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IX TRIESTE WORKSHOP ON
OPEN PROBLEMS IN
STRONGLY CORRELATED SYSTEMS

14 - 25 July 1997

ac HALL EFFECT IN STRONGLY INTERACTING ELECTRON SYSTEMS

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These are preliminary lecture notes, intended only for distribution to participants.

ac Hall Effect in Strongly Interacting Electron Systems

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ac Hall Effect

Introduction

- Hall effect and cyclotron resonance
- Hall angle sum rule
- experimental

Superconductors

Normal State of High T_c Superconductors

- dc magneto-transport - Ong-Anderson model
- magneto-optics - spinon lifetime and mass.
- other models

Superconducting state

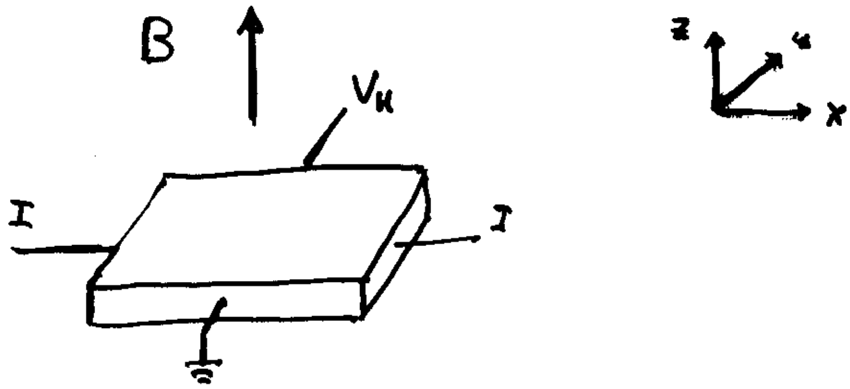
- mixed state of type II superconductors - vortex dynamics
- IR resonances

Implications of Hall Angle Sum Rule

New Experiments

Conclusions

HALL EFFECT



$$E_y = -\rho_{xy} J_x$$

$$E_x = \rho_{xx} J_x$$

$$R_H = \frac{\rho_{xy}}{B}$$ HALL COEFFICIENT

$$\tan\theta_H = \frac{\rho_{xy}}{\rho_{xx}}$$ HALL ANGLE

DRUDE MODEL

$$\rho_{xy} = \frac{1}{nec} \qquad \rho_{xx} = \frac{m}{ne^2}(i\omega + 1/\tau)$$

$$\tan\theta_H = \frac{\omega_c}{i\omega + 1/\tau} \qquad \omega_c = \frac{eB}{mc}$$ CYCLOTRON FREQUENCY

CONDUCTIVITY TENSOR

$$\sigma = \rho^{-1} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

$$\tan\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}$$

CIRCULAR MODES

$$\sigma^{\pm} = \sigma_{xx} \pm i\,\sigma_{xy} \qquad \sigma = \begin{pmatrix} \sigma^{+} & 0 \\ 0 & \sigma^{-} \end{pmatrix}$$

DRUDE

$$\sigma^{\pm} = \frac{ne^2/m}{i(\omega \pm \omega_c) + 1/\tau}$$

MORE GENERALLY

$$\sigma = \sum_i \sigma_i$$

HALL ANGLE SUM RULE

f - SUM RULE

$$\mathrm{Re}\int_0^\infty \sigma_{xx}\,d\omega = \frac{\pi}{2}\frac{ne^2}{m} = \frac{1}{8}\omega_p^2 = \frac{e^2}{4ah}\oint dk_t\,|v|$$

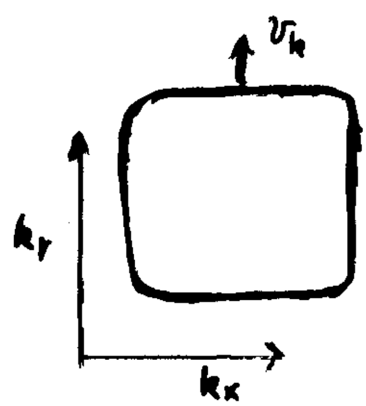
HALL ANGLE SUM RULE

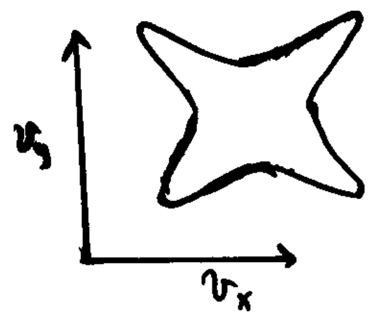
$$\tan\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}$$

$$\mathrm{Re}\int_0^\infty \tan\theta_H\,d\omega = \frac{\pi}{2}\omega_c = \frac{\pi}{2}\omega_H = \frac{\pi}{2}\frac{eB}{\hbar c}\,\frac{\oint dk_t\, v\times dv/dk}{\oint dk_t\,|v|}$$

$$\oint dk_t\, v\times dv/dk$$

$$= \oint dv\times v$$





WHAT DOES ac HALL EFFECT MEASURE ?

$\omega_c \tau \to \infty \qquad R_H = \frac{1}{nec}$

$\omega \tau \to \infty \qquad R_H^{\infty}$

DRUDE $\qquad R_H^{\infty} = \frac{1}{nec}$

n – BAND $\qquad R_H^{\infty} = \frac{1}{ec} \frac{\sum \frac{n_i}{m_i^2}}{\left(\sum \frac{n_i}{m_i}\right)^2}$

FERMI LIQUID $\qquad R_H^{\infty} \propto \frac{\oint dv \times v}{\left(\oint dk |v|\right)^2} \propto \frac{\oint dv \times v}{\omega_p^4}$ AREA OF VELOCITY SURFACE

UNIFORM VELOCITY RENORMALIZATION CANCELS

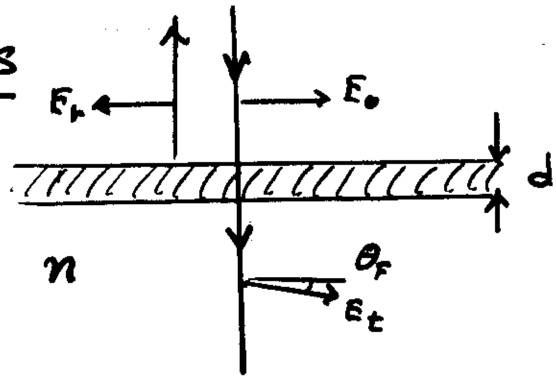
SCALE F.S. $\Rightarrow \quad R_H^{\infty} \sim \left(\frac{1}{\Delta k}\right)^2$

HUBBARD MODEL SHASTRY, SHRAIMAN & SINGH, PRL 70, 2004 (199

$1/\tau << \omega << U$

$R_H^{\infty} = r_0 \left[\frac{1}{4\delta} + \frac{3}{4} - \frac{1}{1-\delta} \right] \qquad \delta = 1 - n$

a.c. HALL MEASUREMENTS – FILMS



$$E_t = E_0 \frac{2n}{1+n+Z_0\sigma} = E_{t0}\frac{1}{1+Z\sigma}$$

$$d < \lambda_L , \quad Z = Z_0/(1+n)$$

$$t = E_t/E_0$$

$B \neq 0$

$$E_t^{\pm} = E_{t0}^{\pm}\frac{1}{1+Z\sigma^{\pm}}$$

$$\frac{t^+}{t^-} = \frac{1+Z(\sigma_{xx}-i\sigma_{xy})}{1+Z(\sigma_{xx}+i\sigma_{xy})} = \frac{1-i\tan\theta_F}{1+i\tan\theta_F}$$

$$\tan\theta_F = \frac{Z\sigma_{xy}}{1+Z\sigma_{xx}}$$

$\boxed{\text{FARADAY ROTATION}} \Rightarrow \mathrm{Re}\,\tan\theta_F$

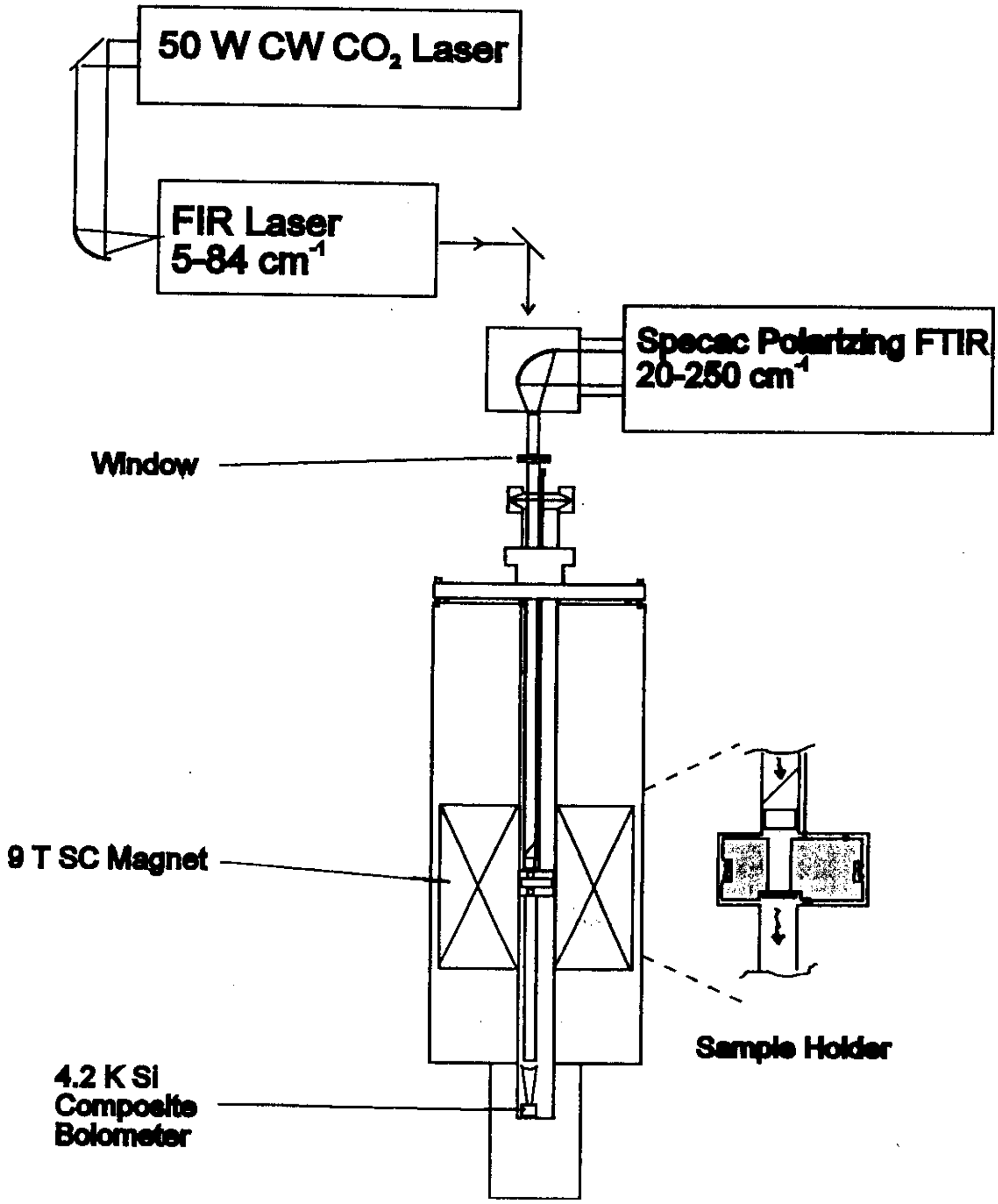
$\boxed{\text{CIRCULAR DICHROISM}} \Rightarrow \dfrac{T^+}{T^-} = \left|\dfrac{t^+}{t^-}\right|^2 \cong 1 + 4\,\mathrm{Im}\,\tan\theta_F$

for $Z\sigma_{xx} >> 1$ $\quad T << 1 \quad \tan\theta_F \cong \tan\theta_H = \dfrac{\sigma_{xy}}{\sigma_{xx}}$

$$R_H = \frac{1}{B}\frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{1}{B}\frac{\tan\theta_H}{\sigma_{xx}}$$

SUM RULE ON θ_F: $\displaystyle\int_0^\infty \tan\theta_F \, d\omega = 0$

FIR Magneto-Transmission



HIGH T_c SUPERCONDUCTORS - REVIEW

NORMAL STATE - ANOMALOUS

$\rho(T) \propto T$ NO SATURATION

$R_H(T) \sim 1/T$

OPTICAL $1/\tau \simeq \omega$ MARGINAL FERMI LIQUID

2-D LUTTINGER LIQUID : P.W. ANDERSON
R. LAUGHLIN

SPINONS & HOLONS → SPIN - CHARGE SEPARATION

SUPERCONDUCTING STATE - NO THEORY

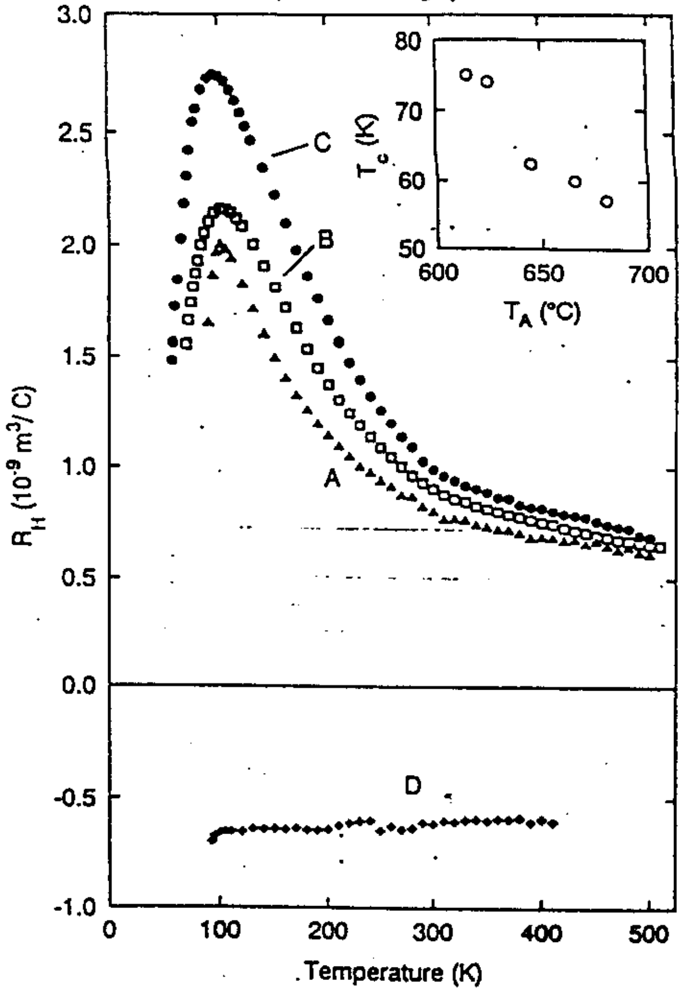
PAIRING - JOSEPHSON EFFECT

d-WAVE PAIRING - SQUID
- MICROWAVE REACTANCE $\delta\lambda_L \sim T$

TYPE II $\xi_c << \lambda_L$

VORTICES
- NEUTRON
- STM
- μ WAVE LOSSES
- d-WAVE ASPECTS ?

M. HARRIS et al., PRB 46 14293 (1998)



T. R. CHIEN et al., PRL 67 2088 (1991)

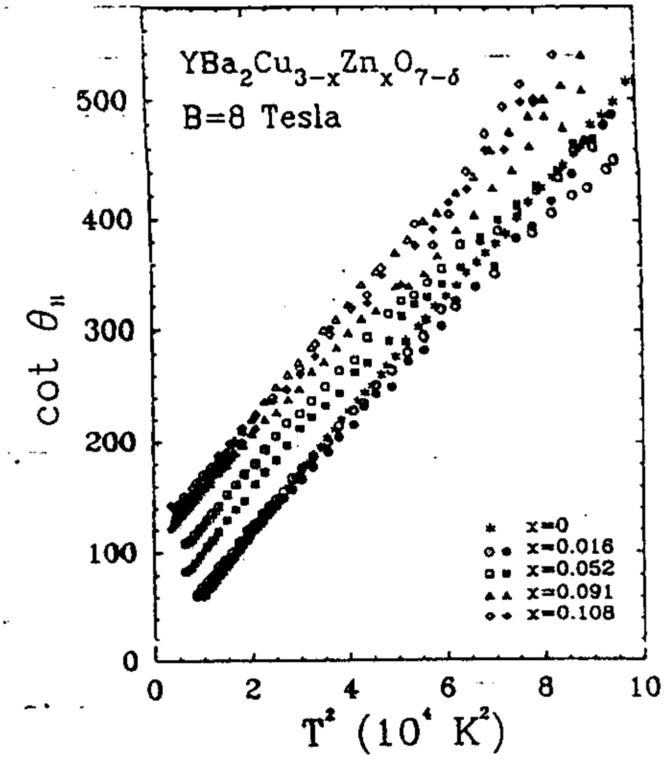
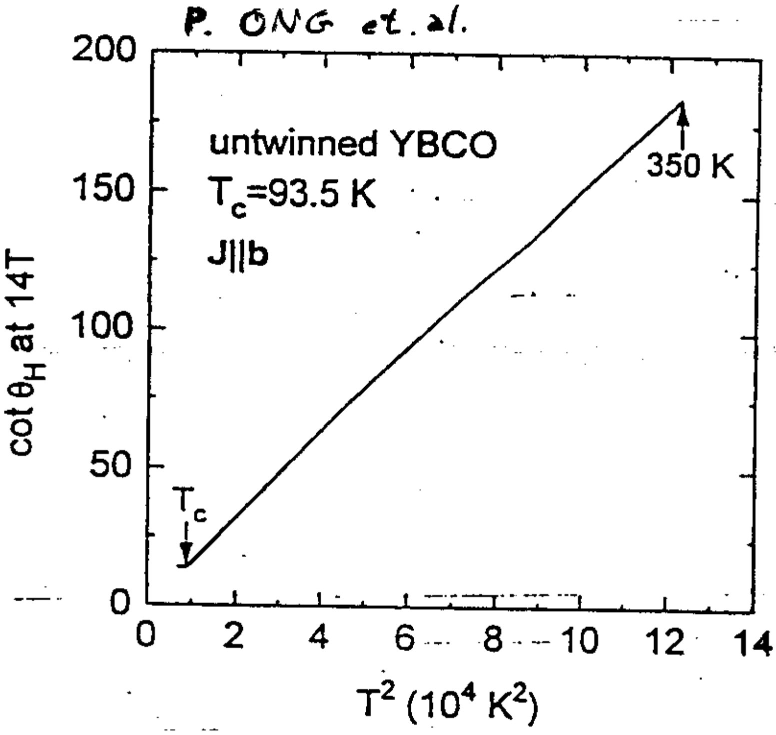


FIG. 2. Temperature dependence of the Hall angle show $\cot\theta_H$ vs T^2 for a series of Zn-doped YBCO crystals. cot computed from ρ_{xy} and ρ_{xx} measured on the same crystal. by $\cot\theta_H = \alpha T^2 + \beta x$ gives $\alpha = 5.11 \times 10^{-3}$.

P. ONG et. al.



$\cot\theta_H = \alpha T^2 + \beta X$

WEAK FIELD MAGNETO-RESISTANCE AND HALL EFFECT

2-D LUTTINGER LIQUID MODEL - P.W. ANDERSON

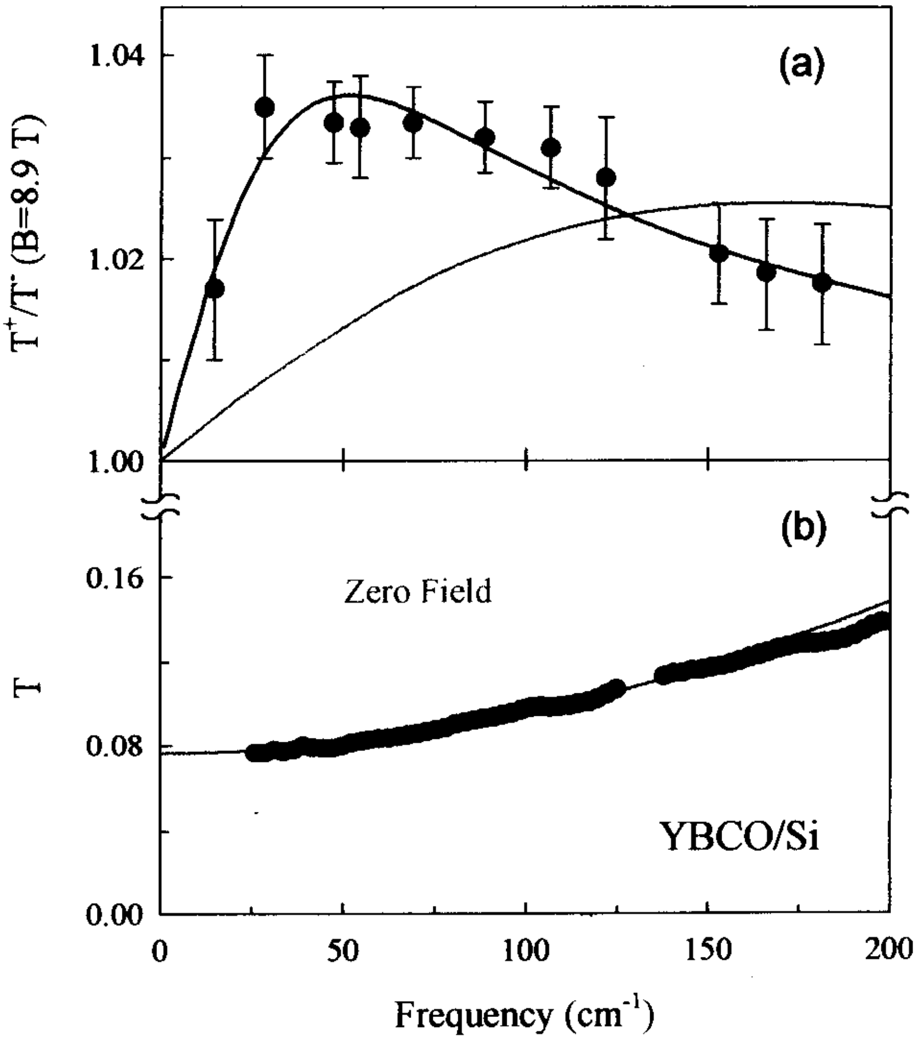
$\tau_{tr}^{-1} \sim T$ (holon)

$\tau_H^{-1} \sim T^2/W_s$ (spinon)

	NON-FERMI	DRUDE
$\sigma_{XX} = \frac{ne^2}{m}\tau_{tr}$	$\sim 1/T$	$\sim 1/T$
$\sigma_{XY} = \frac{ne^2}{m}\tau_{tr}(\omega_H \tau_H)$	$\sim 1/T^3$	$\sim 1/T^2$
$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_{XX}^2} \sim \frac{\tau_H}{\tau_{tr}}$	$\sim 1/T$	$\frac{B}{nec}$
$\cot(\theta_H) = \frac{\sigma_{XX}}{\sigma_{XY}} = \frac{1}{\omega_H \tau_H}$	$\sim T^2$	$\sim T$

$$\omega_H = \frac{eB}{m_H c}$$

Normal State Hall Effect

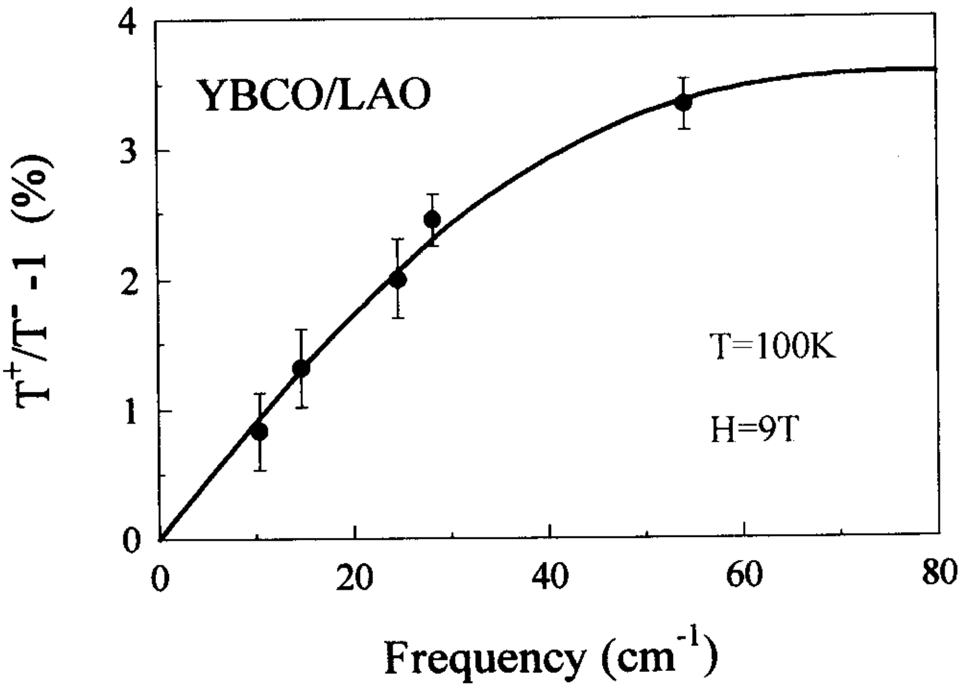


Drude model

$$\sigma(\omega) = \frac{ne^2/m}{i\omega + 1/\tau_{tr}}, \quad T \sim \frac{1}{|\sigma|^2} \sim 1/\tau_{tr}{}^2 + \omega^2$$

$$\sigma^{\pm}(\omega) = \frac{ne^2/m}{i(\omega \pm \omega_C) + 1/\tau_{tr}}, \quad \frac{T^+}{T^-} \sim 1 + \mathrm{Im}\left(\frac{\sigma_{xy}}{\sigma_{xx}}\right)$$

$1/\tau_{tr} \sim 190\ cm^{-1}$
$1/\tau_H \sim 60\ cm^{-1}$



AC Extension

$$\sigma_{xx} = \frac{ne^2}{m^*} \cdot \tilde{\tau}_{tr}$$

$$\sigma_{xy} = \frac{ne^2}{m^*} \cdot \tilde{\tau}_{tr} \cdot (\omega_H \tilde{\tau}_H)$$

$$\begin{cases} 1/\tilde{\tau}_{tr} = 1/\tau_{tr} + i\omega \\ 1/\tilde{\tau}_H = 1/\tau_H + i\omega \end{cases}$$

a.c. GENERALIZATION OF ANDERSON'S MODEL

$$\sigma_{xx} = \frac{ne^2}{m} \tilde{\tau}_{tr} \qquad \tilde{\tau}_{tr}^{-1} = i\omega + 1/\tau_{tr}$$

$$\sigma_{xy} = \frac{ne^2}{m} \tilde{\tau}_{tr}\, \omega_H\, \tilde{\tau}_H \qquad \tilde{\tau}_H^{-1} = i\omega + 1/\tau_H$$

$$\omega_H = \frac{eB}{m_H c}$$

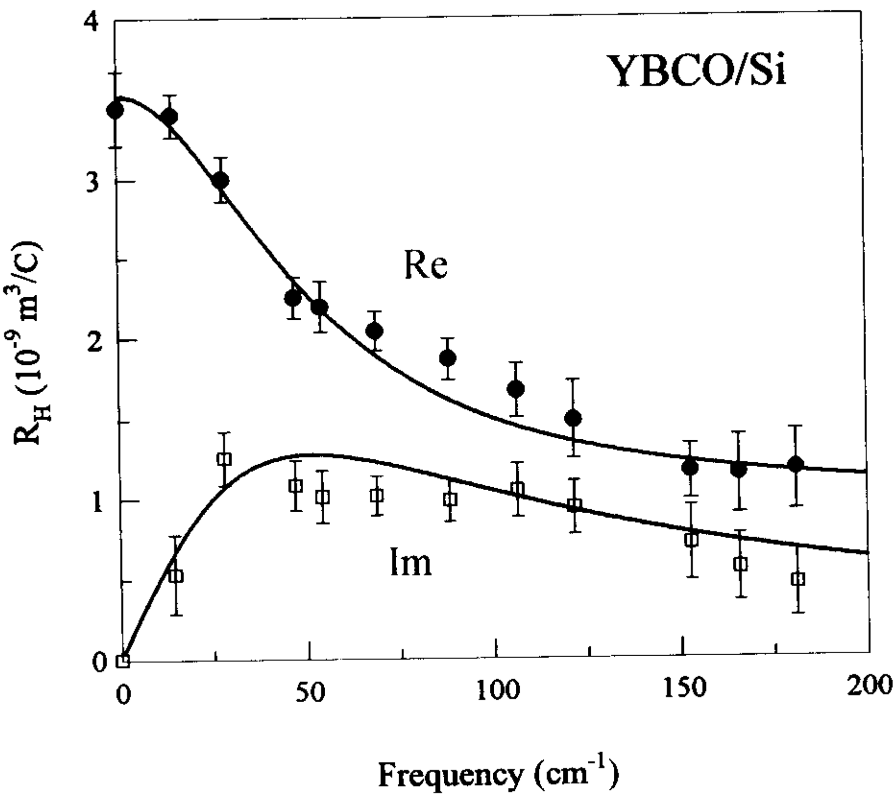
a.c. HALL COEFFICIENT

$$\rho_{xy} \cong \frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{1}{nec} \frac{m_{tr}}{m_H} \frac{i\omega + 1/\tau_{tr}}{i\omega + 1/\tau_H}$$

HALL ANGLE

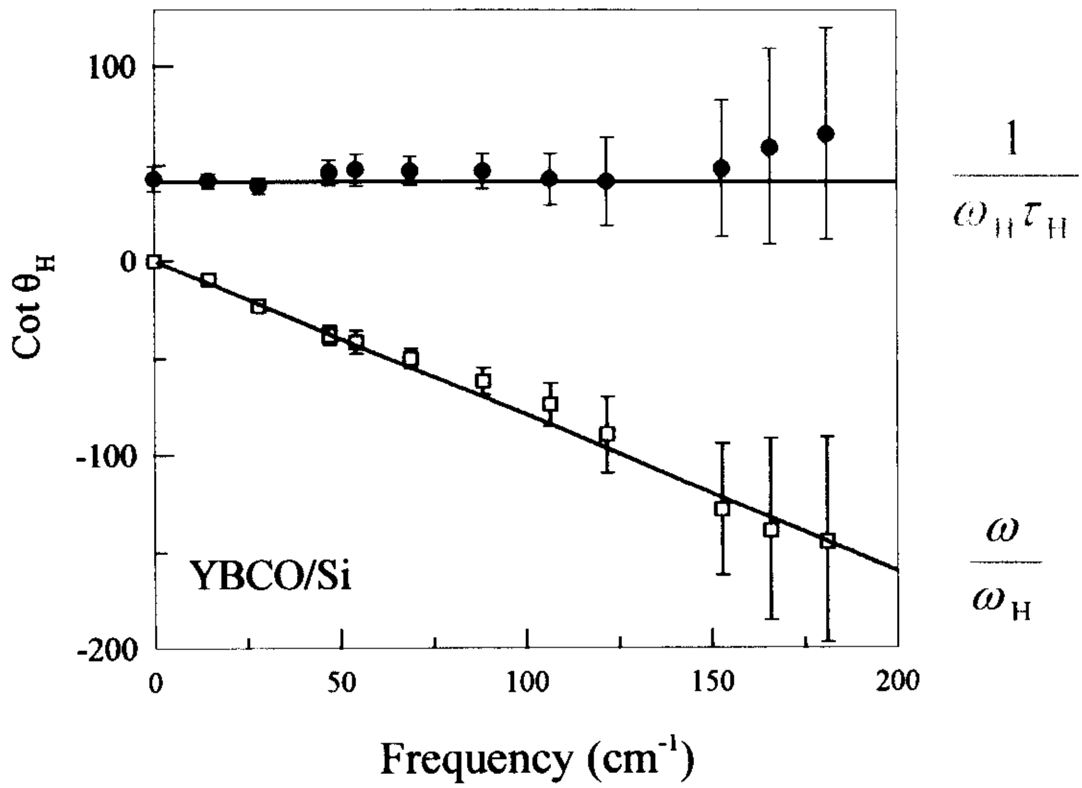
$$\cot(\theta_H) = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_H \tilde{\tau}_H} = \frac{1}{\omega_H \tau_H} + i \frac{\omega}{\omega_H}$$

Hall Coefficient



$$R_H = \frac{\rho_{xy}}{B} \qquad \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2} \sim \frac{\sigma_{xy}}{\sigma_{xx}^2}$$

Hall Angle



$$\cot(\theta_{\mathrm{H}}) \equiv \frac{\sigma_{\mathrm{xx}}}{\sigma_{\mathrm{xy}}} = \frac{1}{\omega_{\mathrm{H}}\tau_{\mathrm{H}}} - i \cdot \frac{\omega}{\omega_{\mathrm{H}}}$$

$$\omega_H \equiv \frac{eB}{m_H}$$

m_H AND τ_H

$$\cot(\theta_H) = \frac{1}{\omega_H \tau_H} + i\frac{\omega}{\omega_H}$$

$$\omega_H \equiv \frac{eB}{m_H c} \quad \Rightarrow \quad m_H \simeq 6\, m_e$$

$$\omega_H \tau_H \simeq 40^{-1} \quad \Rightarrow \quad 1/\tau_H \simeq 60\ \mathrm{cm}^{-1}$$

$$1/\tau_H \simeq T^2/W_s \quad \Rightarrow \quad W_s \simeq 120\ K$$

$$1/\tau = \frac{(\pi T)^2 + (E - E_F)^2}{\pi^2 W_s}$$

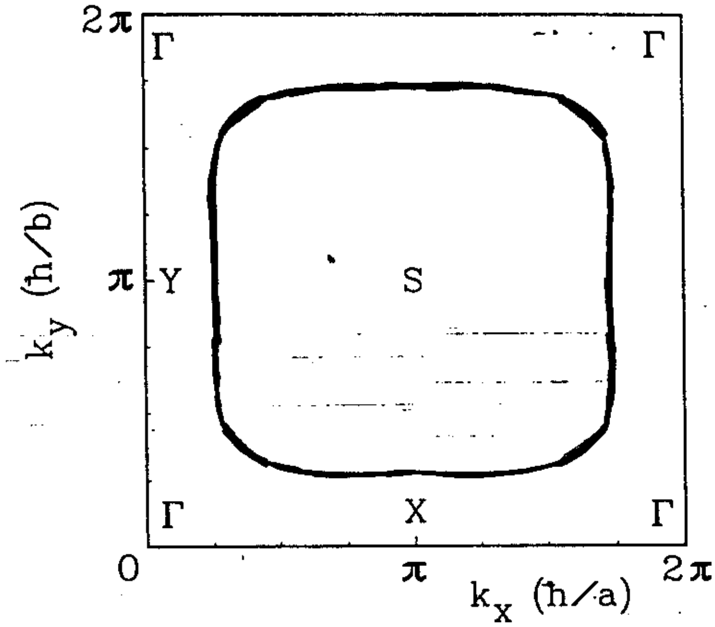
$$\pi^2 W_s \simeq 1200\ K$$

FERMI LIQUID - WEAK FIELD HALL EFFECT

$$\sigma_{xx} = \frac{e^2}{\pi h}\frac{1}{d}\oint dk\,|v|\tilde{\tau} \qquad 1/\tilde{\tau} = i\omega + 1/\tau$$

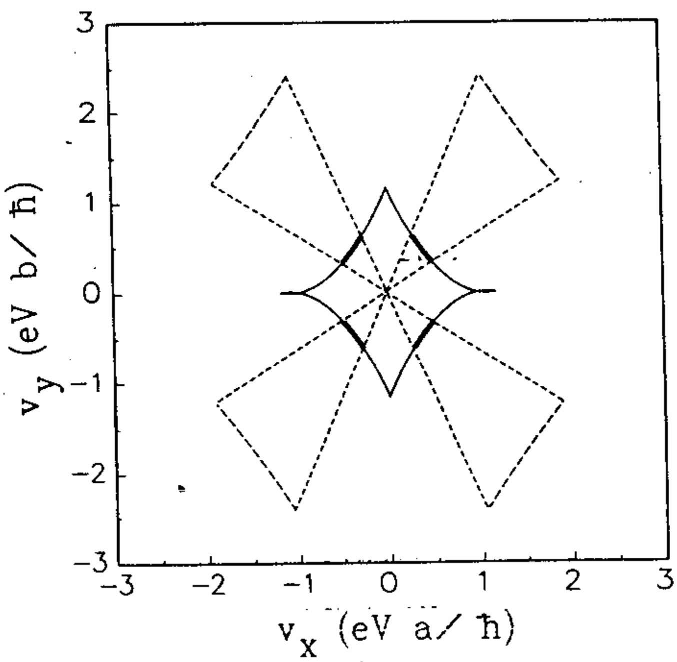
$$\sigma_{xy} = \frac{eB}{\hbar c}\frac{e^2}{h}\frac{1}{d}\oint dk\, v\tilde{\tau}\times d(v\tilde{\tau})/dk \propto \oint d(v\tilde{\tau})\times v\tilde{\tau}$$

FERMI SURFACE



TIGHT BINDING MODEL OF BONDING BAND O.K. ANDERSON et al. PRB 49 4145 (1994) & ARPE - M.C. SCHABEL

VELOCITY SURFACE



HOT SPOTS NEAR $(\pi, 0)$

ac & dc MAGNETO TRANSPORT

B ⊥ ab

B ∥ ab ; N.E. HUSSEY et al PRL 76 122 (1996).

NAFL MODEL

STOJKOVIC & PINES PRB 55 8576 (1997)

FERMI LIQUID – 2τ MODEL

W/ V. YAKOVENKO
A. ZHELEZNYAK

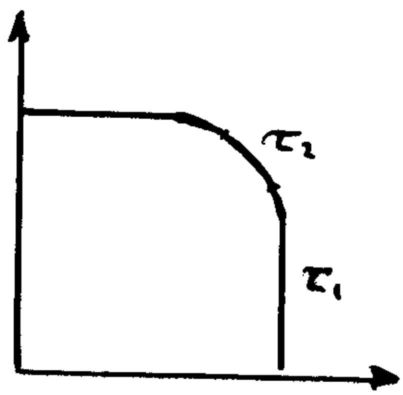
$$\sigma_{xx} = \frac{\omega_p^2}{4\pi}\left[a_1 \tilde{\tau}_1 + a_2 \tilde{\tau}_2\right], \quad a_1 + a_2 = 1$$

$$\tilde{\tau} = \frac{1}{i\omega + 1/\tau}$$

$$\sigma_{xy} = \frac{\omega_p^2}{4\pi}\omega_H\left[b_1 \tilde{\tau}_1^2 + b_2 \tilde{\tau}_2^2\right], \quad b_1 + b_2 = 1$$

$$a \sim \int dk_t\, |v| / \omega_p^2$$

$$b \sim \int dk_t\, v \times dv/dk_t \,/\omega_H$$



GENERAL : 6 PARAMETERS

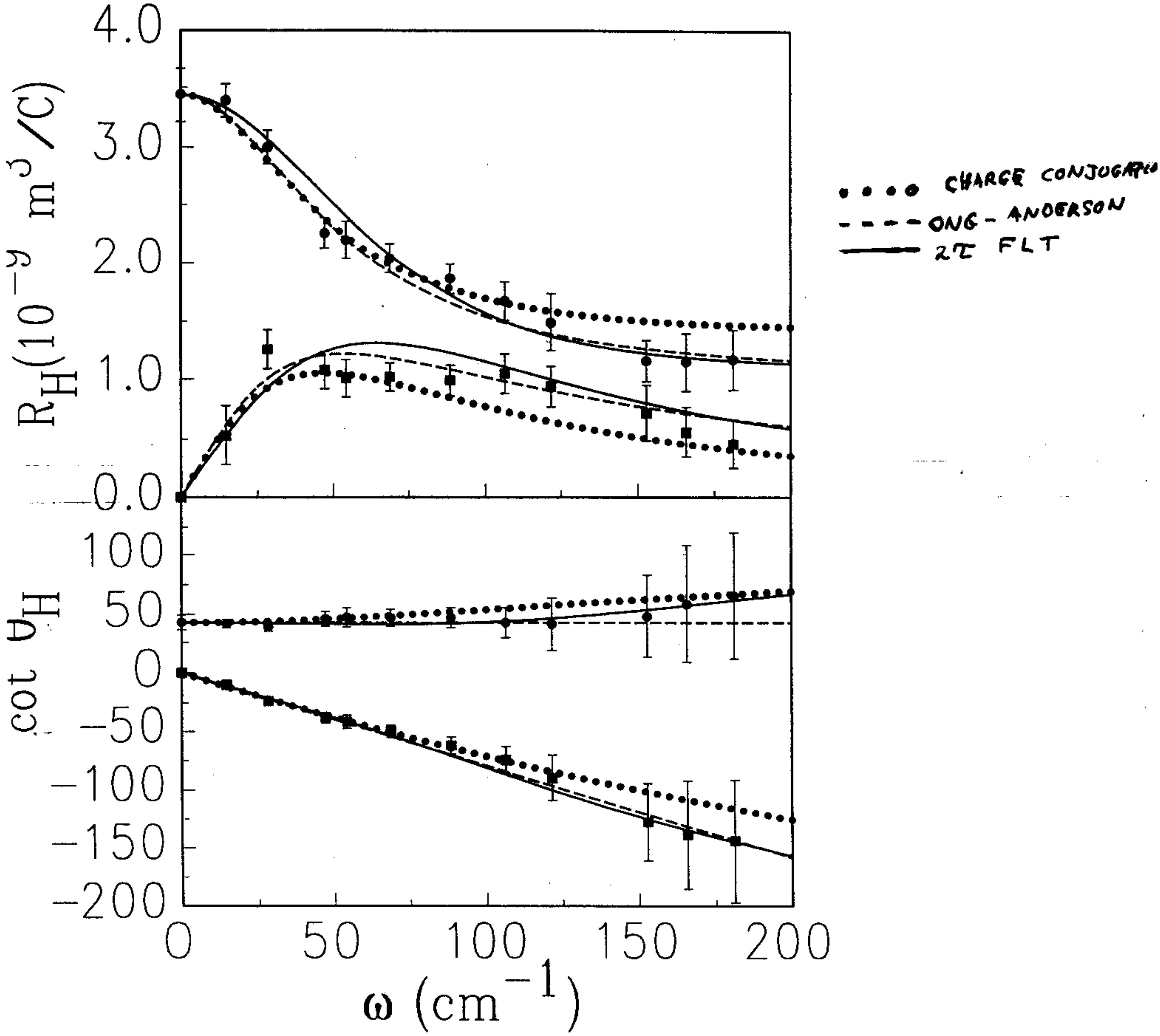
$\omega_p, \omega_H, a_1, b_1, \tau_1, \tau_2$

BAND STRUCTURE $\Rightarrow$ RELATION BETWEEN a_1 & b_1

$$\mathrm{Im}\cot\theta_H \propto \omega \quad \Rightarrow \quad \frac{\tau_2}{\tau_1} = \frac{2 - 1/b_1}{2 - 1/a_1}$$

ONG-ANDERSON MODEL : 4 PARAMETERS ; $\omega_p \; \omega_H \; \tau_1 \; \tau_2$

$\mathrm{Im}\cot\theta_H \propto \omega$


4.0
3.0
2.0
1.0
0.0
$R_H (10^{-9}\ m^3/C)$
100
50
0
−50
−100
−150
−200
$\cot \vartheta_H$
0
50
100
150
200
ω (cm^{-1})
CHARGE CONJUGATION
ONG - ANDERSON
2τ FLT

ADDITIVE 2τ MODEL

SCALE FACTORS: ω_p & ω_H

$\omega_p^2 \propto \oint dk\,|v|$ $\qquad$ $\left(\dfrac{\omega_p^{2\tau}}{\omega_p^{exp}}\right)^2 \simeq 4$

$\omega_H \propto \oint dv \times v$

$$\frac{R_H^{2\tau}}{R_H^{exp}} \simeq \frac{\dfrac{\omega_H^{2\tau}}{\omega_H^{exp}}}{\left(\dfrac{\omega_p^{2\tau}}{\omega_p^{exp}}\right)^2} \simeq 0.6$$

FREQUENCY DEPENDENCE OF $\cot\theta_H$, R_H –

$\cot\theta_H = \dfrac{i\omega + 1/\tau}{\omega_H}$ $\qquad$ ACCIDENTAL

TEMPERATURE DEPENDENCE OF ρ, $\cot\theta$ –

$\cot\theta_H = A T^2$ $\qquad$ ACCIDENTAL

OTHER MODELS

LEE & P.A. LEE (PREPRINT)

SPIN-CHARGE SEPARATION

$$\theta_H = \frac{\bar{n}(T)\, eB/mc}{i\omega + 1/\tau}$$

ONLY PHYSICAL HOLE RESPONDS TO B

$\tau_{hole} \to \infty$ AS $T \to T_c$

θ_H SUM RULE VIOLATED

$R_H(\omega)$ CONSTANT

KOTLIAR, SENGUPTA & VARMA PRB 53 3573 (1996).

SKEW SCATTERING

$$\theta_H = \frac{\omega_c}{i\omega + 1/\tau_{tr}}\left[a + \frac{b}{E_F \tau_s}\right]$$

$\tau_s \propto 1/T$

θ_H SUM RULE VIOLATED

$R_H(\omega)$ CONSTANT

COLEMAN, SCHOFIELD & TSVELIK PRL 76 1324 (1996).

CHARGE CONJUGATION MODEL

$$\sigma_{xx} = \frac{\omega_p^2}{2\pi} \frac{1}{(\tilde{\Gamma}_f(\omega) + \tilde{\Gamma}_s(\omega))}$$

$\tilde{\Gamma}(\omega) = i\omega + \Gamma$

$$\sigma_{xy} = \frac{\omega_p^2}{4\pi} \omega_H \frac{1}{\tilde{\Gamma}_f(\omega)\, \tilde{\Gamma}_s(\omega)}$$

REQUIRES COOPER PAIRING WITH FINITE MOMENTUM

DOESNT FIT DATA AS WELL AS ONG-ANDERSON

STOJKOVIC & PINES PRB 55 8576 (1997)

NAFL MODEL

$\cot\theta_H \sim T^2$ COMES FROM DETAILS OF T DEPENDENCE OF τ_k

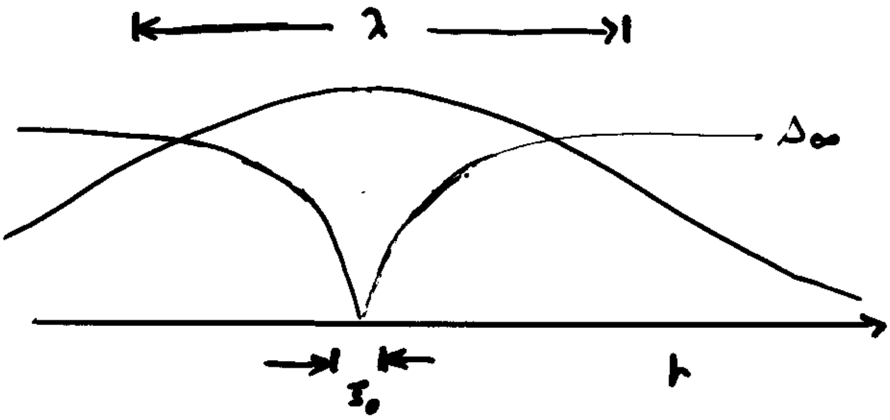
$\sim$ ADDITIVE 2τ MODEL

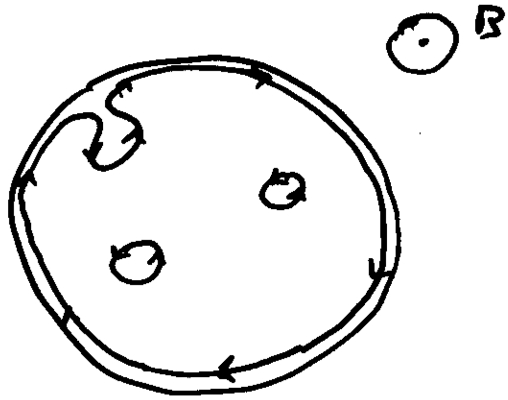
TYPE II SUPERCONDUCTORS $\quad B \neq 0$

COHERENCE LENGTH — radius of Cooper pair

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta} \qquad \xi_0 \simeq 500\,\text{Å} - 15\,\text{Å}$$

TYPE II $\quad \xi_0 \ll \lambda$

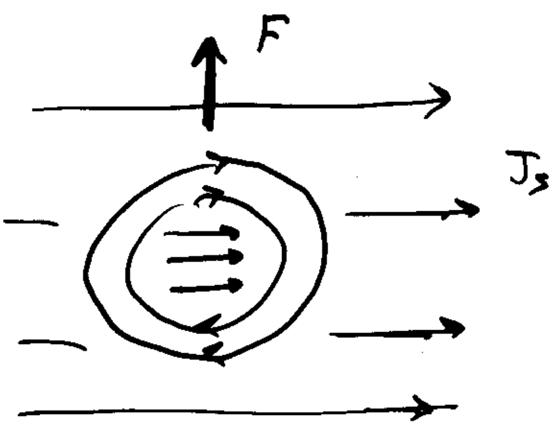




flux line

$$\Phi_0 = \frac{hc}{2e}$$

VORTEX DYNAMICS



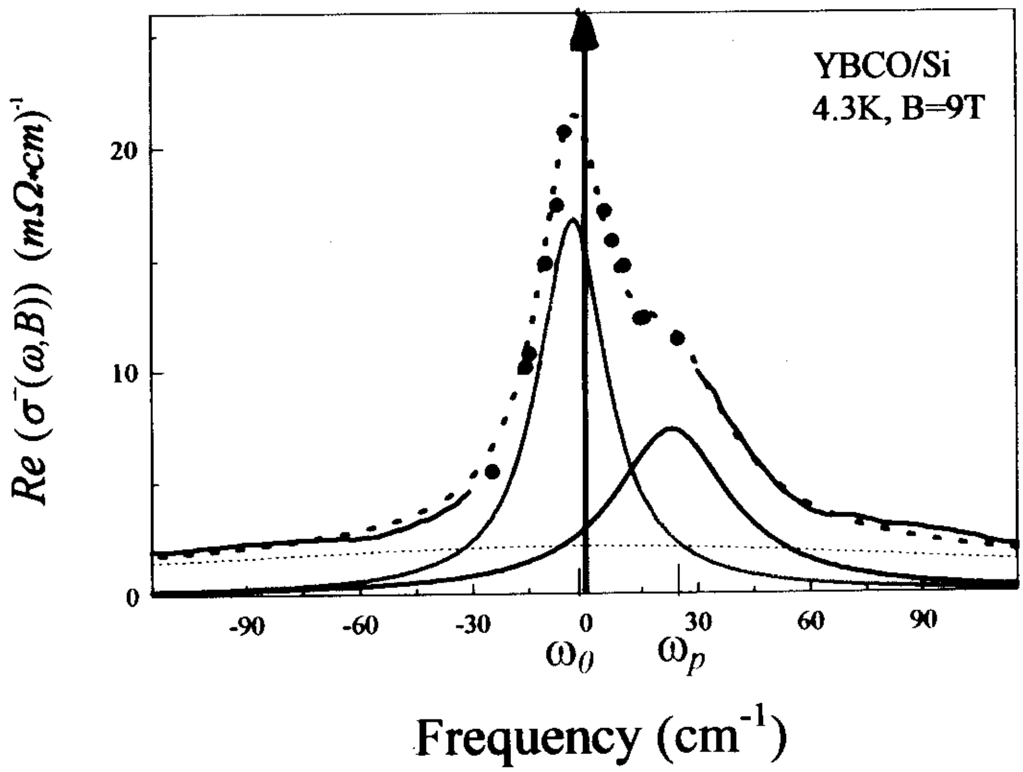
$$F = \frac{1}{c} J_s \times \hat{\Phi}_0 \qquad \text{Lorentz-Magnus}$$

$$E = -\frac{1}{c} v_v \times B \qquad \text{Faraday-Josephson}$$

$$E \cdot J_s \geq 0 \qquad \text{DISSIPATION}$$

Joule heating of core

Magneto-Conductivity



Model: Two Lorentzian oscillators + London term

$$\sigma^{\pm} = \frac{ne^2}{m^*}\left[\frac{f_s}{i\omega} + \frac{f_0}{i(\omega \pm \omega_0) + \Gamma_0} + \frac{f_p}{i(\omega \pm \omega_p) + \Gamma_p}\right]$$

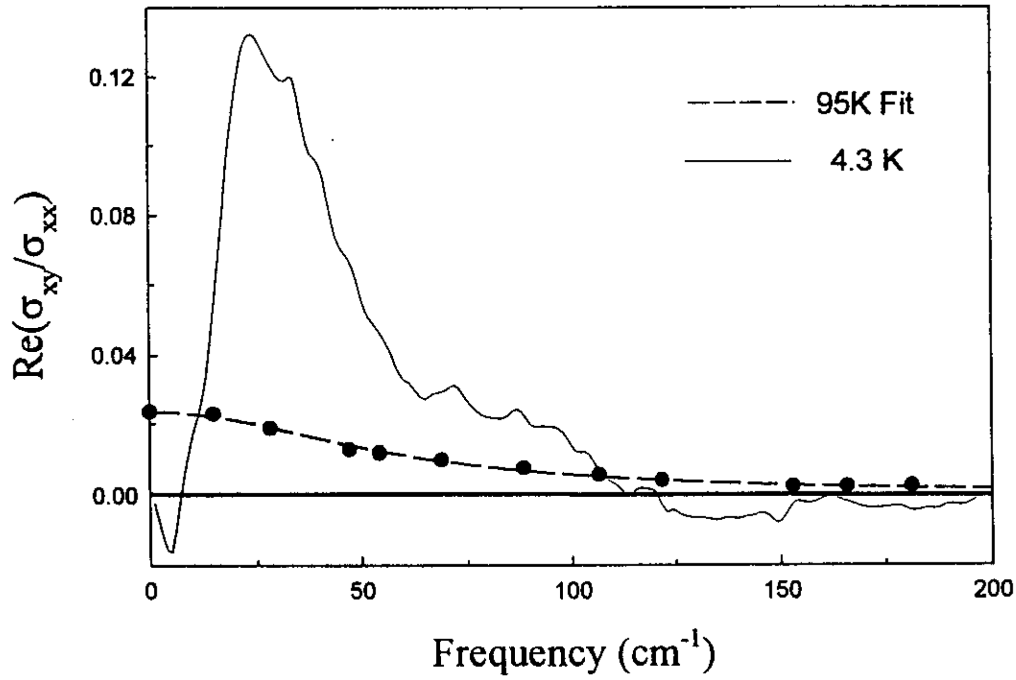
ω_p -- **Pinning frequency ~25cm^{-1}**

Γ_0 -- **Depinning frequency due to damping**

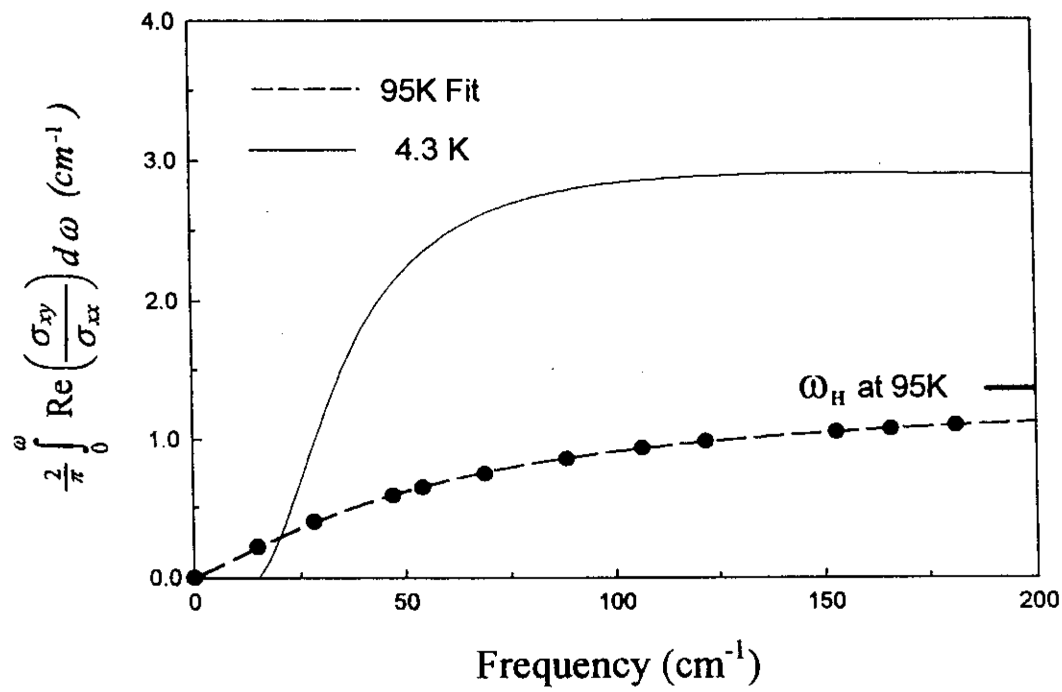
ω_0 -- **Due to Hall force ~ 0 cm^{-1}**

Lihn et al. PRL 76 3810 (1996)

YBCO/Si, H=9T



$$\theta_H' = \frac{\omega_H \tau_H}{1+(\omega\tau_H)^2}$$



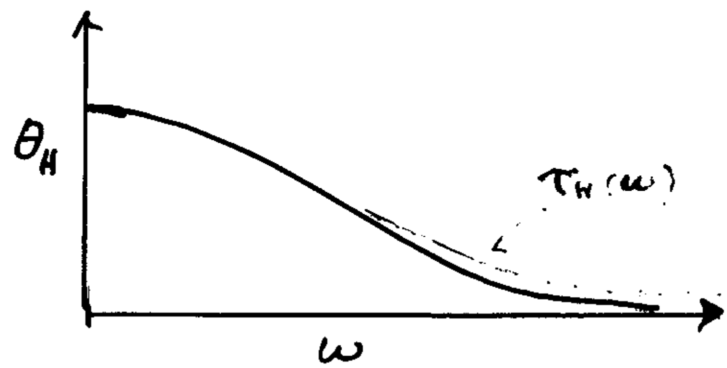
m_H RENORMALIZATION

$$1/\tau_H = \left[(\pi kT)^2 + (\hbar\omega)^2\right]/E_0$$

$\Rightarrow$ high frequency tail in $\theta_H = \dfrac{\omega_H}{i\omega + 1/\tau_H}$

renormalizes m_H

fixes θ_H sum rule ?



$$\theta_H = \frac{\omega_{sc}}{i\omega + M(T,\omega)} \xrightarrow[\omega \to 0]{} \frac{\omega_H}{i\omega + 1/\tau_H}$$

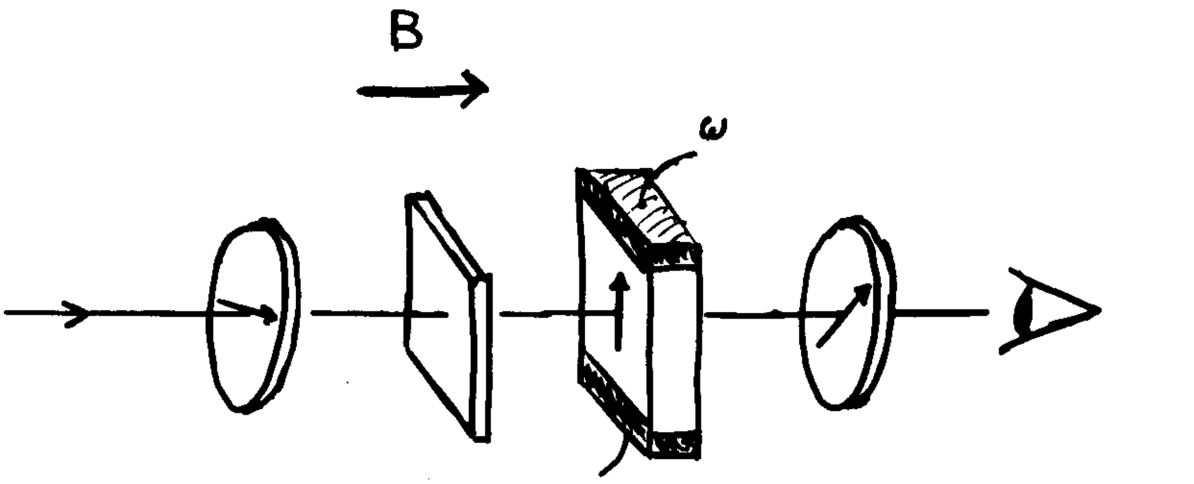
$$M(T,\omega) = \frac{T^2}{W_s} + \lambda_0 \frac{\omega_0 \omega}{\omega - i\omega_0}$$

$$\frac{\omega_{sc}}{\omega_H} = 1 + \lambda_c \qquad \lambda_c \Rightarrow 1.1$$

$$\lim_{\omega \to 0} \frac{M(T,\omega)}{1 + \lambda_c} = \frac{1}{\tau_H} = \frac{(\pi T)^2 + (\hbar\omega)^2}{\pi^2 W_s}$$

$\omega_c \doteq$ [illegible] K

POLARIZATION MODULATION



$$\frac{\Delta T}{T_0} = 2\left[\mathrm{Re}(\tan\theta_F)\, J_2(\beta)\cos 2\omega t + \mathrm{Im}(\tan\theta_F)\, J_1(\beta)\cos\omega t\right]$$

$$\beta = 2\pi \frac{d}{\lambda} \Delta n$$

CONCLUSIONS

1. ac Hall effect provides powerful new probe of strongly interacting electron systems.

2. ac Hall effect discriminates between models of the normal state of the cuprates.

3. A sum rule governs the spectral distribution of the Hall angle.

4. The Hall angle sum rule is found to be violated over the measured frequency range ($\omega \leq 25\, meV$) when the superconducting and normal state responses are compared - implying additional magneto-physics at higher energies.