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Wave Forcing and Cumulus Convection
in the Equatorial Atmosphere

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1 Wave forcing in general

In this section a general introduction is presented atmospheric waves, especially gravity waves.

1.1 Forcing terms in the governing equations

The momentum and thermodynamic equations in the primitive equations in Cartesian coordinate can be written as follows by subtracting their zonal mean:

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)u' - fv' + \Phi'_x = -N'_u + D'_x + F'_x, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)v' + fu' + \Phi'_y = -N'_v + D'_y + F'_y, \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\theta' + w'\frac{\partial\overline{\theta}}{\partial z} = -N'_\theta + D'_\theta + Q', \tag{3}$$

if zonal-mean vertical and meridional wind can be neglected and zonal-mean zonal wind is uniform. Here, over-bar and prime mean zonal mean and the deviation from it, respectively, D expresses dissipation,

$$N'_u = u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial x} + w'\frac{\partial u'}{\partial z} - \overline{\left(u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial x} + w'\frac{\partial u'}{\partial z}\right)}, \tag{4}$$

$$N'_v = u'\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial x} + w'\frac{\partial v'}{\partial z} - \overline{\left(u'\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial x} + w'\frac{\partial v'}{\partial z}\right)}, \tag{5}$$

$$N'_\theta = u'\frac{\partial \theta'}{\partial x} + v'\frac{\partial \theta'}{\partial x} + w'\frac{\partial \theta'}{\partial z} - \overline{\left(u'\frac{\partial v'}{\partial x} + v'\frac{\partial v'}{\partial x} + w'\frac{\partial v'}{\partial z}\right)} \tag{6}$$

are nonlinear terms, F is remaining body force, and Q is diabatic heating.

Combined with linearized continuity and hydrostatic equations, Eqs.(1)-(3) have linear wave solutions when the right-hand sides are set to zero. The wave solutions exhibits possible state of wave motions that can exist. However, this does not mean that the waves do exist in the real atmosphere. They must be exited somehow to appear. Their source, as well as the attenuator of waves, are in the right-hand side of Eqs.(1)-(3).

Forcing in the momentum equations is called **mechanical forcing**, and that in the thermodynamic equation is called **thermal forcing**. However, the separation is not absolute, since the two are interchangeable to some degree as in the following example.

Latent heat release in a small cloud (thermal forcing)
$\Rightarrow$ Entrain ambient air while moving upward
$\Rightarrow$ Nonlinear terms becomes non-zero on a larger scale
$\Rightarrow$ Mechanical as well as thermal forcing on that scale

1.2 Phenomenological classification and examples

1.2.1 Topographic forcing

Gravity waves are forced mechanically when stratified atmosphere flows over mountains. Such waves are called mountain waves. Mountain waves play crucial rolls in the mid- and high-latitude mesosphere and lower thermosphere region to keep mean wind speeds low.

A mountain wave solution can be obtained by assuming that the flow at the lower boundary follows topography. Then, perturbation vertical velocity at the lower boundary is approximated as follows (McFarlane, 1987):

$$w' = \overline{u}\frac{\partial \zeta'}{\partial x}, \tag{7}$$

where ζ' is surface altitude, and $\overline{u}$ here is the mean wind on a horizontal scale larger than that of topography. From the Fourier transformation of Eq.(7) the amplitude of perturbation for each horizontal wave number is determined from the spectrum of the topography. The waves excited have a zero phase speed ($c = 0$), as mountains do not move. Therefore, their intrinsic phase velocity, which is the velocity relative to mean wind, is simply wind velocity times -1. The vertical wavenumber m of waves is derived from the dispersion relationship:

$$\begin{aligned} m &= -\overline{N}/|c - \overline{u}| \\ &= -\overline{N}/|\overline{u}|. \end{aligned} \tag{8}$$

From Eq.(8) and the Fourier transform of Eq.(7), the perturbation vertical velocity field for horizontal wavenumber k is derived as

$$w = \mathrm{Re}\left\{ik\overline{u}\,\zeta'_k\,A(z)\,\exp\left[i\left(kx + \int_0^z m dz\right)\right]\right\}, \tag{9}$$

where ζ'_k is the Fourier coefficient of ζ' for k and $A(z) = \left[\frac{\overline{\rho}(0)\overline{N}(0)\overline{u}(0)}{\overline{\rho}(z)\overline{N}(z)\overline{u}(z)}\right]^{1/2}$ (ρ is density and N is the static stability).

Since most topographically excited gravity waves have horizontal scales smaller than the grid scales of current GCMs. Many of them have the parameterization of these waves. There are a number of mechanisms to excite gravity waves than the topographic effect, but at this moment it is practically the only mechanism that is modeled at some credibility; it is difficult to parameterize other mechanisms in terms of large-scale meteorological variables.

1.2.2 Wave-wave interaction

Superposition of two waves yields a third wave due to the non-linear terms (N) in the Eqs.(1)-(3). Suppose two one-dimensional sinusoidal waves with wavenumber m_1 and m_2. As easily demonstrated with $\sin(m_1x)\sin(m_2x)$, the nonlinear terms that arise form their superposition have wavenumbers m_1+m_2 and m_1-m_2. That is, wave of these two new wavenumbers could be excited. Whether these waves are actually excited or not depends on whether N acts as to force or attenuate them. Its theory can be presented under the framework of three wave interaction, whose details are beyond the scope of this class.

As for the equatorial wave excitation, it has been shown theoretically and observationally the possibility of excitation by forcing from waves that have propagated laterally from mid latitudes into the equatorial region. The idea was first proposed by Mak (1969) for excitation of the mixed Rossby-gravity waves. It was further studied observationally by Zangvil and Yanai (1980) and Magaña and Yanai (1995). Lateral forcing has not been proposed to explain the excitation of other kind of waves.

1.2.3 Turbulent excitation

Turbulence can excite waves through non-linear effects. In this section, the turbulent excitation is demonstrated with a special simple case, namely, the theory of sound wave excitation by Lighthill (1952).

The linearized equation of sound waves is written as follows:

$$\frac{\partial^2 \rho'}{\partial t^2} - c^2 \sum_{i=1}^{3} \frac{\partial^2 \rho'}{\partial x_i^2} = 0, \tag{10}$$

where ρ' is density perturbation and c is the sound speed. When nonlinear terms are not neglected in the momentum equations, the equation above is expressed as

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \sum_{i=1}^{3} \frac{\partial^2 \rho}{\partial x_i^2} = -\sum_{j=1}^{3} \sum_{i=1}^{3} \frac{\partial^2}{\partial x_i \partial x_j} T_{ij}, \tag{11}$$

where T_{ij} is approximated by

$$T_{ij} = \rho u_i u_j, \tag{12}$$

It comes from the non-linear terms $\sum_j \partial(\rho u_j u_i)/\partial x_j$ in the momentum equations. The right-hand side of Eq.(11) act like a quadruple forcing having two point sources and two point sinks because of the second-order derivative. For this simplest parabolic wave equation, the solution obtained under the radiation condition is proportional to the volume integral of $(\partial/\partial t)^2 T_{ij}$ multiplied with a function of relative position to the forcing (here, $(\partial/\partial t)^2$ is originated in the second-order spatial derivative in Eq.(11)). If the turbulence is caused by a turbulent jet flow with a characteristic speed of U, T_{ij} is roughly proportional to U^2. In addition, its characteristic frequency will be proportional to U, so that $(\partial/\partial t)^2 T_{ij}$ varies with U^4. Therefore, the energy of the sound wave varies with its square, U^8. This scaling law has been verified experimentally. See Lighthill (1978) for complete discussion. The Lighthill theory was successfully applied by Ford (1994) to diagnose two dimensional gravity wave excitation by Rossby-wave breaking in the shallow water system.

1.2.4 Geostrophic adjustment

While the topographic excitation is related to boundary condition, geostrophic adjustment is related to initial condition. Geostrophic adjustment refers nothing more than that if there is any "unbalanced" state in the atmosphere at a moment it will spread out as gravity waves. In this theoretical framework, how the initial unbalance, which is the source of the waves, is created before the adjustment starts. Gravity wave excitation due to geostrophic adjustment is thought to be active around the core of jet streams, but it is far from quantified.

The theory of geostrophic adjustment is presented in many standard textbooks of geophysical fluid dynamics and dynamic meteorology such as Holton (1992).

1.2.5 Tidal theory

Atmospheric tides are forced thermally though the term Q' in Eq.(3). The tidal theory was applied by Salby and Garcia (1987) for equatorial wave excitation through large-scale effects of cumulus convection, which will be the subject of the next class. A concise presentation of the tidal theory is found in Andrews et al.(1987).

1.2.6 Excitation by cumulus convection

Cumulus convection excites waves of various scales from O(10) km to O(10^4) km in terms of horizontal wavelength. Convectively excited waves are supposed to dominate in the equatorial middle atmosphere. The remaining part of the text concentrates on this issue.

2 Cumulus convection and wave excitation

In this section, cumulus convection and its interaction with large-scale dynamics are first introduced, in which cumulus parameterization used in GCMs are also introduced. Then, wave forcing due to cumulus convection is introduced briefly.

2.1 Introduction to cumulus convection

Cumulus convection is a generic term to express atmospheric convection related to cumulus and cumulonimbus clouds. These clouds are fueled by latent heat release due to condensation in conditionally unstable atmosphere. The vertical temperature structure is said to be conditionally unstable if it is stable with respect to dry atmosphere but unstable for upward motion of air parcel saturated with water vapor. The conditional instability characterizes most part of the earth's troposphere, but cumulus convection tend to occur frequently where moisture supply is intense such as the tropics. A comprehensive introduction to cumulus convection is found in Emanuel (1994).

Cumulus clouds are turbulent and entrain ambient atmosphere as they ascend, like smoke from a chimney. Cumulonimumbi are bigger cumuli, with "anvil" clouds trailing down-winds at around their levels of neutral buoyancy (LNB) in the upper troposphere. Cumulus convection transports energy received near the surface in the tropics aloft and is in the heart of atmospheric heat engine.

The mechanism of cumulus convection is illustrated with a diagram of the moist static energy whose simplified version is defined as

$$h = s + Lq, \tag{13}$$

where s is dry static energy defined as

$$s = gz + C_pT, \tag{14}$$

g is gravity, C_p is the heat capacity at constant pressure for dry atmosphere, L is latent heat of vaporization, and q is the mixing ratio of water vapor. The moist static energy is conserved in air parcels even when condensation occurs. Contrary, s and the potential temperature θ conserve only when no condensation or evaporation present.

2.1.1 Time averaged views

Cumulus convection is not evenly distributed over the tropics but is active only in limited regions. Main active regions are the tropical West Pacific to Indian Ocean, the intertropical convergence zone (ITCZ), equatorial Africa, and mid to south America. The active regions varies with seasons and year-to-year. The latter includes the well known El Niño-Southern Oscillation.

2.1.2 Hierarchical organization of cumulus convection

Cumulus clouds do not occur randomly like white noise. They tend to be organized in space and time on a wide range of scales from planetary to meso scales. Their space-time distribution, or their wavenumber-frequency distribution in the spectral space, affects largely the waves excited to propagate into the equatorial middle and upper atmospheres.

The largest transient organization of cumulus convection is the Madden-Julian oscillation (MJO), which is dominated by planetary, zonal wavenumber 1 structures though limited in the equatorial region. It propagates eastward slowly to round the equatorial circle by 30-60 days.

Eastward propagation also characterizes organized structures called super-cloud clusters on a few thousand of km, which is smaller than that of the MJO. The term super cloud cluster is coined to mean that they are larger than ordinary cloud clusters on the scale of a few hundred km, which typically move westward (Nakazawa, 1988). It is further known that squall lines and other meso-scale convective systems (MCSs) smaller than cloud clusters sometime moves opposite to their enveloping cloud clusters.

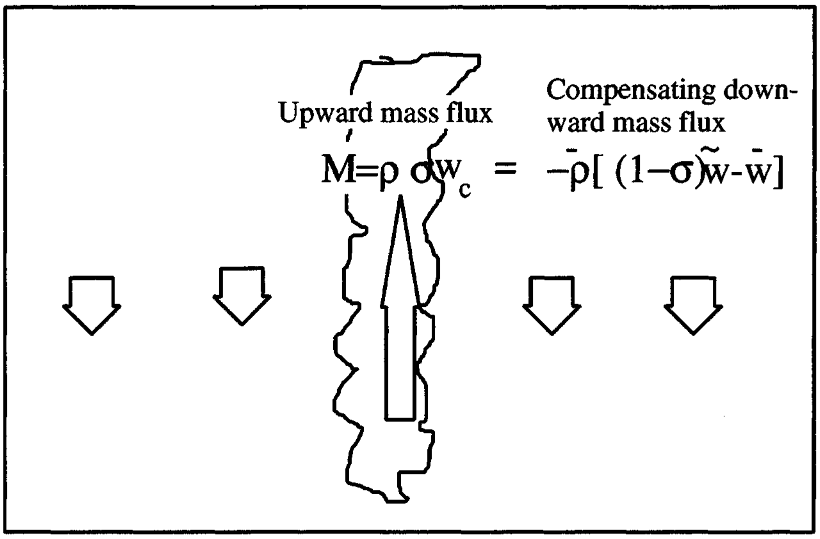

Figure 1: Schematic illustration of upward cumulus mass flux M and its compensating downward mass flux outside the clouds.

2.2 Interaction between cumuli and large-scale dynamics

Stochastic properties of cumuli and cumulonimbi are largely controlled by large-scale atmospheric condition. And the opposite is true; although each of cumuli is small, they change large-scale atmospheric condition by rearranging moist static energy. A typical interaction between the may be illustrated as follows:

- (Large-scale → cumuli):

  Convective available potential energy accumulates by surface heat and moisture fluxes, radiation, and large-scale advection
  → controls the stochastic properties (such as the probability of occurrence) of cumuli and cumulonimbi

- (Cumuli → large-scale):

  Upward mass flux in cumulus towers
  → compensating down-welling of the ambient air + detrainment from the clouds
  → temperature and moisture change on large scale

2.2.1 Fundamentals of cumulus parameterization

Although cumulus convection profoundly influences global atmospheric circulation, general circulation models (GCMs) have grid spacing too coarse to resolve each cumulus clouds. Therefore, cumulus convection is "parameterized" in them, which means that their statistical properties related to large-scale atmospheric condition is somehow modeled deterministically (whether it is possible or not) from the large-scale condition itself. Through parameterization, GCMs bypath subgrid-scale phenomena and close the governing equations in terms of atmospheric properties on their grid scales.

A cumulus parameterization models the interaction between cumuli and large-scale condition mentioned in the previous subsection. Currently, there are a number of different parameterization methods being used in GCMs. It should be noted that a cumulus parameterization is not just a tool for modeling but is a conceptual as well as quantitative model of the interaction. The fact that there are many parameterizations being used implies that scientists do not have a consensus on how the cumulus convection are related to large-scale properties. The calculation methods are diverse, but the output is almost always a combination of change in temperature and moisture (in addition some parameterizations secondarily predict change in momentum). Therefore, when large-scale dynamics are considered, cumulus convection is supposed to act primarily through the thermal forcing term Q' in Eq.(3).

The impact of cumuli on large-scale thermodynamic field can be illustrated with the following approximate equation.

$$\frac{\partial \overline{s}}{\partial t} + \overline{\boldsymbol{v}}_H \cdot \nabla \overline{s} + \overline{w}\frac{\partial \overline{s}}{\partial z} = \frac{M}{\overline{\rho}}\frac{\partial \overline{s}}{\partial z} - Le + Q_{\rm rad} + D_s, \tag{15}$$

where over-bar means horizontal average over a meso-scale domain considered, which for a GCM is a grid cell, $s \equiv C_pT + gz$ is the dry static energy, $\boldsymbol{v}_H$ is horizontal wind, M is the upward mass flux in cumuli, L is latent heat, e is evaporation of raindrops, $Q_{\rm rad}$ is radiative heating or cooling, and D_s is dissipation. To derive Eq.(15), it is assumed that cumulus clouds occupy only a small fraction in the domain so that their fractional area σ is much smaller than 1. Then, the domain average static energy $\overline{s}$ is approximately equal to its average outside of the clouds $\tilde{s}$ $(\overline{s} = (1-\sigma)\tilde{s} + \sigma s_c \sim \tilde{s}$, for $\sigma \ll 1$ and $|s_c - \tilde{s}|/\tilde{s} \ll 1$, where tilde means average outside the clouds and the suffix c means average in the clouds). Therefore, Eq.(15) is derived from the equation for $\tilde{s}$, and the first term on the right-hand side is attributed to downward motion to compensate the cumulus mass flux (See Fig. 1). What a cumulus parameterization parameterizes is the summation of the first term and the second term, namely, effects of compensating downward mass flux and evaporation from rain. The first term is a manifestation of the non-linear term N'_θ in Eq.(3). At the same time, if it mass-weighted and averaged vertically, it should be approximately equal to the vertical integration of diabatic heating due to condensation in cumulus clouds, which indirectly controls compensating mass flux.

The governing equation of moisture is approximated as follows:

$$\frac{\partial \overline{r}}{\partial t} + \overline{\boldsymbol{v}}_H \cdot \nabla \overline{r} + \overline{w}\frac{\partial \overline{r}}{\partial z} = \frac{M}{\overline{\rho}}\frac{\partial \overline{r}}{\partial z} + e + \delta + D_r, \tag{16}$$

where r is mixing ratio, δ is tendency due to detrainment. (A term for detrainment is not included in Eq.(3), since it occurs around the level of neutral buoyancy. That is, s of the air detrained is about the same as the mean s at the level.)

Equations (15) and (16) formulate statistical quantities that affect large-scale fields. What is required to a cumulus parameterization then is to close the equations, namely, to express the quantities as functions only of large-scale quantities. There a number of ways that have been proposed to do it, each of which corresponds to different cumulus parameterization. To explain their details is beyond the scope of this class. It is just noted here that the terms are predicted to be non-zero under certain condition, and when non-zero it generally acts to reduce convective available potential energy (CAPE).

2.3 Wave excitation by cumulus convection

2.3.1 mesoscale gravity waves excited directly by cumulus clouds

Cumulus clouds excite gravity waves as they ascends by their mechanical thrust. A numerical experiment showed that they also excite gravity waves as they oscillate around their levels of neutral buoyancy (Lane et al, 2001). However, their time-space distribution and their relative importance in the equatorial middle atmosphere are far from quantified. Waves excited in this mechanisms inherently have small horizontal scales of ten to a few tens of kilometers. Such small-scale waves have large vertical group velocities and could reach the mesosphere or even to the lower thermosphere. The vertical group velocity of internal gravity waves are expressed as

$$C_z = \pm \frac{Nmk}{(k^2 + m^2)^{3/2}}, \tag{17}$$

which grows with k when $|k|$ is smaller than $|m|/\sqrt{2}$. Contrary, synoptic- to planetary-scale wave are limited within the stratosphere unless they have considerably large vertical wavelengths.

2.3.2 mesoscale waves excited by anvils

Cumulonimbi are accompanied with downwind-trailing stratiform clouds called anvils. These clouds could also excite gravity waves. One of the mechanisms of their wave excitation is called the "moving mountain" effect. If an anvil cloud is sustained against mean flow in the upper troposphere by cumulus clouds or a meso-scale convective complex, it could obstruct the flow like a mountain. This would excite gravity waves. This "mountain" is said to be moving, since convective clouds generally moves with respect to the ground. A caveat on this mechanism though is that the anvil has to be sustained for a certain time by continuous supply of air mass from cumulus clouds to work like a mountain. Otherwise, anvils would be just advected by the mean flow and would not obstruct it to excite waves.

2.3.3 large-scale waves excited indirectly

As discussed in Section 2.2, cumulus clouds indirectly affect large-scale thermal field through the compensating downward motion around the clouds. The large-scale wave excitation is the subject of the next class. As mentioned in the section, the forcing term involved is the non-linear term in the thermo-dynamic equation, which is expressed in the form of diabatic heating by cumulus parameterization. The theory and computational method of tidal excitation by solar diabatic heating has been applied to estimate the large-scale wave excitation by cumulus convection (Salby and Garcia, 1987).

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