



UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL ATOMIC ENERGY AGENCY
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/1001-10

***IX TRIESTE WORKSHOP ON
OPEN PROBLEMS IN
STRONGLY CORRELATED SYSTEMS***

14 - 25 July 1997

***THE FUNDAMENTAL QUESTION OF
METALLIC BEHAVIOR IN 2D***

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These are preliminary lecture notes, intended only for distribution to participants.

FUNDAMENTAL PROBLEMS IN THE $H_1 T_c$ SYSTEM

Acknowledgements

G. Baskaran

S. Strong

D. Kuzshchenko

D. G. Clark

FDM Haldane

S. Sanjay

Experimenters: One, Br. Koss,
Schlesinger, Keimer, etc.

- I. THE MOST BASIC QUESTION:
IS $D \gg 1$ Hubbard a FERMION LIQUID
- (a) Elementary point of view:
Kugel phenomenon, Hartree-
Landau int, PHASE shifts
- (b) Baskaran's Point: How DOES
P.T. FAIL? - ROLE OF ZERO
sound.

II. Some Thoughts about
Quantum Critical Points (and
their absence)

III PHASE DIAGRAM OF H_2O T_c :

CROSSOVER 2D $\rightarrow$ 3D ON A CRITICAL
LINE.

A TEXT BOOK STORY

BAND THEORY : LDF 1-Electron
(electrons and phonons) myth.

10 "STANDARD MODEL" : CORRECT " BY

PERT. THEORY : RENORMALIZED QUASI-
PARTICLES

"BRAIN WASHED BY FERMION" (and
LANDAU

(Gan) Fermi Liquid Theory + B.C.S. (and
Schrieffer)

He³ THE LAST SUCCESS!

(well, maybe weak localization,
1970-75
1978-85

+ 1996!

4 ~~THE~~ 3 DECADES OF FAILURE TO GET
FURTHER Net-Ins Trans
heavy electrons CeCu_2Si_2
organics BEDT-TSF
bucky balls
"bad actors" a_{1g}
+ 1 success ENTIRELY OUTSIDE
G.F.I.T.!

QHE, FQHE : JASTROW (VARIATIONAL)
METHOD, NO PERT. TH.

$$\Psi = \prod_{i,j} (z_i - z_j)^{m_{ij}} e^{-\sum |z_i|^2}$$

$$z = x + iy$$

IV. H1 T_c : A GREAT DEVELOPMENT
NO WAY GFLT;

(INTERLUDE FOR DEMONSTRATION)

III 1D AS MODEL.

TONONAGA, LUTTINGER '50;
 EARLY EXACT SOL.

LIEB-WU (BETHIE, YANG ETC.)
 (HUBBARD MODEL, BETHIE METHOD,

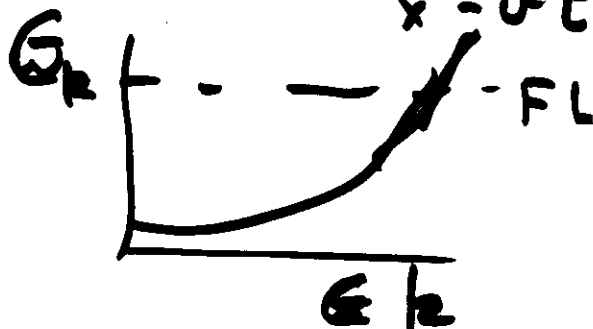
BOSONIZATION (TONONAGA, LUTT. 1970)

What happens and why?

FEYNMAN "PROPAGATOR"

$$G = \langle \psi(0,0) \psi^\dagger(x,t) \rangle$$

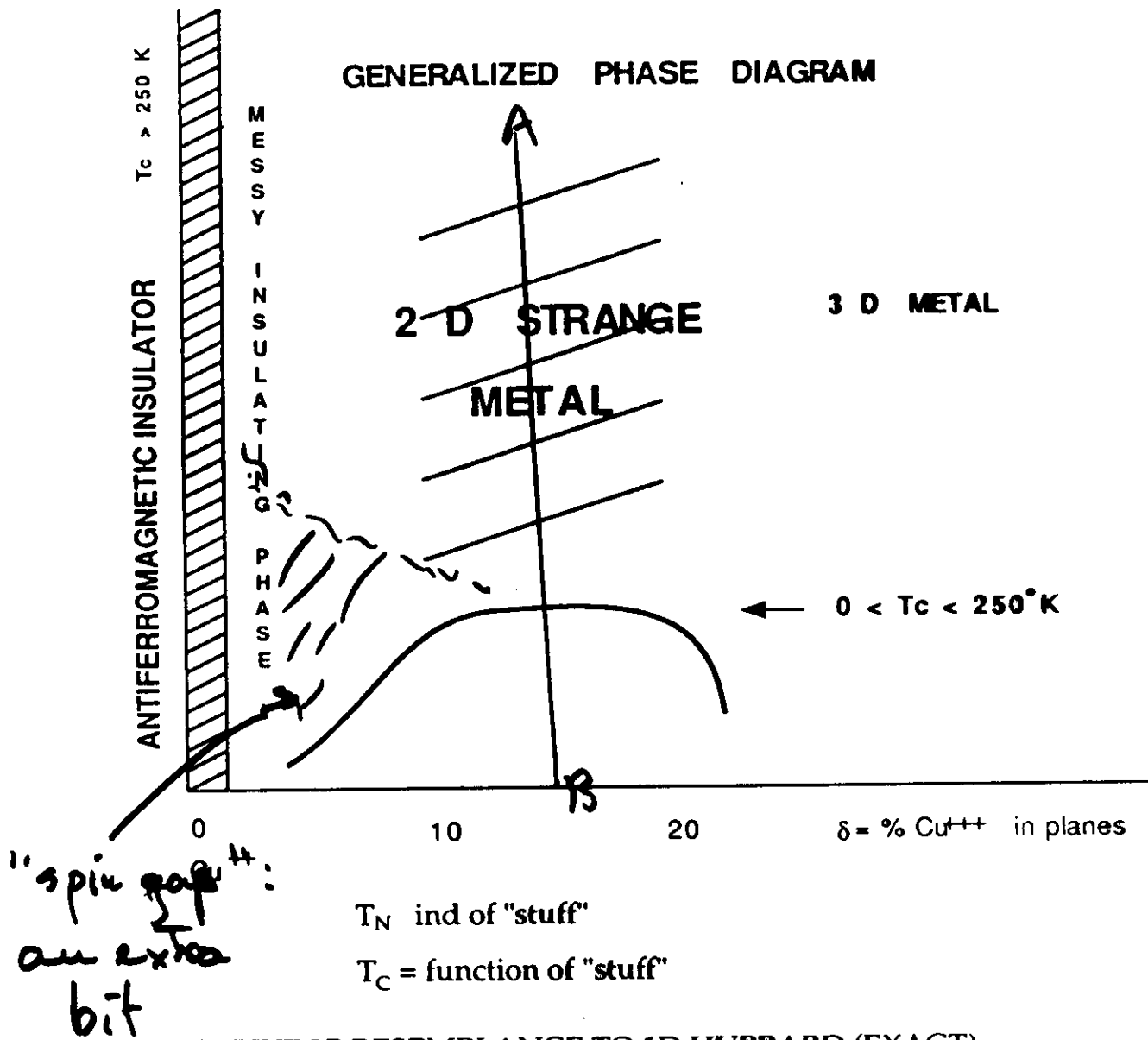
FREE : $\approx \frac{1}{x - vt}$ $\leftrightarrow \frac{1}{\omega - \epsilon_k + i(0)}$



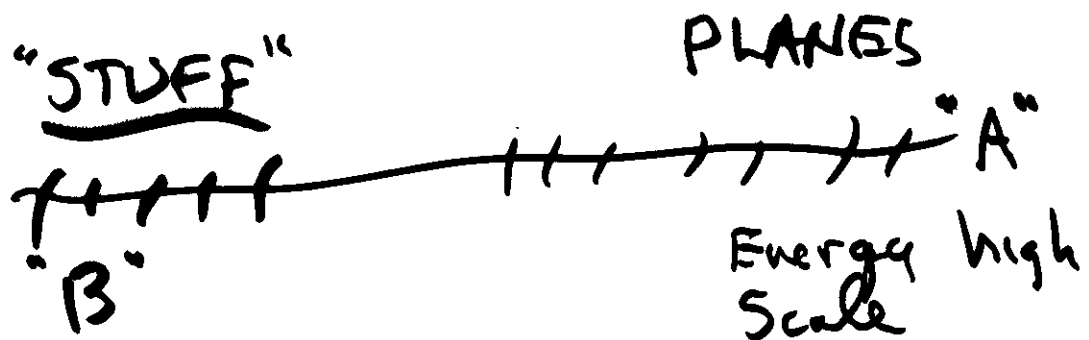
RENORM:

$$\omega - \epsilon - \Sigma(k, \omega)$$

Figure 4

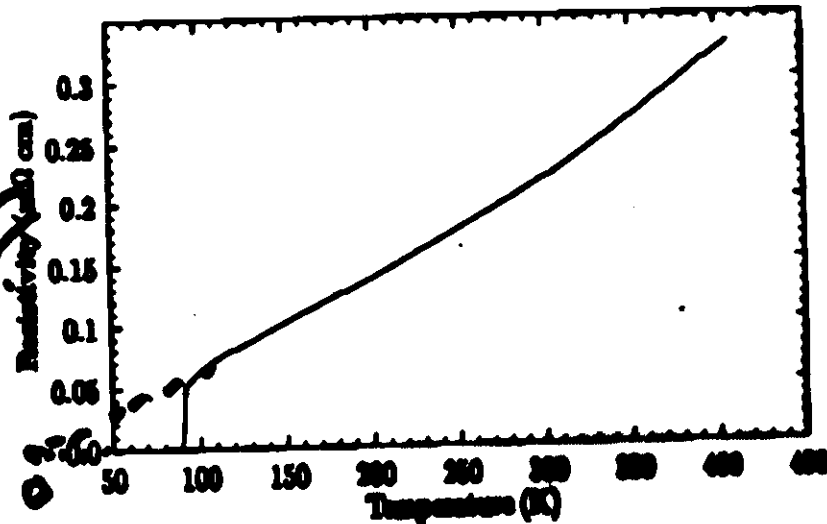


STRIKING RESEMBLANCE TO 1D HUBBARD (EXACT)



$$\frac{1}{\rho} \propto T$$

Resistivity of Single Crystal $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$

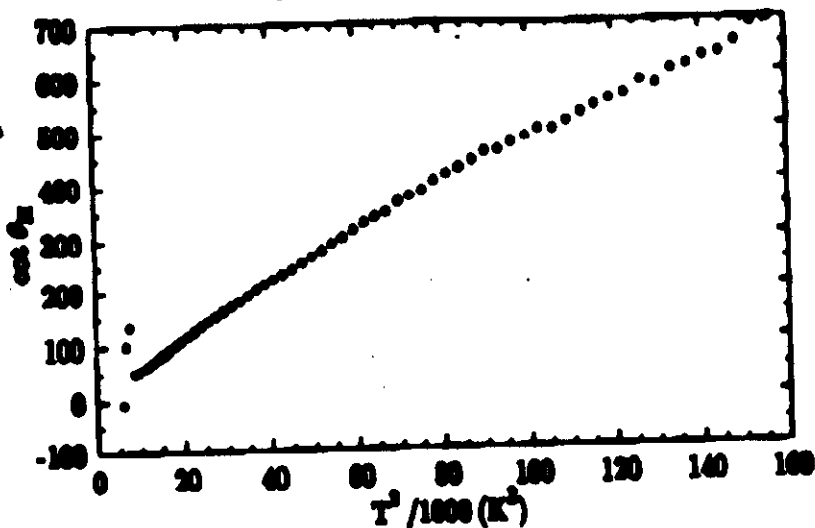


AN
ABSOLUTE
CONTRA-
DICTION!

$$\rho_H \propto \omega_c \tau$$

$$= \frac{l}{l_H}$$

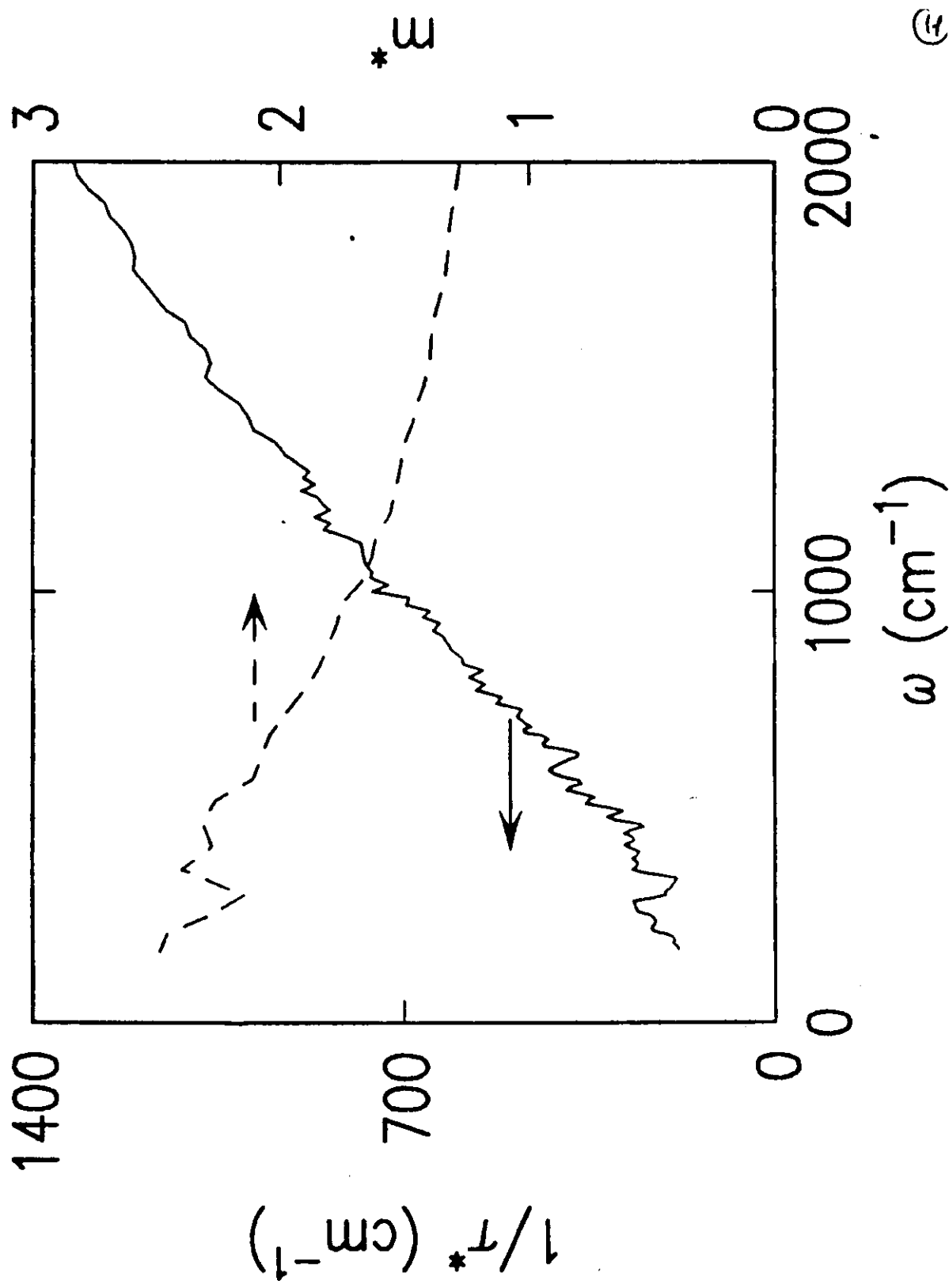
Hall Angle of Single Crystal $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$



$$\rho_H \propto T^2?$$

- (i) The unusual linear resistance of the cuprate superconductors in the normal state
- (ii) The unusual quadratic temperature dependence of the inverse Hall angle. These two pieces of data suggest two types of excitation with linear and quadratic temperature dependences of their scattering rates.

④

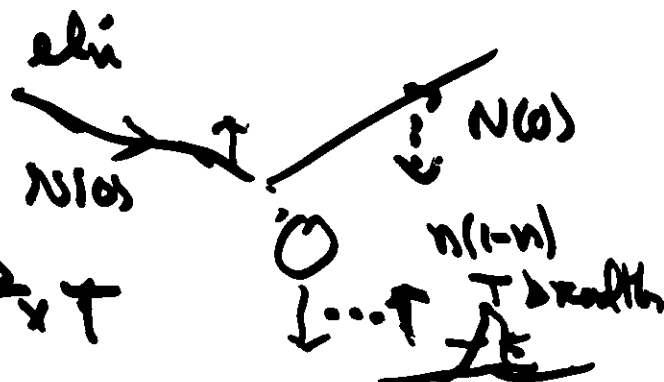


also:
(THERMOCOUPLE : STAFFORD)

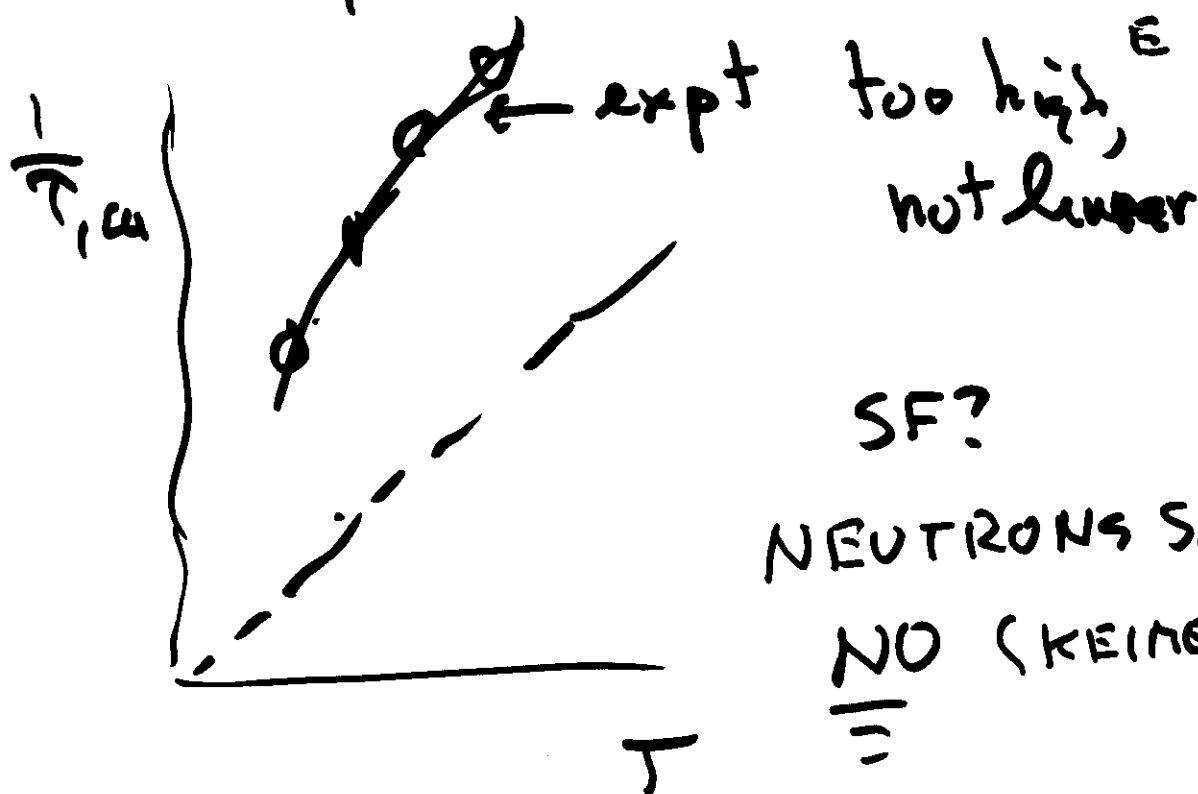
ANOTHER NFL CHARACTERISTIC:

NON-KORRINGA SPIN RELAXATION
FOR CU.

KORRINGA :



$$\frac{1}{T_1} \propto N(0)^2 \times T$$



VERY FUNDAMENTAL: PROVES

- (A) EXISTENCE OF F.S.: NO DIPOLAR ^{etc}
- (B) ANOMALOUS SPIN-SPIN CORR

$$\Sigma (\omega \rightarrow E_F) \rightarrow \boxed{\text{real}} \quad \text{particle lifetime} \rightarrow \infty$$

ACTUALLY, IN 1D NOT SO

> PINNING NFL

$$\frac{1}{(\omega - v_F t)^{1+\alpha}} \leftrightarrow \frac{1}{(\omega - E_F)^{1-\alpha}}$$

(actually $\frac{1}{v \cdot v_F}$ ~~group~~)

ANALYTIC FORM CHANGES!

Σ MEANINGLESS!

BUT STILL FERMION SURF (LUTT) _{TH}
 EVERY RESPONSE HAS NEW POWERLAW.
 (all "vertex corrections" diverge!)

SECOND PHENOM.

SPIN-CHARGE SEPARATION

1 SYMMETRY (U(2) CONS OF PARTICLES;
 $\rightarrow 2$ (U(1) $\times$ SU(2)) OF CHARGE AND SPIN)

Electron BREAKS APART

$$G = \frac{1}{(\omega - v_c t)^{1/2}} \frac{1}{(\omega - v_s t)^{1/2}} \frac{1}{(\omega - v_o t)^{1/2}}$$

(NOTE: DOES THIS MEAN ALL
1D EXPTS UNDERSTOOD?)

NO! WEIRD SOCIOLOGY!

EXPT: "MANY BIZARRE PHENOM."

THEORY: "OUR SOLUTIONS ARE EXACT."

COMMUNICATION: ALMOST ALL

(ONLY NOW BEGINNING (STRONG-
CLARKE)
(except polyacetylene)
and chiral case: QHE)

WHAT GOES WRONG? (IN PARALLEL:
(KRESENKO & BASKARAN) (1D + 2D)

OLD PARADOX DUE TO KRODEL'

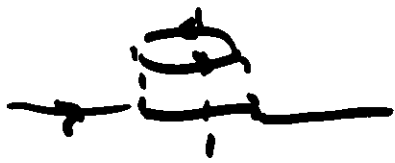
RECENT DISC. BY P NOZIERES ^{et al}

BREAK DOWN OF SIMPLEST PART

OF PERT. THEORY: HARTREE
MEAN
FIELD.

LANDAU FLT: Quasiparticles k, σ
DO NOT DECAY BECAUSE

DENSITY OF FINAL STATES $\rightarrow 0$
at FERNI SURFACE



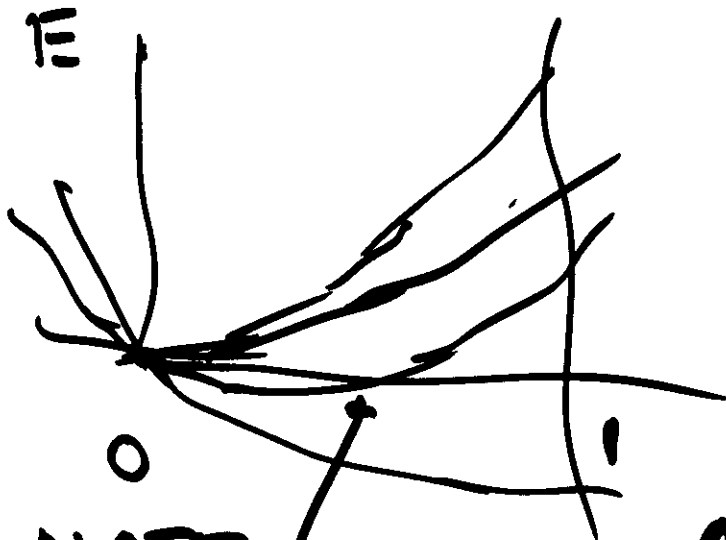
$$\Sigma_{im} \sim \omega^2 = (E - E_F)^2$$

SO ONLY INTERACTION AVERAGE
POTENTIAL (ϕ (random)) in Feynman terms

$$E_{\text{tot}} = \sum_k \epsilon_k n_{k\sigma} + \sum_{k,k'} f_{kk'} n_k n_{k'}$$

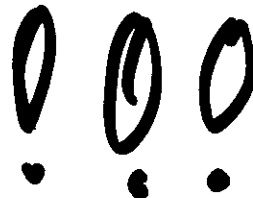
if f_{kk} finite

$$E_{\text{tot}} \sim \epsilon n_k + f_{kk} n_k^2$$



NOTE

$$0 < n_k < 1$$



even worse if $f < 0$! (kholes)

SO HOW DOES P.T. WORK AT ALL?

SWINDLE : LEGISLATE $\sum_{k'} f_{kk'} = 0$
BY ADJUSTING E_F .

NOT LEGITIMATE !!
(doesn't make $f_{kk} = 0$, necessarily!)

IN 1D CANT SET $f_{kk} = 0$

THEOREM (WICKER, DEWITT, ET AL)

$$\delta E = \frac{\eta}{\pi} \times \Delta E$$

on-shell phase shift in channel

separation of levels in channel

IN 1D $\eta \neq 0$

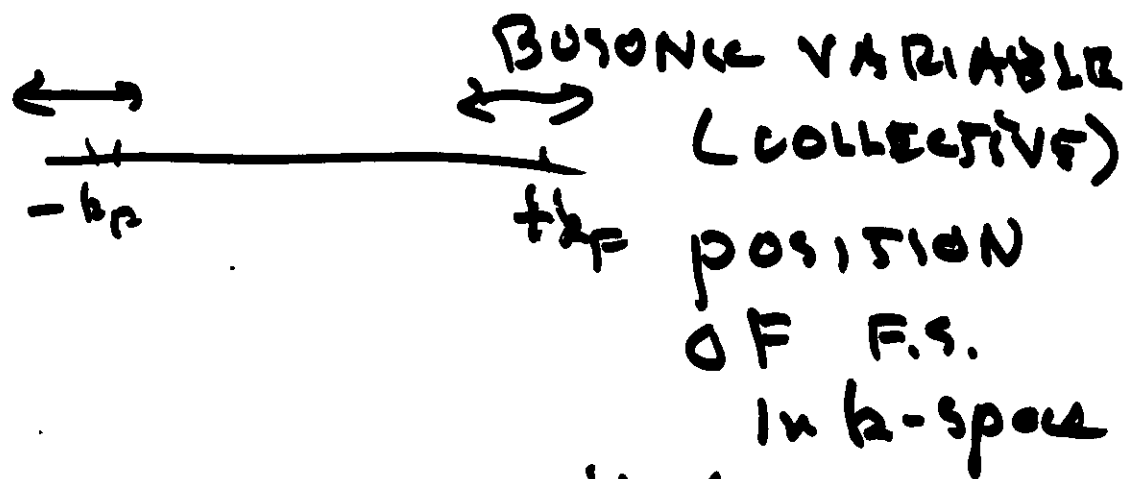
4

always a slope change

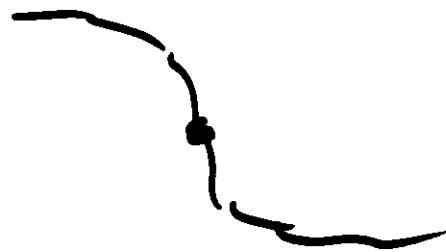
0 $r-r'$

WHAT DO WE DO?

"BOSONIZE" the particles

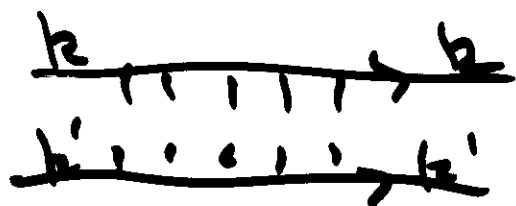


Result as described: QUANTUM
FLUCTUATION OF F.S. DIVER-
GENT BUT CONTROLLED.



WHAT ABOUT HIGHER DIM.? SETTING HARTREE FIELD $= 0$ NEVER
LEGIT, BUT f_{HL} can be $= 0$

for another reason: RECOIL



SOLVE SCHRÖDINGER
EQ'N IN REL.
COORDS.

"Effective Range" in 3D $\eta = Q^2 a$

$a = \text{finite in 3D}$ $\frac{1}{Q \ln Q}$ in 2D

$$Q = k - k'$$



BUT NOT REALLY SO IN $2D$

$2D$: EXCLUSION OF
SOFT RECOILS

EFFECTIVE "MASS" OF
SCATTERER $\rightarrow \infty$: Fixed
IN F.S.

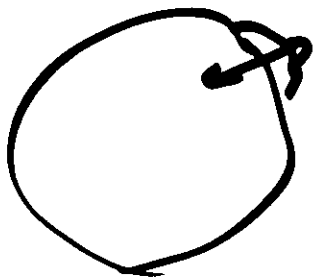
so $\eta_{2D} \neq 0$

$\eta_{3D} = 0$ FOR WEAK SCATT.

SO IN $2D$ MUST BOSONIZE.

TECHNIQUE (LUTHER, ITALDANE,

---)

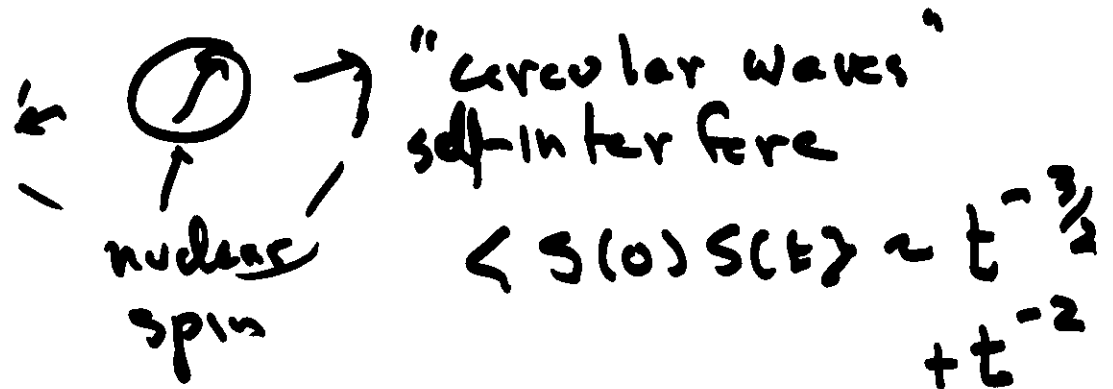


MAKE BOSONS ON
"PITLH" OF F.S.
EFFECTIVE 1D THEORY
FOR SMALL AREA OF F.S.

SO SOLUTION OF S.I. 2D ITINERANT
MODEL IS A NFL

THIS EXPLAINS NFL BEHAVIOR

(1) NON-KORRINGA RELAXATION



TRANSPORT THEORY: SPIN-CHARGE
SEPARATION. 3 ~~degrees of freedom~~ ^{currents} connected
by 1 IDENTITY

spin excitations

charge

"center of mass" = electrons



Added hole. Finally, let us consider states with one hole in the $4\nu + 2$ grounds ($N = N_0 - 1, M = (N - 1)/2$). Then both the I 's and the J 's are integers. energy is minimized choosing consecutive I 's between $-(N - 1)/2$ and $(N - 1)/2$. there are $M + 1$ possibilities for the M J 's. States corresponding to the sequen-

$$\{I_j\} = \{-(N - 1)/2, \dots, (N - 1)/2\} ,$$

$$J_1 = -M/2 + \delta_{\alpha_1, 1} ,$$

$$J_\alpha = J_{\alpha-1} + 1 + \delta_{\alpha_1, \alpha} \quad (\alpha = 2, \dots, M)$$

are shown as the lowest arc between $k = -0.25\pi$ and $k = 0.25\pi$. This is a one-sp

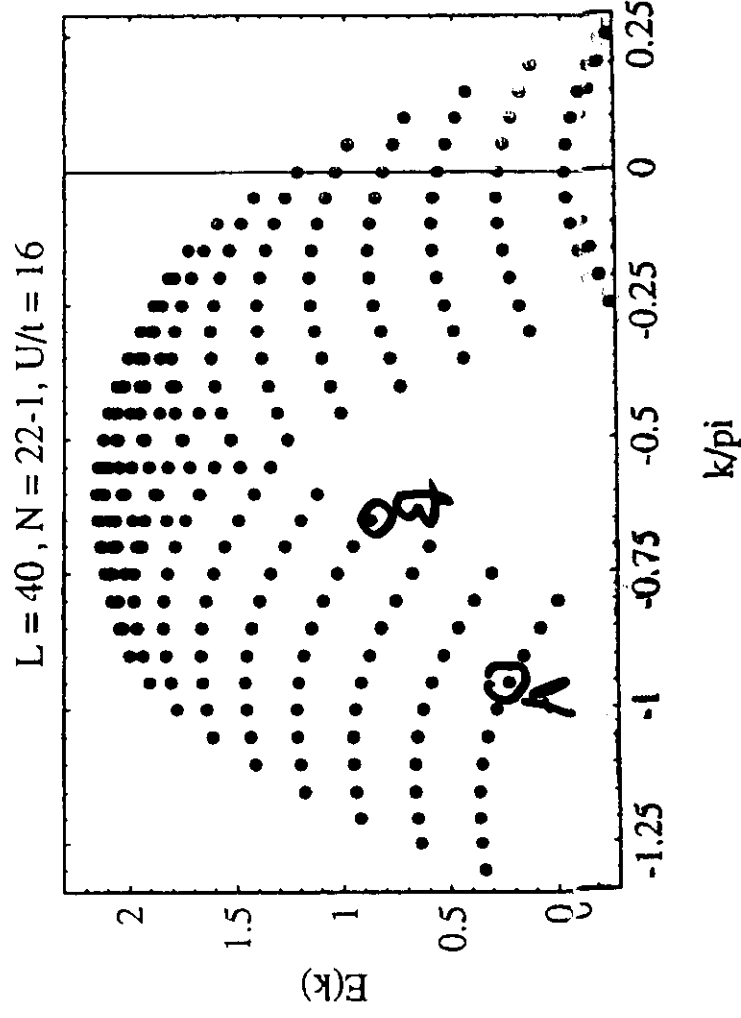


Figure 9: Excitation spectra for an added hole in a Hubbard chain of 40 sites with 16 electrons. The shallow arches correspond to varying α_1 (cf. eq. (1)).

State "A" has spinon but
no holon. BUT IT IS AN
ELECTRON STATE SO CARRIER
CURRENT at v_s

state "B" has holon (charge) but
no spin exc.

§ SO A TRUE TRANSPORT

THEORY MUST ALLOW LARKIN
IDENTITY TO BE MADE UP BY
BACK
COUNTERFLOW IN COLLECTIVE
MOTION. CHARGE + SPIN EXC.

MAY FLOW INDEPENDENTLY.

PROFOUND CONSEQUENCES:

FLUT. DIS DOESN'T WORK.
NO QUASIPARTICLES TO KEEP TRACK
OF DUE TO OF F.S.

(1) $\rho_{xx} \propto T, \omega$: DECAY OF
ELECTRON : PHASE SPACE

QUANTITATIVE

AGREEMENT
WITH I.R.

ω spinon

holon
scatters on impurities:
not dragged

(2) $\Theta_H \propto T^2$

$\Theta_H = \omega_c \tau_H$

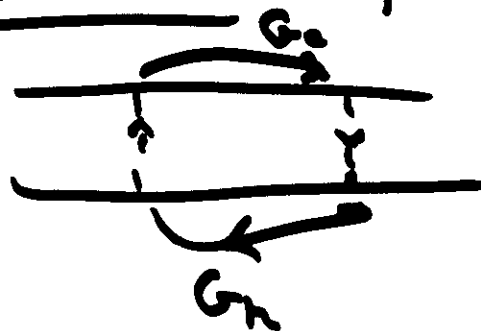
τ_H is $\tau_{\text{spinon}} (\propto \frac{v_c}{v_F} ?)$

$[J_{\Theta}, H_{\text{int}}] = 0$

Larmor precession does
not disturb holon-spin-
electron equilibrium.

(3) $\rho_{zz} \gg \rho_{xx}$: c-axis motion
becomes incoherent by actual
calculation:

JOINT DOS
DOS
minima
 ω



SUPERCONDUCTIVITY : INTERLAYER TUNNELLING MECHANISM



SINGLE-PARTICLE

TUNNELLING
INCOHERENT, BLOCKED

2-particle (JOSEPHSON) TUNN.
STILL COHERENT.

(b) HOW DOES PERT THEORY FAIL?

IDEAS OF BASKARAN :

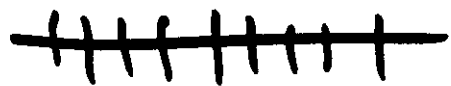
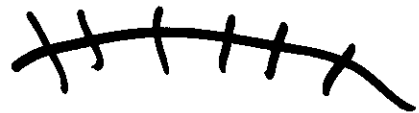
1. QUESTION : DID IT EVER SUCCEED?

ZERO SOUND : $v_s - v_F \propto a^{-\frac{1}{4}}$

density-density correlation not
analytic in U !

2. How does FL $\rightarrow$ NFL as 2D $\rightarrow$ 1D?

Flatten the F.S.



ORTHODOX THEORY SEES NOTHING.

WHAT REALLY HAPPENS : ZERO

SOUND $\rightarrow$ CHARGE BOSONS.

SO CAN MAKE PARTICLE-LIKE
SOLITONS OUT OF ZERO SOUND BOSONS.

MY METHOD

BY EXPLICITLY RETAINING
LUTTINGER THM IN
DYNAMICS, SHOW
 $f_{kk'}$ DIV like (in 2D)

$$\frac{1}{|k-k'|}$$

$$f_{kk} \sim \frac{1}{L}$$

(G.B.)

even with non-sig
 $f_{kk'}$, by keeping
coupling of
tun. graphs
 $f_{kk} \rightarrow \frac{1}{L}$. "RESON"
MAKES WAVE-PACKETS
TRANSVERSELY
LOCALIZED, creates
unique f.

PROBLEM WITH PERT. THEORY:
IT DOES NOT FINITE η BUT
IT DOES NOT CONTAIN BOUNDARY
CONDITIONS HENCE DOES NOT
RETAIN COMPLETENESS AND

ORTHOGONALITY OF BASIS

$$\begin{array}{cc} \vec{k} & \vec{k}' \\ \leftarrow \vec{k} & \leftarrow \vec{k}' \\ k & k' \end{array}$$

$S_k = -S_{k'}$: "IDENTITY
CRUSH"

BUT S_k not free: $f_{kk'}$ is
response of E_k to fixed b .

THIS IS MEANING OF RESONANCE

MY THEORY: (ANDERSON-KYIESCHNIK)

LIKE LIEB-WU (I DO $1D+2D$ in parallel)
introduce int. as BOUNDARY CONDITION
AT ORIGIN $\rightarrow$ BOSON COMMUTATION
RELATIONS,

RESULT: $U_1 \neq U_2$

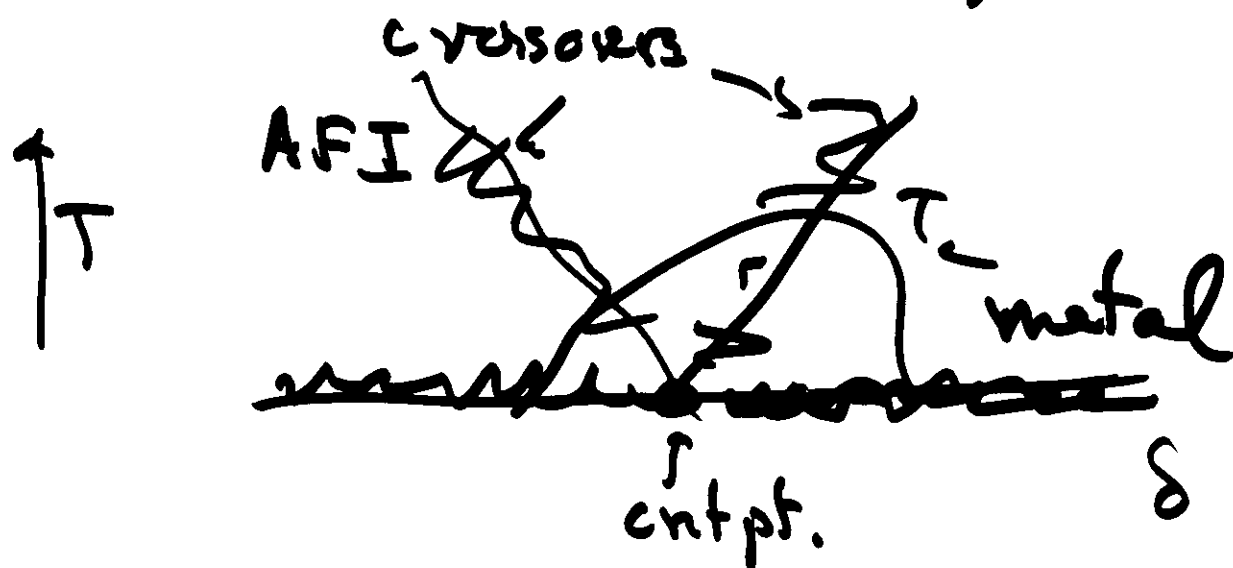
THEN I SATISFY LUTT THAT "BY
HAND" TO GET ANOM. DIMENSIONS.

(G IS BETTER, PROBABLY.)

BOTH WORK.

II Reprise Phase Diagram

Fashionable but wrong idea



cnt pt

various
di Castro
Eun

Zhang

Varma

SF : AF-net.

CDW : ins-met

AFI $\rightarrow$ supercon

"SOS"

GOK

Problem : THE MOTT INSULATOR IS
ONE OF CM P'S MOST WELL-
UNDERSTOOD STATES. IT CANNOT
BE DEFORMED CONTINUOUSLY
INTO A METAL.

NOTT BASIC SYMMETRY

$(SU(2))^N$: ATOMIC SPIN
ROTATIONS

CHARGE DEG OF FREEDOM GAPPED
AT ALL Q.

SLATER FALLACY ~~AF CAUSES
INS~~

PWA : (1959) INS CAUSES AF
J FROM NOTT
not UJ.

NRTEL : $T \neq 0$, $\frac{U}{L^{D-1}} = 0 \rightarrow$ Fe Surface
 $[U(1) \times SU(2)]^{L^{D-1}}$

UNDERLYING HAM. IS K.E.

NO SUPEREXCHANGE

BASIC INT IS SCATTERING OF
MOBILE SPIN-CHARGES.

NO CONTINUOUS DEFORMATION
OF SPACES OF DIFFERING DIM.
AND SYMM INTO EACH OTHER.

GENERIC PICTURE

gas

"solid"



inhomogeneous
phases (Higgs or
Nambu)

(Higgs or
Nambu)

QUANTUM
LIQUID

miscibility
gap: 1st-order
trans.

"S"

not



CP

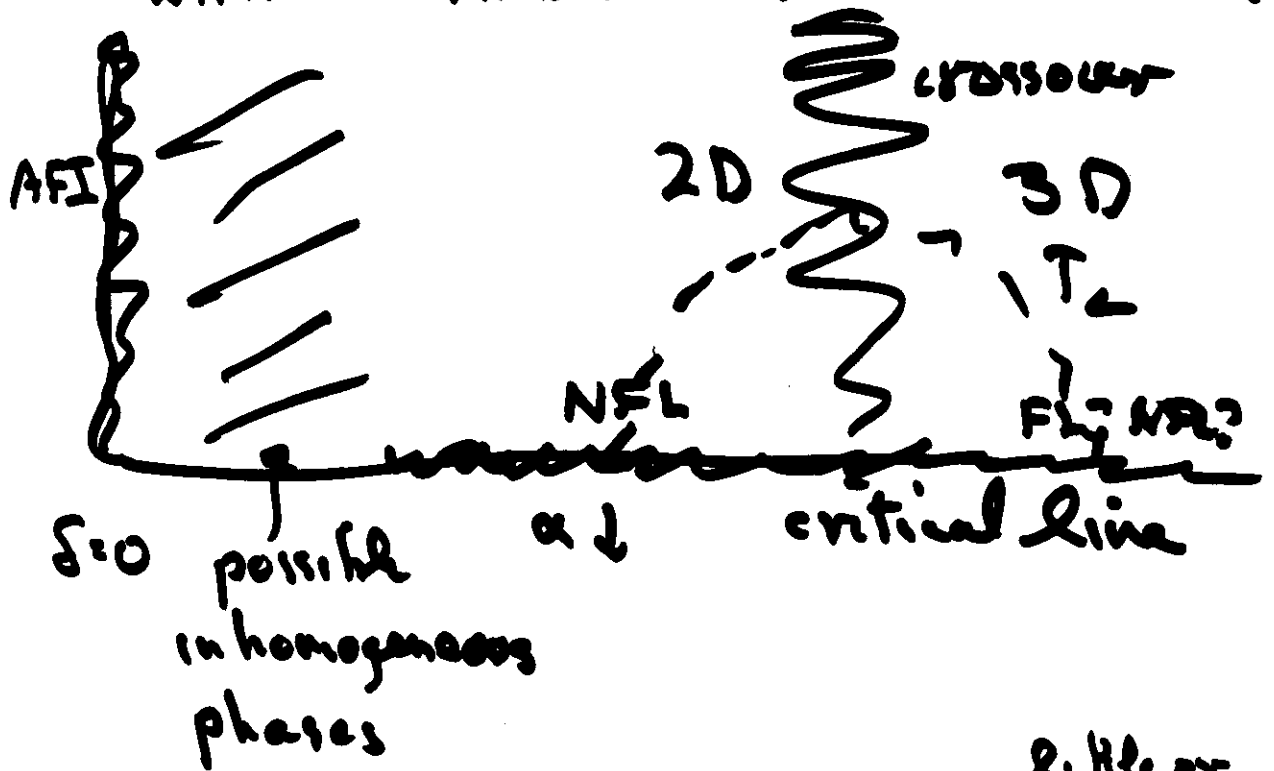
HENCE : NO CRITICAL POINT.

"SOLID" SOLID NOT $\equiv$ CDW.

SIMPLE EXPRESSION : $\Delta U = \text{finite}$ at $n=1$
IN GENERAL : CREATION OF PERMI

SURFACE NOT A CP.

III WHAT THEN IS THE PICTURE?



note : in spin gap phase no c-axis fluctuation conductivity : 2D-like.
 T_c driven by frustrated 3D k.B.

