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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

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Lecture 3

Single Longitudinal Mode Selection

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Lecture 3

Single Longitudinal Mode Selection

Why select a single mode?

1. To obtain a long coherence length, useful for example in holography.

   $\ell_c \sim \dfrac{c}{\Delta\nu}$, where $\Delta\nu$ is the bandwidth of the laser output.

   For a bandwidth limited pulse, duration Δt,

   $\Delta\nu \sim 1/\Delta t$.

   Hence 10 nsec pulse $\rightarrow$ $\Delta\nu \sim 100$ MHz

   $\rightarrow \dfrac{\Delta\nu}{c} \sim 0.003\ \text{cm}^{-1}$

   $\rightarrow \ell_c \sim 300$ cm

2. High resolution in spectroscopic applications.

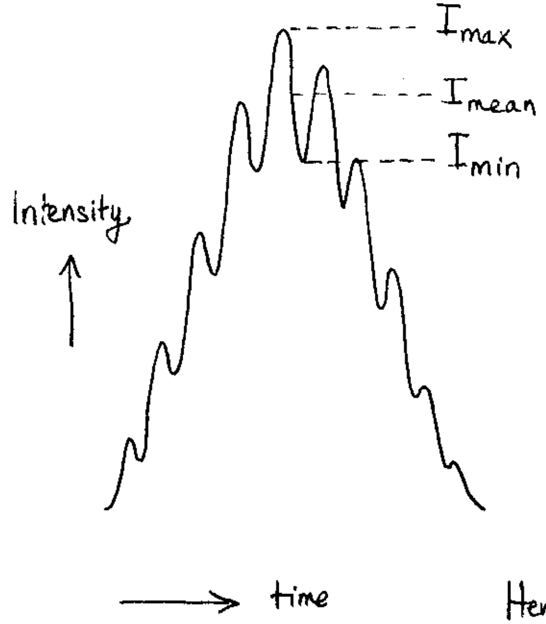
3. To obtain clean temporal behaviour of the pulse. This is necessary for repeatability of results and is also necessary where a precise knowledge of laser intensity is required (e.g. in nonlinear optics)

See Koechner for a general discussion

Criterion defining single-longitudinal mode operation.

The simultaneous presence of two modes in the laser output gives rise to a modulation of the pulse envelope at their frequency difference.

Even if one mode is much weaker than the other, the depth of modulation can be very large, hence this is a very sensitive test of SLM operation.



Specify depth of modulation by index k,

$$k = \frac{I_{max} - I_{min}}{I_{mean}} \qquad (1)$$

Two modes with intensities in the ratio N:1 give modulation

$$k = 4N^{-1/2} \qquad (2)$$

Hence for $k < 0.01$, ie not quite observable on the oscilloscope trace, require $N > 10^5$

Frequency selectivity of etalon used in reflection (resonant reflector) and etalon used in transmission.

If mode m has frequency exactly at RR maximum and mode n is an adjacent mode (displaced by $c/2L$) then the ratio of the reflectivities is

$$\frac{R_m}{R_n} = 1 + \frac{\pi^2 \mu^2 t^2}{L^2}\left(\frac{1-R}{1+R}\right)^2 \qquad (3)$$

where μ, t are refractive index & thickness of etalon. (Hanna et al 1972)

After q round trips of the resonator the mode powers (assumed equal at first) will be in the ratio

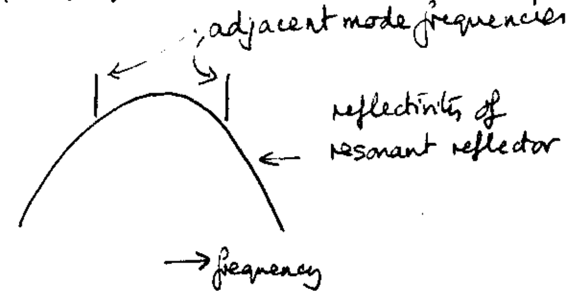
$$\frac{P_m}{P_n} = \left(\frac{R_m}{R_n}\right)^q = \left[1 + \frac{\pi^2\mu^2 t^2}{L^2}\left(\frac{1-R}{1+R}\right)^2\right]^q \qquad (4)$$

The corresponding expression for a transmission etalon (TE) is

$$\frac{P_m}{P_n} = \left[1 + \frac{8\pi^2\mu^2 t^2 R}{L^2(1-R)^2}\right]^q \qquad (5)$$

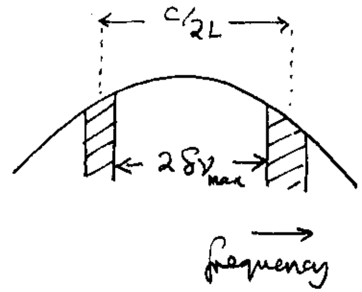
Aim to achieve $P_m/P_n \gtrsim 10^5$, so make μt large, L small, R large (for TE) and most usefully make q as large as possible. The latter can be achieved with a Saturable Absorber Q-switch (Sooy 1965) or its electronic analogue, 'prelase Q-switching (Hanna et al 1972, 1981, 1982)

Even if $P_m/P_n \gg 10^5$ with mode m situated at the peak of the etalon maximum, the situation can arise where the resonator modes drift to the situation depicted below



Then the output will appear as two-mode beating.

In fact there will be some maximum allowable displacement $\delta\nu_{max}$ of the dominant mode from the reflectivity maximum, before the criterion of single mode operation is violated (Hanna & Koo 1982). Thus if one mode falls in one hatched region, the adjacent mode falls in the other symmetrically placed hatched region and the single-mode criterion is no longer met. Over a long period the fraction of shots which are single mode



will be

$$\left(\frac{4L}{c}\right)\delta\nu_{max} = 1 - \frac{\ln N}{q}\cdot\frac{L^2}{\pi^2\mu^2 t^2}\left(\frac{1+R}{1-R}\right)^2 \quad (\text{for RR}) \qquad (6)$$

$$\text{or} = 1 - \frac{\ln N}{q}\cdot\frac{L^2(1-R)^2}{8R\pi^2\mu^2 t^2} \quad (\text{for TE})$$

Thus one can actually predict, using 6 the fraction of laser shots which can be expected to be single-mode (according to the criterion that the dominant mode is at least N times more intense than its weaker neighbour).

On the other hand if the hatched regions are narrow enough one can apply a technique in which the laser frequencies are allowed to sweep themselves out of the hatched region during a pumping pulse and thus ensure that every laser shot gives SLM output (Hanna & Koo 1982)

References on Single Longitudinal Mode Selection

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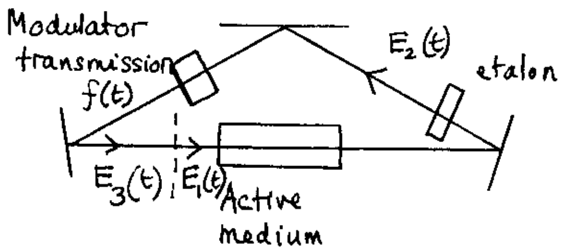
Analysis of Active Mode-Locking.

Kuizenga and Siegman (1970) first analysed the steady-state mode-locked pulse.



Assume Gaussian time-dependence of pulse $E_1(t)$. Transform to frequency domain and then transmit through the active medium and etalon (if any). Transform back to time domain ($E_2(t)$). Effect of modulator in the time domain (e.g. acousto-optic modulator) is to provide a transmission

$$f(t) = \cos(\Theta_m \sin\omega_m t) \quad , \quad \omega_m = 2\pi f_m \qquad (1)$$

Hence find $E_3(t)$.

Now require self-consistency,

$$E_3(t) = E_1(t - \text{round trip time})$$

Result (Kuizenga (1981)) is that the steady-state pulse width τ_{po} is given by

$$\tau_{po} = \frac{(2\ln 2)^{1/2}}{\pi} \frac{1}{\Theta_m^{1/2}} \cdot \frac{1}{f_m^{1/2}} \left(\frac{g}{(\Delta f)^2} + \frac{1}{(\Delta f_e)^2} \right)^{1/4} \qquad (2)$$

where g is the saturated amplitude gain for one round trip, at line centre.

Δf is the gain linewidth (6 cm^{-1} for NdYAG)

Δf_e is an effective bandwidth for the etalon, given by $\Delta f_e = \frac{2c}{\pi h} \frac{1}{n^2-1}$ for an uncoated solid etalon of thickness h and refractive index n.

Typically pulse durations down to ~ 50 ps can be obtained (without the etalon) and these can be increased to ~ 1nsec with appropriate etalons.

To analyse the development of the mode-locked pulse as it evolves towards the steady state, Kuizenga et al (1973) generalised the self-consistency requirement. They still assumed a Gaussian pulse $e^{-\alpha t^2}$ but α was allowed to change by a variable amount $\Delta\alpha$ per round trip. This led to a differential equation for $\Delta\alpha$ which could be solved, thus giving a pulse duration τ_p, after M round trips,

$$\tau_p = \tau_{p0} \Big/ \left[\tanh (M/M_0) \right]^{1/2} \qquad (3)$$

where
$$M_0 = \frac{1}{4\sqrt{g}\,\theta_m} \cdot \frac{\Delta f}{f_m} \qquad (4)$$

Thus τ_p is within 5% of the steady state value τ_{p0} when $M > 1.52\,M_0$, i.e.

$$M > \frac{0.38}{\sqrt{g}\,\theta_m} \cdot \frac{\Delta f}{f_m} \qquad (5)$$

Since $\Delta f/f_m$ is large ($\sim 10^3$) this implies a large number of round trips, and hence tens of μsecs build up.

With an etalon, $\dfrac{\Delta f}{\sqrt{g}}$ is replaced by Δf_e, which can be much smaller, hence steady state is reached much more quickly for longer pulses.

Mode-Locking References

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