INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
34100 TRIESTE (ITALY) - P.O.B. 586 - MIRAMARE - STRADA COSTIERA 11 - TELEPHONES: 224281/2/3/4/5/6
CABLE: CENTRATOM - TELEX 460392-I

SMR/100 - 37

WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

(24 January - 25 March 1983)

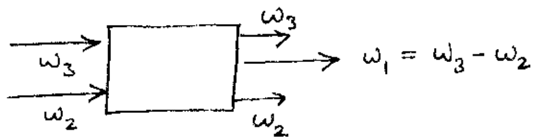
Lecture 5
Difference Frequency Generation

D.C. HANNA

Department of Electronics
University of Southampton
Southampton SO9 5NH
U.K.

Difference frequency generation

Details of the analysis are in Byer (1977) and Svelto (1982)



Assume input waves to nonlinear medium at frequencies ω_3 and ω_2. A wave at the difference frequency $\omega_1 = \omega_3 - \omega_2$ is generated.

Ideally one requires the wavevectors to satisfy the phase-matching condition $k_1 = k_3 - k_2$; in practice one may have $k_1 = k_3 - k_2 - \Delta k$ where Δk $(= k_3 - k_2 - k_1)$ is called the phase mismatch.

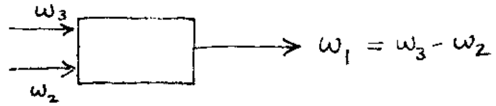
In terms of photons the generation of a photon of energy $\hbar\omega_1$ is achieved by annihilation of a photon of energy $\hbar\omega_3$ (the pump photon) and simultaneous generation of a photon of energy $\hbar\omega_2$

$$\hbar\omega_3 = \hbar\omega_2 + \hbar\omega_1 \quad (\text{equivalent to } \omega_1 = \omega_3 - \omega_2)$$

Thus the waves at frequency ω_1 and ω_2 are amplified at the expense of pump depletion.

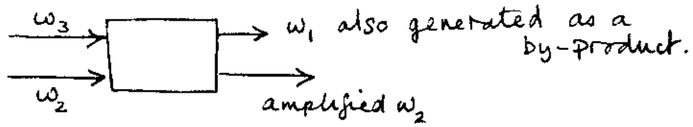
The difference frequency generation process can be exploited in various ways

①



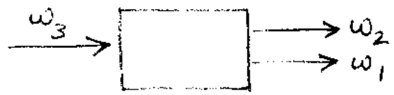
generation of ω_1. Either ω_3 or ω_2 may be tunable

② Optical parametric amplification (OPA)



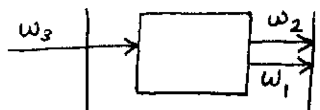
Since amplification of ω_2 involves depletion of pump ω_3, then a large amplification implies ω_3 is a much more powerful source than ω_2.

③ Superfluorescent parametric emission



Pump ω_3 generates very high parametric gain and ω_1, ω_2 grow from noise. ω_1, ω_2 are tuned by changing the phase-matching condition.

④ Parametric oscillation



Again ω_2, ω_1 grow from noise, assisted by feedback from the mirrors. Tuning of ω_1, ω_2 is achieved by changing the phase-match condition.

[illegible] parametric emission and parametric oscillation is given in Byer ([illegible]). We quote the results.

Assuming all waves propagating in the z direction, with fields given by

$$E_1(z,t) = \tfrac{1}{2}\left[E_1(z)\exp i(k_1 z - \omega_1 t) + cc\right]$$

$$E_2(z,t) = \tfrac{1}{2}\left[E_2(z)\exp i(k_2 z - \omega_2 t) + cc\right]$$

and assuming no pump depletion $\left(\frac{dE_3}{dz} = 0\right)$ and no attenuation of waves 1 and 2, the following differential equations are obtained,

$$\frac{dE_1}{dz} = i\left(\frac{\omega_1 d}{n_1 c}\right) E_3 E_2^{*} \exp(i\Delta k z) \qquad (1)$$

$$\frac{dE_2}{dz} = i\left(\frac{\omega_2 d}{n_2 c}\right) E_3 E_1^{*} \exp(i\Delta k z) \qquad (2)$$

Solving these equations for E_1, E_2 it is found that the fields $E_1(\ell)$, $E_2(\ell)$ after propagating through a length ℓ of nonlinear medium (of refractive indices n_1, n_2, n_3 and effective nonlinear coefficient d) are

$$[\text{illegible}] \left[\cosh g\ell - \frac{i\Delta k}{2g}\sinh g\ell\right]$$

$$+ \frac{i\omega_1 d E_3}{n_1 c} E_2^{*}(0)\, e^{i\Delta k\ell/2} \sinh g\ell \qquad (3)$$

$$E_2(\ell) = E_2(0)\, e^{i\Delta k\ell/2}\left[\cosh g\ell - \frac{i\Delta k}{2g}\sinh g\ell\right]$$

$$+ \frac{i\omega_2 d}{n_2 c} E_3 E_1^{*}(0)\, e^{i\Delta k\ell/2} \sinh g\ell \qquad (4)$$

where
$$g = \left[\underbrace{\frac{\omega_1\omega_2 |d|^2 |E_3|^2}{n_1 n_2 c^2}}_{=\Gamma^2} - \left(\frac{\Delta k}{2}\right)^2\right]^{1/2} \qquad (5)$$

With input $E_2(0)$ but no input E_1 (i.e. $E_1(0) = 0$), defining the gain $G_2(\ell)$ of the wave 2 as the fractional increase in intensity, i.e.

$$G_2(\ell) = \frac{|E_2(\ell)|^2}{|E_2(0)|^2} - 1 \qquad (6)$$

we obtain

$$G_2(\ell) = \Gamma^2 \ell^2 \frac{\sinh^2 g\ell}{(g\ell)^2} \qquad (7)$$

For small gain $\Gamma' l$ and for $(\Delta k l/2)^2 \gg \Gamma'^2 l^2$

this gives $G_2(l) = \Gamma'^2 l^2 \,\mathrm{sinc}^2\left(\frac{\Delta k l}{2}\right)$

The generated wave 1 therefore has an intensity

$$I_1(l) = I_2(0) \cdot G_2(l) \frac{\omega_1}{\omega_2}$$

$$= I_2(0) \frac{\omega_1^2}{n_1 n_2 c^2} |d|^2 |E_3|^2 l^2 \,\mathrm{sinc}^2\left(\frac{\Delta k l}{2}\right)$$

$$= \frac{2\omega_1^2 \cdot |d|^2}{\epsilon_0 c^3 n_1 n_2 n_3} I_2(0) I_3 \, l^2 \,\mathrm{sinc}^2\left(\frac{\Delta k l}{2}\right) \qquad (8)$$

This gives the generated intensity $I_1(l)$ for input intensities $I_2(0)$, I_3

Note; the figure of merit of the nonlinear medium is

$\frac{|d|^2}{n_1 n_2 n_3}$,

; it is desirable to work with high input intensities,

; efficient generation is more difficult for longer wavelengths (smaller ω_1)

; a long crystal is desirable provided $\frac{\Delta k l}{2}$ remains small.

Returning to equation (7) we see that in the high gain limit

$$G_2(l) = \tfrac{1}{4} \exp \Gamma l$$

$$= \tfrac{1}{4} \exp \left\{ \frac{8\omega_1 \omega_2 |d|^2 I_3 \, l^2}{\epsilon_0 n_1 n_2 n_3 c^3} \right\}^{1/2}$$

thus implying an exponential growth with distance through the nonlinear medium.

For a detailed discussion of optical parametric amplification, including an analysis of the depleted pump regime see Baumgartner and Byer (1979)

For a description of a very high gain optical parametric amplifier, pumped by picosecond pulses, see Laubereau et al (1974), Seilmeier and Kaiser (1980), and Seilmeier et al (1978)

References ; Difference Frequency Generation.

R.L. Byer , Chapter 2, 'Parametric Oscillators and Nonlinear Materials', in Nonlinear Optics, Proc. of 16th Scottish Universities Summer School in Physics, 1977. Ed. by P.G. Harper and B.S Wherrett, Academic Press 1977

O. Svelto , Principles of Lasers (2nd Edition), Plenum Press, Chap. 8 (1982)

R.L. Herbst, R.N. Fleming and R.L. Byer, Appl. Phys. Lett. 25 (1974) 520-522

S.J. Brosnan and R.L. Byer, IEEE J of Quantum Electron. QE-15, (1979) 415-431

R.A. Baumgartner and R.L. Byer, IEEE J of Quantum Electron. QE-15 (1979) 432- 444

A. Laubereau , L. Greiter and W. Kaiser, Appl. Phys. Letts. 25, (1974) 87-89

A. Seilmeier and W. Kaiser , Appl. Phys. 23 (1980) 113-119 .

A. Seilmeier , K. Spanner, A. Laubereau, and W. Kaiser Opt. Commun. 24 (1978) 237