $w = 0$ at $x = 0$

$$G_n' \sim e^{(\kappa - \frac{1}{2})x} \text{ and } h_n = \frac{H}{1-\kappa}$$

For an isothermal atmosphere with $T_o = 256$K and $H = 7.5$ km then $h_n = 10.5$ km and $\alpha^2 < 0$.

Laplace's Tidal Equation

The θ-dependent part of solution is given by Laplace's tidal equation:

$$\frac{d}{d\mu}\left[\frac{(1-\mu^2)}{(f^2-\mu^2)}\frac{d\Theta_n}{d\mu}\right]-\frac{1}{f^2-\mu^2}\left[\frac{s}{f}\frac{(f^2+\mu^2)}{(f^2-\mu^2)}-\frac{s^2}{1-\mu^2}\right]\Theta_n+\varepsilon_n\Theta_n=0$$

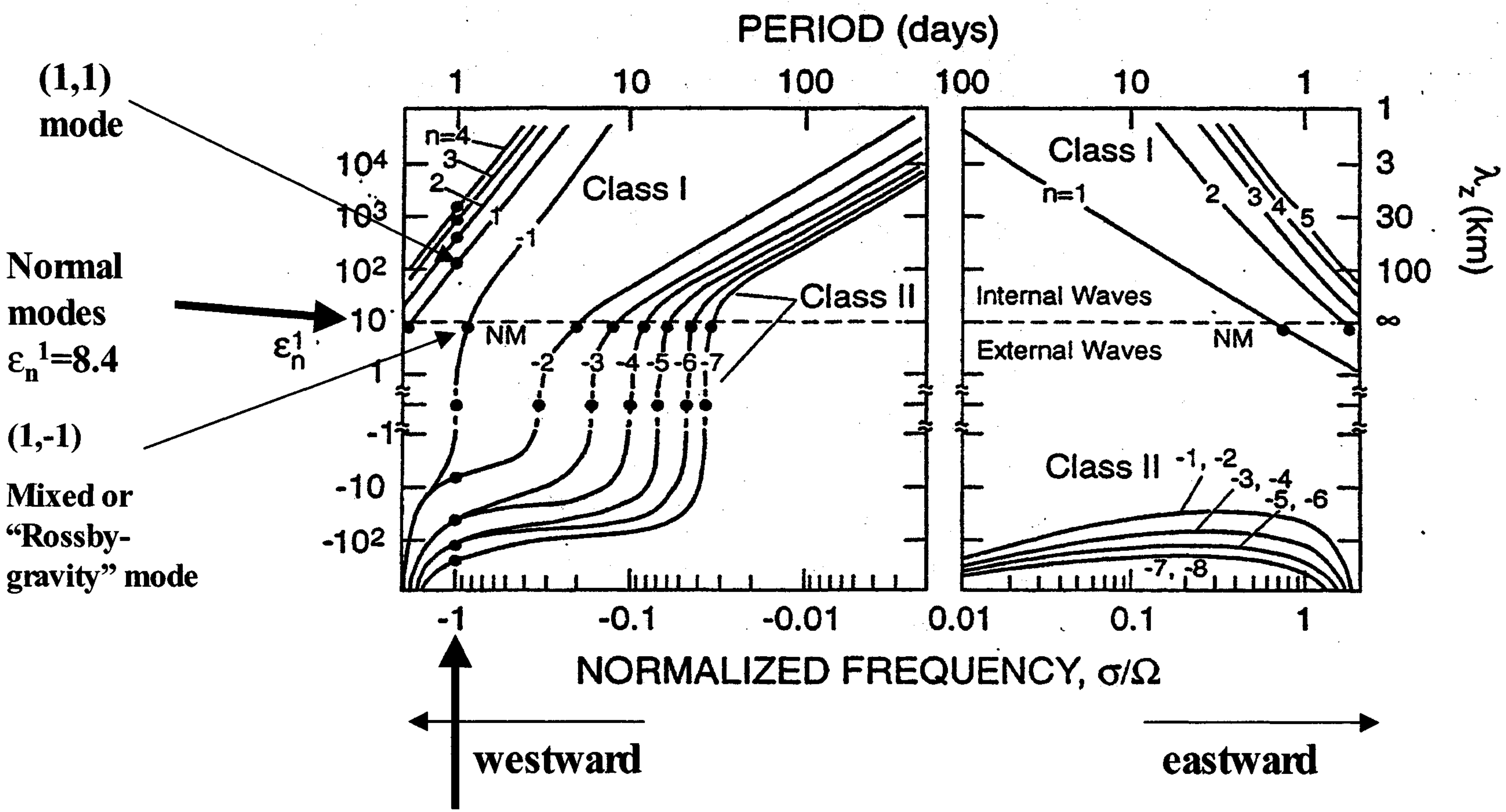
where $\mu = \cos\theta, f = \sigma/2\Omega,$ **and** $\varepsilon_n = (2\Omega a)^2/gh_n$

Often written as $F^{s,\sigma}(\Theta_n^{s,\sigma}) = \varepsilon_n^{s,\sigma}\Theta_n^{s,\sigma} = \frac{4\Omega^2 a^2}{gh_n^{s,\sigma}}\Theta_n^{s,\sigma}$

The solutions for Θ (eigenfunctions) are usually known as "Hough functions"

Each (h_n^s, Θ_n^s) **pair constitute a "mode", referred to as a** (s, n) **mode**

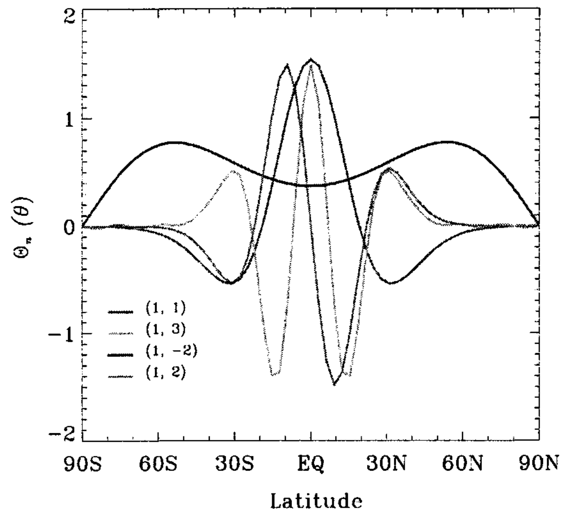
Class I ="gravity modes" **Class II ="Rossby modes"**

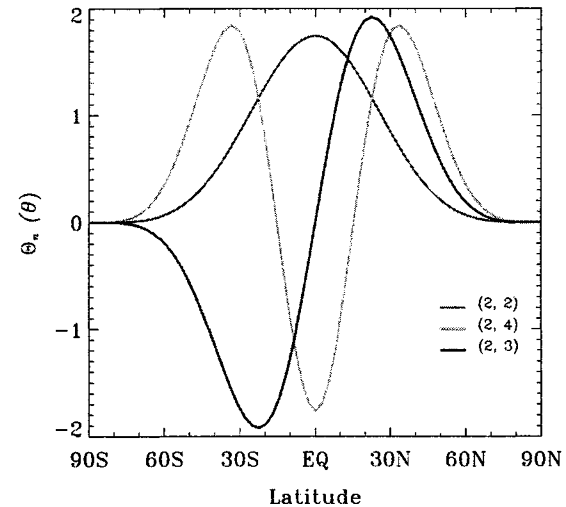


Diurnal tides

S=1 modes

Hough Functions for Diurnal and Semidiurnal Tides





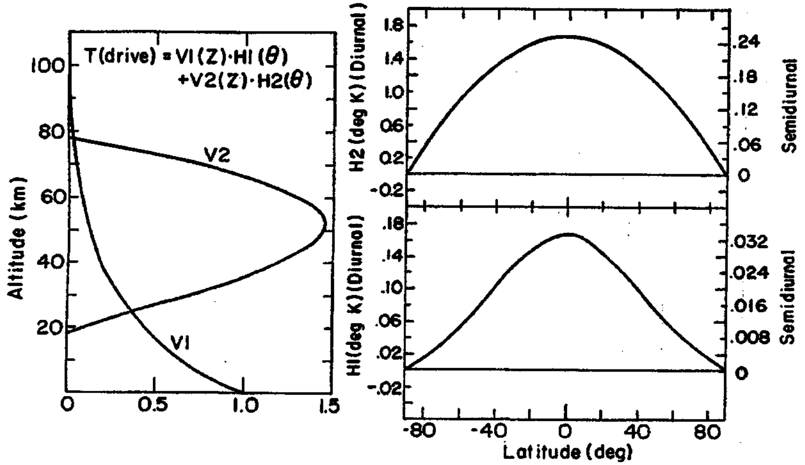
- Θ_n for diurnal (s=1, σ/Ω=-1) and semidiurnal (s=2, σ/Ω=-2) tides
- Latitudinal extent
- Hemispheric symmetry
- Vertical wavelength

Mode	h_n (km)	λ_z (km)
(1, 1)	0.69	≈ 25
(1, -2)	-12.27	Trapped
(2, 2)	7.07	> 100
(2, 4)	1.85	≈ 50

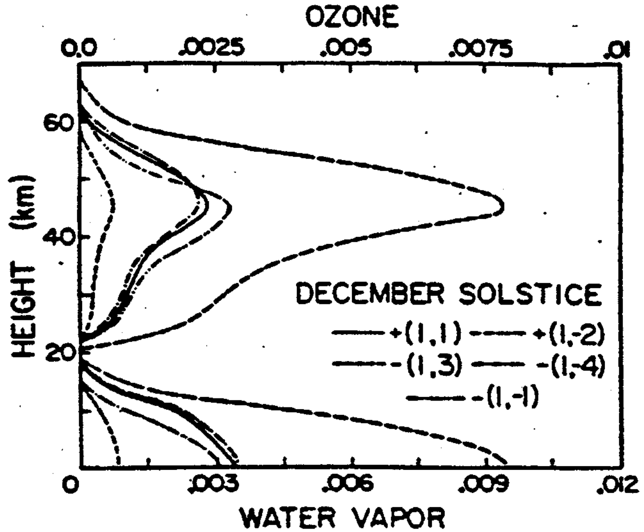
Thermal Excitation: Migrating Tides

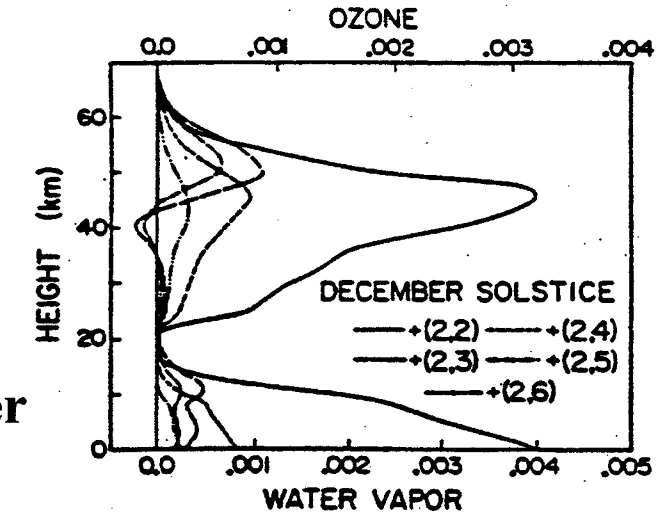
$\sigma = s\Omega$ therefore tides

follow apparent motion of sun



Vertical and latitudinal distributions for water vapour (IR) and ozone (UV) heating (Chapman and Lindzen, 1970)



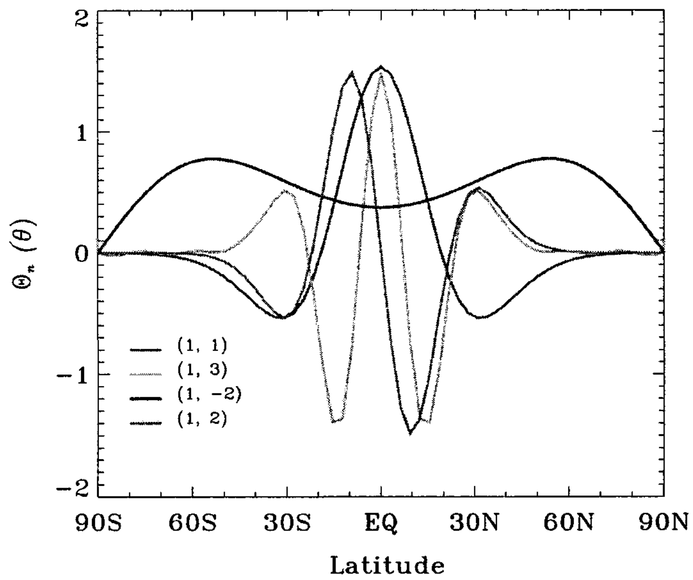


IR and UV heating rates (Jkg^{-1}) for December Solstice (Forbes and Garrett, 1978)

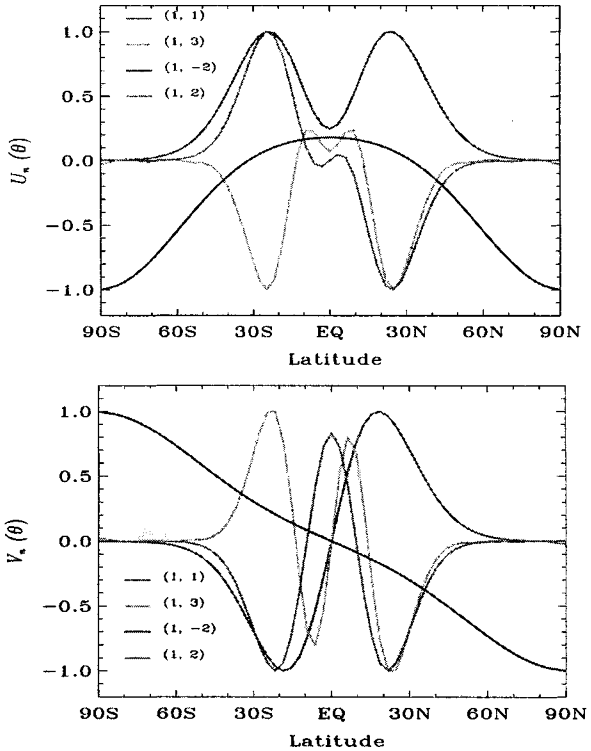
Tidal Polarization Relations

$T_n', w_n', p_n', \rho_n' \propto \Theta_n$

i.e. the values of these quantities for a particular mode in amplitude and phase are proportional to the Hough function

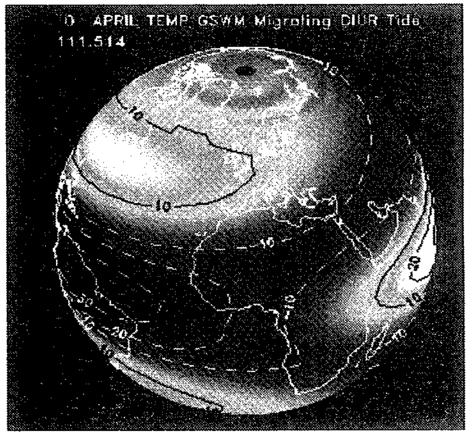


S1 Hough functions



S1 Wind expansion functions

Classical Breakdown



In the presence of

- dissipation
- background winds ($U \neq 0$)
- Latitudinal gradients in background atmospheric fields

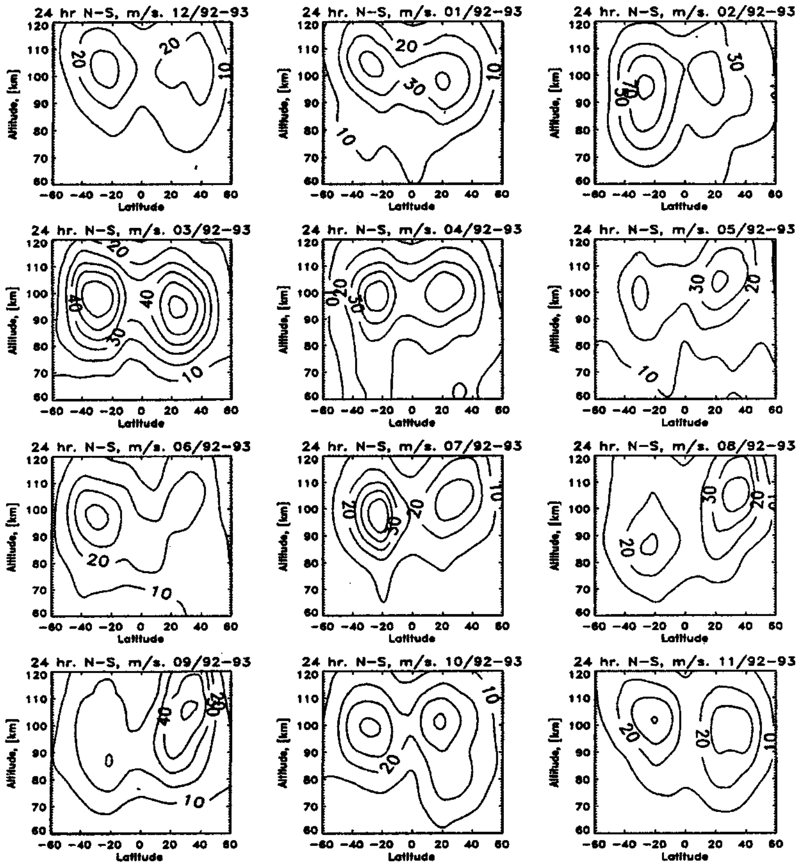
Need to solve the steady-state system equations numerically

e.g. Global-Scale Wave model (GSWM)

Hagan et al (1993), Hagan et al. (1995)

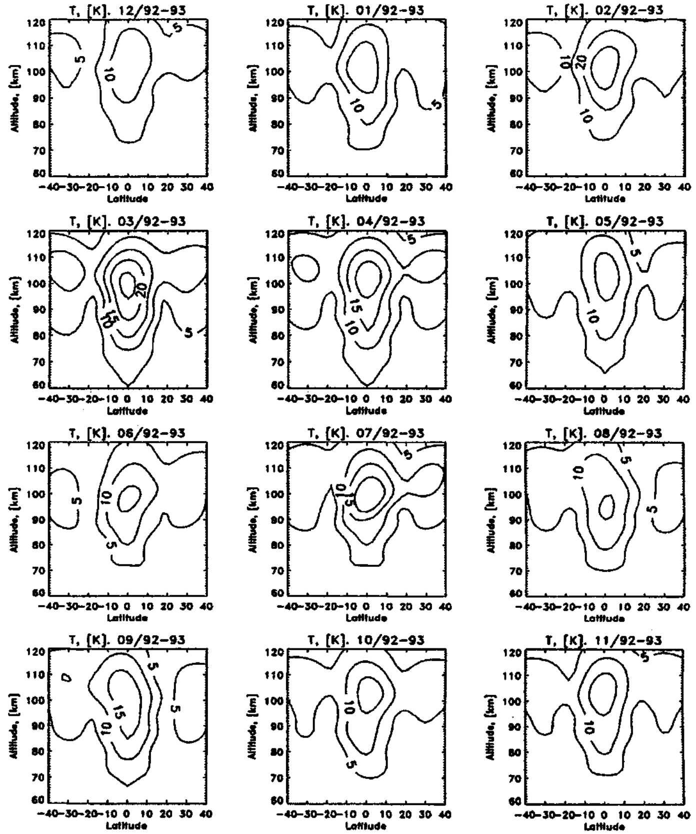
http://www.hao.ucar.edu/public/research/tiso/gswm

Seasonal Variations



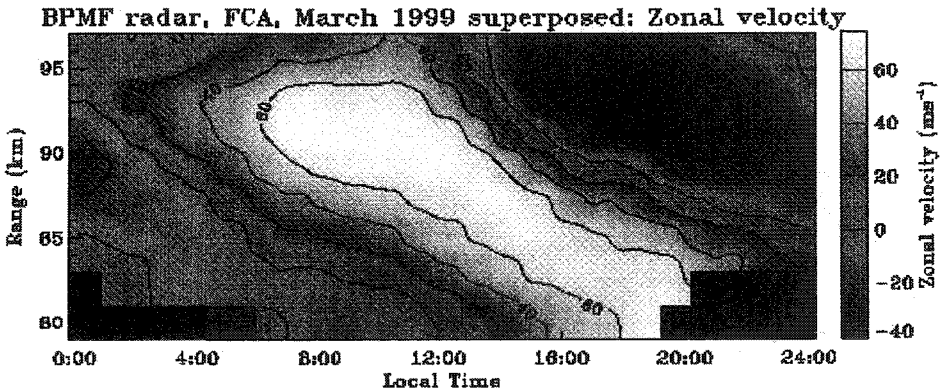
HRDI diurnal winds

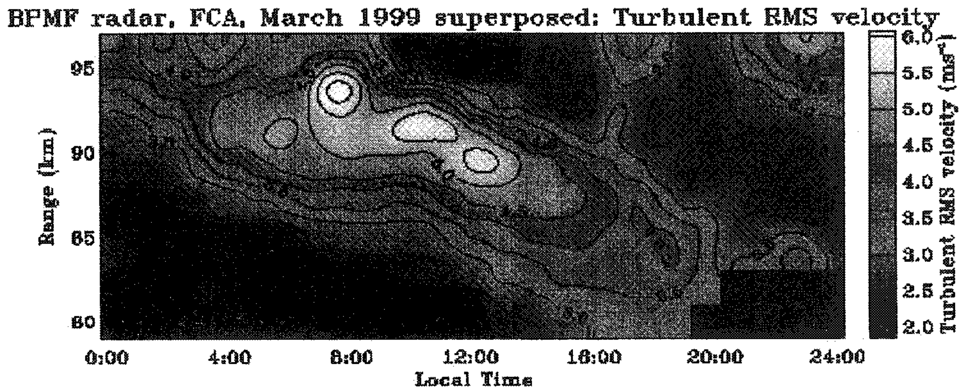
T′

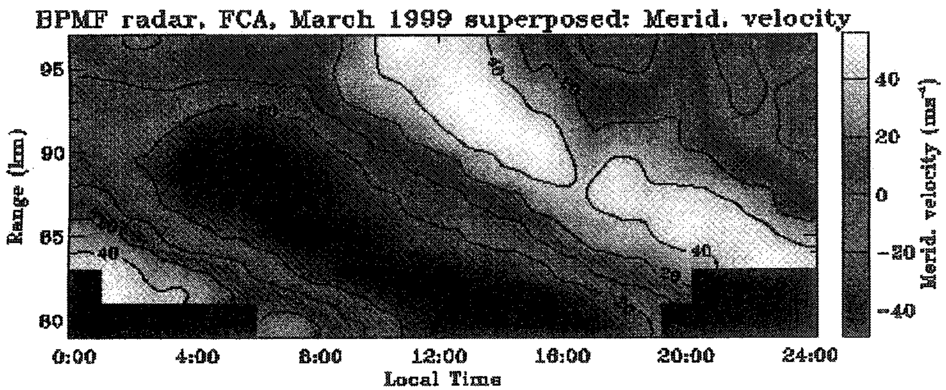


- **Semiannual variation**
- **Equinoctial maxima**

Tidal Effects on the Mean State





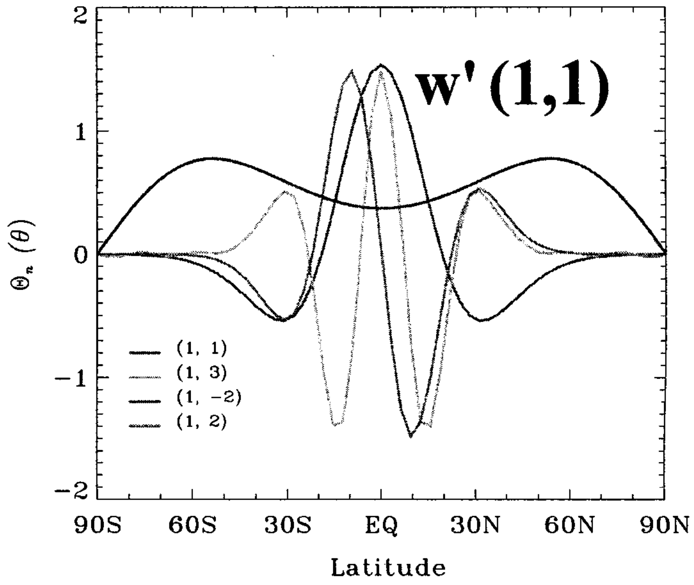


- **Turbulence motions show diurnal variation**
- **In-phase with zonal tidal component**
- **Vertical wavelength (~25 km) consistent with (1, 1) mode**
- **Why does the atmosphere appear to be unstable with 24-hr period?**

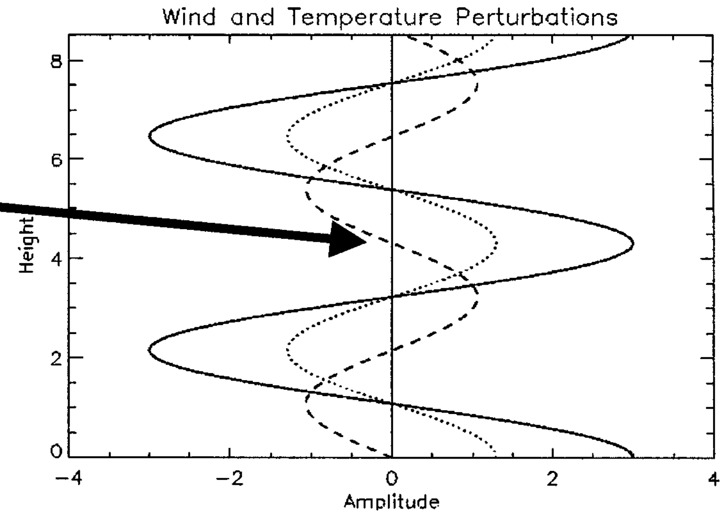
(1,1) Momentum Flux

$\overline{u'w'} < 0$ **for latitudes < 25°**

$\overline{u'w'} > 0$ **for latitudes > 25°**

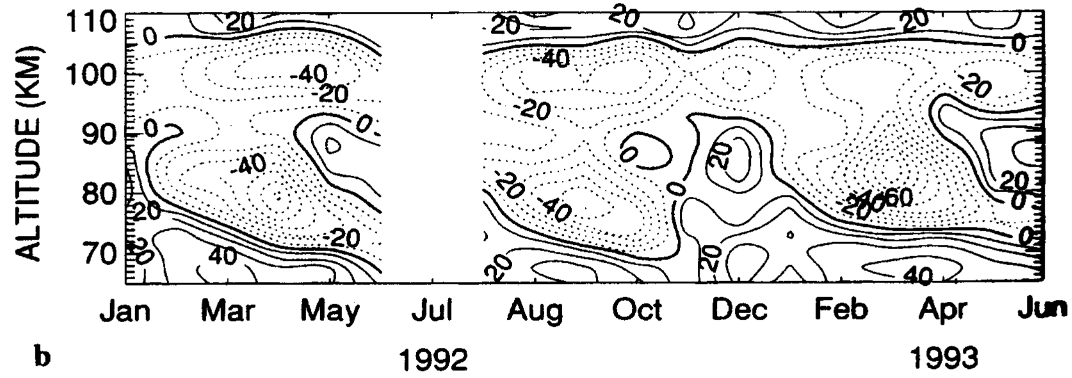


- **Tide is rather like a planetary-scale inertia-gravity wave**
- **Maximum negative temperature gradients (least stable) where *u'* is a maximum**
- **Tide preconditions atmosphere for instability and turbulence**

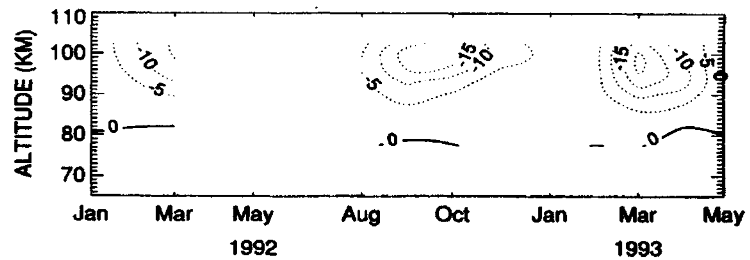


Tide/Mean Flow Interactions

- **Tides transport momentum**
- **Dissipating tides will drive mean zonal flow at equator**

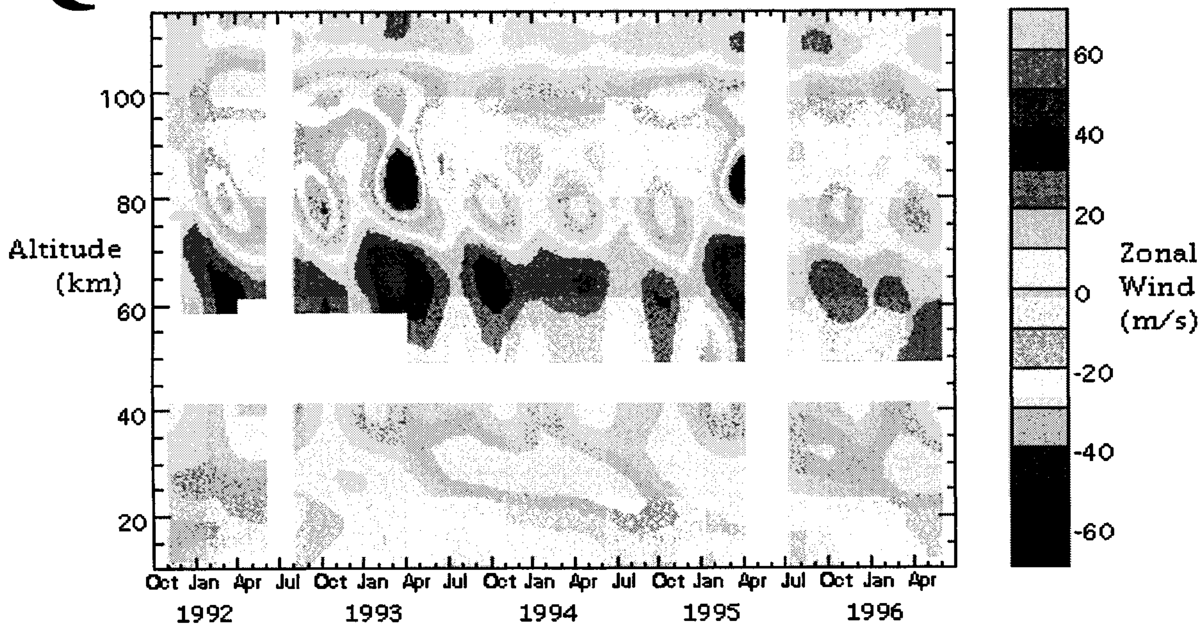


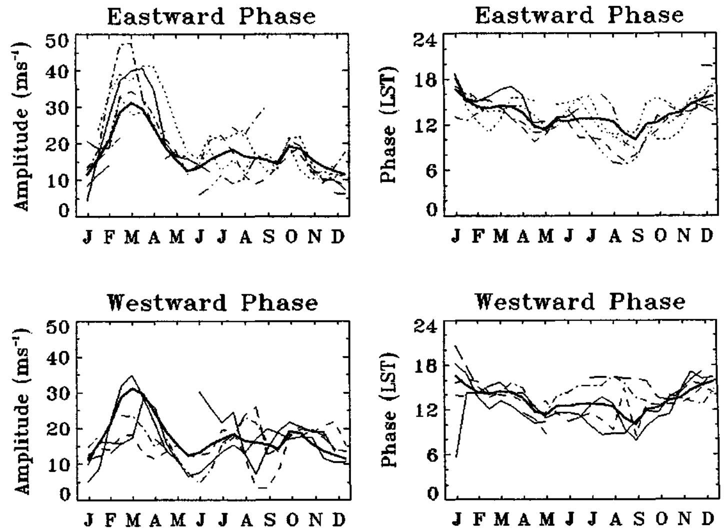
MLT zonal-mean zonal winds at equator



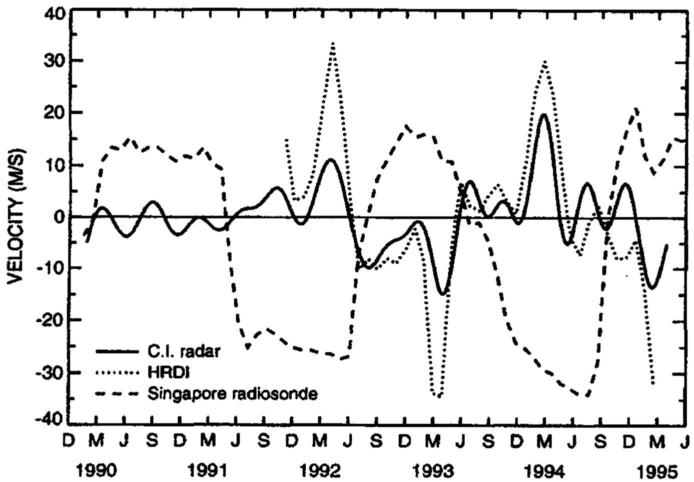
Inferred zonal accelerations by (1,1) mode. Appears to drive time-mean westward winds in 90-105 km height range (Lieberman and Hays, 1994)

QBO Influences





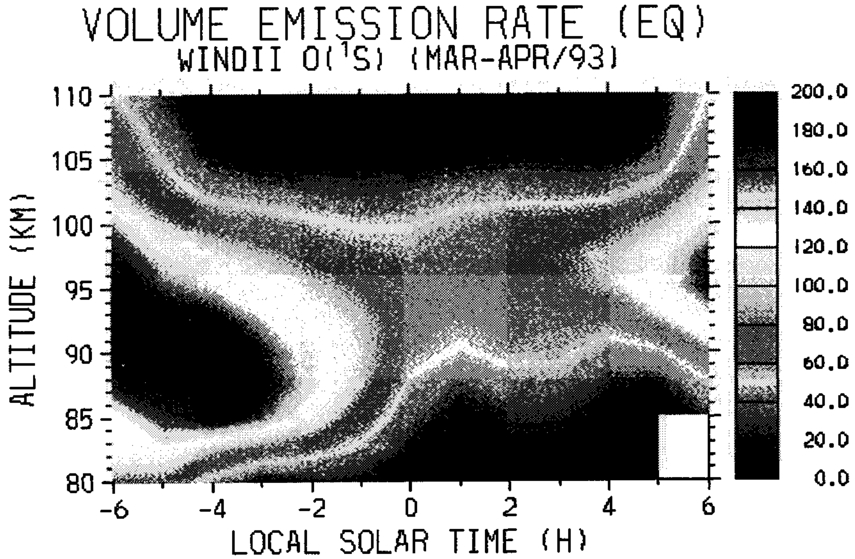
24-hr tide at Adelaide

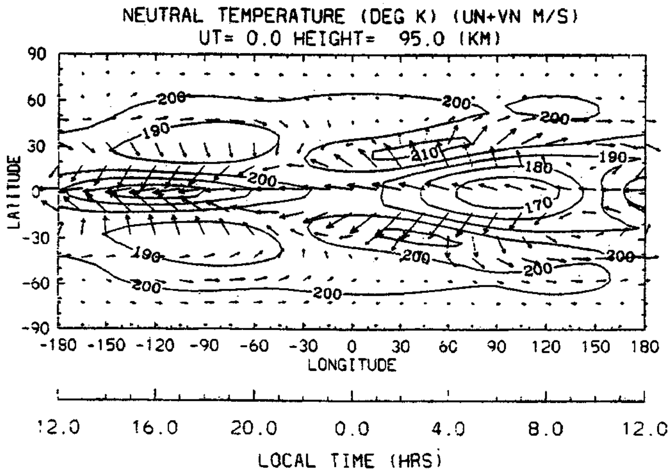


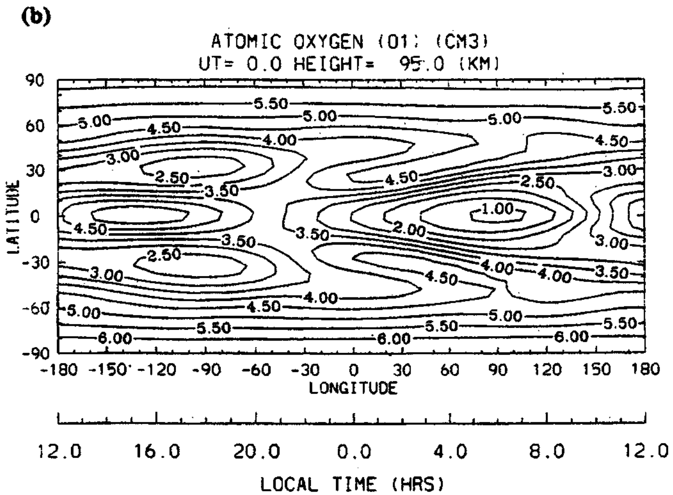
Equatorial zonal winds

- **Interactions between QBO and diurnal tide**
  - **Wave-mean flow interactions?**
  - **Gravity-wave/tidal interactions?**

Tides and Airglow Emissions







TIMEGCM model output

Convergence at equator means downward motions

Downward transport of O-rich air

WINDII observations. Note strong diurnal variation with disappearance of emission near 00 LT

Formation: $O + O + M \rightarrow O_2^* + M$

Density perturbations or vertical transport?