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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

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Lecture 2

Transverse Mode Selection in Solid State Lasers

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Lecture 2

Transverse Mode-Selection in Solid State Lasers.

Résumé of Gaussian beams:

Complex beam parameter: $\frac{1}{q} = \frac{1}{R} - \frac{j\lambda}{\pi w^2}$

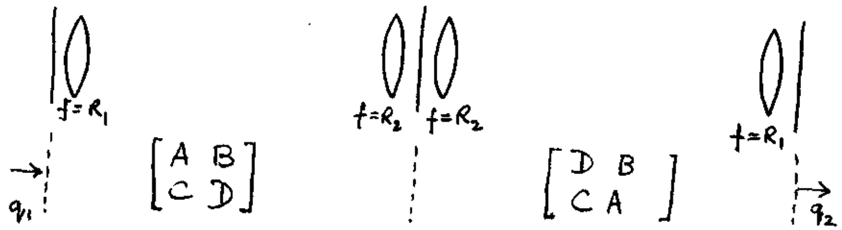
ABCD law: $q_2 = \dfrac{Aq_1 + B}{Cq_1 + D}$

Analysis of resonator (following Baues (1969));

Note, the resonator is not assumed empty, i.e there can be any optical components between the mirrors.

this is equivalent to

Unfolded resonator



$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ $\qquad$ $\begin{bmatrix} D & B \\ C & A \end{bmatrix}$

overall ray matrix $= \begin{bmatrix} D & B \\ C & A \end{bmatrix}\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$= \begin{bmatrix} 2AD-1 & 2BD \\ 2CA & 2AD-1 \end{bmatrix}$ since $\det\begin{bmatrix} A & B \\ C & D \end{bmatrix} = 1$

Hence $q_2 = \dfrac{(2AD-1)q_1 + 2BD}{2CAq_1 + (2AD-1)}$

But self-consistency requires $q_2 = q_1$

Hence find $\left(1/q_1\right)^2 \equiv \left(-\frac{j\lambda}{\pi w_1^2}\right)^2 = \frac{CA}{BD}$

$$\boxed{\frac{\pi w_1^2}{\lambda} = \left(-\frac{BD}{AC}\right)^{1/2}} \qquad (1)$$

Similarly, the spot-size w_2, on mirror 2 is given by

$$\boxed{\frac{\pi w_2^2}{\lambda} = \left(-\frac{AB}{CD}\right)^{1/2}} \qquad (2)$$

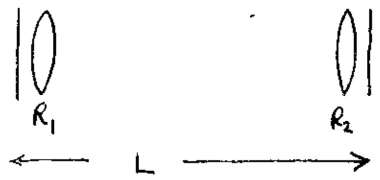
Thus the spot-sizes on the two mirrors can be calculated from the one-way ray transfer matrix elements.

Resonator stability condition (Kogelnik and Li (1966)), requires that the trace of the round-trip matrix lie between -2 and $+2$

i.e. $-2 < 2(2AD-1) < 2$

or $\boxed{0 < AD < 1}$

Now consider the empty resonator



The single-pass ray transfer matrix, from left to right, is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{R_2} & 1 \end{bmatrix}\begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\frac{1}{R_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - L/R_1 & L \\ \frac{L}{R_1 R_2} - \frac{1}{R_1} - \frac{1}{R_2} & 1 - \frac{L}{R_2} \end{bmatrix} = \begin{bmatrix} g_1 & L \\ \frac{(g_1 g_2 - 1)}{L} & g_2 \end{bmatrix}$$

where the g parameters are defined as

$$g_1 = 1 - L/R_1$$

$$g_2 = 1 - L/R_2$$

Hence the spot-sizes are

$$\frac{\pi w_1^2}{\lambda} = L\left(\frac{g_2}{g_1(1 - g_1 g_2)}\right)^{1/2} \qquad ③$$

$$\frac{\pi w_2^2}{\lambda} = L\left(\frac{g_1}{g_2(1 - g_1 g_2)}\right)^{1/2} \qquad ④$$

Ideally we require a resonator design which gives a large spot size in the laser medium, so that energy extraction is efficient.

Assume at first that the laser medium is close to mirror 2, and that any thermally-induced focal length f_R of the laser medium has been incorporated in the curvature R_2.

Now w_2 can be made arbitrarily large by an appropriate choice of R_1, R_2, L.

e.g. with $g_1 = 1$ (i.e. R_1 plane), then as $g_2 \to 1$, so $w_2 \to \infty$ [equ^n 4; with $g_1 g_2 \to 1$]

However it can be shown that

$$\frac{dw_2}{w_2} = \frac{1}{4}\frac{(2g_1 g_2 - 1)}{(1 - g_1 g_2)}\frac{dg_2}{g_2} \qquad ⑤$$

Thus, as $g_1 g_2 \to 1$, the spot size w_2 becomes sensitively dependent on g_2 and hence on f_R. On these grounds it has been argued that reliable large volume TEM_{00} mode operation can never be possible. The argument is incorrect however, since if one chooses $g_1 g_2 = 1/2$ equation ⑤ shows that the spot size w_2 becomes insensitive to variations of g_2. This property was exploited by Steffen et al (1972) in a resonator with two plane mirrors, the laser rod placed close to mirror 2.

Hence $g_1 = 1$, $g_2 = 1 - L/f_R$.

The mirror spacing was chosen as $L = f_R/2$,

hence $g_1 g_2 = 1/2$

and $$w_1^2 = \lambda L/\pi \qquad (6)$$

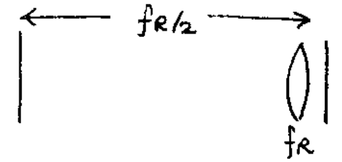
$$w_2^2 = \frac{2\lambda L}{\pi} = \frac{\lambda f_R}{\pi} \qquad (7)$$

Thus if f_R is large, w_2 can be large. One can make f_R arbitrarily large by adding a lens to compensate the rod lens.

Thus w_2 can be made arbitrarily large and yet be insensitive to variation of f_R.

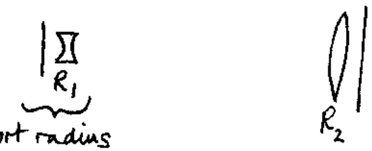
Unfortunately since $L = f_R/2$, this design implies a long resonator.

Note, this resonator is effectively a half confocal resonator.



Two other resonator designs with $g_1 g_2 = 1/2$:

1. Convex-Concave resonator.



Short radius convex mirror

R_1 −ve, $|R_1| \ll L$,
hence $g_1 \gg 1$.
g_2 is +ve but $\ll 1$
Since $g_1 g_2 = 1/2$
Hence R_2 slightly greater than L.

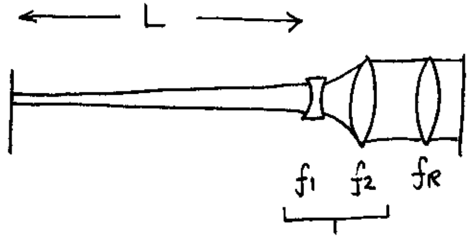
From (3) and (4), find that

$$w_1^2 = \frac{1}{2g_1^2} w_2^2 \qquad (8)$$

and $$w_2^2 = \frac{2\lambda L}{\pi} g_1 \qquad (9)$$

Hence a large w_2 is possible by virtue of a large g_1 (and L can be conveniently small). But (8) shows that w_1 is then very small — this is undesirable in a high power laser.

2. Telescopic Resonator (Hanna et al, 1981)

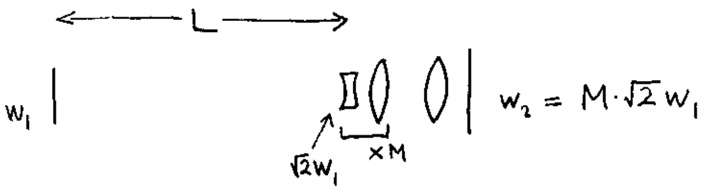


Telescope, magnification $M = -\frac{f_2}{f_1}$

The telescope acts like an adjustable lens, of focal length $f_T = f_2^2/\delta$ where the telescope lens spacing is $f_1 + f_2 + \delta$. The telescope spacing can be adjusted so that the combination f_T, f_R acts as a lens f;
$$\frac{1}{f} = \frac{1}{f_T} + \frac{1}{f_R}$$

The condition for insensitivity of w_2 to variations of f_R, now becomes $f/2 = M^2 L$, (instead of $f/2 = L$ for a resonator without a telescope.)

Thus $w_1^2 = \frac{L\lambda}{\pi}$ (10), $w_2^2 = \frac{2L\lambda M^2}{\pi}$ (11)



These results (10), (11) are easily remembered by thinking of the part of the resonator to the left of the telescope as acting like a half-confocal resonator; spot-size at centre of confocal resonator of length $2L$ is given by $w_1^2 = \frac{2L.\lambda}{2\pi} = \frac{L\lambda}{\pi}$. The spot size at the input to the telescope is then $\sqrt{2}\,w_1$ and after the telescope is $M\sqrt{2}\,w_1$ i.e. $w_2^2 = 2M^2 w_1^2 = \frac{2M^2 L\lambda}{\pi}$

The telescopic resonator gives a good compromise between the long resonator and the short, convex-concave resonator. For the same spot size w_2 the telescopic resonator reduces the resonator length by M^2, although it makes the spot size w_1 smaller by M than for the long resonator.

Design procedure for telescopic resonator.

$$w_2^2 = \frac{2L\lambda M^2}{\pi}$$

Choose w_2 to adequately fill the laser medium, (e.g. we choose $w_2 = 2.5\,\text{mm}$ for an NdYAG rod of 9mm diameter)

Choose L, M to give required w_2, bearing in mind the compromise viz. that a small L (hence large M) implies small w_1 $\left(= (L\lambda/\pi)^{1/2}\right)$.

(Typically we have chosen $L = 0.55\,\text{m}$, $M = 4$.)

When L, M are fixed, then so is f, since $f = 2M^2L$.

Since f is fixed, $\left(\frac{1}{f} = \frac{1}{f_T} + \frac{1}{f_R}\right)$ and f_R is determined by the pumping, then the telescope misadjustment δ must be chosen to give the appropriate f_T and hence f

$$\delta = f_2^2/f_T$$

f_R in our case was given by $f_R(m) = \frac{2.7}{P(\text{kW})}$

Thus one can calculate the appropriate δ for a given pump power P.

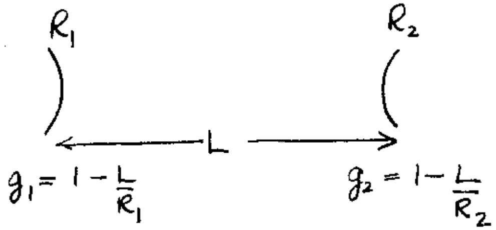
Unstable Resonators

Stability condition

$0 \leq g_1 g_2 \leq 1$ for empty resonator

$0 \leq AD \leq 1$ for a general resonator

A, D are elements of the single-pass ray-transfer matrix of the resonator



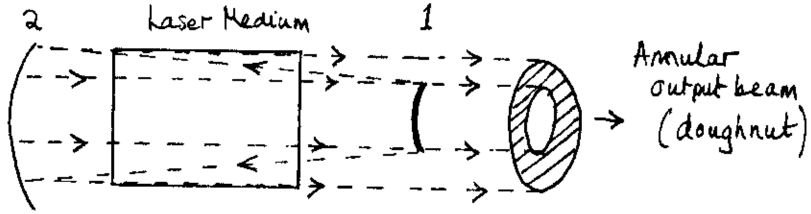
The convex-convex resonator is unstable since $R_1, R_2 < 0$, hence $g_1, g_2 > 1$ and $g_1 g_2 > 1$. It clearly displays the characteristic of any unstable resonator, i.e. the beam passing back and forth between the mirrors progressively grows in transverse extent.

This transverse growth is exploited in two ways

1. It ensures that the beam fills the laser medium
2. It provides output coupling as the beam escapes around the outside of the mirror.

For a detailed discussion of unstable resonators see Steier (1979) and for a short but very useful account see Siegman (1971).

We show below a particular resonator which has been used with Q-switched Nd YAG lasers (Herbst et al 1977, Hanna and Laycock 1978), namely the positive branch confocal resonator.



This gives a collimated beam (plane wave-front) travelling left to right. The central portion is reflected (leaving an annular output to escape around the small mirror 1) and the reflected portion is magnified transversely by a factor M before being converted into a uniform plane wavefront by mirror 2. If the small mirror has radius a_1, then the plane beam striking this mirror has radius Ma_1, and the fraction of power coupled out is $1 - 1/M^2$.

M = 3 to 4 would be typical for Q-switched NdYAG.

A comparison of the unstable resonator technique with the stable resonator technique (telescopic resonator) must take account of the following points.

1. The output energy is ~ factor two greater for the unstable resonator, e.g. 250-300 mJ from a Q-switched NdYAG laser, and ~100-150 mJ for the telescopic resonator.

2. The beam from the unstable resonator has a complex shape, which changes as it propagates, developing a central spot (Poisson spot) and in the far field an Airy disc surrounded by rings. Perhaps 60% only of the energy is in the central disc.

3. The use of Fabry-Perot etalons for frequency selection is less effective in the unstable resonator, since the beam passing through the etalon is only collimated for one direction of travel.

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