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Considering 3D Movements of Blocks in the Model of Block Structure Dynamics

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Abstract. The block model of lithosphere dynamics with 3D movements of blocks is presented. A seismically active region is considered as a system of absolutely rigid blocks separated by infinitely thin plane faults. The system of blocks moves as a consequence of prescribed 3D motion of the boundaries and the underlying medium. Displacements of the blocks are determined so that the system is in quasistatic equilibrium state. Block interaction along the faults is viscous-elastic while the ratio of the stress to the pressure is below a certain strength level. When the level is exceeded for a part of some fault a stress-drop (a failure) occurs in accordance with the dry friction model. The failures represent earthquakes. As a result of numerical simulation a synthetic earthquake catalog is produced. Both simple enough block structures and the structure which roughly approximates the main tectonic elements of the Vrancea region are tested. Common and different features of 3D model and well-known 2D model are discussed.

1. Introduction

Mathematical models of lithosphere dynamics are important tools for study of the earthquake preparation process. An adequate model should reproduce premonitory patterns determined empirically before large events and can be used to suggest and to investigate new patterns that might exist in real catalogs. The basic principles of the model under consideration have been developed by Gabrielov et al. (1986, 1990). These principles essentially lean upon the following assumption. The qualitative stability of lithosphere properties in different seismic regions suggests that the lithosphere can be considered as a large dissipative system. Behaviour of this system depends slightly on the detalization of geostructures and geoprocesses. It is natural that such approach gives, as a rule, low order of approximation to real events in some specific region. Its importance lies in the possibility of establishing and analyzing general patterns and connections by means of studying synthetic earthquake catalogs which may cover very long time intervals (in comparison with the reliable data).

As it is known, on the one hand, 2D block model (for its detailed description, see, for example, Panza et al. (1997) and Gabrielov and Soloviev (1997)) takes into account only plane displacements of lithosphere blocks whereas, on the other hand. there are some essential vertical components in movements of real blocks. Therefore the question arises: how to involve 3D displacements of blocks into the model? Here it is presented an attempt of 3D generalization of 2D block model, where the blocks are assumed to have six degrees of freedom (rather than three). No doubt, 3D model preserves some assumptions adopted for 2D model. In particular, all blocks have the same depth.

As well as in 2D model, a seismically active region is represented as a system of absolutely rigid blocks forming a layer with a fixed thickness between two horizontal planes. Lateral boundaries of blocks consist of segments of tectonic faults intersecting the layer with arbitrary dip angles. The system of blocks moves as a consequence of action of outside forces applied to it and is supposed to be in quasistatic equilibrium state. As the blocks are absolutely rigid, all deformations take place in the fault zones and at the block bottoms. The interaction between the blocks is viscous-elastic ("normal state") while the ratio of the stress to the pressure is below a certain strength level. When this level is exceeded in some part of a fault plane a stress-drop ("a failure") occurs in accordance with the dry friction model. The failures represent earthquakes. Immediately after the earthquake and for some time, the corresponding parts of the faults are in "creep state". This state differs from the normal one because of the faster growing of inelastic displacements and lasts until the stress falls below a given level. As a result of the numerical modelling a synthetic earthquake catalog may be produced.

2. Brief description of the model

Since we use the main constructions and ideas from the works dealing with 2D model, only the brief description of 3D model is presented here (with the emphasis on distinctions between the models).

2.1. Block structure geometry

The definitions and terms used in this section completely correspond to the definitions and terms introduced for 2D model. A layer, with a depth H, limited by two horizontal planes is considered, and a block structure is a limited and simply connected part of this layer. Each lateral boundary of the block structure is defined by portions of parts of planes intersecting the layer. Subdivision of the structure into blocks is also performed by planes intersecting the layer. The parts of these planes which are inside the block structure and its lateral facets are called "fault planes".

The geometry of the block structure is defined by the lines of intersection between the fault planes and the upper plane limiting the layer (these lines are also called "faults"), and by the dip angles of each fault plane. The geometry of the block structure on the lower plane is assumed to be similar to the one on the upper plane. By definition, three or more faults cannot have a common point on the upper (lower) plane, and a common point of two faults is called "vertex". The positions of a vertex on the upper and the lower planes, limiting the layer, are connected by a segment ("rib") of the line of intersection of the corresponding fault planes. The part of a fault plane between two ribs corresponding to successive vertices on the fault is called "segment". The shape of the segment is a trapezium. The common parts of the block with the upper and lower planes are polygons, and the common part of the block with the lower plane is called "bottom".

We assume that the block structure is bordered by a confirming medium, whose motion is prescribed on its continuous parts comprised between two ribs of the block structure boundary. These parts of the medium are called "boundary blocks".

2.2. Block movement

As well as in 2D model, the blocks are assumed to be absolutely rigid. All block displacements are supposed to be infinitely small, compared with block sizes. Therefore the geometry of the block structure does not change during the simulation, and the structure does not move as a whole. The gravitation forces are not essentially changed because of the blocks displacements and, since the block structure is in quasi-static equilibrium state at the initial time moment, it is correct to assume that the gravity does not cause movements of the blocks.

The distinctive feature of 3D model is that the blocks have six degrees of freedom (rather than three) and not all their relative displacements take place along the fault planes separating them. The displacement of each block consists of the progressive and rotation components. The progressive component is determined by translation vector (x, y, z). The rotation component may be described by the following way. Let us assume that the coordinate system with axes X_1, Y_1, Z_1 is strictly connected with the mass center of the block (it coincides under the lack of block displacements with the immovable system with axes X, Y, Z in which we consider all movements of the block). The rotation of the block and of corresponding system (X_1, Y_1, Z_1) with respect to system (X, Y, Z) is described by means of three angles γ, β, φ definition of which is presented in Fig. 1.

The first angle γ is defined as the angle of rotation of axes Y and Z around axis X providing fulfilment of the following condition: if axis Z_2 is the intersection of planes XOZ_1 and YOZ, then axis Z should be mapped into axis Z_2, at that $Y \rightarrow Y_2$. The second angle β is defined as the angle of rotation of axes X and Z_2 around axis Y_2 providing transformation of axis Z_2 into axis Z_1 (it is possible since Z_1 belongs to XOZ_2), at that $X \rightarrow X_2$. And the third angle φ is defined as the angle of such rotation of axes X_2 and Y_2 around axis Z_1 that $X_2 \to X_1$, $Y_2 \to Y_1$.

According to the definition of angles γ, β, φ, at some point (X, Y, Z) of the block the components Δ_x, Δ_y and Δ_z of the displacement are defined by

$$\Delta_x = x - (Y - Y_c)\varphi + (Z - Z_c)\beta,$$

$$\Delta_y = y + (X - X_c)\varphi - (Z - Z_c)\gamma,$$

$$\Delta_z = z - (X - X_c)\beta + (Y - Y_c)\gamma, \qquad (1)$$

where (X_c, Y_c, Z_c) are the coordinates of the mass center of the block, and angles (γ, β, φ) are supposed to be small.

All the values of the components of translation vector and the angles of rotation are found from the condition that the sum of all forces acting on the block and of total moment of these forces have to be zero (at every moment of time the structure is supposed to be in a quasistatic equilibrium state). The interaction of the blocks with the underlying medium takes place along the lower plane. The movements of the boundaries of the block structure (the boundary blocks) and of the underlying medium are assumed to be an external action on the structure. The rates of these movements are considered to be known.

Non-dimensional time is used in 3D model (as well as in 2D model), therefore all quantities that contain time in their dimensions are referred to one unit of non-dimensional time. For example, in the model, velocities are measured in units of length and the velocity of 5 cm means 5 cm for one unit of non-dimensional time. When interpreting the results a realistic value is given to one unit of non-dimensional time. If one unit of non-dimensional time is one year then the velocity of 5 cm, specified for the model, means 5 cm/year.

2.3. Interaction between the blocks and the underlying medium

The elastic force which is due to the relative displacement of the block and the underlying medium, at some point of the block bottom, is assumed to be proportional to the difference between the total relative displacement vector and the vector of inelastic displacement (slippage) at the point.

The elastic force per unit area (f_x^u, f_y^u, f_z^u) (note that component f_z^u is introduced in 3D model) applied to some point (X, Y, Z) of the block bottom ($Z = 0$ corresponds to the upper plane), at some time t is defined by (see (1))

$$f_x^u = K_u(\Delta_x^u - \delta_x^u), \quad f_y^u = K_u(\Delta_y^u - \delta_y^u), \quad f_z^u = K_u^n \Delta_z^u, \tag{2}$$

where (see (1))

$$\Delta_x^u = x - x_u - (Y - Y_c^m)\varphi - (H + Z_c^m)\beta + (Y - Y_c)\varphi_u,$$

$$\Delta_y^u = y - y_u + (X - X_c^m)\varphi + (H + Z_c^m)\gamma - (X - X_c)\varphi_u,$$

$$\Delta_z^u = z - z_u + (X - X_c^m)\beta + (Y - Y_c^m)\gamma + (X - X_c)\beta_u - (Y - Y_c)\gamma_u. \tag{3}$$

Here $(\Delta_x^u, \Delta_y^u, \Delta_z^u)$ is the total relative displacement vector, (X_c^m, Y_c^m, Z_c^m) are the coordinates of the mass center of the block, (X_c, Y_c) are the coordinates of the geometrical center of the block bottom, (x, y, z) and (γ, β, φ) are the translation vector of the block and the angles of its rotation around point (X_c^m, Y_c^m, Z_c^m), (x_u, y_u, z_u) and $(\gamma_u, \beta_u, \varphi_u)$ are the translation vector of the block bottom and the angles of its rotation around point (X_c, Y_c), H is the depth of the layer, (δ_x^u, δ_y^u) is the inelastic displacement vector, the evolution of which is described by the equations

$$\frac{d\delta_x^u}{dt} = W_u f_x^u, \quad \frac{d\delta_y^u}{dt} = W_u f_y^u. \tag{4}$$

The coefficients K_u, K_u^n and W_u in (2) and (4) may be different for different blocks.

2.4. Interaction between the blocks along the fault planes

At time moment t, at some point (X, Y, Z) of the fault plane separating the blocks numbered i and j (the block numbered i is on the left and that numbered j is on the right of the fault) the components Δ_x, Δ_y and Δ_z of the relative displacement of the blocks are defined by (see (1))

$$\Delta_x = x_i - x_j - (Y - Y_c^{mi})\varphi_i + (Y - Y_c^{mj})\varphi_j + (Z - Z_c^{mi})\beta_i - (Z - Z_c^{mj})\beta_j,$$

$$\Delta_y = y_i - y_j + (X - X_c^{mi})\varphi_i - (X - X_c^{mj})\varphi_j - (Z - Z_c^{mi})\gamma_i + (Z - Z_c^{mj})\gamma_j,$$

$$\Delta_z = z_i - z_j - (X - X_c^{mi})\beta_i + (X - X_c^{mj})\beta_j + (Y - Y_c^{mi})\gamma_i - (Y - Y_c^{mj})\gamma_j, \quad (5)$$

where $(X_c^{mi}, Y_c^{mi}, Z_c^{mi})$ and $(X_c^{mj}, Y_c^{mj}, Z_c^{mj})$ are the coordinates of the mass centers of the blocks numbered i and j respectively, (x_i, y_i, z_i), (x_j, y_j, z_j), $(\gamma_i, \beta_i, \varphi_i)$ and $(\gamma_j, \beta_j, \varphi_j)$ are the translation vectors of the blocks i, j and the corresponding angles of its rotation.

Since the relative block displacements in 3D model take place not only along the fault planes (in contrast with 2D model), the displacements along the fault plane and normal to it are connected with Δ_x, Δ_y and Δ_z by the following relations

$$\Delta_t = \Delta_x e_x + \Delta_y e_y, \quad \Delta_l = -\Delta_x e_y cos\alpha + \Delta_y e_x cos\alpha - \Delta_z sin\alpha,$$

$$\Delta_n = -\Delta_x e_y sin\alpha + \Delta_y e_x sin\alpha + \Delta_z cos\alpha. \quad (6)$$

Here Δ_t and Δ_l are the displacements along the fault plane, parallel (Δ_t) and normal (Δ_l) to the fault line on the upper plane, Δ_n is the displacement which is normal to the fault plane, $(e_x, e_y, 0)$ is the unit vector along the fault line on the upper plane, and α is the dip angle of the fault plane.

The elastic force per unit area (f_t, f_l, f_n) (note that component f_n is introduced in 3D model) applied to the point of the fault plane, at some time t is defined by

$$f_t = K_t(\Delta_t - \delta_t), \quad f_l = K_l(\Delta_l - \delta_l), \quad f_n = K_n(\Delta_n - \delta_n). \quad (7)$$

Here δ_t, δ_l, δ_n are the corresponding inelastic displacements, the evolution of which is described by the equations

$$\frac{d\delta_t}{dt} = W_t f_t, \quad \frac{d\delta_l}{dt} = W_l f_l, \quad \frac{d\delta_n}{dt} = W_n f_n. \quad (8)$$

The coefficients K_t, K_l, K_n, W_t, W_l, and W_n in (7) and (8) may be different for different faults.

Formulas (5) are valid for the boundary faults too. In this case one of the blocks separated by the fault is the boundary block. The movement of these blocks is described by their translation vectors and rotation around the origin of coordinates. Therefore the coordinates of the mass center of the block in (5) are zero for the boundary block.

2.5. Equilibrium equations

As mentioned above, the components of translation vectors of the blocks and the angles of their rotation around the mass centers of the blocks are found from the condition that the total force and the total moment of forces acting on each block are equal to zero. This is the condition of quasistatic equilibrium of the system and at the same time the condition of minimum energy.

In accordance with formulas (2), (3), (5)-(7) it is possible to obtain the following system of equations which describes the equilibrium state

$$Aw = \boldsymbol{b}. \tag{9}$$

Here the components of the unknown vector $\boldsymbol{w} = (w_1, w_2, ..., w_{6n})$ are the components of translation vectors of the blocks and the angles of their rotation (n is the number of blocks), i. e. $w_{6m-5} = x_m$, $w_{6m-4} = y_m$, $w_{6m-3} = z_m$, $w_{6m-2} = \gamma_m$, $w_{6m-1} = \beta_m$, $w_{6m} = \varphi_m$ ($m = 1, 2, ..., n$). The moment of forces acting on a block is calculated relative to its mass center.

The elements of matrix A ($6n \times 6n$) and vector $\boldsymbol{b}$ ($6n$) are determined from rather complicated formulas, and, for brevity sake, these formulas are omitted in this paper.

2.6. Discretization

In computational purposes, time discretization is performed by introducing a time step Δt (by full analogy with 2D model). The state of the block structure under consideration is determined at discrete time moments $t_i = t_0 + i\Delta t (i = 1, 2, ...)$, where t_0 is the initial time. The transformation from the state at t_i to the state at t_{i+1} is made as follows: (a) new values of inelastic displacements δ_x^u, δ_y^u, δ_t, δ_l, δ_n are calculated from equations (4) and (8); (b) the translation vectors and the rotation angles at t_{i+1} are calculated for the boundary blocks and the underlying medium; (c) the components of $\boldsymbol{b}$ in system (9) are found, and this system is used to determine the translation vectors and the rotation angles for the blocks. Since the elements of A in (9) do not depend on time, this matrix can be calculated only once, at the beginning of the process.

Space discretization is performed according to the scheme worked out for 2D model. It is defined by parameter ε, and it is applied to the surfaces of the fault segments and to the block bottoms. The discretization of a fault segment is performed as follows. Each fault segment is a trapezium with bases a and b and height $h = H/\sin\alpha$, where H is the depth of the layer, and α is the dip angle of the fault plane. We define

$$n_1 = ENTIRE(h/\varepsilon) + 1, \quad n_2 = ENTIRE(max(a,b)/\varepsilon) + 1$$

(here $ENTIRE$ is the integer part of a number), therefore the trapezium is divided into $n_1 n_2$ small trapeziums which are called "cells". The coordinates X, Y, Z in (5) and the inelastic displacements δ_t, δ_l, δ_n in (7) are supposed to be the same for all points of a cell. It should be noted that the coordinate Z may be calculated by means of X, Y and α for any point of a fault.

The bottom of a block is a polygon. Before discretization it is divided into trapeziums (triangles) by segments passing through its vertices and parallel to axis Y. Discretization of these figures is performed in the same way as in the case of the fault segments. The small trapeziums (triangles) are also called "cells". For all points of a cell the coordinates X, Y and the inelastic displacements δ_x^u, δ_y^u in (2) are assumed to be the same. Notice that the coordinate $Z = -H$ for all points of a block bottom.

In calculations, certain concordance of space and time discretization steps is recommended.

2.7. Earthquake and creep

At every time t_i we calculate (as well as in 2D model) the value of the quantity κ by the following formula

$$\kappa = \frac{\sqrt{f_t^2 + f_l^2}}{P - f_n}, \tag{10}$$

where P is the parameter which may be interpreted as the difference between the lithostatic and the hydrostatic pressure (P has the same value for all faults).

For each fault the three levels of κ are fixed

$$B > H_f \geq H_s.$$

It is assumed that the initial conditions for numerical simulation of block structure dynamics satisfy the inequality $\kappa < B$ for all cells of the fault segments. If, at some time moment t_i, the value of κ in any cell of a fault segment reaches the level B, a failure ("earthquake") occurs. By failure we mean slippage during which the inelastic displacements δ_t, δ_l, δ_n in the cell change abruptly to reduce the value of κ to the level H_f. Note that this procedure for 3D model essentially differs from the analogous procedure for 2D model. The new values of the inelastic displacements in the cell are calculated from

$$\delta_t^e = \delta_t + \gamma^e \xi_t f_t, \quad \delta_l^e = \delta_l + \gamma^e f_l, \quad \delta_n^e = \delta_n + \gamma^e \xi_n f_n, \tag{11}$$

where δ_t, δ_l, δ_n, f_t, f_l, f_n are the inelastic displacements and the components of elastic force vector per unit area just before the failure. The coefficients $\xi_t = K_l/K_t$ ($\xi_t = 0$ if $K_t = 0$) and $\xi_n = K_l/K_n$ ($\xi_n = 0$ if $K_n = 0$) account for nonhomogeneity of displacements along the fault plane (in different directions) and normal to it (they reflect the assumption that the same value of the elastic force per unit area results in different values of rates of changing different inelastic displacements). The coefficient γ^e is given by

$$\gamma^e = \frac{\sqrt{f_t^2 + f_l^2} - H_f(P - f_n)}{K_l\sqrt{f_t^2 + f_l^2} + K_n H_f \xi_n f_n}. \tag{12}$$

It follows from (7), (10)–(12) that after calculation of the new values of the inelastic displacements and the elastic forces the value of κ in the cell is equal to H_f. Here it should be noted the following facts. The new values of the elastic forces calculated according to (7), (11) are to have the same signs as the values just before the failure. Therefore the case when $(1 - K_n\xi_n\gamma_e) < 0$ (and f_n changes its sign) is to be considered in its own right as well as the case when $(1 - K_l\gamma_e) < 0$ (and f_l and f_t change their signs). It may be proved that the second situation is possible only if $f_n < 0$. In the both cases we assume

$$\delta_n^e = \Delta_n, \quad \gamma^e = \frac{\sqrt{f_t^2 + f_l^2} - H_f P}{K_l\sqrt{f_t^2 + f_l^2}}.$$

After calculations described above for all the failed cells, the new components of vector b are computed, and from the system of equations (9) the translation vectors and the angles of rotation for the blocks are found. If for some cell(s) of the fault segments $\kappa > B$, the whole procedure is repeated for this cell (or cells). Otherwise the state of the block structure at time t_{i+1} is determined as follows: the translation vectors, the rotation angles for the boundary blocks and for the underlying medium, and the components of $\boldsymbol{b}$ in (9) are calculated, and then the system (9) is solved.

The cells of the same fault plane in which failure occurs at the same time form a single earthquake. The parameters of the earthquake are defined as follows: (a) the origin time is t_i; (b) the epicentral coordinates and the source depth are the weighted sums of the coordinates and depths of the cells involved in the earthquake (the weight of each cell is given by its square divided by the sum of squares of all cells involved in the earthquake); (c) the magnitude is calculated in accordance with Utsu and Seki (1954) from

$$M = 0.98 \log_{10} S + 3.93, \tag{13}$$

where S is the sum of the squares of the cells included in the earthquake.

Immediately after the earthquake, it is assumed that the quaked cells are in the creep state. It means that, for these cells, in equations (8), which describe the evolution of inelastic displacements, the parameters W_t^s ($W_t^s > W_t$), W_l^s ($W_l^s > W_l$), and W_n^s ($W_n^s > W_n$) are used instead of W_t, W_l, and W_n. They may be different for different faults. The quaked cells are in the creep state as long as $\kappa > H_s$, while when $\kappa \leq H_s$, the cells return to the normal state and hereinafter W_t, W_l, and W_n are used in (8).

2.8. Connection between 2D and 3D models

As mentioned above, here it is presented the attempt of 3D generalization of the well-known 2D block model. The aim of this section is to show that with appropriate choice of parameters (e. g. coefficients in the equations for determination of the elastic forces, inelastic displacements etc.) the results of 3D simulation of dynamics of a block structure are close to the results of 2D simulation of dynamics of the same structure. To achieve it, the following ideas are used: (a) the same displacements in 2D and 3D models should induce the same elastic forces at the bottom and at the faults; (b) the equations for determination of the inelastic displacements should be the same; (c) earthquakes should be processed "in the same manner". The matter is that in 2D model all relative displacements take place only along the fault planes (in contrast with 3D model). The following relations are valid (see Fig. 2):

$$2D: \quad \Delta_l = \Delta_x / \cos\alpha, \quad 3D: \quad \Delta_l^1 = \Delta_x^1 / \cos\alpha, \tag{14}$$

where Δ_l, Δ_l^1 are the relative displacements of the blocks along the fault line (in 2D and 3D cases respectively), Δ_x, Δ_x^1 are the relative displacements along axis x on the upper plane (in 2D and 3D cases respectively), α is the dip angle of the fault plane (for simplicity it is assumed that the displacements take place only along axis x).

From relations (7) and (14) it is easily seen that the conditions providing the equality of the forces f_t, f_l, f_n in 2D and 3D models without considering the inelastic displacements are the following

$$K_t^1 = K, \quad K_l^1 = K/\cos^2\alpha, \quad K_n^1 = K/\cos^2\alpha, \tag{15}$$

where K_t^1, K_l^1, K_n^1 are the coefficients in (7) for 3D model, K is the analogous single coefficient for 2D model. By analogy the conditions providing the equality of the displacements δ_t, δ_l, δ_n may be derived in the form

$$W_t^1 = W, \quad W_l^1 = W\cos^2\alpha, \quad W_n^1 = W\cos^2\alpha, \tag{16}$$

where W_t^1, W_l^1, W_n^1 are the coefficients in (8) for 3D model, W is the analogous single coefficient for 2D model. The corresponding coefficients for the creep state are connected in the same manner.

To provide the equality of the elastic forces and inelastic displacements at the bottom for 2D and 3D models, it should be assumed that

$$K_u^1 = K_u, \quad W_u^1 = W_u, \quad K_u^{n1} = 0, \tag{17}$$

where K_u^1, W_u^1, K_u^{n1} are the coefficients in (2), (4) for 3D model, K_u, W_u are the analogous coefficients for 2D model.

However, there is an insurmountable distinction in calculative parts of 2D and 3D models. It is substantially that in 3D model the moments of rotation about axes X and Y determining angles γ and β are necessarily generated and taken into account in the system (9) whereas in 2D model, according to its definition, these moments are ignored but they really exist because of the fact that the forces acting on the faults applied to points of different depth. It is therefore concluded that the results of simulation of the block structure dynamics by means of 3D model with the values of parameters (15)–(17) and if the depth of the layer H tends to zero (or another condition providing the moments of rotation about axes X and Y to be small is fulfilled) should tend to the results of simulation of the same block structure dynamics by means of 2D model in the following sense:

$$\max_i |w_i^1 - w_i| \to 0,$$

where $w^1 = (x^1, y^1, z^1, \gamma^1, \beta^1, \varphi^1)$ represents the translation vector and the rotation angles of the block structure for 3D model, $w = (x, y, 0, 0, 0, \varphi)$ represents analogous characteristics for 2D model.

It is not difficult to show that under concordance of parameters (15), (16) the procedures processing earthqukes in 2D and 3D models work almost "in the same manner" (see (10)–(12)), moreover the less is the value of H, the closer are the results.

Thus, there are no such relations between parameters that dynamics of 3D model may be exactly reduced to that of 2D model. However, there exist the conditions providing closeness of 3D movements of blocks to 2D movements of the same blocks.

3. Numerical simulation of 3D block structure dynamics

On the basis of 3D model an interactive program was created. The program enables to specify arbitrary block structures and to obtain synthetic earthquake catalogs as a result of simulation. Both simple enough block structures and the structure which approximates the main tectonic elements of the Vrancea (Romania) region were tested by means of the program to find the existence of premonitory patterns and features detected in real catalogs. Here only the first simulation results obtained in accordance with the scheme worked out for testing 2D model are presented.

To study clustering of earthquakes in the model and dependence of frequency-magnitude relation on structure geometry the dynamics of block structures BS1, BS2, and BS3 shown in Fig. 3 was simulated. On the upper plane each of them is a square with a side of 320 km divided by faults into smaller squares. Structures with the similar geometry were considered, for example, by Bariere and Turcotte (1994) and by Keilis-Borok et al. (1997). The depth of the layer is 20 km. The same movement of boundaries was specified for all the structures. The sides of the largest squares move progressively with the constant velocities (for their directions, see Fig. 3). The underlying medium does not move. The numerical simulation was made with time step $\Delta t = 0.001$, spatial step $\varepsilon = 5$km, with zero initial conditions (zero displacement of boundary blocks and the underlying medium and zero inelastic displacements for all cells).

Investigation of clustering of earthquakes is exceptionally important for comprehension of seismicity and specifically for earthquake prediction. This phenomenon was studied by many researchers, e.g. Kagan and Knopoff (1978). It is vital, is clustering caused by specific features of tectonics of a region under consideration or it is a phenomenon which reflects general features of systems of interacting blocks of the seismoactive lithosphere. Clustering of earthquakes in synthetic catalogs arising from modelling of dynamics of simple block structures is in favour of the second supposition (Maksimov and Soloviev, 1996). The moments of earthquakes in structure BS1 are shown as vertical lines in Fig. 4 for the segment and for the whole structure for the interval of 3 units of non-dimensional time which begins at $t = 75$. Clustering of earthquakes appears clearly on the most of fault segments. The picture for the whole structure looks likely, however, groups of earthquakes can be identified. Clustering of earthquakes for other time intervals is not substantially different from that presented in Fig. 4.

The accumulative frequency magnitude relations obtained for the synthetic catalogs for the same time period are shown in Fig. 5. It follows that the number of earthquakes with magnitude less than 5.2 increases while the structure complexity increases. Number of earthquakes with magnitudes from 6.1 to 6.7 is maximum for BS2, and with magnitudes greater than 6.7— for BS1. The curve for BS3 is more straight than the other two and provides the Gutenberg—Richter low fulfillment better. The histograms of the number of events for different magnitude intervals and time periods were obtained (see Tables 1–3). They show that the seismic flow does not have a long-term trend, and the annual number of events is stable after some interval.

Fig. 6 shows another type of regular behaviour. There is a fault segment which is divided into cells. Empty cells are in normal condition, black colored cells denote earthquake and dashed cells are in creep state. A sequence of the segment states for rather close moments of non-dimensional time is presented. In this sequence one can identify existence of foreshocks, main shock, and aftershocks. A main shock happened at $t = 7.65$. The rest events are foreshocks and aftershocks belonging to the group of earthquakes. A series of groups of events like this occurs on the segment approximately every 3 units of non-dimensional time, and this corresponds to the concept of the seismic cycle. For all active segments the existence of periods of post-seismic relaxation and inter-seismic stress accumulation between the strongest earthquakes is clearly observed. Every segment has its own characteristic time interval between these earthquakes.

The simulation of dynamics of the block structure approximating the tectonic structure of the Vrancea region was also carried out. Vrancea is a relatively small seismoactive region with a high level of seismic activity, mainly occurring at intermediate depth. In accordance with Arinei (1974) the main structural elements of the Vrancea region are: (1) the East-European plate, (2) the Moesian, (3) the Black Sea, and (4) the Intra-Alpine subplates (Fig. 7). The main directions of the movement of the structural elements are also shown. This information is sufficient to define a block structure which can be considered as a rough approximation of the region for numerical simulation of its dynamics. The present work continues studies for Vrancea which were made by Panza et al. (1997) with 2D block model, and some values of the model parameters and the data on the observed seismicity of the region were taken from the paper mentioned above.

The synthetic earthquake catalog is obtained as the result of the simulation for the period of 200 units of non-dimensional time, starting from the initial zero conditions. The synthetic catalog contains 31650 events with magnitude between 5.3 and 8.5. The minimum value of magnitude corresponds, in accordance with (13), to the minimum square of one cell. Space distribution of the number of events shows that the most synthetic events occur on faults 6 and 9 (see Fig. 7) which correspond to the subduction zone of Vrancea where the most part of the observed seismicity is concentrated. The distribution is similar to that for 2D block model. The frequency-of-occurrence curve for the synthetic catalog is presented in Fig. 8. It is almost linear, and it has approximately the same slope as the proper curve for the observed seismicity. It should be noted that here the preliminary results of the simulation of the dynamics of the Vrancea region are presented. But some features of the synthetic catalog which are similar to those of the real one make possible to hope for some adequacy of 3D model to the real seismic process.

4. Conclusion

Thus, an attempt of considering 3D movements of blocks in the model of block structure dynamics is presented. The brief description of 3D model is given with the emphasis on distinctions between 3D and 2D models. Relations between parameters connecting 3D and 2D models are discussed. Some preliminary results of the numerical simulation of dynamics of different block structures are obtained.

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Table 1 Histogram of the dependence of the number of events on magnitude intervals for the catalog obtained for structure BS1;

$\Delta M = 0.2$, $\Delta t = 5$ years.

Time	5.10		5.50		5.90		6.30		6.70		7.10		7.50	
1900	.	.	.	.	.	.	.	.	.	.	.	.	.	0
	9	6	6	7	1	4	1	.	.	.	2	6	.	42
1910	105	22	34	14	3	6	2	1	2	.	6	3	.	198
	173	54	49	20	8	8	4	5	1	4	3	1	.	330
1920	273	48	48	25	11	10	4	3	.	2	3	.	.	427
	470	74	78	20	12	8	4	2	1	4	4	.	.	677
1930	465	72	61	18	11	9	7	.	.	2	2	.	.	647
	839	115	79	17	12	8	9	1	.	2	2	.	.	1084
1940	566	77	57	16	15	11	5	1	.	5	3	.	.	756
	736	81	73	18	12	8	1	1	1	2	1	.	.	934
1950	646	61	55	16	15	8	4	1	.	2	2	.	.	810
	811	79	81	21	12	8	2	.	2	2	1	.	.	1019
1960	967	93	67	21	14	9	.	1	.	1	1	.	.	1174
	946	107	92	25	18	12	2	.	2	.	2	.	.	1206
1970	731	72	60	22	14	6	2	.	1	.	1	.	.	909
	845	92	62	20	11	7	2	.	1	.	1	.	.	1041
1980	804	69	63	23	16	4	1	.	2	.	2	.	.	984
	931	106	71	26	21	3	1	.	1	.	1	.	.	1161
1990	974	112	83	22	20	3	4	.	1	1	1	.	.	1221
	894	89	65	13	23	7	2	.	2	1	.	.	.	1096
2000	.	.	.	.	.	.	.	.	.	.	.	.	.	
	12185	1429	1184	364	249	139	57	16	17	28	30	10	0	15716

Table 2 Histogram of the dependence of the number of events on magnitude intervals for the catalog obtained for structure BS2;
$\Delta M = 0.2$, $\Delta t = 5$ years.

Time \ Magnitude	5.10		5.50		5.90		6.30		6.70		7.10		7.50	
1900	.	.	.	.	.	.	.	.	.	.	.	.	.	0
	47	18	5	17	3	4	5	.	5	2	1	3	.	110
1910	183	43	32	28	15	4	5	5	5	1	2	2	.	325
	308	72	32	29	4	6	2	4	4	.	2	.	.	463
1920	533	82	46	17	13	7	2	2	4	3	1	.	.	710
	482	84	59	20	12	7	6	6	7	2	.	.	.	685
1930	559	83	33	23	15	5	4	4	4	1	.	.	.	731
	639	87	44	14	11	8	1	7	2	2	.	.	.	815
1940	764	105	38	22	10	6	2	4	4	.	.	.	.	955
	717	90	37	19	13	5	2	3	4	.	.	.	.	890
1950	718	100	41	25	7	6	.	6	6	.	.	.	.	909
	881	101	40	26	11	5	.	4	3	.	.	.	.	1071
1960	915	124	43	22	15	1	.	7	3	.	.	.	.	1130
	776	108	44	27	13	3	.	4	3	.	.	.	.	978
1970	791	106	41	27	10	4	.	4	4	.	.	.	.	987
	796	85	40	19	11	9	1	3	8	.	.	.	.	972
1980	816	85	39	24	9	5	.	2	5	.	.	.	.	985
	714	98	38	25	14	3	.	4	5	.	.	.	.	901
1990	821	97	34	21	6	1	.	1	6	.	.	.	.	987
	753	96	44	25	8	3	.	1	7	.	.	.	.	937
2000	.	.	.	.	.	.	.	.	.	.	.	.	.	
	12213	1664	730	430	200	92	30	71	89	11	6	5	0	15541

Table 3 Histogram of the dependence of the number of events on magnitude intervals for the catalog obtained for structure BS3;

$\Delta M = 0.2$, $\Delta t = 5$ years.

Magnitude

Time	5.10		5.50		5.90		6.30		6.70		7.10		7.50	
1900	.	.	.	.	.	.	.	.	.	.	.	.	.	0
	116	33	18	25	13	1	2	3	3	.	.	4	.	218
1910	231	50	40	35	19	1	5	4	2	.	2	2	.	391
	328	72	28	22	10	11	3	3	4	.	2	.	.	483
1920	605	80	43	27	11	7	1	2	3	1	2	.	.	782
	535	73	41	26	8	9	4	.	2	1	2	.	.	701
1930	772	94	37	20	9	3	4	2	2	1	1	.	.	945
	808	94	45	20	10	5	4	1	2	2	.	.	.	991
1940	825	85	33	18	8	7	3	.	3	1	.	.	.	983
	868	91	43	17	11	7	5	2	1	1	.	.	.	1046
1950	866	98	36	13	8	5	3	1	.	2	.	.	.	1032
	979	97	38	17	9	1	1	1	.	1	.	.	.	1144
1960	969	112	34	15	12	.	.	1	.	2	.	.	.	1145
	997	93	35	14	7	2	.	1	.	1	.	.	.	1150
1970	996	103	37	10	4	1	1	.	1	2	.	.	.	1155
	1007	99	38	17	3	2	.	.	1	2	.	.	.	1169
1980	994	93	36	14	9	1	.	.	1	2	.	.	.	1150
	1008	101	31	17	4	2	1	.	1	2	.	.	.	1167
1990	1000	105	37	16	4	2	.	.	2	2	.	.	.	1168
	1006	116	41	13	10	1	.	.	1	2	.	.	.	1190
2000	.	.	.	.	.	.	.	.	.	.	.	.	.	
	14910	1689	691	356	169	68	37	21	29	25	9	6	0	18010

Figure captions

Figure 1 Definition of rotation angles γ, β, φ.

Figure 2 Relations between displacements in 2D and 3D models.

Figure 3 The block structures under consideration a— BS1, b— BS2, c— BS3.

Figure 4 The moments of earthquakes (vertical lines) a— for the segment and b— for the whole structure BS1 for the time interval of 3 units.

Figure 5 Frequency-of-occurence plots for the synthetic catalogs for structures 1— BS1, 2— BS2, 3— BS3.

Figure 6 The sequence of cell states for the segment of BS3.

Figure 7 The Vrancea region: the main structural elements and the block structure used in the simulation with the numbers of the faults (1–9). The arrows outside and inside the structure indicate the movement of boundary blocks and the underlying medium respectively.

Figure 8 Frequency-of-occurence plots for the synthetic catalog for block structure of the Vrancea region.

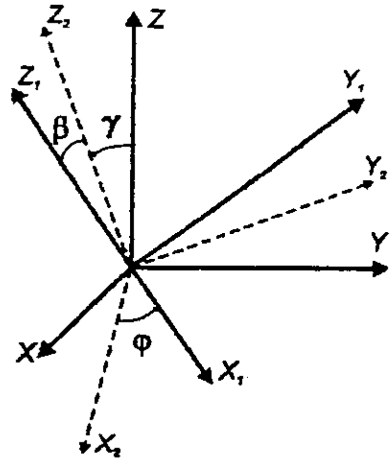

Z₂
Z
Z₁
β
γ
Y₁
Y₂
Y
X
φ
X₁
X₂

Fig. 1

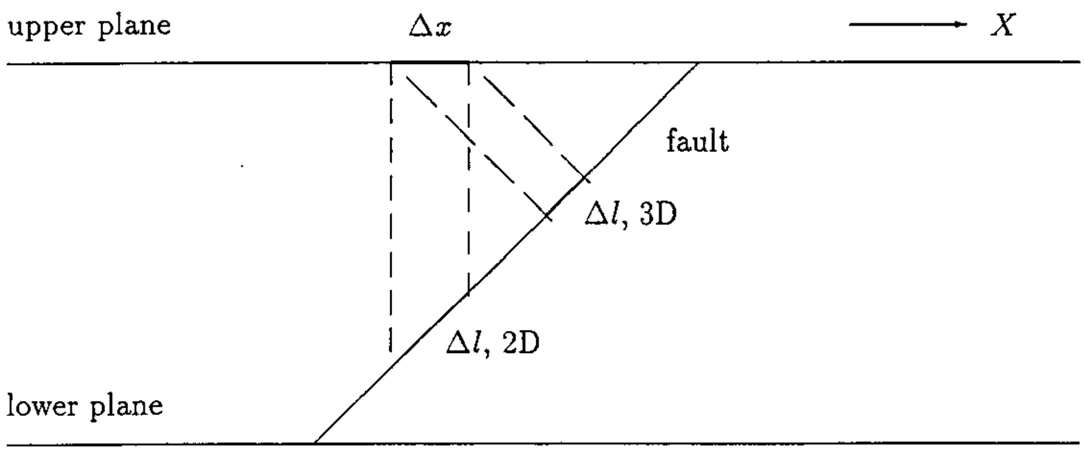

upper plane
Δx
X
fault
Δl, 3D
Δl, 2D
lower plane

Fig. 2

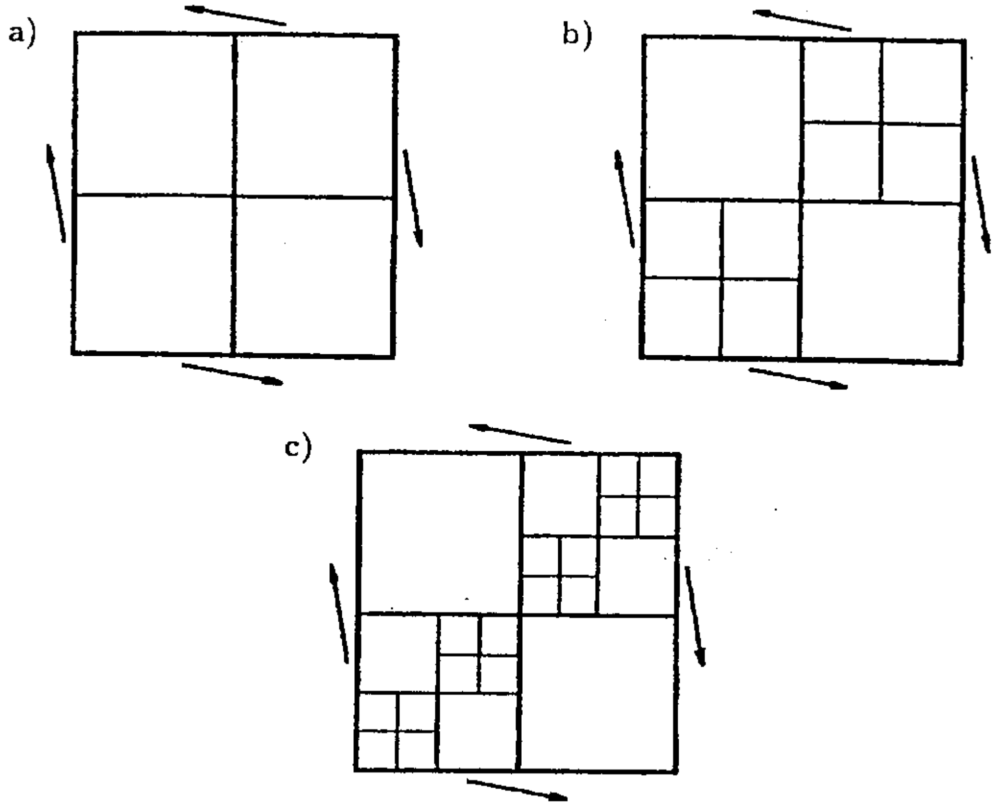
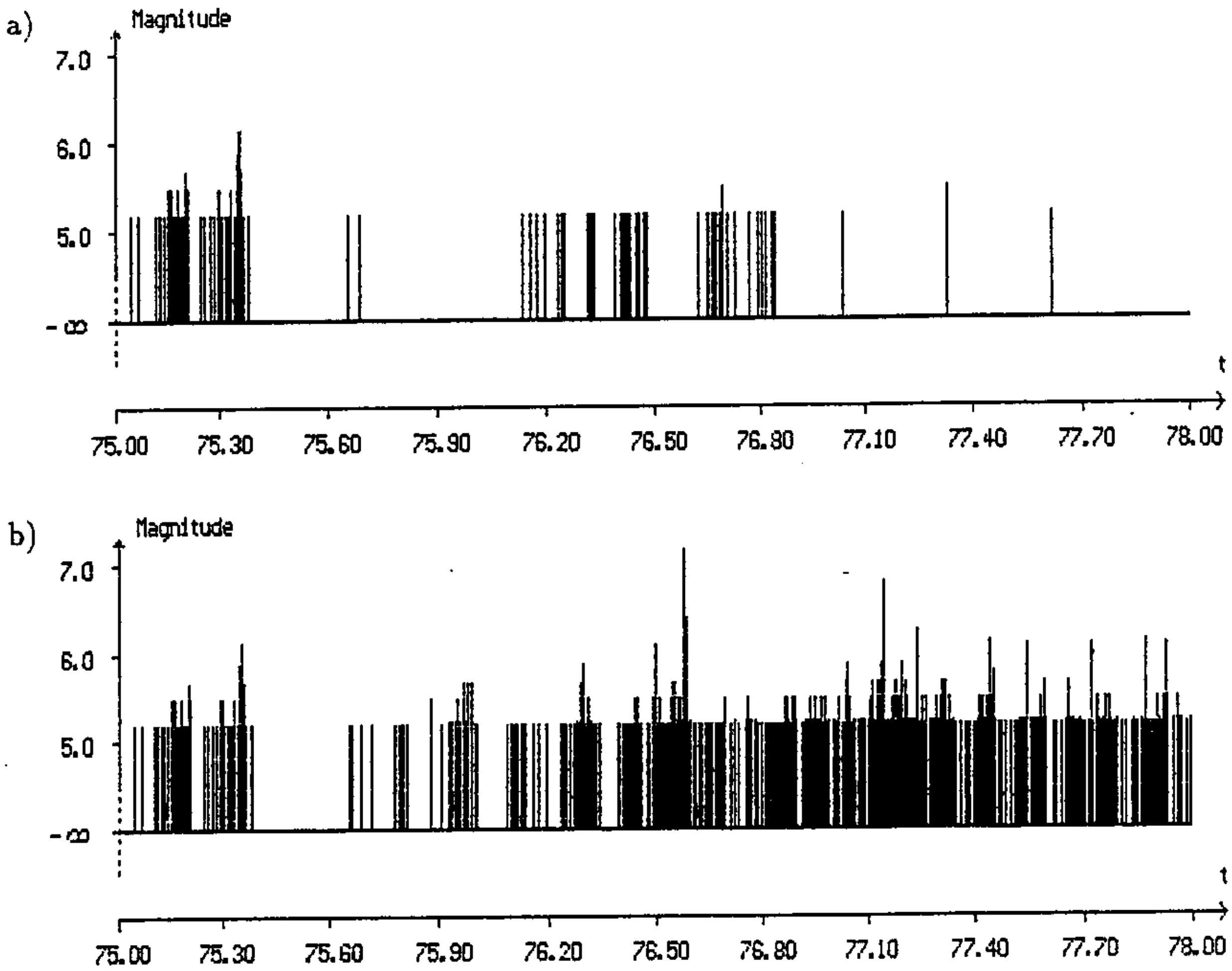
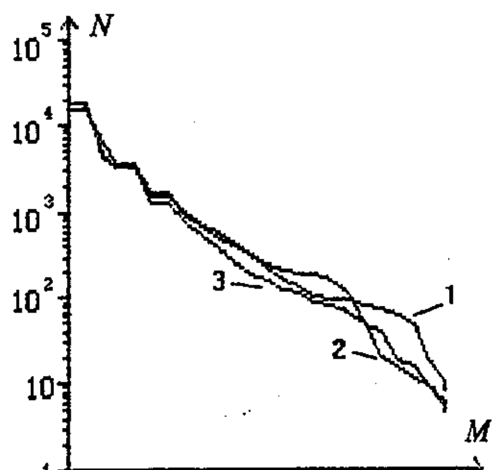


Fig. 3



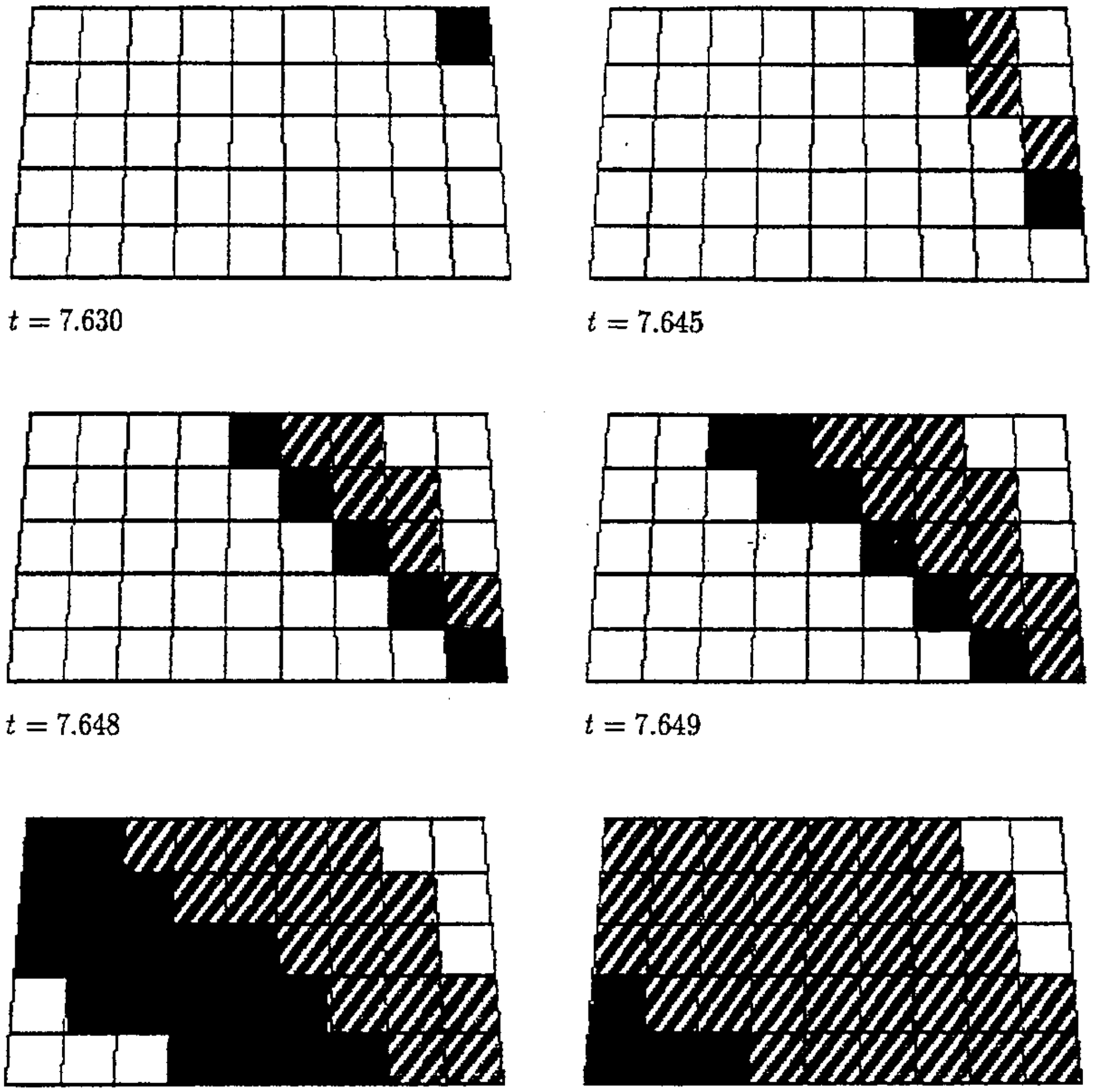
a)
Magnitude
7.0
6.0
5.0
-∞
t
75.00
75.30
75.60
75.90
76.20
76.50
76.80
77.10
77.40
77.70
78.00
b)
Magnitude
7.0
6.0
5.0
-∞
t
75.00
75.30
75.60
75.90
76.20
76.50
76.80
77.10
77.40
77.70
78.00

Fig. 4



N
M
1
2
3
5.0
5.5
6.0
6.5
7.0

Fig. 5



$t = 7.630$ $t = 7.645$

$t = 7.648$ $t = 7.649$

$t = 7.650$ $t = 7.651$

Fig. 6

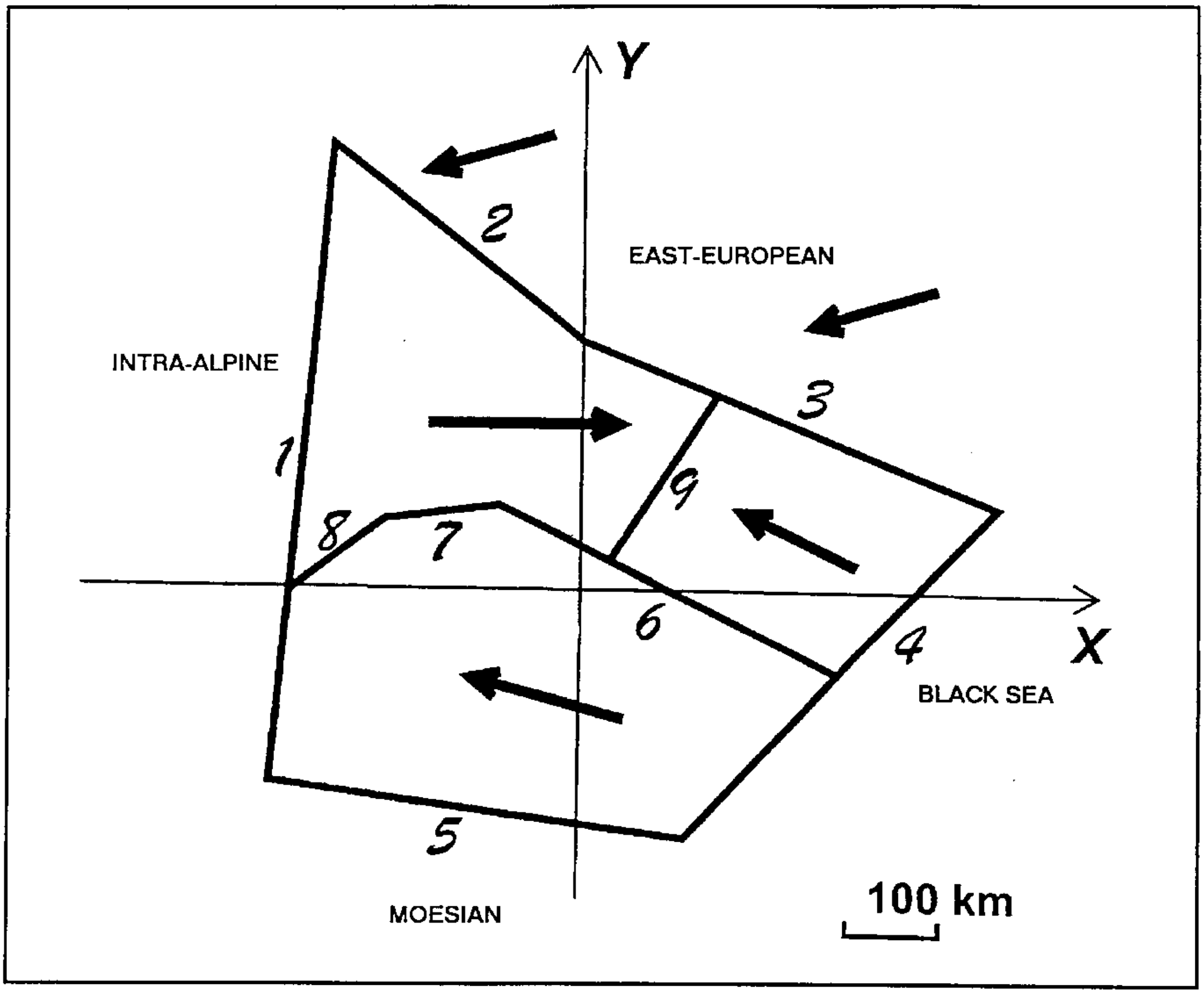

Y
EAST-EUROPEAN
INTRA-ALPINE
BLACK SEA
MOESIAN
X
100 km
1
2
3
4
5
6
7
8
9

Fig. 7

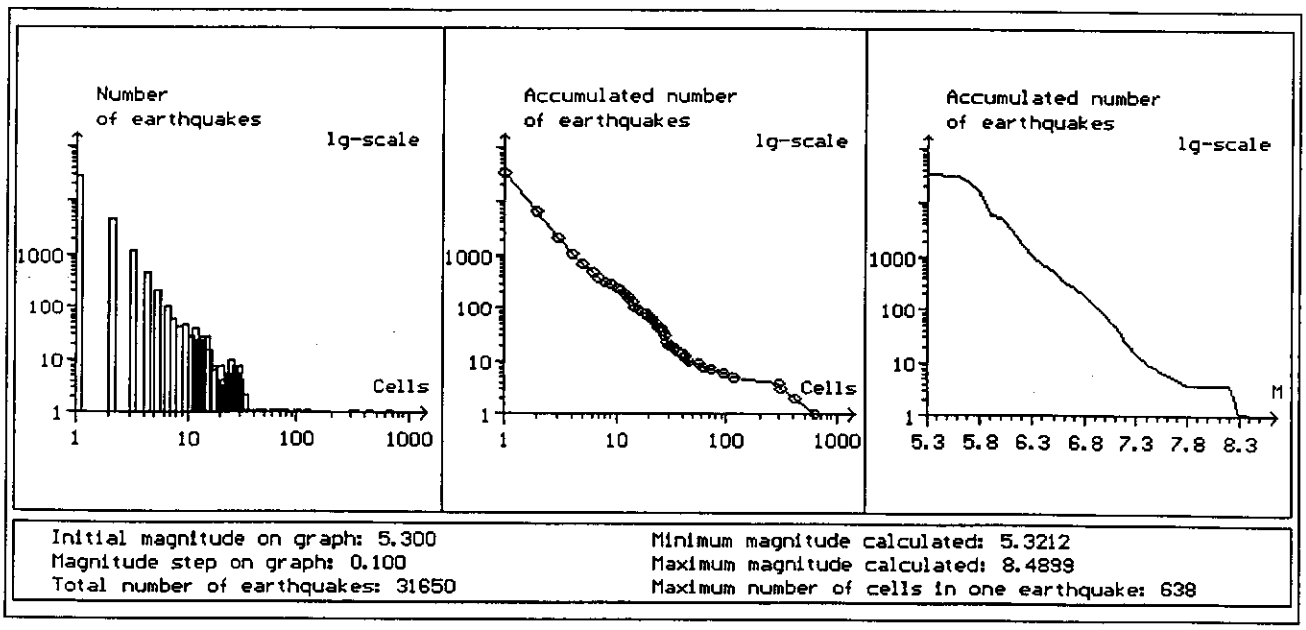

Number
of earthquakes
lg-scale
Cells
Accumulated number
of earthquakes
lg-scale
Cells
Accumulated number
of earthquakes
lg-scale
M
Initial magnitude on graph: 5.300
Magnitude step on graph: 0.100
Total number of earthquakes: 31650
Minimum magnitude calculated: 5.3212
Maximum magnitude calculated: 8.4899
Maximum number of cells in one earthquake: 638

Fig. 8