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School on the Physics of Equatorial Atmosphere

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Appendix to

Gravity Waves

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Gravity Waves

International School on the

Physics of the Equatorial Atmosphere and Ionosphere

ICTP Trieste

September 2001

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Basic Wave Parameters

- The following information is required to assess the importance of gravity waves in determining the state of the tropical atmosphere
  - Wave amplitudes or energy as a function of time and space
  - Propagation directions in both the horizontal and vertical
  - Relative importance of different wave sources
  - Energy and momentum fluxes as a function of frequency and wavenumber
- How do we obtain this information from basic observations of wind, temperature and other observed quantities?

Wave Energy

- Filter wind or temperature field to obtain wave perturbations or fluctuations
- Compute mean square amplitudes and hence energy

$$U = (u, v, w) \text{ where } u = \overline{u} + u' \text{ etc}$$

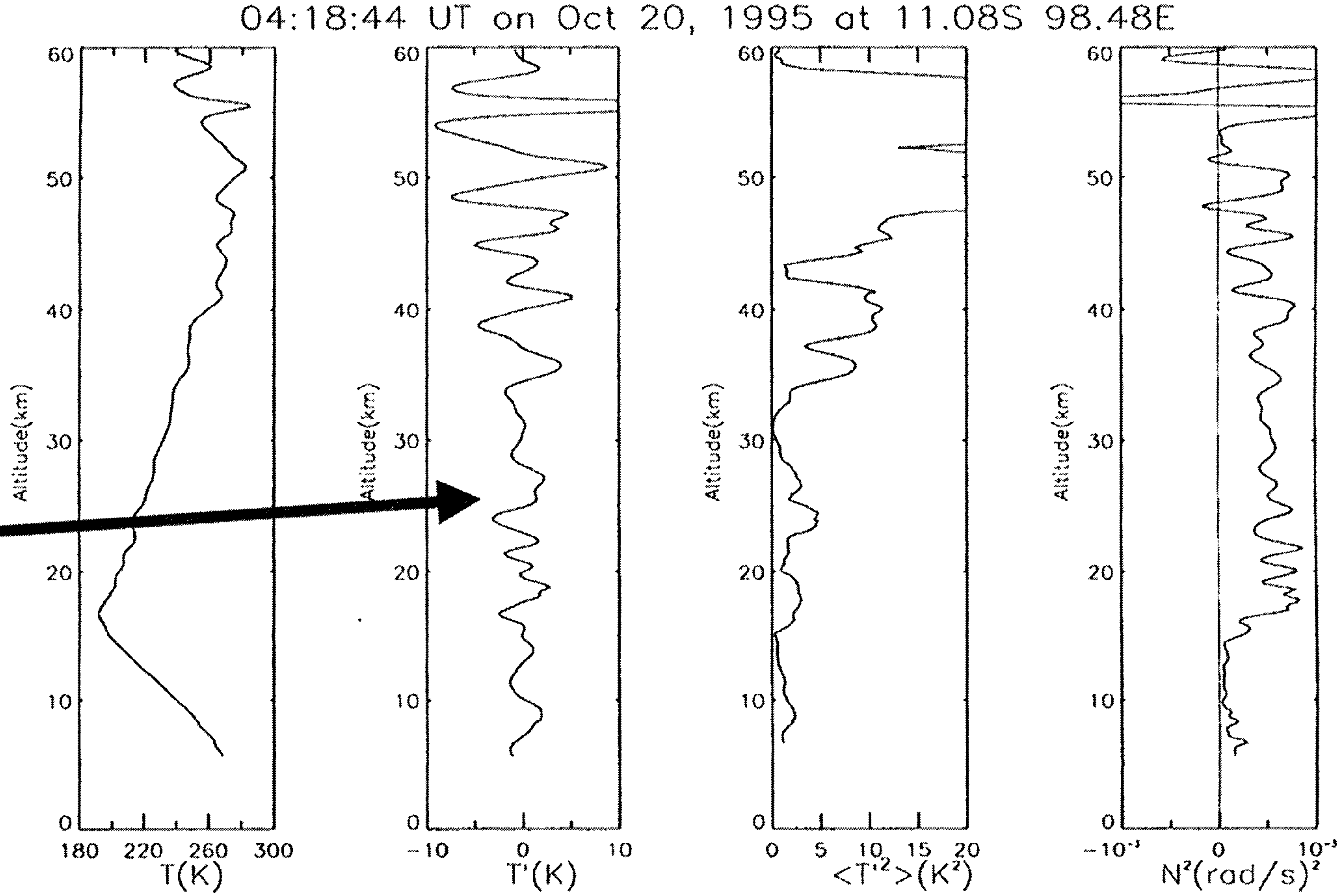
$$E = E_k + E_p$$

$$E = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2} + \frac{g^2}{N^2}\overline{\hat{T}^2} \right)$$

$$\text{where } \hat{T} = \frac{T'}{\overline{\overline{T}}}$$

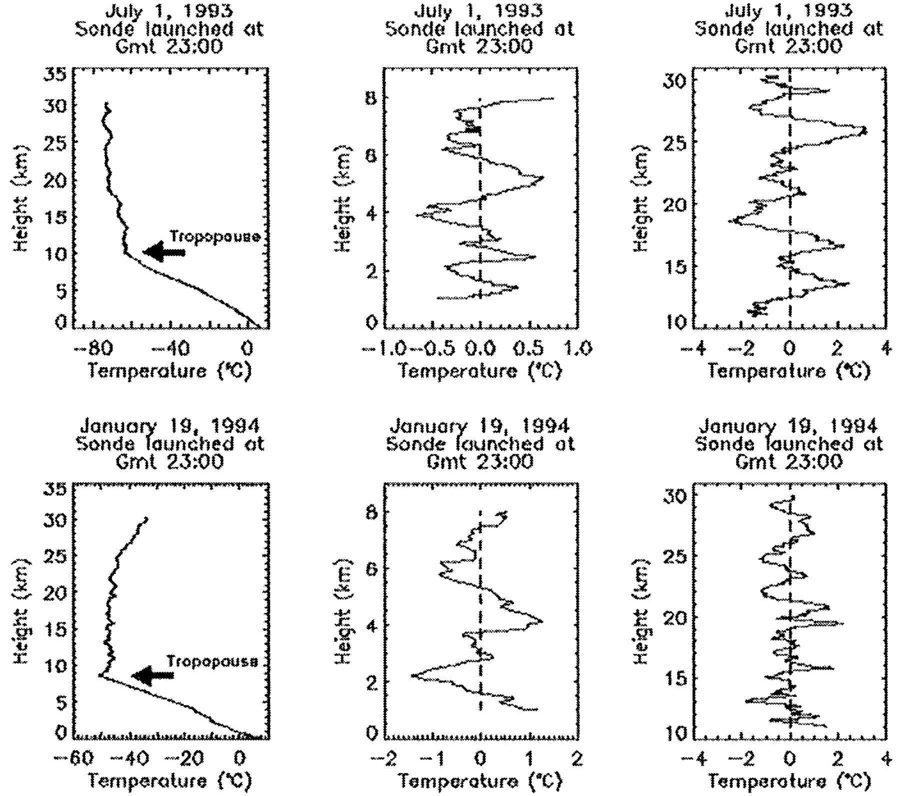
Example from GPS/MET observations (Tsuda et al, 2000)

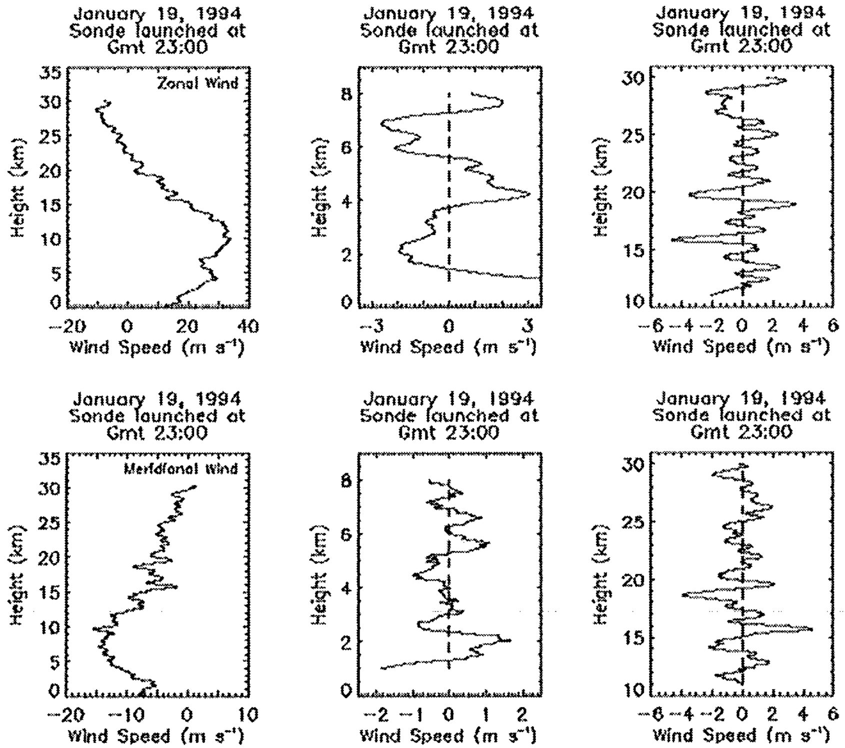
High pass spatial filter with 10 km cut-off



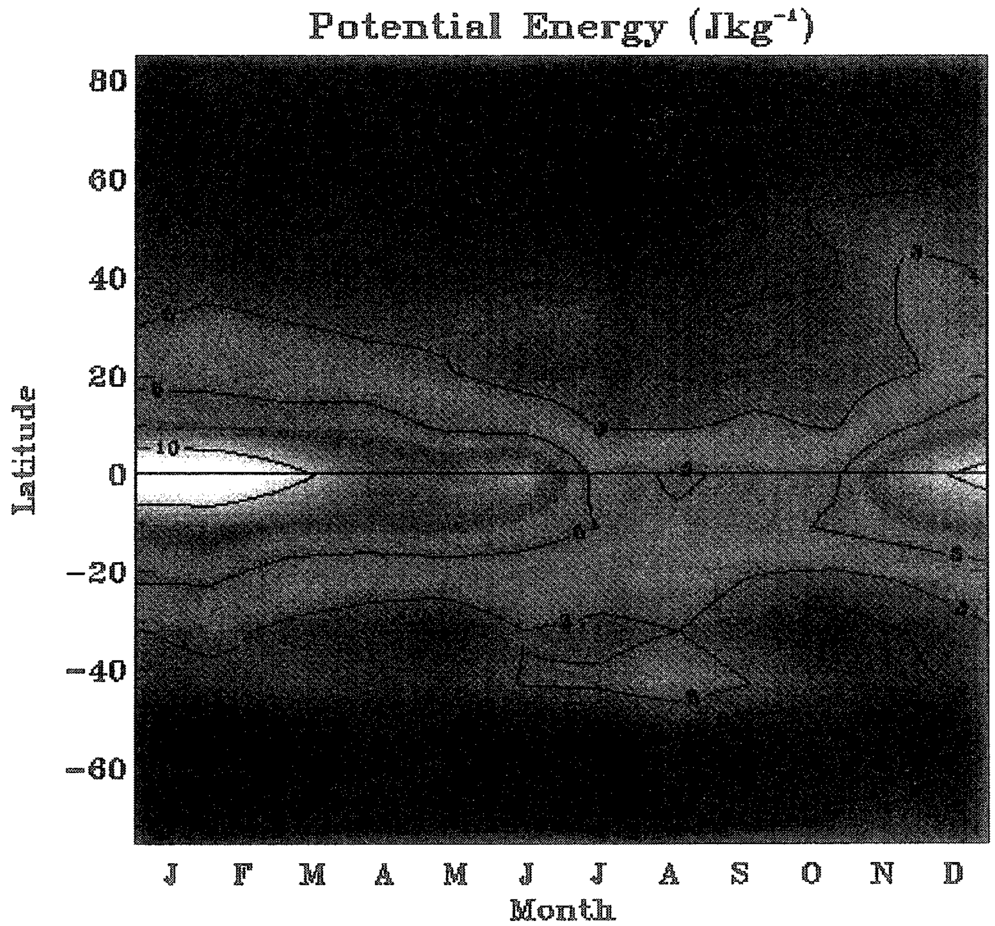
T $\quad T'$ $\quad \overline{T'^2}$ $\quad N^2$

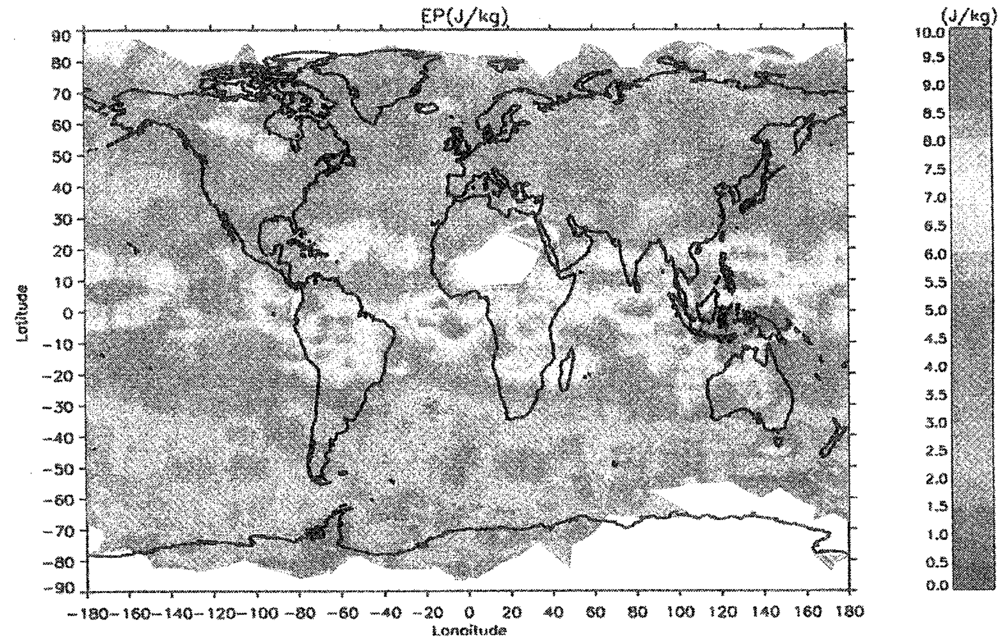
- **Synoptic radiosonde observations are a valuable resource**
- **Fit polynomial to remove background**





E_p Climatologies





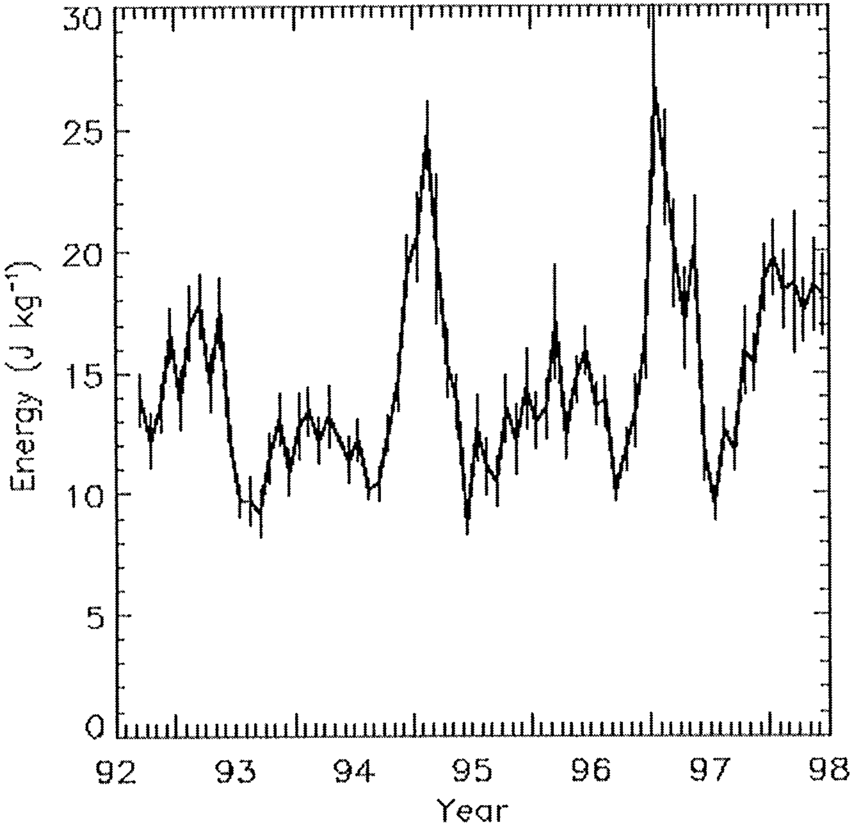
GPS/MET Nov-Feb 20-30 km

SPARC radiosonde climatology

17-24 km

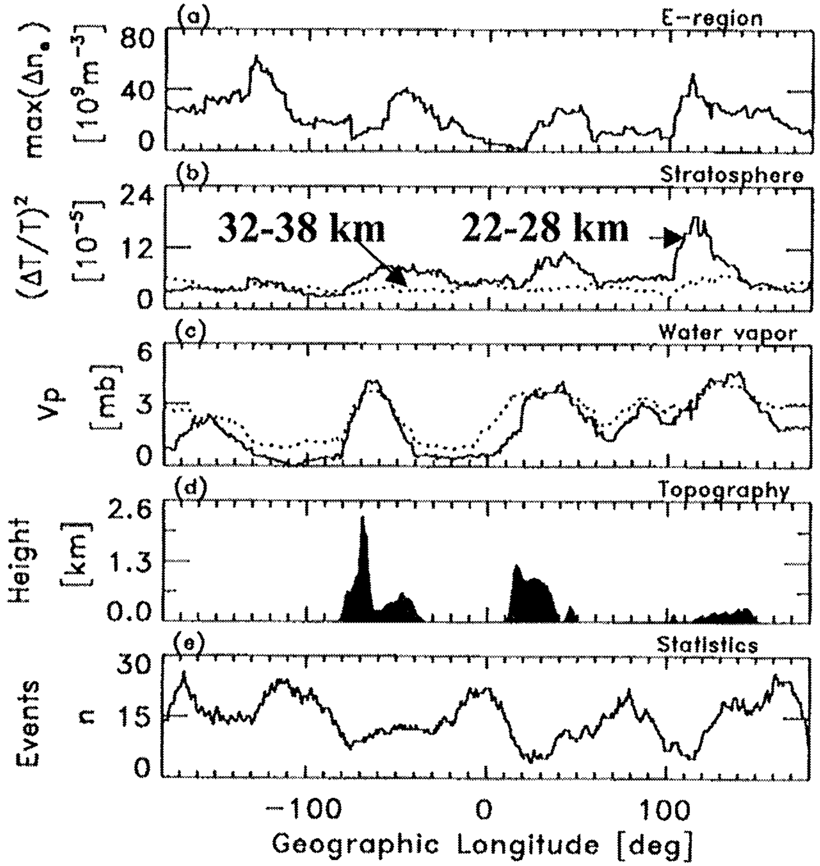
Note large equatorial amplitudes. Source effect or observational bias?

Tropical GW



Six year time series of *E* in 18-25 km range at Cocos Is (12°S). Note annual and QBO-like cycles. Source or wind filtering effects?
(Vincent and Alexander, 2000)

GPS/MET observations
(Hocke and Tsuda, 2001



Note apparent geographic relation between location of enhanced fluctuations in *N*, *T*, and humidity (convection?).

Definitions

To determine direction of propagation in both the vertical and horizontal use the polarization relations that relate the perturbation quantities, u', v', w', and T'. Assume that we are dealing with harmonic waves in a rotating fluid

$$u', v', w', T' \propto \rho^{-1/2} e^{i(kx+ly+mz-\hat{\omega}t)}$$

$$v' = \frac{\hat{\omega}l - ifk}{\hat{\omega}k + ifl} u'$$

$$\hat{\omega} = \omega - \vec{k}_h \cdot \vec{U}$$

$$m^2 = \frac{N^2 - \hat{\omega}^2}{\hat{\omega}^2 - f^2} k_h^2 - \frac{1}{4H^2}$$

$\hat{\omega}$ = **intrinsic frequency**

ω = **ground - based (observed) frequency**

f = **Coriolis parameter**

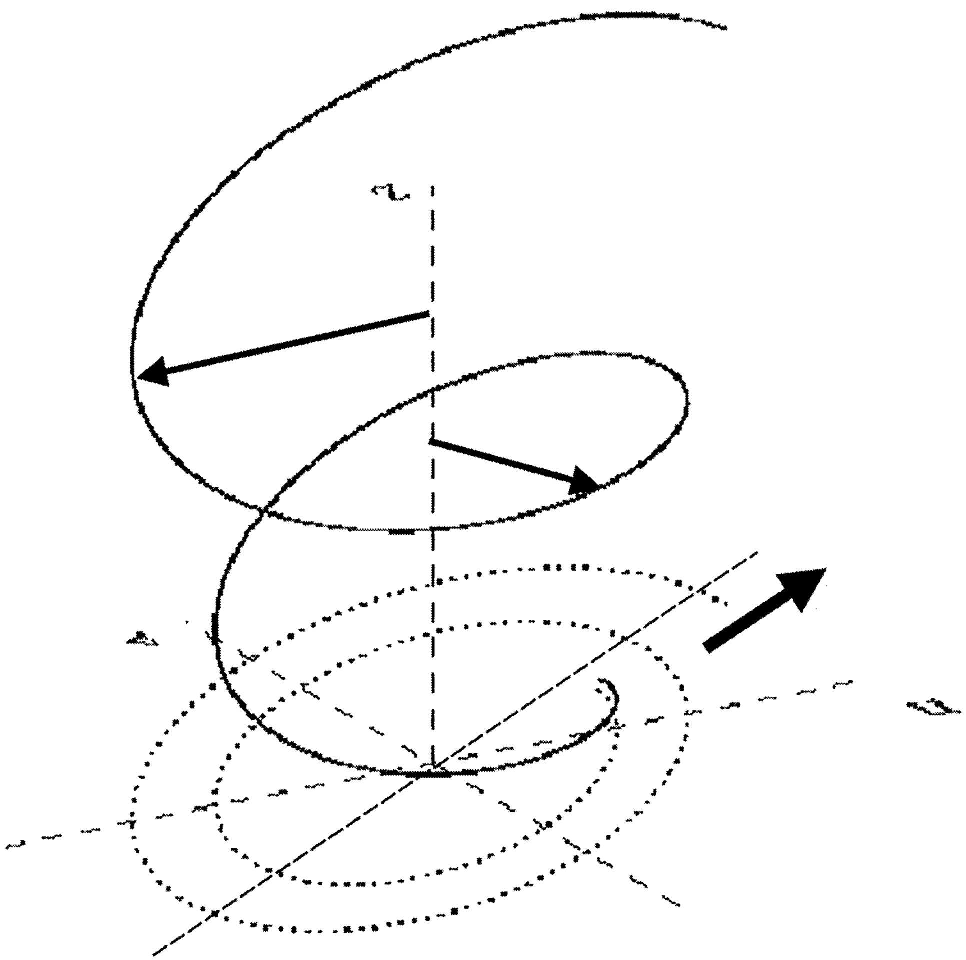
$\hat{c} = (\hat{\omega}/k, \hat{\omega}/l)$ = **intrinsic phase speed**

For many waves it is possible to assume

$$m^2 >> \frac{1}{4H^2}$$

$$m = \pm\sqrt{\frac{N^2 - \hat{\omega}^2}{\hat{\omega}^2 - f^2}} k_h$$

Propagation Directions: Anisotropy



- Hodograph analysis
- For upward energy propagation
  - Clockwise rotation in NH ($f > 0$)
  - Anticlockwise in SH
- Circular polarization as $\hat{\omega} \to f$
- Linear polarization as $\hat{\omega} \to N$

Vertical Propagation

- **Rotary decomposition provides a statistical approach to hodograph analysis**

$$\mathrm{V}(z) = u'(z) + iv'(z) = ae^{-imz}$$

$$\mathrm{V}(z) \leftrightarrow \Psi(m)$$

Clockwise component $C(m) = \frac{1}{2}\Psi(-m)\Psi^*(-m)$

$$\mathrm{V}(z) \leftrightarrow \Psi(m)$$

Anticlockwise component $A(m) = \frac{1}{2}\Psi(m)\Psi^*(m)$

Total Energy, $E(m) = C(m) + A(m)$

In NH (SH) fraction of upward energy $r_N = \frac{\overline{C(m)}}{\overline{E(m)}} \quad \left(r_S = \frac{\overline{A(m)}}{\overline{E(m)}} \right)$

Horizontal Direction

- If axes are aligned parallel and perpendicular to major and minor axes of hodograph ellipse then

$$v'_{\perp} = -i\frac{f}{\hat{\omega}}u'_{\parallel}$$

Axial ratio, $AR = \dfrac{u'_{\parallel}}{v'_{\perp}} = \dfrac{\hat{\omega}}{f}$

- Hodograph gives
  - direction of propagation (with 180° ambiguity)
  - intrinsic frequency.

Statistical approach is better.

Stokes Parameters

- Note similarity between gravity wave and electromagnetic wave
- Both are transverse oscillations that are in general elliptically polarised
- Stokes parameters are relations that determine sense and degree of polarisation

$I = \overline{u'^2} + \overline{v'^2}$ — I = total intensity

$D = \overline{u'^2} - \overline{v'^2}$ — D = intensity difference

$P = \overline{2u'v'\cos\delta}$ — P = Linear polarization parameter

$Q = \overline{2u'v'\sin\delta}$ — Q = circular polarization parameter

Stokes Parameters II

- Q gives sense of rotation
  - $Q > 0 \rightarrow$ Anticlockwise polarisation
  - $Q < 0 \rightarrow$ Clockwise polarisation
- Degree of polarisation

$$d = \frac{(D^2 + P^2 + Q^2)^{1/2}}{I}$$

$d = 1 \Rightarrow$ perfect polarisation

$d = 0 \Rightarrow$ unpolarised

Polarisation Ellipse

- **Polarisation ellipse parameters**

$$\delta = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$2\phi = \tan^{-1}\left(\frac{P}{D}\right)$$

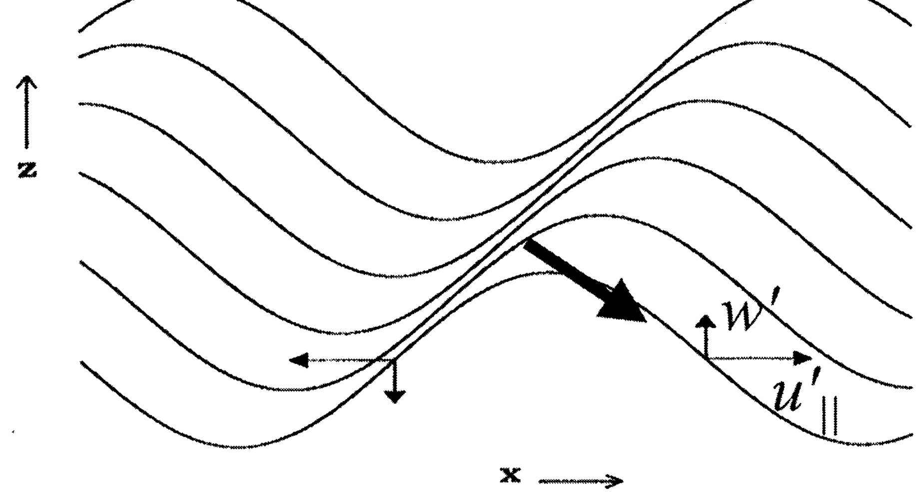
$$AR = \cot\varepsilon \;\; \text{where } 2\varepsilon = \sin^{-1}\left(\frac{Q}{d\,I}\right)$$

Apply in frequency domain using radar winds data acquired at a single height

Apply in vertical wavenumber domain using rocket or radiosonde wind profiles

Horizontal Directions

- Require extra information to resolve 180° directional ambiguity from hodograph
- Use either vertical velocity or temperature



$$w' = -\frac{k_h}{m} u'_{\|}$$

w' **in-phase with** u' **for** $k_h > 0$ **and** $m < 0 \Rightarrow \overline{u'_{\|} w'} > 0$

w' **out-of-phase with** u' **for** $k_h < 0$ **and** $m < 0 \Rightarrow \overline{u'_{\|} w'} < 0$

Horizontal Directions II

- **Temperature polarisation relation**

$$\hat{T}' = \mp i \frac{N^2}{g\hat{\omega}} \frac{k_h}{m} u'_{\parallel}$$

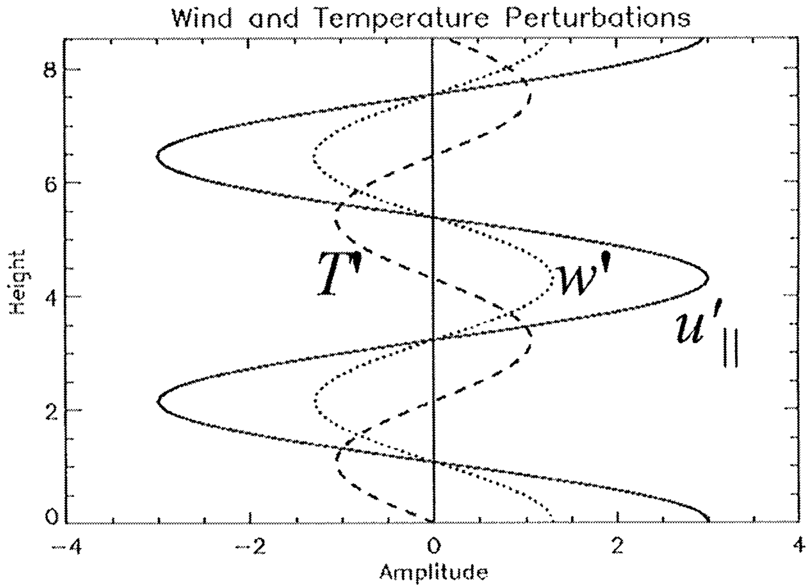
$-$ sign chosen when $km > 0$

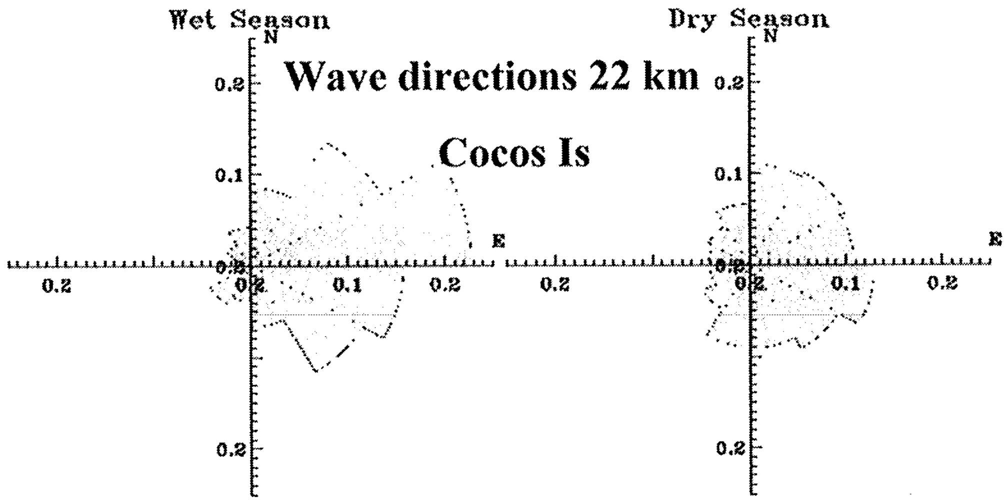
$+$ sign chosen when $km < 0$

Note 90° phase shift between T and u

$$\overline{u'_{\parallel} w'} = \frac{\hat{\omega} g}{N^2} \overline{u'_{\parallel} \hat{T}'_{+90}} \quad \text{where } \hat{T}'_{+90} = i\hat{T}'$$

Sign of $\overline{u'_{\parallel} \hat{T}'_{+90}}$ determines direction of propagation





Momentum Fluxes

$\overline{u'w'}, \overline{v'w'}$

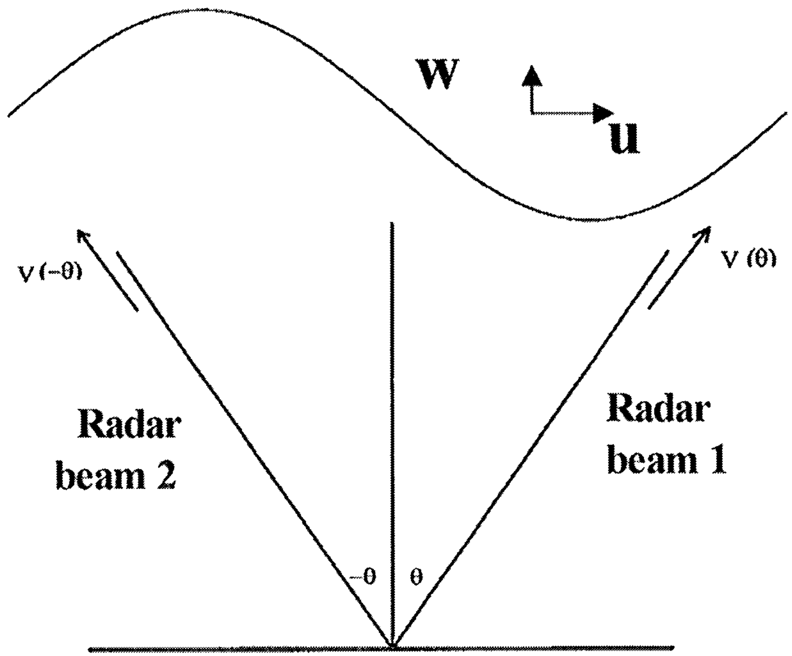
- **Important quantities, but difficult to measure directly** $(w' << u', v')$
- **Indirect estimates using temperature require knowledge of** $\hat{\omega}$

If $\overline{U} = 0$ **then** $\hat{\omega} = \omega$**, the ground - based frequency**

Radar Measurements

$\overline{u'w'}, \overline{v'w'}$

- **Dual-beam Doppler technique**
- **Measure radial velocity at range, *R*, along beams 1 and 2**



$$v_1(\theta, R) = u' \sin\theta + w' \cos\theta$$
$$v_2(-\theta, R) = u' \sin\theta - w' \cos\theta$$
$$v_1^2(\theta, R) = u'^2 \sin^2\theta + 2u'w' \cos\theta \sin\theta + w'^2 \cos^2\theta$$
$$v_2^2(-\theta, R) = u'^2 \sin^2\theta - 2u'w' \cos\theta \sin\theta + w'^2 \cos^2\theta$$

Subtract and rearrange

$$\overline{u'w'}(z) = \frac{\overline{v_1^2(\theta, R)} - \overline{v_2^2(-\theta, R)}}{2 \sin 2\theta}$$

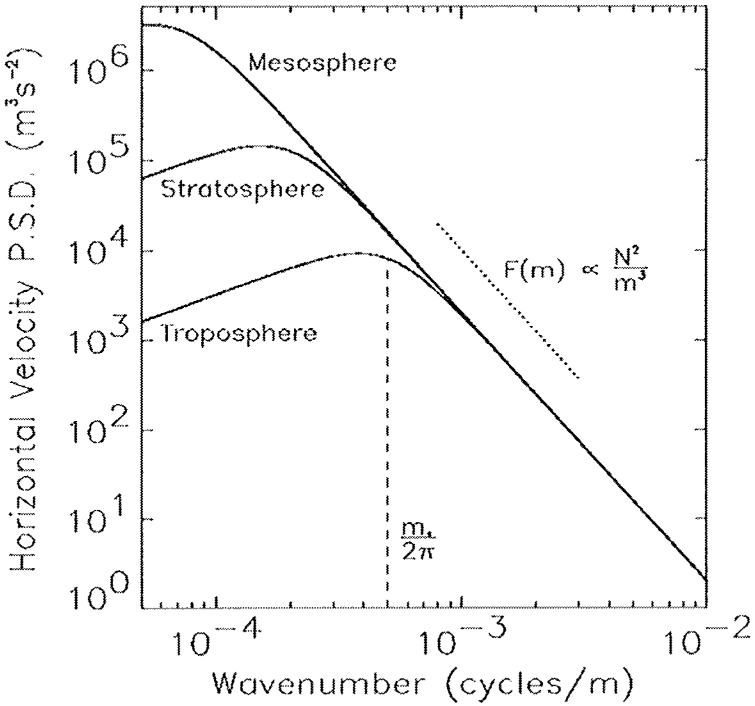
Strengths:
- **Robust**
- **Direct**

Limitations:
- **Require accurate beam angles**
- **Small differences**

Spectra

- **Frequency and wavenumber spectra are a convenient way of summarizing wave information**
- **Suggestion that there is a "universal" wave spectrum**
- **Spectral shapes and amplitudes set by instability processes**
- **Many different theories for producing spectral features**
- **Observations show that**
  - **Frequency spectra derived from ground-based radar and lidar observations of horizontal winds show $S_u(\omega) \propto \omega^{-p}$ with p~5/3**
  - **Recent** *constant pressure* **balloon observations provide first** *intrinsic frequency spectra* $S_u(\hat{\omega}) \propto \hat{\omega}^{-2}$
  - **Vertical wavenumber spectra derived from radiosonde, rocket, and radar observations of winds and temperatures show $S_{u,T}(m) \propto m^{-t}$ with $t \sim 3$ for large m**

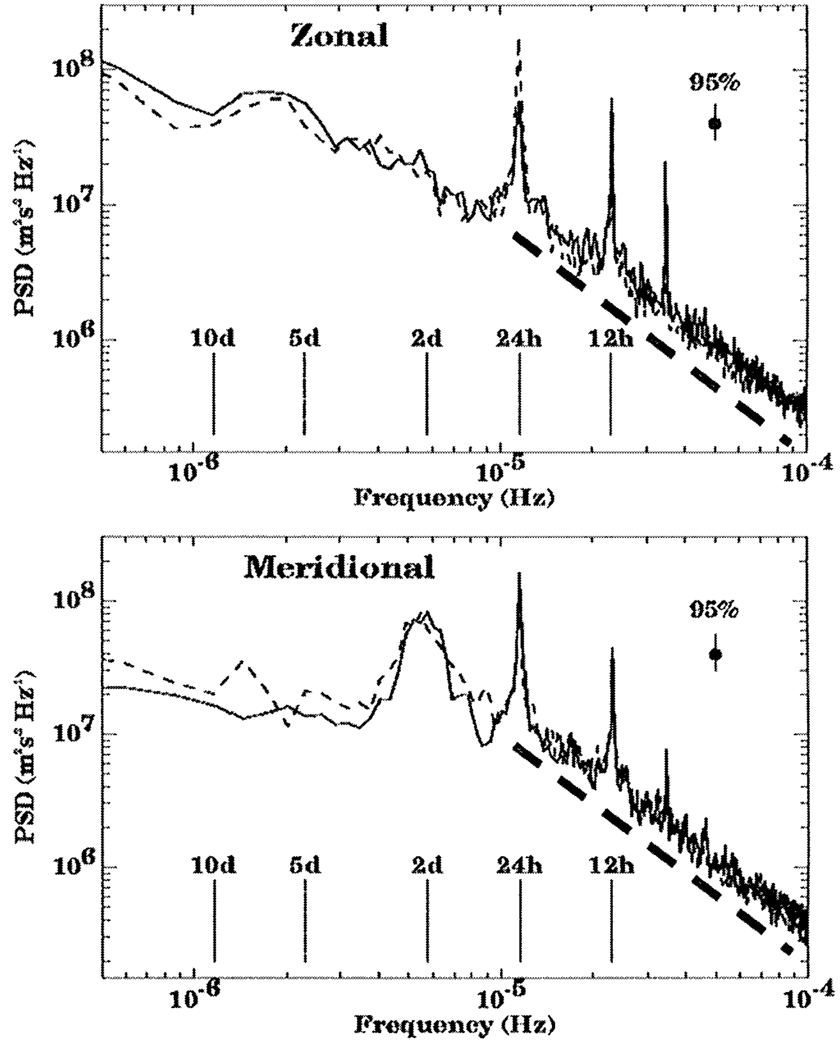
Spectra: Examples



Modified Desaubies spectrum

$$F(\mu) = \frac{\mu^s}{1+\mu^{s+t}}$$

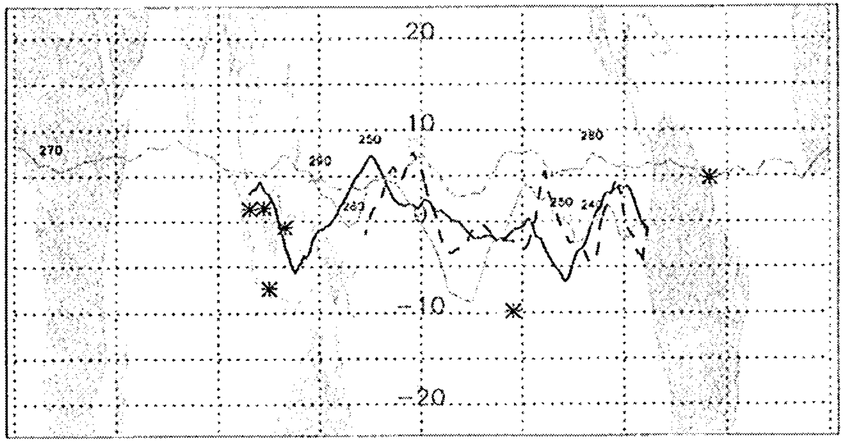
$$\mu \equiv \frac{m}{m_*}, t \sim 3, s \sim 1(?)$$

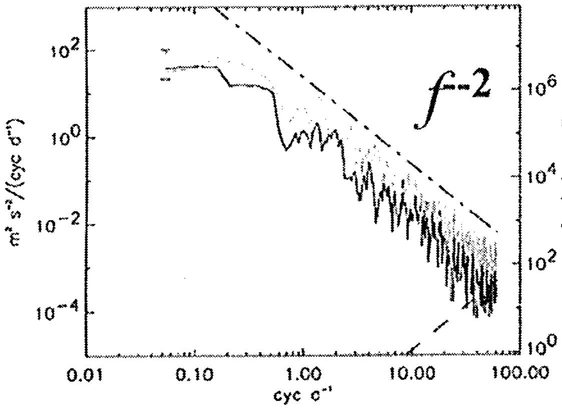


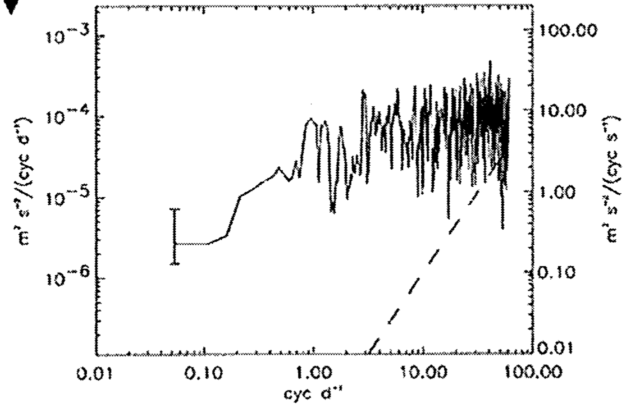
Equatorial mesospheric frequency spectra

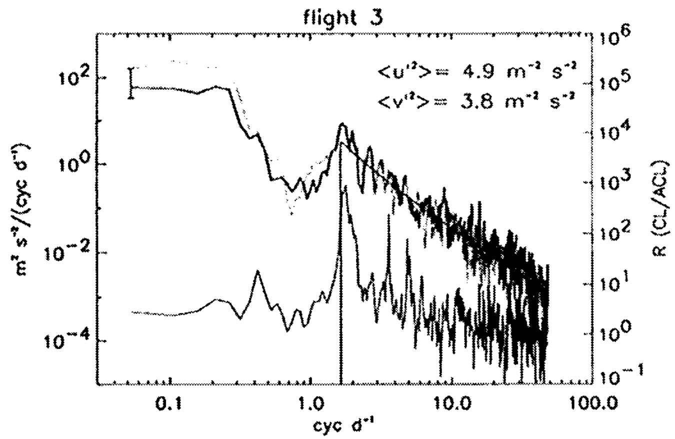
Intrinsic Frequency Spectra

- Trans-Pacific constant-pressure balloon flights
- 19-20 km float altitude
- Zonal, meridional, vertical velocity spectra





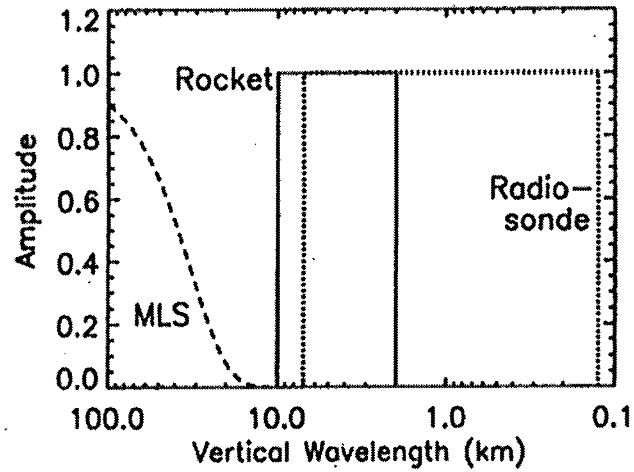




Circumpolar spectra

Observational Selection

- All observational techniques have some bias in which part of the wave spectrum they can observe
- No one technique can cover the complete wave spectrum
- Important to recognize and quantify the possible biases



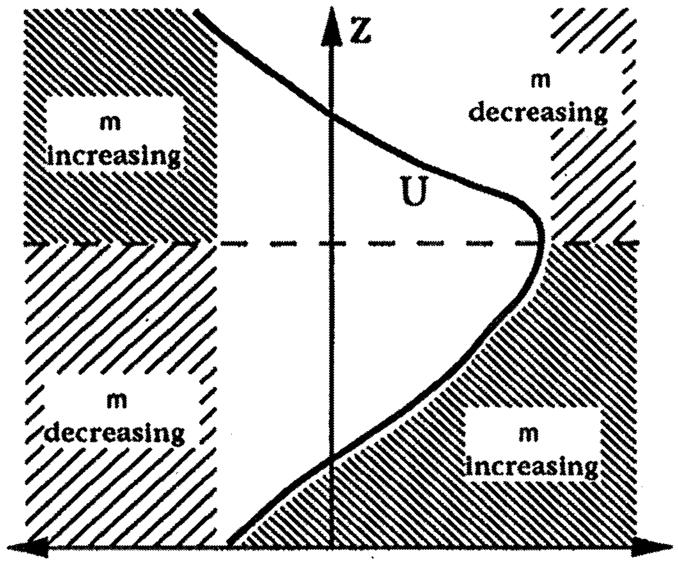
Observational filters for MLS, rockets, (GPS/MET), radiosondes

Modelling of observations helps with interpretation

e.g. Alexander and Vincent (2000); McLandress et al (2000)

Simple dispersion relation

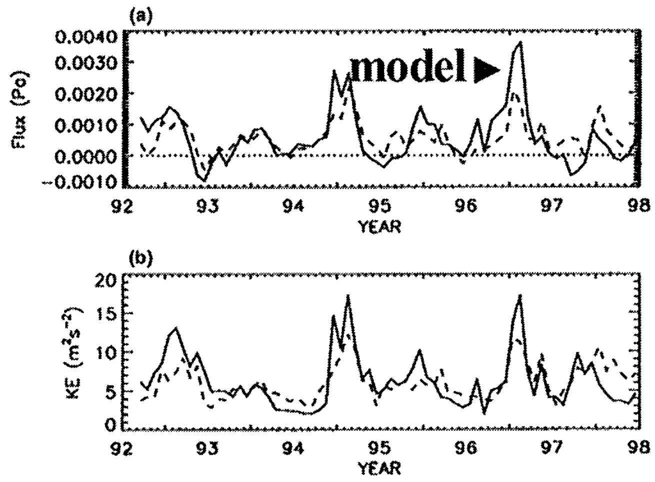
$$|m| \sim \frac{N}{\hat{\omega}} k_h \sim \frac{N}{|\bar{u} - c|}$$



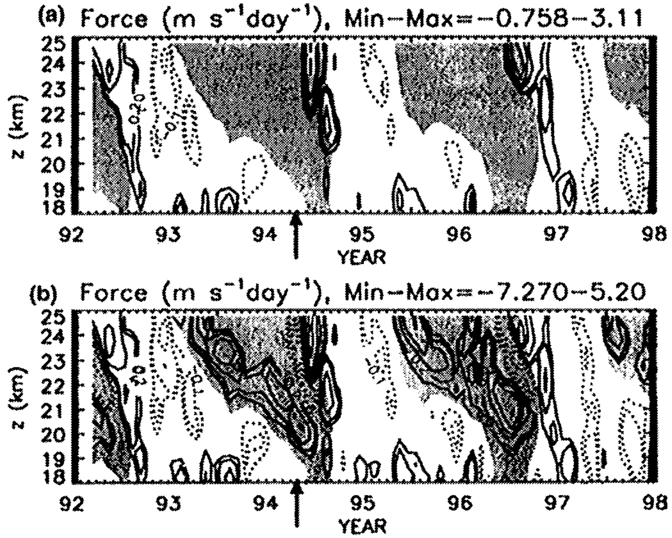
Waves move in or out of each "window" as *m* changes due to height-varying background winds

Modelling of observations helps with interpretation

e.g. Alexander and Vincent (2000); McLandress et al (2000)



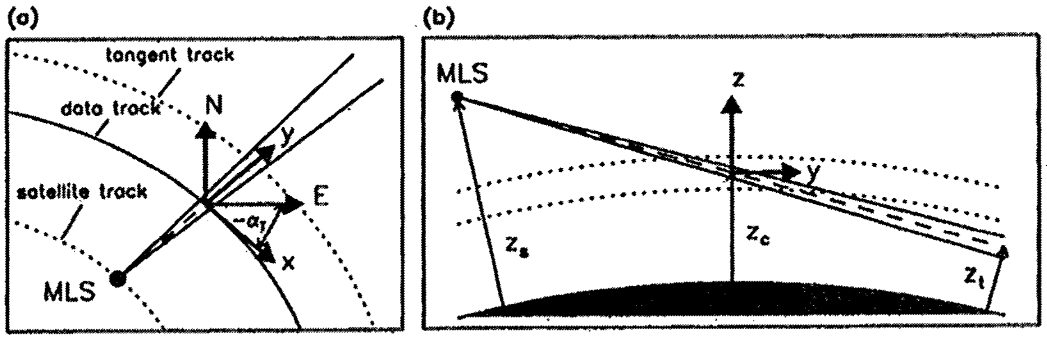
Radiosonde observations constrain model



Gravity-wave driven force

Force estimate from "best-fit" model

Force estimated without allowing for observational selection



Understanding MLS