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H4-SMR 1012/1

AUTUMN COLLEGE ON PLASMA PHYSICS

13 October - 7 November 1997

Plasma Computer Experiments Laboratory

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University of California, EECS Department

These are lecture notes, intended for distribution to participants.

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PLASMA COMPUTER EXPERIMENTS LABORATORY

by

C. K. Birdsall, V. P. Gopinath, and J. P. Verboncoeur

Memorandum No. UCB/ERL M96/86

20 December 1996

ELECTRONICS RESEARCH LABORATORY

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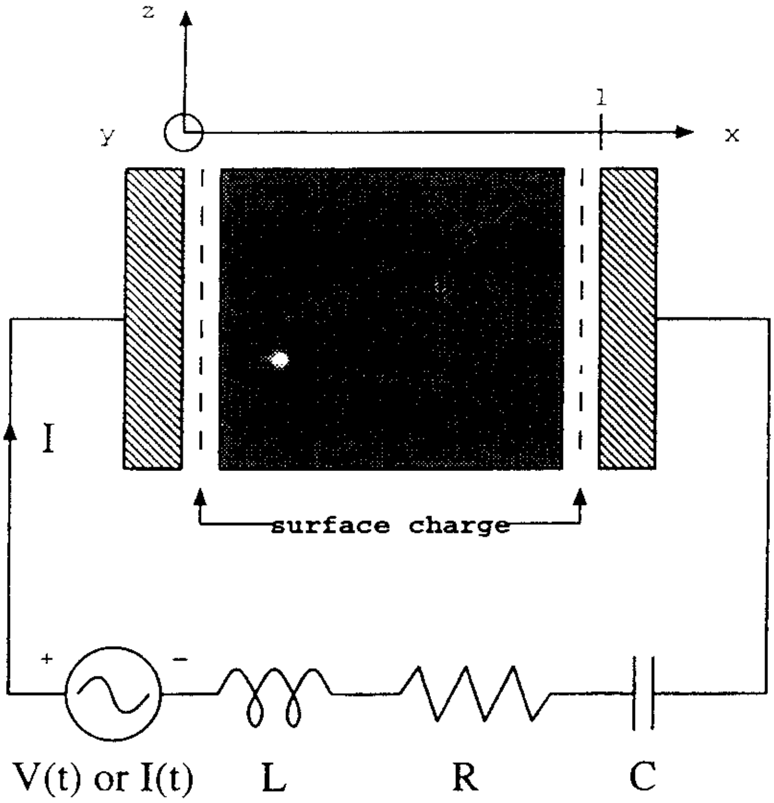

Plasma Computer Experiments Laboratory

Prof. C. K. Birdsall, Dr. V. P. Gopinath and Dr. J. P. Verboncoeur
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Berkeley, CA 94720

Frontispiece to:

PLASMA COMPUTER EXPERIMENTS LABORATORY

In 1995 we designed this lab to complement the lectures given by Prof. Michael A. Lieberman for his course using the text *Principles of Plasma Discharges and Materials Processing* by himself and Prof. Allan J. Lichtenberg (Wiley, 1994). For 15 weeks we followed his analytic modeling in class with related computer experiments in our lab. We had a weekly one-hour demonstration-discussion period, with homeworks, computer experiments, assigned, due the next week. Two of our students, Igor Kouznetsov and Regina Soufli turned in minor Masters theses each week. The rest did the experiments, eventually. Each student who put some effort into the homeworks gained considerable insight into the plasma physics of discharges used in plasma processing.

In 1996, we repeated the lab, again accompanying Prof. Lieberman's lectures. This time, at the 1995 students' suggestion, we made the meetings two hours, with the first used for demonstrations, and the second used for discussions. Future offerings should require the lab.

What to do about these notes? We have given them away to many professors with like-minded purposes. We have thought of publishing a monograph. We have talked with Professors Lieberman and Lichtenberg about incorporating these into their second edition,now in preparation, appearing as homeworks in most of their chapters (we like this idea).

For a stop-gap, we are bringing the present notes out as an ERL report, to be distributed on our mailing list of some 200+ researchers.

We advocate the use of computer simulations and computer experiments in ALL physical and chemical sciences and engineering courses. We propose that freshman (or high school) students learn about finite difference equations, which can immediately be used in computer simulations, along-side of infinitesimals (for ode's and pde's). Students would then use fde's in ALL science and engineering courses, and have their own codes. When they graduate, all of them will be using a great number of codes.

This report is dedicated to Drs. Vahid Vahedi and John Verboncoeur who, as graduate students, wrote our Plasma Devices series of codes and their graphics which made these labs possible, as well as originating many of the input files used. The codes are so robust that we could run the first five weeks before we said anything about accuracy and stability, allowing the students to run simply by editing only the input files, which were 95% physical and only 5% numerical.

CKB, VPG, JPV, end 1996

PLASMA COMPUTER EXPERIMENTS LABORATORY(1995)

PLASMA THEORY AND SIMULATION GROUP (PTSG)
PROF. C.K. BIRDSALL, DR. V.P. GOPINATH, DR. J.P. VERBONCOEUR
EECS DEPARTMENT, CORY HALL,
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FOREWARD
PREFACE
ACKNOWLEDGMENTS
CONTENTS

- I Course description and instructions
- II XPDP1, bounded plasma basics, sheaths
  - **maxwella:** Short circuited diode, demonstrates rapid loss of non-neutralized ions.
  - **maxwelle:** Decay of warm Maxwellian plasma, fixed ions, demonstrates sheath.
  - **maxwello:** Open circuit equivalent of maxwelle
  - **maxwellep:** Electron-Proton short circuited plasma, no collisions.
  - **bmaxwell:** Magnetized plasma diode, demonstrates negative potential formation.
  - **childhe:** Emulates collisionless RF driven child law sheath
  - **os100m:** RF Sheath with oscillating sheath width.
  - **diffusion:** Simple diffusion of zero charge particles, neutral collisions included.
  - **ambipolar:** Demonstrates ambipolar diffusion of charged particles.
- III XES1, Oscillations and Instabilities
  - **coldplas:** Oscillations in a cold electron plasma with neutralizing immobile ions.
  - **hybrid:** Demonstrates hybrid plasma oscillations.
  - **beamplas:** Demonstration of cold beam instability.
  - **2stream:** Study of 2 stream instability.
- IV XPDP1, RF discharges, Planar
  - **rfdanc:** Voltage driven RF plasma; no collisions.
  - **rfdhomog:** Study of the ion matrix sheath model.
  - **rfda:** Argon plasma, both species with electron and ion-neutral collisions.
- V XPDC1, cylindrical RF discharges
  - **maxwellea:** Demonstration of a cylindrical plasma diode.
  - **rfda2:** Argon plasma, demonstrates asymmetrical sheath and heating.
- VI XPDP1, mixed
  - **ecr:** Simple ECR demonstration.
  - **piiia:** Plasma Ion Immersion Implantation example.
  - **dch:** DC discharge example.
- VII PTSG Plasma Device Codes Available, How to Obtain (free), Updates
- VIII List of Publications using Plasma Device Codes from PTSG
- IX Acknowledgments to Berkeley Code and Graphics Developers
- X List of articles reprinted for class

Videotapes of Spring 1995 may be viewed in 205 McLaughlin. Also available for $279; contact Pam Atkinson, atkins@coe.berkeley.edu (Ask for EECS 298-9 tapes, Spring 1995 **Course Notes** available: Contact birdsall@eecs.berkeley.edu
Codes obtainable from WWW: http://ptsg.eecs.berkeley.edu

PREFACE (C.K. Birdsall)

(a) Objectives, relation to L&L text, quantitative, from first-principles, dynamic, etc., not just pictures.

(b) Spring 1995 trial, with help from PG on every homework, assistance from JV on several homeworks, consulting with MAL and AJL.

(c) current input files developed in doing research, by many people; will try to put authors and dates on .inp files, and on updates.

(d) the graphics make simple the understanding of numerical experiments and extracting physics and chemistry numbers.

(e) Contributions, suggestions, ideas from outside PTSG are most welcome and will be acknowledged.

The Spring 1995 lab was an experiment itself, to see whether a lab would materially aid in teaching of Professor Lieberman's EECS 239 course, from his new text (with Professor Lichtenberg). We have requested critiques from the 8 or so students, asking various general questions, so as to aid in planning repeating this lab (or similar) in the future.

We (mostly Dr. V.P. Gopinath and me) prepared the lab demos and assignments week by week. We (mostly me) seldom had the homework ready to hand out (on e-mail) at the meetings of the lab. Hence, there were logistics criticisms, understood. (Such will not occur again, as we have the record from this term.)

One student pointed out that it would be helpful, in the last few minutes of the lab/lecture, to review the assignment, parts of which were spread throughout the hour lecture. Accepted.

Taking a cue from Prof. R.M. White's "First Course", we also might consider adding an hour long discussion session (after the lab assignment is made) to go over the various methods of doing the computer experiment and in making the measurements from the experiment. To be run by the professor. Done in 1996.

The "real" experiment was in NOT spending much time at HOW the computer experiments do the numerics. Indeed, we gave the codes to the students on the first day, showed them how to enter the numbers to an input file, told them how to run the computer experiment, and how to use the graphics, and made the first assignment. Not until about the fourth lab, when Professor Lieberman was doing chemistry (for which we had no simulations), did we tell them, briefly, about the numerical methods used, with Δt and Δx , accuracy and stability, plus pitfalls. We used a basic 1d3v periodic code for this (XES1), with assignments on basic plasma physics, as in the Birdsall and Langdon text, Chapter 5. To all indications from the students, this approach appears to have worked all right, to our great relief (as the 239 course and our lab were not meant to emphasize numerical methods per se.)

The L&L book is a major contribution to plasma discharges and materials processing. The big step forward is contributing *modeling* and *analysis*, much of it original with L&L, or added to, and putting such all together.

Our contribution is to add plasma simulations, with many displays, adding further insight, and quantitatively confirming the L&L modeling and analysis.

An overall objective of both contributions is to encourage more modeling, analysis, and simulation in the semiconductor equipment manufacturing industry and also in semiconductor manufacturers, as well, of course to provide students skilled in *modeling, analysis* , and *simulation* to these industries (both now highly empirical, highly driven by time-to-market pressures, and highly successful!)

The level of the homeworks, this first time through, is below the quantitative capabilities of the computer experiments. This level will be raised the next time through. Suggestions are most welcome.

Videos are available of both Professor Lieberman's 45 lectures (EE239) and of Professor Birdsall's 15 lectures and demonstrations (EE298-9), Spring 1995. The purchase prices are $975 and $279 respectively. Contact: T.V. office, tel. (510)642-5776, fax (510)643-5877, atkins@coe.berkeley.edu.

Clearly, processes missing for the current simulations (containing only electrons and ions, with a much denser cool neutral gas) are volume chemistry and surface effects involving charges and neutral particles. There are attempts to include such. Perhaps a "platform" will evolve uniting plasma simulation, volume chemistry, and surface effects, a true companion to the L&L text.

Please submit your Homework Assignments as

COMPUTER EXPERIMENT LAB REPORTS

with the following format:

First page:

Your name and date of your experiment(s); title of the computer experiment (FILENAME); Copy of
e-mail assignment questions, suggestions.

Second page(s):

INPUT FILE(s) which you used. Circle with colored pen or highlight changes which you made from the input file supplied.

Next pages:

Printer outputs from your computer experimental runs.
Answer the assignment questions on your output sheets including ALL calculations, indicating times, lengths, frequencies, etc. directly on the plots, making good use of the cross hair to increase your accuracy of measurement.
Please do NOT make printer outputs of ALL possible diagnostics, which simply wastes paper. Still, make as many as you like.

Last page(s):

Your comments on the computer experiment(s)
done, such as:
improvements which you made;
improvements you would like to see included (e.g., new graphics)
any “strangeness” in your observations, numerical or physical (such as why you see series resonance with no smoothing, but do not see it with smoothing).

The first intention is to give you a good record of what you did, one that you might want to come back to (who knows when?)

The second intention is to make grading of your computer experiments easier for me as I found that I was spending 1/2 hour per homework paper, trying to find everything.

Thank you, Ned Birdsall 16 March 1995.

Dear Class Members:

We are slowly getting your homeworks read and graded. We feel rewarded that some of you have really gotten into the spirit of doing the computer experiments and then comparing these numbers with numbers from the analyses presented by Prof. Lieberman or from models suggested in the Homework Assignments. It is these quantitative comparisons that are most valuable.

Please follow the instructions on formatting your Homeworks, given on e-mail of March 16.

This week and next, we want to see each of you for an informal chat on how you are progressing with the Homeworks (bring them along) and on how we might help you. These chats are wholly ungraded. Sign-up sheets are on my door (191M Cory) and PG and John's door (187M Cory).

At the April 26 class, please hand in a course evaluation, commenting on anything you like. What has helped most, or least, in understanding what is being given by Prof. Lieberman. What would you like to have added, or taken out. Comments on content. Comments on style. Should there be a one hour lecture-demonstration only, (as is now the case) or should there be an additional hour of discussion?

(Please realize that this computer experiments lab has been an experiment for all of us, with no time for advanced planning and preparation, that the next time will benefit from this experience and from your suggestions.)

The FINAL EXAM will be oral, one hour, with both examination of your Homeworks and questions on your results, and on your skills at the terminal. For a satisfactory or pass grade, a minimum of half the Homeworks should be done and done correctly. (Next time, the minimum will be much higher.) We will set up times very soon, for the period May 5-11 (before formal finals begin). (Not done; projects demonstrated instead.)

Ned Birdsall

13 April 1995

Dear Class:

You are receiving one or two or more assignments each week, to run various input files, to vary the input parameters and to answer questions on what you observe.

Once you have gotten into a routine, we think that you should find the work rather straightforward. The biggest mistake is to let it go, not turning in your work weekly.

Please keep your homeworks in a binder. We will try to return them to you weekly.

At mid-term time, we plan to set up one hour "informal exams", looking at your binders with you, asking you (and answering) questions, and having you show us your ability at running our codes, on one of our workstations. (Done)

At final exam time, we will do the same, for the problems run in the second half of the term. (Note done)

C.K. Birdsall

6 February 1995

MAXWELLA.INP

One objective of our Laboratory is to obtain meaningful numbers directly from the simulations, and to compare these with analytic (theoretical) answers.
On these first examples,we will look more for estimates rather than three place accuracy, with some exceptions.
Your encouraged to make plots from your computer runs, including the input file. You may write directly on the plots, if this makes answering the questions easier, or more directly.

MAXWELLA questions:

The model is as diode, two parallel plates separated 0.1 m (10 cm), with the planes tied together by an external short circuit; take both plates to be at zero potential. The space between is uniformly filled with argon ions of mass $M = 80,000 \times m_e$. (There are no other particles; this is a non-neutral plasma.) The ions are cold, and have no velocity at $t = 0$.

(1) Solve Laplace's equation for the potential in the diode, $\phi(x)$. This should be a parabola.

(2) Write an expression for the maximum potential, ϕ_{max}.

(3) Use an initial density of $n_{io} = 10^{16}/m^3$. What is ϕ_{max}? (Should be 226,000 volts.)

(4)Estimate the time for a typical ion to leave the system; should be on the order of some tens of nanoseconds.

(5) Check your answers for (2, 3, 4) with your computer run. Agree?

Conclusion: hard to keep a single species, non-neutral, from escaping.
You might try electrons, which escape sqrt of the mass ratio faster.

More tomorrow on MAXWELLE questions.
C.K. Birdsall 26 Jan 1995

MAXWELLA.INP (IN MKS UNITS)

Cold argon ions in a **short circuited** diode, to demonstrate rapid loss of **non-neutralized** ions.

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 1    100   1e11  5e-10     0.1         0.01         1.0        0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 0.0                0.0                0.0     0          0.0     1e10    0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0          v        0.0           0.0                0.0         0.0     0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0              0                   0            0        256       0            0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                  0.2            2                 5e-2              .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19            0.0             0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
1.0e-20             12.0            50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20             13.6           60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19           0.0                     2.0e-19       0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 66.8e-27     0.0               0.             1e16

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0            0.            0.0       0.0             0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0             50        0             3e5       50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                        100       0.01        15.0          .045       .055
```

MAXWELLE

This model has warm (temperature 1eV) mobile electrons and fixed ions, between two parallel planes which are connected by a short circuit. Take both planes to be held at $\phi = 0$. The object is to observe and measure the formation of a sheath at the edge of the plasma, and make as many other observations as you care.

(1) Observe ϕ_{mid}, the potential at the middle of the diode as a function of time. What is the approximate rise time (to a few times kT_e)? Compare this time with the transit time across the diode of an electron at thermal velocity. (Show that at 1eV the thermal velocity is 4.2×10^5 m/s.)

(2) Observe the oscillations of the mid-potential in time. Measure the frequency (either by using the cross-hairs to measure a cycle time; frequency = 1/period), or use the FFT of ϕ_{mid}, labelled "Mag ϕ_{mid}", and use the cross hair to get f. Compare your measured frequency with the plasma frequency found from the density (measured in the center). Using the approximate expression $f_{plasma} = 9\sqrt{n_e/m^3}$. Note that a density of $10^{14}/m^3$ produces $f_{plasma} = 90MHz$.

(3) Observe the sheath width, say, from either wall in to where the electric field is roughly zero. A common way is look at the time average E, and draw a straight line along E at the left wall up to where the line crosses E = 0. For the time being, let this be defined as the sheath width, s. Compare this width with the Debye length, defined by $\lambda_d = V_{thermal}/\omega_{pe}$ frequency. (Write a simple expression for λ_D, equal to a constant times the $\sqrt{T_e/n}$; find the constant, using MKS units to give meters, and using CGS units to give centimeters.) We expect that s is a few Debye lengths.

(4) Observe the current and measure the oscillation frequency, which is a fraction of the plasma frequency, called the series resonance frequency (where the diode total admittance of the diode goes to infinity). $f_{series} = f_{plasma}\sqrt{2s/L}$.

(5) Turn the trace on on the vx-x phase space and note the electron trajectories. After the sheath forms, the electrons remain trapped by the repelling fields of the sheaths, and escape hardly at all. The fastest electrons have escaped and charged the walls negatively. See whether the velocity of the fastest remaining electrons corresponds the the average phi-mid (i.e., $1/2\ mv^2 = q\ \phi_{mid}$). Note that the original distribution function, with full Maxwellian tail (out to about 3 or 4 $V_{thermal}$) becomes truncated as the tail gets absorbed. (There are no collisions, which would replenish the tail.)

(6) Explain how to obtain the quantity (on the input file) called rhoback. It is the uniforma and constant ion background charge density, set up to equal the initial uniform electron charge density, but opposite in sign. Provide an analytic expression for rhoback.

(7) The time step is given to be 3×10^{-10} sec. The code uses a leap-frog mover to advance the velocity and then the position; this mover is numerically stable if $(\omega_{pe}\Delta t < 0)2$, and usually sufficiently accurate for ($\omega_p\Delta t$) less than 0.2. What is the value of $\omega_{pe}\Delta t$ in this model? Is it stable and accurate? (You are invited to increase Δt to make the model numerically unstable.)

(8) Look at the diagnostic J.E, which is the rate of work done by E on the current J, with units of watts/volume (power density). Explain what you see, which is a negative power density, in the sheath. We will meet this quantity again (both + and -) in looking at RF driven plasmas, where the electrons may be heated in the sheath, as well as in the bulk.

Try ALL of the diagnostics available. Note the self consistency of all of them. For experimentalists in the class, note tha all diagnistics here are non-invasive; we are not putting probes in of finite area, or drawing current with wires. This ability to make intimate measurements without interference is a very real plus for simulation.

C. K. Birdsall
27 January 1995

MAXWELLE.INP (IN MKS UNITS)

Decay of a warm electron maxwellian distribution and **fixed ions** in a short cicuited diode, to demonstrate sheath formation

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 1    200   2.5e7  3e-10     0.05     0.01        1.0       0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 1.602e-5          0.0              0.0     0          0.0     1e10     0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0           v        0.0           0.0             0.0        0.0      0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0              0                 0              0      256       0           0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                  0.2         2                5e-2               .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19           0.0            0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20           12.0           50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20           13.6           60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19          0.0                   2.0e-19      0.0


SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31    0.0              0.0              1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0           0.         4.19e5     4.19e5       0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0          150       0           10        50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100     0.01       15.0         .02      .03
```

MAXWELLO.INP

These are to follow what you have done for maxwella and maxwelle.

This is the same as MAXWELLE.INP, except that the diode external circuit is an open circuit; the C external was made zero. This model was given in the L&L text, Figure 1.11, pages 12-15.

(At the time that was done, we had the left hand electrode at zero potential; this is now changed, with the right hand electrode held at zero potential, accomodating our cylindrical and spherical codes, with the outer electrode at zero potential, appearing at the right.)

Repeat the exercises suggested with MAXWELLE.INP. One decided difference will be that I = 0, so that there is no series resonance.

ENJOY!

Ned Birdsall
March 10, 1995

```
                         MAXWELLO.INP
DECAY OF A MAXWELLIAN DIST.(IN MKS UNITS)
Same as maxwelle.inp but with Open circuit.

-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 1    200    2.5e7   3e-10     0.05       0.01            1.0          0.0          0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 1.602e-5               0.0                   0.0       0          0.0        0.0       0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
0                 v           0.0                0.0                  0.0            0.0      0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
0                  0                     0         0           256       0                 0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                        0.2           2                      5e-2                  .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19               0.0              0.0               10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
1.0e-20                12.0                                   50.0   100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20               13.6              60.0              110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19               0.0                               2.0e-19            0.0

SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.0                     0.                  1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0              0.              4.19e5       4.19e5         0.           0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0                      0.0            150          0              10            50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                               100         0.0          15.0            .02          .03
```

Dear Class:

Some scheduling first.

Critique of this experimental course, as requested 13 April, is due April 26. Thank you.

The FINAL EXAM will be informal, in the style of the "interviews" this past week.

12. Bring ALL of your EECS298-9 HOMEWORKS, a binder, in the format requested 16 March.

13. Bring your ideas on what HOMEWORKS you would have liked us to prepare (but did not) to cover a topic presented by Prof. Lieberman in EECS 239. Prepare a FILE-NAME.INP and a typed sheet on HOMEWORK, with objectives and questions (one page). Bring sample runs and your numerical checks with Lieberman analyses. We may run such during the exam. Your grade on such will depend on the sophistication of the problem chosen; no Mickey Mouse, please.

14. A sign up sheet will be prepared to do these "interviews" Monday, Tuesday, Wednesday, May 8,9,10.

15. An alternative to individual "interviews" is to have everyone attend a three-hour session on May 8 or 9 or 10 (the usual exam length of time), with each of you presenting your answer to 2. and running such. A valiant attempt (imperfect results, due to time limitation now to May 8) will be all right. Also, it is all right for two of you to work together. We will "vote" on this alternative in the next class meeting (April. 26).

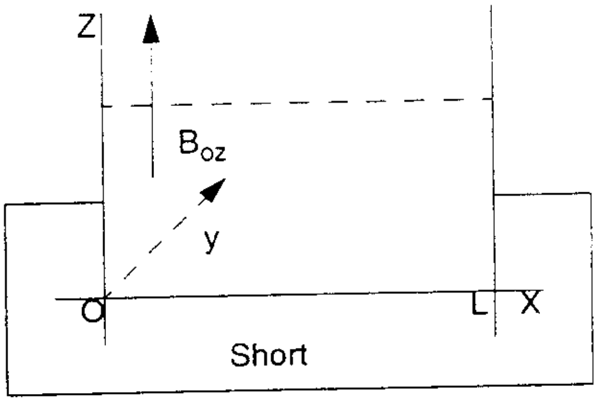
16. Depending on whether 4. is adopted (which day), we will have a pizza party at LaVal's after the end of the final, tentatively set for 5p.m. Wednesday May 10 (my treat).

MAXWELLOXY.INP (IN MKS UNITS)

3 Species, attempt to simulate Fig 10.3, pg 324 of L and L , density ratios 0.2:0.8:1.0 Decay of an **electron:-O::O2+** plasma, warm Maxwellian distribution, short circuited diode, to study sheath formation and particle loss in an electronegative plasma.

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 3      500  1e6  1.5625e-10  0.03    0.01       1.0       0.0        0.0
-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]----q0[C]-
 0.0               0.0              0.0    0         0.0      1.0      0.0
-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0          v       0.0          0.0              0.0        1e7      0.0
--secondary---e_collisional---i_collisional--reflux---nfft--nsmoothing--ntimestep--
 0             0               0              0        256    5            0
--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                 0.2          2              5e-2               .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19          0.0            0.0            10.0
--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20          12.0           50.0           100.0
--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20                         13.6           60.0 110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19         0.0                     2.0e-19      0.0
SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31  0.              0.                0.2e14
--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.         4.19e5     4.19e5      0.         0.
---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0           150      0.0         5.0        50000
-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       200      0.0        5.0         .0035 .0065
SPECIES 2
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 2.672e-26 0.              0.                0.8e14
--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.           0.         3.69e2     3.69e2      0.0        0.0
---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0           150      0           5.0        50000
-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       200      0.0        5.0         .0035     .0065
SPECIES 3
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 5.344e-26 0.               0.                1e14
--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.           0.         2.60e2     2.60e2      0.0        0.0
---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0           150      0           5.0        50000
-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
 200                   0.0      5.0        .0035       .0065
```

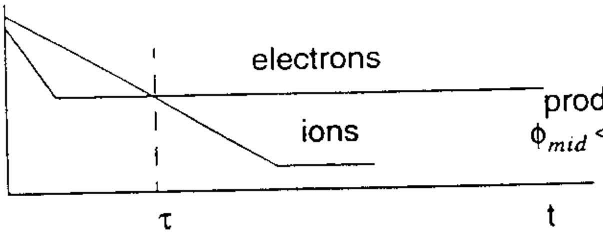
b maxwell



1. Calculate electron and ion gyroradii, r_{ce},r_{ci}

2. What are: $\frac{L}{r_{ce}}, \frac{L}{r_{ci}}, f_{ci}, f_{ce}$?

3. Note number decay time

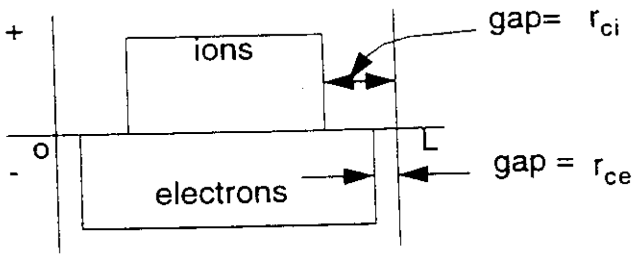


produces net negative plasma, $\phi_{mid} < 0$ for $t > \tau$ totally trapped

Relate τ to τ_{ci}, roughly time to capture edge ions.

4. Observe ϕ_{mid} (t); relate the fast and slow oscillations to f_{ce} and f_{pe}.

5. Provide an explanation for $\phi_{mid} \cong -0.3$ volts and not the 3 to 5 kT_e for a positive plasma, $B_0 = 0$. Consider ϕ_{mid} for density as:



Or, invent your own explanation.

6. Change ψ from 90° to 90° - δ; try $\delta < \sqrt{\frac{m_e}{m_i}}$, $\delta > \sqrt{\frac{m_e}{m_i}}$. Particles lost?

1. Magnetized Plasmas

- Equations of motion
- Simple Harmonic Motion
- E × B drifts
- Hybrid oscillations
- Homework

1.1. Equations of Motion

$$m\frac{d\mathbf{v}}{dt} = q\left[\mathbf{E}(\mathbf{r},t) + \mathbf{v} \times \mathbf{B}(\mathbf{r},t)\right] \tag{1.1}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}(t) \tag{1.2}$$

- For $\mathbf{B} = B_0\hat{\mathbf{z}}$ and $\mathbf{E} = 0$, we obtain the cyclotron motion:

$$\frac{d^2 v_x}{dt^2} = -\omega_c^2 v_x \tag{1.3}$$

where the cyclotron frequency is given by $\omega_c = qB_0/m$. Gyroradius is $r_c = v_{\perp 0}/\left|\omega_c\right|$.

1.2. E × B drifts

Let $\mathbf{B} = B_0\hat{\mathbf{z}}$ and $\mathbf{E} = E_{\perp 0}\hat{\mathbf{x}} + E_{z0}\hat{\mathbf{z}}$.

$$\mathbf{v}(t) = v_z(t)\hat{\mathbf{z}} + \mathbf{v}_E + \Re\left(\mathbf{v}_{c0}\exp\left(j\omega_c t\right)\right) \tag{1.4}$$

where the perpendicular drift is

$$\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B_0^2} \tag{1.5}$$

Independent of mass and charge!

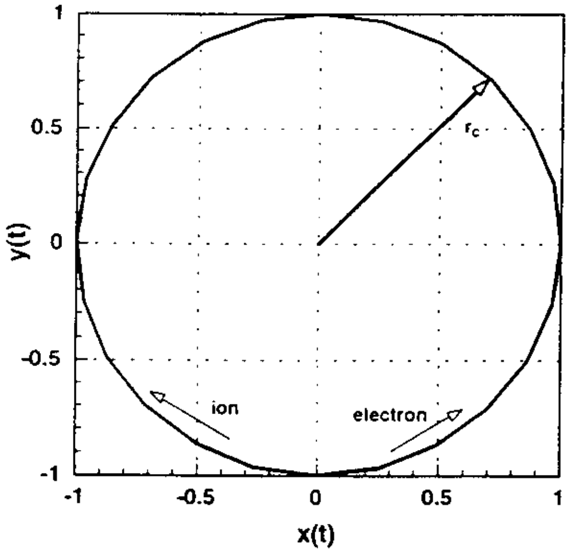


Figure 1.1: Cyclotron motion

1.3. Hybrid oscillations

Recall cold plasma oscillations

$$\omega_p = \sqrt{\frac{ne^2}{\epsilon m}} \tag{1.6}$$

Letting $kT_e = \mathbf{E}_0 = \mathbf{v}_0 = 0$; n_0 and $\mathbf{B}_0$ constant, and $M \gg m$, linearized electron equations become:

$$m\frac{\partial \mathbf{v}_1}{\partial t} = -e\left(\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0\right) \tag{1.7}$$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \mathbf{v}_1 = 0 \tag{1.8}$$

$$\epsilon_0 \nabla \cdot \mathbf{E}_1 = -en_1 \tag{1.9}$$

Choose $\mathbf{k}$, $\mathbf{E}_1$ in $\hat{\mathbf{x}}$, $\mathbf{B}$ in $\hat{\mathbf{z}}$. Then Eq. 1.6 becomes:

$$-i\omega m v_{x1} = -eE_1 - ev_{y1}B_0 \tag{1.10}$$

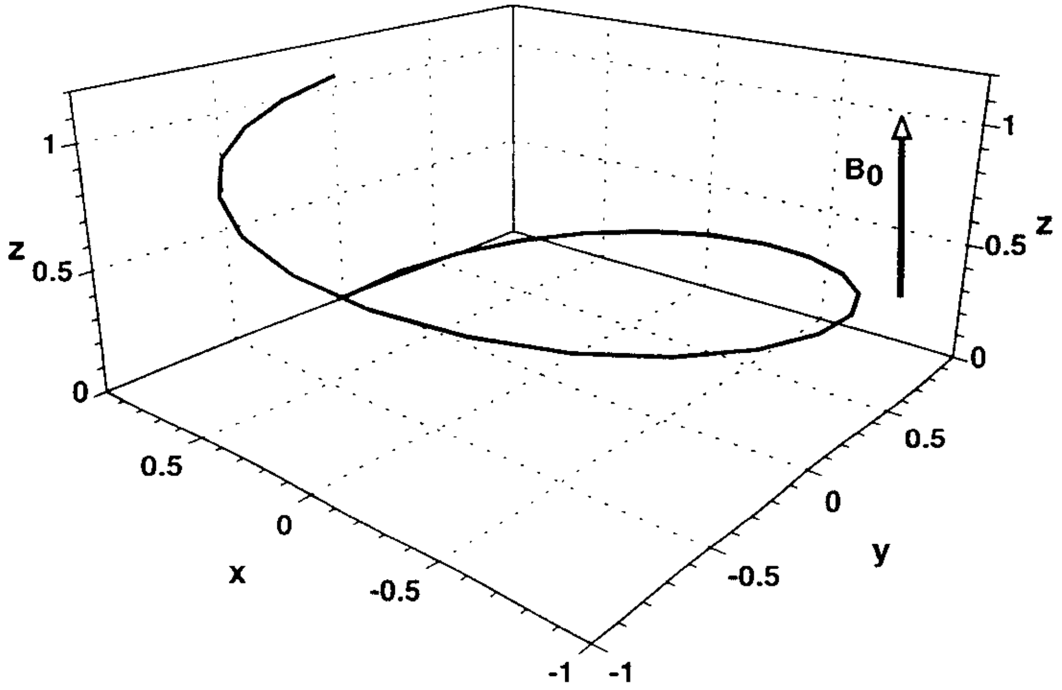


Figure 1.2:

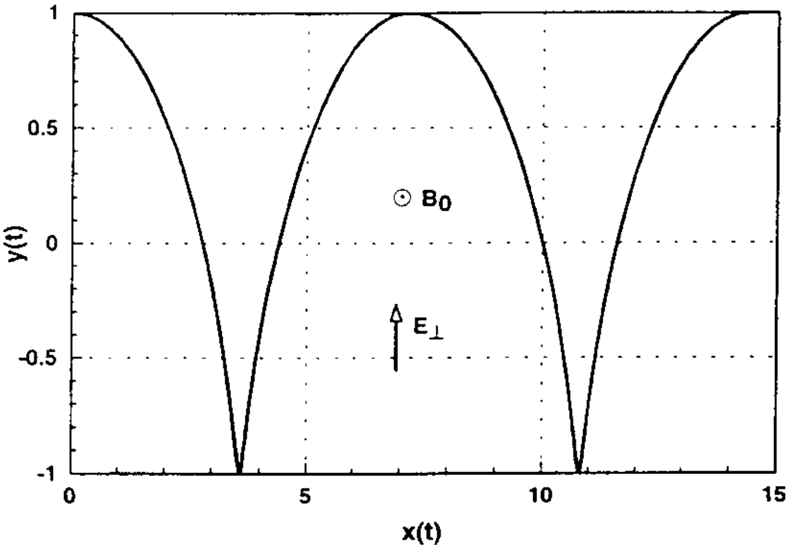


Figure 1.3: $E \times B$ drift

$$-i\omega m v_{y1} = e v_{x1} B_0 \tag{1.11}$$

$$-i\omega m v_{z1} = 0 \tag{1.12}$$

Then:

$$v_{x1} = \frac{eE_1/im\omega}{1 - \omega_c^2/\omega^2} \tag{1.13}$$

$$n_1 = \frac{k}{\omega} n_0 v_x \tag{1.14}$$

The dispersion relation becomes:

$$\omega^2 = \omega_h^2 = \omega_p^2 + \omega_c^2 \tag{1.15}$$

BMAXWELL.INP

(1) Given a 1d system containing equal densities of ion and electrons, with $M_i/m_e = 40$, and $T_e = T_i = 0.5$ eV, determine the angle psi (of the magnetic field, in XPDP1) at which the electron and ion loss rates are equal. The electrons are more mobilealong the field lines, but the ions are more mobile across the field lines. Show how this angle varies with mass ratio M_i/m_e.

(2) The input file is bmaxwell.inp. The angle of the magnetic field can be varied by editing the file and changing PSI (in degrees). (The original BMAXWELL.INP has psi = 90°, along the z axis.)

(3) A collisional variation of this exercise could also be tried. Add a background gas of Argon, for example, at some pressure like 10-100 mTorr. Then the mobilities and diffusion coefficients can be calculated (see the L&L text, Chapter 5), and compared to the simulation results.

To do the collisional part, cross sections are to be copied from RFDA.INP. Setting up this one may be a bit difficult.

Proposed by Dr. John Verboncoeur

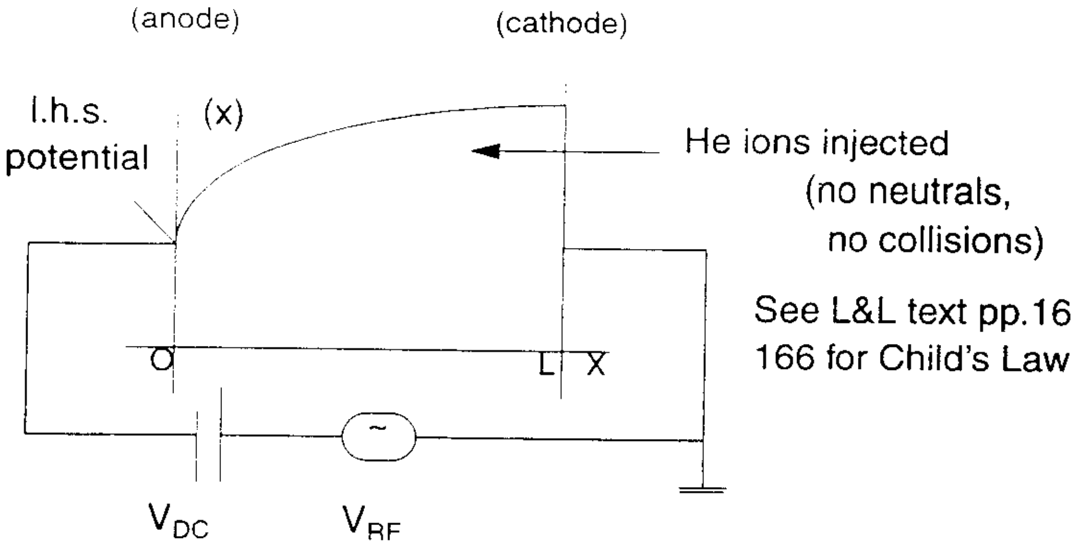
Edited by Ned Birdsall
10 March 1995

BMAXWELL.INP

Magnetized plasma diode, Short Circuit, **mass ratio 40**, demonstrates negative potential formation and trapping of both species

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 2    100    1e8    1e-10      1e-2     1.0         1.0       1.6e-2    90.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]--
 0.0               0.0                0.0    0.0       0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0          v       0.0        0.0               0.0        1e7      0.0

--secondary---e_collisional---i_collisional---reflux---nfft---nsmoothing--ntimestep--
 0              0               0              0        512     2           0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                 0.2          2               5e-2               .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19          0.0            0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20          12.0           50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20          13.6           60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19         0.0                     2.0e-19      0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31    0.0             0.                1e13

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.          4.2e5      4.2e5        0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 6.7e5            0.0            50       0.          10.        50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100     0.01       15.0        .045      .055

SPECIES 2

----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 3.644e-29    0.0             0.                1e13

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.           0.          6.66e4     6.66e4       0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 1.06e5           0.0            50       0.          10.        50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100     0.01       15.0        .045      .055
```

Child He.inp



See L&L text pp.164-166 for Child's Law

$V_0 = V_{DC} + V_{RF}\sin\omega_{RF} t$

This homework is in two parts (a) V_{DC}only, $V_{RF} = 0$.

(b) both V_{DC} and V_{RF}

Objective of (a) is to observe Child's Law (derived in 1911) with $\Phi(x) \sim x^{4/3}$, $E(x) \sim x^{1/3}$, $\rho(x) \sim x^{-2/3}$, using an injection velocity very small compared with v_{anode} (Like Problem 6.1, p. 89).

Objective of (b) is to emulate RF driven sheath and observe the ion energy distribution ("IED"), using initial velocity, $\upsilon_{x=L}\big|_{Bohm} \equiv \sqrt{\frac{kT_e}{m_i}}$ as expected from acceleration from the bulk plasma to the sheath edge (across the pre-sheath, L&L text, pp. 154-161).

CHILDHE.INP

Emi Kawamura demonstrated this for you in class on 15 February.

Child's Law was published in 1911, showing what current density should be expected from a copious electron emitter. (Child was a chemist, working with various oxides, seeking to obtain low work function coatings). The derivation is in the L&L text, pp 164-166.
We are interested in this Law, as in the sheath of an RF capacitively coupled discharge, there are very few electrons and the ions follow the time-average acceleration to the walls, much as in Child's Law. Now there is an emission velocity, taken to be the Bohm velocity, but usual acceleration is to to much higher velocities, so that much of the simple Child's Law (no initial velocity) holds.
The objective here is to replicate parts of Emi's results, for applying a DC voltage plus an RF voltage to a diode, in order to observe the Ion Energy Distribution, the IED, as you have in the handout. We will be interested in these results when we do the fully collisional, RF driven discharge, with ionization et al.
Here we have a simple way of seeing what can be done analytically and with a simple simulation (a few minutes).
The approximation that is not so good here is that the sheath width is held constant, that of our model diode. This assumption is taken out in the next model, OS100M.INP

The homework is to emulate Emi's results, part of such, say, at one frequency, looking at the IED.

Ned Birdsall
10 March 1995

```
                    CHILDHE.INP    CHILD LAW DIODE (IN MKS UNITS)
emulates collisionless RF driven sheath capacitive coupling Voltage-driven ion diode
(helium atom)

-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr---B[Tesla]---PSI[D]--
 1    100    3e7    1e-11      0.001    0.016       1.0          0.0          0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]---extL[H]---extC[F]---q0[C]-
 0.0                  0.0                  0.0        0.0          0.0        1.0       0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0               v          -100              0.0                 0              5e8      0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0                 0                   0          0           256       0             2000000

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]-
 0.0                  0.0            1                    5e-3               0.026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19             0.0              0.0              10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20             12.0             50.0             100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20             13.6             60.0             110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19            0.0                            2.0e-19           0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 3.34e-27    0                     38.5              0

--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.0          1e1          0            0            0.0          0.0

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]--max-np--
 0                   0.0           2000       0.0          200        60000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                             2000       0.0         200            .000000    .0000005
```

Figure1: IEDs(He gas)
Frequencies(13,33,50,100,200,500MHz)

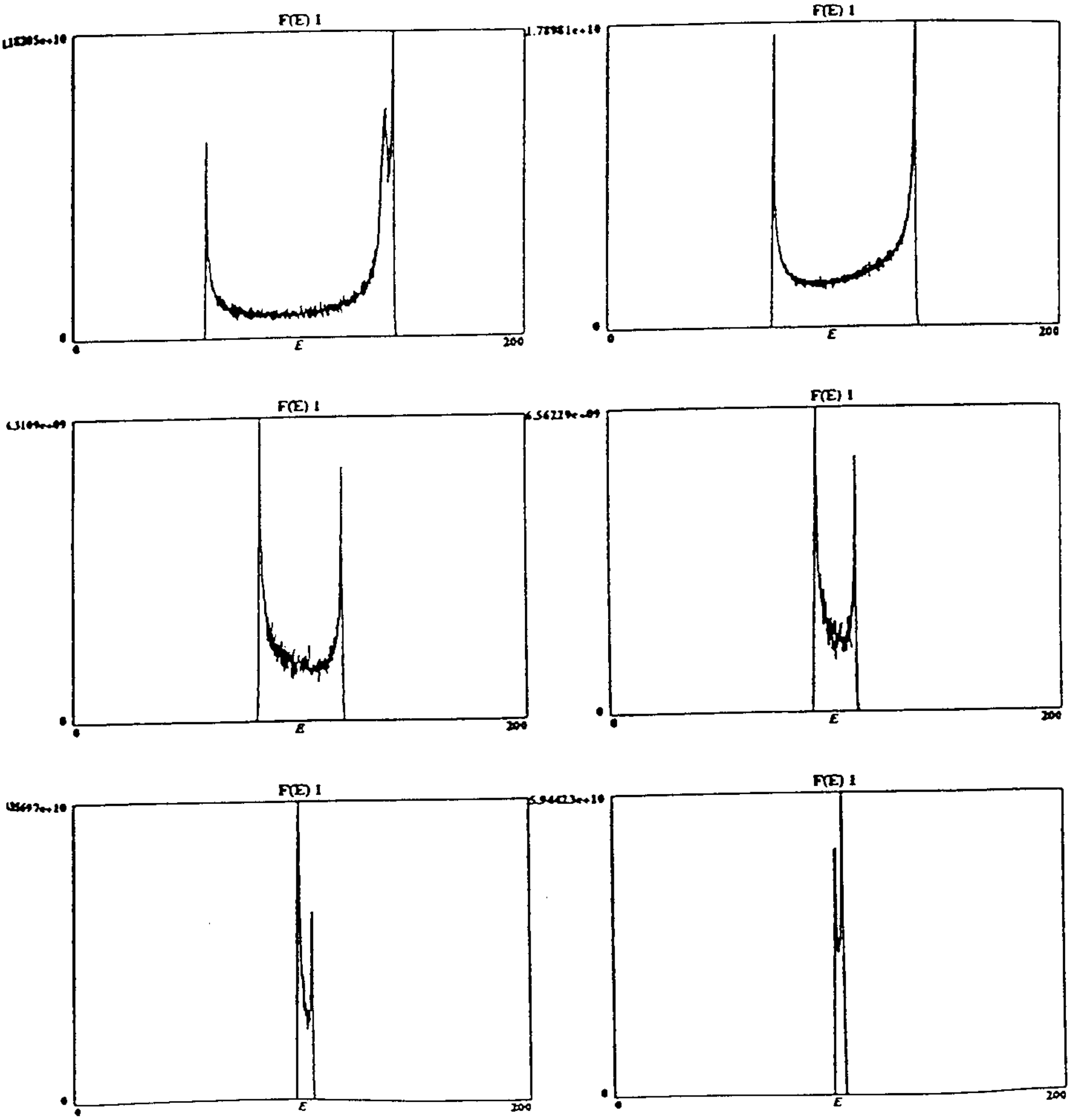


Figure 2: IEDs at 100MHz (He gas)
(left figure includes thermal velocities)

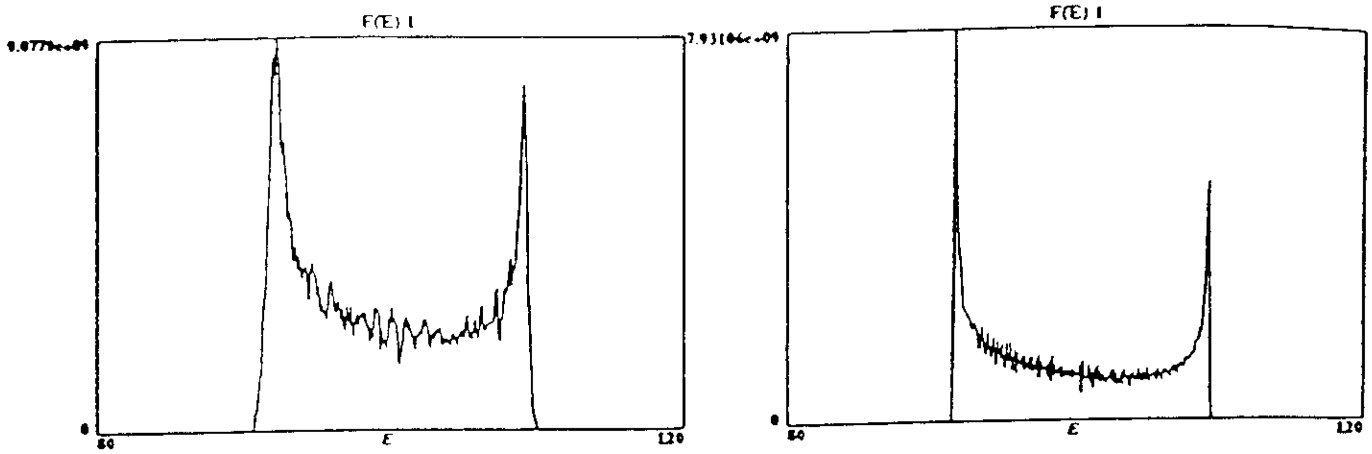


Figure 3: Diagnostics for Child Law Diode(He gas)
Frequency = 300MHz

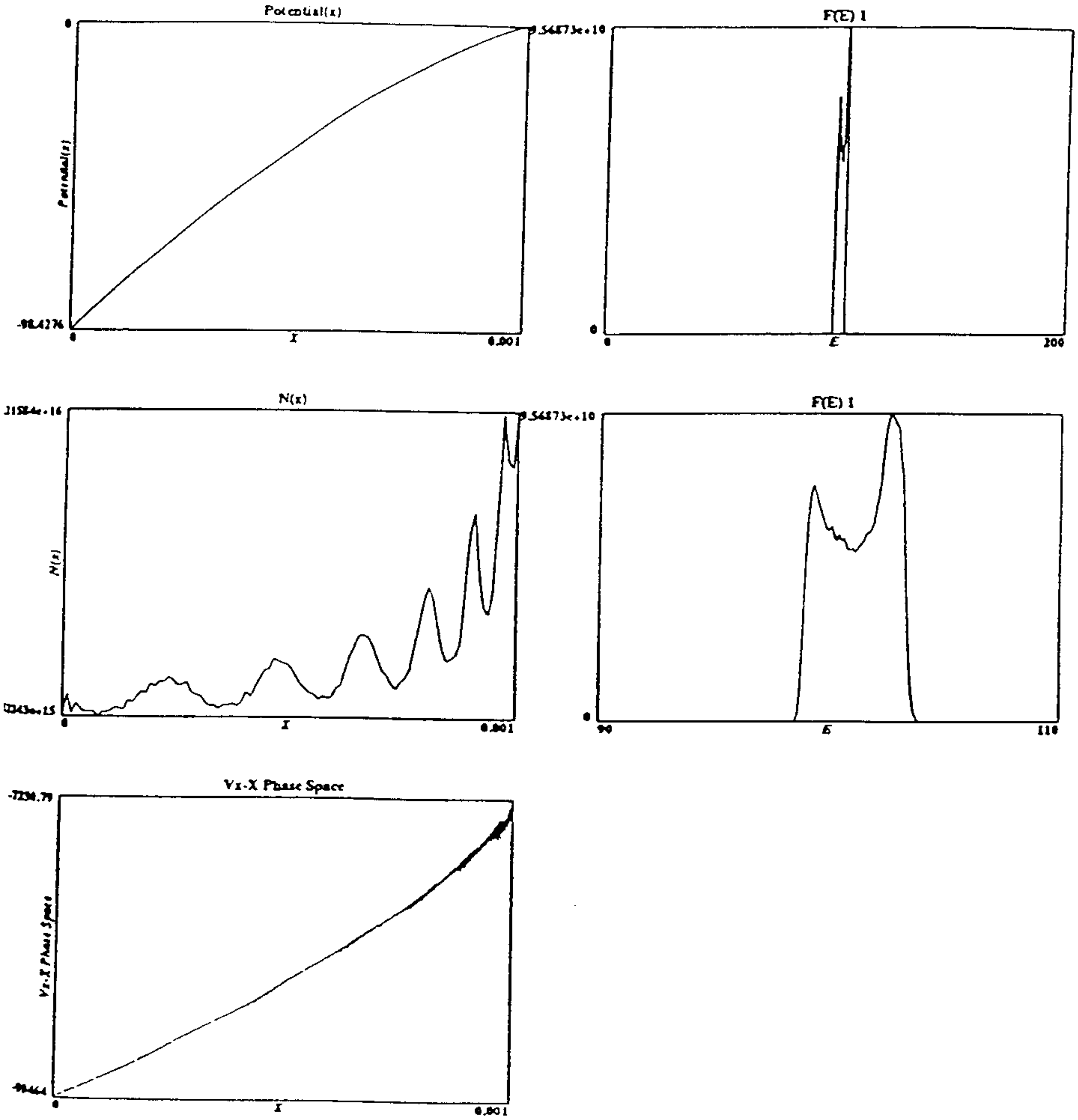
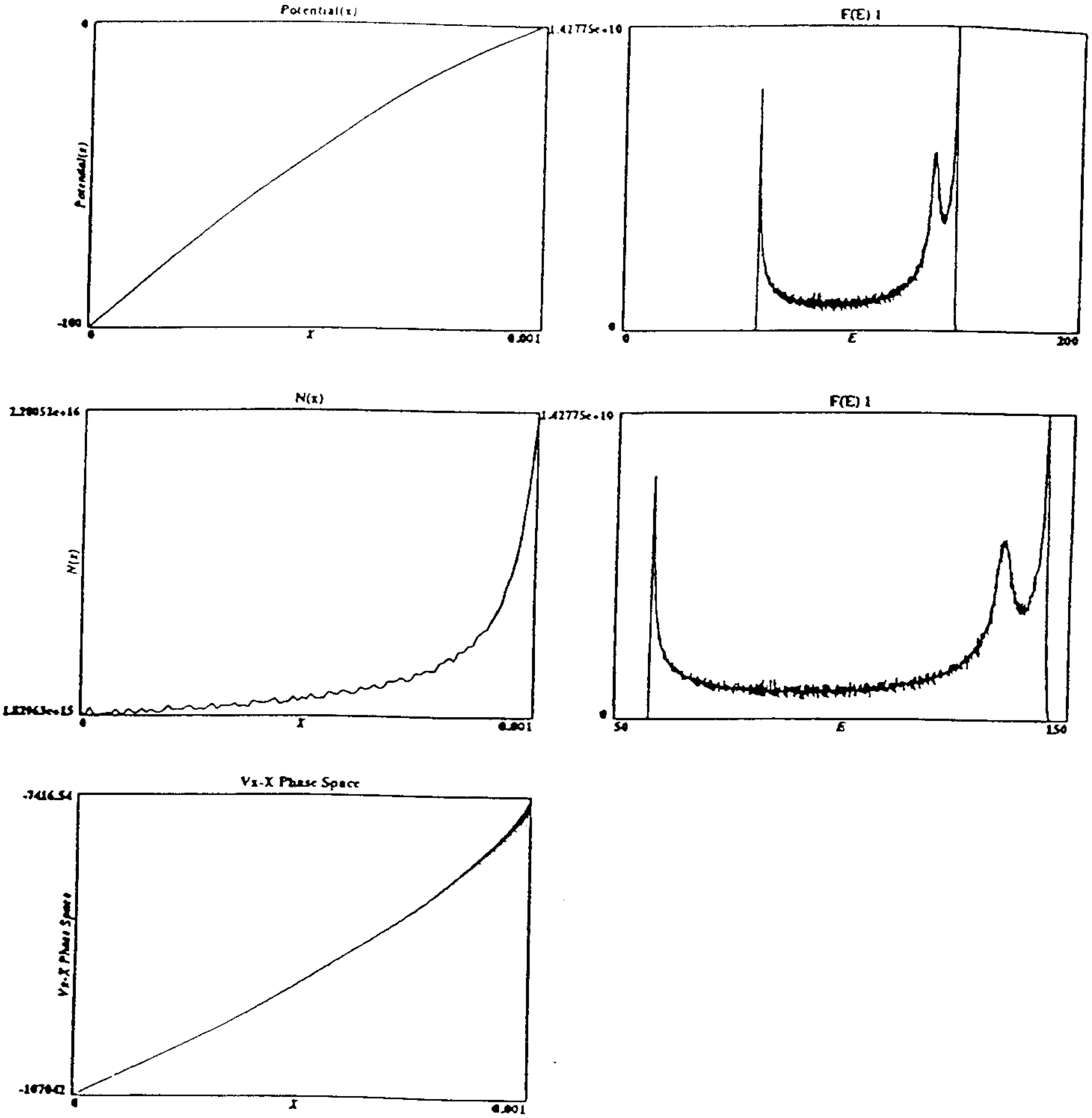


Figure 4: Diagnostics for Child Law diode(He gas)
Frequency = 10MHz



OS100M.INP

This is also Emi Kawamura's model for getting at the IED in an RF discharge, but, here, the sheath width may vary in time, naturally (as in the "real" RF discharge).

(This model was suggested by Dr. Vahid Vahedi.)

Let the diode be current driven at 100MHz, with injection of a half Maxwellian of helium ions, plus the same for electrons, at the right hand electrode. (This is a so-called Q-machine like source.)

1)Near the injection plane, a relatively small source sheath will form. Observe this potential drop and compare with the overall diode potential drop; is it "realtively small"?
2) Note the behavior of the electrons and ions at the source sheath region. (Are many electrons returned and all ions transmitted?)
3) Observe the behavior at the left electrode, where electrons are repelled and ions are accelerated. Observe the oscillating sheath width. Observe the IED, (f(E)1) Compare with similar results from CHILDHE.INP.
4) Change to a lower frequency, like 10MHz and observe that the ionms now can follow the RF, so will produce a much broader ion energy distribution, IED.
5) Make a plot of the maximum and minimum ion energy as a function of RF frequency, as well as ΔE(the energy spread). Mark where the RF frequency equals the ion plasma frequency (ω_{pi}); make comments on this (is there a change at ω_{pi}, and if so, why?).

Add whatever Emi suggested you try as well.

Ned Birdsall
10 March 1995

os100M.inp

RF Sheath with oscillating sheath width (**100MHz**), **Current driven**, 2 Species, **right hand wall injection**. Objective: observe Ion energy and Ion angular distributions at LHS

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 2    100    3e8    1e-11     0.005     0.016       1.0        0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 0.0               0.0               0.0      0.0        0.0       1.0       0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0           i         0.0            0.0               5.75          1e8      0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0                 0               0              0       256        0             0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                 0.0            1                 5e-3               .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19            0.0             0.0             10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
1.0e-20             12.0            50.0            100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20            13.6            60.0            110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19             0.0                     2.0e-19         0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 3.34e-27       0.0                22.4               0.0

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.0          1.4e3        1.4e3          0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 1.4e3              0.0           200        0           200       50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       200        0.0         200           .0025      .003

SPECIES 2

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31    0.0                9600               0.0

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.0          6e5          6e5            0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 6e5                0.0           200        0           200        50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       200        0.0         200           .0025      .003
```

Figure 5: Diagnotics for Current-Driven RF Sheath(He gas)
Oscillating Sheath Width, f = 100MHz

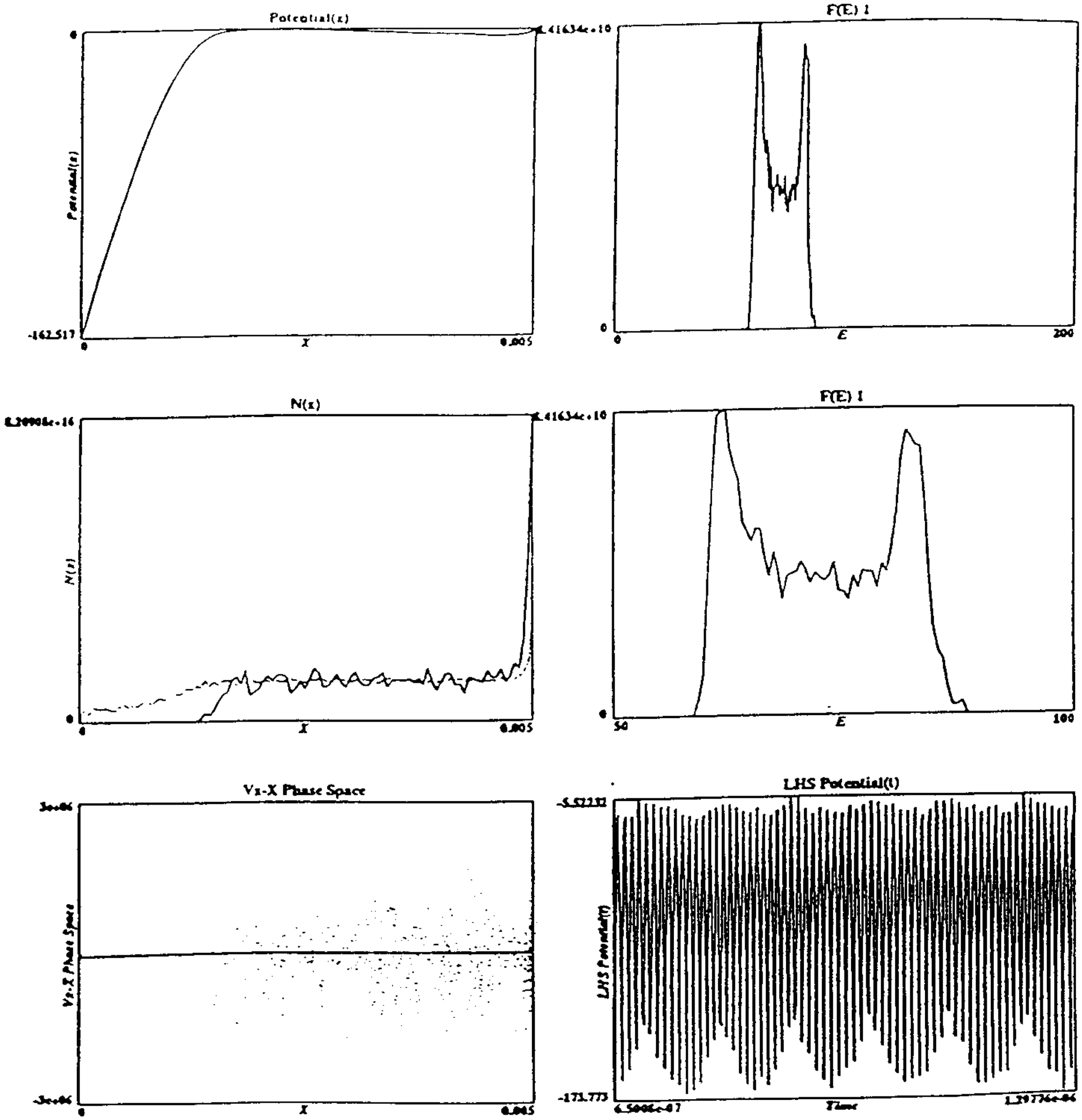
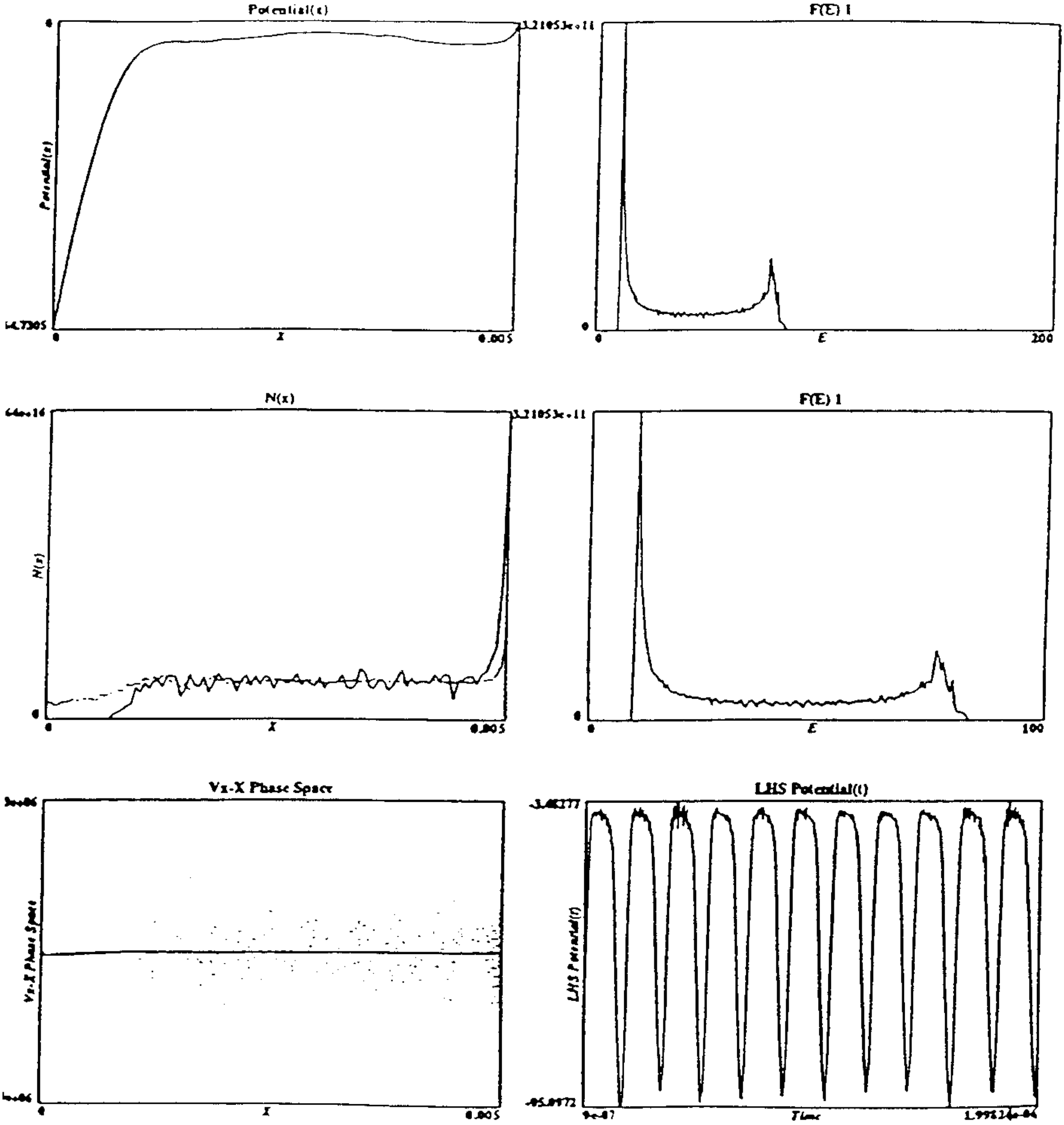


Figure 6: Diagnostics for Current-Driven RF Sheath(He gas)
Oscillating Sheath Width, f = 10MHz



EECS 298-9 HOMEWORK DIFFUSION

February 15, 1996

due February 22, 1996

ABSTRACT

The purpose of this problem set is to provide the student with an opportunity to simulate a number of one-dimensional diffusion problems. The problems describe below model collisional, uncharged diffusion, ambipolar diffusion, and diffusion in a uniform magnetic field.

1. **diffusion.inp**: These problems are performed using the input file, diffusion.inp. Note that PDP1 loads initial densities uniformly in space, requiring a Fourier expansion of the solution to the 1d diffusion equation. This is described in L&L, Sect. 5.2, for the time-dependent case. In this input file, the particles are neutral – there are no electric forces.
   (a) **Collisionless diffusion**. Initially, run diffusion.inp with collisions turned off (e_collisional = i_collisional = 0). Quantify the rate of loss at early times, observing N(t). Hint - look for exponential decay using the log scale on N(t). Estimate the flux at the edge at t=0 by integrating over a half Maxwellian distribution, $\Gamma = c\int_0^\infty v f(v) dv$. Describe the distribution $f(x = \delta, v)$ as a function of time for $\delta \ll l$, by recognizing the shear which occurs in $v_x - x$ phase space.
   (b) **Collisional diffusion**. Edit diffusion.inp to add collisions for electrons and ions. The parameters are configured for a constant electron scattering cross section, $\sigma_{el} = 2 \times 10^{19}$ m^2. This leads to an velocity-dependent collision frequency, $\nu_m = n_g \sigma v$. Observe the decay of N(t), and $n_e(x)$, and compare these to analytic calculations at 10 ns, 100 ns and 1000 ns. Compute T_e, T_i.
   (c) The analytic solution includes all the odd terms A_i in Eq. 5.2.8 in L&L. Using N(t), estimate the rates of decay for $i = 7, 5, 3, 1$. It is helpful to note that the higher order terms decay faster than the lower order terms (see Eq. 5.2.10 in L&L). Look for the lowest order term, $\cos(\pi x/l)$, in the limit of t large.

2. **ambipolar.inp**. In this set of problems, the particles are charged, resulting in electric fields, potentials, etc. The same comment applies regarding loading particles uniformly – analytic results must include a sum of the Fourier terms.

(a) Observe the rate of loss of the particles in N(t). Compare to the uncharged case.

(b) Quantify the reason for the modified flux by computing the diffusion coefficients for the charged (ambipolar) and uncharged cases. Estimate the electron and ion diffusion coefficients, $D = kT/m\nu$, by approximating the collision frequency at a reasonable energy, and then computing the ambipolar diffusion coefficient. Measure the diffusion time, τ, from N(t) and compare to the above.

3. **Crossed-field diffusion**. Collisions can result in diffusion across field lines, as discussed in Sect. 5.4 of L&L. Modify ambipolar.inp to include a magnetic field sufficient to make $r_{ce}, r_{ci} \ll l$, so that both species are magnetized. Consider the diffusion rates at $\psi = 90$ by observing N(t) and n(t). Compute $D_\perp$ and $D_\parallel$ for electrons and ions, and the resultant ambipolar diffusion coefficient, D_a. Next, compute the angle at which $\Gamma_i = \Gamma_e$ for these parameters, and demonstrate this with PDP1.

DIFFUSION.INP

You have the class handout of 8 February to do. The input file you can obtain from the Master Account.

Ned Birdsall
10 March 1995

DIFFUSION.INP

Simple diffusion of a maxwellian distribution of Zero charge particles. The particles are scattering off the background

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 2   100    3e7   1e-10     0.1      0.01       1.0        0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 0.0                0.0              0.0       0          0.0      1e10   0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0           v        0.0          0.0                0.0        0.0      0.0

-secondary--e_collisional--i_collisional--reflux--nfft--nsmoothing--ntimestep--
 0               1                2         0   256         5               0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                  0.2           2                1e-2            .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 2.0e-19             0.0           0.0          100.0
--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 0.0                 12.0          50.0         100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 0.0                 13.6          60.0         110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19             0.0                      2.0e-19          0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 0.0        9.11e-31      0.0              0.                1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0         0.          2e6         2e6          0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0          50        0          10       50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100      0.01       15.0          .045     .055

SPECIES 2

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 0.0        3.6e-29       0.0              0.                1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0         0.          9.0e3       9.0e3        0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0          50        0          10       50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100      0.01       15.0          .045     .055
```

diffusion.inp

Diffusion is well treated in Chapter 5 L&L text. Our input file starts with uniform density, so that there is no density gradient (at t=0). Hence, initially, there is no diffusive flux, $\Gamma_D = -D\nabla n$. We ask that you run the input file,

(a) with $q_e = 0 = q_i$ (no charges, hence, $E = 0 = \Phi$) with collisions turned off also (edit the file). Of course, the particles will still go to the walls. Observe the number decay N(t); using the log scale, is the decay exponential? What is the rate of loss (is it exp - t/τ)? what is τ?) Account for the (initial) rate of loss by calculating the (half) Maxwellian flow to each electrode ($\Gamma = n_0\bar{\upsilon}$); Check with the simulation (near t = 0). L&L p. 39.

(b) Keep $q_e = 0 = q_i$ ($E = 0$, $\phi = 0$, $\rho = 0$) but turn on the electron-neutral collisions (as in the file originally). Observe the number decay, as in (a). Is it now larger, due to collisions, $\upsilon_{\text{collision}} = n6\sigma\upsilon \approx (1.2e8/\text{sec})$, at 10m Torr.?

(c) In the L&L text, Sec. 5.2, the diffusion eqn. is solved. Here, we begin with n uniform in x, meaning all odd terms (all A_i) exist in 5.2.8, with Ai ~ $\frac{1}{i}$. Observe N(t). See if you can identify i = 7 (fast), 5, 3, 1 (slow) rates. See whether n(x) takes on the (advertised) cosine (πx/l) shape for $t \to \infty$.

AMBIPOLAR.INP

You have the class handout of 8 February,
with the input file available from the Master Account.

Ned Birdsall
10 March 1995

AMBIPOLAR.INP

Simple diffusion of a maxwellian distribution of particles. The particles of both species are scattering off the background Same as diffusion.inp, but with charges turned **on**!

```
-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 2    100    3e7    1e-10    0.1      0.01        1.0      0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 0.0               0.0              0.0   0          0.0      1e10    0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0           v       0.0        0.0              0.0        0.0      0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0                1                2            0       256     5           0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                 0.2          2              1e-2              .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 2.0e-19          0.0            0.0            100.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 0.0              12.0           50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 0.0              13.6           60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19          0.0                    2.0e-19      0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.6e-19 9.11e-31      0.0             0.                1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.          2e6        2e6         0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0           50      0            10         50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100      0.01       15.0        .045     .055

SPECIES 2

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.6e-19  3.6e-29       0.0             0.                1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.          9.0e3      9.0e3       0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0               0.0           50      0            10         50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100      0.01       15.0        .045     .055
```

ambipolar.inp

This is an extension of diffusion.inp. We have put back in q_e and q_i, so there is now ρ, E, Φ, leading to ambipolar diffusion, as in the L&L text p. 131.

Again there are the same comments, about starting with $n_i = n_e$ at $t = 0$, uniform (not cosine). Report observing done in diffusion.inp.

Calculate T_e (something like 30eV) and T_i (something like room temperature, 1/40 eV).

(a) Do you find that the flux is now the same or much larger, as given (D is given, not Γ) in Eq.(5.1.14). $D_i \left(1 + \frac{T_e}{T_i}\right) >> D_i$?

Add your own observations.
Is there a better set of choice of parameters?

Possibly try $\frac{m_i}{m_e}$ for protons (1836 m_e) or for helium (4 x 1836 m_e).

COLDPLAS.INP

The object is to view plasma oscillations in a uniform plasma, using XES1, a periodic electrostatic code.

The reference is Birdsall and Langdon (Plasma Physics via Computer Simulation, Adam-Hilger, IOP, 1991), Section 5-3, pages 86-92.

(Further reference to this text will be as: B&L.)

[This input file was made up decades ago, when computers were painfully slow. So, feel free to put in lots more particles, as the results will have lower noise.

Hint: for ES1, use the number of particles as some integral multiple of the number of cells/grids. And, the number of cells or grids must be a power of two, as the field solver uses an FFT with base 2.]

Run the code, looking at all of the diagnostics. Make sure that you understand each of these, and how they "check" on each other (as, electric field is the derivative of potential, charge density is the second derivative of potential, etc.)

QUESTIONS:

1) Is the measured frequency ω_p, as put in?
2) What is $\omega_p \Delta t$?
3) Try $\omega_p \Delta t$ just a little smaller than 2.0 (Stable?) Try is just a little larger that 2 - should be unstable; is it?
4) Mode 1 is excited by a small particle displacement; how much smaller in field energy is mode 2, mode 3, etc. (how many dB down; expect something like many dB down)? Try larger/smaller excitations, with X1; comment. Instead of X1, position perturbation, try V1 excitation, a velocity perturbation (turn X1 off). What value of either X1 or V1 causes particles to cross; want expression in terms of ω_p, system length, number of cells, etc.
5) Expand the x axis in the phase space plot, x vs. v, in order to look at the trajectory of one particle, which should be elliptical. Is it?

Ned Birdsall
21 March 1995

COLDPLAS.INP COLD PLASMA OSCILLATIONS

Oscillations can be observed in a cold electron plasma in a neutralizing nonmobile ion background. The oscillations are initiated by perturbing the density of the electrons in the fundamental mode.

```
nsp-----------l---------dt----------nt------mmax-----l/a----accum
1    6.283185307 0.5         150       14     0          0
ng----iw--ec--epsi-------a1------a2-------E0-------w0
32   2     0   1.00   0.00     0.00   0          0

SPECIES 1: Cold Electron Plasma

n----nv2---nlg---mode
64   0     1     1
wp----wc-----qm-----vt1----vt2---v0
1.00 0.00   -1.00  0.00 0.00   0.00
x1----v1---thetax--theta
0.001 0.0 0.00   0.00
--nbins---vlower---vupper 50 0.0 0.
```

HYBRID.INP

The object is to view hybrid plasma oscillations, called $\omega_H \equiv \sqrt{\omega_p^2 + \omega_c^2}$, equal to the square root of the squares of omega-p and omega-c. We will set the two ω 's equal to 1, and look for $\omega_H = \sqrt{2}$.

The reference is B&L,section 5-5, pages 92-93.
Run the code, looking at all diagnostics, especially at the transverse velocity space, υ_x vs. υ_y.
What shape is the trajectory of one particle? (Use the trace.)
More or less repeat the questions for COLDPLAS.

We will come across hybrid oscillations again when we do ECR, electron cyclotron resonance, driving the hybrid resonance, to obtain heating.

Ned Birdsall
21 March 1995

```
                        0
HYBRID.INP              HYBRID OSCILLATIONS

A cold electron plasma is loaded in a uniform magnetic field to observe oscillations at
the hybrid frequency. Mode 1 is excited at small amplitudes by velocity modulation,
small enough so that the gyro orbits do not cross.

nsp-----------l---------dt----------nt----mmax----l/a----accum
 1   6.283185307 0.10         150    3       0      0
ng----iw-ec---epsi-------a1------a2------E0------w0
512    1  0  1.00    0.00   1000.00   0       0

SPECIES 1: Cold Electron Plasma

n----nv2---nlg---mode
128  0       1      1
wp----wc-----qm-----vt1----vt2---v0
1.00 -1.00  -1.00    0.0  0.00 0.00
x1----v1---thetax--thetav
0.00 0.02   0.00     0.00
--nbins---vlower---vupper
50          0.0         0.0
```

2STREAM.INP

The object is to view a very strong plasma instability (one of a large number), measure the growth rate, and observe the final state.

The model has two cold electron streams shot at each other, in an immobile ion neutralizing background. As noted in lecture, when the streams slip past each other one wavelength in one plasma period, perturbations on one stream should reinforce perturbations on the other stream, hence, grow exponentially in time. This leads to expecting maximum growth rate at $k\upsilon_0/\omega_p = 0.5$. The simulation is set up near that point. The theoretical maximum growth rate (from linear theory, when the perturbations are still small) is $\omega_i = \frac{\omega_p}{2}$, as large as any known plasma instability (usually growth rates are much smaller).

The reference is B&L, Sections 5-6 to 5-9, pages 94-109.

1) Run the code, looking at all of the diagnostics. In x-v phase space, note the growth of the initial perturbation. Note that the growth (in ESE, electrostatic energy) ends just about when each stream has some electrons just at v = 0.
2) Measure the growth rate, $\omega_{\text{imaginary}}$, on the field energy semi-log plot. The slope (where a straight line) is $2\omega_{\text{imaginary}}$, as the plot is of a quadratic quantity, ρ_ϕ.
3) After the maximum in field energy is reached, the beams are trapped, with particles bouncing; the bounce frequency is seen in the field energy plot. You may calculate this bounce frequency from the potential plot, and check with the observed frequency.
4) Look at the evolution of the distribution function, f(v). Note that a Maxwellian (Gaussian) is not achieved by this instability. Putting in collisions will lead f(v) to a Maxwellian.

Ned Birdsall
22 March 1995

```
                         2STREAM.INP            TWO STREAM INSTABILITY
Two cold electron beams are drifting in opposite directions with a neutralizing ion background. Both
beams are perturbed in the lowest frequency mode for the periodic system (the fastest growing mode).

nsp-----------l---------dt---------nt-----mmax----l/a---accum
2     6.283185307 0.10              300     5           0       0
ng----iw--ec--epsi-------a1------a2------E0-----w0
512    1    0    1.00      0.00      0.00     0       0

SPECIES 1: Cold Electron Beam

n----nv2---nlg---mode
2048 0      1       1
wp----wc-----qm-----vt1----vt2---v0
1.00  0.00    -1.00   0.00    0.00  1.00
x1--------v1---thetax--thetav
0.0001 0.00        0.00    0.00
--nbins---vlower---vupper
 100        -2.0         2.0

SPECIES 2: Cold Electron Beam

n----nv2---nlg---mode
2048 0      1       1
wp----wc-----qm-----vt1----vt2---v0
1.00  0.00     -1.00  0.00     0.00  -1.00
x1---------v1---thetax--thetav--
-0.0001 0.00       0.00    0.00
--nbins---vlower---vupper
 100        -2.0         2.0
```

BEAMPLAS.INP

The object is to view the growth and saturation of the beam-plasma instability, where the beam density is very small compared with the plasma density. Here it is set to be 1% of the plasma density.

Reference is B&L, Sections 5-10 to 5-12, pages 110-121.

Such an instability might arise from secondary electrons accelerated from a wall, back into a bulk plasma, by the sheath field, and interacting with the bulk plasma.

This instability is related generically to the two-stream instability, but grows much more slowly, yet still leads to trapping of the beam particles, in their drift frame, a slowing and heating. So, look for this.

Run the code and all of the diagnostics.

1) Obtain the growth rate from the field energy plot, as done with the 2stream.inp run. Look for about $\frac{\omega_p}{6}$. Obtain the maximum field energy and compare it with the total energy - should be quite small.
2) Follow f(v) in time to observe the beam heating and slowing.
3) Edit the input file so as to have the beam at zero velocity and the plasma at a drift velocity of -1. That is, view the action in the beam frame, so as to see the trapping more clearly.

Ned Birdsall
22 March 1995

BEAMPLAS.INP BEAM PLASMA INSTABILITY

A cool electron beam is drifting through a cold unperturbed plasma. The small temperature of the beam avoids the non-physical cold beam instability, so the only phenomenon is the growth of the physical beam plasma instability.

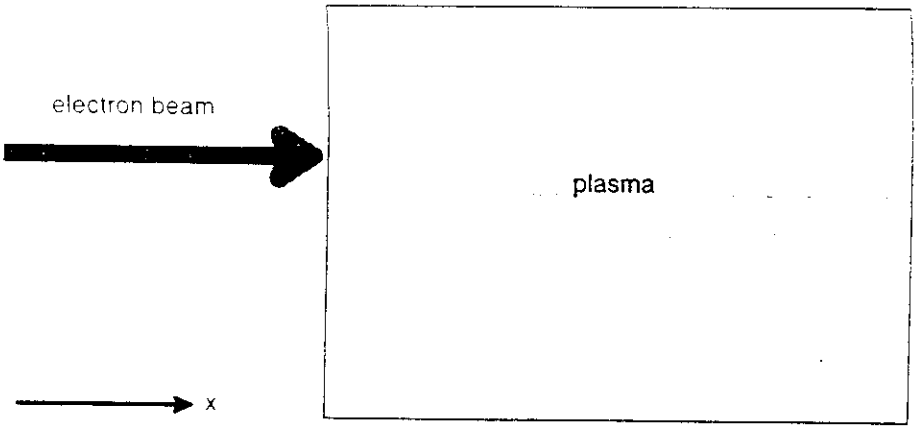
```
nsp---------l---------dt---------nt----mmax-----l/a----accum
2 6.283185307 0.20 1000 3 01
ng----iw--ec--epsi-------a1------a2-----E0-----w0
64 2 0 1.00 0.00 0.00 0 0

SPECIES 1: Cool Electron Beam
n----nv2---nlg---mode
512 0 1 0
wp----wc-----qm-----vt1----vt2---v0
0.1 0.00 -1.00 5e-4 0.0 0.00
x1----v1---thetax--thetav
0.0 0.00 0.00 0.00
--nbins---vlower---vupper
50 -1.0 2.0

SPECIES 2: Cold Electron Plasma
n----nv2---nlg---mode
256 0 1 0
wp----wc-----qm-----vt1----vt2---v0
1.00 0.00 -1.00 0.00 0.00 -1.00
x1----v1---thetax--thetav
0.00 0.00 0.00 0.00
--nbins---vlower---vupper
50 -1.0 1.0
```

Beam Plasma Instability

EECS 298-9
April 11, 1996
J. P. Verboncoeur
Dept. EECS
University of California
Berkeley, CA 94720-1770

Outline

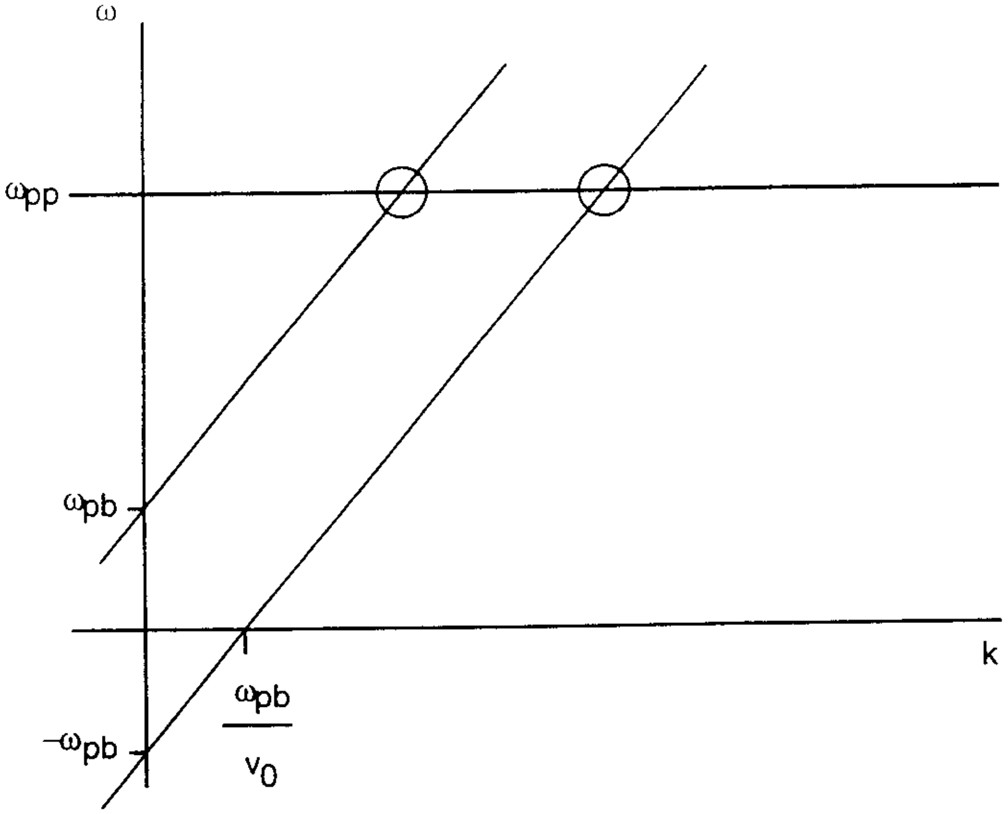
1. Introduction
2. Linear analysis
   (a) Weak beam
   (b) Strong beam
3. Approximate nonlinear analysis
4. Simulation



1.Schematic of the beam plasma instability.

Introduction

- Similar to two-stream instability
- Beam: electrons drifting at $v = v_0$
- Plasma: electrons and ions, $M_i/m_e \gg 1$
- Can be bounded or periodic model
- One-dimensional effect
- Beam may be from cathode, secondary emission, etc.



2.Dispersion relation for weak beam-plasma interaction. Coupling is indicated by circles.

Linear Analysis

Early analysis by Boyd *et. al.*(1958) [1]. See also Briggs (1996) [2] and Birdsall and Langdon [3].

The dispersion relation for a cold plasma and cold

---

[1] G. Boyd, L. M. Field and R. Gould, "Excitation of plasma oscillations and growing plasma waves", *Phys. Rev.* **109**, 1393 (1958).

[2] R. J. Briggs, *Electron Stream Interaction with Plasma*, M.I.T. Press, Cambridge, MA (1964).

[3] C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulation*, Adam-Hilger (1985).

beam is

$$\frac{\varepsilon(\omega, k)}{\varepsilon_0} = 1 - \frac{\omega_{pp}^2}{\omega^2} - \frac{\omega_{pb}^2}{(\omega - \mathbf{k} \cdot \mathbf{v}_0)^2} = 0,$$

where ω_{pp} and ω_{pb} are the plasma frequencies of the background plasma and the beam, respectively, and v_0 is the velocity of the cold beam. The dispersion relation for the weak beam-plasma interaction, $\omega_{pb} \ll \omega_{pp}$, is shown in Figure 2. For a strong interaction, the dispersion diagram is shown in Figure 3. The dispersion relation can be written in a convenient dimensionless form:

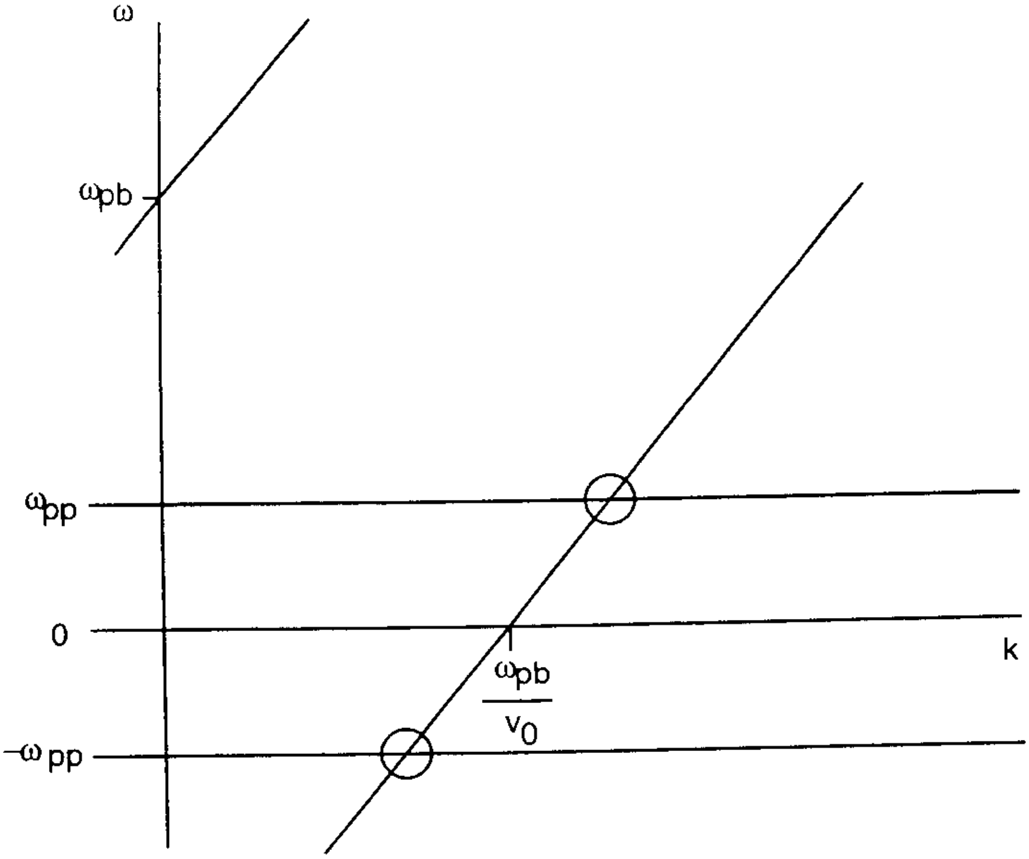
$$1 - \frac{1}{W^2} - \frac{R}{(W - K)^2} = 0,$$

where $W \equiv \omega/\omega_{pp}$, $K \equiv (\mathbf{k} \cdot \mathbf{v}_0)/\omega_{pp}$, and $R \equiv (\omega_{pb}/\omega_{pp})^2$. This can be solved for k real and ω complex, as shown in Figure 4.

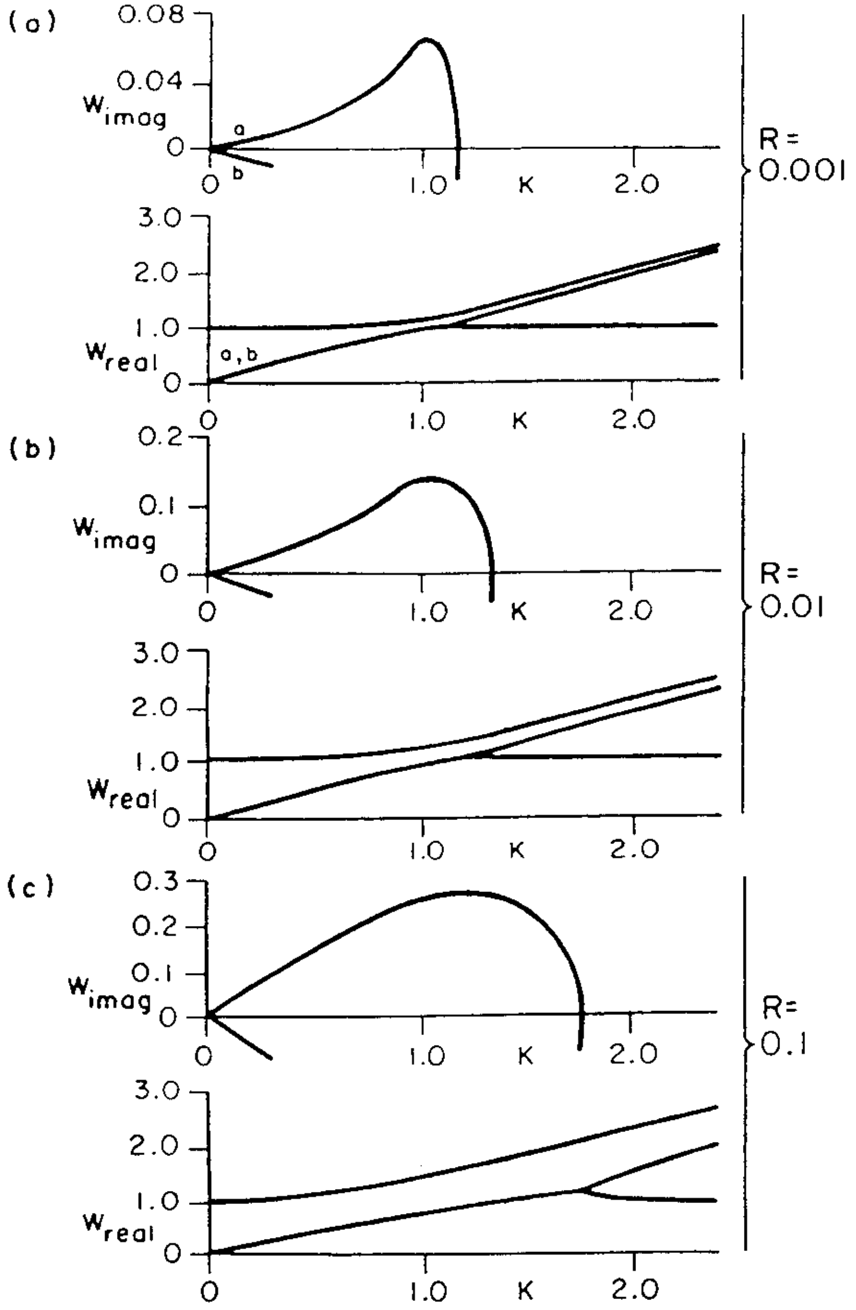
The maximum growth rate can be plotted for a number of normalized beam densities, as shown in Figure 5. The growth rate in the weak beam region is characterized by

$$\frac{(\omega_i)_{\max}}{\omega_{pp}} \sim R^{1/3},$$

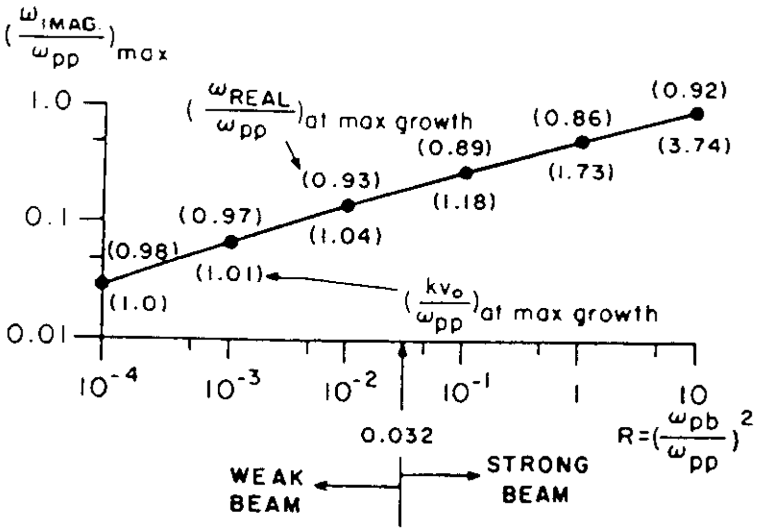
while in the strong beam region, the growth rate is



3.Dispersion relation for strong beam plasma interaction.



4.Roots of the dispersion relation for a number of beam density ratios. From B&L.



5.Maximum normalized growth rate for a cold beam-plasma instability. The dilineation between strong and weak beams is just a characterization. From B&L.

given by

$$\frac{(\omega_i)_{\max}}{\omega_{pp}} \sim R^{1/4}.$$

In the strong beam regime, the system is no longer neutral, with ion plasma density given by

$$\frac{n_{pi}}{n_{pe}} = \frac{1+R}{1-m_e/M_i}.$$

Nonlinear Analysis

Treated by Drummond *et. al.* 1970 [4], Gentle and Lohr 1973 [5] and [6], Hasegawa (1975) [7] and Kainer *et. al.* 1972 [8], among many others.

A number of effects can be seen in the non-linear analysis:

- trapping of the beam
- saturation
- stationary potential in frame of reference of beam

Relationship of perturbations:

$$\left(\frac{n_1}{n_0}\right)_p \approx \left(\frac{v_1}{v_0}\right)_p \approx \delta\left(\frac{v_1}{v_0}\right)_b \approx \delta^2\left(\frac{n_1}{n_0}\right)_b$$

where $\delta^3 = -R/2$ for $\delta \ll 1$ in the dispersion $\omega = \omega_{pp}(1-\delta)$. Then velocity modulations are small, even for $n_{1b} \approx n_{0b}$.

Potential is sinusoidal (even well into the nonlinear

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[4] W. E. Drummond, J. H. Malmberg, T. M. O'Neil, and J. R. Thompson, "Nonlinear development of the beam plasma instability", *Phys. Fluids* **13**, 2422 (1970).

[5] K. W. Gentle and J. Lohr, "Phase-space evolution of a trapped electron beam", *Phys. Rev. Lett.* **30**, 75 (1973).

[6] K. W. Gentle and J. Lohr, "Experimental determination of the nonlinear interaction in a one-dimensional beam-plasma system", *Phys. Fluids* **16**, 1464 (1973).

[7] A. Hasegawa, *Plasma Instabilities and Nonlinear Effects*, Springer-Verlag, Berlin (1975).

[8] S. Kainer, J. M. Dawson and R. Shanny, "Interaction of a highly energetic electron beam with a dense plasma", *Phys. Fluids* **15**, 493 (1972).

regime):

$$\Phi_1 \cos(kx - \omega t) = \Phi_1 \cos \omega_{pp} \left[\frac{x}{v_0} - t(1 - \delta_r) \right]$$

with an envelope, $\exp(\omega_{pp}\delta_i t)$ for the growth. Particles are then trapped in the stationary (in the frame of the beam) potential, $\Phi_1 = \bar{\Phi} \cos(kx)$. The energy of the ith particle can then be written

$$\frac{1}{2} m_i v_i^2 + q_i \bar{\Phi} \cos(kx_i) = c_i,$$

where c_i is the total energy of the ith particle. Then low energy particles, $c_i < q_i\bar{\Phi}$ are trapped.

Implementation of a Secondary Electron Emission Model in PIC-MCC Codes

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Abstract

The present model of secondary creation and the energies of electron generated secondary electrons present in our PDX1 codes is extremely simple. The current implementation emits a fraction (< 1) irrespective of the energy and incidence angle of the primary electron. Further, the velocity of emitted electron is calculated using primary electron input file parameters. A more realistic implementation of electron generated electron secondaries which models the generation taking into account incident energy, incident angle and velocity distribution of the emitted secondaries is described.

1 Background

The subsequent sections will discuss the impact on secondary emission and the properties of the emitted electrons due to the following:-

- Dependence of number of secondaries emitted on primary electron energy.
- Change in number of secondaries emitted due to primary electron incidence.
- Velocity distribution of the emitted secondaries.

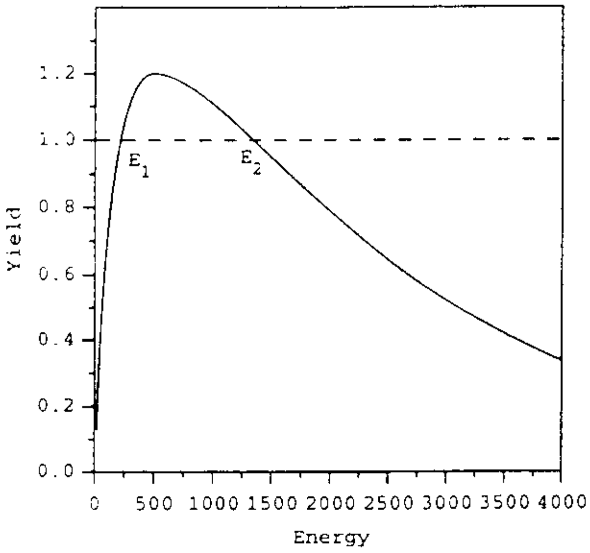

Figure 1: A Typical Metal Secondary Yield vs. Normal Incidence Energy Curve [1]

2 Energy Dependence

Figure 1 shows a typical secondary electron yield vs. incident electron energy for a metal substrate (note: yield can change from > 1 for a smooth surface to < 1 for a rough one) It can be seen the the yields peaks at an incident energy of approximately 500 V and then decreases steadily after that. This is due to the fact that the emitted secondaries are electrons present in the atoms on the substrate surface and a relatively low energy incidence electron may not free the secondary while a relatively higher energy primary will be in contact with the surface for a shorter time [1]. It is also of interest to note the two points E_1 and E_2 on Figure 1. These are low and high energy points between which the yield of secondaries is greater than unity. The region between them can cause the multipactor effect under certain circumstances [2].

Vaughan [3] has modeled this curve as

$$\sigma = \sigma_{max}(we^{1-w})^k \tag{1}$$

where

$$w = \frac{(E_i - E_0)}{(E_{max} - E_0)} \tag{2}$$

and E_0 is the minimum threshold energy. E_{max} is the maximum yield energy and σ_{max} is the corresponding yield. k is a curve fit parameter given by

$$\begin{array}{l} k = k_1 = 0.62 \quad w < 1 \\ k = k_2 = 0.25 \quad w > 1. \end{array} \tag{3}$$

Shih *et. al* [4] have conducted experiments on polished molybdenum and find that the above theory shows good agreement with their results.

3 Angular Dependence

The secondary yield normally increases with the incidence angle (0° signifies normal incidence). The theory of variation of yield with incidence is described in [1] and more recently in [3] and [4]. The simplest theory for angular dependence is given by [1] as

$$\sigma = \sigma_0 \exp k'(1 - \cos\theta) \tag{4}$$

where σ_0 is the maximum secondary yield at normal incidence at any energy and σ is the yield at angle θ. k' is a parameter which fits the curve to experimental data. The value of k' for Ni at 400V is 0.55. It should be noted here that the value of k' depends on incident energy.

A more accurate modeling of angular dependence is reviewed in Vaughan [3]. This model accounts for the variation of k' and σ_{max} with energy and adds a "smoothness" parameter k_s in order to model the characteristic of the surface. This model modifies the values of σ_{max} and E_{max} as

$$\sigma_{max\theta} = \sigma_{max0}(1 + k_s\theta^2/\pi) \tag{5}$$

and

$$E_{max\theta} = E_{max0}(1 + k_s\theta^2/2\pi) \tag{6}$$

before using equations (1) and (2). The value of k_s lies between 0 (rough) and 2 (smooth).

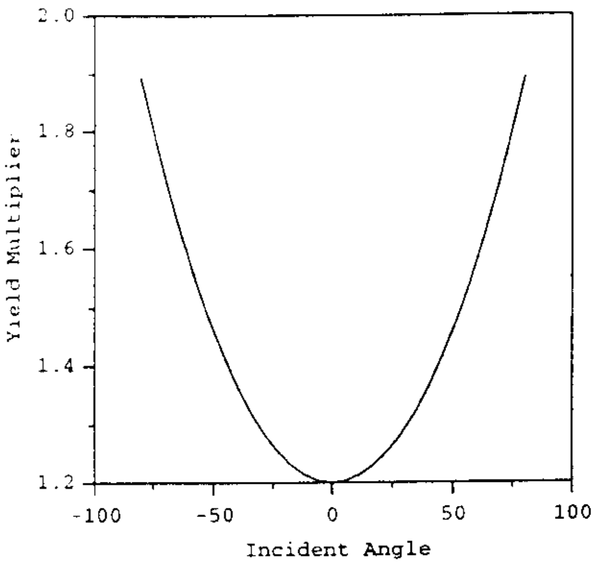

Figure 2: Equation (4) with $k' = 0.55$

4 Secondary Velocity Distribution

There are few references available regarding the velocity spread of the emitted secondaries. Spangenberg [1] mentions that irrespective of the incident electron velocity spread, secondary electrons are emitted with a isotropic in velocity. The velocity distribution of the emitted electrons, shown in Figure 4, are distributed as follows

4.1 Low Energy (I)

90% of the emitted electrons fall in this category with energies below 20V peaking around 10 V. These are implemented in the simulation model by picking a random energy between 0-20 V and distributing them into the three velocity dimensions to provide an isotropic distribution.

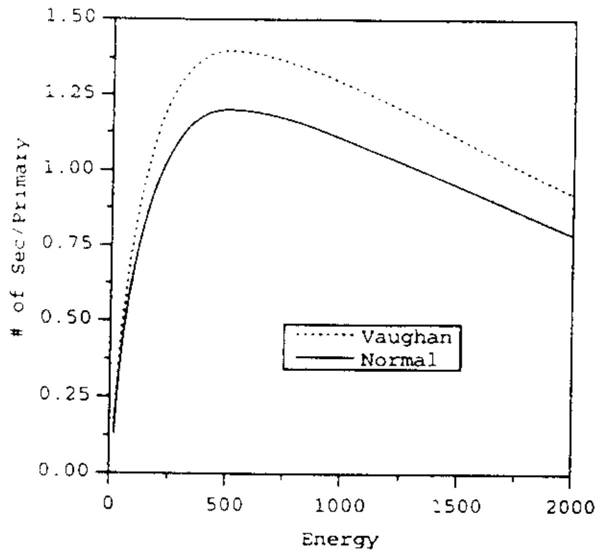

Figure 3: Angular Dependence Plot, $k_s = 1$ in eqns (5), (6), $k' = .38$ in eqn. (4)

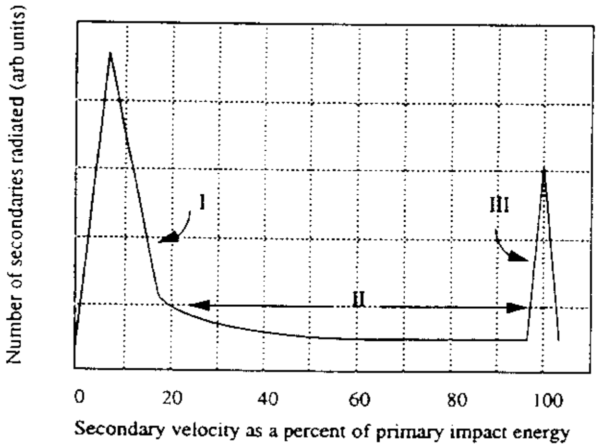

Figure 4: Relative velocity distribution of electrons, adapted from Spangenberg ([1] fig. 4.17, pg 52).

4.2 Medium Energy (II)

7% of the emitted particles lie in this energy range with energies ranging from 20 V to 98% of the incident electron energy. The velocity distribution in the simulation is calculated the same way as in the low energy case.

4.3 High Energy (III)

These are not really secondary electrons but reflected primaries. 3% of the secondaries lie in this energy range which peaks around 99% of the incident electron energy. The proposed simulation model reflects the incident normal velocity for this case. It should be noted here that even though at high energies, the yields are relatively lower, these secondaries, when emitted, have a significant impact on the system. These electrons make their presence "felt" since they have enough energy to travel into the system and interact there for a significant amount of time.

5 Acknowledgments

Thanks are due to Y. Y. Lau, D. Chernin and A. Shih for discussions on finer details of secondary emission properties.

References

[1] Karl R. Spangenberg, *Vacuum Tubes*, McGraw-Hill, New York, 1948

[2] R. Kishek and Y. Y. Lau, "Interaction of Multipactor Discharge and RF Circuit," To be Published.

[3] J. R. M. Vaughan, "A New Formula for Secondary Emission Yield," IEEE Trans. **ED-36**, 1963 (1989).

[4] A. Shih and C. Hor, "Secondary Emission Properties as a Function of the Electron Incidence Angle" IEEE Trans. **ED-40**, 824 (1993).

Primary Emission

EECS 298-9

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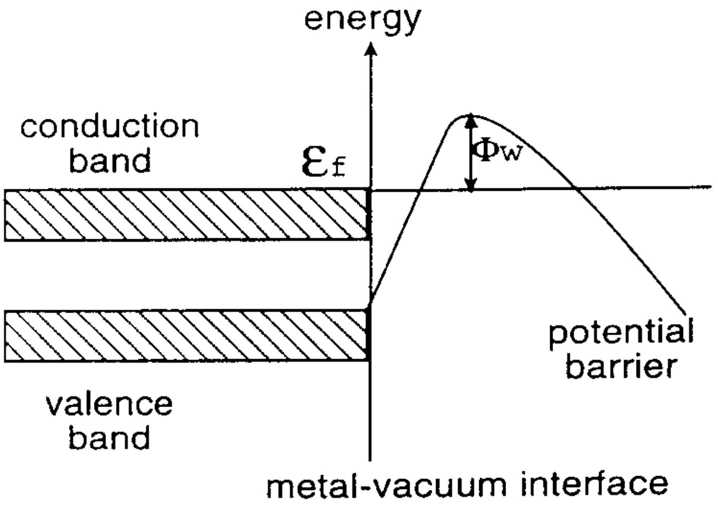
Outline

1. Photoelectric Emission
2. Thermionic Emission
3. Field Emission
4. Injection Parameters in PDP1
5. Current Injection in PDP1
6. Cold Child-Langmuir Law Emission
7. Virtual Cathode Oscillations

Photoemission



Potential energy schematic at a metal surface.

- Work function: $\Phi_w = \Phi(E = 0) - \mathcal{E}_F \approx 2 - 5$ eV.
- Irradiation energy $\hbar\omega \geq \Phi_w$
- Ejected electron carries away additional energy $\mathcal{E} = \hbar\omega - \Phi_w$

Thermionic Emission

- Electrons are heated to $\mathcal{E} > \mathcal{E}_F + \Phi_w$
- Current given by Richardson-Dushman Equation[1]:

$$J = 120T^2 \exp\left(-e\Phi_w/kT\right) \ \text{A/cm}^2$$

- $T =$ absolute temperature of cathode.
- Typical current densities $J \lesssim 100$ A/cm^2.

---

[1] S. Dushman, *Rev. Modern Phys.* **2**, 381 (1930).

Field Emission

- Fowler Norheim Law[2]:

$$J_{FN} = \frac{AE^2}{\Phi_w t^2(y)} \exp\left(\frac{-Bv(y)\Phi_w^{3/2}}{E}\right) \text{ A/m}^2$$

- E = normal component of electric field at surface
- $B = 6.8308 \times 10^9$
- $A = 1.5414 \times 10^{-6}$
- $t^2(y) \approx 1.1$
- $v(y) = 0.95 - y^2$
- $y = 3.79 \times 10^{-5} E^{1/2} / \Phi_w$

---

[2] R. H. Fowler and I. W. Nordheim, "Electron Emission in Intense Electric Fields", *Proc. Roy. Soc.* **A119**, 173 (1928).

Injection Parameters in PDP1

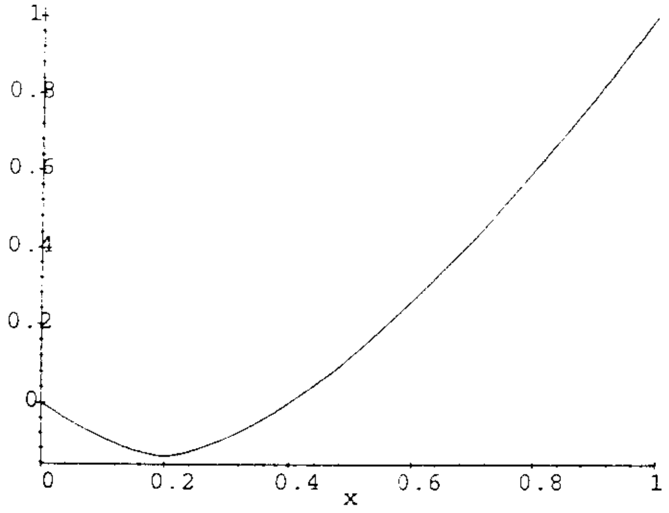
name	unit	description
j0L	A/m^2	Current injected from the left boundary.
j0R	A/m^2	Current injected from the right boundary
v0L	m/s	Drift velocity in the x-direction for particles injected from the left wall.
v0R	m/s	Drift velocity in the x-direction for particles injected from the right wall.
vtL	m/s	Thermal spread in the x-direction for particles injected from the left wall.
vtR	m/s	Thermal spread in the x-direction for particles injected from the right wall.
vcL	m/s	Velocity cutoff in the x-direction for particles injected from the left wall.
vcR	m/s	Velocity cutoff in the x-direction for particles injected from the right wall.
vperp0	m/s	Drift velocity perpendicular to the x-direction.
vperpt	m/s	Thermal velocity perpendicular to the x-direction.

Current Injection in PDP1

- v_x chosen by inverting cumulative distribution function $F(v) = \int_0^v f(v')dv'$
- $f(v) = \exp\left(\frac{1}{2}\left(v - v_0\right)^2 / v_t^2\right)$
- $v_\perp = v_{\perp 0} + v_{\perp t}\sqrt{-2\ln R_1}$
- $0 < R_i \leq 1$ is a random number
- $\theta = 2\pi R_2$
- $v_y = v_\perp \sin\theta$, $v_z = v_\perp \cos\theta$
- for nth particle of N, $\delta t_n = n/N$
- account for E and time center:

$$v_x' = v_x + \frac{q}{m}E\left(\delta t - \frac{1}{2}\right)\Delta t$$

Child-Langmuir Emission



Formation of virtual cathode.

- Child-Langmuir Current Limit:

$$J_{CL} = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V^{3/2}}{d^2} \text{ A/m}^2$$

RFDANC.INP

The object is to observe an RF driven argon plasma with no collisions, (no ionization, etc.), in order to obtain the essence of a strongly driven plasma, as in an RF discharge. Of course, after a few RF cycles (f = 13.56MHz, T = 73.7 nanosec.), the ions and electrons are absorbed by the walls; there is still time to observe the general plasma behavior. (We will add e and i sources in the next steps.)

The initial loading is 1eV electrons and 1/40ev ions, at an initial density of $10^{15}/\text{m}^3$. The argon mass ratio is used ($M_i/m_e = 73300$). The RF voltage is 500 volts.

As usual, observe all of the diagnostics.

1) From NUMBER, note that the electrons tend to be trapped after about a quarter RF cycle; estimate this time from the electron thermal velocity, the length of the system, etc.
2) As dissussed in lecture, the ions are accelerated by the time average potential into the walls, as the ions cannot follow the 13.56MHz oscillations. This statement is true for ω_{pi} less than the RF frequency; is this so? (Calculate ω_{pi} .)
3) View the maximum ion velocity at the walls and see whether the ion kinetic energy, $\frac{1}{2}m_i\upsilon_i^2$, is equal to the time average potential (energy) drop, $q_i(\phi_{\text{mid}} - \phi_{\text{wall}})$. Note the value of this potential drop, less than about half the applied voltage, as in L&L Fig. 11.3. p.335. Run the TRACE on the potential plot and eyeball the time average.
4) As discussed in L&L text, Section 6.3. p. 164 the high voltage sheath width, s, is much larger than a few Debye lengths, given by $\upsilon_{\text{max}}/\omega_p$ (see lecture today), where υ_{max} is obtained from the time average potential drop, mid to wall. Check this value of s with that measured on the time average E field plot (also as shown in class today).

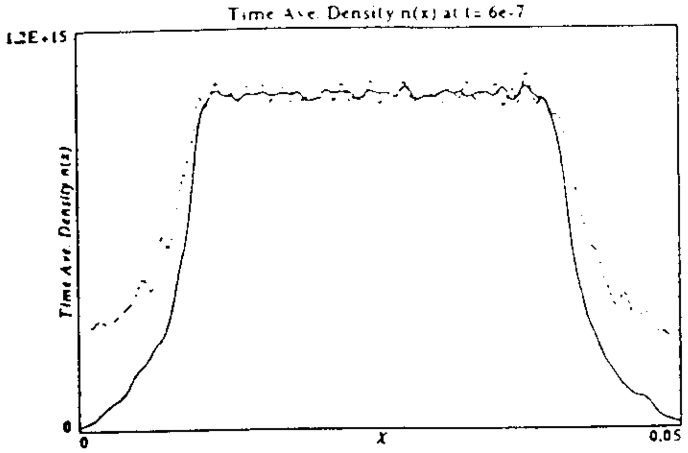
Make any more measurements you think interesting, along with analytic estimates of what you observe.

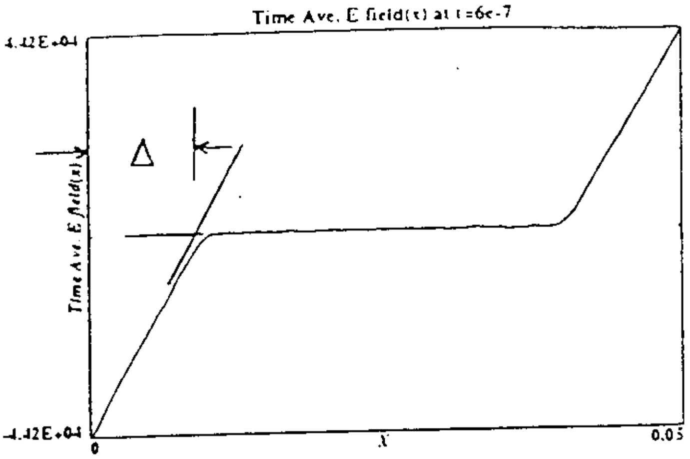
Comments welcome.

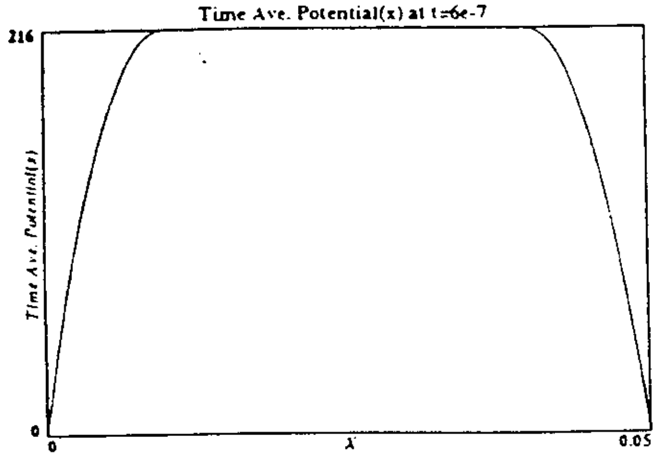
Ned Birdsall
22 March 1995

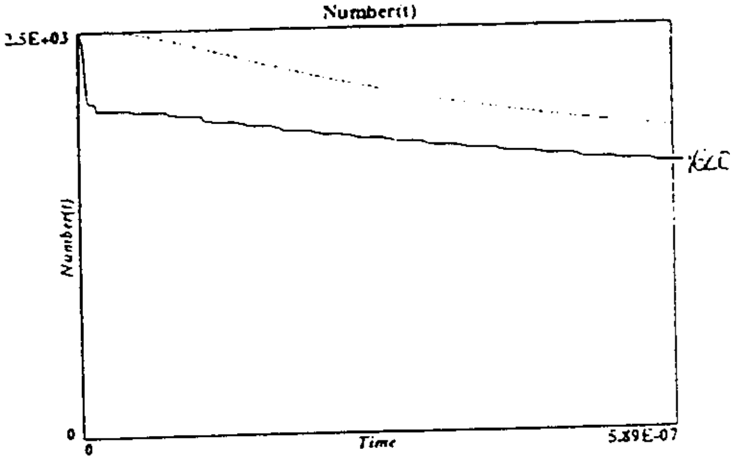
```
                 RFDANC.INP (RF DISCHARGE(IN MKS UNITS))
Voltage-driven without collisions (Argon atom)
-nsp---nc---nc2p---dt[s]--------length[m]--area[m^2]--epsilonr---B[Tesla]---PSI[D]--
 2    200    2e8    7.201788e-11 0.05    0.01         1.0         0.0      0.0
-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]---extL[H]---extC[F]---q0[C]-
 0.0               0.0              0.0     0.0        0         1.0      0.0
-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0            v       0.0           0.0             500.0     13.56e6  0.0
-secondary--e_collisional--i_collisional--reflux--nfft--nsmoothing--ntimestep--
 0              0               0          0     1024       2             0
--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0               0.2         2                  5e-2             0.026
ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.2e-19           0.3            15.0           20.0
--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 7.0e-21           11.55          30.0           100.0
--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 3.0e-20           15.76          30.0           100.0
ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 2.0e-19          5.5e-19                 1.8e-19       4.0e-19
SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.               0.              1e15
--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.0          0.          4.2e5       4.2e5       0.          0.
---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0              0.0          150      0.0         10.0       50000
-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100      0.0        10.0         .020     .030
SPECIES 2
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 6.68e-26      0.               0.              1e15
--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.           0.          2.45e2      2.45e2      0.          0.
---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0              0.0          50       0.0         500.0      50000
-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100      0.0        1.0          .020     .030
```

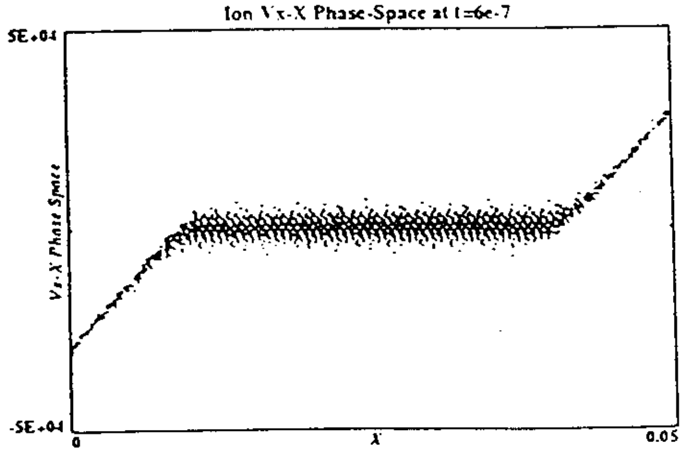
Voltage Driven RF Discharge with Mobile Electrons and Ions, without Electron and Ion-Neutral Collisions





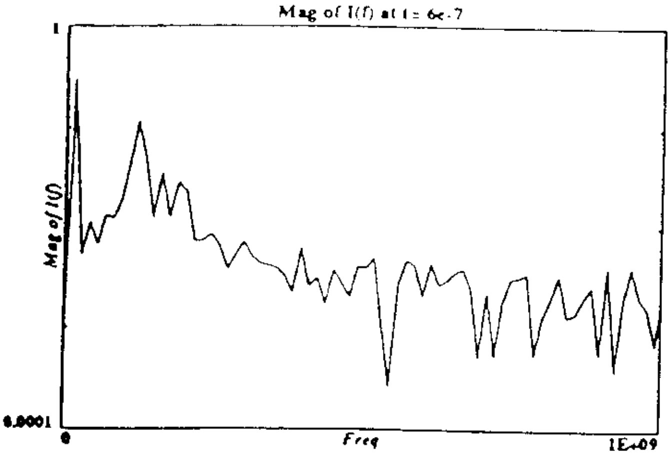


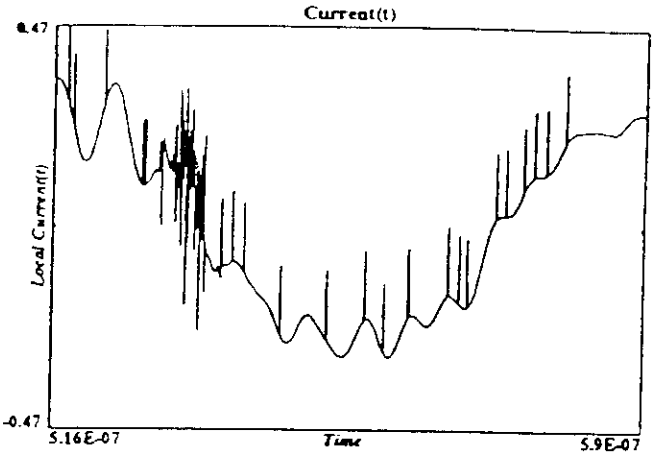


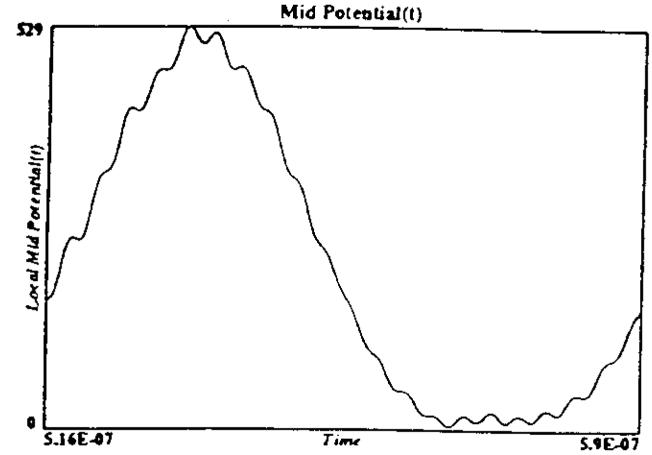


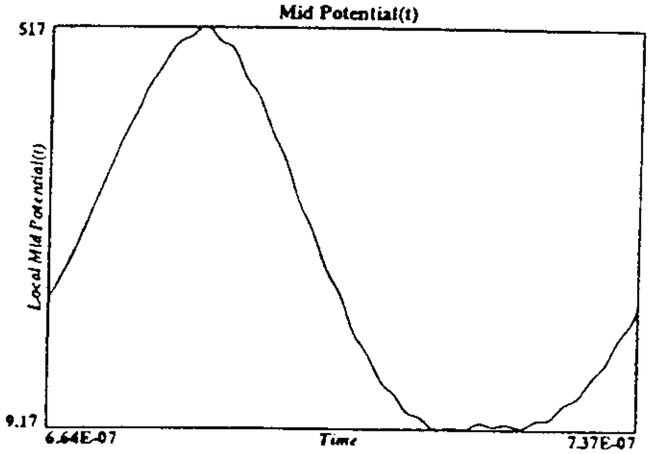
The argon ions now move, producing the time-average densities $n_e(x)$ (solid curve), $n_i(x)$ (dashed curve), field $E_x(x)$ (now much closer to zero in the bulk), and potential $\phi(x)$. Also now the ions escape (as shown) so the electrons also leave. The ions, with $f_{pi} = 1.05$ MHz $\ll f_0 = 13.56$ MHz move with the time average $E_x(x)$, accelerated to both electrodes. f_{series} and f_{plasma} appear in $I(f)$, $V_{mid}(f)$, but are much less pronounced.

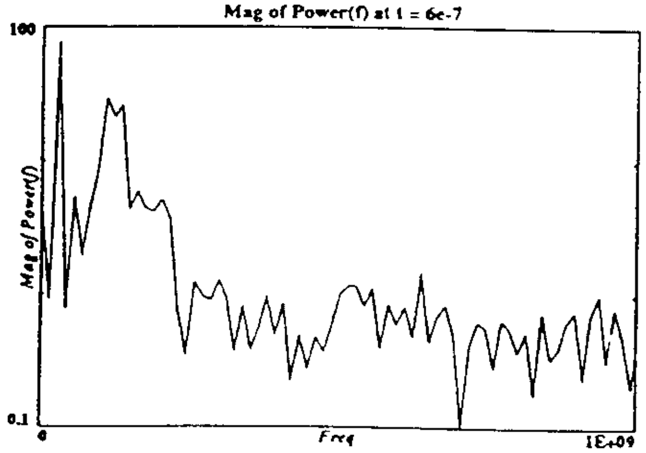
Voltage Driven RF Discharge with Mobile Electrons and Ions, without Electron and Ion-Neutral Collisions

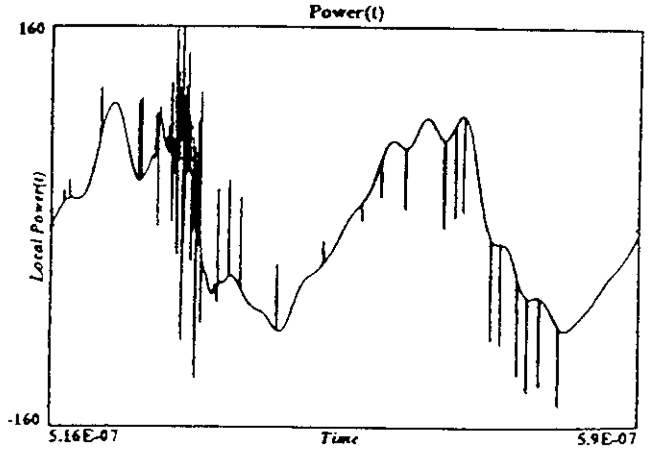












RFDHOMOG.INP

The object is to devise as close a model as possible to that in the L&L text,Section 11.1, called HOMOGENEOUS MODEL, or matrix sheath (immobile ions) model, or the Godyak model.

Your code running and observations are to confirm and complement the text analysis, meaning using your ingenuity as to what you choose to corroborate.

Here are my questions.

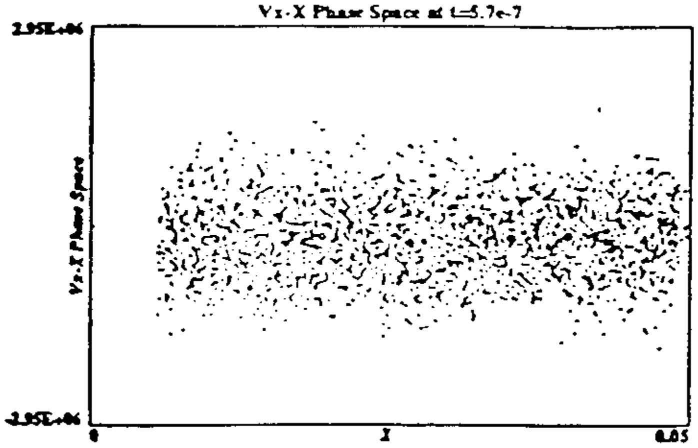
1) The faster electrons are RF driven/ (or drive themselves) out in about 15 ns; relate this time to, say $2v_{thermal}$, a quarter cycle of the RF (13.56MHz) or transit time, say of or faster electrons across the system or sheath (is there a sheath at that early time?). Why are the electrons effectively trapped after that time?
2) Observe the current, I(t). The dominant frequency is, it is claimed, the series resonant frequency, on the order of 74MHz, not a harmonic of the drive. Let's check out the series resonant frequency The analytic value for the series resonance is $\omega_p \times \sqrt{2s/L}$ where s is the sheath width and L is the electrode spacing. One value for s is Eq 6.3.5, on p. 164 in L&L; you will need the $\phi_{mid} - \phi_{wall}$ potential drop, from the time average potential plot, to put into this expression. One computer experimental value of s is obtainable the plot of time-average E, measuring s as where the asymptote of E-bar crosses the E=0 line (as done in lecture). So, is the 74Mhz the series resonance? What are your values of s? What is your value for $\frac{s}{\lambda_d}$?
3) Observe the frequency spectra of current, of mid-potential, of power (and anything else). $I(\omega)$ and ϕ_{mid} will have strong fundamental components, and the latter will have a second harmonic (as expected from L&L). $I(\omega)$ also will have the 74 MHz signal. Explain, if you can why both ϕ_{mid} and power have a double peak, centered on ω_{series}, with the peaks separated by 2f/RF. Look for ω_p peaks; we found such at higher density, which you migh try. (The peak in ϕ_{mid} at about 147MHz might be the second harmonic of the series resonance; do you believe that? Why does the first peak in the power spectra appear at twice the RF frequency (don't miss this one or you flunk EE!).
4) Was this exercise worthwhile for you, in gaining understanding of the electron dynamics of an RF discharge?

Further comments welcome.
Ned Birdsall
2 March 1995

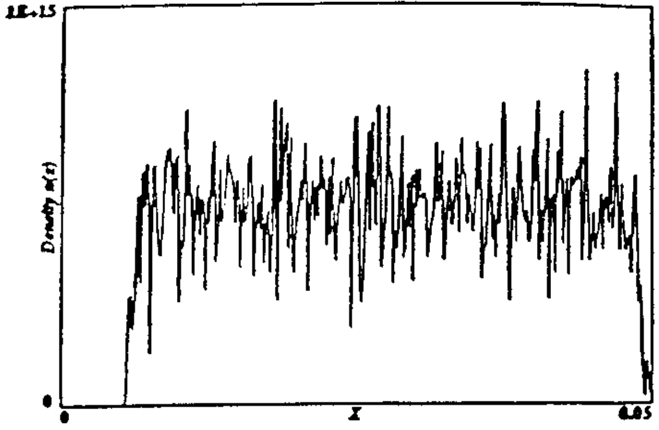
```
                   RFDHOMOG.INP (RF DISCHARGE(IN MKS UNITS))
Homogenous Background of postive charge to simulate infinitely massive ions 1eV
Electrons, 13.56 MHz voltage Drive at 100 V. Matrix Sheath Model (Godyak)

-nsp---nc---nc2p-------dt[s]-----length[m]--area[m^2]--epsilonr---B[Tesla]---PSI[D]--
 1    200   1e8   1.44036e-10 0.05     0.01          1.0         0.0      0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]-
 8.01e-5         0.0                0.0     0.0       0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0          v       0.0            0.0             100      13.56e6     0.0

-secondary-e_collisional-i_collisional---reflux---nfft--nsmoothing-ntimestep--
 0             0             0            0      1024      0          0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                 0.2           2               3e-2              .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19           0.0           0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20           12.0          50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20           13.6          60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 3.0e-19          0.0                     2.0e-19       0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.              0.                5e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.          4.2e5      4.2e5        0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0           150      0.0          50.0      50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100      0.0        15.0         .020      .030
```

A Simple Voltage Driven RF Discharge with Mobile Electrons and Fixed Uniform Ions (Godyak Homogeneous Model)

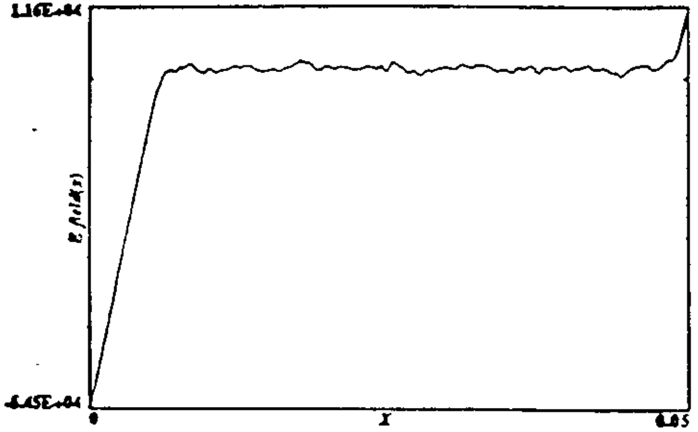
Vx-X Phase Space at t=5.7e-7



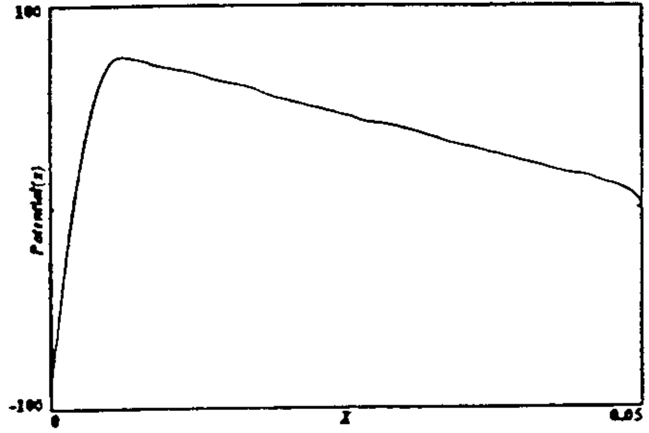
Density n(x) at t=5.77e-7



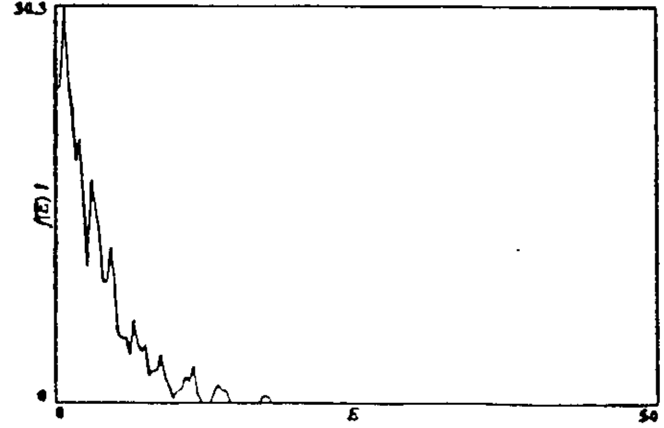
E field(x) at t=5.7e-7



Potential(x) at t=5.7e-7

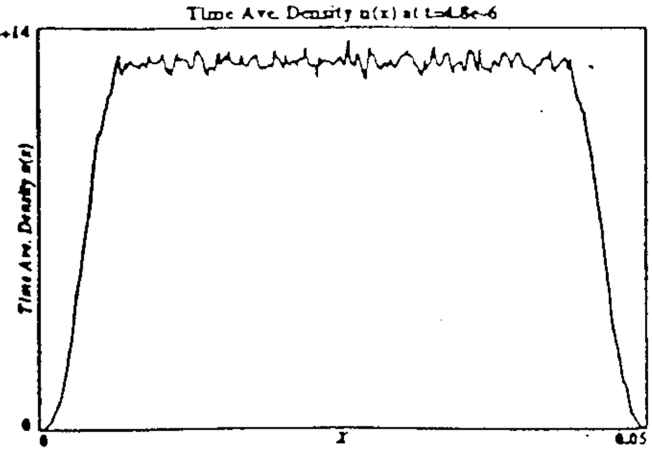


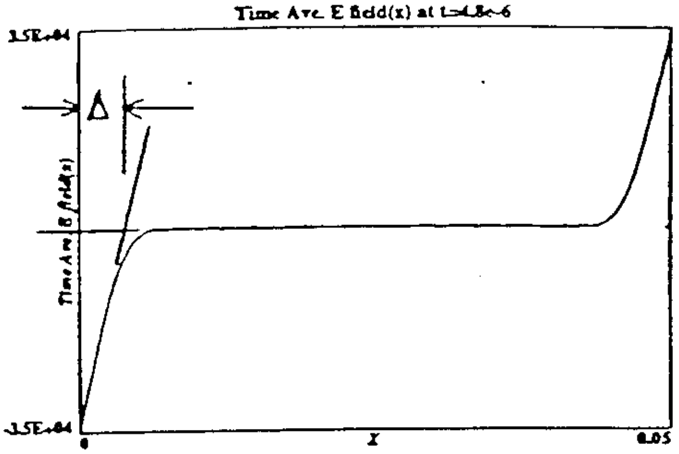
fe(E) at t=5.7e-7

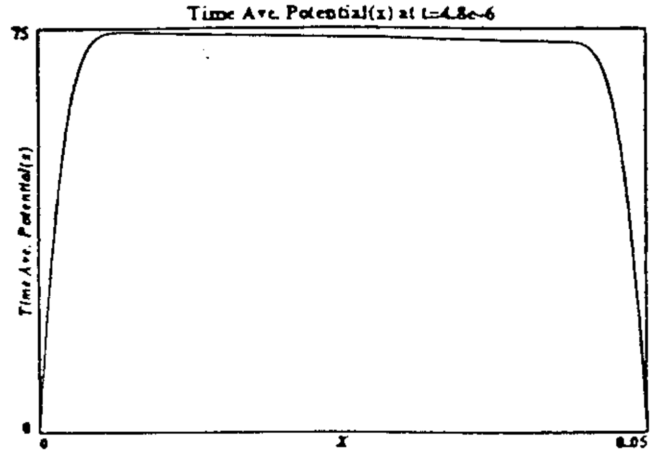


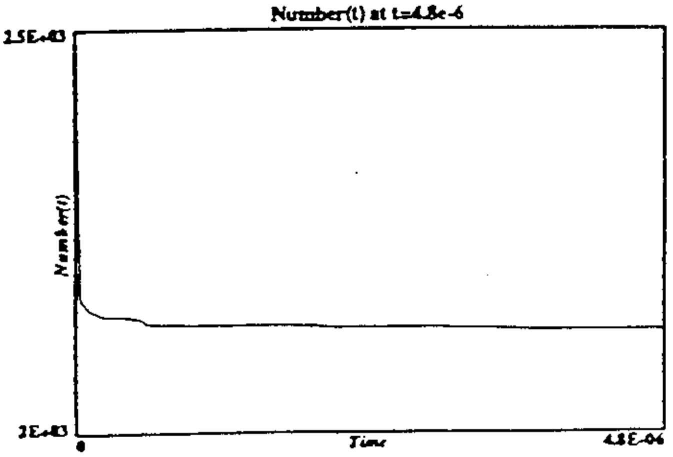
These are snapshots of $v_x - x$ phase space, density $n(x)$, $E_x(x)$, $\phi(x)$ and $f_e(E)$ at the left (target) electrode. Initially $T_e = 1eV$, $n_o = 5 \times 10^{14}/m^3$. RF drive is 100 Volts at $f_o = 13.56$ MHz. Electrode spacing is 0.05 m. With uniform immobile ions, $E_x(x)$ is larger than with mobile ions.

A Simple Voltage Driven RF Discharge with Mobile Electrons and Fixed Uniform Ions (Godyak Homogeneous Model)







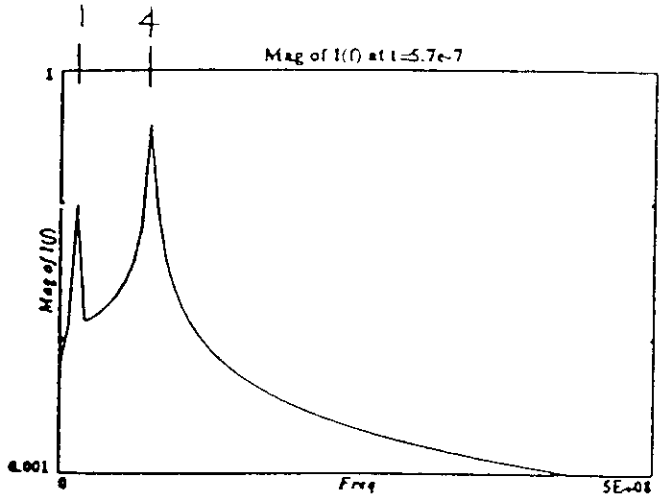


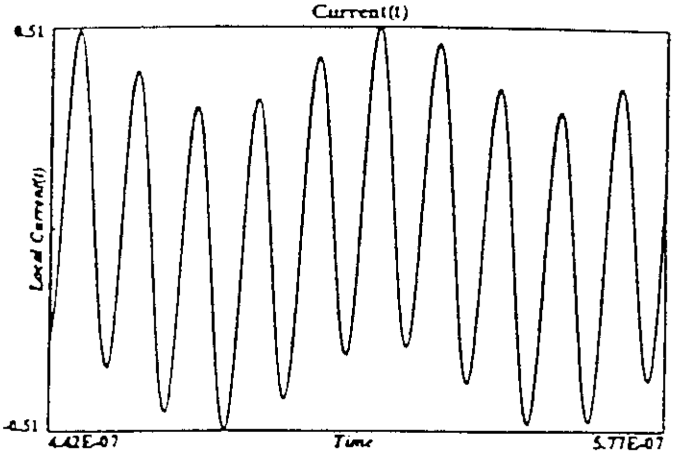
Three of these are time averaged plots, of n(x), $E_x(x)$ and $\phi(x)$. The "vacuum gap," Δ, is estimated from the slope of $E_x(0)$, to be 0.0035 m (roughly equal to 10 λ_D) which predicts $f_{series} = f_{plasma}\sqrt{2\Delta/L_x} = 75.6$MHz. The total number of (computer) electrons decreases from 2500 at t = 0 to 2126 after about one RF cycle; the electrons are trapped thereafter.

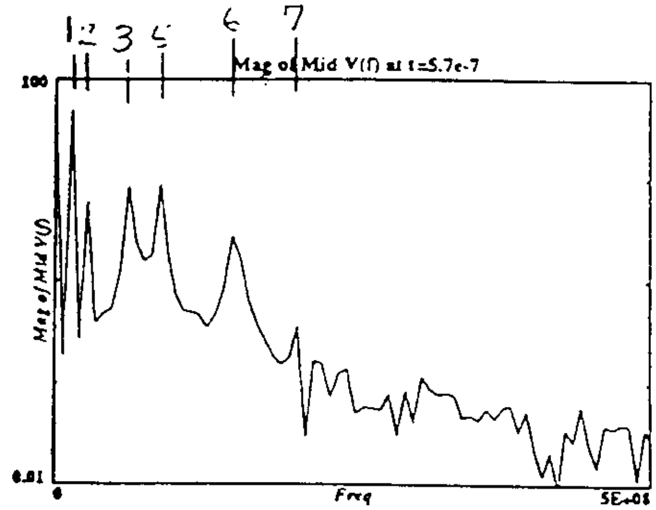
On the next page, the spectra of I(f) shows peaks at f_0 and f_{series}, noted as 1,4. $V_{mid}(f)$ shows peaks at f_0, $2f_0$, $f_{series} \pm f_0$, $2f_{series}$, and f_{plasma}, (peaks 1, 2, 3, 5, 6, 7). Power (f) has peaks at $2f_0$ and $f_{series} \pm f_0$ (peaks 2, 3, 5).

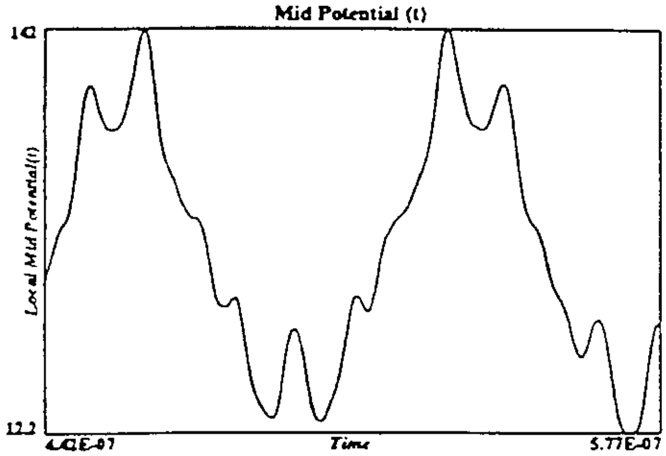
Raising n_0 to $5 \times 10^{15}/m^3$ changes these spectra to some extent, namely with a more distinct response in $V_{mid}(f)$ at f_{plasma}.

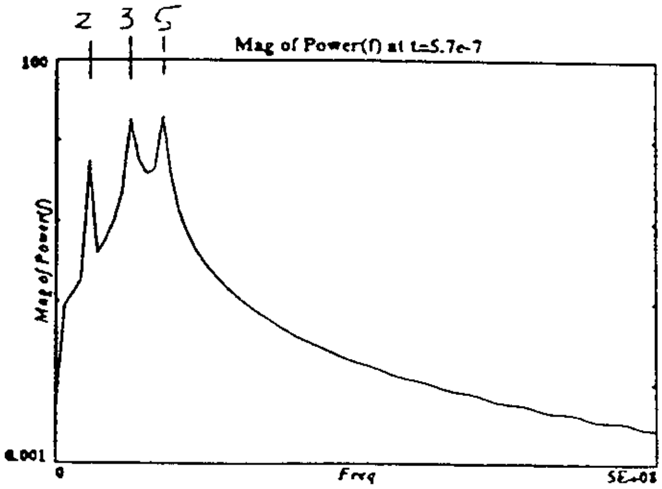
A Simple Voltage Driven RF Discharge with Mobile Electrons and Fixed Uniform Ions (Goydak Homogeneous Model)

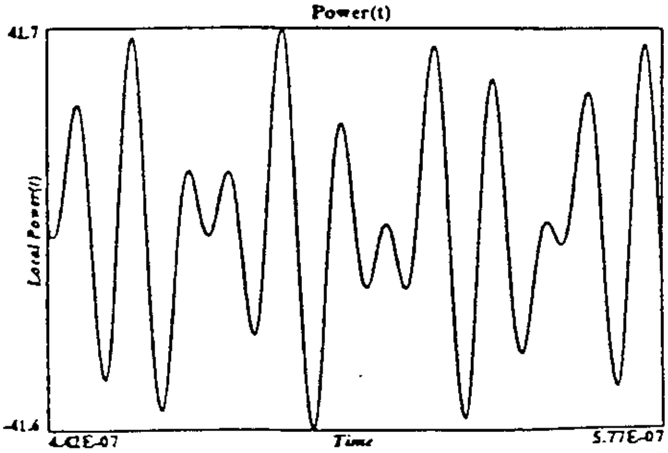












f_1 = 13.56e6, drive frequency, f_0
$f_2 = 2f_0$
f_3 = 6.11e7
f_4 = 7.5e7 $(\cong f_3 + f_0, \cong f_5 - f_0) \cong .38 f_p$

f_5 = 8.8e7
f_6 = 1.5e8
f_7 = 2e8 $\cong 9\sqrt{n}$, n = 5e14 (f_p)

RFDA.INP

The object is to run as close to an argon RF discharge as we can, with real mass ratio, real collision cross sections, real gas pressures, real dimensions. There are many objects, like learning the physics, but engineering objectives as well, such as learning the ion energy and angular distributions at the target (the left hand electrode), which are f(E)2 and angle.
The references for learning about the PIC-MCC technique are the Birdsall article in the Reading Materials (an introduction) and the Vahedi and Surendra report (soon to be published). We will discuss the MCC technique in lecture.

See, in V&S, Section 4 and Figures 15-17 for computer experimental results in argon, compared with laboratory experiments (the two-temperature results), for a current driven model. You may try current drive also (after running RFDA.INP), as all of the necessary dimensions et al. are in V&S.

We are making a BIG change here, putting in collisions, as, at low pressures, the collision frequency can be, by far, the lowest frequency, longest time, tied to diffusion time scales. This means that we have to run RFDA for many RF cycles, many 1,000's of time steps in order to reach some form of equilibrium.
Hence, start running NOW; keep your program running (e.g.,in the
background) for quite a few hours.

Run the code. Look at all of the diagnostics.

1) Keep NUMBER on, so that you will see the number of electrons and ions continuously increasing toward some final state (maybe 10,000 particles each). Estimate the run time to get there. You might increase the background pressure to increase the collision frequency, and shorten the time to equilibrium.
2) For pressures below about 200mT, you should see the two-temperature electron energy distribution evolve (as in V&S, noted above). For pressures above about 300mT, you will find a single temperature distribution. (I will show and give out these results in class.)
3) Observe the spectra of current, of mid-potential, of power, and comment. Do you see the harmonic structure found analytically in L&L?
4) Observe J dot E, power delivered by the fields to the charges, for both electrons and ions. The ions have positive J.E, as they are simply accelerated the the time average E field in the edge sheaths; explain the ripples (they drift to the wall), confirmed by the f(E)2 at the target and the (enlarged) v-x traces of the particle trajectories. The electrons have regions of both signs of J.E; comment on this (read L&L pp344-346 on heating mechanisms).

I will copy some more articles on simulations much like this, ones
that ran very long times.

Ned Birdsall
23 March 1995

RFDA.INP RF DISCHARGE(IN MKS UNITS)

Voltage-driven with **electron and ion-neutral** collisions (Argon atom)

```
-nsp---nc---nc2p-------dt[s]---length[m]--area[m^2]--epsilonr---B[Tesla]---PSI[D]--
 2    300    1e9  7.20179e-11 0.05        0.01         1.0         0.0         0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]---extL[H]---extC[F]---q0[C]-
 0.0                 0.0               0.0      0.0        0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0            v        0.0              0.0              500.0    13.56e6    0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0                  1                   2          0         1024     6               100

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                    0.2           2                     5e-2            0.026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----

--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.2e-19               0.3             15.0            20.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 7.0e-21               11.55            30.0            100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 3.0e-20               15.76            30.0            100.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----

---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 2.0e-19               5.5e-19                   1.8e-19            4.0e-19

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31      0.                  0.                1e15

--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.0           0.            5.9e5          5.9e5          0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0                 0.0          150        0.0          50.0       50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100       0.0          15.0          .020       .030

SPECIES 2

----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 6.68e-26      0.                  0.                1e15

--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.            0.            2.19e3         2.19e3         0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0                 0.0          50         0.0          200.0      50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100       0.0          15.0          .020       .030
```

PLASMA COMPUTER EXPERIMENTS AS AIDS TO MODELING DISCHARGES

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Abstract. Plasma computer experiments are simulations of complete devices, with ''internal'' plasma and ''external'' circuits. Such are intended to complement analytic modeling and laboratory experiments. We begin with the numerics which we use to describe the physics and the chemistry of many-particle codes. We work from first principles (classical mechanics, electromagnetics) and seek self consistency, producing particle-in-cell Monte-Carlo collision (PIC-MCC) computer experiments. For orientation, demonstrations will be presented of particle simulations: undriven collisionless plasma; RF driven plasma, with mobile electrons but fixed ions; full RF driven collisional discharge. Comparison will then be made between our simulations and the laboratory experiments of V. Godyak and R.B. Piejak (1991); good agreement is obtained with most quantities.

An example of a design challenge is increasing the frequency of an RF driven discharge, done by observing simulation results for 13.56, 30 and 50 MHz. ''Short runs'' show density increases (as f_0^2), sheath width decreases (as f_0^{-1}) which allows use of lower voltage drive (less target damage) and produce a more uniform, normal flux.

Recent extensions have been to simulations with electronegative gases (in this case, oxygen), and to 2d3v models, where we are finding differences from the widely used 1d3v models, such as asymmetric discharges (large and small end plates).

Comparison of Computer Experiments and Laboratory Experiments

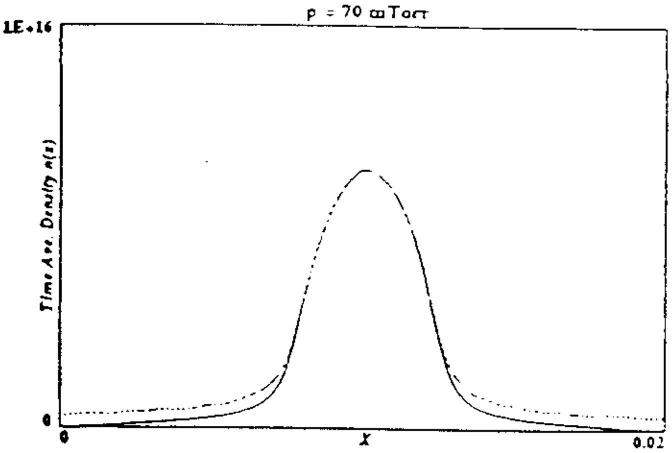
Argon
Pressure ranges from 70 mTorr to 500 mTorr
Parallel plane electrodes, gap = 2 cm
Current driven, I = 0.3 A rms, 13.56 MHz

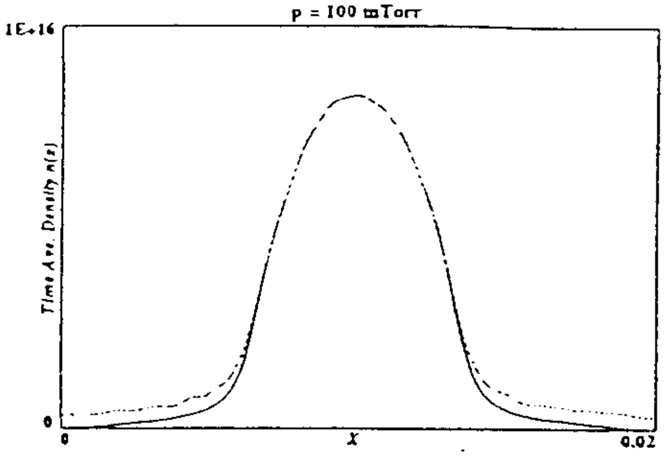
Let us look at time average spatial profiles from the 1d computer experiments:

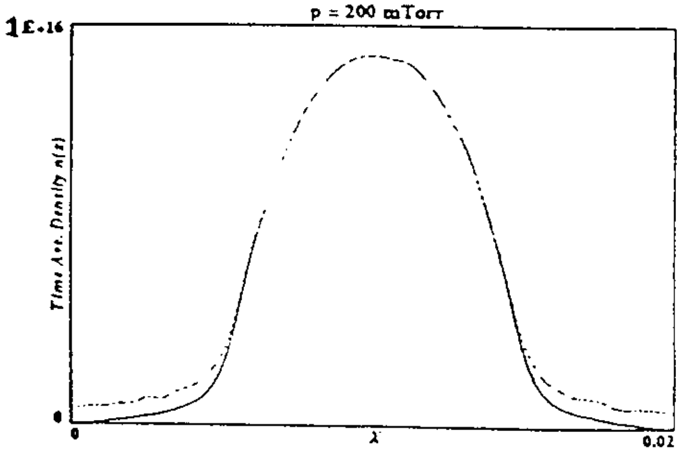
density	$n(x)$
potential	$\phi(x)$
power deposition	$(\mathbf{J} \cdot \mathbf{E})(x)$
ionization	

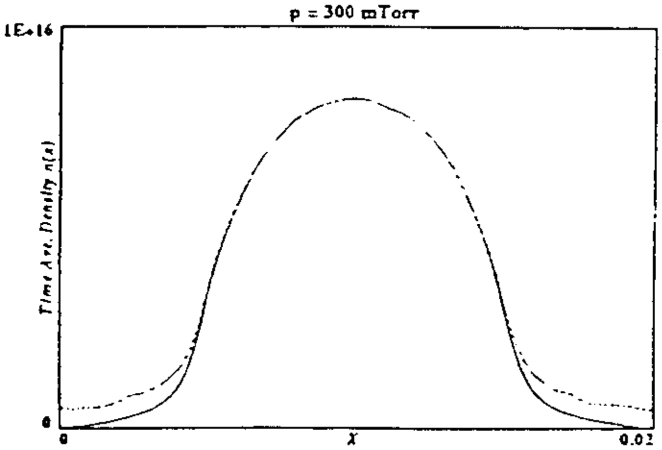
Many details are also available, such as the ion phase space distribution (beaming).

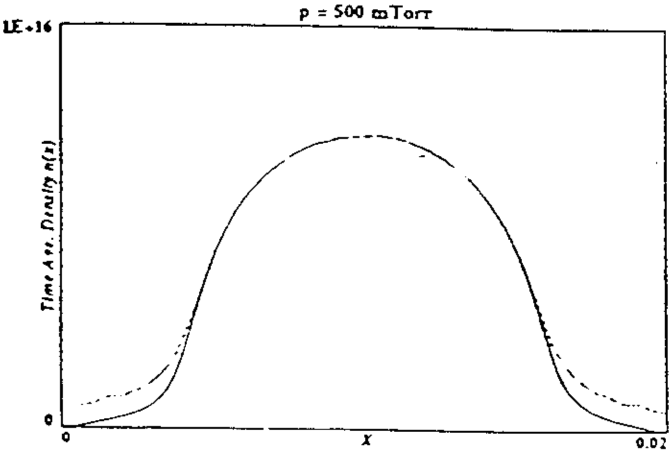
Particle Density of Electrons and Ions in an Argon Current-Driven RF Discharge (Gap= 2cm, I= 0.3 A rms)







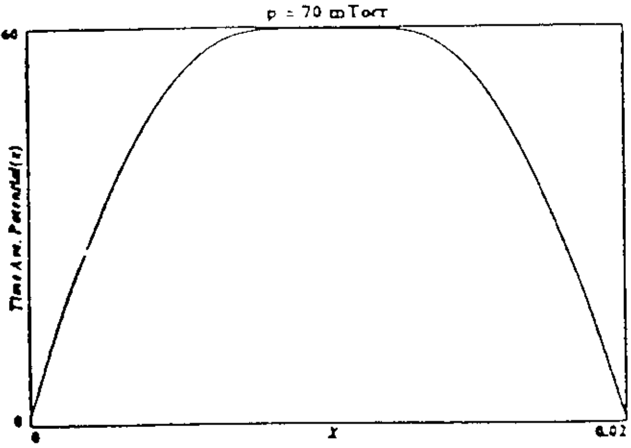


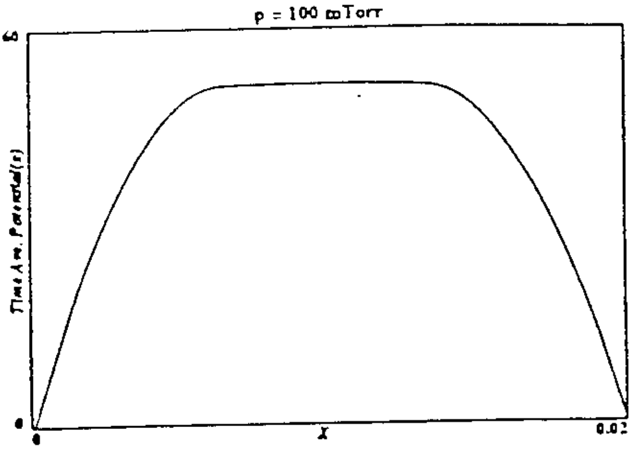


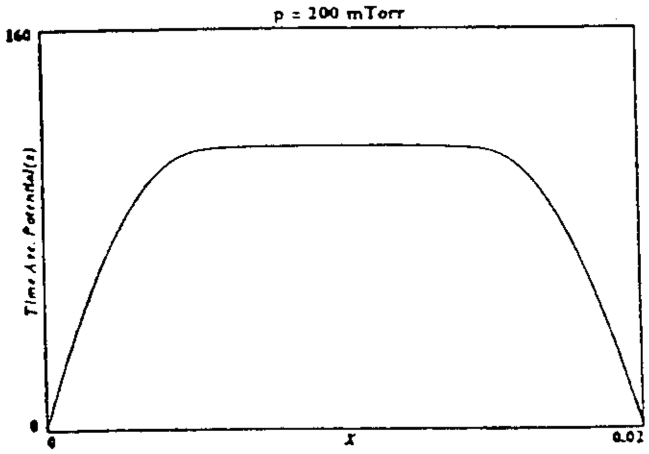
Next, electron and ion collisions with neutrals (scatter, excitation, ionization, charge exchange) are included. The examples shown are current driven at 13.56 MHz at 70, 100, 200, 300 and 500 milliTorr pressure. The discharge is self-sustaining.

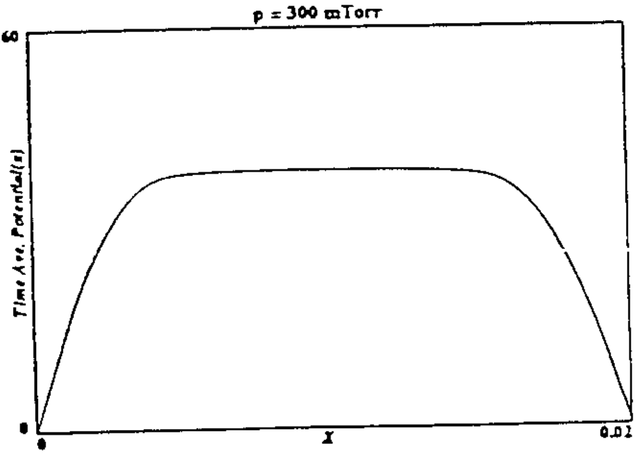
This set of time-average density n(x) plots shows the ion density reaching both walls (dashed line), with much smaller electron density; dynamically $n_e(x,t)$ shifts from left to right at f_0.

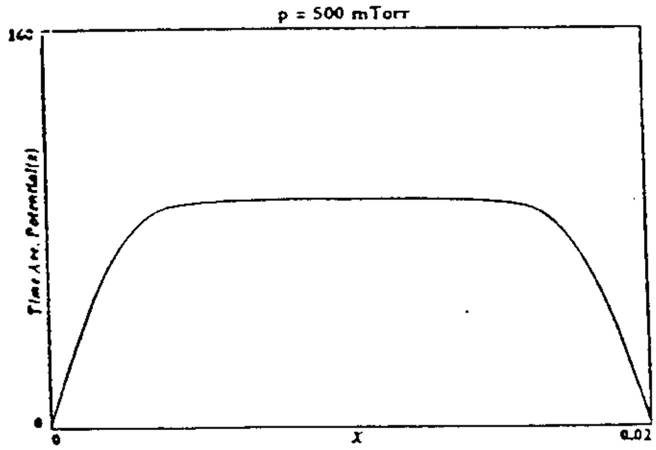
Time Averaged Potential in an Argon Current-Driven RF Discharge (Gap= 2cm, I= 0.3 A rms)





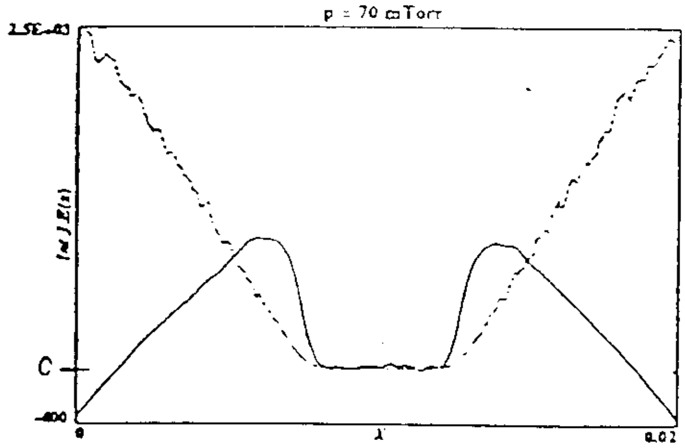


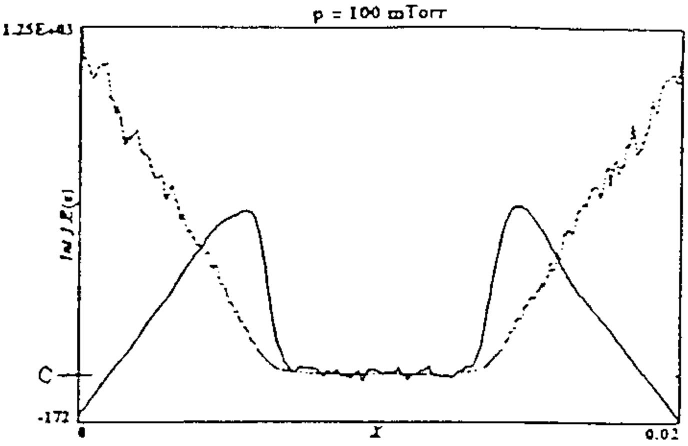


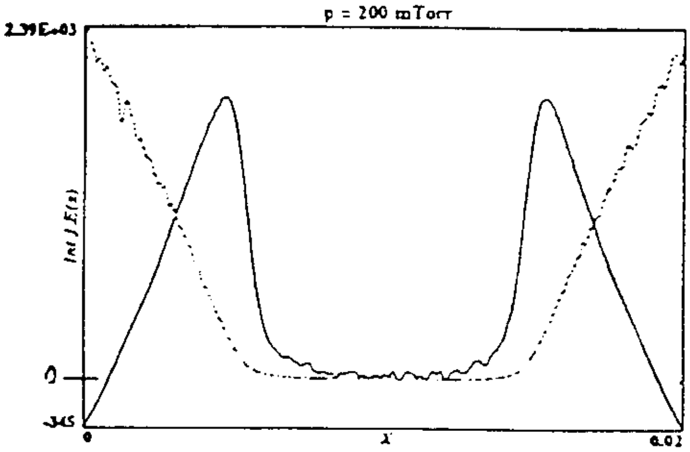


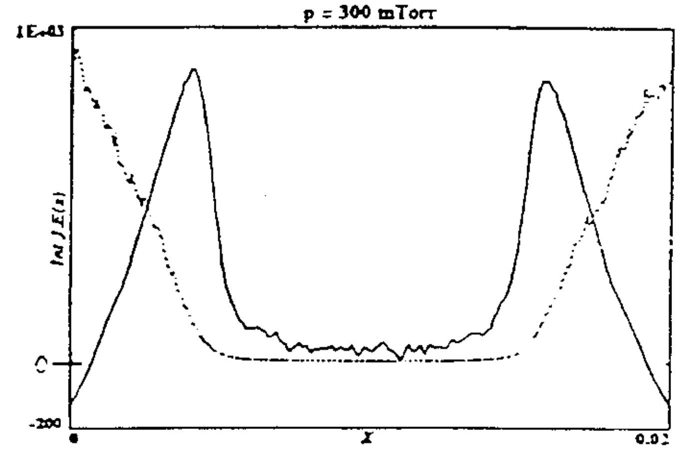
The time-average $\phi(x)$ shows similar behavior, with large E(x) in the sheaths, and E(x) very near zero in the bulk.

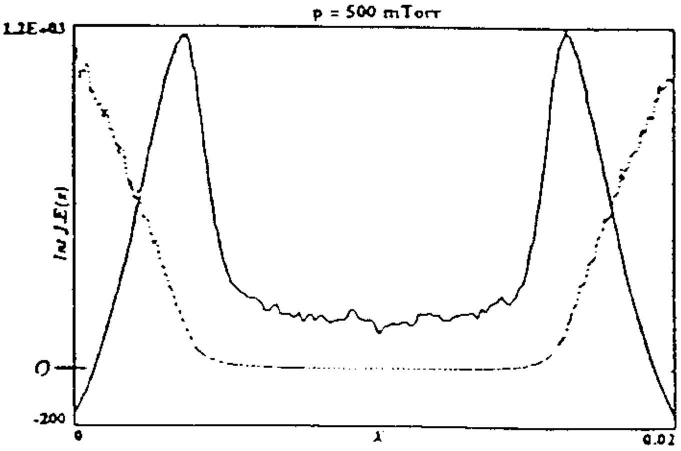
Spatial Profile of Average J.E in an Argon Current-Driven RF Discharge (Gap= 2cm, I= 0.3 A rms)





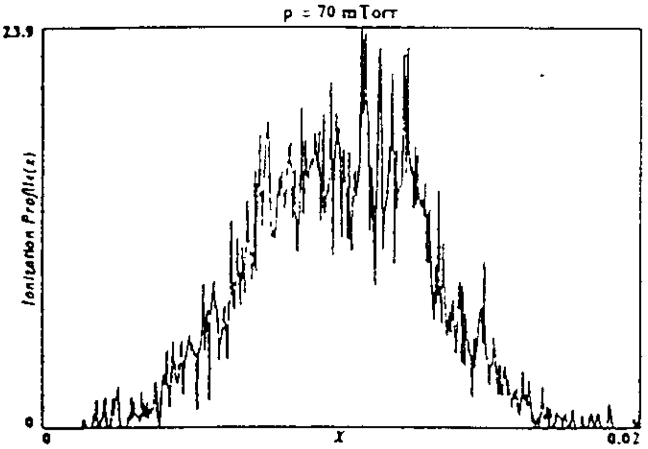


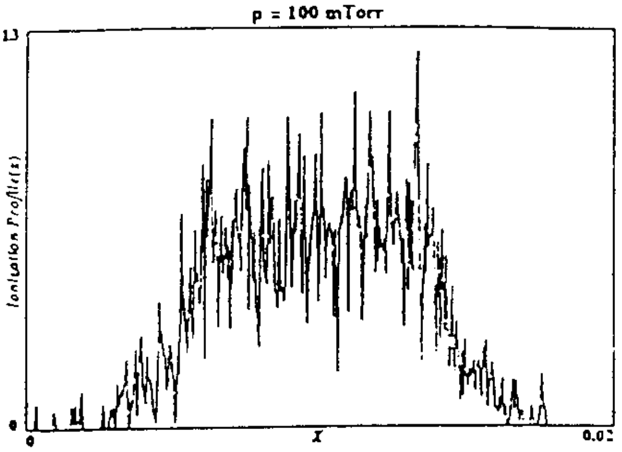


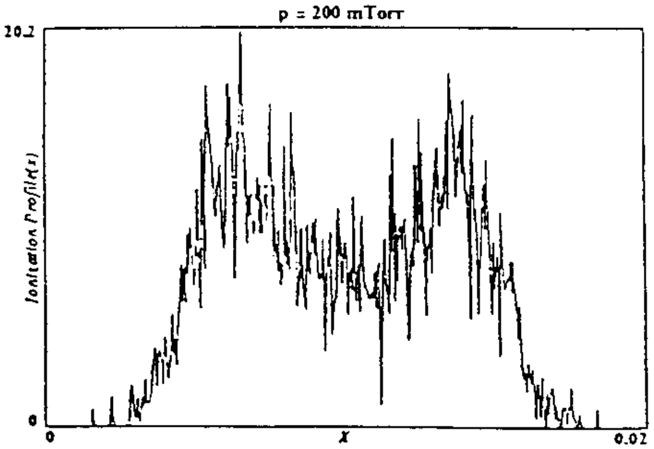


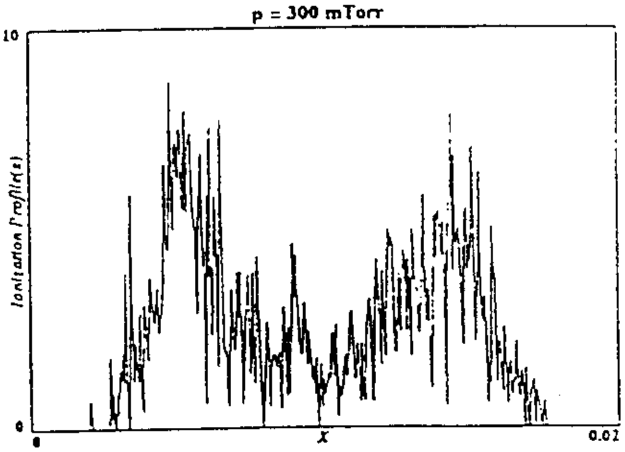
These plots are the time-average values of $\underline{J} \cdot \underline{E}$, the power delivered by the field to the particle (work done by $\underline{E}$ on $\underline{J}$), over the RF diode. The ions (dashed line) are clearly accelerated in the sheaths, with no work done on the ions in the bulk plasma. The electrons are heated in (and by) the sheaths, with little work done in the bulk plasma at low pressures, but with increased (ohmic) heating done in the bulk at high pressures.

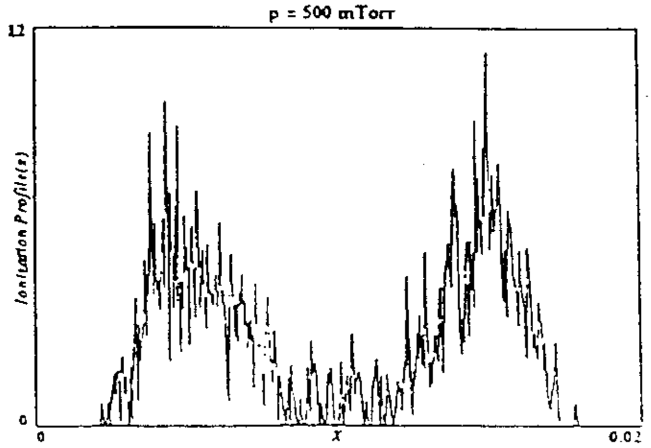
Ionization Profile in an Argon Current-Driven RF Discharge (Gap= 2 cm , I = .3 A rms)











The ionization profiles follow the density profiles at low pressure, but peak at about the sheath edges at high pressures.

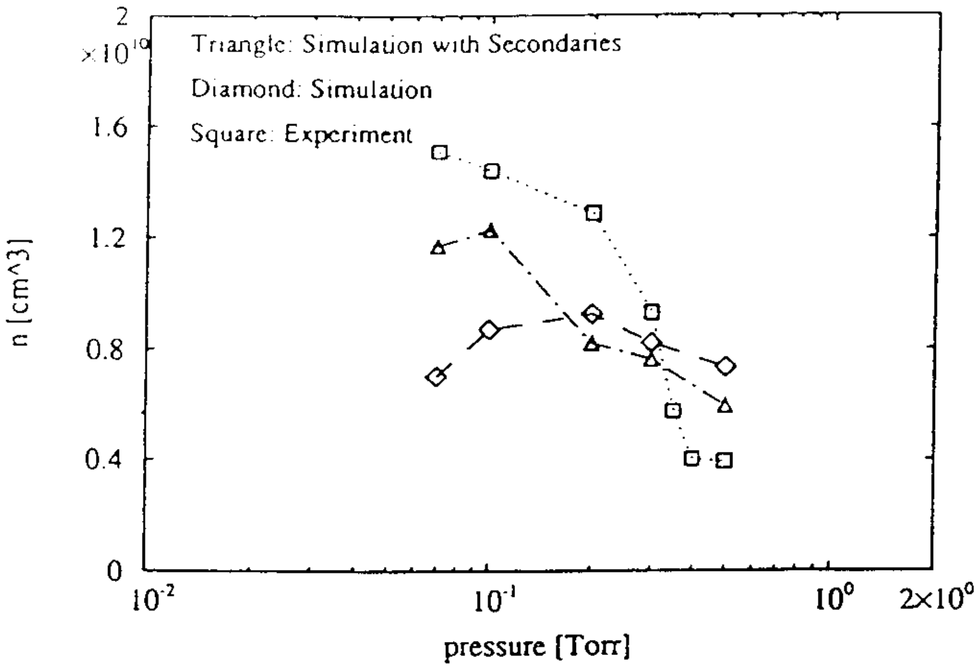
Next, let us look at the dependence on pressure of our computer experiments and the laboratory experiments of Godyak and Piejak (1990).

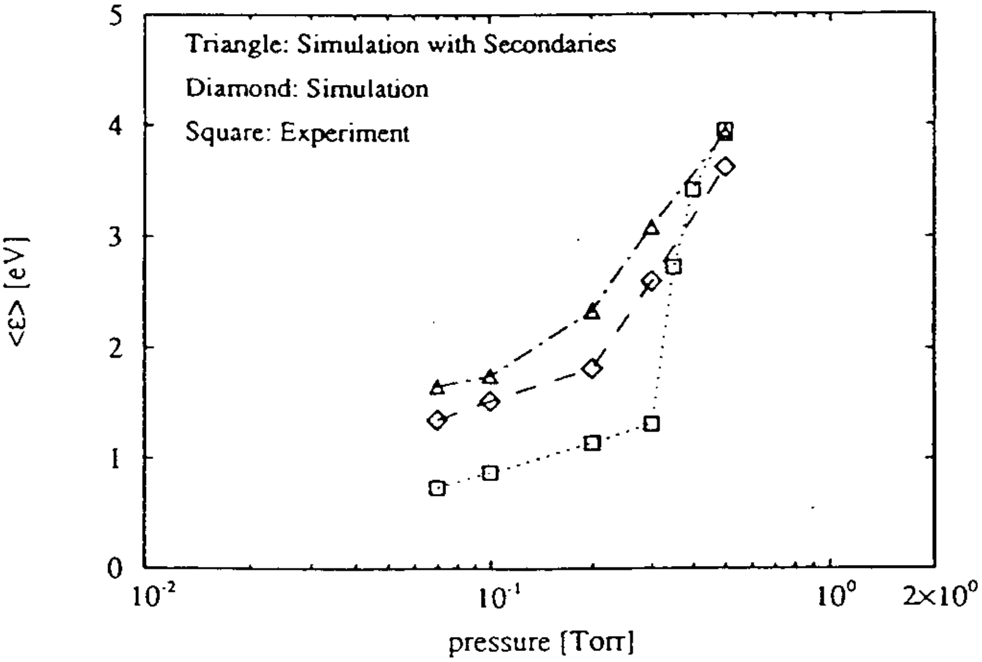
First: Time averaged density
The lab result is a drop in density of about a factor of 4. Our initial simulation result was almost constant density. We then added secondary emission due to ion bombardment; the agreement improves, with the simulation density dropping by about a factor of 2. (More work is needed here.)

Second: The average kinetic energy in the system fits rather well at higher pressures.

Third: Let us look at the time average electron energy distribution $f(E)$. At pressures below 200 mTorr, the simulation and the laboratory results both show two temperatures, a cool bulk, plus a hot tail. Indeed, the values of the low and high temperatures observed agree very well.

Fourth: We will show a video of the simulation EVDF, resolved as $f(x,v,t)$, which shows that the hot tail moves from one side to the other, from $+v$ to $-v$. Hence, a two-temperature model is misleading; the apparent bi-Maxwellian is *not* time independent.


Time Averaged Density in the bulk


Average Electron Energy in the system

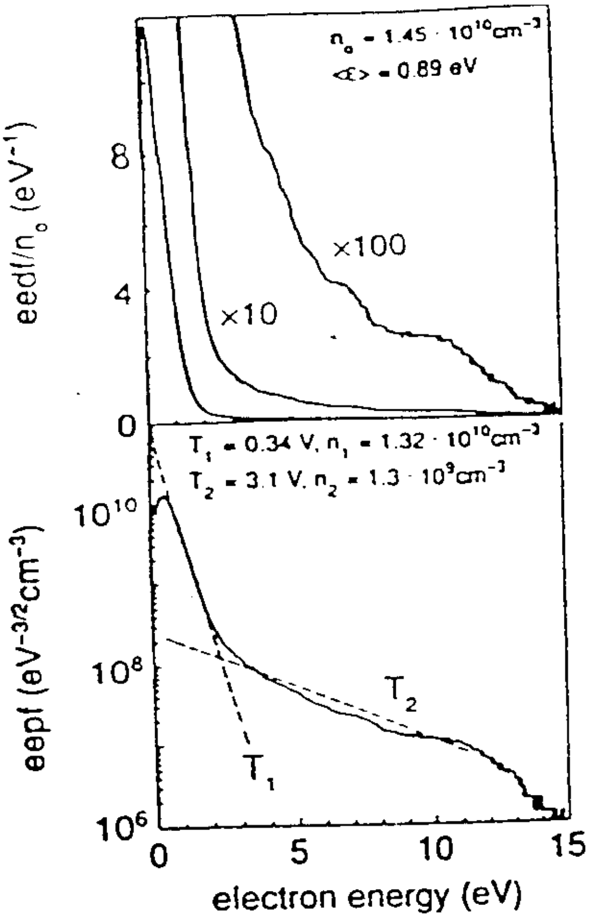


FIG. 2. The EEPF (lower) and normalized EEDF (upper), $F(\varepsilon)/n_0$, obtained for $p = 0.1$ Torr and $I_d = 0.3$ A rms.*

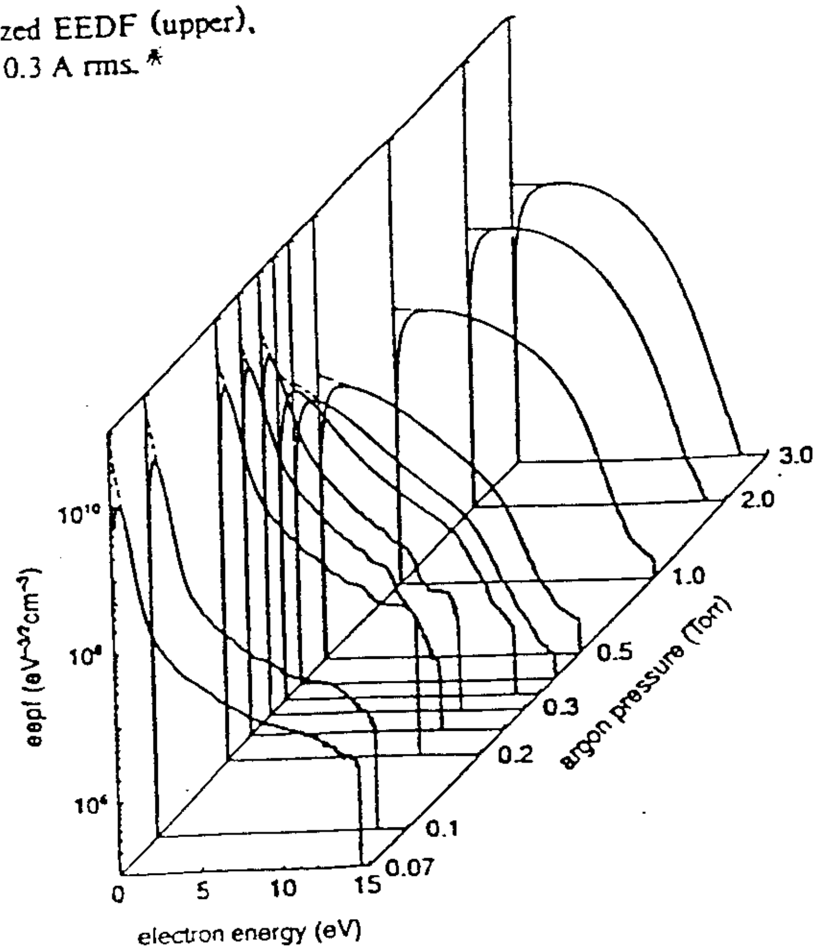
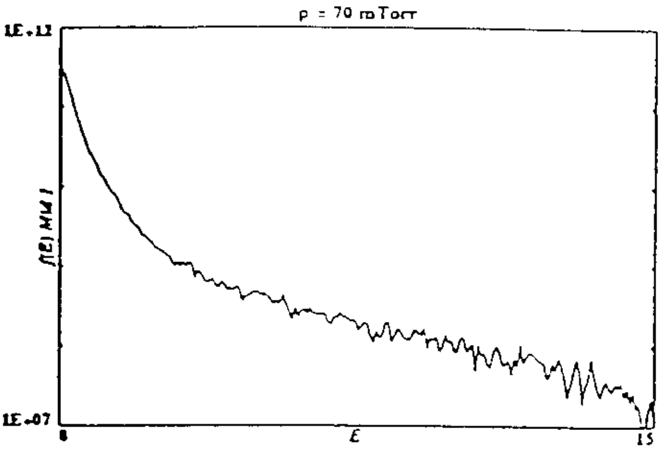


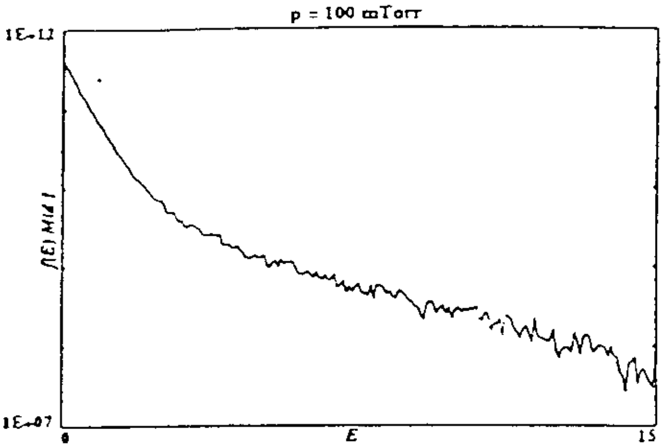
FIG. 3. The EEPF evolution with changing argon pressure, $I_d = 0.3$ A rms.*

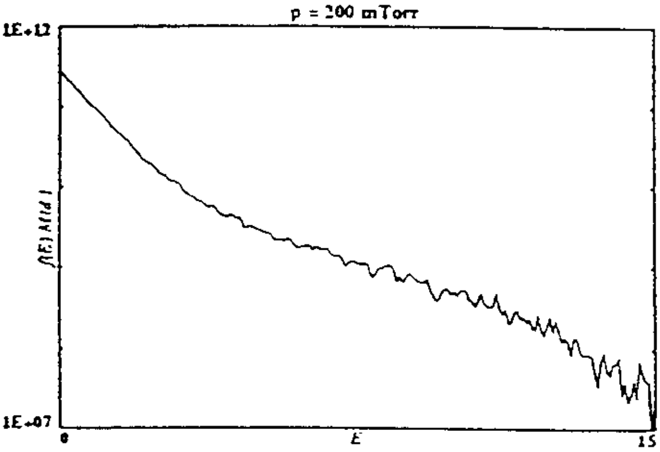
* V. A. Godyak & R. B. Piejak,
Phys. Rev. Lett., 65 996 (1990)

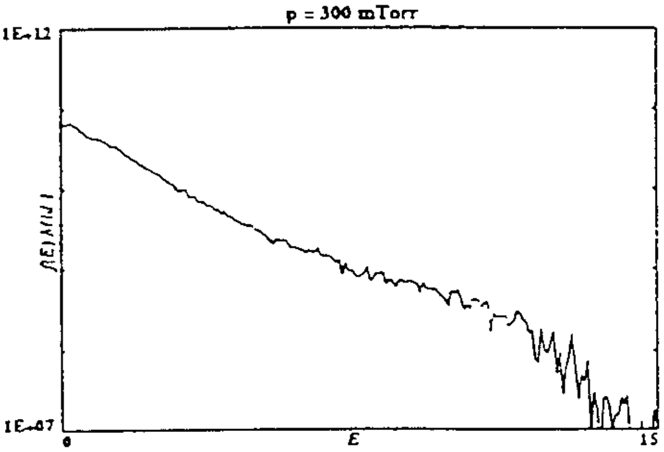
⑦

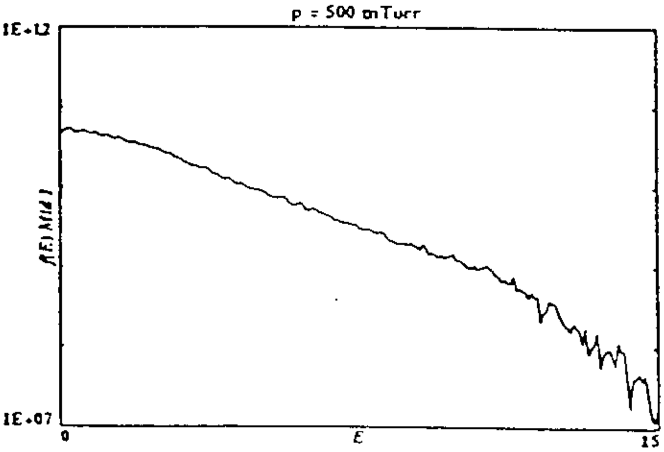
Electron Energy Distribution Function in an Argon Current-Driven RF Discharge (Gap= 2 cm, I= 0.3 A rms)





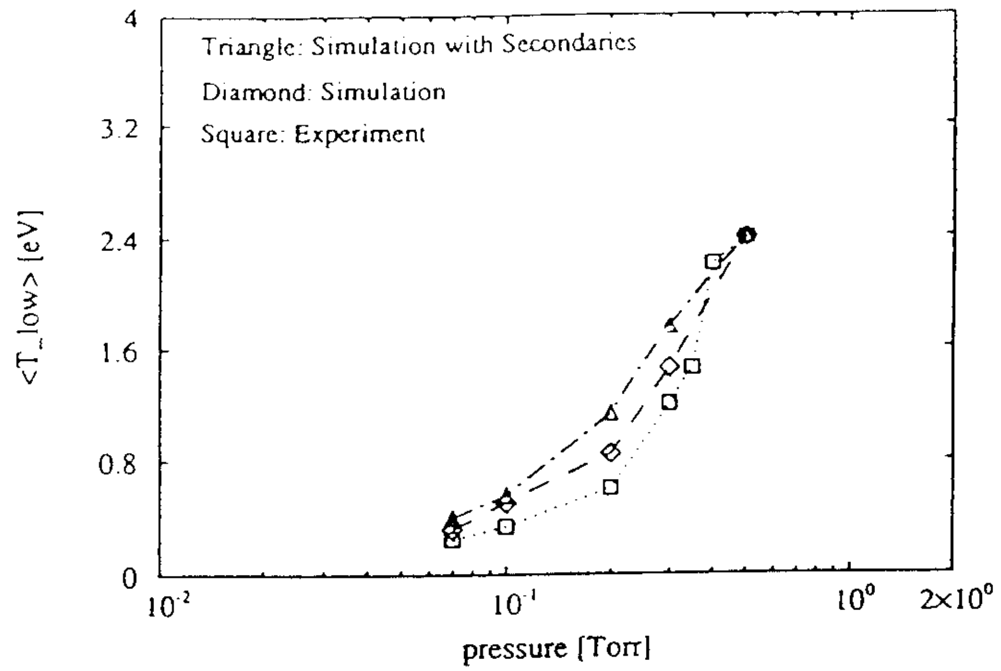




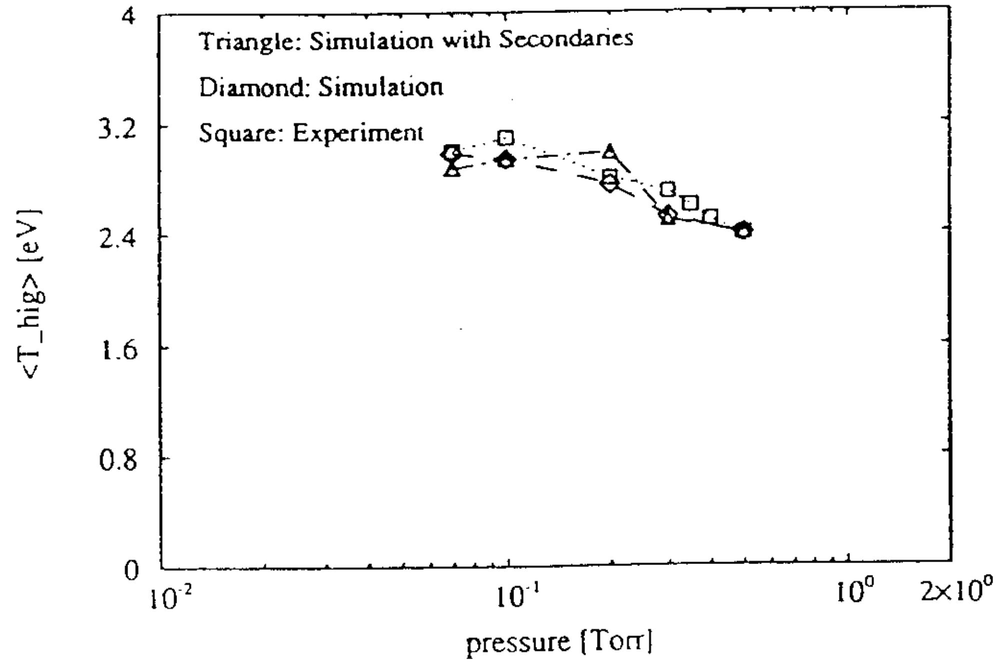


The electron energy distribution function, $f_e(E)$, shows a cool bulk temperature (below 1 eV) and a much warmer tail (few eV), at low pressures, but a single T_e at higher pressures, consistent with the $\underline{J} \cdot \underline{E}$ picture, increasing bulk heating at 500 mT. Such behavior is also observed in laboratory experiments.[3] Dynamically, the hot tail appears at the left-hand sheath ($v_x < 0$) and then at the right-hand sheath ($v_x > 0$), with velocity sign lost on these plots of $E = \frac{1}{2}mv^2$. In addition, at low pressures, one observes fast electrons (above ionization energies) having beam like behavior in the bulk, bouncing between the sheaths.[4]

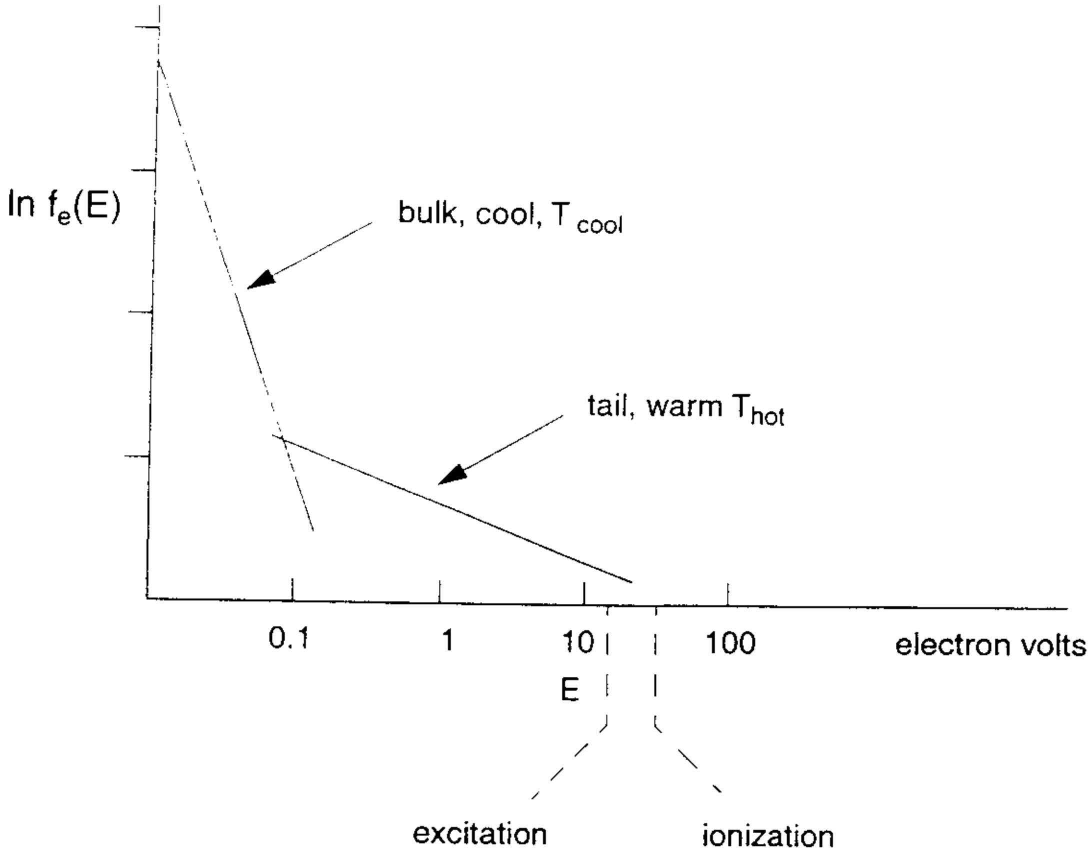
Low Electron Temperature in the Bulk Plasma



High Electron Temperature in the Bulk Plasma



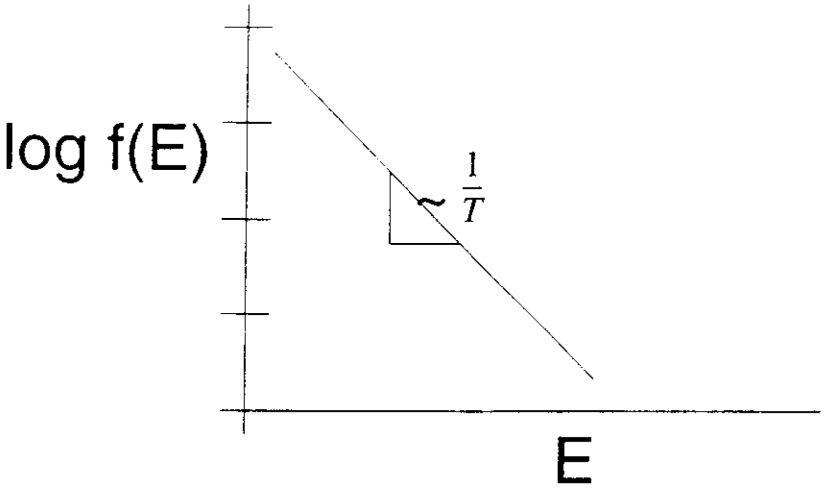
Importance of fast electrons



EEPF

electron energy probability function

f(E) = A exp $\left(\frac{E}{kT}\right)$



In particle simulations, we wish to count the number of particles with kinetic energies ($\frac{1}{2}mu^2 = E$) between E and E + dE, called f(E)dE. This density is obtained from

$$\delta n = \int_{dE} f(E) d\upsilon^3$$

Now $dv^3 \sim d(E^{3/2}) \sim \sqrt{E}$ dE, so

$$\delta n \sim \int_{dE} f(E)\sqrt{E}\ \mathrm{d}E$$

Hence, $f(E)dE \sim \frac{\delta n}{\sqrt{E}} \sim \frac{\delta n}{\upsilon}$, the density desired, and plotted.

MAXWELLEA.INP

HOMEWORK IN CYLINDRICAL MODEL, USING XPDC1, CONCENTRIC CYLINDERS:

This is a simple model, two concentric cylinders, both grounded, initially filled with 1eV electrons and 1/40 eV ions (argon), no collisions with neutrals. The inner electrode has radius of 0.01 m and the outer cylinder has radius of 0.05 m. We chose to use argon, rather than $M_i/m_e = 400$ as in MAXWELLEP.INP in your file, in order to be more realistic.

The object is to observe and measure the difference between this ASYMMETRIC MODEL and the symmetric planar model, as done earlier with Maxwelle, Maxwellep, Maxwello. (Note that the planar model, with equal area electrodes, should be thought of as "driven" push-pull, to be rigorously symmetric.) The most obvious difference that you should find is that the inner, smaller radius, electrode has the more pronounced sheath, with much larger radial E field. (You have seen such already in the video of our 2d3v simulations of a small area driven electrode and a much larger area grounded electrode). The two sheath capacitances are not the same, so the the voltage divisions are not equal.

Observe the same quantities as in the planar models done earlier. Measure the oscillation frequency of the midpotential; compare with plasma frequency obtained from the measured mid-density.

Measure the oscillations in current, possibly the series resonance. Obtain an analytic expression for the series resonance frequency, by setting the series impedance equal to zero. Use the model of vacuum inner sheath + plasma bulk + vacuum outer sheath, as done in lecture, with the usual cold plasma dielectric. The expression will have lots of log terms; simplify as much as possible. Obtain a numerical value for omega-series from your expression, using the inner sheath amd outer sheath radii as measured from the Er field extrapolation as shown in lecture earlier (or, semi-justified by the discussion of Section 6.3 in the L&L text, p.164). Check your analytic value (using your computer experimental values of sheath radii) for series resonance frequency with that of the observed current oscillation frequency.

As presented in EE 239 lectures, and in the L&L text, Section 11.4, for capacitively coupled RF discharges, the ratio of the inner sheath potential drop to the outer sheath potential drop depends in some fashion to the ratio of outer electrode area to inner electrode area. Now, here, we do NOT have a discharge, but an after-glow. However, the sheath drop ratio can be obtained and related to the ratio of outer r to inner r, by running MAXWELLEA.INP for different radii. Try this, keeping r/outer - r/inner fixed (the electrode separation); plot the ratio of the V drops vs radii ratio on lag-log paper in order to obtain the the value of "q", as in Eq 11.4.8 in L&L, p.370.
BE VERY CAREFUL WHERE YOU MEASURE THE SHEATH DROP; INITIAL SUGGESTION IS TO OBTAIN THE SHEATH EDGE AT THE SHEATH RADIUS DETERMINED FROM THE TIME AVERAGE E-FIELD PLOT.

Ned Birdsall
19 April 1995

```
                    MAXWELLEA       DECAY OF A MAXWELLIAN DISTRIBUTION
Argon, real mass ratio, dimensions same as rfda2

-nsp---nc---nc2p---dt[s]------r0[m]----r1[m]---height[m]--epsilonr--Bz[Tesla]-
 2   100    6e7  1.5625e-10 0.01      0.05        0.1          1.0         0.0

-rhoback[C/m^3]-backj[Amp/m^2]---dde---extR[Ohm]--extL[H]---extC[F]--q0[C]-
 0.0                 0.0            0.0       0.0        0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0              v      0.0             0.0                0.0         1e7      0.0

--secondary----e_collisional----i_collisional----reflux---nfft--
 0                0                    0          0       256

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                    0.2               2                .1           .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19              0.0            0.0             10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20              12.0           50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20              13.6           60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2*V^1/2]-------ascat[m^2]--bscat[m^2*V^1/2]---
 3.0e-19                   0.0              2.0e-19       0.0

SPECIES 1

----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.0               0.0                   1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]--vtperp[m/s]--
 0.0           0.0         4.2e5          4.2e5       0.             0.            4.2e5

---nbin----Emin[eV]----Emax[eV]---max-np---
 50              0           10             50000

SPECIES 2

----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 6.690e-26    0.0              0.0                   1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]--vtperp[m/s]--
 0.            0.          245.786      245.786     0.             0.            245.786

---nbin----Emin[eV]----Emax[eV]---max-np---
 50              0           5              50000
```

HOMEWORK ON XPDC1 (cylindrical diode) RFDA2

The object is to run a cylindrical RF discharge model in order to observe and measure the similarities and differences with the planar model already run (RFDA on XPDP1). The inner electrode has radius 0.01m and the outer, 0.05m; the gas is argon, at a pressure of 100 mT; the drive is 250 volts at 13.56Mhz.

The cylindrical model, with driven inner cylinder and grounded outer cylinder, with a blocking capacitor, is called ASYMMETRIC, which will be seen clearly in this model, with the (inner sheath potential drop)/(outer sheath potential drop) roughly equal to $\frac{r_{\text{inner}}}{r_{\text{outer}}}$, the area ratio to the power (about) 1.

The references are the L&L text Section 11.4 and the 1991 article handed out in class, by Alves et al. ("Sheath voltage ratio for asymmetric RF discharges"), left out of the L&L refs., unintentionally. (It was Maria Virginia Alves' Ph.D. thesis.)

1) Run RFDA2.INP in order to observe the asymmetric sheaths, with much larger Er at the inner cylinder that at the outer cylinder. (Let the run go on for many RF cycles, as the ionization comes slowly - or, change parameters to accelerate such, like increasing pressure.) Run the TRACE on the potential to note the imbalance in inner/outer electrode sheaths and in the DC bias; lookat the time average potential for the same view, as well as time average Er Our running shows that the equilibrium takes many RF cycles, so we ran it for about a half day and have put it into a dump file, which you may start from. Here are the instructions for getting the dump file and for starting from the dump:

```
xpdc1hp -i inp/rfda2.inp -d inp/rfda2.dmp
```

2) When you start from the dump, before running, look at all of the time-average plots (saved from the initial run), which are lost after one Dt into the run from the dump file. Record these if you like. The new time average plots will start from the beginning of the dump.

The J.E values (work done on the particles, Watts/m3) are calculated from the increase in the kinetic energies of the particles (J is not calculated), per unit volume (which, of course increases with r). You should observe a much larger J.E at the driven inner electrode, than at the outer electrode, both for the ion acceleration into the electrodes and for the electron heating.The driven electrode, in this model, is the smaller area electrode, or the target electrode for ion etching, ion implantation, ion doping, ion sputtering (or other); hence, having the larger ion acceleration done there is most desirable, with negligible ion acceleration, bombardment of the grounded electrode.

3)For numerical checks, compare the ratio of sheath voltage drops with the electrode area ratio. How close do you come to the results in the Alves et al. 1991 article?

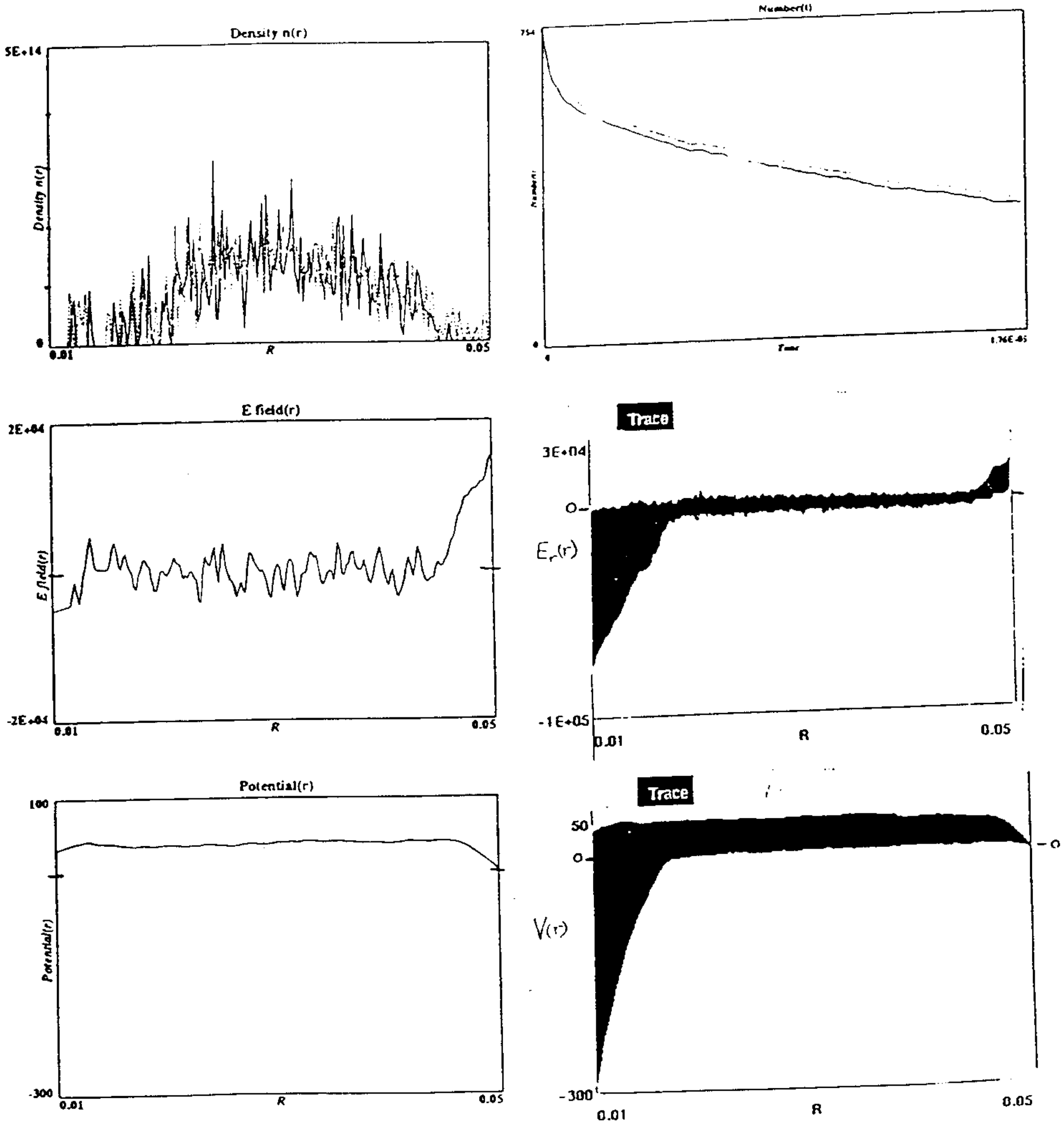
4) We find that the left-hand side potential varies from about +50 volts to about -260 volts. Turn on the trace on the phi(x,t) plot and look also at the time-average phi, time-average E. Explain all of the various potential drops, in terms of the circuit in Fig. 11.18, p.369, L&L text. You may assume the matrix sheath model in order to obtain the two sheath vacuum capacitances, from extrapolating E on the average E plot, as usual (and as you were asked to do using MAXWELLEA on XPDC1).

5) Observe the time average density plot, with quite different shape that for the planar model. Observe the various frequency spectra and comment.

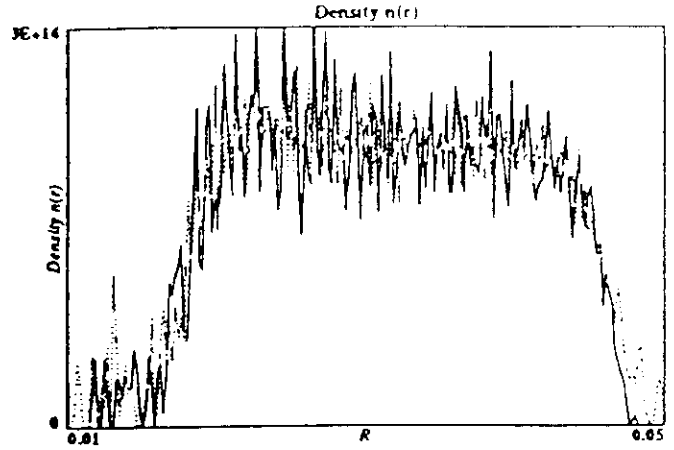
Ned Birdsall
24 April 1995

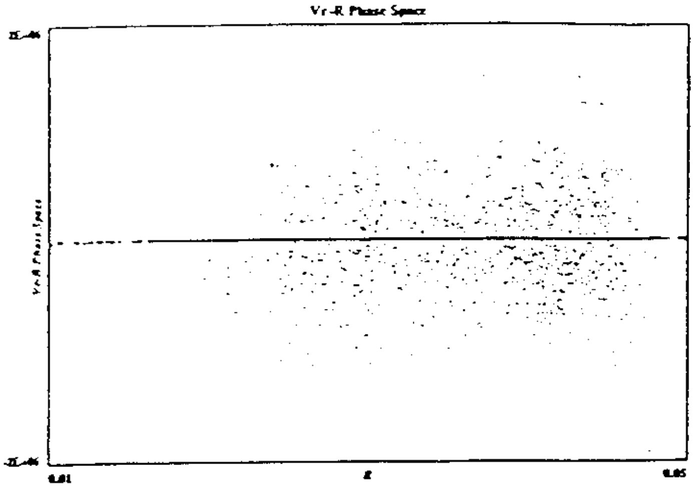
```
                         RFDA2 RF Discharge, Argon
-nsp---nc---nc2p---dt[s]--------r0[m]----r1[m]---height[m]--epsilonr--Bz[Tesla]-
 2    200   6e8  7.20179e-11 0.01    0.05        0.1          1.0         0.0

-rhoback[C/m^3]-backj[Amp/m^2]---dde---extR[Ohm]--extL[H]---extC[F]--q0[C]-
 0.0                0.0               0.0      0.0         0.0       2e-11   0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0               v          0.0              0.0            250.0     1.356e7  0.0

--secondary----e_collisional----i_collisional----reflux---nfft--
 0                  1                   2              0        1024

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                  0.2          2                  0.1                 .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.2e-19            0.3             15.0            20.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 7.0e-21            11.55           30.0            100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 3.0e-20            15.76           30.0            100.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2*V^1/2]-------ascat[m^2]--bscat[m^2*V^1/2]---
 2.0e-19            5.5e-19                   1.8e-19        4.0e-19

SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.0              0.0                   2e15

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]--vtperp[m/s]-
 0.0          0.0        3e5          3e5          0.          0.          3e5

---nbin----Emin[eV]----Emax[eV]---max-np---
 50          0            100          50000

SPECIES 2

----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 6.690e-26     0.0              0.0                   2e15

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]--vtperp[m/s]--
 0.           0.         3e4          3e4          0.0         0.0         3e4

---nbin----Emin[eV]----Emax[eV]---max-np---
 50          0            100          50000
```

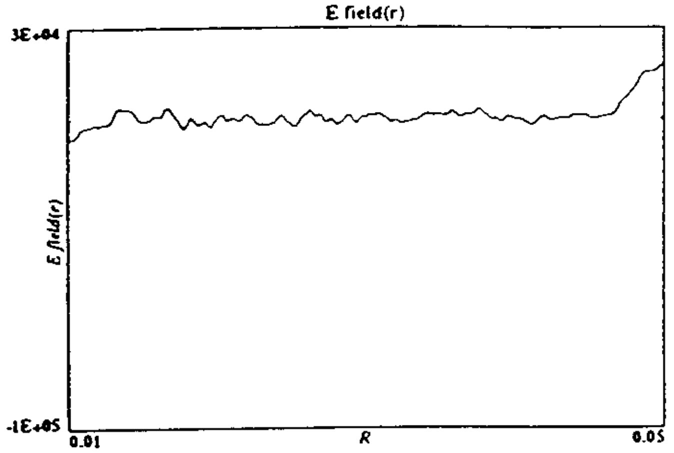
13.56 Cylindrical Radio Frequency Discharge
10mT , 250V, Argon Gas, 4cm gap

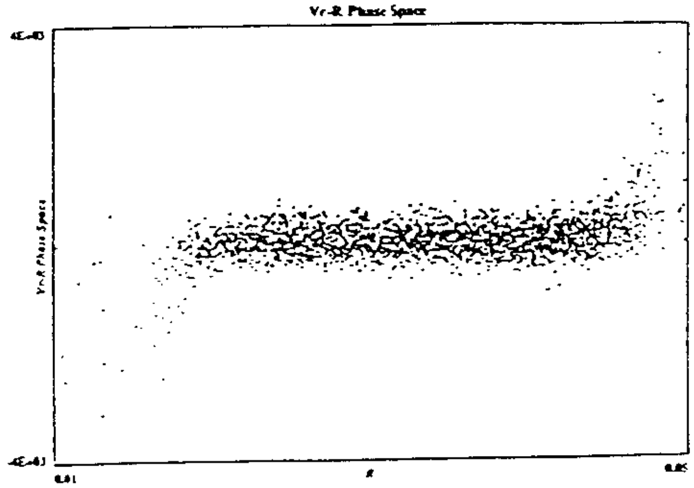


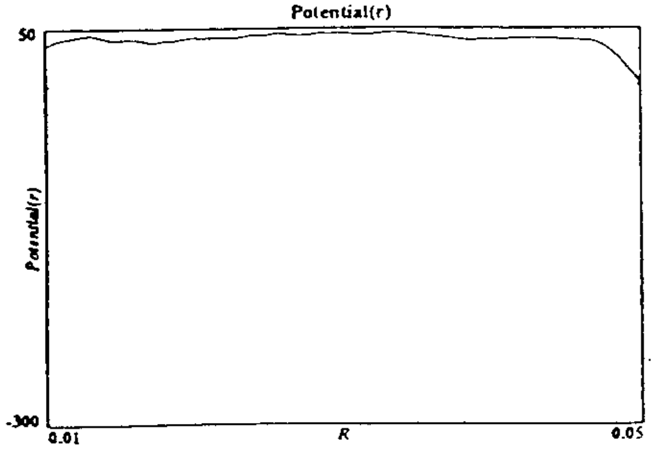
13.56Mhz Cylindrical Radio Frequency Discharge
100mT, 250V, Argon Gas, 4cm gap

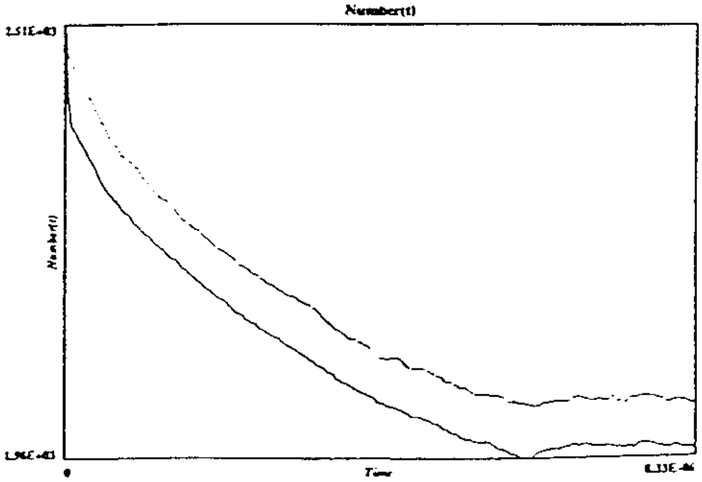












ECR.INP

In spite of the limitations of this simple ECR simulation, many interesting characteristics of ECR plasmas can be observed. This HW will study the behavior with respect to density, driving voltage and pressure.

Main reference: Chap 13 of L and L:

Some General Guidelines for doing this problem set.

1) In order to get a fair comparison, run all the sets for the same number of time steps.
2) In order to compare with the base set; do not quit the base simulation. This will help you compare the runs visually.
3) For the runs that require a higher initial density, increase nc2p from 1e4 to 1e6, otherwise the program will exit with "too many particles" error
4) *** This hw will most certainly generate a lot of plots. please use the command ps6to1 (present in ~ee298i/xgrafix/src) which will allow you to put 6 postscript plots on one page. So one may have the PS plots of J.E, time ave E, Time ave n, ionization profile, Bfield vs x (~ee298i/xpdp1/inp/ecrB.ps) and any other plot of you choice.
5) The exectuable is named xpdp1hp.ecr, use this in conjunction with ecr.inp ONLY!

0. Base Simulation:
   initn = 1e14/m3; V = 50V; P = 5mTorr all these runs will be done at 2.45GHz driving frequency in particular note location and magnitude of peak in J.E, |E| and ne at this location. Also pick the value of B field at this location from B vs. X plot.
1. VS. Density
   ECR heating is known to occur at the Hybrid frequency $\sqrt{\omega_c^2 + \omega_p^2}$. Since the driving frequency is held constant, an increase in ω_p (ne) should force ω_c (B) to a lower value. i.e. The location of the resonance will no longer be at 875 Gauss but at a lower value.
   Test this by
   increase initn to **1e16**, (increase nc2p to 1e6). Compare this run to base run.
   Compare values of J.E, E, ne etc. and comment on the same.
2. VS. Pressure
   The paper by Asmussen indicates that ECR is most efficient at relatively lower pressures and that at lower pressure, even smaller |E| leads to larger heating.
   To test this claim
   Keeping all else the same, in the base run input deck, change pressure to **50 mT**. This has the effect of increasing collision frequency. Compare the plots with the base run. In particular compare the value of J.E and |E| with the base case. In addition check the ionization profile, it will be sharply peaked near the resonance.
3. VS. input Power
   Fig. 13.8 of L and L indicates a certain power-pressure threshold for ECR to kick in. test this by reducing the driving voltage (and hence the power input) to **10V**. alternatively, this test can also be looked at as the variation of heating w.r.t to input power.
   Comment on what you observe
4. One can theoretically couple power into the ions, using Ion Cyclotron Resonance. However, this is hardly ever discussed. List some practical issues which would hinder such a source?

Magnetized plasma diode. . attempt to simulate ECR!

```
-nsp---nc---nc2p------dt[s]-----length[m]--area[m^2]--epsilonr--B[Tesla]---PSI[D]--
 2    100    1e4    1.5944e-11 1e-2        1.0e-4     1.0       1.6e-1      90.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]--extL[H]---extC[F]---q0[C]--
 0.0               0.0               0.0    0.0        0.0     1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0          v        0.0           0.0            50.0       2.45e9  0.0

--secondary---e_collisional---i_collisional---reflux---nfft---nsmoothing--ntimestep--
 0               1               2            0        256       2              0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0               0.2          2                5e-3               .026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.2e-19           0.3            15.0            20.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 7.0e-21           11.55          30.0            100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 3.0e-20           15.76          30.0            100.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 2.0e-19            5.5e-19              1.8e-19         4.0e-19

SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31    0.0              0.                 1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0          0.         4.2e5      4.2e5        0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 6.7e5             0.0          50        0.          20.         500000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100       0.01        15.0         .0045     .0056

SPECIES 2
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 6.68e-26     0.0              0.                 1e14

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.           0.         6.66e4     6.66e4       0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 1.06e5            0.0          50        0.          10.        500000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                       100       0.01        15.0         .0045     .0055
```

Dear Class:

We are a bit ahead of Prof. Lieberman's lecturing, as we will do plasma immersion ion implantation today, so-called PIII, to be done in argon. See the L&L text pp. 526-538. Today we will show some sketches in class on the general configurations used. The simulation model to be used attempts to model a semi-infinite plasma, by using a plasma source at the right-hand-side electrode (both warm electron and ion injection) to maintain the plasma at near the initial uniformly loaded plasma ($n = 10^{13}$/m3). The gas is argon at 20mT, with many ion charge exchange mean free paths for the length of both plasma and sheath, hence quite collisional. The left hand side electrode is to be the target for the ions, accelerated by a large negtive voltage applied at t = 0; the electrons are accelerated away from the target, setting up (initially) a matrix sheath (uniform ion density).

1) You may check the "semi-infinite" modeling by observing the behavior near the rhs source plate. Note the oversupply of electrons causes formation of a source sheath which accelerates all ions, to about Bohm velocity. Measure these potentials and velocities and compare them with those at the lhs electrode, due to the large applied voltage: should be negligible.

2) Run the model up to about the end of the input pulse ramp (0.5μ sec) and become accustomed to the general physics occuring. Note the max/min values so that you may re-scale in the next step.

3) We found that there are a number of general observations made quite easily using the TRACE on a number of diagnostics, starting from t = 0, such as: the time average quantities (density, field, potential) the f(E)2, the ion energy distribution deposited on the lhs electrode the vx-x phase space plot, enlarged to observe the ion acceleration and charge exchange.

4) The time average plot traces occur every DT times the nfft number, here $10^{-10} \times 128 =$ 12.8 ns. Such allows you to obtain the position of the sheath edge in time, hence the sheath edge velocity, to check against the analysis in the text. Do so. Consider Problem 16-5 in L&L. View phi-mid(t) and I(t) to see the oscillatory behavior at about 12MHz, which is lower that the plasma frequency of about 28MHz. Explain analytically, using the measured sheath width (big hint).

5) Repeat the above in two ways.

(a) The electron collisions were turned off as being unimportant here; was this justified? (Put such back in.)

(b) Turn off the ion collisions, to see whether the ions at the target can be made nearly monoenergetic; or, is this an idealization, ignoring the transient voltage rise and finite transit time of the ions, and the moving sheath edge. Comment, from your observations and from analysis. We do use 3-axis plots, with time as one axis and can rotate these on the monitor to look a plots versus time. these are not on your codes. When the matrix sheath begins to allow ion current, then the model moves to a Child Law like sheath. Check on the behavior in x, measured from the sheath edge, at x to various powers for n, E, phi.

Ned Birdsall
26 April 1995

Plasma-Immersion Ion Implantation

- Conventional Ion Implantation
  - Carried out in vacuum environment with an ion source to create beam of ions.
  - Beam accelerated through high voltages.
  - Due to small spot size. mechanical and electrostatic scanning required.
  - Beam optics limitations lead to relatively low beam currents, leading to high costs for large dosage.
  - Scaling all the above with wafer size is not trivial.
- PIII
  - Wafer immersed in a plasma environment.
  - Wafer is pulsed to a large negative potential.
  - Negative potential repels electrons exposing ions.
  - This potential accelerated ions to the wafer.
  - Potential relaxes from matrix sheath to a child's law sheath.
  - Costs less than convention ion implantation.

Piii.inp

- Planar simulation. short circuited diode filled with electrons and argon ions.
- Demonstrates plasma-immersion ion implantation, effects of collisions on IEDF of ions hitting target.

Observe

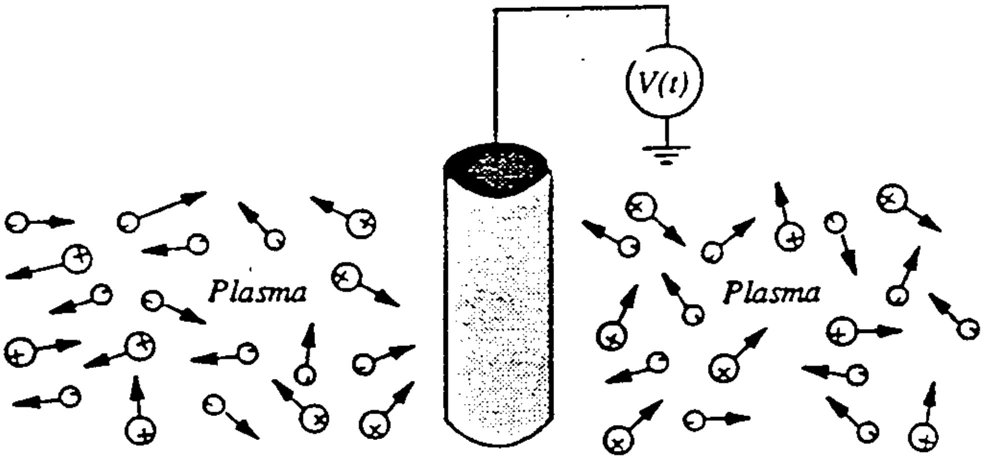
- The electrons get repelled from the target on a time scale proportional to ω_{pe}^{-1}.
- The ions react to the field on an ion plasma frequency scale ($\approx \omega_{pi}^{-1}$).
- Observe the relaxation of the potential.
- This example simulates a single pulse.
- Observe the effects when ion collisions are turned on. Especially in the *Vx-X* and *F(E) 2* diagnostics.

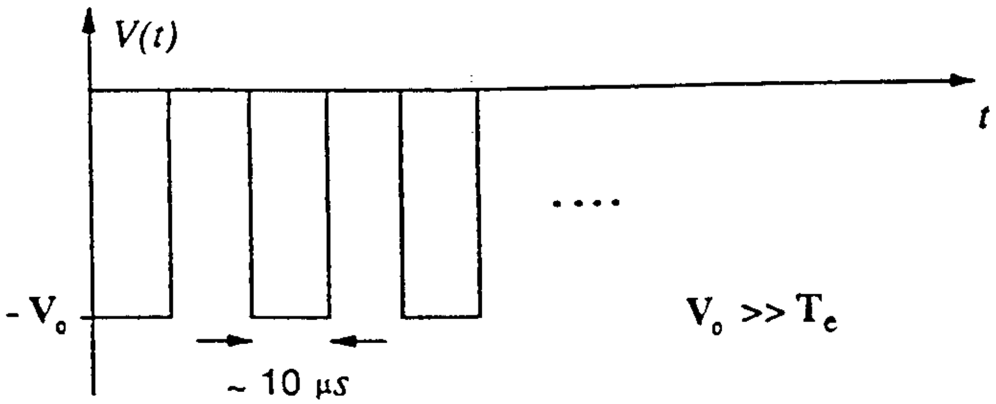
PIIIA.INP

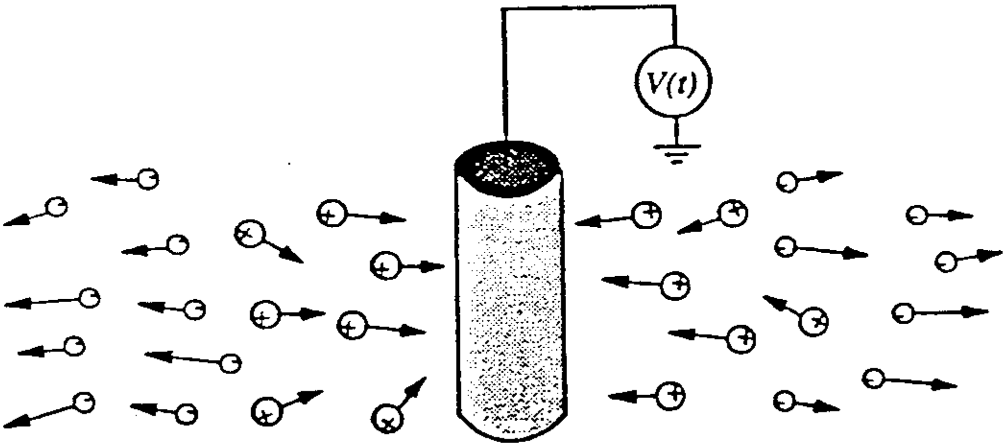
```
Plasma Immersion Ion Implantation (IN MKS UNITS) With ion-neutral collisions (Argon
atom)

-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]---epsilonr--B[Tesla]---PSI[D]--
 2   100   1e7   1e-10    0.3     0.01        1.0       0.0       0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde---extR[Ohm]---extL[H]---extC[F]--q0[C]-
 0.0              0.0               0.0     0.0       0.0      1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 1           v       -500         -1e9                0.0         1e6     0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 0              0               2               0      128      2            0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                0.2          2              2e-2              .026


ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.2e-19           0.3            15.0           20.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
7.0e-21            11.55          30.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 3.0e-20           15.76          30.0           100.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2/V^1/2]-------ascat[m^2]--bscat[m^2/V^1/2]---
 2.0e-19          5.5e-19        1.8e-19                 4.0e-19

SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31    0.0               .4               1e13

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.0         0.          4.2e5       4.2e5        0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0          50       0          10         50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100      0.01       10.0         .014      .016

SPECIES 2
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 66.8e-27     0.0       1.477e-3             1e13

--v0L[m/s]---v0R[m/s]---vtL[m/s]---vtR[m/s]----vcL[m/s]---vcR[m/s]-
 0.          0.          2e2         2e2          0.         0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[ev]---max-np--
 0.0              0.0          50       0          500        50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100      0.01       15.0         .014      .016
```

The next example is a bounded plasma, with one bounding electrode quickly driven negative. First, the drive voltage will be small, a few kT_e, looking for double layer(s), on average. Then the drive will be made much larger, seeking ion acceleration into the driving electrode; this model behavior will be given in detail, both for small and for large collisionality. Current drive will also be described.

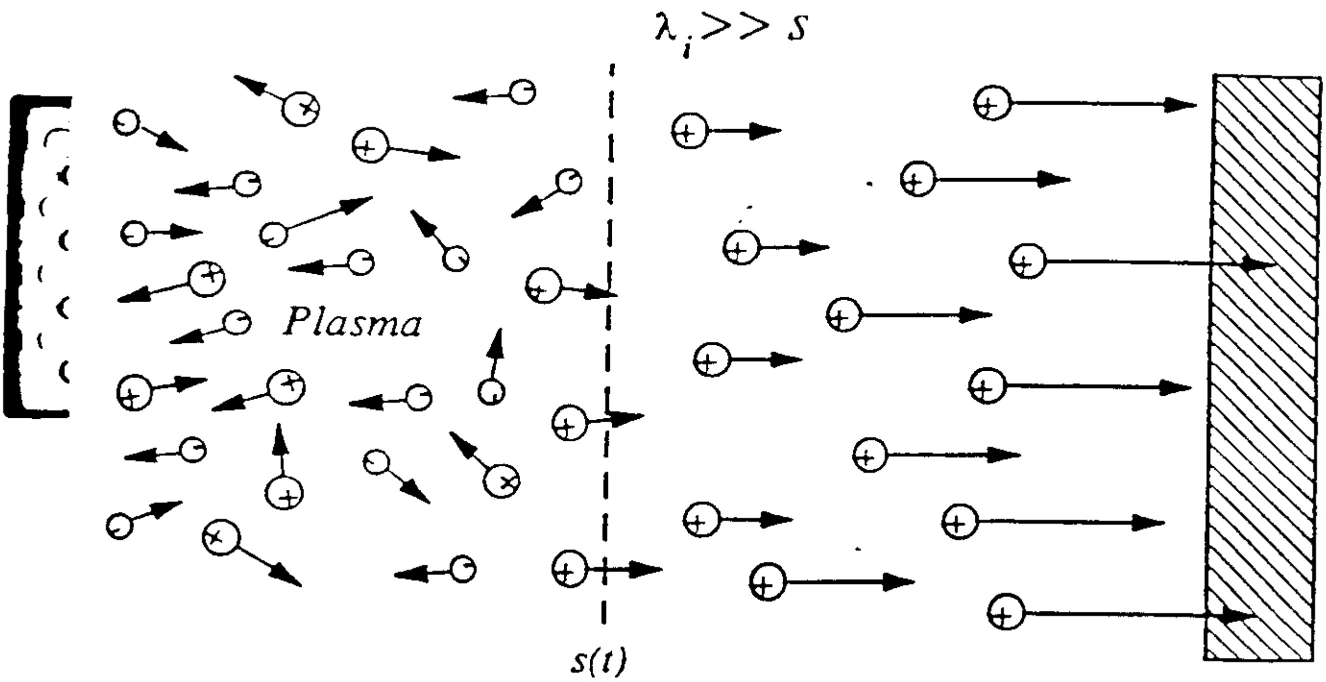
PLASMA IMMERSION ION IMPLANTATION



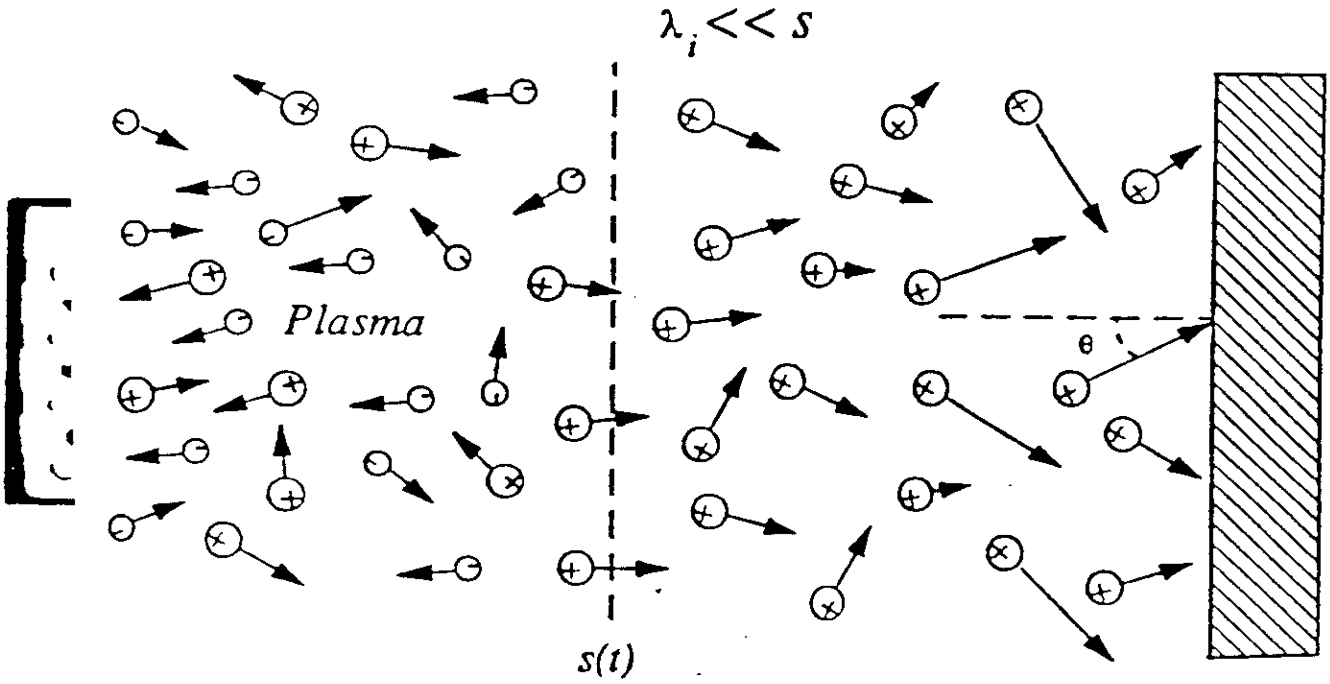




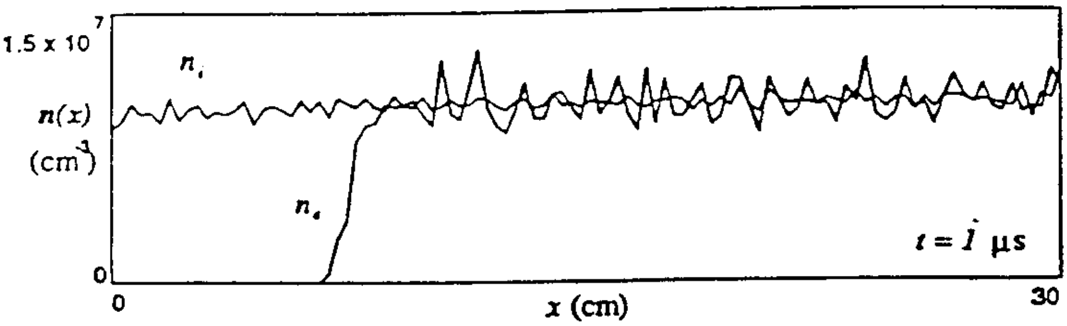
LOW NEUTRAL PRESSURE (COLLISIONLESS IONS)

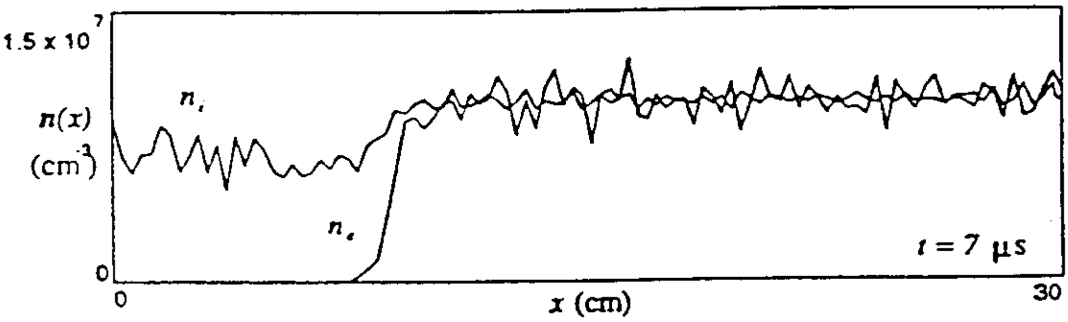


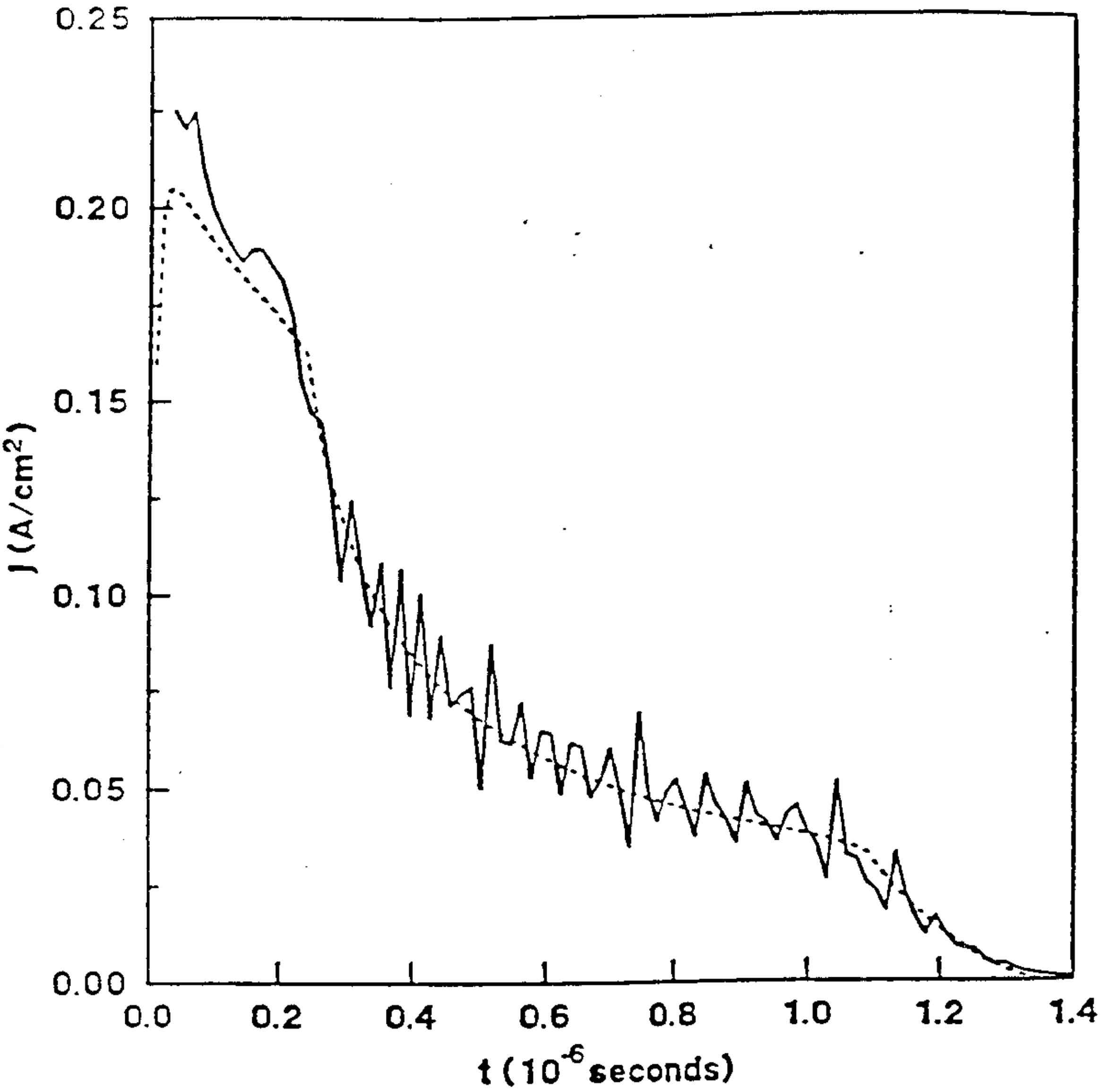
HIGH NEUTRAL PRESSURE (COLLISIONAL IONS)



Particle (PIC-MCC) simulations







Current density versus time for $\beta = 0.1$, $V_o = 30$ kV, $t_r = 0.2$ μs, $t_p = 0.8$ μs, $t_f = 0.3$ μs, and $n_o = 5.5\text{x}10^{11}$ cm^{-3}, $M = 40$ amu (— numerical solution; --- analytical solution)

G. Emmert, fluid code

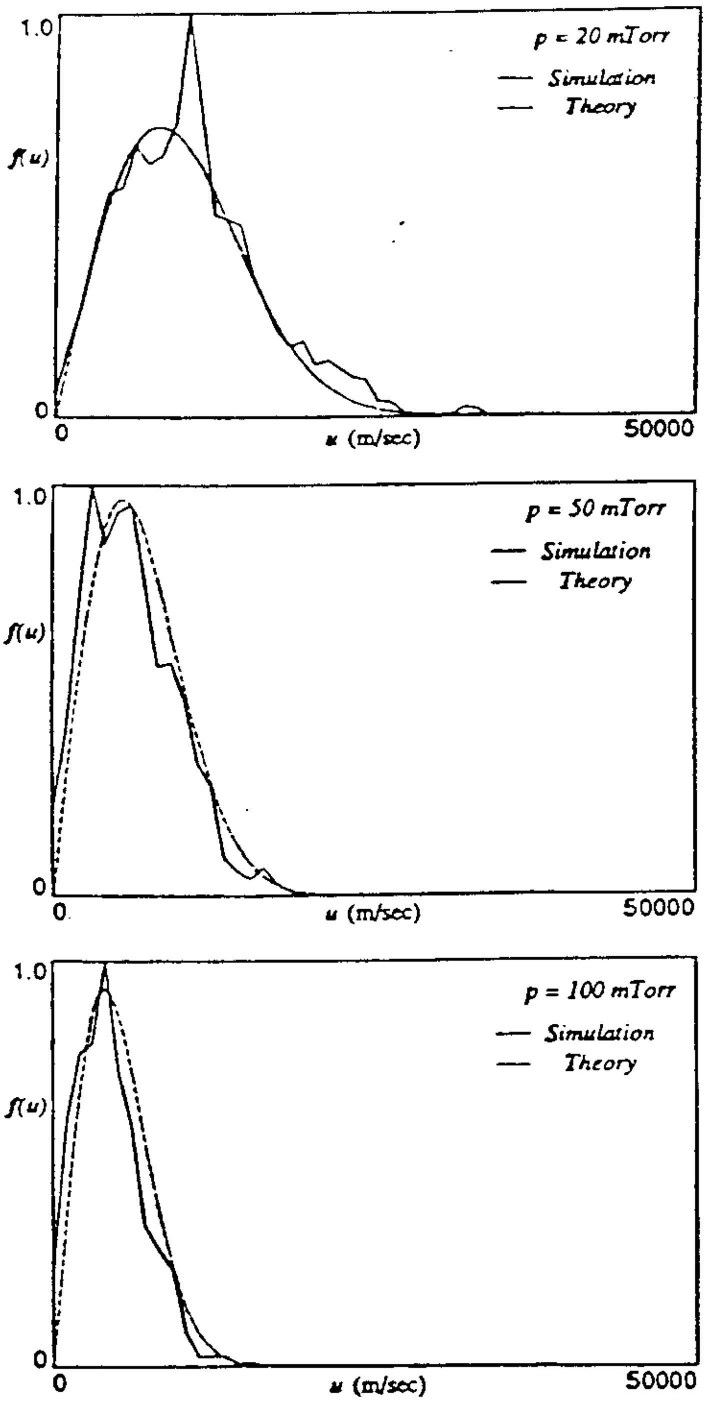
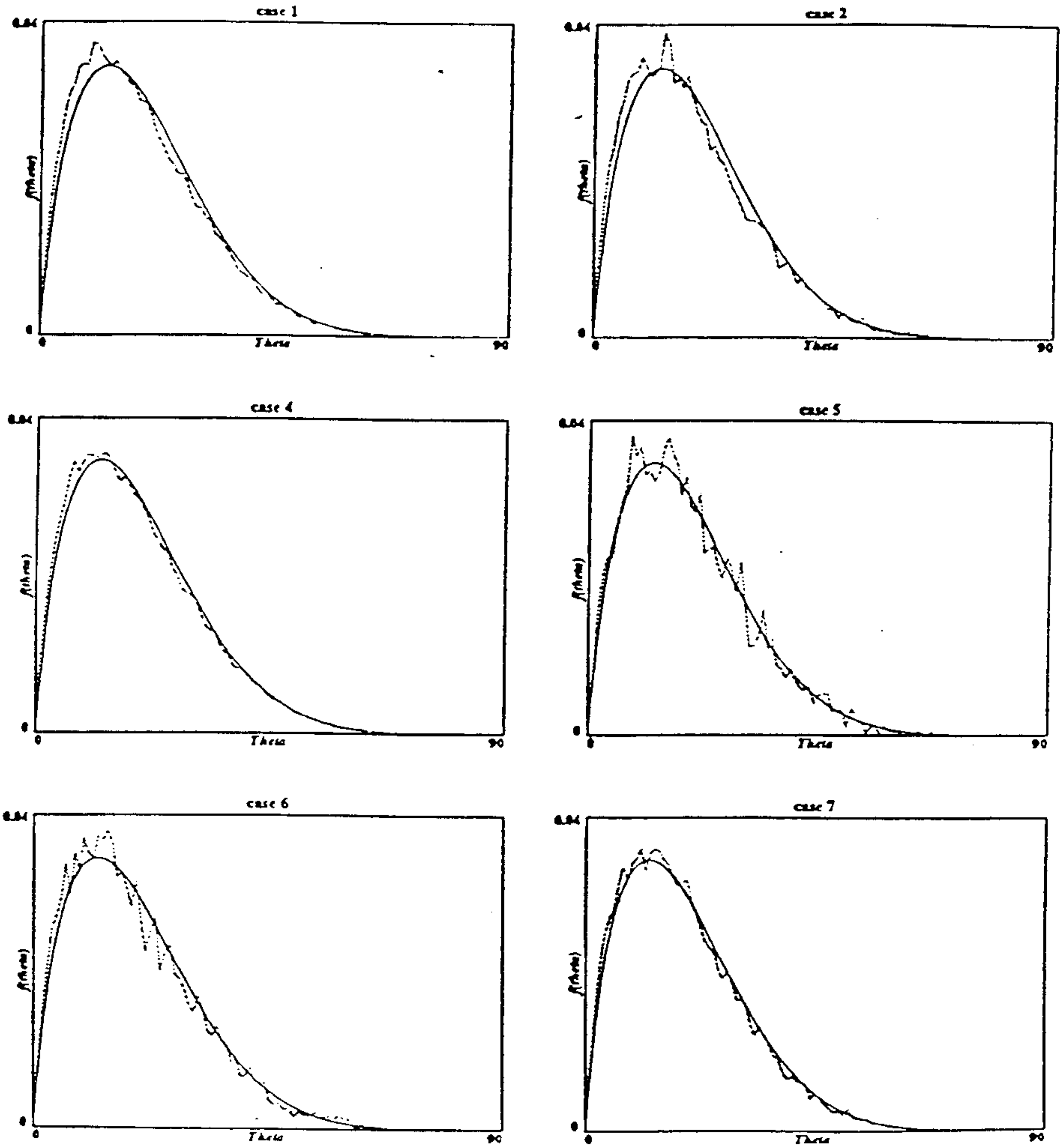
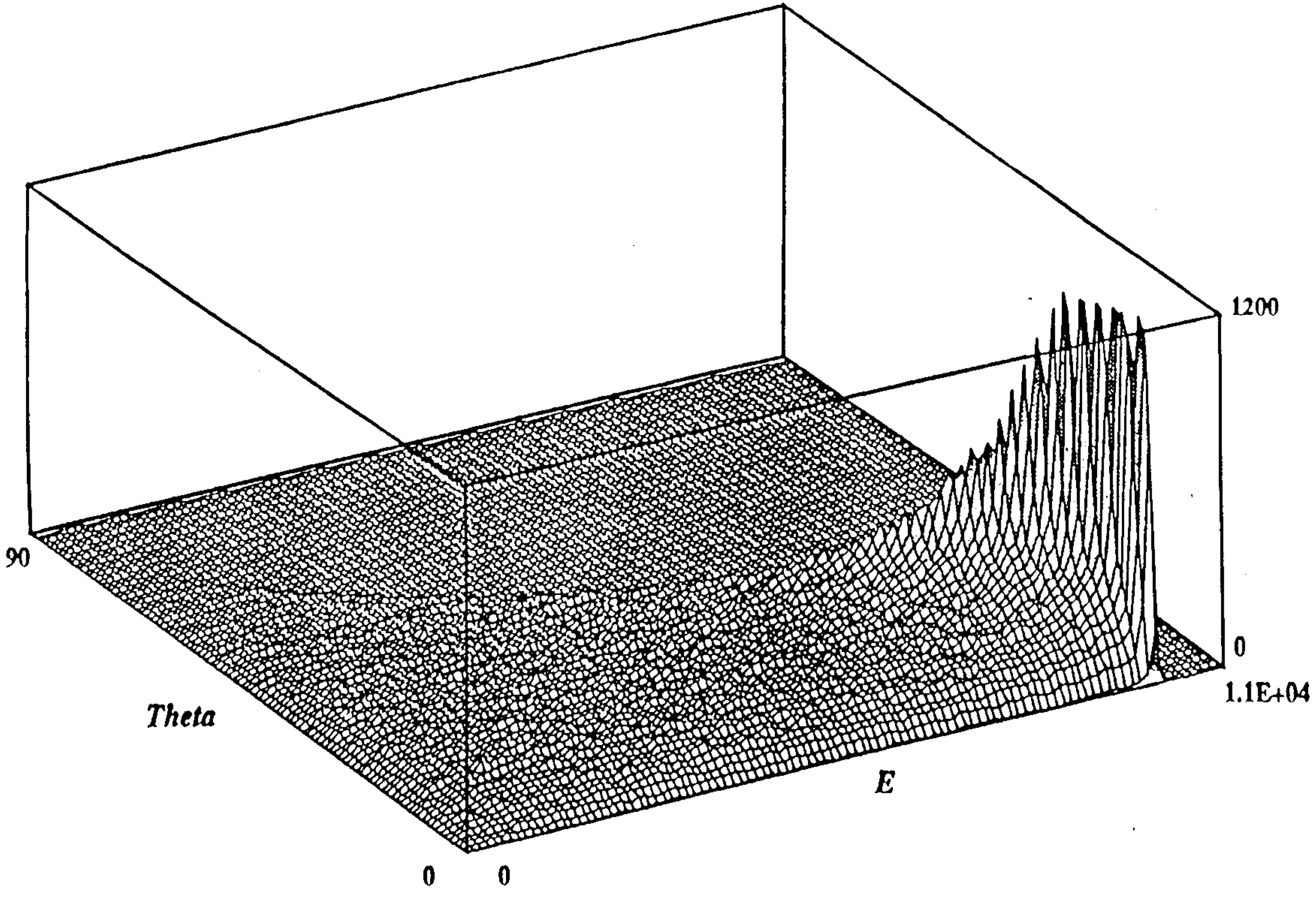


Fig 16.8 p535 LtL text

Angular distribution of the ion flux at the target



p= 5 mTorr, V= 10 kV, s= 10 cm, Argon Type Cross Sections
Energy and Angular Distribution of the Ion Flux at the Target



EECS 298-9 Homework
DC Discharges
April 25, 1996

This homework uses the input file, dch.inp (DC discharge in Hydrogen), and the near-equilibrium dump file, dch.dmp.

1. Estimate the ionization, excitation and scattering mean free paths and collision frequencies analytically using the cross sections from the input file and some reasonable velocity distribution.
2. From Lieberman and Lichtenberg section 14.2, make modifications appropriate for a 1d diode with axial diffusion. Solve the diffusion equation (ambipolar) to obtain the electron density, $n(x)$, and compare with simulation. Estimate the diffusion velocity and velocity at the sheath edge for the electrons, and compare with simulation. Also compute the Bohm velocity; which velocity should apply at the sheath edge?
3. Estimate the electron temperature for this configuration, and compare to simulation. Use the simplification that $n = 0$ at the cathode (although this forces $v \to \infty$ for finite flux Γ). The temperature from the simulation should be obtained from the fmid1 diagnostic, with appropriate start and end points. Also compare this with a temperature estimated (very roughly) from phase space using the rule of thumb, $v_{tx} \approx v_{\max}/3$.
4. Measure E, n_i, and T_i in the sheath. Using these values, Kirchhoff's law, and the ion momentum transfer frequency, predict the current required for this discharge.
5. Perform a power balance analysis similar to L&L section 14.2. Compare this to the simulation.

```
                DCH.INP    DC DISCHARGE(IN MKS UNITS)
Current-driven with electron and ion-neutral collisions (atomic Hydrogen)

-nsp---nc---nc2p---dt[s]---length[m]--area[m^2]--epsilonr---B[Tesla]---PSI[D]--
 2   200    2e8   1e-10      0.2        0.01        1.0         0.0        0.0

-rhoback[C/m^3]---backj[Amp/m^2]---dde--extR[Ohm]---extL[H]---extC[F]---q0[C]-
 0.0              0.0              0.0     0.0        0.0       1.0      0.0

-dcramped--source--dc[V|Amp]--ramp[(V|Amp)/s]---ac[V|Amp]---f0[Hz]--theta0[D]-
 0           i      -0.005       0.0               0.0       13.56e6    0.0

--secondary---e_collisional---i_collisional---reflux---nfft--nsmoothing--ntimestep--
 1              1               2              0      1024      6           0

--seec(electrons)---seec(ions)---ion species----Gpressure[Torr]---GTemp[eV]---
 0.0                  0.2           2                50e-3           0.026

ELECTRON-NEUTRAL COLLISIONAL PARAMETERS-----
--selsmax[m^2]---elsengy0[eV]---elsengy1[eV]---elsengy2[eV]-
 1.0e-19           0.0             0.0            10.0

--sextmax[m^2]---extengy0[eV]---extengy1[eV]---extengy2[eV]---
 1.0e-20           12.0            50.0           100.0

--sionmax[m^2]---ionengy0[eV]---ionengy1[eV]---ionengy2[eV]---
 1.0e-20           13.6            60.0           110.0

ION-NEUTRAL COLLISIONAL PARAMETERS-----
---achrgx[m^2]--bchrgx[m^2*V^1/2]-------ascat[m^2]--bscat[m^2*V^1/2]---
 3.0e-19          0.0                    2.0e-19       0.0

SPECIES 1
----q[C]-------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 -1.602e-19 9.11e-31     0.                0.                5e14

--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.0            0.           5.9e5      5.9e5          0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0               0.0           150       0.0         100.0      50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100       0.0       100.0         0         0.5

SPECIES 2
----q[C] ------m[Kg]---j0L[Amp/m^2]---j0R[Amp/m^2]----initn[m^-3]--
 1.602e-19 1.67e-27      0.               0.                 5e14

--vx0L[m/s]---vx0R[m/s]---vxtL[m/s]---vxtR[m/s]--vxcL[m/s]--vxcR[m/s]---
 0.             0.          2.19e3     2.19e3          0.          0.

---vperpt[m/s]---vperp0[m/s]---nbin----Emin[eV]----Emax[eV]---max-np--
 0.0               0.0           50        0.0         200.0      50000

-For-Mid-Diagnostic---nbin----Emin[eV]---Emax[eV]----XStart--XFinish--
                      100       0.0       100.0         0         .05
```