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WINTER COLLEGE ON LASERS, ATOMIC AND MOLECULAR PHYSICS

(24 January - 25 March 1983)

Coherence Properties of Lasers

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These are preliminary lecture notes, intended only for distribution to participants.
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COHERENCE AND CORRELATIONS

- Introduction
- Random variables and random processes
- Correlation functions
- Experimental techniques

  Laser and chaotic source

  Laser pulses

  Scattered fields

The electric field associated with a e.m. wave having central frequency ω_0 is, in complex notation,

$$E(\vec{r}, t) = A\, e^{-i(\omega_0 t - \vec{k}\cdot\vec{r} + \varphi)} \qquad (1)$$

A and φ real quantities

If A and φ do not depend on $\vec{r}$ and t, (1) is a <u>monochromatic</u> <u>plane wave</u>: <u>fully</u> <u>coherent</u> in the sense that, given $E(t)$ at $\vec{r}_0, t_0$, $E(t)$ is completely predictable at any point $\vec{r}_0 + \vec{s}$, $t+\tau$. If A and φ depend randomly on t (intrinsic noise or external disturbance), the field is <u>partially</u> <u>coherent</u> because predictability of $E(t)$

① [illegible]

Similarly for space dependence.

Quantitative measurement of coherence: correlation functions. The simplest is the field correlation function

$$G_1(t, \tau) = \langle E^*(t)\, E(t+\tau) \rangle$$

$*$: complex conjugate

$\langle\ \rangle$: ensemble average

classically → average over a probability density

quantum mech. → $\mathrm{Tr}\{\rho \cdots\}$ $\quad$ ρ: density matrix

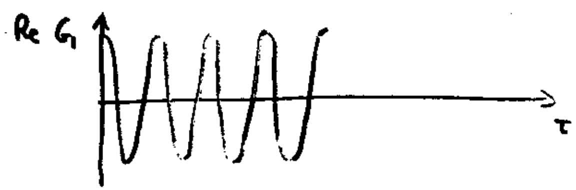
Many practical cases: stationary and ergodic process. Therefore G_1 depends only on τ and ensemble average can be substituted by time average (much simpler from an experimental point of view).

$$G_1(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E^*(t)\, E(t+\tau)\, dt$$

Monochromatic wave:

$$G_1(\tau) = I\, e^{i\omega_0 \tau}$$

$I = E^*(t)\, E(t)$



$$G_1(\tau) = e^{i\omega_0\tau} \langle A(t)A(t+\tau)\, e^{i[\varphi(t)-\varphi(t+\tau)]} \rangle =$$

$$= \langle I \rangle\, e^{i\omega_0\tau}\, \gamma(\tau)$$

$$\gamma(0) = 1 \qquad ; \qquad \gamma(\infty) = 0$$

$|\gamma|$: degree of temporal coherence

$$|\gamma| \sim e^{-\tau/\tau_c}$$

τ_c : coherence time

Similarly for spatial coherence.

Fourier integral

$$E(t) = \int_0^\infty E(\omega)\, e^{-i\omega t}\, d\omega$$

Optical spectrum $\quad S(\omega) = \langle |E(\omega)|^2 \rangle$

$$\boxed{G_1(\tau) = \int_0^\infty S(\omega)\, e^{-i\omega\tau}\, d\omega}$$

Since $\quad \langle I \rangle = \int S(\omega)\, d\omega$

spectral density $\quad s(\omega) = \dfrac{S(\omega)}{\langle I \rangle}$

$$\gamma(\tau) = \int_0^\infty s(\omega)\, e^{-i(\omega-\omega_0)\tau}\, d\omega$$

If $s(\omega)$ is symmetrical around ω_0, $\gamma(\tau)$ is real

Lorentzian spectrum $\qquad s(\omega) = \dfrac{\Gamma/\pi}{\Gamma^2 + (\omega-\omega_0)^2}$

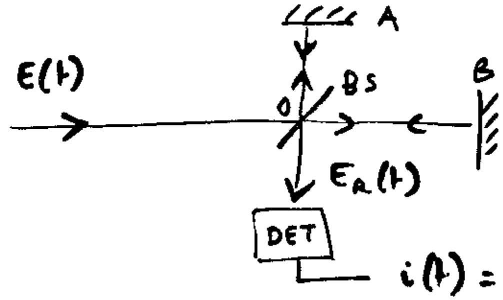
$$\rightarrow \gamma(\tau) = e^{-\Gamma|\tau|}$$

$\tau_c = \Gamma^{-1}$ coherence time

$\Gamma \to 0$: monochromatic field $\quad \tau_c \to \infty$

$\Gamma \to \infty$: white spectrum $\quad \tau_c \to 0$

Measurement of $\gamma(\tau)$: Michelson interferometer



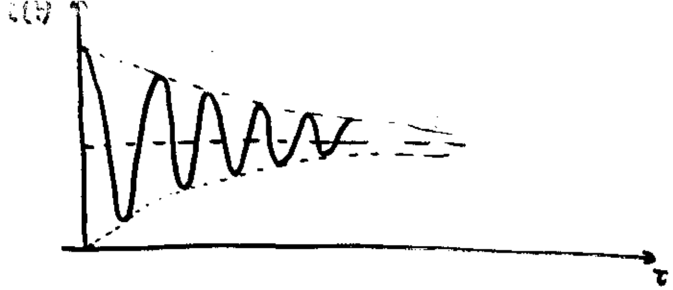
$$i(t) = \alpha\, I_R(t)$$

$$E_R(t,\tau) = \frac{1}{2}\left[E(t) + E(t+\tau)\right]$$

$$\tau = 2\, \frac{OB - OA}{c}$$

$$I_R = |E_R|^2 = \frac{1}{4}\left[I(t) + I(t+\tau) + E(t)E^*(t+\tau) + c.c.\right]$$

$$\langle i(t) \rangle = \alpha \langle I_R \rangle = \frac{\alpha}{2}\left[\langle I \rangle + \mathrm{Re}\{G_1(\tau)\}\right]$$

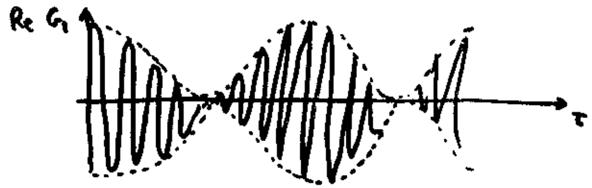


Alternatively: measure $S(\omega)$ by a Fabry-Perot interferometer.

$|\gamma(\tau)| \to 0$ for $\tau \to \infty$ because spectrum is continuous. Take

$$E(t) = E_1 e^{-i(\omega_1 t + \varphi_1)} + E_2 e^{-i(\omega_2 t + \varphi_2)}$$

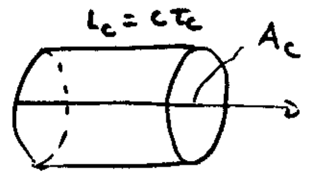
$$\to G_1(\tau) = E_1^2 e^{i\omega_1 \tau} + E_2^2 e^{i\omega_2 \tau}$$



$$\mathrm{Re}\, G_1 \sim \cos\frac{\omega_1+\omega_2}{2} \cos\frac{\omega_1-\omega_2}{2}$$

Space dependence: coherence area

Space + time: coherence volume



Random variables

Continuous variable x

Probability density $W(x)$

$$\int W(x)\, dx = 1$$

$$\langle f(x) \rangle = \int f(x)\, W(x)\, dx$$

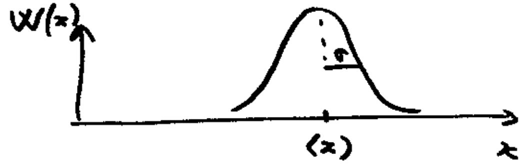
Moments:

$$M_k = \langle x^k \rangle = \int x^k\, W(x)\, dx$$

$$M_1 = \langle x \rangle \quad , \quad M_2 = \langle x^2 \rangle$$

$$v = \langle x^2 \rangle - \langle x \rangle^2 = \langle \Delta x^2 \rangle \qquad \text{variance}$$

$$\sigma = v^{1/2} = \langle \Delta x^2 \rangle^{1/2} \text{ standard deviation}$$



skewness $\quad s = \langle (x - \langle x \rangle)^3 \rangle$

If $s \neq 0$, $W(x)$ is asymmetric with respect to $\langle x \rangle$

Characteristic function:

$$Q(\lambda) = \langle e^{-i\lambda x} \rangle = \int e^{-i\lambda x}\, W(x)\, dx$$

$$Q(\lambda) = \sum_{k=0}^{\infty} (-1)^k \frac{(i\lambda)^k}{k!} M_k$$

Moment generating function:

$$M_k = (-i)^k \frac{d^k}{d\lambda^k} Q(\lambda) \Big|_{\lambda=0}$$

Discrete variable: $n = 0, 1, 2, \ldots$

$$\sum_{n=0}^{\infty} p(n) = 1$$

$$M_k = \langle n^k \rangle = \sum n^k p(n)$$

Factorial moments:

$$F_k = \langle n(n-1)\cdots(n-k+1) \rangle$$

Multidimensional continuous variable:

$$\vec{x} = x_1, \ldots, x_N$$

Joint probability density

$$W_N(x_1, \ldots, x_N)$$

Electric field is a bidimensional variable:

$$W_2(x_1, x_2)$$

where x_1 and x_2 are amplitude and phase (or real and imaginary part)

Conditional probability:

$$W_c(x_1/x_2)$$

probability density to find x_2 given x_1.

$$W_c(x_1/x_2) = W_2(x_1, x_2) / W_1(x_1)$$

Statistically independent variables:

$$W_2(x_1, x_2) = W(x_1)\, W(x_2)$$

$$W_c(x_1/x_2) = W(x_2)$$

Joint moments

$$M_{kl} = \langle x_1^k x_2^l \rangle = \iint x_1^k x_2^l\, W_2\, dx_1\, dx_2$$

$$M_{10} = \langle x_1 \rangle, \quad M_{01} = \langle x_2 \rangle$$

$M_{11} = \langle x_1 x_2 \rangle \leftarrow$ lower order correlation function

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EXAMPLES

Deterministic variable $\quad W(x) = \delta(x - x_0)$

$$\langle x \rangle = x_0 \quad ; \quad \langle x^k \rangle = x_0^k \quad ; \quad v = 0$$

Gaussian variable

$$W(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-(x-x_0)^2 / 2\sigma^2\right]$$

$$\langle x \rangle = x_0 \quad ; \quad \langle (x-x_0)^{2k+1} \rangle = 0 \quad ; \quad v = \sigma^2$$

Poisson distribution

$$W(n) = \frac{1}{n!} x_0^n e^{-x_0} \qquad n \text{ integer} \geq 0$$

$$\langle x \rangle = x_0 \;, \quad v = x_0 \;, \quad F_k = x_0^k$$

Bose-Einstein distribution

$$W(n) = \frac{x_0^n}{(1+x_0)^{n+1}} \qquad n \text{ integ.} \geq 0$$

$$\langle x \rangle = x_0 \;, \quad v = \langle \Delta n^2 \rangle = x_0 + x_0^2$$

[illegible] process

$x(t)$

If statistical properties do not depend on t: stationary process.

Random process is specified by joint probability distributions:

$$W_2\,(x(t_1), x(t_2))$$

$$W_3$$

$$\vdots$$

$$W_N$$

Given W_N, all W_K with $K < N$ are given.

Time correlation function:

$$\langle x(t_1)\, x(t_2)\rangle = \iint dx_1\, dx_2\; x_1 x_2\, W_2(x_1, x_2)$$

If the process is stationary, it depends only on the time delay $\tau = t_2 - t_1$

Markov processes (short memory):

$$W_c\,(x_1(t_1), x_2(t_2), \dots, x_{n-1}(t_{n-1}) / x_n(t_n)) = $$

$$= W_c\,(x_{n-1} / x_n)$$

Markov process is specified only by W_c

(9) [illegible] field is a complex random process

$$E(t) = A\, e^{i\omega_0 t + i\varphi}$$

Correlation functions:

$$G_{nm} = \langle E^*(t_1) \cdots E^*(t_n)\, E(t_1') \cdots E(t_m')\rangle$$

Since detectors of optical signals are square-law detectors (energy detectors), n is always equal to m, therefore we use

$$G_{nn} \longrightarrow G_n$$

G_1: field correlation function

$$G_2 = \langle E^*(t_1)\, E^*(t_2)\, E(t_2)\, E(t_1)\rangle = \langle I(t_1)\, I(t_2)\rangle$$

intensity correlation function

Quantum definitions

$$E = E^+ + E^-$$

$$E^+ = \sum a_k\, e^{-i\omega_k t}$$

$$E^- = \sum a_k^+\, e^{i\omega_k t}$$

$\boxed{a}$: creation operator

$\boxed{a^+}$: annihilation operator

$$[a, a^+] = aa^+ - a^+a = 1$$

Coherent states are eigenstates of a

$$a\,|\alpha\rangle = \alpha\,|\alpha\rangle \qquad \alpha: \text{c-number}$$

$$|\alpha\rangle = \exp\left[-\tfrac{1}{2}|\alpha|^2\right] \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Coherent state is a superposition of $|n\rangle$ states

$$|\langle n|\alpha\rangle|^2 = p(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \qquad \underline{\text{Poisson}}$$

α states are <u>complete</u> but <u>non orthogonal</u>

$$\rho = \int d^2\alpha \; P(\alpha) \, |\alpha\rangle\langle\alpha| \qquad \text{diagonal representation}$$

$P(\alpha) \geq 0$ is a <u>quasi-probability density</u>

$$G_n = \mathrm{Tr}\left\{\rho \, E^{(-)}(x_1) \cdots \; E^{(+)}(x'_n)\right\}$$

<u>Normal ordered</u> correlation functions are computed by using $P(\alpha)$ representation.

Note that $\langle a^+ a \, a^+ a\rangle = \langle n^2\rangle =$

$$= \langle a^+ a^+ a a\rangle + \langle a^+ a\rangle = \langle n(n-1)\rangle + \langle n\rangle$$

$$= F_2 + M_1$$

Difference between $\langle a^+ a \, a^+ a\rangle$ and $\langle a^+ a^+ a a\rangle$ is $O\left(\frac{1}{n}\right)$: <u>many-photon field is classical.</u>

$$\boxed{p(n) = \iint d^2\alpha \; P(\alpha) \; e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}}$$

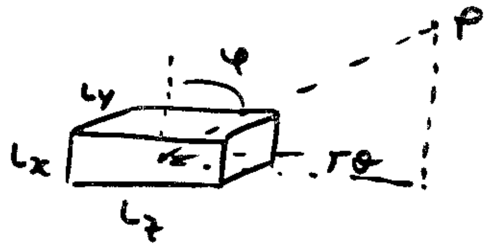
Poisson Kernel

if $P(\alpha)$ is $\delta(\alpha - \alpha_c) \to p(n)$ Poisson

is Gaussian $\to p(n)$ Bose-Einstein

(11) EXAMPLES:

CHAOTIC SOURCES



Rectangular box.

$$E(t) = \sum E_j \, e^{-i\omega_j t + i\vec{k}_j \cdot \vec{r}}$$

Take a single mode :

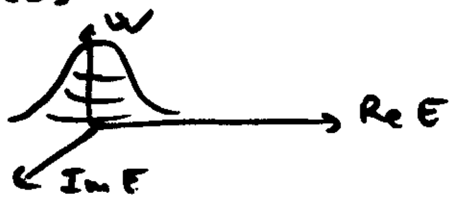
<u>spatial coherence depends on the geometry</u>

$$\Omega_c(\theta,\varphi) = \frac{\lambda^2}{L_y L_z \sin\theta\cos\varphi + L_x L_z \sin\theta\sin\varphi + L_x L_y \cos\theta}$$

<u>temporal coherence depends on atomic dynamics</u>

The field in P is linear superposition of many independent emission events (two-dim. random walk) $\to$ GAUSSIAN PROBAB. DENSITY

$$W(E) = \frac{1}{\pi\langle I\rangle} \exp\left[-|E|^2/\langle I\rangle\right]$$



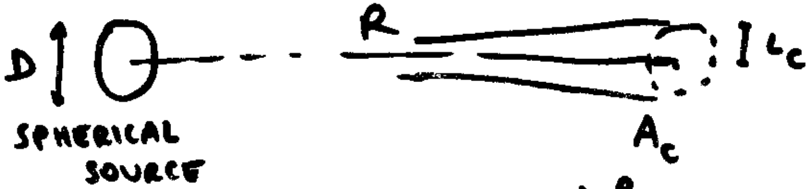
$$G_1(\tau) = \langle I \rangle e^{i\omega_0 \tau} e^{-\tau/\tau_c}$$

$$G_2(\tau) = \langle I \rangle^2 \left(1 + e^{-2\tau/\tau_c}\right)$$

$$\boxed{= G_1^2(0) + |G_1(\tau)|^2}$$

TIME CORRELATIONS : SAMPLE WITHIN A COHERENCE AREA.

SPACE CORRELATIONS : SAMPLE WITHIN A COHERENCE TIME.



$$L_c \sim \lambda \frac{R}{D}$$

<u>Measurement</u> of <u>L_c</u> <u>gives</u> D if <u>R is known</u>

The source may be a star.

SUN $R = 150 \times 10^6$ Km

$D = 1.4 \times 10^6$ Km

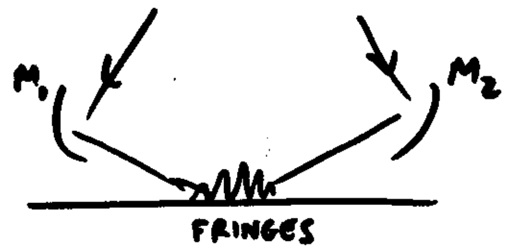
$\lambda = 0.5\ \mu m \rightarrow L_c \simeq 50\ \mu m$

PROXIMA CENTAURI

$R = 4.2$ light-years

$\theta = D/R \sim 0.01'' \rightarrow L_c \simeq 10$ m

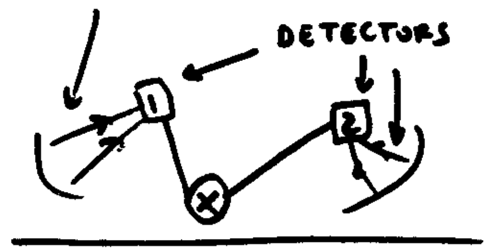
STELLAR INTERFEROMETER



If $L_c > 10$ m it is difficult to eliminate air turbulence effects → intensity interferometer (HANBURY BROWN & TWISS)

Gaussian field : G_1 and G_2 are related

In general G_1 cannot be obtained from G_2 because G_2 contains only amplitude fluctuations, whereas G_1 contains both amplitude and phase fluctuations



GLAUBER DEFINITION OF COHERENCE AT ANY ORDER

$$|G_n| = \prod_{k=1}^{n} G_1^{1/2}(x_k) \prod_{k=1}^{n} G_1^{1/2}(x'_k)$$

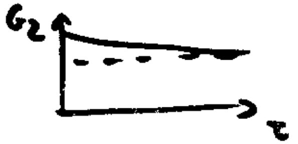
FULL COHERENCE : NO FLUCTUATIONS

This definition includes as a particular case classical definition :

$$|G_1| = |\langle E^*(t_1)\, E(t'_1)\rangle| \to \left(I(t_1)\, I(t'_1)\right)^{1/2}$$

stationary $= \langle I \rangle$

$$G_2 = \langle I(t_1)\, I(t_2)\rangle \quad \to \quad \langle I\rangle^2$$



If there are intensity fluctuations,

$G_2(0) > G_2(\infty)$

Poisson distribution :

$$G_2(0) = F_2 = \langle n\rangle^2 = G_2(\infty) \qquad !!$$

PAY ATTENTION : also the superposition of many uncorrelated modes may give a Poisson distribution!

Photon counting technique

Detectors of e.m. waves at optical frequencies work through the absorption of e.m. energy in quantised steps (absorption of photons) and the consequent excitation (or emission) of electrons. Photon counting techniques rely on a photoelectric detector, the photomultiplier tube (PMT), which is able to give a measurable electric pulse at the output after the absorption of a single photon.

If the optical signal is so weak that it is very unlikely to detect more than one photon within the response time of the photodetector, the output electric current consists of a random train of non-overlapping photoelection pulses.

The statistical distribution $p(m,T)$ of m photoelection counts per interval T is connected with $W_1(E)$ as follows

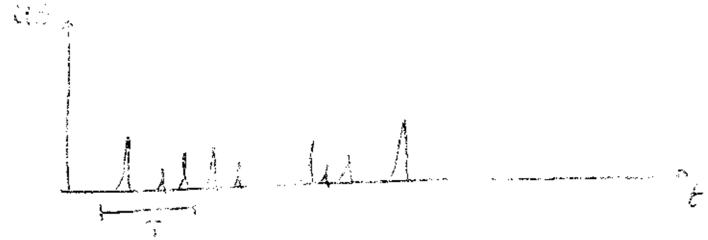
$$p(m,T) = \int \underbrace{\frac{(\eta |E|^2)^m}{m!}\, e^{-\eta |E|^2}}_{\text{Poisson kernel}} W_1(E)\, d^2E$$

T should be much smaller than τ_{2c}, the correlation time for intensity fluctuations

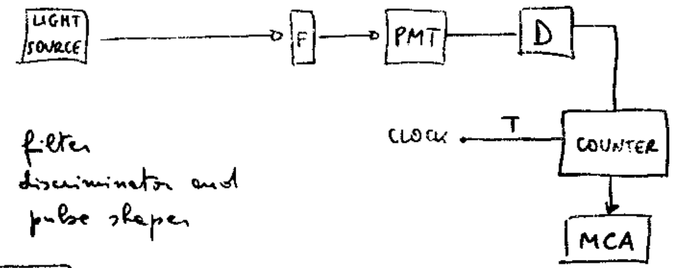
Examples : laser field (Poisson) $\quad p(m,T) = \dfrac{\langle m\rangle^m}{m!} e^{-\langle m\rangle}$

chaotic source (Bose-Einstein) $\quad p(m,T) = \dfrac{\langle m\rangle^m}{(1+\langle m\rangle)^{m+1}}$

$$p(m) = \sum p(n) \underbrace{\binom{n}{m} \eta^m (1-\eta)^{n-m}}_{\text{BERNOULLI}} \qquad \eta = \frac{\langle m\rangle}{\langle n\rangle}$$



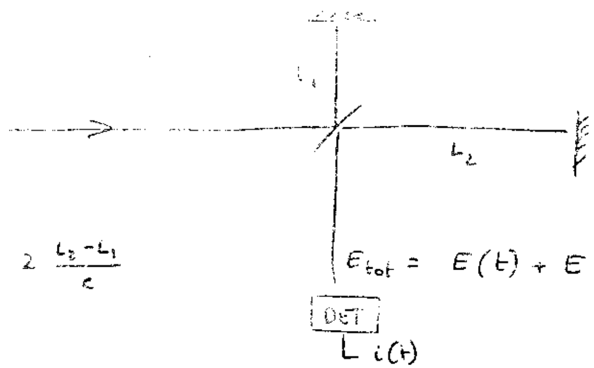
Photoelection pulses are not identical because the PMT is a statistical device. The experimental apparatus includes a discriminator + standardizer of pulses before the counter. The output of the counter is sent to a multichannel analyzer (MCA).



F: filter
D: discriminator and pulse shaper

Experimental problems: uncorrelated noise
dead-time corrections

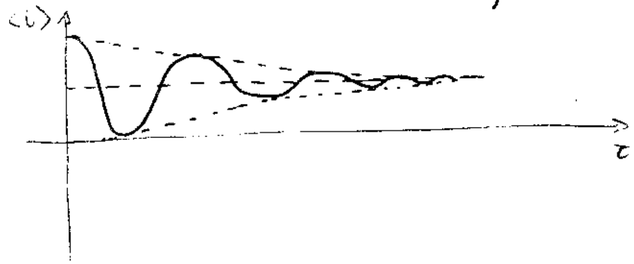
Also the joint photocount distribution $p_2(n_1 t_1 T;\ n_2 t_2 T)$ can be measured.



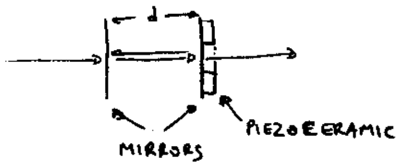
$$\tau = 2\,\frac{L_2 - L_1}{c} \qquad E_{tot} = E(t) + E(t+\tau)$$

$$i(t) \propto I(t) = |E_{tot}|^2 = 2\,|E|^2 + 2\,\mathrm{Re}\{E(t)E^*(t+\tau)\}$$

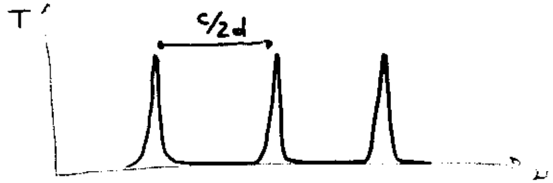
$$\langle i(t)\rangle \propto 2\langle I\rangle\,\{1 + f(\tau)\cos\omega_0\tau\}$$



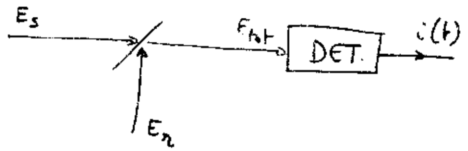
Fabry-Perot interferometer $\rightarrow S_1(\omega)$



T: F.P. transmission as a function of frequency ν.



Intensity correlation [illegible] measured by using the reference-beam technique, also called heterodyne. This method is completely general, that is does not rely on the assumption that the field is Gaussian.



E_s is the field we want to study, E_r is a reference field

$$E_{tot}(t) = E_s(t) + E_r(t)$$

$$I_{tot} = |E_{tot}|^2 = I_s(t) + I_r(t) + 2\,\mathrm{Re}\left\{E_s(t)\,E_r^*(t)\right\}$$

$$G_{2\,tot} = G_{2s} + G_{2r} + 2\,\mathrm{Re}\left\{G_{1s}\,G_{1r}^*\right\}$$

By putting $E_s(t) = E_{0s}(t)\,e^{i(\omega_{0s}t + \varphi_s(t))}$

$$E_r(t) = E_{0r}\,e^{i(\omega_{0r}t + \varphi_r)}$$

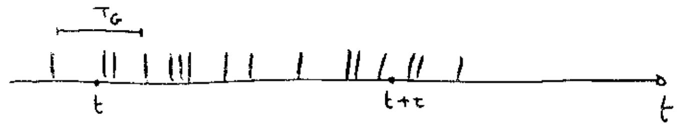
with E_{0r}, φ_r deterministic values

$$G_{2\,tot} = \langle I_s\rangle^2 (1 + g_s(\tau)) + \langle I_r\rangle^2 + 2\langle I_s\rangle\langle I_r\rangle \cos\left[(\omega_{0s} - \omega_{0r})\tau\right] f(\tau)$$

If $\langle I_s\rangle \ll I_r$, the first term can be neglected, and the time dependent part of $G_{2\,tot}$ is proportional to $\mathrm{Re}\{G_{1s}(\tau)\}$ with a frequency shift ω_{0r}.

Digital correlators

The signal coming from the PMT is already in digital form



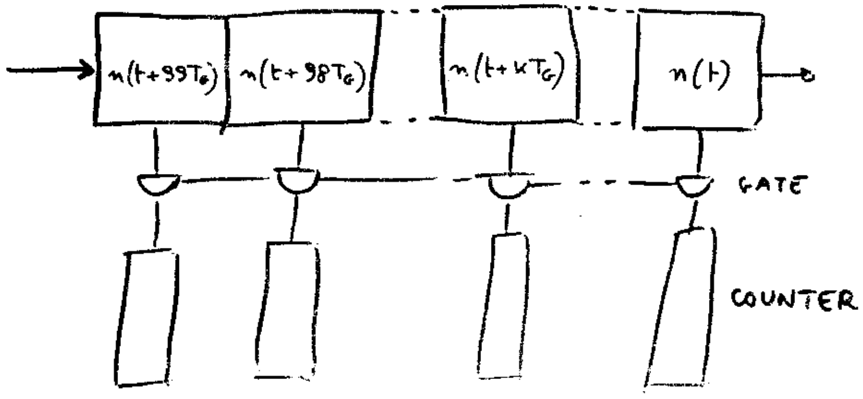
$n(t, T_G)$ is the number of photoelectron pulses counted in an interval T_G centered around time t. The autocorrelation function of the random variable n is

$$R(\tau, T_G) = \langle n(t, T_G)\, n(t+\tau, T_G)\rangle$$

If $T_G \ll \tau_c$, decay time of $G_2(\tau)$, then

$$R(\tau, T_G) \propto G_2(\tau)$$

The maximum number of photoelectron pulses n recorded in the interval T_G is limited by the deadtime of the apparatus. If real time ~~operation~~ is ~~desired~~ at high sampling frequencies, another limit is ~~set~~ by the fact that large numbers require long multiplication and registration times.



$k = 0, 1, 2, \ldots, 99$

$n(t)$ before being kicked out by the sample $n(t+100\,T_G)$ is multiplied by all the 99 samples $n(t+kT_G)$. The result of each multiplication is stored into the appropriate counter. Present state of the art : 4 bits, 20 MHz.

In the low frequency range it is possible to build "software correlators" which feed directly into a computer without intermediate hardware.

MEASUREMENT OF HIGHER-ORDER CORREL.

1. Electronic processing of the output of the PMT
2. Direct measurement by multiphoton absorption or by harmonic generation (picosecond pulses, see next lecture)

BEAM THROUGH A MIRROR

$$p(n) \longrightarrow \underset{R}{|} \longrightarrow p(m) \qquad \frac{\langle m\rangle}{\langle n\rangle} = 1-R$$

$p(m) = p(n) *$ BERNOULLI DISTRIB.

Poisson remains Poisson

B. E. " B. E.

SUMMARY.

$E(t)$ is random bidimensional process (single mode)

$E(t)$ is characterized by a hierarchy of joint probab. distrib. or by nth order correlation functions

$$\left.\begin{array}{l} P(\alpha) \to p(n) \\ G_1 \\ G_2 \end{array}\right\} \text{measurement techniques}$$

Full coherence: no fluctuations

single mode: $p(n)$ is Poisson

STATISTICAL THEORY OF THE SINGLE-MODE LASER

Damped harmonic oscillator

$$\alpha \to \alpha e^{-i\omega_0 t}$$

$$\dot{\alpha} + \gamma\alpha = \Gamma(t) \qquad \text{Langevin Eq.}$$

$\Gamma(t)$: complex stationary gaussian process

$$\langle\Gamma(t)\rangle = 0 \quad ; \quad \langle\Gamma^*(0)\Gamma(t)\rangle = Q\,\delta(t)$$

α : Markov process

Derive conditional probability

$$W_c(\alpha_0, 0\, ; \alpha, t)$$

Fokker-Planck Eq :

$$\frac{\partial W_c}{\partial t} - \gamma\, \mathrm{div}_\alpha(\alpha W_c) = \frac{Q}{4}\nabla_\alpha^2 W_c$$

Solution :

$$W_c = \frac{1}{\pi\sigma^2(t)} \exp\left[-\frac{|\alpha - \alpha_0 e^{-\gamma t}|^2}{\sigma^2(t)}\right]$$

$$\sigma^2(t) = \frac{Q}{2\gamma}\left[1 - \exp(-2\gamma t)\right]$$

[illegible]

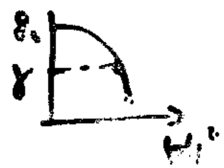
W_2 is also Gaussian

α: single mode of a classic field with a Lorentz spectrum

<u>Simple mode laser</u>

$$\dot{\alpha} + (\gamma - G)\alpha = \Gamma(t)$$

$G = g_0 - p|\alpha|^2$ saturable gain



$$\dot{\alpha} - p(d - |\alpha|^2)\alpha = \Gamma(t)$$

$$d = (g_0 - \gamma)/p$$

F. P. equation

$$\frac{\partial W_c}{\partial t} + p \, \mathrm{div}_\alpha \left[(d - |\alpha|^2)\alpha W_c \right] = \frac{Q}{4} \nabla_\alpha^2 W_c$$

$$\alpha = r e^{i\varphi}$$

Stationary solution:

$$W(r, \varphi) = N \exp\left[-\frac{p}{Q} r^4 + \frac{2pd}{Q} r^2 \right]$$

Pump parameter:

$$a = d(4p/Q)^{1/2}$$

Threshold $g_0 = \gamma \rightarrow d = a = 0$

Far below threshold $a < -10$

$$W = N \exp\left(-\frac{2p|d|}{Q} r^2 \right) \quad \text{Gaussian}$$

$$\langle n \rangle = \langle r^2 \rangle = Q/2p|d|$$

Far above threshold $a > 10$

$$W = N \exp\left[-\frac{4pd}{Q} (r - \sqrt{d})^2 \right] \quad \text{displaced Gaussian}$$

W does not depend on φ

For below threshold (Gaussian field)

$$|G_1(\tau)| = \frac{Q}{2p|d|} e^{-p|d|\tau}$$

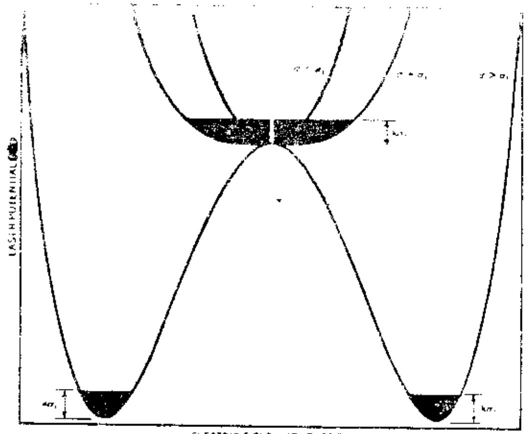
$$G_2(\tau) = \langle|\alpha|^2\rangle + \left(\frac{Q}{2p|d|}\right)^2 e^{-2p|d|\tau}$$

For above threshold

$$|G_1(\tau)| = d\, e^{-(Q/4d)\tau} + \frac{Q}{8pd} e^{-(2pd + Q/4d)\tau}$$

$$G_2(\tau) = \langle|\alpha|^2\rangle + \frac{Q}{2p} e^{-2pd\tau}$$

Above threshold the main noise is phase diffusion noise



The laser potential $G(E)$, the minima of which give the steady states of the laser. The three cases shown are: below threshold (unsaturated population inversion σ less than its threshold value σ_t), at threshold and above it. The magnitudes of the fluctuations of the laser field E at steady state are found by considering energy fluctuations $k\sigma_t$ (see equation 4) above the minima of $G(E)$ and deriving the corresponding changes in the field amplitude.

FIGURE 1

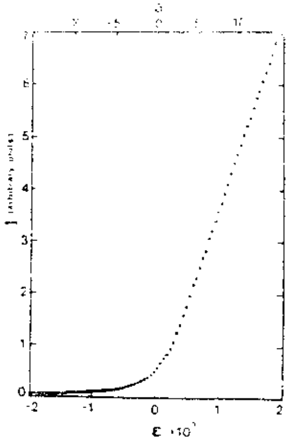
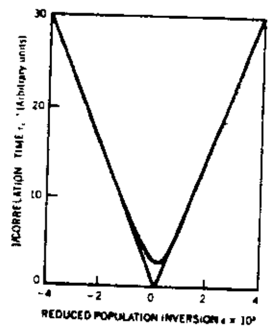


FIG. 2 The average laser intensity as a function of the normalized net gain (lower scale) and of the pump parameter (upper scale). Errors are smaller than the dot size. The theoretical curve is not distinguishable from the line interpolating the experimental points.



The reciprocal of the correlation time τ_c of laser-intensity fluctuations plotted versus ϵ. Apart from the threshold region ($\epsilon \simeq 0$), where rounding off occurs because of the finite laser volume, the correlation time follows a simple power law, which has an analog in the Landau ferromagnetism theory. The measurements are described in reference 18.

FIGURE 3

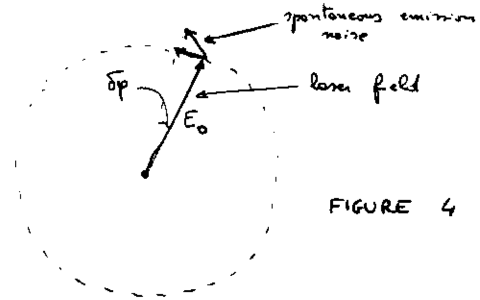


FIGURE 4

$\delta\varphi$: phase shift due to single spontaneous emission event $\quad \delta\varphi \div \frac{1}{E_0} \div \frac{1}{P_{out}^{1/2}}$

Random walk : $\langle\Delta\varphi^2\rangle = N\langle\delta\varphi^2\rangle =$

$\div\ \sigma t \frac{1}{P_{out}}$

Coherence time $\tau_c \rightarrow \langle\Delta\varphi^2\rangle^{1/2} \sim \pi$

$$\tau_c \div \frac{P_{out}}{\sigma}$$

$M_k = \sum n^k p(n)$

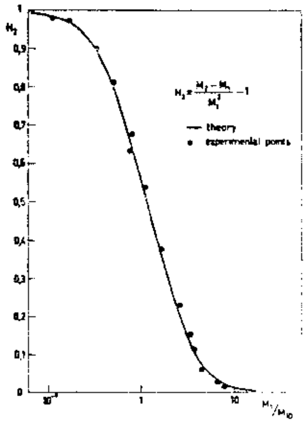


Fig. 17. The reduced second-order factorial moment H_2 as a function of the normalized laser intensity M_1/M_{10}. M_{10} denotes here the average intensity at threshold. The solid line represents theoretical values, the dots represent the experimental results (Arecchi et al. 1967a).

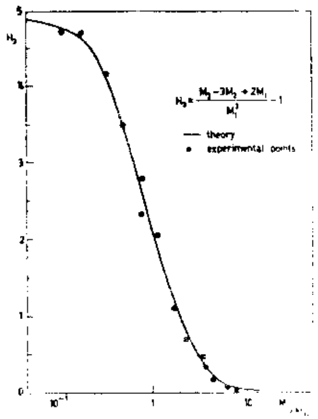
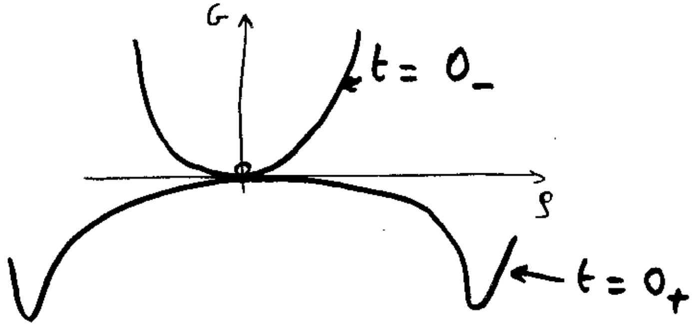


Fig. 18. Plot of H_3 versus M_1/M_{10}. Explanations are [illegible] 17

Instability transients (Q-switching)

Fluctuations may be large even far away from threshold: decay of an unstable state characterized by the absence of systematic forces.



Decay initiated by a fluctuation

Linear amplification → saturation → steady state

Single-mode laser: Van der Pol equation

$$\dot{I} = \alpha I - 2\beta I^2$$

$$I = |E|^2 \quad , \quad \alpha = 2(g_0 - \gamma)$$

Peak of the variance in correspondence with the inflection point of the average intensity transient (Arecchi, Degiorgio, Querzola Phys. Rev. Lett 1967)

Phenomenological model (Arecchi - Degiorgio Phys. Rev. 1971): deterministic evolution from a statistically defined initial condition.

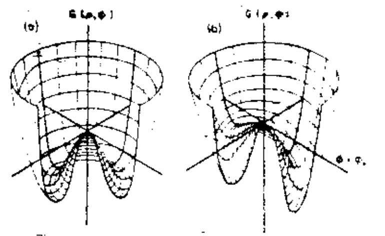


Fig. 5 (a) Effective free energy, $G(\rho, \phi)$ of laser. (b) Effective free energy, $G(\rho, \phi)$ of laser subjected to symmetry breaking signal having phase ϕ_0.

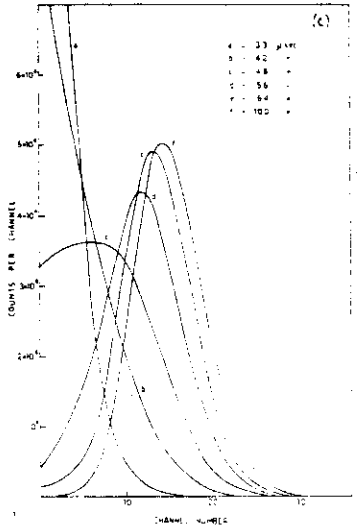


FIG. 6 Experimental statistical photocount distributions obtained with different delays with respect to the switching time. The solid lines join the experimental points which are not shown, to make the figure clearer. All distributions are normalized to the same area.

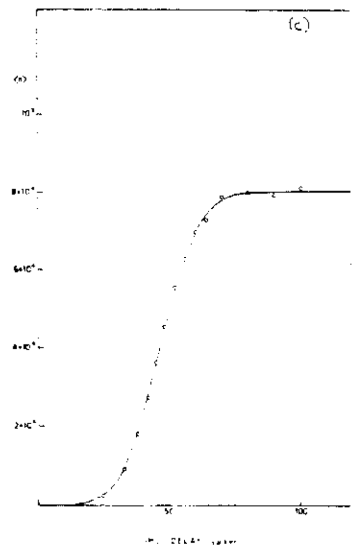


FIG. 7 Evolution of the average photon number $\langle n \rangle$ inside the cavity as a function of the time delay τ. Solid lines represent best-fit results computed from the theory of Scully, Lamb, and Sargent.

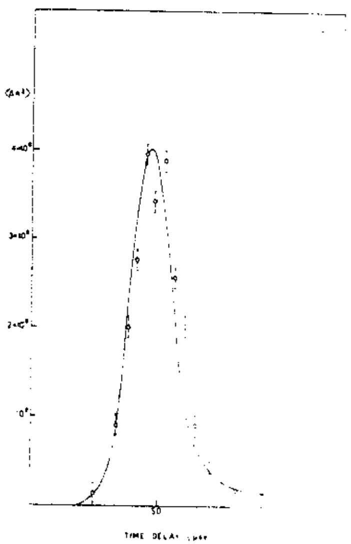
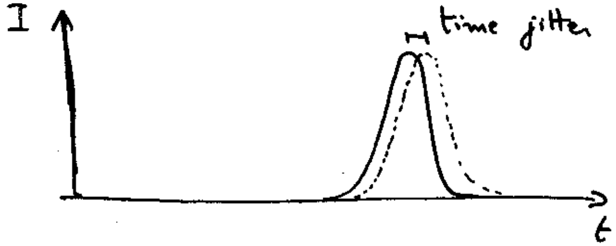


FIG. 8 Evolution of the variance (Δ- statistical distribution of photons inside a function of the time delay. Solid lines theoretical results



Q-switching pulse : fluctuations in shape and area are negligible (ideal laser), there is only a time jitter (fluctuation in time position) due to the initial statistics : number of photons which are present in the cavity at the Q-switching instant is a random variable with a Bose-Einstein distribution. Time jitter is comparable to pulse duration!

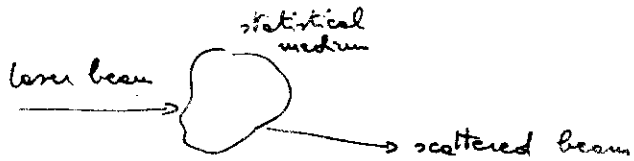
Conclusions about intrinsic noise :

- c.w. single mode laser : far above threshold (normal operation) fluctuations are negligible
- pulsed laser : fluctuations only in the time position of the peak
- multimode laser : depends on mode interaction

External noise :

- pump fluctuations → affect laser amplitude
- cavity length fluctuations → affect laser frequency
- mirror parallelism fluctuations → affect laser amplitude, may introduce transverse modes

1. Light scattering experiments



$$E_s(t) = a(t)\, E_0(t)$$

Linear process ; $a(t)$ is a scattering amplitude which contains information about fluctuations in the scattering medium. Measurement of statistical properties of $E_s(t)$ allows to obtain $a(t)$. Usually only lower order correlations are needed.

~~2.~~
2. Nonlinear field-matter interaction : harmonic generation, multiphoton absorption (photochemistry, ultrashort pulse measurements)

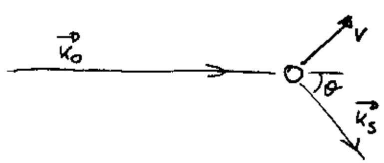
Average velocity profiles and velocity fluctuations can be measured in a flowing fluid by analyzing the power spectrum (or the autocorrelation function) of laser light scattered by particles suspended in the fluid (either naturally occurring or specially introduced).

Seeding particles should not be too large, otherwise they cannot faithfully follow the motion of the medium. Particle diameter : $\leq 10\,\mu m$ for liquid flows

$\leq 0.5\,\mu m$ for supersonic gas flows

Put $\delta\varepsilon(\vec{r},t) = \sum_{j=1}^{N} \Delta\varepsilon \; \delta(\vec{r} - \vec{r}_j(t))$

Doppler effect : $E_s = E_0 \sum a_j \, e^{i\vec{k}\cdot\vec{r}_j(t)} \, e^{i\omega_0 t}$

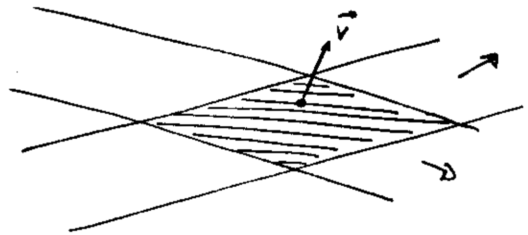


$$\vec{r}_j(t) = \vec{r}_j(0) + \vec{v}\, t$$

$$E_s = E_0 \sum a_j \, e^{i[(\vec{k}\cdot\vec{v}) + \omega_0]t} \, e^{i\vec{k}\cdot\vec{r}_j(0)}$$

Doppler frequency shift $\Delta\omega_D = \vec{k}\cdot\vec{v}$ (depends on θ)

Crossed beams : $\Delta\omega_1 = (\vec{k}_s - \vec{k}_1)\cdot\vec{v}$; $\Delta\omega_2 = (\vec{k}_s - \vec{k}_2)\cdot\vec{v}$



Total shift : $\Delta\omega_{12} = \Delta\omega_1 - \Delta\omega_2 = (\vec{k}_2 - \vec{k}_1)\cdot\vec{v}$ (independent of θ)

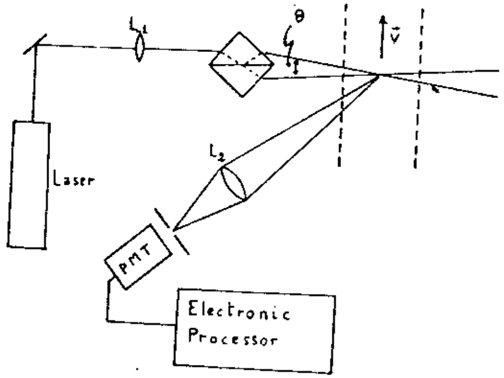


Fig. 1 - Basic Doppler difference setup

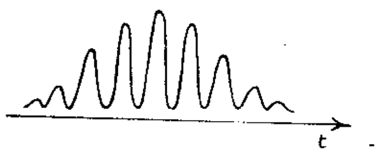


Fig. 2 - Typical Doppler signal; its finite duration reflects the finite dimension of the intersection region.

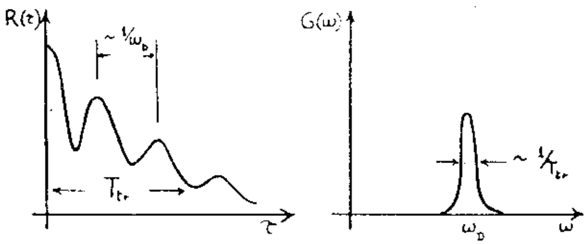


Fig. 4 - An example of a correlation function and of a frequency spectrum.

Fringe spacing $s = \dfrac{\lambda}{2 \sin \theta/2} \rightarrow \Delta\nu_D = \dfrac{u}{s}$; $u = \dfrac{(\vec{k}_2 - \vec{k}_1)\cdot\vec{v}}{|\vec{k}_2 - \vec{k}_1|}$

Example: $\theta = 5°$, $\lambda = 0.5\ \mu m$ (argon laser), $v = 1$ m/s

$\rightarrow \Delta\nu_D \simeq 200$ KHz

Signal processing : spectrum analyzer
frequency tracker
photon correlator → for weak signals

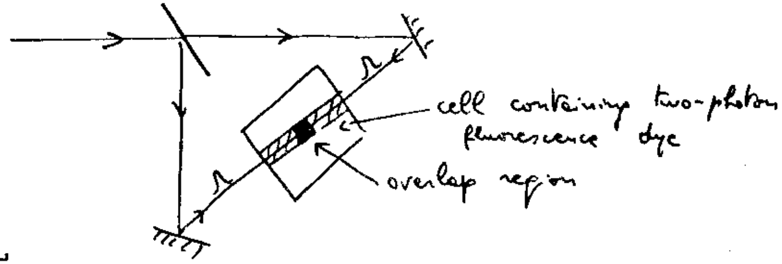
Example (from Pike): one 1 μm particle, illuminated by 1 mW of laser light at $\lambda = 0.6\ \mu m$, focused down to 100 μm, a photodetector 1 m away with a collection aperture of 10 cm in diameter, records about 10^8 photoelectrons per second. If the particle stays in the beam 10^{-4} s (i.e. the fluid velocity is 1 m/s) there are 10^4 photoevents for each particle crossing. If the particle diameter is 0.2 μm, the same calculation gives 1 photoevent per particle crossing.

$$R(\tau) = A \exp\left(-\frac{u^2\tau^2}{r^2}\right)\left[1 + \frac{1}{2}\cos\left(\frac{2\pi u}{s}\tau\right)\right]$$

Applications : hydrodynamics (laminar and turbulent flows)
aerodynamics (wind tunnels)
"in vivo" blood flow
electrophoresis

Note that $R(\tau)$ does not depend on the sign of u. To get also velocity direction, one should have a frequency shift between the two incident beams

Measurement of ultrashort pulse duration: the two-photon fluorescence method



$\tau = 2x/c$

Fluorescence signal:

$$F(\tau) = k \left\{ G_2(0) + 2G_2(\tau) + \text{rapidly varying terms} \right\}$$